

About electroweak domain walls in Majoron models

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Some time ago it was claimed in “Spontaneous breaking of lepton number and cosmological domain wall problem” [Lazarides *et al.* *Phys. Rev. Lett.* **122**, 151301 (2019)] that nonperturbative instantons of the weak interaction $SU(2)_W$ lead to the formation of domain walls in Majoron models owing to the anomaly of the spontaneously broken global lepton number L symmetry $U(1)_L$ with respect to $SU(2)_W$. We point out that it has long been known that this effect can be completely rotated away unless there is a source of explicit $B + L$ breaking present, where B denotes the baryon number. We further estimate the tiny instanton-induced Majoron mass from $B + L$ breaking and analyze the cosmological impact of such domain walls. In general, this scenario does not lead to a cosmological catastrophe, and we demonstrate that the tiny instanton-induced mass can act as a bias term to collapse walls induced by a larger source of lepton number breaking. Alternatively, this electroweak Majoron could act as dynamical dark energy.

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I. INTRODUCTION

Domain walls from the spontaneous breaking of a discrete symmetry [1] are perhaps the most well-known topological defects (consult, e.g., Ref. [2] for an overview over this vast subject) and have haunted particle cosmologists for more than 50 years. Numerical simulations in Refs. [3–6] indicate that domain walls reach a scaling regime, where their size reaches the scale of the cosmological horizon $1/H$ in terms of the Hubble rate H , and there would be an $\mathcal{O}(1)$ number of walls inside our Hubble volume. As the energy density in domain walls redshifts in a radiation dominated background as $1/R^2$, where R is the cosmic scale factor, domain walls are expected to eventually dominate over both radiation ($1/R^4$) and matter ($1/R^3$). A domain wall-dominated universe would evolve with a scale factor that grows with cosmic time as t^2 [1] and such power-law inflation would affect primordial nucleosynthesis, the decoupling of cosmic microwave photons, or the formation of large scale structure [7]. Moreover the presence of even a single horizon-sized domain wall could potentially ruin the cosmological principles of homogeneity and isotropy, which hold to a very high degree as is evident from the observed anisotropic temperature

fluctuations $\Delta T/T \simeq 10^{-5}$ of the cosmic microwave background (CMB) [8].

In order to avoid such cosmological disasters, one typically has to either avoid the use of discrete symmetries altogether or use inflation to dilute the defects [9]. Recent simulations indicate that a statistical population bias for the distribution of field values from inflationary fluctuations does not lead to the collapse of the domain walls due to superhorizon correlations [10]. Low temperature symmetry restoration [11–14] is another remedy. One can also render the discrete symmetry to be only approximate via the introduction of explicit breaking terms colloquially known as “bias terms”, which allow for the destruction of the walls [12,15,16]. Recently, such collapsing domain walls have become en vogue again, since they could offer an explanation (see, e.g., [17]) for the correlated stochastic gravitational wave signal detected by various pulsar timing collaborations [18–22].

One class of models that are affected by the domain wall problems are global $U(1)$ symmetries that are explicitly broken to a $\mathbb{Z}_N$, $N \in \mathbb{N}$. The most prominent offender is the QCD axion [23,24] originating as the pseudo-Nambu-Goldstone boson (pNGB) of a global Peccei-Quinn (PQ) symmetry [25,26] that is anomalous and hence not a symmetry of the full quantum theory. Nonperturbative QCD instanton effects generate a potential for the QCD axion around the time of the QCD phase transition, and domain walls appear once the axion has relaxed to one of the N minima of its potential. On top of that, cosmic strings are produced at the much earlier PQ phase transition [27–29] and become the boundaries of the domain walls.

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For $N = 1$, it was shown that the resulting hybrid defect will collapse on its own as the strings can cut the walls into smaller slices [30,31] or via the nucleation of expanding closed string loops [28]. The bias term solution can work here as well [16], but faces pressure from the axion quality problem [32]. Alternatively, a second interim contribution to the axion potential, that switches off before the QCD phase transition, can lead to a uniform axion field value in all Hubble patches [33–35].

Global lepton number $U(1)_L$, that can explain the smallness of Majorana neutrino masses via the fermion-mediated Type I/III seesaw mechanisms [36–42] or the electroweak triplet scalar-mediated Type II seesaw [43–47], is another popular choice of symmetry with its own associated pNGB called the Majoron [48–50]. While the model in which the Majoron originates from the scalar triplet [50] is excluded by the observed decay width of the Z -boson at LEP 1 [51], one can construct viable models via the inclusion of an additional scalar singlet [52–54] that predominantly houses the Majoron. An overview over the Majoron phenomenology and its potential as a dark matter candidate can be found in Refs. [55–59]; the connection to proton decay is discussed in Ref. [60].

Some time ago, Ref. [61] claimed that due to the well-known ABJ anomaly [62,63] of the lepton number with respect to the weak gauge interaction $U(1)_L \otimes SU(2)_W^2$ domain walls should be formed in Majoron models via weak interaction instanton processes that typically break the lepton number down to a $\mathbb{Z}_3$. The authors of Ref. [61] then focused on building Majoron models that either feature $N = 1$ for a collapsing string-wall network or embed the remaining $\mathbb{Z}_N$ into a gauge group [64,65], which makes all N minima physically equivalent and thus prevents the formation of domain walls in the first place. This work then stimulated further research into building Majoron models with $N = 1$ and their cosmic implications [66,67].

In this paper, we demonstrate that the conclusions of Ref. [61] are flawed because the authors overlooked the fact that electroweak instantons do not generate a potential for the Majoron in the absence of *additional* explicit breaking of *both* B and L in the linear combination $B + L$, where B denotes the baryon number. Furthermore, even if such a breaking was present, we show that the correct Majoron potential from electroweak instantons depends on an *additional* coupling constant and does not necessarily lead to cosmological problems, even when considering finite temperature effects. The symmetry argument we provide has already been made in various forms throughout the years [57,68–73] (see also Refs. [74–77] for examples from the QCD axion literature and Refs. [78–80] from the dark energy literature).¹ While this line of

reasoning is surely well known to experts, we think that a pedagogical summary will be useful to the particle physics and cosmology communities at large. In the next section, we argue why the Majoron does not receive a potential from electroweak instantons as long as $B + L$ is not explicitly broken, and further, we estimate the Majoron mass in the presence of $B + L$ breaking from a dimension six operator. We then investigate the cosmological consequences of the Majoron mass and the associated domain walls. If there exists another less suppressed source of explicit lepton number breaking, that gives rise to a larger Majoron mass, the comparatively tiny contribution from the electroweak instanton can be used as a bias term to make the domain wall network collapse. In case the electroweak instanton is the leading contribution to the Majoron potential, then the Majoron can play the role of dynamical dark energy. We delineate the parameter space for both scenarios.

II. THE ARGUMENT

The topological term of the weak interaction is given by

$$-\frac{g_W^2 \theta_{EW}}{32\pi^2} W_{\mu\nu} \tilde{W}^{\mu\nu}, \quad (1)$$

where g_W is the gauge coupling, and θ_{EW} is an angle. We can rephase the lepton doublet L (quark doublet Q) by a lepton number $U(1)_L$ [baryon number $U(1)_B$] transformation $L \rightarrow e^{-i\alpha_L} L$ ($Q \rightarrow e^{-i\alpha_B/3} Q$) which induces a shift in the electroweak vacuum angle via the $U(1)_{B+L} \otimes SU(2)_W^2$ anomaly [71],

$$\theta_{EW} \rightarrow \theta_{EW} + N_g(\alpha_L + \alpha_B), \quad (2)$$

where $N_g = 3$ is the number of Standard Model (SM) fermion generations. The electroweak topological term can be removed for $\alpha_L + \alpha_B = -\theta_{EW}/N_g$. Now, suppose we break lepton number spontaneously via the vev of a complex scalar σ with a $U(1)_L$ charge of minus two in a single generation toy model based on, e.g., the Type I seesaw framework² with a sterile neutrino N [48,49],³

$$Y_N \sigma N N + Y_L L H N + \text{H.c.}, \quad (3)$$

where we decompose σ in terms of its radial (angular) mode $h_\sigma(j)$ and vacuum expectation value (vev) v_L ,

$$\sigma = \frac{h_\sigma + v_L}{\sqrt{2}} e^{i\frac{j}{v_L}}. \quad (4)$$

²The choice of high-scale seesaw and number of generations is just for illustration and none of our conclusions depend on it.

³We utilize two component spinor notation [82] throughout this work.

¹In our analysis, we do not take gravity into account; for more information, consult Refs. [70,81].

Without loss of generality, we take Y_N to be real and carry out an anomaly-free redefinition of the gauge singlet neutrinos N as $N \rightarrow N e^{-i\alpha_N}$ with $\alpha_N = j/(2v_L)$, which moves the Majoron into the Y_L Yukawa interaction and generates derivative couplings of j to N . A field redefinition of L with $\alpha_L = \text{Arg}[Y_L] - j/(2v_L)$ moves the Majoron and the phase of the coupling Y_L into the topological term and generates derivative couplings of j to L . Since baryon number B is unbroken, we have the freedom to completely remove this term via a redefinition of Q , which also picks up a derivative coupling to j , with $\alpha_B = -(\theta_{\text{EW}} + \text{Arg}[Y_L]) + j/(2v_L)$. Thus, the Majoron coupling to $W\tilde{W}$ is unphysical, and it cannot become massive via $\text{SU}(2)_W$ instantons. The situation changes once we switch on a term that also breaks baryon number and specifically $B + L$ like, e.g., (see Refs. [75,77,83,84] and references therein for more operators),

$$-\mathcal{L}_{B+L} = \frac{c_L^{ijkl}}{M_{\text{UV}}^2} L_i Q_j Q_k Q_l, \quad (5)$$

which can arise in, e.g., $\text{SU}(5)$ grand unified theory (GUT [85] from the exchange of gauge bosons or colored Higgses, where c_L would be given by a gauge or Yukawa coupling squared, and the cutoff scale M_{UV} would be identified with the mass of the exchanged boson. In the following, we assume that $\text{SU}(2)_W$ is not unified with $\text{SU}(3)_c$ in the $\text{SU}(5)$ GUT, because the $\text{U}(1)_L \otimes \text{SU}(2)_W^2$ anomaly would then also imply an anomaly with QCD (see also the discussion in Ref. [73]). This would render the Majoron to be a QCD axion with its own distinct phenomenology [86–89]. Nevertheless unification can still be realized if $\text{U}(1)_L$ is an emergent accidental symmetry that only appears below the unification scale. Exotic processes like proton decay can be avoided by an appropriate flavor structure in the couplings of Eq. (5), but the details are not relevant for our discussion.

Redefining Q with the aforementioned value for α_B just moves the Majoron from the topological term into $\mathcal{L}_{B+L}$, and hence, the Majoron dependence cannot be redefined away. For this to occur, *both* baryon and lepton numbers need to be violated, and the relevant linear combination $B + L$ is picked out by the anomaly structure of $\text{SU}(2)_W$ [71]. In this case, the coupling of the Majoron to $W\tilde{W}$ is physical, and it will be affected by $\text{SU}(2)_W$ instantons. This is entirely analogous to the case of a QCD axion a with massive quarks q , where chiral field redefinitions just move the axion back and forth between the topological term $a/f_a G\tilde{G}$ and the chiral symmetry breaking mass term $m_q \bar{q} q$. As usual, only *relative* phases are physical and for the Majoron $\mathcal{L}_{B+L}$ plays the role of $m_q \bar{q} q$ [71]. It is well known that the QCD axion mass vanishes in the presence of at least one massless quark. The underlying reason is that in this case the axion is aligned with an anomaly-free linear

combination of $\text{U}(1)_{\text{PQ}}$ and the restored chiral symmetry of the massless quark $\text{U}(1)_q$. The analogous observation for the Majoron is that in the absence of $B + L$ breaking there is an anomaly free linear combination of $\text{U}(1)_L$ and $\text{U}(1)_{B+L}$ that the Majoron is aligned with, and this linear combination is nothing more than $\text{U}(1)_{B-L}$. This also removes any potential confusion about whether one should impose a global $\text{U}(1)_L$ or $\text{U}(1)_{B-L}$ when building Majoron models. Therefore, the Majoron potential must depend on a combination of $B + L$ violating couplings such as c_L in Eq. (5).

We first discuss the zero temperature mass of the Majoron and later focus on finite temperature effects. Using the methods of Ref. [77] for estimating the effect of a 't Hooft determinant in a one-instanton-background [90,91], we can estimate the Majoron mass induced by the nonperturbative $\text{SU}(2)_W$ effects in the presence of explicit $B + L$ breaking. Throughout this paper, we do not consider other sources of explicit symmetry breaking for a lepton number. The limits for the integral over the instanton size ρ are the ultraviolet (UV) cutoff $1/M_{\text{UV}}$ of our effective theory as well as the infrared (IR) cutoff $1/(g_W v_H)$, where $v_H = 246$ GeV denotes the Higgs vev. The IR cutoff can be understood from the formalism of “constrained instantons” [92] since due to the spontaneous breaking of $\text{SU}(2)_W$ there appears a suppression factor of $e^{-8\pi^2 \rho^2 v_H^2}$, which effectively suppresses instantons larger than $1/(g_W v_H)$. Reference [77] obtained that the integral for a $\text{SU}(2)$ gauge theory is always dominated by the UV cutoff, and these UV instantons are also known as “small size” instantons. This is different from the case of the QCD axion, whose potential is dominated by IR effects and not expected to be captured by a one instanton calculation as it is connected to confinement (see Ref. [77] and references therein). Assuming the same renormalization running as in the Standard Model with the one-loop beta function $\beta_W = -b_0 g_W^3/(16\pi^2)$ and $b_0^{\text{SM}} = 19/6$, the estimate for the Majoron mass at zero temperature turns out to be [77],⁴

$$m_j^2 \simeq (5 \times 10^{-29} \text{ eV})^2 |c_L|^3 \cos(\theta_{\text{EW}} + 3\delta_L) \times \left(\frac{g_W(10^{16} \text{ GeV})}{g_W(M_{\text{UV}})} \right)^8 \left(\frac{e^{-\frac{8\pi^2}{g_W^2(M_{\text{UV}})}}}{10^{-137}} \right) \times \left(\frac{10^8 \text{ GeV}}{v_L} \right)^2 \left(\frac{M_{\text{UV}}}{10^{16} \text{ GeV}} \right)^4, \quad (6)$$

where we used that $g_W(10^{16} \text{ GeV}) \simeq 0.523$ and the smallness can be understood from the exponential suppression of

⁴Here, we rewrote Eq. (4.7) of Ref. [77] by using the definition of the renormalization group equation invariant scale in Eq. (2.7) of the same reference, which explains the appearance of the exponential factor and the power of M_{UV} .

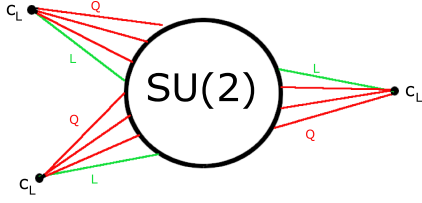


FIG. 1. Diagrammatic representation of the single instanton 't Hooft vertex with the fermion zero modes for each generation of $L(Q)$ in green (red) closed up by three insertions of the $B + L$ breaking effective operator in Eq. (5) with coefficient c_L . Adapted from Ref. [77].

the instanton $\sim \exp(-8\pi^2/g_W(M_{UV})^2)$. It is evident from Fig. 1 that the Majoron mass requires three insertions of the operator in Eq. (5), where for simplicity's sake we assumed flavor-blind couplings and defined $\delta_L \equiv \text{Arg}(c_L)$. Including a viable flavor structure to, e.g., suppress proton decay will lead to additional suppression factors and might introduce a dependence on quark or lepton mixing angles as noted in Ref. [78], but a detailed analysis of these implications is beyond the scope of this work. If one uses operators built purely from $SU(2)_W$ singlet fields instead of Eq. (5), the estimate for m_j will be further reduced by products involving all SM Yukawa couplings [77]. Note that due to the UV sensitivity this result is expected to change when $SU(2)_W$ is embedded in a larger group or in the presence of additional charged matter. The impact of adding additional scalar multiplets is discussed around Eq. (15). The putative UV completion of the effective operator in Eq. (5) could also change this estimate. Here, we chose $v_L > 10^8$ GeV to comply with astrophysical bounds from stellar cooling [57] and neutrino masses from, e.g., a Type I/III seesaw can be realized with natural values for the Yukawa couplings Y_L for v_σ as large as 10^{15} GeV.

III. COSMOLOGY

First, we show that the instanton-induced mass in Eq. (6) by itself only leads to domain walls that are not in conflict with standard cosmology. For $m_j < \mathcal{O}(10^{-28}$ eV), coherent oscillations of the Majoron [93–95] occur after recombination so j cannot be the dark matter. We also checked that the corresponding relic density would be more than subdominant. The Majoron begins to oscillate when $H \simeq 3m_j$, which implies $T \simeq \sqrt{m_j M_{\text{Pl}}}$. during radiation domination. Once the Majoron relaxes to the minimum of its effective potential, the $\mathcal{Z}_{N_g} = \mathcal{Z}_3$, that is preserved by the weak instantons (see Ref. [96] for an overview over the anomaly free discrete symmetry of the SM), is spontaneously broken, and domain walls appear as defects in the Majoron field. We estimate the surface tension σ_{DW} of a single Majoron domain wall as [97,98],

$$\sigma_{\text{DW}} \simeq 9m_j v_L^2, \quad (7)$$

and take the energy density of this horizon sized defect to be $\rho_{\text{DW}} \simeq \sigma_{\text{DW}} H$. The present day energy density fraction of a single domain wall is determined to be

$$\Omega_{\text{DW}} h^2 \simeq 2 \times 10^{-16} h |\tilde{c}_L|^{\frac{3}{2}} \left(\frac{v_L}{10^8 \text{ GeV}} \right) \left(\frac{M_{\text{UV}}}{10^{16} \text{ GeV}} \right)^{\frac{5}{12}}, \quad (8)$$

where we defined $|\tilde{c}_L| = |c_L| \cos(\theta_{\text{EW}} + 3\delta_L)^{1/3}$, suppressed the dependence on g_W purely for notational brevity, and took $H_0 = h \times 100$ km/(s Mpc). We checked numerically that the corresponding energy density is too small to ever dominate the energy budget of the Universe in the past or today. If the domain walls are present during big bang nucleosynthesis (BBN), their energy density can modify the rate of the background expansion similar to an additional species of dark radiation. Note that the equation of state parameter for a domain wall ranges between $\omega = -2/3$ for a static wall [1] and $\omega = 1/3$ for a ultrarelativistic wall [7], which means that its energy density can redshift differently from radiation. We recast limits on the abundance of dark radiation for a domain wall population [97], and the corresponding number of additional effective neutrinos reads

$$\Delta N_{\text{eff}}(T_{\text{BBN}}) \simeq 7 \times 10^{-33} |\tilde{c}_L|^{\frac{3}{2}} \left(\frac{v_L}{10^8 \text{ GeV}} \right) \left(\frac{M_{\text{UV}}}{10^{16} \text{ GeV}} \right)^{\frac{5}{12}}, \quad (9)$$

where we set $T_{\text{BBN}} = 1 \text{ MeV}$. At the time of recombination with $T_{\text{rec}} = 0.1$ eV, we obtain $\Delta N_{\text{eff}} \simeq 4 \times 10^{-19}$ for the same parameter choice. The presence of domain walls breaks the isotropy and homogeneity of the cosmological background. Such walls affect the propagation of photons via gravitational lensing, which can be expressed as a fluctuation of the cosmic background temperature $\Delta T/T \simeq \sigma_{\text{DW}}/(HM_{\text{Pl}}^2)$ [1,99], and one needs to check that this effect is not larger than the observed CMB temperature anisotropies of $\Delta T/T \simeq 10^{-5}$. Numerically, we obtain a present day value of

$$\frac{\Delta T}{T} \simeq 9 \times 10^{-17} |\tilde{c}_L|^{\frac{3}{2}} \left(\frac{v_L}{10^8 \text{ GeV}} \right) \left(\frac{M_{\text{UV}}}{10^{16} \text{ GeV}} \right)^{\frac{5}{12}}, \quad (10)$$

and it is obvious from Eqs. (8), (9), and (10) that the contribution of the Majoron domain wall to the energy budget of the Universe at late times is completely negligible, unless m_j is significantly enhanced by tens of orders of magnitude from the aforementioned UV effects.

In Ref. [61], it was claimed that thermal sphaleron transitions instead of the quantum tunneling encoded in the instanton effect were responsible for the formation of the topological defect, and it was assumed that $\sigma_{\text{DW}} \simeq m_W v_L^2$. This choice was justified by identifying the domain wall's width with the width of a sphaleron $\rho \sim 1/m_W$ [100] [see also the discussion about the IR cutoff for a constrained instanton above Eq. (6)]. On the one hand,

this relation for the surface tension would imply $m_j \simeq m_W$, which is missing the required insertion of the $B + L$ violating couplings such as c_L , and numerically, this guess is in stark disagreement with our estimate in Eq. (6) by around 40 orders of magnitude. On the other hand, impact of QCD sphalerons on the axion dynamics was studied in Ref. [101], where it was found that the sphaleron effect does not alter the axion potential, but rather manifests itself as an additional source of friction in the equation of motion for this field. Hence, for the Majoron, we also do not expect sphalerons to play a role when it comes to its potential and the domain wall formation.

IV. BIAS TERM

In case another source of explicit lepton number symmetry breaking (e.g., quantum gravity [102–111]) is responsible for significantly larger Majoron masses and thus domain wall tensions, we could use the comparatively tiny contribution from the weak anomaly in Eq. (5) as a bias term to make these walls collapse [112–117]. This ansatz can be compared to the case of the QCD axion, where the bias term arises from additional explicit breaking by hand or from quantum gravity, whereas the dominant contribution to the potential is due to QCD instantons. For the Majoron, the roles are reversed since the bias term will come from electroweak instantons, and the leading contribution to the potential originates from explicit breaking by hand or quantum gravity. Moreover, the strength of the bias term for the QCD axion is limited from above (see, e.g., Ref. [32]) by the shift of the potential minimum $\bar{\theta}$ away from zero, which should be $\bar{\theta} < 10^{-10}$ to still solve the strong CP problem, whereas no such constraint exists for the Majoron as no known observable depends on the rephasing invariant electroweak vacuum angle $\theta_{EW} + 3\delta_L$.

First, we denote the dimensionless Majoron field as $\theta_j \equiv j/(2v_L)$ due to the residual $\mathcal{Z}_2$ symmetry of the seesaw models [see the redefinition of N below Eq. (4)]. The electroweak potential is $\propto \cos(3\theta_j)$ due to the $U(1)_{B+L} \otimes SU(2)_W^2$ anomaly for $N_g = 3$ generations of SM fermions. We parametrize the additional and dominant source of lepton number breaking with an operator of the form

$$V_n = c_n \frac{\sigma^n}{M_{Pl}^{n-4}} + \text{H.c.}, \quad (11)$$

which gives rise to a potential $\propto \cos(2n\theta_j + \text{Arg}(c_n))$, and we take the mass $M_j^2 \simeq 2c_n v_L^2 (v_L/M_{Pl})^{n-4} \sin(\text{Arg}(c_n))$ originating from this term to dominate over the instanton-induced one $M_j \gg m_j$. In this scenario, the domain wall surface tension now reads approximately $9M_j v_L^2$. As long as n is not coprime⁵ with 3, the electroweak contribution

will bias the previously degenerate minima of V_n . As long as $m_j/M_j < 0.2$, the different vacua will percolate sufficiently [7,97,118]. The expansion of the energetically lower and therefore true vacuum will push the domain walls toward each other, eventually leading to the collapse of the network. In order for this to occur before the walls dominate the energy density of the Universe, we have to demand that

$$m_j > 5.7 \times 10^{-23} \text{ eV} \left(\frac{M_j}{10^{-18} \text{ eV}} \right) \left(\frac{v_L}{10^{13} \text{ GeV}} \right). \quad (12)$$

The energy density of the domain wall is then released in the form of an abundance of Majorons with [58]

$$\Omega_{\text{bias}} h^2 \simeq 0.12 \left(\frac{M_j}{10^{-18} \text{ eV}} \right)^{\frac{3}{2}} \left(\frac{1.7 \times 10^{-21} \text{ eV}}{m_j} \right) \times \left(\frac{v_L}{10^{13} \text{ GeV}} \right)^2. \quad (13)$$

Here, we focus on ultralight Majorons, that can be good dark matter candidates, and their mass range is constrained by various astrophysical and cosmological considerations (see Ref. [119] for an overview). Structure formation bounds from the Lyman- α forest rule out $M_j < 2 \times 10^{-20} \text{ eV}$ [120]. Some more stringent limits come from angular momentum loss of rotating black holes via the superradiance effect. Observations of spinning supermassive black holes exclude masses in the window $7 \times 10^{-19} \text{ eV} < M_j < 10^{-16} \text{ eV}$ [121]. However, these limits should be interpreted carefully due to uncertainties in the measurements of the black hole spins [122]. Furthermore the previous bounds assumed no self-interactions, which constrains the decay constant for a conventional QCD axion to be above 10^{14} GeV [123]. Since the Majoron potential scales differently with its decay constant $\sim v_L$ compared to a QCD axion, and we typically consider $v_L < 10^{15} \text{ GeV}$ for the seesaw mechanism, we do not impose this bound and only consider the constraint from the Lyman- α forest. It is important to emphasize that this ultralight mass range is difficult to realize for both a field-theoretic⁶ and a string-theoretic QCD axion [128], which provides further motivation for our Majoron-based setup.

In order for the Majorons produced in the domain wall collapse to constitute the dark matter, we should impose that they are produced before matter-radiation equality at the temperature $T \simeq 0.76 \text{ eV}$, which implies

$$m_j > 5.9 \times 10^{-23} \text{ eV} \sqrt{\frac{M_j}{10^{-18} \text{ eV}}}. \quad (14)$$

⁶The nonminimal model of Refs. [124,125] is a well known field-theoretic exception that however suffers from other drawbacks [126,127].

⁵That means the greatest common divisor of n and 3 is 1.

Note that since the Majoron only couples very weakly to matter, there is no obstruction to having the domain walls decay around or after the time of BBN [58]. Equations (12)–(14) illustrate that the instanton contribution in Eq. (6) is too small for our purposes if we only include which the contribution of the SM degrees of freedom to the $SU(2)_L$ β -function. This can be remedied by changing the running of weak gauge couplings, so that it decreases less in the UV, which softens the exponential suppression of m_j . Adding N_S additional scalar $SU(2)_W$ representations at the scale $M < M_{UV}$ changes the one-loop β -function from $19/6$ to $b_0^{BSM} < 19/6$ and thereby increases [77],⁷

$$m_j^2 \rightarrow \left(\frac{M_{UV}}{M}\right)^{\frac{19}{6}-b_0^{BSM}} m_j^2. \quad (15)$$

We can increase the bias term for $M \ll M_{UV}$ and note that generically the new intermediate scale M should lie above some TeV due to laboratory bounds on the new scalars. The change in the one-loop beta function for N_S mass degenerate scalars in a d -dimensional $SU(2)_W$ representation reads [129]

$$\frac{19}{6} - b_0^{BSM} = N_S \frac{d(d^2 - 1)}{36}, \quad (16)$$

which vanishes for a gauge singlet as expected. For $N_S = 1$, tree level perturbative unitarity of $SU(2)_W$ gauge interactions require $d < 8(9)$ for a complex (real) scalar [130], and the $SU(2)_W$ gauge couplings remain perturbative below the Planck scale as long as $d < 7$ [131]. Of course there exist other constraints on additional scalar representations such as electroweak precision observables or Higgs vacuum stability, but here we are merely interested in a proof of concept that demonstrates the existence of a suitable UV theory.

We depict the parameter space subject to the previous considerations for two values of $M_j = 10^{-18}$ eV, 10^{-17} eV in Fig. 2. For both benchmarks, we find that 10^{12} GeV $< v_L < 10^{15}$ GeV and that the cutoff M_{UV} can remain safely below the Planck scale, possibly indicating a GUT origin for the effective operator in Eq. (5). For $|c_L| \cos(\theta_{EW} + 3\delta_L)^{1/3} = 1$, the aforementioned Majoron masses require, e.g., a single generation of either a quadruplet ($d = 4$) with $M = 5$ TeV or a quintuplet ($d = 5$) with $M = 20$ TeV in order to have an early enough

collapse and to explain the observed dark matter relic abundance. In case the bias term is further suppressed by $|c_L| \cos(\theta_{EW} + 3\delta_L)^{1/3} < 1$, or due to the flavor structure of c_L , we have to increase either d , N_S or decrease M . For $v_L = 10^{13}$ GeV, one can realize $M_j = 10^{-18}$ eV via an operator with dimension $n = 17$ in Eq. (11). As usual when dealing with the explicit breaking of a symmetry via *ad hoc* higher dimensional operators, we have to assume a mechanism to protect the required quality of the potential in Eq. (11) by preventing the appearance of all lower dimensional operators with $n < 17$ or suppressing their coefficients [102–111].

One can deduce that the Majoron yield from the domain wall destruction in Eq. (13) dominates over the misalignment contribution [93–95] with a patch-averaged misalignment angle of $\theta_i \simeq 2.1$ [98] for $v_L < 10^{15}$ GeV. Since the energy density of gravitational waves radiated from the collapsing network scales with $\sigma_{DW}^2 \sim M_j^2$ [133], we do not expect a detectable gravitational wave spectrum. Furthermore, the collapse typically happens long after the QCD crossover, so the peak frequency of the spectrum would be too small to account for the signal observed in pulsar timing arrays [18–22].

V. DARK ENERGY

If the electroweak instanton-induced potential is the sole or leading contribution to the Majoron mass, this scale is suggestively close the value of the Hubble rate today $H_0 = \mathcal{O}(10^{-33}$ eV), which implies that the Majoron could act as dynamical dark energy, also known as quintessence [134–137], which was previously considered in Refs. [78–80, 138]. pNGB quintessence models [78, 139–143] with [144] $m_j \simeq H_0$ typically require decay constants $f_j \equiv 2v_L/3 \lesssim M_{Pl}$. to explain the present day energy density about $0.7 \times 3H_0^2 M_{Pl}^2 / (8\pi)$ and such large v_L would suppress the mass from Eq. (6) far below H_0 for M_{UV} at or below the Planck scale. This outcome can be avoided by again modifying the $SU(2)_W$ β -function via Eq. (16), which increases m_j parametrically via the relation in Eq. (15). Another way to come to the same conclusion is to note that the Majoron's potential for $M_{UV} \simeq M_{Pl}$ reads $V_j = \mathcal{O}(10^{-132} M_{Pl}^4)$ [80], which is much smaller than the required energy density of the Universe today of roughly $\mathcal{O}(10^{-123} M_{Pl}^4)$ [79]. Here, the caveats from the quality problem [102–111] apply as well, and we assume some mechanism that guarantees that all other unwanted contributions to the Majoron potential are absent.

In fact, recent data from the first two releases of the DESI Collaboration, which measure baryon acoustic oscillations, point toward a hint for time-dependent dynamical dark energy [145–149], whose significance ranges between about $(3\text{--}4)\sigma$ depending on the supernova datasets used in the combined analysis. Attempts to explain this hint in terms of thawing [150] pNGB quintessence

⁷Here, we do not include additional fermions following Ref. [77], because they typically lead to additional zero modes in the one instanton background and thus can change the integral over the instanton radius. The insertions to close of the additional zero modes can even cancel the enhancement from the change of the running, which was found to occur in supersymmetric theories in Refs. [78, 80]. Moreover, the change in the prefactor g_W of Eq. (6) due to the additional scalars is a two-loop effect [77].

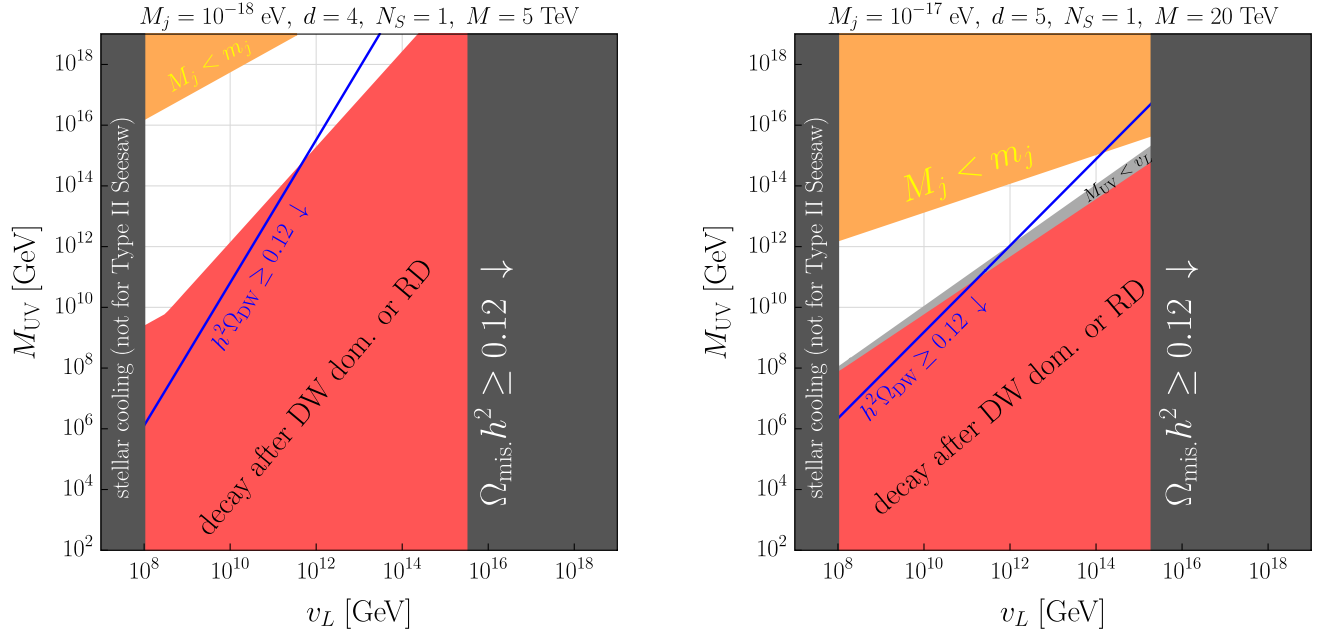


FIG. 2. Parameter space for the production of ultralight Majoron dark matter from domain wall decay. Here, we depict a Majoron mass of $M_j = 10^{-18}$ eV together with a single scalar quadruplet ($d = 4$) with mass $M = 5$ TeV contributing to the one-loop β -function of $SU(2)_W$ for the electroweak instanton-induced bias term m_j (left) and $M_j = 10^{-17}$ eV together with a quintuplet ($d = 5$) of mass $M = 20$ TeV (right). On the blue line, one can reproduce the dark matter relic abundance solely by domain wall decay [see Eq. (13)]. The red region is ruled out because the domain walls would either decay after they dominate the Universe's energy budget [see Eq. (12)] or after matter-radiation equality [see Eq. (14)]. In the orange slice, the bias term contribution m_j becomes larger than M_j . The small gray region is excluded because the lepton number breaking scale v_L would need to be larger than the cutoff M_{UV} of our EFT. Stellar cooling bounds from the Majoron's coupling to electrons typically rule out $v_L < 10^8$ GeV for the Type I/III seesaw [57], but this can be avoided for the Type II seesaw [132]. For larger values of $v_L > 10^{15}$ GeV, the energy density from coherent oscillations from the misalignment mechanism [93–95] becomes the dominant source of the dark matter relic density. In all plots, we take $|c_L| \cos(\theta_{EW} + 3\delta_L)^{1/3} = 1$.

were undertaken in Refs. [149,151–155]. The analysis of Ref. [149] found the following parameter ranges by combining the DESI data with CMB data from Planck [156,157] and ACT [158,159] with various supernova datasets: $\log_{10}(m_j/\text{eV}) = -32.67^{+0.23}_{-0.25}$ and $\log_{10}(f_j/M_{\text{Pl}}^{\text{red}}) = -0.13^{+0.33}_{-0.29}$ for Pantheon+ [160], $\log_{10}(m_j/\text{eV}) = -32.50^{+0.28}_{-0.30}$ and $\log_{10}(f_j/M_{\text{Pl}}^{\text{red}}) = -0.29^{+0.63}_{-0.35}$ for Union3 [161], as well as $\log_{10}(m_j/\text{eV}) = -32.63^{+0.26}_{-0.30}$ and $\log_{10}(f_j/M_{\text{Pl}}^{\text{red}}) = -0.09^{+0.66}_{-0.40}$ for DESY5 [162]. Here, $M_{\text{Pl}}^{\text{red}}$ denotes the reduced Planck mass.

From the parameter space depicted in Fig. 3, we find that the hint for dynamical dark energy for each supernova dataset be accounted for by, e.g., the addition of a single triplet ($d = 3$) scalar with a mass of $M = 10$ TeV for $|c_L| \cos(\theta_{EW} + 3\delta_L)^{1/3} = 1$. Note that the required value $v_L = \mathcal{O}(10^{18})$ GeV is close to, or even above, the reduced Planck scale. String theory-motivated conjectures such as the weak gravity conjecture [163] and the swampland distance conjecture [164] seem to rule trans-Planckian decay constants. To be more precise, the weak gravity conjecture demands for a given instanton action S_{inst} that [163]

$$f_j < \frac{M_{\text{Pl}}^{\text{red}}}{S_{\text{inst}}}. \quad (17)$$

Pushing the cutoff M_{UV} to the reduced Planck scale, we find for the case of pure SM $S_{\text{inst}} = 8\pi^2/g_W^2(M_{\text{Pl}}^{\text{red}}) = \mathcal{O}(100)$, and for the aforementioned modification of the running from a triplet with mass 10 TeV, we numerically obtain an instanton action of the same order of magnitude. Hence, the lepton number breaking scale should be $v_L < \mathcal{O}(10^{16})$ GeV, and we conclude that parameter space for dark energy is incompatible with these conjectures. A supersymmetric solution of this problem can be found in Ref. [80]. Furthermore, the preferred regions of parameter space in Fig. 3 are located at the edge of the regime of validity ($v_L < M_{UV}$) for our effective treatment in terms of the operator in Eq. (5). On the other hand, this could also be interpreted as pointing toward Planck scale origin of Eq. (5), via explicit quantum gravitational breaking of $U(1)_{B+L}$.

The previous discussion involved a vanishing cosmological constant and a quintessence field, whose equation of state parameter ω_j never crosses over into the so called

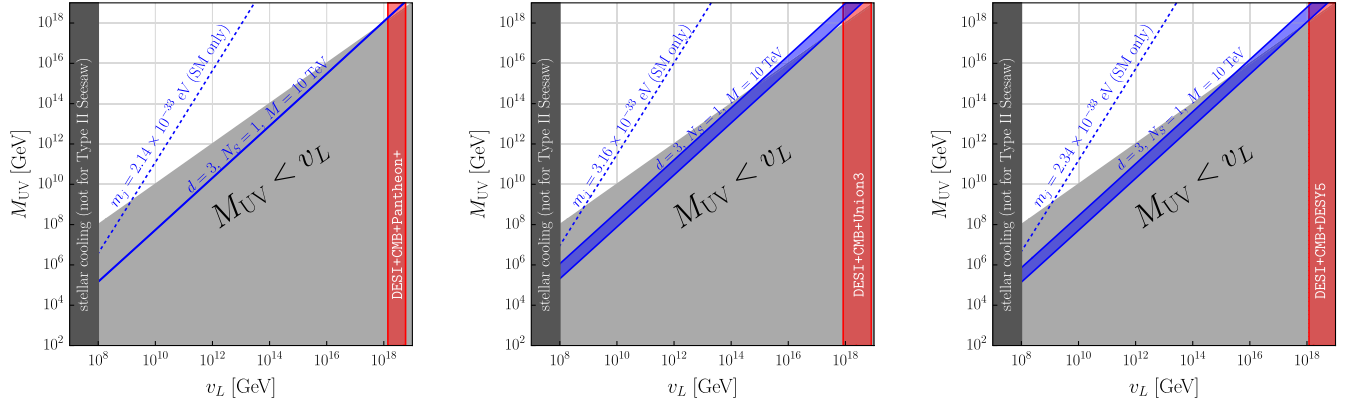


FIG. 3. Parameter space that reproduces the preferred range of Majoron masses and decay constants $f_j \equiv 2v_L/3$ for the hint of thawing quintessence in the combined data from DESI [145–149], the CMB surveys Planck [156,157], and ACT [158,159], as well as the supernova catalogs Pantheon+ [160] (left), Union3 [161] (middle), and DES+5 [162] (right) found by the analysis in Ref. [149]. The blue bands correspond to the required range of m_j , and the red bands indicate the needed range of v_L . In the gray region, the lepton number breaking scale is larger than the UV cutoff M_{UV} of the assumed effective field theory. In all plots, we take $|c_L| \cos(\theta_{EW} + 3\delta_L)^{1/3} = 1$ and include the effect of a single triplet ($d = 3$) scalar with a mass of $M = 10$ TeV on the one-loop β -function of $SU(2)_W$.

“phantom” regime $\omega_j < -1$ [165,166]. While some authors (see, e.g., Ref. [167]) claim that the apparent preference for a phantom crossing found in the first DESI data release could be attributed to the phenomenological Chevallier-Polarski-Linder parametrization [168,169] of the equation of state used in many fits, nonparametric reconstruction methods also seem find a phantom behavior [149]. One can accommodate this feature via the assumption of a negative kinetic term for the quintessence field, which however leads to a violation of the null energy condition and various problems such as, e.g., vacuum decay [170] (see Ref. [171] for a review). Modified gravity theories that equip the quintessence field with a nonminimal coupling to gravity [172–174] can describe the phantom crossing without running into the aforementioned pathologies. Another approach is to consider multiple fields, where each component has an equation of state above -1 , but the *effective* equation of state for the multifield system can cross the phantom divide. Typical examples consider a coupling between quintessence and dark matter [175,176], see Refs. [177,178] for recent microphysical implementations of this idea. In Ref. [179], a very minimal realization of this “phantom mirage” is presented: Here, one considers a cosmological constant Λ together with a Majoron of the mass $\log_{10}(m_j/\text{eV}) = -32.5$ with a present day energy fraction of $r_j = \rho_j/(\rho_j + \rho_\Lambda) \simeq (8\text{--}12)\%$. Initially, the Majoron will be frozen, and it acts as a component of dark energy ($\omega_j \gtrsim -1$) at redshifts $z \geq 1$. Due to $m_j \gtrsim H_0$, it begins its coherent oscillations before the present time $z = 0$, so that it acts as dark matter today ($\omega_j = 0$). The interplay of Majoron’s dynamics with the cosmological constant realizes the phantom crossing of their effective equation of state. Due to the smaller energy fraction

$r_j \simeq (8\text{--}12)\%$, one requires a somewhat smaller lepton number breaking scale of $v_L \simeq (4.7\text{--}5.8) \times 10^{17}$ GeV compared to the quintessence scenario without a cosmological constant. This makes it easier to accommodate the mirage scenario in our effective field theory approach, but we note that this scale is still in violation of the weak gravity conjecture [163] by about an order of magnitude.

If the dark energy is realized in terms of our electroweak Majoron, we do not expect it to be able to explain the observed birefringence in the CMB [180–183], because $U(1)_{B-L}$ is anomaly free with respect to electromagnetism so that the Majoron-photon coupling will be suppressed by the smallness of the Majoron mass [57,184].

VI. CONCLUSIONS

We critically reexamined previous claims about the formation of domain walls in theories of a spontaneously broken global lepton number due to explicit breaking from nonperturbative effects of the weak interaction [61]. Based on a host of previous arguments [57,68,69,71,72], we reiterated that the Majoron can only couple to the topological term of the weak interaction if $B + L$ is explicitly broken by *additional* interactions. Using the methods of Ref. [77], we estimated the Majoron mass, which heavily depends on the details of the UV theory. Typically, the energy density of the resulting Majoron potential is completely negligible in the early Universe. Consequently, even if $B + L$ is explicitly broken, the domain walls in Majoron models do *not* automatically lead to cosmological disasters, and the model building complications suggested in Ref. [61] are not necessarily required.

The ultralight electroweak Majoron could be relevant for dark energy in the late Universe [78–80] and might explain the hint for dynamical dark energy in the recent DESI data [145–149]. Alternatively, if there exists a larger source of explicit lepton number breaking apart from the weak instantons, we can utilize the instanton-induced potential as a bias term to destabilize the wall network, which can produce the correct dark matter relic density in terms of ultralight Majorons. Thus, the weak anomaly of the lepton number could actually be a *virtue* instead of disaster.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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