

Fully-strange tetraquarks: Fall-apart decays and experimental candidates

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We present a systematic analysis of the fall-apart decays for the 1S, 1P, and 2S-wave fully-strange tetraquark states. It shows that most of the fully-strange tetraquark states have a relatively narrow fall-apart decay width of $\mathcal{O}(10)$ MeV. The newly observed axial-vector state $X(2300)$ at BESIII may favor the low-lying 1S-wave 1^{+-} state $T_{(4s)1^{+-}}(2323)$, while the $X(2500)$ resonance observed in the earlier BESIII experiment may favor the low-lying 1P-wave 0^{-+} state $T_{(4s)0^{-+}}(2481)$. Some fully-strange tetraquark states predicted in theory can be searched for in their dominant fall-apart decay channels in experiments, such as $\phi\phi$, $\phi\phi(1680)$, $\eta^{(\prime)}\phi$, $\eta^{(\prime)}h_1(1415)$, and $\phi f'_2(1525)$, to which they have relatively large couplings.

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I. INTRODUCTION

Searching for genuine exotic hadrons beyond the conventional quark model has been one of the most important initiatives since the establishment of the quark model in 1964 [1,2]. Recently, an exotic resonance $X(6900)$ was observed in the di- J/ψ invariant mass spectrum by the LHCb Collaboration [3], which was also confirmed by the CMS and ATLAS Collaborations three years later [4,5]. In addition, in the lower-mass region, the CMS measurements show that exotic resonances $X(6600)$ and $X(7100)$ can also be established in the di- J/ψ spectrum. These exotic resonances may be evidence for genuine fully-charmed tetraquark states. As an analogy, one may expect to observe stable fully-strange tetraquark $T_{ss\bar{s}\bar{s}}$ states as well in experiments though some ambiguities may be inevitable due to the light mass of the strange quark.

Over the past 20 years, several $T_{ss\bar{s}\bar{s}}$ candidates have been observed in experiments. For example, the $f_0(2200)$ and $f_2(2340)$ resonances listed in the *Review of Particle*

Physics (RPP) [6] may favor the 1S-wave $T_{ss\bar{s}\bar{s}}$ states according to the mass analysis in the literature [7,8]. In 2016, the BESIII Collaboration observed a new resonance $X(2500)$ in the di- ϕ invariant mass spectrum of the $J/\psi \rightarrow \gamma\phi\phi$ process [9]. The $X(2500)$ may be explained as the 1P-wave $T_{ss\bar{s}\bar{s}}$ state with $J^{PC} = 0^{-+}$ from the point view of the mass [7,10]. Very recently, a new axial-vector state with $J^{PC} = 1^{+-}$, denoted as $X(2300)$, was observed in the $\phi\eta$ and $\phi\eta'$ invariant mass spectra by the BESIII Collaboration [11]. With the interpretation of the conventional $s\bar{s}$ state $h_1(3P)$, the observed mass is about 100 MeV lower than the theoretical predictions [12]. However, it is interesting to find that the observed mass of $X(2300)$ is consistent with our prediction for the $T_{(ss\bar{s}\bar{s})1^{+-}}(2323)$ [7]. To further confirm these candidates as $T_{ss\bar{s}\bar{s}}$ states, in addition to the mass spectrum, a systematic study of the fall-apart decay properties is also crucial.

In the present work, we further systematically explore the fall-apart decay mechanism of the $T_{ss\bar{s}\bar{s}}$ states by combining our previous predictions of the mass spectrum [7]. Although there are many studies on the $T_{ss\bar{s}\bar{s}}$ mass spectrum from various models and approaches, such as QCD sum rules [10,13–21], quark models [8,22–24], and the constituent quark model combined with the complex scaling method [25], detailed calculations of the decay properties of $T_{ss\bar{s}\bar{s}}$ are still limited. Some efforts are made on the fall-apart decays [20,22] and radiative transitions [23] based on different solutions for the wave functions. In this work,

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we focus on the $1S$ -, $1P$ -, and $2S$ -wave $T_{ss\bar{s}\bar{s}}$ states and look for candidates for the $T_{ss\bar{s}\bar{s}}$ states from the decay properties. In particular, we show that some decay channels may be useful for further identifying the $T_{ss\bar{s}\bar{s}}$ states in experiments.

The paper is organized as follows. In Sec. II, the theoretical framework is introduced. In Sec. III, numerical results concerning the properties of the fully-strange states are presented and discussed. Finally, a brief summary is provided in Sec. IV.

II. THEORETICAL FRAMEWORK

In our previous work [7], the mass spectra of the $1S$ -, $1P$ -, and $2S$ -wave $T_{ss\bar{s}\bar{s}}$ states were systematically studied within a nonrelativistic quark model. By using the masses and wave functions obtained from Ref. [7], in this work, we further evaluate the fall-apart decays of the $T_{ss\bar{s}\bar{s}}$ states within a quark-exchange model [26]. In this model, the quark-quark interactions as part of the Hamiltonian are considered to be the sources of the fall-apart decays of multiquark states via the quark rearrangement, which is shown in Fig. 1.

For the decay process $A \rightarrow BC$, the decay amplitude $\mathcal{M}(A \rightarrow BC)$ is described by

$$\mathcal{M}(A \rightarrow BC) = -\sqrt{(2\pi)^3} \sqrt{8M_A E_B E_C} \left\langle BC \left| \sum_{i < j} V_{ij} \right| A \right\rangle, \quad (1)$$

where A stands for the initial multiquark state, and BC stands for the final hadron pair. M_A is the mass of the initial state, while E_B and E_C are the energies of the final states B and C in the initial-hadron-rest system, respectively. While V_{ij} stands for the interactions between the inner quarks of final hadrons B and C (note that $ij \neq 13, 24$ or $ij \neq 14, 23$), they are taken the same form as that of the nonrelativistic potential model adopted in Ref. [7]. The details of the potentials V_{ij} and the parameters are given in Eq. (2) and Table I of Ref. [7], respectively.

Then, the partial decay width Γ for the decay process $A \rightarrow BC$ is given by

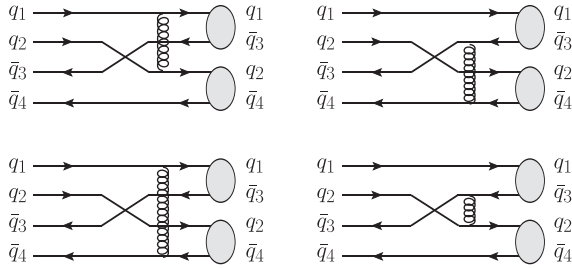


FIG. 1. The fall-apart decays of a $T_{ss\bar{s}\bar{s}}$ state induced by the interactions V_{ij} ($ij \neq 13, 24$ or $ij \neq 14, 23$) between inner quarks of final hadrons B and C .

$$\Gamma = \frac{1}{s!} \frac{1}{2J_A + 1} \frac{|\mathbf{p}|}{8\pi M_A^2} |\mathcal{M}(A \rightarrow BC)|^2, \quad (2)$$

where $|\mathbf{p}|$ stands for the magnitude of the three momentum for the final states B and C in the rest system of the initial A state. The term $1/s!$ represents a statistical factor that accounts for the indistinguishability of particles. In scenarios where the final state contains two or more identical particles, it is necessary to divide by the number of permutations among these particles to avoid overcounting, as they are indistinguishable from one another. This phenomenological quark-exchange model has achieved a good description of the low-energy S -wave phase shift for the $I = 2$ $\pi\pi$ scattering at the quark level [26,27]. Recently, this model has also been extended to study the fall-apart decays of multiquark states in the literature [28–34], and a lot of inspiring results are obtained.

In the present work, the masses and wave functions of the initial $T_{ss\bar{s}\bar{s}}$ states are adopted from the numerical results obtained in our previous work [7]. For the final B and C meson states, for simplicity, the wave functions are adopted in a single harmonic oscillator (SHO) form. Their SHO parameters are determined by fitting the root mean square radii of the $s\bar{s}$ states, which are obtained from the potential model calculations using the same Hamiltonian as in Ref. [12]. Our determined SHO parameters for the final meson states are collected in Table I. For the well-established meson states, the masses are adopted from the PDG averaged values [35]; otherwise, the meson masses are adopted the potential model predictions. The masses of the final meson states are collected in Table I as well. It should be mentioned that the η and η' mesons are mixed states between $s\bar{s}$ and $n\bar{n}$ ($n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$), i.e.,

$$\begin{aligned} \eta &= n\bar{n} \cos \phi_P - s\bar{s} \sin \phi_P, \\ \eta' &= n\bar{n} \sin \phi_P + s\bar{s} \cos \phi_P, \end{aligned} \quad (3)$$

TABLE I. The masses (MeV) and effective SHO parameters β (MeV) of the final meson states. The η and η' mesons are mixed states via $n\bar{n}$ - $s\bar{s}$ flavor mixing; their effective SHO parameters are estimated by using the pure $s\bar{s}$ configuration. The missing 2^1S_0 $s\bar{s}$ state is denoted by $\eta'(2S)$. There is still controversy for both the $f_0(1370)$ and $f_1(1420)$ resonances as the assignments of $s\bar{s}$ states, which is marked with a superscript “?”.

State	Meson	$M_{\text{exp}}[35]$	$M_{\text{th}}[12]$	β
1^1S_0	η/η'	548/958	...	466
1^3S_1	$\phi(1020)$	1019	1017	388
2^1S_0	$\eta'(2S)$	...	1619	367
2^3S_1	$\phi(1680)$	1680	1699	330
1^3P_0	$f_0(1370)^?$	1400 ± 100	1373	365
1^1P_1	$h_1(1415)$	1409	1462	334
1^3P_1	$f_1(1420)^?$	1428	1492	327
1^3P_2	$f_2'(1525)$	1517	1513	308

where the mixed angle is taken to be $\phi_P = 41.2^\circ$, which is determined by fitting the η' photo-production data in Ref. [36]. The SHO parameters, $\beta = 466$ MeV, for the η and η' mesons are estimated by using the pure $s\bar{s}$ configuration.

III. RESULTS AND DISCUSSIONS

The decay results are presented in Table II. For the ground state, the spin-spin interactions play a crucial role for the fall-apart decays of a multiquark state. However, for some excited states, the linear confinement potential and/or Coulomb potential also have significant contributions. For most of the $T_{ss\bar{s}\bar{s}}$ states, the width contributed by the fall-apart decays is no more than 50 MeV, while several excited states possess much larger widths due to the opening of specific channels. Based on the mass spectrum and fall-apart decay properties, some discussions about the $T_{ss\bar{s}\bar{s}}$ states are given as follows.

A. 1S states

According to our quark model predictions, there are four 1S states $T_{(4s)0^{++}}(2218)$, $T_{(4s)0^{++}}(2440)$, $T_{(4s)1^{++}}(2323)$, and $T_{(4s)2^{++}}(2378)$ [7]. Their masses are significantly greater than the $\phi\phi$ threshold, which allows them to decay into the $\eta\eta$, $\eta\eta'$, $\eta'\eta'$, and/or $\phi\phi$ channels through quark rearrangement.

1. $J^{PC} = 0^{++}$ state

The $T_{(4s)0^{++}}(2218)$, as the lowest $T_{ss\bar{s}\bar{s}}$ state, has a fall-apart width of $\Gamma \simeq 19$ MeV, which is mainly contributed by the $\eta\eta'$, $\eta'\eta'$, and $\phi\phi$ channels, and also has sizeable decay rates into the $\eta\eta$ channel. The partial width ratios between these channels are estimated to be

$$\Gamma[\eta\eta] : \Gamma[\eta\eta'] : \Gamma[\eta'\eta'] : \Gamma[\phi\phi] \simeq 1.0 : 3.4 : 2.7 : 4.3. \quad (4)$$

While the high-lying 1S state $T_{(4s)0^{++}}(2440)$ also has a fall-apart width of $\Gamma \simeq 18$ MeV, which is nearly saturated by the $\phi\phi$ channel, the decay rates into the $\eta\eta'$ and $\eta'\eta'$ channels are estimated to be $\mathcal{O}(1\%)$.

Some evidences of the ground 0^{++} $T_{ss\bar{s}\bar{s}}$ states may have been observed in experiments. In Ref. [37], Kozhevnikov carried out a dynamic analysis of the resonance contributions to $J/\psi \rightarrow \gamma X \rightarrow \gamma\phi\phi$ by using the data from BESIII [9]. This analysis indicates that there may exist two 0^{++} resonances with masses ~ 2.2 GeV and ~ 2.4 GeV in the $\phi\phi$ invariant mass spectrum. The two scalar resonances extracted from the data of $J/\psi \rightarrow \gamma X \rightarrow \gamma\phi\phi$ are likely candidates of the states $T_{(4s)0^{++}}(2218)$ and $T_{(4s)0^{++}}(2440)$ predicted in the quark model. Furthermore, the analysis of the decay data of $B^0 \rightarrow J/\psi\phi\phi$ [38] also shows an evidence of a scalar resonance around 2.2 GeV in the $\phi\phi$ mass spectrum [39].

It should be mentioned that the evidences of the scalar resonance around 2.2 GeV observed in the $\phi\phi$ channel may be consistent with the resonance $f_0(2200)$ listed in RPP [6]. The mass of $f_0(2200)$ cannot be explained with a conventional $s\bar{s}$ state, however, is consistent with the ground $T_{ss\bar{s}\bar{s}}$ state predicted in the quark model [8]. For the $f_0(2200)$ resonance, the $K\bar{K}$ and $\eta\eta$ channels have been observed in experiments. To confirm the nature of $f_0(2200)$ and establish the 0^{++} 1S-wave $T_{ss\bar{s}\bar{s}}$ states, more observations of the $\eta\eta'$, $\eta'\eta'$ and $\phi\phi$ channels are expected to be carried out in future experiments.

Finally, it should be mentioned that the full-strange tetraquark states $T_{(4s)0^{++}}(2218)$ and $T_{(4s)0^{++}}(2440)$ are flavor partners of the fully-charmed tetraquark states $T_{(4c)0^{++}}(6455)$ and $T_{(4c)0^{++}}(6550)$ predicted in our previous works [34,40], respectively. Some evidence of these two scalar fully-charmed tetraquark states may have been observed in recent LHC experiments as well. The $T_{(4c)0^{++}}(6455)$ and $T_{(4c)0^{++}}(6550)$ may be the contributors to the bump structures around 6.4–6.6 GeV observed in the di- J/ψ invariant mass spectrum at LHC [3,4].

2. $J^{PC} = 2^{++}$ state

For the 1S state $T_{(4s)2^{++}}(2378)$, the fall-apart width is a very small value of $\Gamma \simeq 1$ MeV. It is interesting to note that the decay channel into $\phi\phi$ is forbidden. The prohibition of the decay of the 1S state $T_{(4s)2^{++}}(2378)$ into $\phi\phi$ originates from the exact cancellation among the dynamical transition matrix elements. Specifically, under the configuration of the wave function $\psi_{000}^{1S} \chi_{2m_s}^{11} |\bar{3}3\rangle^c$, the color matrix elements exhibit opposite signs, satisfying the identity,

$$\begin{aligned} \langle \bar{3}_{12} 3_{34} | \lambda_1 \cdot \lambda_2 | 1_{13} 1_{24} \rangle &= \langle \bar{3}_{12} 3_{34} | \lambda_3 \cdot \lambda_4 | 1_{13} 1_{24} \rangle \\ &= -\langle \bar{3}_{12} 3_{34} | \lambda_1 \cdot \lambda_4 | 1_{13} 1_{24} \rangle \\ &= -\langle \bar{3}_{12} 3_{34} | \lambda_2 \cdot \lambda_3 | 1_{13} 1_{24} \rangle. \end{aligned} \quad (5)$$

Meanwhile, the associated 1S spatial wave function ensures that the interaction integrals for all quark pairs are identical. Consequently, the two primary components of the decay amplitude, $\langle B_{13} C_{24} | V_{12} + V_{34} | A \rangle$ and $\langle B_{13} C_{24} | V_{14} + V_{23} | A \rangle$ (or the corresponding terms for the $B_{14} C_{23}$ channel), will cancel each other exactly. This leads to a complete nullification of all potential terms driving the decay—including the confinement, Coulomb, spin-spin, and tensor interactions—when taking the summation of V_{12} , V_{34} , V_{14} , and V_{23} . This dynamical prohibition, rooted in the symmetry of the wave function, also explains why the rearrangement decay widths of $ss\bar{s}\bar{s}$ systems are generally suppressed. The partial widths for the main fall-apart channels $\eta\eta$, $\eta\eta'$, and $\eta'\eta'$ are predicted to be $\mathcal{O}(100)$ keV.

In Refs. [8,23], the tensor resonances $f_2(2300)$ and $f_2(2340)$ listed in RPP [6] were suggested to be candidates of the 1S state $T_{ss\bar{s}\bar{s}}$ with $J^{PC} = 2^{++}$ according to the mass

TABLE II. Partial fall-apart decay widths (MeV) of the $T_{s\bar{s}\bar{s}}$ states. In this table, “o” denotes calculated values less than 0.05 MeV, and “...” denotes a forbidden decay channel. The 2^1S_0 and 2^3S_1 $s\bar{s}$ states involving in the fall-apart decay channels are denoted by $\eta'(2S)$ and $\phi(2S)$, respectively. While the $1P$ $s\bar{s}$ states with $J^{PC} = 0^{++}, 1^{+-}, 1^{++}, 2^{++}$ are denoted by f_0, h_1, f_1 , and f'_2 , respectively.

State	$\eta\eta$	$\eta\eta'$	$\eta'\eta'$	$\phi\phi$	ηf_0	$\eta' f_0$	ηf_1	$\eta' f_1$	ηf_2	$\eta' f_2$	$\eta' f'_2$	ϕh_1	$\eta\eta'(2S)$	$\eta'\eta'(2S)$	$\phi\phi(2S)$	$f_0 f_0$	$f_0 f_1$	$f_0 f'_2$	$f_1 f_1$	$f_1 f'_2$	$h_1 h_1$	$f'_2 f'_2$
$M_{f_1} + M_{f_2}$	1096	1506	1916	2038	1921	2331	2040	2450	2065	2475	2475	2428	2167	2577	2699	2746	2865	2890	2984	3009	2818	3034
$T_{(4s)0^{++}(2218)}(1S)$	1.6	5.5	4.4	6.9	...	...	...	...	...	...	...	...	0.1	...	...	...	...	...	...	...	...	...
$T_{(4s)0^{++}(2440)}(1S)$	0.1	0.3	0.2	17.8	...	...	...	...	...	...	...	...	0	...	...	...	...	...	...	...	...	...
$T_{(4s)2^{++}(2378)}(1S)$	0.2	0.4	0.2	...	...	...	...	...	...	...	...	...	0	...	...	...	...	...	...	...	...	...
$T_{(4s)0^{++}(2798)}(2S)$	0.6	2.4	2.3	0.7	...	...	0.2	0.4	...	...	...	8.8	7.0	23.9	13.1	0.1	...	...	...	...	...	...
$T_{(4s)0^{++}(2876)}(2S)$	o	o	o	21.1	...	...	0.3	0.5	...	...	...	2.1	0.1	0.5	22.0	3.8	...	...	...	...	191.9	...
$T_{(4s)0^{++}(2954)}(2S)$	0.4	1.3	1.1	1.3	...	...	0.5	0.9	...	...	...	3.2	1.3	3.4	36.8	2.1	...	0	...	...	241.1	...
$T_{(4s)0^{++}(3155)}(2S)$	o	0.1	0.1	1.5	...	...	0.1	0.3	...	...	...	0.8	1.6	4.4	19.3	0.3	...	1.1	7.3	4.6	14.8	23.0
$T_{(4s)1^{++}(2943)}(2S)$	...	...	...	o	o	o	0.1	0.1	0.1	0.1	0.1	0.4	...	...	0.2	...	0.1	...	...	...	0.1	...
$T_{(4s)2^{++}(2854)}(2S)$	0.2	0.4	0.2	3.7	...	...	0.2	0.2	0.1	o	o	0.3	0.1	0.1	43.2	o	...	...	...	...	o	...
$T_{(4s)2^{++}(2981)}(2S)$	o	0.1	o	6.3	...	...	0.2	0.5	1.5	2.6	2.6	0.6	0.1	0.1	69.6	2.8	41.8	169.8	...	...	0.1	...
$T_{(4s)0^{++}(2481)}(1P)$	...	...	...	3.7	5.3	5.2	...	...	o	...	...	0.6	...	...	...	...	...	...	...	...	...	...
$T_{(4s)0^{++}(2635)}(1P)$	...	...	...	14.5	2.8	6.3	...	...	0.6	0.2	0.2	3.4	...	...	...	...	...	...	...	...	...	...
$T_{(4s)0^{++}(2761)}(1P)$	...	...	...	2.5	9.7	22.2	...	...	2.7	1.9	1.9	0.4	...	...	o	...	...	...	...	...	...	...
$T_{(4s)1^{++}(2564)}(1P)$	...	...	...	0.1	...	...	1.3	1.9	0.2	o	o	1.0	o	...	...	...	...	...	...	...	...	...
$T_{(4s)1^{++}(2632)}(1P)$	...	...	...	2.6	...	...	0.1	o	0.1	o	o	16.6	...	...	...	...	...	...	...	...	...	...
$T_{(4s)1^{++}(2778)}(1P)$	...	...	...	o	...	...	4.9	10.0	o	o	o	o	o	...	...	...	...	...	...	...	...	...
$T_{(4s)2^{++}(2537)}(1P)$	...	...	...	0.1	0.1	0.1	0.1	o	1.3	1.9	1.9	11.1	...	...	...	...	...	...	...	...	...	...
$T_{(4s)2^{++}(2669)}(1P)$	...	...	...	0.1	0.3	0.2	0.3	0.1	2.1	3.8	3.8	14.8	...	...	...	...	...	...	...	...	...	...
$T_{(4s)2^{++}(2837)}(1P)$	...	...	...	o	0.7	0.6	1.0	0.8	1.0	0.5	0.5	1.6	...	...	o	...	...	...	...	...	...	...
State	$\eta\phi$	$\eta'\phi$	$\eta'\phi$	ηh_1	$\eta' h_1$	$f_0 h_1$	$f_1 h_1$	$f_2 h_1$	$f_1 \phi$	$f'_2 \phi$	$\eta\phi(2S)$	$\eta'\phi(2S)$	$\phi\eta(2S)$	2638	2638	2782	2901	2926				
$M_{f_1} + M_{f_2}$	1567	1977	2367	1957	2367	2392	2511	2536	2511	2536	2228	2638	2638	2638	2638	2782	2901	2926				
$T_{(4s)1^{++}(2323)}(1S)$	3.3	5.0	...	...	...	...	...	...	...	...	0.1	...	...	...	...	...	...	...	...	...	...	...
$T_{(4s)0^{++}(2891)}(2S)$	...	...	...	...	0.2	1.2	2.4	5.5	2.4	5.5	...	...	...	...	...	...	...	...	...	...	...	...
$T_{(4s)0^{++}(2967)}(2S)$	...	...	...	1.8	3.0	0.1	0.4	0.4	0.4	0.4	...	...	...	...	...	...	0.1	...	o	...	...	...
$T_{(4s)1^{++}(2835)}(2S)$	3.3	6.1	...	o	0.1	0.1	0.5	o	0.5	o	0.5	2.2	14.4	11.3	...	...	...	...	...	...	...	...
$T_{(4s)1^{++}(2950)}(2S)$	0.2	0.5	...	0.5	0.9	0.4	1.8	5.5	1.8	5.5	3.6	12.8	47.9	31.1	387.7	719.8	...	...	...	...	...	...
$T_{(4s)2^{++}(2965)}(2S)$	...	o	...	o	0.1	o	0.1	0.3	0.1	0.3	...	o	o	o	o	0.2	0.4	...	...	...	...	...
$T_{(4s)0^{++}(2507)}(1P)$	2.3	3.0	...	...	...	...	...	...	...	...	0.1	...	...	...	...	...	...	...	...	...	...	...
$T_{(4s)0^{++}(2821)}(1P)$	0.7	1.7	...	...	...	...	47.2	0.6	47.2	0.6	0.4	0.3	o	0.2	...	...	...	...	...	...	...	...
$T_{(4s)1^{++}(2445)}(1P)$	0.1	o	...	o	o	8.6	...	...	...	...	o	...	...	...	...	...	...	...	...	...	...	...
$T_{(4s)1^{++}(2567)}(1P)$	2.3	3.4	...	1.0	0.8	2.3	9.9	6.5	9.9	6.5	o	...	...	...	...	...	...	...	...	...	...	...

(Table continued)

TABLE II. (Continued)

State	$\eta\phi$	$\eta'\phi$	ηh_1	$\eta' h_1$	$f_0\phi$	$f_1\phi$	$f_2\phi$	$\eta\phi(2S)$	$\eta'\phi(2S)$	$\phi\eta(2S)$	$f_0 h_1$	$f_1 h_1$	$f_2 h_1$
$M_{f_1} + M_{f_2}$	1567	1977	1957	2367	2392	2511	2536	2228	2638	2638	2782	2901	2926
$T_{(4s)1^{+-}}(2627)(1P)$	0	0.1	1.3	1.6	0.7	1.6	3.0	0	...	...	...	...	...
$T_{(4s)1^{+-}}(2766)(1P)$	0.2	0.2	0.9	0.7	27.6	0.1	0.7	0.1	0.1	0	...	...	...
$T_{(4s)1^{+-}}(2984)(1P)$	0.5	0.7	7.0	13.8	0.7	9.8	25.4	0	0	0	5.1	4.3	5.8
$T_{(4s)2^{+-}}(2446)(1P)$	0	0	0	...	0	...	...	0.1	...	...	...	...	...
$T_{(4s)2^{+-}}(2657)(1P)$	0.1	0.2	0.2	0.1	0	20.4	15.8	0.1	0	...	...	...	...
$T_{(4s)2^{+-}}(2907)(1P)$	0.7	1.2	0	0	0.8	14.5	1.6	0.1	0.1	0	9.6	0.4	...
$T_{(4s)3^{+-}}(2719)(1P)$	0.1	0.1	0.1	0.1	0.7	1.0	64.2	0	...	...	...	...	...

analysis. Both the $f_2(2300)$ and $f_2(2340)$ have been seen in the $\phi\phi$ channel with a broad width of several hundred MeV. While the $f_2(2340)$ has been seen in the $\eta\eta'$, $\eta'\eta'$, and $\phi\phi$ channels with a broad width of $\Gamma \simeq 300$ MeV.

However, in assigning $f_2(2300)$ or $f_2(2340)$ as the $1S$ state $T_{(4s)2^{++}}(2378)$, we face two challenges: (i) the observed $\phi\phi$ channel cannot be explained. According to our analysis, the $\phi\phi$ channel is forbidden; (ii) the broad width cannot be well understood. The predicted partial widths for the $\eta\eta'$ and $\eta'\eta'$ channels are $\mathcal{O}(100)$ keV, which is negligibly small compared with the measured widths of $f_2(2300)$ and $f_2(2340)$. It should be mentioned that from the point of view of mass, the $f_2(2300)/f_2(2340)$ may be consistent with a tensor glueball predicted within lattice QCD [41,42].

3. $J^{PC} = 1^{+-}$ state and $X(2300)$

The $1S$ state $T_{(4s)1^{+-}}(2323)$ has a narrow fall-apart decay width of $\Gamma \simeq 8$ MeV, which is saturated by the $\eta\phi$ and $\eta'\phi$ channels. The partial width ratio between these two channels is predicted to be

$$\Gamma[\eta\phi] : \Gamma[\eta'\phi] \simeq 1.0 : 1.5. \quad (6)$$

Recently, a new 1^{+-} state $X(2300)$ with a width of $\Gamma_{\text{exp}} = 89 \pm 15 \pm 26$ MeV was observed in the $\phi\eta$ and $\phi\eta'$ invariant mass spectra by using the $\psi(3686) \rightarrow \phi\eta\eta'$ decay at BESIII [11]. If assigning $X(2300)$ as the conventional $s\bar{s}$ state $h_1(3P)$, the observed mass is notably smaller than the quark model predictions, while the measured width is too broad compared to the theoretical expectation [43]. However, it is interesting to find that the observed mass, decay channels, and quantum numbers of the new resonance $X(2300)$ favor the assignment of the fully-strange tetraquark state $T_{(4s)1^{+-}}(2323)$ predicted within the quark model [7].

It should be pointed out that if the $X(2300)$ is a $T_{ss\bar{s}\bar{s}}$ state indeed, the two fall-apart channels $\phi\eta$ and $\phi\eta'$ cannot saturate its decay width, and their branching fractions are estimated to be $\sim 7\%$ and $\sim 11\%$, respectively. The $X(2300)$ may have large potentials to be observed in the other channels, such as $\bar{K}K^* + c.c.$

Finally, it should be mentioned that if $X(2300)$ corresponds to the low-lying fully-strange tetraquark state with $J^{PC} = 1^{+-}$ indeed, its partner in the fully-charmed sector $T_{(4c)1^{+-}}(6500)$ predicted in the quark model [40] may have a large potential to be observed in the $\eta_c J/\psi$ channel.

B. $2S$ states

There are 12 $2S$ states predicted in the quark model. Their masses scatter in the range of ~ 2.8 – 3.2 GeV [7], which is significantly greater than the $\phi\phi(1680)$ threshold. Thus, many fall-apart decay channels as listed in Table II are opened for these states.

1. $J^{PC} = 0^{++}$ states

According to predictions, there are four $2S$ states with $J^{PC} = 0^{++}$: $T_{(4s)0^{++}}(2798)$, $T_{(4s)0^{++}}(2876)$, $T_{(4s)0^{++}}(2954)$, and $T_{(4s)0^{++}}(3155)$ [7].

The low-lying $2S$ state $T_{(4s)0^{++}}(2798)$ has a fall-apart decay width of $\Gamma \simeq 60$ MeV and mainly decays into the $\eta'\eta'(2S)$ and $\phi\phi(1680)$ channels with branching fractions $\sim 40\%$ and $\sim 22\%$, respectively.

While for the high-lying $2S$ state $T_{(4s)0^{++}}(3155)$, the fall-apart decay width is estimated to be $\Gamma \simeq 78$ MeV. This state has a large decay rate into the $\phi\phi(1680)$ channel with a branching fraction of about 25%.

Both the $T_{(4s)0^{++}}(2876)$ and $T_{(4s)0^{++}}(2954)$ states may be too broad to be observed in experiments. Their partial widths into the $h_1(1415)h_1(1415)$ channel are estimated to be ~ 200 – 250 MeV due to a large overlap of the spatial wave functions of initial and final states. Except for the dominant $h_1(1415)h_1(1415)$ channel, they also have significant decay rates into the $\phi\phi(1680)$ channel with a partial width of $\mathcal{O}(10)$ MeV.

To search for the 0^{++} $2S$ $T_{ss\bar{s}\bar{s}}$ states, the $\phi\phi(1680)$ channel is worth observing in future experiments.

2. $J^{PC} = 2^{++}$ states

According to our quark model predictions, there are two $2S$ states with $J^{PC} = 2^{++}$: $T_{(4s)2^{++}}(2854)$ and $T_{(4s)2^{++}}(2981)$ [7].

The low-lying state $T_{(4s)2^{++}}(2854)$ has a fall-apart width of $\Gamma \sim 49$ MeV, which is mainly contributed by the $\phi\phi(1680)$ channel with a branching fraction of $\sim 89\%$. While the partial widths for the $\eta\eta$, $\eta\eta'$, $\eta'\eta'$, $\eta\eta(2S)$, and $\eta'\eta(2S)$ channels are predicted to be $\mathcal{O}(100)$ keV.

The high-lying $2S$ state $T_{(4s)2^{++}}(2981)$ may be a broad state with a very large fall-apart decay width of $\Gamma \simeq 296$ MeV. It is mainly contributed by the $f_0(1370)f_2'(1525)$ channel, while the contributions from the $\phi\phi$ and $\phi\phi(1680)$ channels are also sizeable. The partial width ratios between these main channels are predicted to be

$$\Gamma[\phi\phi] : \Gamma[\phi\phi(1680)] : \Gamma[f_0(1370)f_2'(1525)] \simeq 1 : 11 : 27. \quad (7)$$

To establish the 2^{++} $2S$ -wave $T_{ss\bar{s}\bar{s}}$ states, the $\phi\phi(1680)$ invariant mass spectrum around the 2.8–3.0 GeV region is worth observing in future experiments.

3. $J^{PC} = 1^{+-}$ states

According to our quark model predictions, there are two $2S$ states with $J^{PC} = 1^{+-}$: $T_{(4s)1^{+-}}(2835)$ and $T_{(4s)1^{+-}}(2950)$ [7].

For the $T_{(4s)1^{+-}}(2835)$ state, the fall-apart width is estimated to be $\Gamma \simeq 38$ MeV. This state has large decay rates into the $\eta'\phi$ and $\phi\eta'(2S)$ channels. The partial width

ratio between these two main channels is predicted to be

$$\Gamma[\eta'\phi] : \Gamma[\phi\eta'(2S)] \simeq 1.0 : 2.4. \quad (8)$$

The $T_{(4s)1^{+-}}(2835)$ also has sizeable decay rates into the $\eta\phi$ and $\eta'\phi(1680)$ channels with a partial width in the range of ~ 4 – 7 MeV. To search for the $T_{(4s)1^{+-}}(2835)$ state, the $\eta'\phi$ and $\eta'\phi(1680)$ channels are worth observing in future experiments.

The high-lying state $T_{(4s)1^{+-}}(2950)$ may be too broad to be observed in experiments. The partial widths of $f_1h_1(1415)$ and $f_2'(1525)h_1(1415)$ are estimated to be very large values of ~ 388 MeV and ~ 720 MeV, respectively.

4. $J^{PC} = 0^{+-}, 2^{+-}$ states

According to quark model predictions, there are two 0^{+-} states, $T_{(4s)0^{+-}}(2891)$ and $T_{(4s)0^{+-}}(2967)$, and one 2^{+-} state, $T_{(4s)2^{+-}}(2965)$ [7].

For the $T_{(4s)0^{+-}}(2891)$ state, the fall-apart width is estimated to be $\Gamma \simeq 10$ MeV. This state may have large decay rates into the $f_0(1370)\phi$, $f_1\phi$, and $f_2'(1525)\phi$ channels with partial width ratios,

$$\Gamma[f_0(1370)\phi] : \Gamma[f_1\phi] : \Gamma[f_2'(1525)\phi] \simeq 1.0 : 2.1 : 4.8. \quad (9)$$

The high-lying 0^{+-} state $T_{(4s)0^{+-}}(2967)$ also has a narrow the fall-apart width of $\Gamma \simeq 11$ MeV, which is mainly contributed by the $\eta h_1(1415)$ and $\eta' h_1(1415)$ channels with branching fractions $\sim 31\%$ and $\sim 54\%$, respectively.

The 2^{+-} state $T_{(4s)2^{+-}}(2965)$ has a very narrow fall-apart width of $\mathcal{O}(100)$ keV, which is mainly contributed by the $f_2'(1525)\phi$, $f_2'(1525)h_1(1415)$, and $f_1h_1(1415)$ with comparable decay rates.

To search for the $2S$ -wave states with $J^{PC} = 0^{+-}$ and 2^{+-} , the $\eta^{(\prime)}h_1(1415)$ and $\phi f_2'(1525)$ channels are worth observing in future experiments.

C. $1P$ states

There are 20 $1P$ states predicted in the quark model. Their masses scatter in the range of ~ 2.4 – 3.0 GeV [7]. The fall-apart partial decay widths are evaluated and presented in Table II.

1. $J^{PC} = 0^{-+}$ states and $X(2500)$

According to quark model predictions, there are three $1P$ -wave states with 0^{-+} : $T_{(4s)0^{-+}}(2481)$, $T_{(4s)0^{-+}}(2635)$, and $T_{(4s)0^{-+}}(2761)$ [7].

The low-lying 0^{-+} state $T_{(4s)0^{-+}}(2481)$ has a relatively narrow fall-apart width of $\Gamma \simeq 15$ MeV. The main fall-apart decay channels are $\phi\phi$, $\eta f_0(1370)$, and $\eta' f_0(1370)$. The partial width ratios are predicted to be

$$\Gamma[\phi\phi]:\Gamma[\eta f_0(1370)]:\Gamma[\eta' f_0(1370)] \simeq 1.0:1.4:1.4. \quad (10)$$

For the middle mass state $T_{(4s)0^{++}}(2635)$, the fall-apart width is estimated to be $\Gamma \simeq 28$ MeV, which is mainly contributed by the $\phi\phi$ channel with a branching fraction of $\sim 52\%$.

In contrast, in the high-lying state $T_{(4s)0^{++}}(2761)$, the fall-apart width is estimated to be $\Gamma \simeq 39$ MeV, which is mainly contributed by the $\eta' f_0(1370)$ and $\eta f_0(1370)$ channels with branching fractions $\sim 25\%$, $\sim 57\%$, respectively. There is a sizeable decay rate into the $\phi\phi$ channel with a branching fraction $\sim 6\%$.

Some evidence of $T_{(4s)0^{++}}(2481)$ may have been observed in experiments. In 2016, the BESIII Collaboration observed a new resonance $X(2500)$ with a broad width of $\Gamma \simeq 230$ MeV in the $\phi\phi$ invariant mass spectrum from the $J/\psi \rightarrow \gamma\phi\phi$ process [9]. The spin-parity numbers seem to more favor 0^{++} . The observed mass, spin-parity numbers, and decay mode of $X(2500)$ are consistent with those of $T_{(4s)0^{++}}(2481)$ predicted in theory. The $X(2500)$ resonance is also suggested as a $0^{++} T_{ss\bar{s}\bar{s}}$ candidate based on the mass analyses within the QCD sum rules [10,19] and the relativized quark model [23]. However, the $X(2500)$ was explained as a conventional $s\bar{s}$ state in the literature [12,44–46]. To confirm the nature of $X(2500)$, more studies are needed in both theory and experiments.

Finally, it should be mentioned that the $X(2370)$ listed in RPP [35] is explained as a $J^{PC} = 0^{-+} T_{ss\bar{s}\bar{s}}$ state in the literature [10,19]. This resonance was observed in the $\pi\pi\eta'$ and $K\bar{K}\eta'$ invariant mass spectra via the J/ψ decays at BESIII [47,48]. Recently, the spin-parity numbers are determined to be 0^{-+} by the BESIII Collaboration [49]. Although the quantum numbers and decay modes are consistent with the $0^{-+} T_{ss\bar{s}\bar{s}}$ states, the measured mass of $X(2370)$ is too low to be comparable with our quark model predictions. The $X(2370)$ may favor a glueball-like particle [50–52] as that predicted by the lattice QCD [42,53].

2. $J^{PC} = 1^{--}$ states

According to our quark model predictions, there are five $1P$ -wave states with 1^{--} : $T_{(4s)1^{--}}(2455)$, $T_{(4s)1^{--}}(2567)$, $T_{(4s)1^{--}}(2627)$, $T_{(4s)1^{--}}(2766)$, and $T_{(4s)1^{--}}(2984)$ [7]. The fall-apart partial decay widths are presented in Table II.

The low-lying 1^{--} state $T_{(4s)1^{--}}(2455)$ has a fall-apart decay width of $\Gamma \simeq 9$ MeV, which is nearly saturated by the $\phi f_0(1370)$ channel.

The second 1^{--} state $T_{(4s)1^{--}}(2567)$ has a fall-apart decay width of $\Gamma \simeq 26$ MeV. This state has large decay rates into the $\eta\phi$, $\eta'\phi$, $\phi f_0(1370)$, $\phi f_1(1420)$, and $\phi f_2'(1525)$ channels. The partial width ratios between these channels are predicted to be

$$\Gamma[\eta\phi]:\Gamma[\eta'\phi]:\Gamma[\phi f_0(1370)]:\Gamma[\phi f_1(1420)]:\Gamma[\phi f_2'(1525)] \simeq 1.0:1.5:1.0:4.3:2.8. \quad (11)$$

The third 1^{--} state $T_{(4s)1^{--}}(2627)$ has a relatively narrow fall-apart width of $\Gamma \simeq 8$ MeV. The partial widths for the main fall-apart channels, $\eta' h_1(1415)$, $\phi f_1(1420)$, and $\phi f_2'(1525)$, are estimated in the range of ~ 1 – 3 MeV.

The fourth 1^{--} state $T_{(4s)1^{--}}(2766)$ has a fall-apart width of $\Gamma \simeq 30$ MeV, which is mainly contributed by the $\phi f_0(1370)$ channel with a branching fraction $\sim 91\%$.

The high-lying 1^{--} state $T_{(4s)1^{--}}(2984)$ has a broad fall-apart decay width of $\Gamma \simeq 72$ MeV, which is mainly contributed by the $\eta^{(\prime)} h_1(1415)$, $\phi f_1(1420)$, and $\phi f_2'(1525)$ channels. The partial width ratios between these channels are predicted to be

$$\Gamma[\eta h_1(1415)]:\Gamma[\eta' h_1(1415)]:\Gamma[\phi f_1(1420)]:\Gamma[\phi f_2'(1525)] \simeq 1.0:2.0:1.4:3.6. \quad (12)$$

To search for the $1P$ -wave $T_{ss\bar{s}\bar{s}}$ states with $J^{PC} = 1^{--}$, the $\phi f_0(1370)$, $\phi f_1(1420)$, and $\phi f_2'(1525)$ channels are suggested to be observed in future experiments.

Some evidence of the $1P$ -wave $T_{ss\bar{s}\bar{s}}$ states with 1^{--} may have been observed in experiments. For example, in the $\phi f_0(980)$ invariant mass spectrum from the BABAR [54], Belle [55,56], and BESIII [57] observations, there exists a vague peak structure around 2.4 GeV (denoted by $X(2400)$). This structure may be contributed by the low-lying 1^{--} state $T_{(4s)1^{--}}(2455)$. The $X(2400)$ is also explained as the $J^{PC} = 1^{--} T_{ss\bar{s}\bar{s}}$ state based on the analysis of QCD sum rules [19,20]. To confirm the nature of $X(2400)$, some other channels, such as $\phi f_0(1370)$ and $\phi f_2'(1525)$, are worth observing in future experiments.

Finally, it should be mentioned that the well-known vector resonance $\phi(2170)$ listed in RPP [35] is explained as a $1^{--} T_{ss\bar{s}\bar{s}}$ state based on the mass analysis of QCD sum rules [13–17,19,20] and the flux-tube model [24]. However, according to our quark model predictions, we find that the measured mass of $\phi(2170)$ is about 270 MeV smaller than the lowest $J^{PC} = 1^{--}$ state $T_{(4s)1^{--}}(2455)$. The $\phi(2170)$ may favor other assignments, such as a conventional $s\bar{s}$ state [58–60], a hidden-strange baryonium or hexaquark state [61–66], a dynamically generated state [67–72], or a strangeonium hybrid state [73], as suggested in the literature.

3. P -wave states with other quantum numbers

There is no experimental evidence for the other $1P$ -wave $T_{ss\bar{s}\bar{s}}$ states with quantum numbers 1^{++} , 2^{++} , 0^{--} , 2^{--} , and 3^{--} . The fall-apart decay properties have been evaluated and are presented in Table II.

These states may have large potentials to be observed in their main fall-apart decay channels. For example, the

$T_{(4s)0^-}$ (2507) state with $J^{PC} = 0^{--}$ and the $T_{(4s)1^-}$ (2632) state with $J^{PC} = 1^{-+}$ predicted in the quark model may be likely to be established in the $\eta^{(\prime)}\phi$ and $\phi h_1(1415)$ channels, respectively, while the two 2^{--} states, $T_{(4s)2^-}$ (2657) and $T_{(4s)2^-}$ (2907), and the 3^{--} state, $T_{(4s)3^-}$ (2719), may be first observed in the $\phi f'_2(1525)$ channel.

IV. SUMMARY

Based on the mass spectra of the $1S$ -, $1P$ -, and $2S$ -wave fully-strange tetraquark states obtained in the previous work of our group [7], we continue to study their fall-apart decays within a quark-exchange model. Most of the fully-strange tetraquark states have a relatively narrow fall-apart decay width of $\mathcal{O}(10)$ MeV. Our theoretical predictions indicate that some evidence of fully-strange tetraquark states may have been observed in the previous experiments.

Some key points are emphasized as follows:

- (i) The axial-vector state $X(2300)$ newly observed at BESIII favors the low-lying $1S$ -wave $T_{ss\bar{s}\bar{s}}$ state with $J^{PC} = 1^{+-}$, $T_{(4s)1^{+-}}$ (2323).
- (ii) The $X(2500)$ observed in the $\phi\phi$ invariant mass spectrum of the $J/\psi \rightarrow \gamma\phi\phi$ process at BESIII may be a candidate of the low-lying $1P$ -wave $T_{ss\bar{s}\bar{s}}$ state with $J^{PC} = 0^{-+}$, $T_{(4s)0^{-+}}$ (2481).
- (iii) Evidence of the $1S$ -wave $T_{ss\bar{s}\bar{s}}$ states with $J^{PC} = 0^{++}$ may have been observed in the $\phi\phi$ invariant mass spectrum around 2.2 GeV and 2.4 GeV.
- (iv) Evidence of a $1P$ -wave $T_{ss\bar{s}\bar{s}}$ state with $J^{PC} = 1^{--}$ may have been observed in the $\phi f_0(980)$ invariant mass spectrum around 2.4 GeV.
- (v) The $1S$ state $T_{(4s)2^{++}}$ (2378) is prohibited from decaying into $\phi\phi$ due to an exact cancellation mechanism. The $\phi(2170)$, $X(2370)$, $f_2(2300)$, and $f_2(2340)$ resonances listed in RPP cannot be explained with $T_{ss\bar{s}\bar{s}}$ states.

We show that more S -wave $T_{ss\bar{s}\bar{s}}$ states can be searched for in the $\phi\phi$ and $\phi\phi(1680)$ channels. It is also strongly recommended that more $1P$ -wave states can be investigated in the $\eta^{(\prime)}\phi$, $\eta^{(\prime)}h_1(1415)$ and $\phi f'_2(1525)$ channels in future experiments at BESPCII/BESIII and Belle-II. It is noteworthy that some of these states with strong S -wave couplings to the nearby open threshold may serve as a short-distance component in the formation of the physical states of which the long-distance components are hadronic molecules. In the future studies, a combined analysis would allow us to have a better understanding of the underlying dynamics.

Finally, to examine the validity of treating the initial tetraquark states as pure $ss\bar{s}\bar{s}$ configurations, we further investigate the possible flavor-configuration mixing between the $ss\bar{s}\bar{s}$ and $s\bar{s}n\bar{n}$ configurations in the Appendix. As a whole, the mixing between the $ss\bar{s}\bar{s}$ and $s\bar{s}n\bar{n}$ configurations is not significant. Therefore, within the accuracy considered

in the present work, it is reasonable to approximately treat the initial tetraquark states as pure $ss\bar{s}\bar{s}$ configurations.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: FLAVOR-CONFIGURATION MIXING IN THE $1S$ -WAVE TETRAQUARK STATES

In the present work, the initial tetraquark states are treated as pure $ss\bar{s}\bar{s}$ configurations. If the $ss\bar{s}\bar{s}$ configurations significantly mix with $s\bar{s}n\bar{n}$ ($n = u, d$), the mass spectrum predicted in our previous work [7] may be affected, and hence, the strong-decay calculations in the present work as a continuation may also be affected. Therefore, we further examine the possible $ss\bar{s}\bar{s}$ - $s\bar{s}n\bar{n}$ mixing in the $1S$ -wave tetraquark states.

First, we should mention that in the $n\bar{n}$ and $s\bar{s}$ spectrum, only the $IJ^P = 00^-$ mesons η and η' have obvious $n\bar{n}$ - $s\bar{s}$ mixing, which is induced by the $U(1)$ anomaly. The mixing angle has been well determined in experiments. In the states with other J^P numbers, such as the ω and ϕ mesons and their excitations, the mixing is negligibly small. Since the η contains a significant $s\bar{s}$ component through this mixing, some $ss\bar{s}\bar{s}$ states can decay into final states containing η mesons, such as $\eta\eta$, $\eta\eta'$, $\eta\phi$, and so on. This is the main reason why we consider the $n\bar{n}$ - $s\bar{s}$ mixing for the η and η' mesons.

We also evaluate the $ss\bar{s}\bar{s}$ - $s\bar{s}n\bar{n}$ mixing in the $1S$ -wave tetraquark states. To study such an effect, one requires a mechanism for evaluating the off-diagonal mixing matrix elements in theory. In Ref. [74], the $n\bar{n} \leftrightarrow s\bar{s}$ mixing is considered a consequence of the K -meson exchange potential, with which the η - η' mixing angle can be reasonably explained. Thus, we evaluate the $ss\bar{s}\bar{s}$ - $s\bar{s}n\bar{n}$ mixing for the tetraquark states by using the K -exchange potential.

For example, we consider the $IJ^{PC} = 01^{+-}$ configurations, which are also related to the candidate $X(2300)$ recently observed at BESIII. For the $s\bar{s}n\bar{n}$ system, there are four configurations, while for the $ss\bar{s}\bar{s}$ system, there is only one configuration. They are given in Table III.

We adopt the same nonrelativistic quark potential model as that used in our previous work [7]. In this model, the Hamiltonian is given by

$$H = \left(\sum_{i=1}^4 m_i + T_i \right) - T_G + \sum_{i<j} V_{ij}(r_{ij}), \quad (\text{A1})$$

TABLE III. Configurations of the $1S$ -wave $J^{PC} = 1^{+-}$ states for the $ss\bar{s}\bar{s}$ system and the $I = 0$ $sn\bar{s}\bar{n}$ system.

Configuration	Wave function	
$ ss\bar{s}\bar{s}\rangle_1$	$ 6\bar{6}\rangle^f \chi_{11}^{11}$	$ \bar{3}3\rangle^c$
$ sn\bar{s}\bar{n}\rangle_1$	$ \bar{3}3\rangle^f \chi_{11}^{11}$	$ 6\bar{6}\rangle^c$
$ sn\bar{s}\bar{n}\rangle_2$	$ 6\bar{6}\rangle^f \chi_{11}^{11}$	$ \bar{3}3\rangle^c$
$ sn\bar{s}\bar{n}\rangle_3$	$\frac{1}{\sqrt{2}}(3\bar{6}\rangle^f \chi_{11}^{10} - 63\rangle^f \chi_{11}^{01})$	$ 6\bar{6}\rangle^c$
$ sn\bar{s}\bar{n}\rangle_4$	$\frac{1}{\sqrt{2}}(3\bar{6}\rangle^f \chi_{11}^{01} - 63\rangle^f \chi_{11}^{10})$	$ \bar{3}3\rangle^c$

where m_i and T_i stand for the constituent quark mass and kinetic energy of the i th quark, respectively, and T_G stands for the center-of-mass kinetic energy of the tetraquark system. The effective potential $V_{ij}(r_{ij})$ is written as

$$V_{ij}(r_{ij}) = V_{ij}^{\text{conf}}(r_{ij}) + V_{ij}^{\text{sd}}(r_{ij}), \quad (\text{A2})$$

where the confinement potential adopts the standard form of the Cornell potential,

$$V_{ij}^{\text{conf}}(r_{ij}) = -\frac{3}{16}(\lambda_i \cdot \lambda_j) \left(b_{ij} r_{ij} - \frac{4}{3} \frac{\alpha_{ij}}{r_{ij}} + C_0 \right), \quad (\text{A3})$$

and the spin-dependent potential is given by

$$V_{ij}^{\text{sd}}(r_{ij}) = V_{ij}^{SS} + V_{ij}^T + V_{ij}^{LS}. \quad (\text{A4})$$

The explicit forms of these interactions can be found in Ref. [7].

Using the model parameters (see Table IV) determined by fitting the well-established meson spectrum (see Table V), we first calculate the masses and wave functions for the $s\bar{s}n\bar{n}$ and $ss\bar{s}\bar{s}$ configurations given in Table III within the nonrelativistic quark model.

Then, by using the obtained wave functions and masses for the $ss\bar{s}\bar{s}$ and $s\bar{s}n\bar{n}$ configurations with $J^{PC} = 1^{+-}$, we further investigate the flavor-configuration mixing induced by the K -exchange interaction. The form of the K -exchange potential and the relevant parameters are taken from Ref. [75]. The K -exchange potential is given by

$$V_{ij}^K(r_{ij}) = \frac{g_K^2}{4\pi} \frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2} \frac{m_K^3}{3m_i m_j} \times \left[Y(m_K r_{ij}) - \frac{\Lambda_K^3}{m_K^3} Y(\Lambda_K r_{ij}) \right] (\mathbf{S}_i \cdot \mathbf{S}_j) I_K, \quad (\text{A5})$$

where

$$Y(x) = \frac{e^{-x}}{x}, \quad I_K = \sum_{a=4}^7 \lambda_i^a \lambda_j^a. \quad (\text{A6})$$

TABLE IV. The fitted parameters of the light mesons.

Channel	m_q (GeV)	b (GeV ²)	C_0 (GeV)	σ (GeV)	α_s
$n\bar{n}$	0.3500	0.1350	-0.5245	0.6911	0.8183
$n\bar{s}(s\bar{n})$		0.1350	-0.4997	0.6390	0.8602
$s\bar{s}$	0.6000	0.1350	-0.5190	0.6000	0.7700

TABLE V. The calculated masses and experimental values of the light mesons used in the parameter fitting. The masses are in units of GeV.

Quark content	State	J^{PC}	Ours	Expt.
$n\bar{n}$	π	0^{-+}	0.135	0.135
	ρ	1^{--}	0.763	0.775
	$\pi(1300)$	0^{-+}	1.276	1.300
	$\rho(1450)$	1^{--}	1.514	1.450
$n\bar{s}$	K	0^{-}	0.498	0.498
	K^*	1^{-}	0.892	0.892
	$K(2S)$	0^{-}	1.472	...
	$K^*(2S)$	1^{-}	1.621	...
$s\bar{s}$	$1S$	0^{-+}	0.797	...
	ϕ	1^{--}	1.017	1.020
	$\eta_s(2S)$	0^{-+}	1.619	...
	$\phi(1680)$	1^{--}	1.699	1.672

The coupling constant, cutoff parameter, and the K -meson mass are also taken from Ref. [75].

With the following basis:

$$(|s\bar{s}\bar{s}\rangle_1, |s\bar{s}\bar{n}\rangle_2, |s\bar{s}\bar{n}\rangle_3, |s\bar{s}\bar{n}\rangle_4, |ss\bar{s}\bar{s}\rangle_1), \quad (\text{A7})$$

the total mixing Hamiltonian is given by

$$H_{\text{mix}}^{(1^{+-})} = \begin{pmatrix} 1947 & -30 & -112 & 87 & 0 \\ -30 & 2032 & 80 & -16 & -68.6 \\ -112 & 80 & 2029 & 115 & 0 \\ 87 & -16 & 115 & 2010 & 0 \\ 0 & -68.6 & 0 & 0 & 2323 \end{pmatrix} \text{ MeV}. \quad (\text{A8})$$

After diagonalizing the above mixing Hamiltonian, the obtained eigenvalues are

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix} = \begin{pmatrix} 1778.71 \\ 1971.50 \\ 2083.62 \\ 2167.20 \\ 2339.96 \end{pmatrix} \text{ MeV}. \quad (\text{A9})$$

The corresponding eigenstates are expanded as

$$\begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \\ \Psi_5 \end{pmatrix} = \begin{pmatrix} 0.621 & -0.143 & 0.564 & -0.524 & -0.018 \\ -0.492 & -0.807 & 0.279 & -0.057 & -0.158 \\ 0.526 & -0.340 & 0.049 & 0.772 & -0.097 \\ 0.306 & -0.394 & -0.771 & -0.355 & -0.174 \\ 0.039 & -0.239 & -0.078 & -0.005 & 0.967 \end{pmatrix} \begin{pmatrix} |sn\bar{s}\bar{n}\rangle_1 \\ |sn\bar{s}\bar{n}\rangle_2 \\ |sn\bar{s}\bar{n}\rangle_3 \\ |sn\bar{s}\bar{n}\rangle_4 \\ |ss\bar{s}\bar{s}\rangle_1 \end{pmatrix}. \quad (\text{A10})$$

It can be seen that Ψ_5 is governed by the $|ss\bar{s}\bar{s}\rangle_1$ configuration, whose component is

$$|0.967|^2 \simeq 93.52\%. \quad (\text{A11})$$

This demonstrates that in the states with $J^{PC} = 1^{+-}$ the $ss\bar{s}\bar{s}$ configuration has less mixing with the $s\bar{s}n\bar{n}$ configurations.

Similarly, we have examined the flavor-configuration mixing in the $J^{PC} = 0^{++}$ and 2^{++} states and obtained the

same conclusion. As a whole, the mixing between $ss\bar{s}\bar{s}$ and $s\bar{s}n\bar{n}$ configurations is not significant. In fact, it is easy to understand the tiny mixing. Due to the heavy s quark, the mass difference between $ss\bar{s}\bar{s}$ and $s\bar{s}n\bar{n}$ is a rather large value, $\mathcal{O}(300)$ MeV, while the mixing matrix elements are relatively small. These two points suppress the $ss\bar{s}\bar{s} - s\bar{s}n\bar{n}$ mixing.

Therefore, within the accuracy considered in the present work, it is reasonable to approximately treat the initial tetraquark states as pure $ss\bar{s}\bar{s}$ configurations.

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