

Irreducible Bhabha background in the detection of muonium-antimuonium conversion

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Experiments such as MACS and the proposed MACE study muonium-antimuonium conversion by the energies of the final state $e^\pm$. The e^+ and e^- from an antimuonium decay tend to be nonrelativistic and relativistic, respectively, and vice versa for muonium. However, these $e^\pm$ can exchange their energies by hard Bhabha scattering, causing muonium to fake an antimuonium decay signal. We compute the rate for this background and find that, while negligible for MACE, it will become larger than the signal for conversion probabilities less than 10^{-18} .

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I. INTRODUCTION

The conversion of muonium (a bound state of a μ^+ and an e^-) to antimuonium (μ^-e^+) [1] is a clean probe of new physics, being a purely electromagnetic bound state free from hadronic uncertainties. This idea has long been pursued both theoretically [1–12] and experimentally [13–18]. The most recent search for such conversion, the MACS experiment [18] at PSI in 1999, observed no events in $\sim 10^{11}$ muonium decays. There is also a proposed experiment MACE [19] that expects to produce $\sim 10^{14}$ muonia.

Experiments such as MACS and MACE attempt to distinguish the decays of muonium ($\mathcal{M} = \mu^+e^-$) and antimuonium ($\overline{\mathcal{M}} = \mu^-e^+$) by the kinematics of the final-state leptons. A typical $\mathcal{M}$ decay produces a fast positron e_f^+ (from $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$ with $E_{e^+} \gg m_e$), a slow electron e_s^- (the bound electron, with kinetic energy $\sim m_e\alpha^2 \ll m_e$, with $\alpha = e^2/4\pi$), and two neutrinos. We denote this final state by $f \equiv e_f^+e_s^-\bar{\nu}_\mu\nu_e$. A typical $\overline{\mathcal{M}}$ decay yields instead a slow positron e_s^+ , a fast electron e_f^- , and two neutrinos: $\bar{f} \equiv e_s^+e_f^-\nu_\mu\bar{\nu}_e$. An irreducible background

arises when the e_f^+ and e_s^- from an $\mathcal{M}$ decay undergo hard Bhabha scattering so that e_f^+ and e_s^- become e_s^+ and e_f^- , mimicking $\bar{f}$. We denote such final state by $\tilde{f} \equiv e_s^+e_f^-\bar{\nu}_\mu\nu_e$. Note that $\tilde{f}$ is by definition indistinguishable from $\bar{f}$, provided that the experiment neither distinguishes between $\bar{\nu}_\mu\nu_e$ and $\nu_\mu\bar{\nu}_e$ nor measures the helicity of e_f^- (which is left-handed in $\bar{f}$). Assuming this is the case, as in MACS and MACE, the process $\mathcal{M} \rightarrow \tilde{f}$ constitutes an *irreducible* background to the signal $\mathcal{M} \rightarrow \overline{\mathcal{M}} \rightarrow \bar{f}$.

In this work, we compute, for the first time, the rate for $\mathcal{M} \rightarrow \tilde{f}$. We find the relation between the number of muonia and the lowest $\mathcal{M} - \overline{\mathcal{M}}$ conversion probability that can be probed. Our results show that, while negligible at the proposed MACE experiment, this Bhabha background will impact the reach of future experiments. In particular, for $\mathcal{M} - \overline{\mathcal{M}}$ conversion probabilities smaller than 10^{-18} , the Bhabha background is larger than the signal, and hence can no longer be neglected.

Bhabha scattering between e_f^+ and e_s^- was considered by Feinberg and Weinberg [3], where it was required that a large energy of more than 10 MeV be transferred from the e_f^+ to e_s^- . However, they did not impose the final e^+ energy to be at the atomic energy scale, $\sim m_e\alpha^2$. Thus, their background does not mimic $\bar{f}$ and is reducible.

The Bhabha background considered in this work should not be confused with the *accidental* Bhabha background discussed in [18,19], where the e_f^+ from an $\mathcal{M}$ decay scatters with an e^- in the detector (i.e., not the e_s^- from the $\mathcal{M}$ decay) and produces a fast e^- that may be mistaken as coming from an $\overline{\mathcal{M}}$ decay. Such accidental Bhabha background is reducible by experimental design. For example,

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MACE plans to use a pulsed muon beam with late-time windows, tight time-of-flight and energy selection for e_s^+ and transverse-momentum requirements on e_f^- to reduce this accidental background. The dominant background at MACE is expected to be “internal conversion” [19], where a μ^+ in the beam decays as $\mu^+ \rightarrow e_f^+ e_s^- e^+ \bar{\nu}_\mu \nu_e$ with the extra e^+ going undetected. We assume this background will be reduced in future experiments, for example, by detecting the extra e^+ .

II. DECAY RATES OF MUONIUM AND ANTIMUONIUM

The typical kinematics of $\overline{\mathcal{M}}$ decay, which defines the signal region of phase space, f , is shown in Fig. 1. Here, the e_s^+ coming from the bound state has kinetic energy $\sim m_e \alpha^2$ while the e_f^- coming from the μ^- is relativistic with kinetic energy $\sim m_\mu$. To be concrete, we consider the case of pseudoscalar muonium. To calculate the $e\mathcal{M}\mu$ vertex (the gray blob in Fig. 1), we follow the treatment of bound states in the Appendix of [12], where, instead of $\mathcal{O}^\mu = \bar{\mu}\gamma^\mu P_L e$, we use $\mathcal{O} = \bar{\mu}\gamma_5 e$ to create/annihilate (anti-)muonium. Working in the muonium rest frame [$P = (m_\mu, \mathbf{0})$ in Fig. 1], and keeping only the leading term in m_e/m_μ and/or α , we find that the spin-summed squared amplitude is

$$\langle |\mathcal{A}(\overline{\mathcal{M}} \rightarrow \bar{f})|^2 \rangle = 2^{13} \pi m_e G_F^2 a_0^3 \frac{(p_1 \cdot p_f)(p_2 \cdot P)}{(1 + a_0^2 \mathbf{p}_s^2)^4}, \quad (1)$$

where $a_0 \equiv 1/m_e \alpha$ is the Bohr radius of muonium, hence $a_0 |\mathbf{p}_s| \sim 1$. The m_e and a_0 dependence in the prefactor can be understood as follows: the spinor for the e_s^+ contributes a factor of $\sqrt{m_e}$, while the momentum space wave function of muonium contributes $a_0^{3/2}$ to the amplitude, leading to the $m_e a_0^3$ dependence in the squared amplitude.

Of interest in this work is the case when $\mathcal{M}$ decays before conversion and undergoes hard scattering that

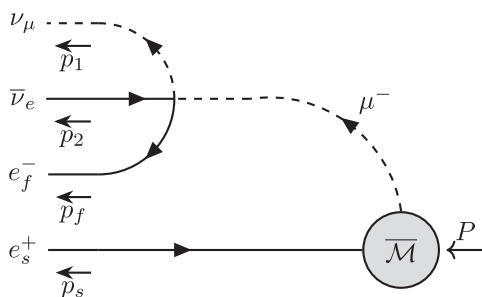


FIG. 1. Diagram for the antimuonium ($\overline{\mathcal{M}}$) decay part of the signal, not including the conversion ($\mathcal{M} \rightarrow \overline{\mathcal{M}}$) part. Dashed lines carry muon number, while solid lines electron number. The momenta carried by the outgoing fermion lines are labeled next to the line, and P is the four-momentum of the $\overline{\mathcal{M}}$. Crucially, e_{f}^- and e_{s}^+ are relativistic and nonrelativistic, respectively.

causes the $e^\pm$ to have the “wrong” kinematics, that is, $\mathcal{M} \rightarrow \tilde{f}$. Figure 2 shows three different 1-loop QED diagrams that contribute to this process. Figures 2(a) and 2(b) depict the t - and s -channel Bhabha diagrams, respectively. The third “non-Bhabha” diagram, shown in Fig. 2(c), is much smaller than the Bhabha diagrams, as we will explain below. Then, ignoring the non-Bhabha diagram, we find that the spin-summed squared amplitude for the irreducible Bhabha background is given by

$$\langle |\mathcal{A}(\mathcal{M} \rightarrow \tilde{f})|^2 \rangle = 512\pi G_{\text{F}}^2 \alpha^3 \frac{(p_1 \cdot p_{\text{f}})(p_2 \cdot P)}{E_{\text{f}}^2 + a_0^2(\mathbf{p}_{\text{f}} \cdot \mathbf{p}_{\text{s}})^2}, \quad (2)$$

where $E_f \gg m_e$ is the energy of the e_f^- .

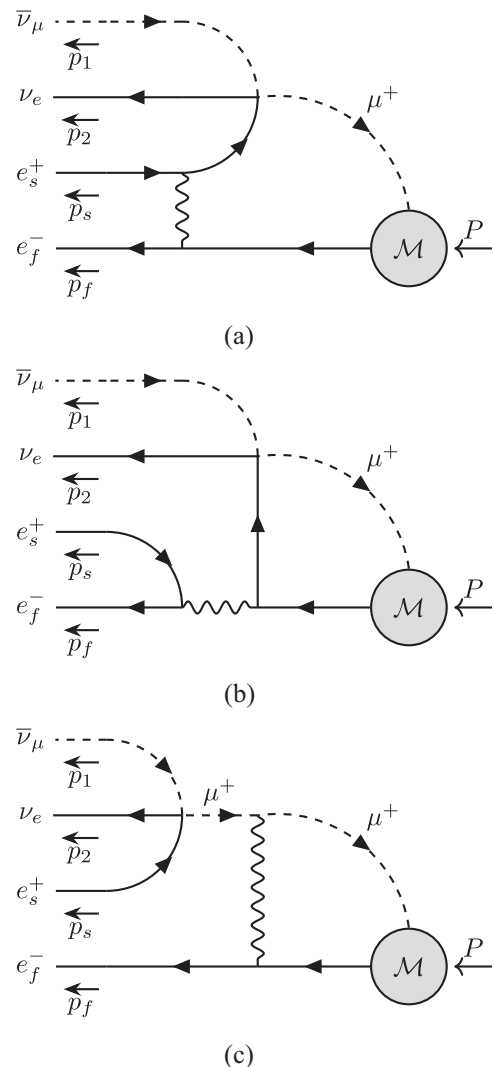


FIG. 2. Diagrams for the hard photon exchange background. Note that the final state e_s^+ and e_f^- have the same four-momenta p_s and p_f , respectively, as those in the signal (Fig. 1), and hence this final state is kinematically indistinguishable from that of the signal. (a) t -channel Bhabha diagram. (b) s -channel Bhabha diagram. (c) non-Bhabha diagram.

The amplitude for the non-Bhabha process is suppressed by a factor of $\sim m_e \alpha / m_\mu$ with respect to the t - and s -channel Bhabha amplitudes. To see why this is the case, note that in both the t - and s -channel Bhabha diagrams, there is a positron propagator inside the loop which goes as the inverse loop momentum. However, because the bound-state wave function cuts the integral off at loop momenta $\sim 1/a_0 = m_e \alpha$, the positron propagator contributes a factor of $\sim 1/m_e \alpha$ to the amplitude. In the non-Bhabha diagram, on the other hand, there is a muon propagator outside of the loop instead, which contributes a factor of $\sim 1/m_\mu$ to the amplitude. Hence, the non-Bhabha amplitude is smaller than the t - and s -channel Bhabha by $\sim m_e \alpha / m_\mu$.

Using (1) and (2), the differential decay rates for the signal and background are

$$\frac{d^3\Gamma(\overline{\mathcal{M}} \rightarrow \bar{f})}{dx d\mathcal{E}_f d\mathcal{E}_s} = \frac{m_\mu^5 G_F^2}{4\pi^4} \mathcal{E}_f^2 \left(1 - \frac{2}{3} \mathcal{E}_f\right) \frac{\sqrt{\mathcal{E}_s}}{(1 + \mathcal{E}_s)^4} \quad (3)$$

and

$$\frac{d^3\Gamma(\mathcal{M} \rightarrow \tilde{f})}{dx d\mathcal{E}_f d\mathcal{E}_s} = \frac{m_\mu^5 G_F^2 \alpha^6}{16\pi^4} \left(\frac{m_e}{m_\mu}\right)^2 \left(1 - \frac{2}{3} \mathcal{E}_f\right) \frac{\sqrt{\mathcal{E}_s}}{1 + \mathcal{E}_s x^2}, \quad (4)$$

respectively, where $\mathcal{E}_f \equiv E_f/(m_\mu/2)$, $\mathcal{E}_s \equiv E_s/(m_e \alpha^2/2)$, $E_s \sim m_e \alpha^2$ is the kinetic energy of the e_s^+ , and $x \equiv \cos \theta_{fs}$ with θ_{fs} being the angle between $\mathbf{p}_f$ and $\mathbf{p}_s$. As a sanity check, the total decay rate $\Gamma(\overline{\mathcal{M}} \rightarrow \bar{f})$ is given by

$$\begin{aligned} \Gamma(\overline{\mathcal{M}} \rightarrow \bar{f}) &= \int_{-1}^1 dx \int_0^\infty d\mathcal{E}_s \int_0^1 d\mathcal{E}_f \frac{d^3\Gamma(\overline{\mathcal{M}} \rightarrow \bar{f})}{dx d\mathcal{E}_f d\mathcal{E}_s} \\ &= \frac{G_F^2 m_\mu^5}{192\pi^3} \equiv \Gamma_0, \end{aligned} \quad (5)$$

which is the total decay rate of the muon, as expected. We have extended the upper bound of the $\mathcal{E}_s$ integral to infinity because the integrand dies off quickly for $\mathcal{E}_s \gtrsim 1$. Similarly, the lower bound on the $\mathcal{E}_f$ integral has been extended from $2m_e/m_\mu$ to 0. These extensions are consistent with our approximation of working at leading order in m_e/m_μ .

III. RESULTS

Using (3) and (4), the differential branching fractions BR_{sig} and BR_{bkg} for the signal ($\mathcal{M} \rightarrow \overline{\mathcal{M}} \rightarrow \bar{f}$) and irreducible Bhabha background ($\mathcal{M} \rightarrow \tilde{f}$), respectively, are given by

$$\frac{d^3\text{BR}_{\text{sig}}}{dx d\mathcal{E}_f d\mathcal{E}_s} \equiv P_C \frac{d^3\Gamma(\overline{\mathcal{M}} \rightarrow \bar{f})}{\Gamma_0 dx d\mathcal{E}_f d\mathcal{E}_s} \quad (6)$$

and

$$\begin{aligned} \frac{d^3\text{BR}_{\text{bkg}}}{dx d\mathcal{E}_f d\mathcal{E}_s} &\equiv (1 - P_C) \frac{d^3\Gamma(\mathcal{M} \rightarrow \tilde{f})}{\Gamma_0 dx d\mathcal{E}_f d\mathcal{E}_s} \\ &\simeq \frac{d^3\Gamma(\mathcal{M} \rightarrow \tilde{f})}{\Gamma_0 dx d\mathcal{E}_f d\mathcal{E}_s}, \end{aligned} \quad (7)$$

where $P_C \equiv P(\mathcal{M} \rightarrow \overline{\mathcal{M}}) \ll 1$ is the $\mathcal{M} - \overline{\mathcal{M}}$ conversion probability.

Experimentally, to ensure the e_s^+ and e_f^- are sufficiently slow and fast, respectively, cuts are imposed on $\mathcal{E}_s$ and $\mathcal{E}_f$. Specifically, we require $0 \leq \mathcal{E}_s \leq \mathcal{E}_s^{\text{max}}$ and $\mathcal{E}_f^{\text{min}} \leq \mathcal{E}_f \leq 1$. Integrating (6) and (7) over $\mathcal{E}_s$ and $\mathcal{E}_f$ in these ranges and $x \in [-1, 1]$, we find the branching fractions are

$$\text{BR}_{\text{sig}} = \frac{2}{\pi} P_C \bar{\mathcal{F}}(\mathcal{E}_f^{\text{min}}) \bar{\mathcal{S}}(\mathcal{E}_s^{\text{max}}) \quad (8)$$

and

$$\text{BR}_{\text{bkg}} = \frac{8\alpha^6}{\pi} \left(\frac{m_e}{m_\mu}\right)^2 \tilde{\mathcal{F}}(\mathcal{E}_f^{\text{min}}) \tilde{\mathcal{S}}(\mathcal{E}_s^{\text{max}}), \quad (9)$$

where

$$\begin{aligned} \bar{\mathcal{F}}(z) &\equiv (1 - z)(1 + z + z^2 - z^3), \\ \bar{\mathcal{S}}(z) &\equiv \arctan \sqrt{z} - \frac{\sqrt{z}(1 + \frac{1}{3}z)(1 - 3z)}{(1 + z)^3}, \end{aligned} \quad (10)$$

and

$$\begin{aligned} \tilde{\mathcal{F}}(z) &\equiv (1 - z)(2 - z), \\ \tilde{\mathcal{S}}(z) &\equiv (1 + z) \arctan \sqrt{z} - \sqrt{z}. \end{aligned} \quad (11)$$

Setting the lower limit of $\mathcal{E}_s$ to 0 as above is a good approximation experimentally (e.g., see [19,20]), and we assume that this will still be the case for future experiments. Similarly, we have integrated over all θ_{fs} (i.e., all x), assuming that it is not measured, as in MACS and MACE. Since the θ_{fs} dependence of the background, as seen in Eq. (4), is very mild, imposing cuts on θ_{fs} will not change our conclusion.

Now, using (8) and (9), the ratio of the signal to the irreducible Bhabha background is given by

$$\begin{aligned} \mathcal{R} &\equiv \frac{\text{BR}_{\text{sig}}}{\text{BR}_{\text{bkg}}} = \frac{P_C}{4\alpha^6} \left(\frac{m_\mu}{m_e}\right)^2 \frac{\bar{\mathcal{F}}(\mathcal{E}_f^{\text{min}}) \bar{\mathcal{S}}(\mathcal{E}_s^{\text{max}})}{\tilde{\mathcal{F}}(\mathcal{E}_f^{\text{min}}) \tilde{\mathcal{S}}(\mathcal{E}_s^{\text{max}})} \\ &= \frac{P_C}{1.4 \times 10^{-17}} \frac{\bar{\mathcal{F}}(\mathcal{E}_f^{\text{min}}) \bar{\mathcal{S}}(\mathcal{E}_s^{\text{max}})}{\tilde{\mathcal{F}}(\mathcal{E}_f^{\text{min}}) \tilde{\mathcal{S}}(\mathcal{E}_s^{\text{max}})}. \end{aligned} \quad (12)$$

Because $\bar{\mathcal{F}} \bar{\mathcal{S}} / \tilde{\mathcal{F}} \tilde{\mathcal{S}}$ has a maximum value of 16, $\mathcal{R} \leq (1.1 \times 10^{18}) P_C$. Thus, for conversion probabilities smaller than 0.88×10^{-18} the branching fraction for the Bhabha background becomes greater than that of the signal, regardless of the cuts $\mathcal{E}_f^{\text{min}}$ and $\mathcal{E}_s^{\text{max}}$.

Armed with an expression for the branching fraction of the irreducible Bhabha background, we now investigate

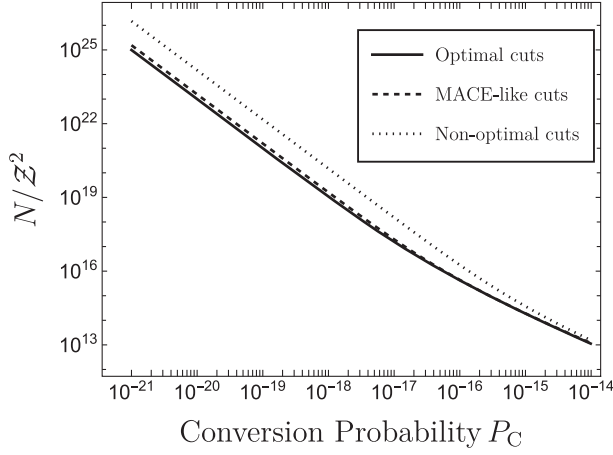


FIG. 3. N/Z^2 as a function of P_C for different scenarios. The solid line uses the optimal cuts, as shown in Fig. 4, i.e., $N = N_{\min}$. The dashed line uses the fixed cuts similar to those proposed by MACE [19] ($\mathcal{E}_f^{\min} = 0.4$ and $\mathcal{E}_s^{\max} = 1.5$). The dotted line uses poorly chosen fixed cuts $\mathcal{E}_f^{\min} = 0.1$ and $\mathcal{E}_s^{\max} = 9$ to illustrate that nonoptimal choices do lead to a higher N , but not significantly higher.

how many muonia are needed in an experiment to probe a desired P_C . Let us consider an $\mathcal{M} - \overline{\mathcal{M}}$ conversion experiment with N muonia. The expected number of signal events is $N \text{BR}_{\text{sig}}$, while the expected number of irreducible Bhabha background events is $N \text{BR}_{\text{bkg}}$. We adopt the expected discovery significance $\mathcal{Z}$ given by [21]

$$\mathcal{Z} = \sqrt{2N \text{BR}_{\text{bkg}} [(1 + \mathcal{R}) \ln(1 + \mathcal{R}) - \mathcal{R}]} \quad (13)$$

We can use (13) to find the number of muonia N as a function of $\mathcal{E}_f^{\min}$, $\mathcal{E}_s^{\max}$, and P_C for a given $\mathcal{Z}$. Minimizing N with respect to the cuts yields the minimum number $N_{\min}$ of muonia necessary to be sensitive to a conversion probability of P_C at significance $\mathcal{Z}$. This is plotted as a solid line

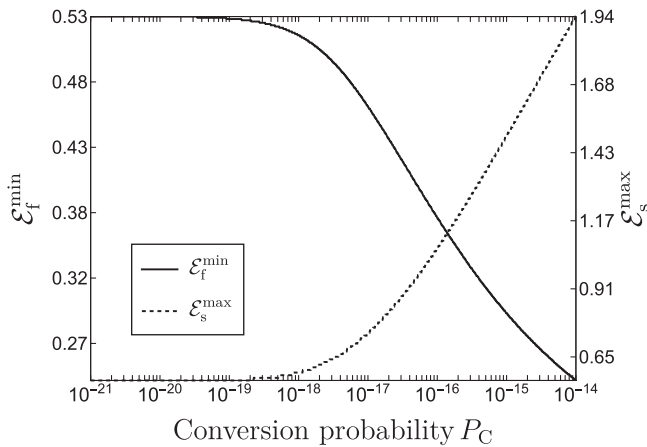


FIG. 4. The optimal values of $\mathcal{E}_f^{\min}$ (solid) and $\mathcal{E}_s^{\max}$ (dashed) that yield $N = N_{\min}$ for a given P_C .

in Fig. 3 which shows N/Z^2 vs P_C . The optimal cuts (as functions of P_C) that yield $N_{\min}$ are plotted in Fig. 4. While these cuts do minimize the number of muonia required, it is worthwhile to note that, for reasonable cuts similar to those used in MACS and the proposed MACE (plotted as a dashed line in Fig. 3), the dependence of N on these cuts is very mild. Even a poor choice of cuts (the dotted line in Fig. 3) does not significantly diminish the sensitivity. The takeaway here is that one cannot use kinematical cuts to reduce the Bhabha background.

IV. IS FURTHER DISCRIMINATION POSSIBLE?

In principle, there are two ways. One way is to detect the flavors of the final-state neutrinos, because $\mathcal{M}$ decays to $\bar{\nu}_\mu \nu_e$ while $\overline{\mathcal{M}}$ decays to $\nu_\mu \bar{\nu}_e$. This would eliminate the Bhabha background but is not realistic in practice. The other possibility is to measure the helicity of the e_f^- . When we sum over the e_f^- helicities in (2), the entire contribution is from a right-handed e_f^- at this order in m_e/m_μ and α . The left-handed e_f^- contribution completely cancels between the t - and s -channel diagrams. Whereas for the signal, the e_f^- , which comes from a weak interaction, is purely left-handed. Then, in principle, an *event-by-event* helicity measurement of the e_f^- could enhance the signal-to-background ratio by a factor of at least $\sim (m_\mu/m_e)^2 \sim 10^4$. However, to the best of our knowledge, such a measurement can only be made using the weak interaction itself, which is at least as difficult as detecting neutrino flavors. On the other hand, *statistical* discrimination of electron helicities by methods such as those in Refs. [22,23] would only offer mild reduction of the Bhabha background like the cut on θ_{fs} . Therefore, increasing the number of muonia seems to be the most effective way to increase the experimental reach.

V. CONCLUSION

We have computed the rate for the irreducible Bhabha background to muonium-antimuonium conversion. While Bhabha scattering can give rise to both reducible and irreducible backgrounds, our results focus on the irreducible part of Bhabha scattering where the final state $e^\pm$ are in the same region of the phase space as those of the signal. We obtained the minimum number of muonia necessary to probe a given P_C , and found this to be only mildly sensitive to the cuts $\mathcal{E}_f^{\min}$ and $\mathcal{E}_s^{\max}$ on the energies of the $e^\pm$. Furthermore, we find that for $P_C \leq 10^{-18}$, the Bhabha background is larger than the signal, irrespective of the cuts $\mathcal{E}_f^{\min}$ and $\mathcal{E}_s^{\max}$, confirming the irreducibility of this background in the absence of neutrino and helicity detection. This also motivates us to explore new detection schemes that do not suffer from this background. For example, perhaps direct detection of the conversion is possible by ionizing $\overline{\mathcal{M}}$ into $\mu^- e^+$ to distinguish from the μ^+ from $\mathcal{M}$ or the μ^+ beam.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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