

Polyakov loops tame phase transitions

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We estimate the impact of Polyakov loop (PL) contributions on electroweak phase transitions (PTs). We show that the PL, which is unavoidable in thermal gauge field theory, tends to tame thermal contributions, thereby softening electroweak PTs and affecting bubble dynamics, nucleation, and the related gravitational-wave spectrum. Including this contribution in perturbative approaches results in a thermal effective potential that disfavors first-order PTs over either second-order PTs or smooth crossovers. This feature is universal for both fermionic and bosonic contributions to the effective potential.

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I. INTRODUCTION

Electroweak phase transitions (PTs) and related dynamics are key drivers for the phenomenology of our early universe. They are particularly relevant when considered in conjunction with the known shortcomings of the Standard Model (SM) of particle physics, in which first-order electroweak PTs are critical for, e.g., electroweak baryogenesis to fulfil Sakharov's criteria [1]. The available tools for analyzing the electroweak thermal evolution often rely on perturbatively improved effective potential calculations [2–14]. Computations using lattice approaches are considered in parallel [15–22], and are also compared to higher-order improved perturbative methods in e.g. [23–26]. By computing the “bounce action” [27–37], statistical ensembles are then constructed to identify the correct vacuum configuration and gauge the transition rate. In the context of beyond-the-SM (BSM) scenarios, such an analysis forms a critical part of baryon-number generation that is additionally informed by bubble dynamics, CP violation, and sphaleron conversion and washout (for reviews see e.g. [38,39]).

Currently available tools [2–14] allow one to study the PT in terms of a classical scalar (Higgs) background field, relying on the effective potential and analysing the respective local and global minima. These approaches, primarily based

on perturbative methods in diagrammatic and functional techniques, do not, in general, capture all relevant effects. A pioneering example of the latter is the so-called Polyakov loop (PL) [40–43] (a Wilson loop in thermal field theory). In Ref. [43], the PL is identified as an order parameter of (de)confinement in QCD, and it is demonstrated that the vanishing trace of the PL implies that the system is in the confined phase; otherwise, it is in the deconfined phase. Loop-improved effective actions for the PL in pure gauge theories have been discussed in [44–46] using constraint potential techniques [47,48]. Thermal Wilson lines, more broadly, characterize how colored excitations propagate in the quark-gluon plasma (see e.g. [49–51]). It has been noted that the thermal partition function for a spin system requires a nonzero imaginary chemical potential that ensures the removal of unphysical degrees of freedom [52]. This imaginary chemical potential (μ_I) is related to the gauge charges and is quantized [52,53]. The nontrivial presence of the PL is mandatory for consistent quantization in thermal gauge field theory and for compatibility with periodic boundary conditions for fields in the thermal partition function [43]. Furthermore, given the results of [54,55], the PL in thermal gauge field theory (in the absence of dynamical matter fields) replicates the behavior of the imaginary chemical potential $\beta\mu_I$ of spin chains, see Refs. [43,52,53,56,57].

In this work, we consider modifications to the perturbative effective potential by including PL contributions and develop a framework for their inclusion in publicly available codes. Considering a toy scenario, we trace the relevance of the PL to phenomenological implications such as the strength of the first-order PT and its interplay with bubble nucleation rates, as well as expected gravitational-wave spectra. This paper is organized as follows: In Sec. II, we discuss how the effective potential is modified by the

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inclusion of the PL. We develop a practical way to include these effects by modifying the well-known characteristic functions of the $T > 0$ domain. Subsequently, in Sec. III, we perform a case study inspired by existing analyses in the literature to highlight the phenomenological relevance of the PL in a motivated toy scenario. Implications for bubble nucleation and the resulting gravitational-wave phenomenology are discussed. We conclude in Sec. IV.

II. POLYAKOV LOOPS AND THEIR IMPRINT ON THE EFFECTIVE POTENTIAL

We start with a brief review of the relevance of PLs [40,41] in thermal field theory and how they lead to corrections to the effective potential used to analyze PT dynamics. At zero temperature, quantum field theory provides a framework for computing transition probabilities. In the early Universe at nonzero temperature, however, the background state is a thermal bath, and the dynamics are described within thermal field theory. The transition to this setting is implemented by Euclideanizing Minkowski time, $t \rightarrow i\beta$, with inverse temperature $\beta = 1/T$ [58–60]. Green's functions reflect this transition through the Kubo-Martin-Schwinger [61,62] relation. In the imaginary time formalism that can be used to tackle practical calculations [58], this imposes (anti)periodicity conditions on the fields of the theory. Put differently, thermal field theory lives on $S^1 \times \mathbb{R}^3$ where the compactification radius is determined by β . This space is topologically nontrivial as $\pi_1(S^1 \times \mathbb{R}^3) = \mathbb{Z}$. Just as for $T = 0$, nontrivial topology can have nontrivial non-perturbative implications. In thermal gauge theory, these are encoded in the holonomy around the Euclidean time circle, the PL matrix, i.e., a thermal Wilson loop

$$\mathbf{L}(\mathbf{x}) = \mathbf{T} \left(\exp \left\{ - \int_0^\beta d\tau A_0(\mathbf{x}, \tau) \right\} \right), \quad (2.1)$$

where $\mathbf{T}$ defines Euclidean time path-ordering and A_0 is the temporal component of the Euclidean gauge field. The PL is the thermal equivalent of a Wilson line that is wrapped around Euclidean time on the circle S^1 . A gauge-invariant diagnostic is the traced (normalized) PL

$$\Omega(\mathbf{x}) = \frac{1}{N} \text{Tr} \mathbf{L}, \quad (2.2)$$

(the trace is evaluated on the gauge indices), which serves as an order parameter for (de)confinement in pure $SU(N)$ gauge theory [43]. Perturbative calculations typically assume a specific gauge choice, but A_0 cannot be removed globally, because it is determined by holonomy.

We consider a scalar (φ) quantum field theory with a background $SU(N)$ gauge theory. Here, we employ the heat-kernel method, defined in Euclidean space, to

compute the one-loop effective potential after integrating out the quantum fluctuations associated with the scalar field.¹The one-loop effective Lagrangian in $(d+1)$ -dimensional Euclidean spacetime² $\mathbb{R}^{d+1}$ is given by [54,55]

$$\mathcal{L}^{1L} = c_s \text{Tr} \int_0^\infty \frac{dt}{t} \int \frac{d^{d+1}p}{(2\pi)^{d+1}} e^{-m^2 t} \langle x | e^{-(D^2+U)t} | p \rangle \langle x | p \rangle, \quad (2.3)$$

where $\langle x | p \rangle = \exp\{ix_\mu p^\mu\}$, $c_s = 1/2(1)$ for real (complex) scalars and -1 for Dirac fermions. $D_\mu = \partial_\mu + A_\mu$ denotes the covariant derivative with A_μ ³ as the Euclidean gauge field in $\mathbb{R}^{d+1}$, and $D^2 = D_\mu D^\mu$. We define $U = \delta^2 V^{\text{tree}} / \delta \varphi^2$, where V^{tree} is the tree-level potential of φ . From this we can obtain thermal field theory on $S^1 \times \mathbb{R}^d$ with bosonic and fermionic fields that satisfy periodic and antiperiodic boundary conditions on S^1 , respectively [54,63,64], leading to

$$\mathcal{L}^{1L} = c_s \frac{1}{\beta} \int_0^\infty \frac{dt}{t} \text{Tr} \sum_{p_0} \int \frac{d^d p}{(2\pi)^d} e^{-m^2 t} e^{-((D_t + p_t)^2 - Q^2 + U)t}, \quad (2.4)$$

where $\langle \mathbf{p}, p^0 | \mathbf{k}, k_0 \rangle = (2\pi)^d \beta \delta(\mathbf{p} - \mathbf{k}) \delta_{p_0, k_0}$, and $p_0 = 2\pi n / \beta$ with $n \in \mathbb{Z}$ is the quantized momentum along S^1 (referred to as the Matsubara modes [58]). We further define $Q^2 = |D_0 + ip_0|^2 = |\partial_0 + A_0 + ip_0|^2$ with A_0 ⁴ as the Euclidean (imaginary) temporal gauge field and $A_0 = \text{const.} \neq 0$ being a consistent choice [43]. With this, the PL for an $SU(N)$ gauge theory reads

$$\mathbf{L} = \exp\{-\beta A_0\} \Rightarrow \Omega = \frac{1}{N} \text{Tr} \exp\{-\beta A_0\}, \quad (2.5)$$

which leads to

$$\begin{aligned} Q^2 &= \left| i \frac{2n\pi}{\beta} + A_0 \right|^2 = \left| i \frac{2n\pi}{\beta} - \frac{\log \mathbf{L}}{\beta} \right|^2 \\ &= \left(\frac{2\pi}{\beta} \right)^2 \left[n + i \frac{A_0 \beta}{2\pi} \right]^2. \end{aligned} \quad (2.6)$$

Now, within this thermal field theory framework with gauge fields in the background, the gauge-invariant and gauge-parameter-independent one-loop effective potential, including the PL contribution, can be computed using Eq. (2.3) as [54,55,65]

¹It is worth mentioning that, as the gauge fields are in the background, there are no gauge quantum fluctuations, and thus they do not contribute to the one-loop effective potential directly.

²We work with an all-negative metric.

³The respective gauge coupling is absorbed in the gauge field in this convention.

⁴By convention, we absorb the coupling constant into the gauge field, $A_0 = g A_{0,j} T^j$, where g is the gauge coupling, T_j are the generators in the fundamental representation, where $j = 1, \dots, (N^2 - 1)$ for an $SU(N)$ gauge theory.

$$\begin{aligned}
-V_{\text{eff}}^{\text{IL},S}(m, \beta, \tilde{n}) &= \frac{c_s}{\beta} \text{Tr} \int_0^\infty \frac{dt}{t} \sum_{p_0} \frac{e^{-m^2 t}}{(4\pi t)^{\frac{d}{2}}} e^{Q^2 t} \\
&= \frac{c_s}{\beta} \sum_n \frac{\mu^{4-d}}{(4\pi)^{d/2}} \Gamma(-d/2) \text{Tr}[m^2 + Q^2]^{d/2} \\
&= \frac{c_s}{\beta} \sum_n \frac{\mu^{4-d}}{(4\pi)^{d/2}} \Gamma(-d/2) [m^2 + \langle Q^2 \rangle]^{d/2} \\
&= \frac{c_s}{\beta} \sum_n \frac{\mu^{4-d}}{(4\pi)^{d/2}} \left[m^2 + \left(\frac{2\pi}{\beta} \right)^2 (n + \tilde{n})^2 \right]^{d/2} \Gamma(-d/2) \\
&= \frac{1}{2} \left[\frac{m^4}{32\pi^2} \left(\log \left(\frac{4\pi\mu^2}{m^2} \right) + \frac{3}{2} - \gamma_E \right) + \frac{m^2}{\pi^2 \beta^2} \sum_{n=1}^\infty \frac{\cos(2\pi n \tilde{n})}{n^2} \mathbf{K}_2(m\beta n) \right], \quad (2.7)
\end{aligned}$$

where $\mathbf{K}_2$ is the modified Bessel function of the second kind, m is the background field-dependent mass of the scalar field, μ is the renormalization scale, and γ_E is the Euler constant. Furthermore,

$$\langle Q^2 \rangle = \left(\frac{2\pi}{\beta} \right)^2 \left\langle \left[n + i \frac{A_0 \beta}{2\pi} \right]^2 \right\rangle = \left(\frac{2\pi}{\beta} \right)^2 [n + \tilde{n}]^2. \quad (2.8)$$

In practice, the connection of $\tilde{n}$ and Ω , i.e., $\langle Q^2 \rangle$ is established via the eigenvalues of the untraced PL, which we can bring to the maximal torus of the group [43]. Assuming, for simplicity, that the underlying gauge symmetry is $SU(2)$, we can express a representative gauge field, in Polyakov gauge, $A_0 = \hat{A}_{0,3} \mathbb{I}_3$ with the diagonal isospin generator $\mathbb{I}_3$ of the $SU(2)$ algebra in the considered representation [66]. For an $SU(2)$ gauge theory with spin J representation ($R = 2J + 1$), the PL is

$$\Omega = \frac{1}{2} \text{Tr}[\text{diag}\{\exp(-J\beta\hat{A}_{0,3}), \dots, \exp(J\beta\hat{A}_{0,3})\}]. \quad (2.9)$$

The PL contribution to the Matsubara sum, i.e., $\tilde{n}$ in Eq. (2.7), for pure gauge theory, is related to the PL as

$$\tilde{n} = \frac{i\beta}{4\pi} \hat{A}_{0,3}, \quad (2.10)$$

cf. Eq. (2.6).⁵ In the absence of dynamical matter fields, i.e., pure gauge theory, the PL is the order parameter of

⁵The additional factor of $1/2$ appears due to the normalization $N = 2$ for $SU(2)$. In the case of a semisimple gauge theory $\otimes_i \mathcal{G}_i$, with different gauge groups $\mathcal{G}_i$, we will have PL contributions for each gauge group if the scalar and/or fermion fields contain all gauge charges. In that case, $\tilde{n} = \mathcal{F}(\tilde{n}_1, \dots, \tilde{n}_n)$, and the explicit form of this function $\mathcal{F}$ is model dependent.

(de)confinement, and confinement is characterized by the vanishing trace of the PL, i.e., $\Omega = 0$. This corresponds to the “Weiss minima,” which leave the $\mathbb{Z}_2$ center symmetry invariant.⁶ The corresponding solutions also guarantee the statistical removal of the unphysical (negative norm) degrees of freedom from the physical spectrum in the thermal gauge field theory. It is interesting to note that these solutions, see Table I, replicate the imaginary chemical potential solutions that are the result of the Popov-Fedotov trick for algebraic removal of unphysical degrees of freedom of spin chains [52,53].

We highlight that the presence of a background gauge field enters the effective potential only through the PL via $\tilde{n}$. In the case of a scalar gauge theory, where the gauge fields, like the scalars, are dynamical, we must integrate out the gauge-field fluctuations over their respective backgrounds. This leads to an additional contribution to the effective potential that contains the PL effect only for the non-Abelian case. This result has been generalized to include the contributions from dynamical gauge fields for Abelian scalar QED, and the PL-corrected one-loop effective potential has been computed in [55]. We base our work on this result.

As in the conventional diagrammatic approach that sums all one-particle-irreducible vertex functions, Eq. (2.7) expresses the effective potential as a sum over Matsubara modes⁷ and recovers the Coleman-Weinberg potential [69] for $T = 0$. For $\tilde{n} = 0$, this sum can be expressed in a closed analytical form related to the dimensionless characteristic function

⁶In the context of $SU(N)$ gauge theory, it is $\mathbb{Z}_N$.

⁷It is worth highlighting that infrared problems associated with the zero modes present for bosons are ameliorated when $\tilde{n} \neq 0$, as the PL contribution shifts all modes by a real-valued, noninteger number $\tilde{n}$ [54], i.e. the PL gives rise to a nonperturbative contribution to the Debye mass (see also [67,68] for related discussions in QCD).

TABLE I. Values of $\hat{A}_{0,3}$ for different representations (R) of $SU(2)$ in the confined phase, $\Omega = 0$. Note that the effective potential is independent of the *sign* of $\tilde{n}$.

R	2	3	4
$\hat{A}_{0,3}$	$\pm \frac{i\pi}{\beta}$	$\pm \frac{2i\pi}{3\beta}$	$\pm \frac{2i\pi}{4\beta}, \pm \frac{6i\pi}{4\beta}$
$\tilde{n} = \mp \frac{i\beta\hat{A}_{0,3}}{4\pi}$	$\mp \frac{1}{4}$	$\mp \frac{1}{6}$	$\mp \frac{1}{8}, \mp \frac{3}{8}$

$$J_B(m, \beta, \tilde{n} = 0)$$

$$= \int_0^\infty dx x^2 \log \left(1 - \exp \left\{ -\sqrt{x^2 + m^2 \beta^2} \right\} \right), \quad (2.11)$$

see [70,71]. This forms the starting point of perturbative numerical investigations using publicly available tools [2–5,8–11,13,14]. Including PL effects, we define (see also [54])

$$J_B(m, \beta, \tilde{n}) = -\pi^2 \beta^4 S_\Omega^\tilde{n}(m, \beta), \quad (2.12)$$

with

$$S_\Omega^\tilde{n}(m, \beta) = \frac{m^2}{\pi^2 \beta^2} \sum_{n=1}^\infty \frac{1}{n^2} \cos(2\pi n \tilde{n}) \mathbf{K}_2(mn\beta), \quad (2.13)$$

which yields Eq. (2.11) for $\tilde{n} = 0$. As numerical tools are built around Eq. (2.11) in a modular fashion (we discuss fermions further below), comparing Eqs. (2.11) and (2.12) offers an economical strategy for incorporating PL corrections into existing frameworks.

In our analysis, we also consider dynamical matter fields. Thus, it is not necessarily the case that $\Omega = 0$ satisfies the dual minimization of the effective potential $V_{\text{eff}}^{\text{IL}}(\varphi, \tilde{n})$ with respect to φ and $\hat{A}_{0,3}$. Therefore, for a given theory, the choice of $\tilde{n}$ in the effective potential should be consistent with $\delta V_{\text{eff}}^{\text{IL}}/\delta \varphi = \delta V_{\text{eff}}^{\text{IL}}/\delta \hat{A}_{0,3} = 0$. In this work, our main objective is to display the impact of the PL on a potential PT. Thus, we include the impact of temporal gauge fields through the PL by assigning phenomenological values to $\tilde{n}$ instead of restricting to a confining gauge background. In other words, we will treat $\tilde{n}$ as a free constant⁸ parameter that parametrizes the plasma through this connection (see [50,72] for similar discussions in the context of the semi-quark-gluon plasma in QCD).

The effective potential after integrating out the chiral fermions in an $SU(N)$ background gauge theory is given by [54]

⁸We choose it to be a (dimensionless) real number < 1 , motivated by the solutions of $\Omega = 0$ in Table I.

$$\begin{aligned} -V_{\text{eff}}^{\text{1L},F} = & -\frac{d}{2} \left[\frac{m^4}{32\pi^2} \left(\log \left(\frac{4\pi\mu^2}{m^2} \right) + \frac{3}{2} - \gamma_E \right) \right. \\ & + \frac{m^2}{2\pi^2 \beta^2} \sum_{n=1}^\infty \frac{\cos(2\pi n \tilde{n})}{n^2} \mathbf{K}_2(2m\beta n) \\ & \left. - \frac{m^2}{\pi^2 \beta^2} \sum_{n=1}^\infty \frac{\cos(2\pi n \tilde{n})}{n^2} \mathbf{K}_2(m\beta n) \right], \quad (2.14) \end{aligned}$$

where $d = 3$. Similarly to the bosonic case, we can define an extended characteristic temperature-dependent function

$$J_F(m, \beta, \tilde{n}) = \pi^2 \beta^4 [2S_\Omega^\tilde{n}(m, 2\beta) - S_\Omega^\tilde{n}(m, \beta)], \quad (2.15)$$

with $S_\Omega^\tilde{n}(m, 2\beta)$ given in Eq. (2.13). Again, this reduces to the well-known form

$$\begin{aligned} J_F(m, \beta, \tilde{n} = 0) \\ = \int_0^\infty dx x^2 \log \left(1 + \exp \left\{ -\sqrt{x^2 + m^2 \beta^2} \right\} \right), \quad (2.16) \end{aligned}$$

and a relatively straightforward replacement of the characteristic J_F function in existing numerical tools is possible.

Further details on the $\tilde{n} \neq 0$ functions are given in Appendix, where we also present a closed integral form based on an imaginary chemical potential $\mu_I = \mu_I(\tilde{n})$, which might prove helpful in future applications.

III. MODIFIED CRITICAL BEHAVIOR

A. A toy model and its effective potential

To examine the impact of the PL contribution transparently, we trace its relevance for an $SU(2)$ toy model⁹ that nevertheless captures relevant bosonic features of the SM (for instance, a first-order PT for light Higgs masses). Concretely,

$$-\mathcal{L} = m^2 |\Phi|^2 + \lambda |\Phi|^4 - (D^\mu \Phi)^\dagger D_\mu \Phi + \frac{1}{4} W^{a\mu\nu} W_{\mu\nu}^a, \quad (3.1)$$

with the Higgs field in the fundamental representation

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho + i\eta \\ \zeta + \omega + i\psi \end{pmatrix}. \quad (3.2)$$

The covariant derivative is given by

$$D_\mu = \partial_\mu - ig W_\mu^a \frac{\sigma^a}{2}, \quad (3.3)$$

⁹We emphasize that our intention is not to propose a realistic model for a first-order electroweak PT, but to define a suitable scenario to highlight the PL's role in electroweak PTs clearly with minimal technical complexity.

where the three Pauli matrices are denoted by σ^a ($a = 1, 2, 3$) as usual. This model, modulo rephasing, admits trivial $|\langle\Phi\rangle| \equiv \omega/\sqrt{2} = 0$ and/or nontrivial $\omega \neq 0$ vacuum configurations at finite temperature.¹⁰ At zero temperature, we can identify

$$\omega^2 \xrightarrow{T \rightarrow 0} v^2 = -\frac{m^2}{\lambda} > 0, \quad (3.4)$$

analogous to the SM Higgs sector. Thus, this toy model provides a motivated candidate theory for investigating the impact of the PL contribution and its phenomenological consequences.

In the following, we investigate the finite-temperature dynamics employing the BSMP code [3,5,8]. This aligns with conventional choices in the literature, e.g. those recently used in Refs. [73,74]. Concretely, we consider the one-loop effective potential¹¹

$$V_{\text{eff}}^{\text{1L, toy}} \equiv V^{\text{tree}} + V^{\text{CW}} + V^{\text{CT}} + V^\beta + V^{\text{daisy}}, \quad (3.5)$$

where the tree-level potential is given by Eq. (3.1),

$$V^{\text{tree}} = m^2|\Phi|^2 + \lambda|\Phi|^4. \quad (3.6)$$

The zero-temperature Coleman-Weinberg potential [69] reads

$$V^{\text{CW}} = \frac{1}{64\pi^2} \sum_{X=S,G} S_X m_X^4 \log\left(\frac{m_X^2}{\mu^2} - k_X\right), \quad (3.7)$$

with $S_X = \{1, 3\}$ and $k_X = \{3/2, 5/6\}$ for $X = \{S, G\}$. The masses m_X are the eigenvalues of the scalar (S) and gauge-boson (G) mass matrices, i.e. the states that run in the loop. For the toy model of Eq. (3.1), these are given by

$$m_{\chi,\eta,\psi}^2 = \lambda(\omega^2 - v^2), \quad m_\zeta^2 = 3\lambda\omega^2 - \lambda v^2, \quad m_{W^a}^2 = \frac{g^2}{4}\omega^2, \quad (3.8)$$

such that $M_h^2 = 2\lambda v^2$ for the physical Higgs boson. The finite-temperature part of the potential, including the PL contribution, following the discussion in Sec. II and Ref. [55], can be written as

$$V^\beta = \sum_{X=S,G} \frac{S_X}{2\pi^2\beta^4} J_B(m_X, \beta, \tilde{n}). \quad (3.9)$$

¹⁰The vacuum is additionally characterized by $\langle\rho\rangle, \langle\eta\rangle, \langle\zeta\rangle, \langle\psi\rangle = 0$.

¹¹In Refs. [65,75,76], it has been shown that a gauge-invariant (and gauge-fixing parameter-independent) realization of the one-loop effective potential is identical to functional methods using the Landau gauge.

To reflect the PL contributions as detailed above, the $\tilde{n}$ -modified finite-temperature functions have been implemented in the BSMP code.¹² A range of cross-checks have been performed to evaluate the finite-temperature $\tilde{n}$ -corrected effective potential. In practice, we find that truncating the series at $n \simeq 600$ in Eq. (2.13) provides an accurate description of the PT phenomenology without compromising numerical efficiency. In particular, the “standard” $\tilde{n} = 0$ results using the closed form of the J functions are accurately reproduced. We therefore opt for the truncated series definition in our numerical simulation.

The daisy corrections, collected in V^{daisy} , are derived from the one-loop thermal masses $\bar{m}_X^2$

$$V^{\text{daisy}} = -\frac{1}{12\pi\beta} \sum_{X=S,G} [(\bar{m}_X^2)^{3/2} - (m_X^2)^{3/2}]. \quad (3.10)$$

Daisy corrections are critical in the PL-free case, where they improve the infrared behavior of the zero Matsubara modes. Polyakov contributions, however, as mentioned in Sec. II, act as additional regulators. Lastly, the counterterm potential V^{CT} is included to fix the vacuum expectation value (VEV) and the masses at zero temperature to their leading-order values.

B. Phase transition dynamics for $\tilde{n} \neq 0$

As the Universe cools, it can undergo PTs, depending on the underlying theory we use to describe its dynamics. In particular, first-order electroweak PTs, which are shown to be realized within a variety of BSM models, allow for promising avenues to explain baryogenesis and open new observational prospects with the advent of space-based gravitational-wave (GW) observatories, cf. [39,79,80] for recent reviews and references therein. The strength of a first-order PT is conventionally estimated by the ratio

$$\xi_c \equiv \frac{\omega_c^{\text{true}} - \omega_c^{\text{false}}}{T_c}, \quad (3.11)$$

i.e., it is given by the difference in VEVs of the scalar fields between the true and false minimum at the critical temperature $\omega_c = \sqrt{2}\langle\Phi\rangle(T_c)$ (when the nontrivial second minimum that emerges during the cooling of the Universe and the high-temperature minimum are degenerate, and separated by a potential barrier), measured in units of the critical temperature T_c .¹³ For a sufficiently strong first-order PT with

¹²Modified Bessel functions with generally complex arguments are evaluated by employing the C++ wrapper [77] of the original routines of [78].

¹³The gauge-dependence of this measure and its implications have given rise to alternative considerations [24,81,82], see also [83,84]. It is worth highlighting that the gauge dependencies are absent in the background Fermi gauges at all orders, as demonstrated in [65,75,76]. We will not consider this further in the present work.

$\xi_c \gtrsim 1$, sphaleron washout in the broken phase is considered to be reasonably suppressed inside the forming true-vacuum bubble to satisfy the conditions required for, e.g., baryogenesis in more realistic theories than the one considered in this work. The key to a first-order PT is the formation of a potential barrier separating the false from the true vacuum, with $\omega \neq 0$, and $\omega \xrightarrow{T \rightarrow 0} v$. Tunnelling between the vacua proceeds via the so-called bounce action [27,28,31,85], which triggers the nucleation of expanding true vacuum bubbles, cf. [86] for a review. This creates the out-of-equilibrium condition required by Sakharov's criteria.

In the following, we will focus primarily on the behaviour of ξ_c as a function of $\tilde{n}$ to understand how the Polyakov contribution affects the PT dynamics. That said, we will also investigate the impact of $\tilde{n}$ on the dynamics at the nucleation (the formation of one bubble per Hubble volume) and percolation (the formation of a network of bubbles and the large-scale conversion of 34% of the comoving volume to the true minimum [87]) stages of the PT. At the latter stage, we will focus on the effect of $\tilde{n}$ on the sourced GWs. Comparing these stages with their $\tilde{n} = 0$ counterparts is relevant because it provides insights into how $\tilde{n}$ can change the PT timeline and lead to direct phenomenological implications. We furthermore stress that our choice of $\tilde{n}$ is driven solely by phenomenological interests, to demonstrate how the inclusion of the PL contribution affects the order of the PT and related phenomena.

1. Phase transitions with bosonic degrees of freedom

Within the toy model detailed above, such a (strong) first-order PT can be observed for the $\tilde{n} = 0$ potential toward smaller values of the scalar mass $M_h \in [30 \text{ GeV}, 60 \text{ GeV}]$ and larger values of the zero-temperature VEV

$v \in [160 \text{ GeV}, 200 \text{ GeV}]$ for $g = 1.2$. The motivation for this parameter choice is to maximize the parameter space compatible with a first-order PT. In doing so, we deliberately ignore other theoretical and experimental constraints of this toy model. This choice aligns with our aim to display and highlight the impact of the PL contribution on the parameter space compatible with a first-order PT. We first consider the purely bosonic finite-temperature potential, then comment on the relevance of fermions below. We begin by discussing the impact of $\tilde{n}$ on the representative benchmark point (BP) with

$$M_h = 46 \text{ GeV}, \quad v = 180 \text{ GeV}, \quad g = 1.2. \quad (3.12)$$

In the left panel of Fig. 1, we compare contours along the ζ direction of the effective potential at the critical temperature as a function of $\tilde{n}$. Starting with $\tilde{n} = 0$, i.e. with the “standard” perturbative calculation, we observe a potential barrier that yields a first-order PT with $\xi_c \simeq 1.37$. Increasing $\tilde{n}$ reduces the barrier height significantly compared to the $\tilde{n} = 0$ case, and in addition leads to a shift of the low-temperature true minimum to lower field values and to an increase of the critical temperature. We find critical temperatures of $T_c = \{94.18, 96.22, 110.33, 115.01\} \text{ GeV}$ and resulting ξ_c values of $\xi_c = \{1.37, 1.30, 0.72, 0.07\}$ for $\tilde{n} = \{0, 0.02, 0.05, 0.0548\}$. A value $\tilde{n} = 0.0548$ is found to be the largest possible value that, for this BP, still yields a potential barrier. This result is further illustrated by the right panel of Fig. 1, where we show the temperature dependence of the found high- and low-temperature minima, as a dashed magenta and a solid blue line, respectively, for $\tilde{n} = \{0, 0.05, 0.0548, 0.0549\}$. For $\tilde{n} = 0.0549$, depicted by a solid black line, we find no barrier, but a continuous connection, compatible with a

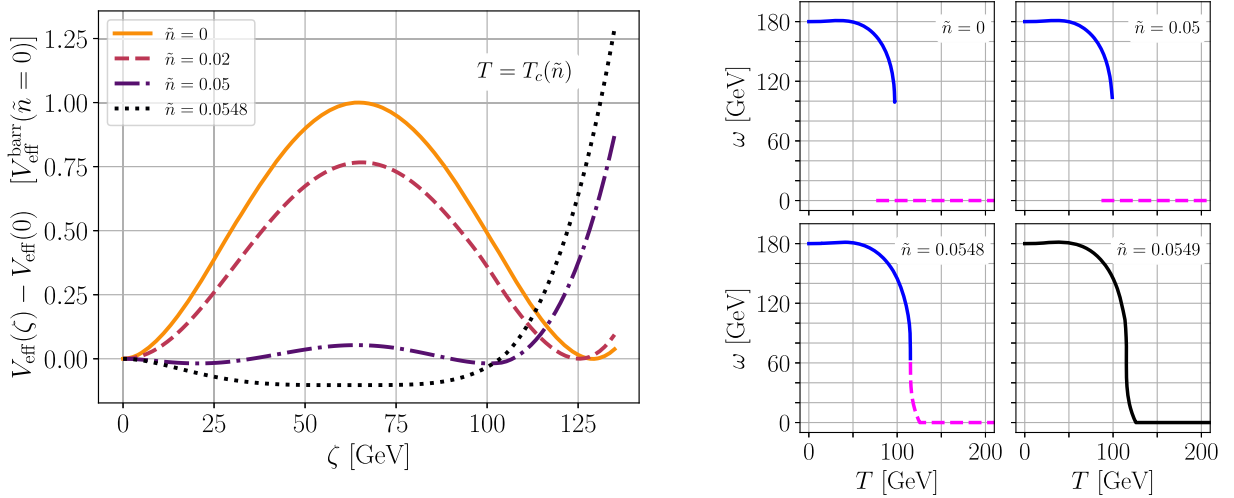


FIG. 1. Impact of different values of $\tilde{n}$ on the BP of Eq. (3.12). Left: Potential contours at the critical temperatures normalized to the barrier height for $\tilde{n} = 0$. Right: Minima as a function of the temperature. Note that the discontinuity in the minima is reduced when $\tilde{n}$ is increased, which clearly reflects the smoothing of the barrier in the left panel.

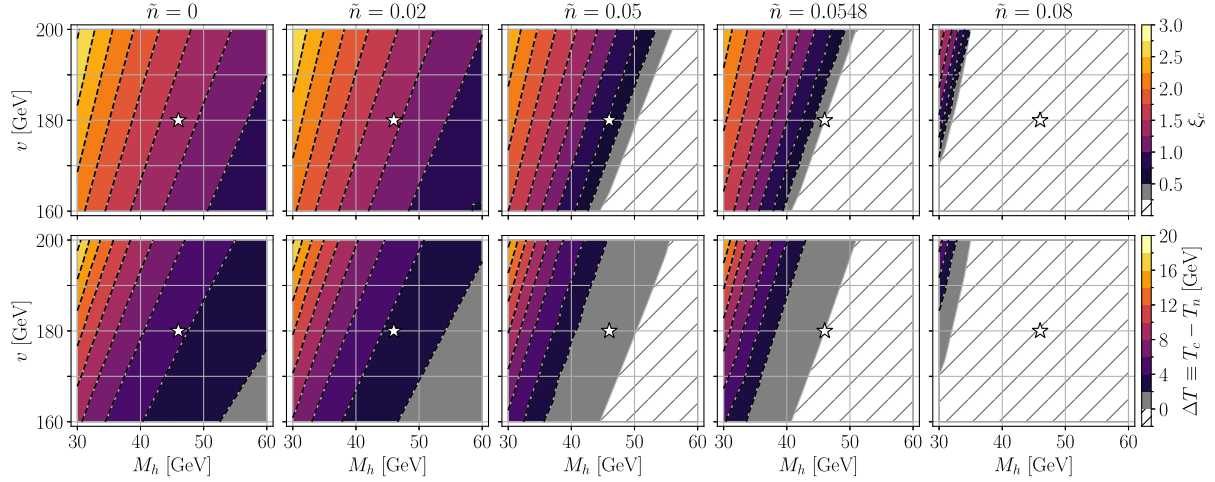


FIG. 2. Strength of the PT according to Eq. (3.11) and the difference ΔT between the critical and the nucleation temperature (second row) in the parameter space defined by M_h and v for values of the PL $\tilde{n} = \{0, 0.02, 0.05, 0.0548, 0.08\}$ (columns) and for $g = 1.2$. The hatched area corresponds to the parameter space regions for which a second-order PT (potentially a smooth cross-over) is found. The BP: $M_h = 46$ GeV, $v = 180$ GeV, $g = 1.2$, given in Eq. (3.12), is marked with an asterisk.

second-order PT.¹⁴ As also visible in Fig. 1, $\tilde{n}$ changes the shape of the potential around the origin via odd monomials of the field-dependent mass, which consequently moves the high-temperature false minimum away from the origin to $\omega \neq 0$. These monomials can be traced to the analytical properties at small $\tilde{n}$ that we detail in the Appendix.

From the in-depth analysis of the BP, we conclude that increasing $\tilde{n}$ not only softens the first-order PT, but can completely remove the barrier and turn the $\tilde{n} = 0$ expectation of a strong first-order PT into a second-order PT. The monotonic weakening of the PT with increasing $\tilde{n}$ is a universal feature and is expected to persist across models, rather than being specific to the toy model considered here. In this sense, although our analysis is carried out in a simplified setting, the qualitative behavior we identify is expected to be broadly applicable.

How, then, do the findings for the BP of Eq. (3.12) translate to the parameter space spanned by M_h and v ? In Fig. 2, we illustrate this parameter space for $g = 1.2$ with $30 \text{ GeV} \leq M_h \leq 60 \text{ GeV}$ and $160 \text{ GeV} \leq v \leq 200 \text{ GeV}$ and for $\tilde{n} = \{0, 0.02, 0.05, 0.0548, 0.08\}$ (columns) with the colored contours corresponding to the value of ξ_c (top row) and the difference between the critical and nucleation temperature, $\Delta T \equiv T_c - T_n$ (bottom row). We find that our observation for the chosen BP of Eq. (3.12), marked with an asterisk in Fig. 2, directly translates to a larger parameter space. Larger values of $\tilde{n}$ lead to a decrease in ξ_c that shifts the region of viable (strong) first-order PTs toward even smaller M_h and larger v . Consequently, regions with a (strong) first-order PT in the conventional perturbative treatment with $\tilde{n} = 0$ can show a weak first-order or

second-order PT (hatched area) for $\tilde{n} > 0$. In addition, the gap between the critical and nucleation temperatures, as displayed in the bottom row of Fig. 2, is observed to be reduced for a fixed location in the (M_h, v) -plane (correlated with the weakened PT), while the total area covered by the ΔT contours also decreases.¹⁵ Consequently, the Polyakov contribution not only drives (strong) first-order PTs toward second-order ones, but also, in line with expectations, reduces supercooling and bubble sizes.

As part of a wider program to track electroweak first-order PTs in the early universe, the relation of such transitions to a stochastic gravitational-wave background is increasingly exploited to connect particle physics with GW astronomy [88–93]. In the following, we estimate the effect of $\tilde{n}$ on first-order PT-sourced GW spectra. Using BSMPTv3.2.1, we assume GWs to be sourced at the percolation temperature with a wall velocity of $v_w = 0.95$. Thus, the two key parameters for GWs are the PT strength α ¹⁶ and inverse PT duration β/H , cf. Ref. [8] for details on the implementation. Finally, as an estimate of the detectability of the predicted GWs at the Laser Interferometer Space Antenna [98–101], we obtain signal-to-noise ratios (SNRs), see Ref. [8] and references therein for details.¹⁷

¹⁵We find a similar behavior for the gap between nucleation and percolation temperature.

¹⁶This parameter should not be confused with our earlier definition of ξ_c ; it measures the released vacuum energy density relative to the energy density of a radiation-dominated universe. For the purpose of this study, we estimate α via the trace anomaly [94,95], but refer the reader to [96,97] for the more accurate definition in terms of the pseudotrace.

¹⁷For the chosen toy scenario with $v_w = 0.95$, the dominant source of GWs are sound waves, which we model as a double broken power law [102], as further detailed in Ref. [8].

¹⁴The distinction between second-order PT and smooth cross-over is beyond the scope of this work.

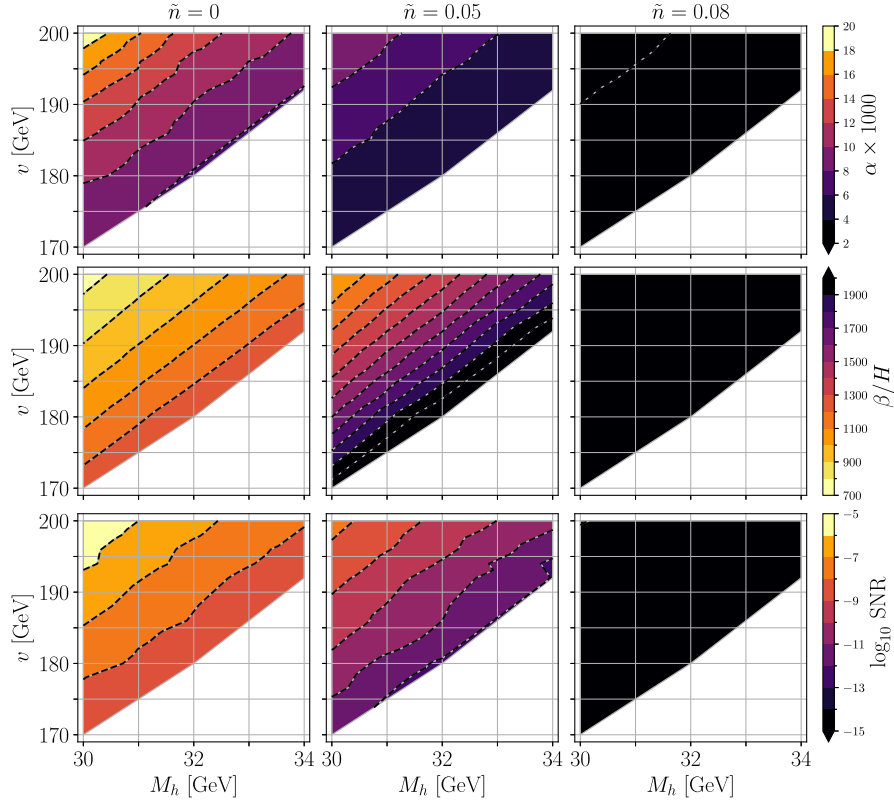


FIG. 3. PT strength α , inverse PT duration β/H and signal-to-noise ratio (SNR) for detection by LISA (assuming a three-year data-taking period) for $g = 1.2$ and $\tilde{n} = \{0, 0.05, 0.08\}$ in the parameter space region where we still find a first-order PT even for $\tilde{n} = 0.08$.

The results for our toy model are presented in Fig. 3. In line with our observations discussed above, increasing $\tilde{n}$ decreases the PT strength α , and increases the inverse PT duration β/H , thereby yielding more rapid PTs. These effects conspire to drastically reduce the SNR, suggesting that Polyakov contributions may be critical for analyzing GW signatures of PTs in more realistic models.¹⁸

2. Fermion-induced effects in phase transitions

We conclude our analysis by discussing potential modifications to the critical behavior outlined in the previous section, arising from the inclusion of chiral fermions that acquire mass via the Higgs mechanism. To this end, we extend the zero-temperature potential by adding

$$V \supset y_t \bar{Q}_L \Phi^c t_R + y_b \bar{Q}_L \Phi b_R + \text{H.c.}, \quad (3.13)$$

with $Q_L = (t_L, b_L)$ and the charge-conjugated field $\Phi^c = i\sigma^2 \Phi^*$, and

$$y_{t,b} = \frac{\sqrt{2}}{v} m_{t,b}, \quad (3.14)$$

where flavor mixing is incorporated through the Cabibbo–Kobayashi–Maskawa (CKM) [103,104] element V_{tb} , in analogy with the SM. Additionally, we assume a flavor symmetry among the fermions, to mimic the effect of the SM’s color charge without including QCD gauge boson degrees of freedom.¹⁹ This leads to a new contribution to the zero-temperature Coleman–Weinberg potential

$$V^{\text{CW}} \supset \sum_{F=\{t,b\}} \frac{S_F}{64\pi^2} m_F^4 \log \left(\frac{m_F^2}{\mu^2} - k_F \right), \quad (3.15)$$

with $S_F \equiv -2$ and $k_F = 3/2$. The temperature-dependent effective potential receives the additional contributions

$$V^\beta \supset \sum_{F=\{t,b\}} \frac{S_F}{2\pi^2\beta^4} J_F(m_X, \beta, \tilde{n}), \quad (3.16)$$

¹⁸We note that a more realistic choice of v_w is expected to lead to lower SNR values, already at $\tilde{n} = 0$. However, as our focus here is an illustration of the relative suppression of the SNR due to $\tilde{n} \neq 0$, we choose an overly optimistic v_w to show that even the strongest possible GWs are drastically damped by $\tilde{n} \neq 0$.

¹⁹Strictly speaking, an additional fermion doublet is necessary to avoid global anomalies [105]. If, however, the mass of this fermion is small, it will be phenomenologically irrelevant for the PT as its contribution decouples from the effective potential for a vanishing Yukawa coupling.

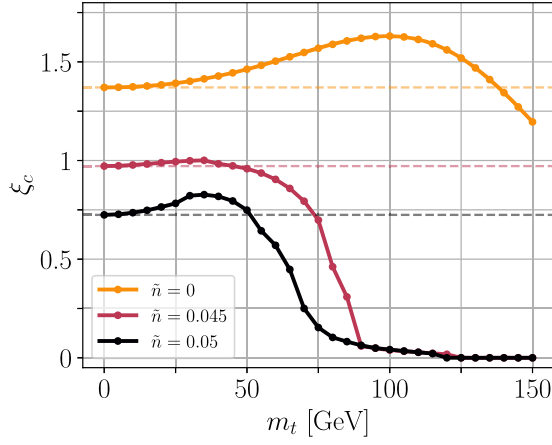


FIG. 4. Left: ξ_c vs m_t for different choices of $\tilde{n}$ for the BP with $g = 1.2$ defined in Eq. (3.12). Right: Values of ξ_c for the same BP, but for different choices of the gauge coupling: $g = 1.2$ ($g = 1$ in brackets). Second-order PTs, e.g. for $g = 1$ with $\tilde{n} = 0.05$, are indicated with $\xi_c = 0$.

and the background-field-dependent masses are

$$m_F = \frac{y_F}{\sqrt{2}} \omega, \quad F \in \{t, b\}. \quad (3.17)$$

As there are no fermionic zero modes, no daisy corrections are included.

The combined impact of $\tilde{n}$ and a nonzero fermion mass on the first-order PT strength ξ_c for our BP of Eq. (3.12) is shown in Fig. 4. For any value of $\tilde{n}$, the inclusion of a nonzero m_t initially yields a stronger first-order PT.²⁰ As m_t is increased further, ξ_c first reaches a maximum and then decreases. For sufficiently large m_t and a relatively light scalar mass, the one-loop correction may turn the scalar quartic coupling negative, calling into question the stability of the effective potential. This caps the maximum value of m_t in our analysis. Near this upper limit on m_t ,²¹ the first-order PT is found to be weaker than for the purely bosonic case with $m_t = 0$ (for the same $\tilde{n}$). The increase in first-order PT strength for an intermediate fermion mass range cannot, in general, overcome the weakening effect of the PL. A finite $\tilde{n}$ is found to push the maximal possible ξ_c to lower m_t and to yield weaker first-order or second-order PTs, for all values of m_t .

In addition, we investigate the impact of the gauge coupling g . For the relatively small value $g = 1$, as illustrated

m_t	ξ_c		
	$\tilde{n} = 0$	$\tilde{n} = 0.02$	$\tilde{n} = 0.05$
0	1.370 (0.972)	1.302 (0.840)	0.724 (0)
5	1.371 (0.973)	1.303 (0.840)	0.727 (0)
25	1.392 (0.983)	1.319 (0.839)	0.783 (0)
50	1.463 (1.026)	1.380 (0.839)	0.748 (0)
75	1.569 (1.098)	1.482 (0.768)	0.153 (0)
100	1.631 (1.140)	1.546 (0.610)	0.042 (0)

by the values in brackets in the table on the right side of Fig. 4, we generally find weaker first-order PTs, compared to the case with $g = 1.2$ (numbers not in brackets), which are further weakened if a finite PL and finite fermion mass are considered. For a choice of $g = 0$, which corresponds to a theory with only scalars and fermions, we generically find very weak first-order, $\xi_c \lesssim 0.1$, or second-order PTs.

IV. SUMMARY AND CONCLUSIONS

Electroweak phase transitions in the early Universe provide a phenomenologically rich arena for connecting particle physics with phenomena that remain unexplained by the Standard Model. Given the absence of direct hints of new physics, this research direction has attracted growing attention. This includes work on conceptual problems such as gauge dependence, higher-order corrections, and especially effective-field-theory formulations in thermal field theory [26,55,65,109–116]. In this work, we study the Polyakov loop [40,41], a topologically rooted feature of thermal field theory, and its relation to the effective thermal potential that governs electroweak phase-transition histories. We show that this contribution can have direct implications for associated phenomenological signatures, such as gravitational waves. (For recent analyses in QCD-type setups, yet with similar observations, see also [117,118].)

We derive the PL term in the effective Higgs potential using the heat-kernel method. Treating the PL as a phenomenological parameter, we develop a practical prescription for incorporating these effects into existing perturbative calculations through modifications of the characteristic finite-temperature functions $J_{B/F}$. The standard results are recovered directly in the limit of a vanishing A_0 when the PL becomes a trivial identity operator. In this sense, our work provides a first step toward incorporating nontrivial plasma correlations into standard perturbative frameworks [2–14].

²⁰This aligns with observations in “standard” perturbative studies, cf. e.g. [106,107]. The inclusion of $m_b \neq 0$ at a fixed $\tilde{n}$ further amplifies the effect of $m_t \neq 0$. For the inclusion of nonzero mixing, this amplification is weakened. For the purpose of our qualitative study, we limit our following discussion to the case with $m_b = 0$. For a more realistic follow-up, we refer to [108].

²¹For the sake of simplicity, we limit our analysis to scenarios where the zero-temperature global NLO minimum coincides with the EW minimum of 246.22 GeV. Of course, a sufficiently metastable vacuum also constitutes a physical vacuum.

We apply this approach to a toy model: an $SU(2)$ gauge theory with a complex scalar field in the fundamental representation. This enables us to transparently assess the impact of the PL on the components of the effective potential that control the PT. Starting from regions of the toy-model parameter space that exhibit a first-order PT in the absence of PL contributions, our primary goal is to demonstrate the impact of the PL on the PT dynamics. Given the close relationship between our scenario and the SM, we expect that many of the implications found in this work will qualitatively extend to the SM. This includes the smoothing of the effective-potential barrier. Overall, phase transitions are tamed by sizeable PL contributions.

We extend our discussion to fermions to clarify the role of heavy quarks as encountered in the SM. To this end, we analyze the intertwined effects of PL contributions and fermion masses in the presence of additional bosonic degrees of freedom. We find that the change in the order and strength of the first-order PT induced by fermionic degrees of freedom follows a pattern closely analogous to the bosonic case, apart from some intermediate oscillatory behavior.

Throughout our analysis, we consider the PL values as effective phenomenological variables. For a given theory, the PL is fixed by minimising the effective potential with respect to the background scalar and gauge fields. Our analysis suggests that models containing additional non-SM matter fields with nonzero gauge charges will acquire significant contributions from PLs. This can result in the exclusion of large parts of the parameter space that would otherwise be compatible with a (strong) first-order PT. This suggests that such scenarios should be revisited once PL contributions are included consistently. Such effects can

alternatively be captured through thermal effective operators where the PL contribution is encoded in the Wilson coefficients [115]. We further expect that higher-order corrections to the thermal effective potential and the inclusion of running parameters in the presence of PL contributions might play important roles in PTs. We leave such a targeted follow-up study to future work [108].

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: FURTHER DETAILS ON $\tilde{n} \neq 0$ FUNCTIONS

The phenomenology that we discuss in this work can be put into an analytical context through an expansion of Eq. (2.13) as [119]

$$\begin{aligned}
 S_{\Omega}^{\tilde{n}}(m, \beta) &= \frac{m^2}{\pi^2 \beta^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos(2\pi n \tilde{n}) \mathbf{K}_2(mn\beta) \\
 &= \frac{m^2}{\pi^2 \beta^2} \left[\frac{m^2 \beta^2}{16} \left\{ \log\left(\frac{m\beta e^{\gamma_E}}{4\pi}\right) - \frac{3}{4} \right\} - \frac{1}{2} \left\{ \frac{1}{4} \phi^2 - \frac{\pi}{2} \phi + \frac{\pi^2}{6} \right\} + \frac{2}{m^2 \beta^2} \left\{ -\frac{1}{48} \phi^4 + \frac{\pi}{12} \phi^3 - \frac{\pi^2}{12} \phi^2 + \frac{\pi^4}{90} \right\} \right. \\
 &\quad \left. + \frac{\pi}{2m^2 \beta^2} \sum_{\substack{l \in \mathbb{Z} \\ l \neq 0}} \left\{ \frac{1}{3} [m^2 \beta^2 + (\phi - 2\pi l)^2]^{3/2} - \frac{1}{3} |\phi - 2\pi l|^3 - \frac{1}{2} |\phi - 2\pi l| m^2 \beta^2 - \frac{m^4 \beta^4}{16\pi |l|} \right\} \right], \quad (A1)
 \end{aligned}$$

where $\phi = [2\pi \tilde{n}] \bmod (2\pi)$ with $\tilde{n} < 1$. Furthermore, a closed integral representation can be obtained for $S_{\Omega}^{\tilde{n}}$ by introducing an imaginary chemical potential, reminiscent of the Popov-Fedotov trick [52] (see also [119–121]). Concretely, we can define

$$\mu(\tilde{n})\beta = 2\pi i \tilde{n}, \quad (A2)$$

and understand $J_B(m, \beta, \tilde{n})$ as the real part of an analytically continued Eq. (2.11)

$$J_B(m, \beta, \tilde{n}) = \int_0^{\infty} dx x^2 \operatorname{Re} \left\{ \log \left(1 - \exp \left\{ -\sqrt{x^2 + m^2 \beta^2} + \mu(\tilde{n})\beta \right\} \right) \right\}. \quad (A3)$$

$S_{\Omega}^{\tilde{n}}$ and J_F follow from Eq. (A3) via Eqs. (2.12) and (2.15).

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