

Dynamical determination of the cut-off scale in loop-induced neutrino mass models with noninvertible symmetry

Hiroshi Okada^{*} and Jia-Jun Wu[†]

Department of Physics, *Henan Normal University*, Xinxiang 453007, China



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We propose our framework as an effective field theory valid below the cutoff scale Λ , in which we explain the tiny scale of neutrino masses by integrating a noninvertible symmetry with the dynamical determination of the cutoff scale. In our model, we introduce three families of $SU(2)_L$ quintet fermions (Σ_R) and a quartet scalar (ϕ_4), both of which are charged under the Fibonacci fusion rule (FFR). A central feature of this construction is that the vacuum expectation value (VEV) v_4 of ϕ_4 is induced at the one-loop level via dynamical symmetry breaking. To resolve the inherent arbitrariness of the cutoff scale Λ in loop-induced VEV models, we identify Λ with the scale at which the $SU(2)_L$ gauge coupling g_2 encounters the renormalization group evolution (RGE) of g_2 , naturally fixing the physical cutoff within the range of approximately 10^5 to 10^7 GeV. Quantitatively, this framework yields $v_4 \sim 0.07$ – 0.1 GeV, which in turn leads to neutrino Yukawa couplings on the order of 10^{-3} . This result provides a significantly more natural explanation for the neutrino mass hierarchy compared to standard seesaw models, which typically require couplings as small as 10^{-6} or less. Notably, our approach maintains a relatively simple particle content and does not necessitate additional (gauge) bosons for symmetry breaking.

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I. INTRODUCTION

A wide range of theoretical frameworks beyond the Standard Model (BSM) have been proposed to explain neutrino masses. Among them, numerous ideas have been advanced to address the so-called neutrino hierarchy problem—namely, the challenge of explaining, from the perspective of naturalness, why neutrino masses are extremely small compared to those of other fermion sectors. One representative example is the radiative seesaw mechanism, in which neutrino masses are generated through loop effects. Classical realizations include the Zee model [1] and the Zee–Babu model [2], where neutrino masses arise at the one- and two-loop levels, respectively. More recently, the Ma model [3] has provided a particularly attractive framework that simultaneously links neutrino mass generation with dark matter. The central philosophy of such approaches is that neutrinos do not directly couple to the SM Higgs boson but instead interact with new particles such as dark matter candidates. As a result, neutrino masses are generated exclusively at the

loop level, acquiring a suppression factor of $\sim 1/(4\pi)^2$ per loop, which naturally explains their tiny scale.

In addition, models in which neutrino masses originate from suppressed vacuum expectation values (VEVs) have attracted attention as compelling approaches to naturally account for the tiny neutrino mass scale. Within this context, the simplest construction is the Type-II seesaw model [4], which introduces an $SU(2)_L$ triplet scalar with hypercharge $Y = 1$. Neutrino masses are generated directly through the triplet VEV, v_Δ . Precision measurements constrain this VEV to be of order a few GeV, approximately 2 orders of magnitude smaller than the Standard Model–like Higgs VEV. However, even with such a suppressed v_Δ , explaining the observed neutrino masses requires Yukawa couplings of order 10^{-10} , which is unsatisfactory from the standpoint of naturalness. A natural extension to address this is to further reduce the VEV in order to enhance the Yukawa couplings, but it is natural to seek a theoretical mechanism that provides a structural explanation rather than relying on *ad hoc* parameter tuning.

To address this issue, several attempts have been made to induce v_Δ radiatively by introducing additional symmetries or fields [5,6]. Typically, such models impose discrete or continuous symmetries to forbid tree-level contributions. More recently, however, a novel theoretical framework has emerged in which noninvertible symmetries are employed to generate loop effects via dynamical symmetry

^{*}Contact author: hiroshi3okada@htu.edu.cn

[†]Contact author: wujiajun@htu.edu.cn

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breaking [7–16].¹ The advantage of this approach lies in its ability to explain the neutrino mass hierarchy without requiring new (gauge) bosons for symmetry breaking, thereby maintaining a relatively simple particle content. Despite its promise, the application of noninvertible symmetries raises theoretical challenges, particularly the appearance of divergences in loop calculations. To regulate these divergences, a cutoff scale Λ has to be introduced. Previous studies [28] have typically adopted phenomenologically reasonable values such as $\Lambda \sim 10^5$ GeV that corresponds to the maximal energy scale accessible at current or near-future experiments such as CEPC-SPPC [29]. But the absence of a fundamental principle to fix Λ has remained a significant obstacle. In cases where the divergence is logarithmic, the weak dependence of results on Λ has further discouraged detailed discussion.

In this work, we propose a scenario in which the cutoff scale Λ is determined dynamically from the renormalization group (RGE) flow of the $SU(2)_L$ gauge coupling g_2 . While in the SM, g_2 decreases at high energies due to near-asymptotic freedom, the introduction of higher $SU(2)_L$ multiplets alters the RGE behavior such that g_2 grows at high energies and eventually encounters a Landau pole. We define the scale at which g_2 reaches the strong coupling regime as the physical cutoff Λ . In this way, the dominant contributions to loop integrals arise from the perturbative regime where g_2 remains small, while the upper bound of the integration is naturally fixed by the internal dynamics of the theory, yielding our framework as an effective field theory valid below Λ .

In the specific model proposed in this paper, we introduce the following new $SU(2)_L$ multiplet particles²:

- (i) Three families of $SU(2)_L$ quintet fermions Σ_R with hypercharge $Y = 0$, and
- (ii) An $SU(2)_L$ quartet scalar ϕ_4 with hypercharge $Y = 1/2$.

Neutrino masses are generated via a seesaw mechanism mediated by the small VEV v_4 of ϕ_4 . This VEV is induced at the one-loop level through dynamical symmetry breaking under a noninvertible symmetry governed by the Fibonacci fusion rule. In this construction, the higher multiplets, (Σ_R and ϕ_4), predominantly govern the determination of the cutoff scale Λ through the renormalization-group running of the $SU(2)_L$ gauge coupling g_2 . The finite cutoff scale thus obtained radiatively induces an effective quartic coupling at one loop, leading to a naturally suppressed vacuum expectation value v_4 . Since v_4 directly controls the

neutrino-mass scale, the framework simultaneously accounts for the experimentally favored hierarchy $v_4 \ll v_H$ and a theoretically well-motivated finite cutoff scale Λ , all without the introduction of extra (gauge) bosons. Here, $v_H \approx 246$ GeV is VEV of the SM Higgs. Moreover, these multiplet states are expected to be within the discovery reach of the Large Hadron Collider when their masses lie at the TeV scale, particularly around 1 TeV. A related discussion has previously appeared in Ref. [35].

This paper is organized as follows. In Sec. II, we introduce our model, derive the active neutrino mass matrix, and discuss the typical order of magnitude of the Yukawa couplings and relevant mass scales. We conclude and discuss in Sec. III.

II. MODEL SETUP

In this section, we review our model. As we already mentioned in the introduction, we introduce three isospin quintet right-handed fermions Σ_R that have zero hypercharge and add an isospin quartet scalar ϕ_4 with hypercharge $Y = 1/2$ in addition to the SM Higgs denoted by H . Furthermore, we impose τ to all leptons and bosons except H under the Fibonacci fusion rule (FFR). The particle contents and their charges are shown in Table I. Fibonacci fusion algebra consists of two elements $\mathbb{I}$ and τ , which is known as the minimal fusion rule. They satisfy the following relations:

$$\mathbb{I} \otimes \mathbb{I} = \mathbb{I}, \quad \mathbb{I} \otimes \tau = \tau \otimes \mathbb{I} = \tau, \quad \tau \otimes \tau = \mathbb{I} \oplus \tau. \quad (1)$$

Following these multiplication rules, $H^\dagger \phi_4^* H^T H$ quartic coupling is not allowed by the FFR even though it is allowed by the $SU(2)_L \otimes U(1)_Y$. In details, we will discuss in the subsection of Higgs potential. We define that neutral components of H and ϕ_4 have nonzero VEVs, which are, respectively, denoted by $v_H/\sqrt{2}$ and $v_4/\sqrt{2}$. Then, VEVs are constrained by the following ρ parameter at tree level [35,36]:

$$\rho = \frac{v_H^2 + 7v_4^2}{v_H^2 + v_4^2}, \quad (2)$$

where the experimental bound should be $\rho = 1.00038 \pm 0.00020$ at 1σ confidence level [37]. Combining the

TABLE I. Contents of fermion and scalar fields and their charge assignments under $SU(2)_L \times U(1)_Y$ and Fibonacci fusion rule (FFR).

	Lepton fields			Scalar fields	
	L_L	e_R	Σ_R	H	ϕ_4
$SU(2)_L$	2	1	5	2	4
$U(1)_Y$	$-\frac{1}{2}$	-1	0	$\frac{1}{2}$	$\frac{1}{2}$
FFR	τ	τ	τ	$\mathbb{I}$	τ

¹The other phenomenological applications are discussed in the following references: quark–lepton texture analyses [17–21], strong CP physics [22–24], dark matter models [7], suppression of flavor changing neutral currents [25], realization of family-independent matter symmetries [26], and generalized CP constructions [27].

²The other intriguing neutrino mass ideas are found in Refs. [30–34], introducing such multiplet particles.

deviation of ρ with $v_{\text{SM}} = \sqrt{v_H^2 + v_4^2} \simeq 246$ GeV, we find the upper bound as follows:

$$v_4 \lesssim 2.8 \text{ GeV}, \quad (3)$$

where we allow the deviation up to 2σ and $v_H \approx 245.984$ GeV. The relevant Lagrangian under these symmetries is given by

$$\begin{aligned} -\mathcal{L}_Y = & (y_\ell)_{ii} \overline{L_{L_i}} H e_{R_i} + (y_\nu)_{ij} [\overline{L_{L_i}} (i\tau_2) \phi_4^* \Sigma_{R_j}] \\ & + (M_R)_i [\overline{\Sigma_{R_i}} \Sigma_{R_i}] + \text{H.c.}, \end{aligned} \quad (4)$$

where $i = 1-3$, $j = 1-3$, τ_2 is the second Pauli matrix, and y_ℓ and M_R can be taken to be diagonal without loss of generality. y_ℓ generates the SM charged-lepton mass eigenvalues $m_\ell \equiv y_\ell v_H / \sqrt{2}$ after the electroweak symmetry breaking. Note that inside bracket “ $[\]$,” the $SU(2)_L$ indices are contracted so that it makes singlet where we omit details here. We work on the basis that all the coefficients are real and positive for simplicity.

A. Fermion quintet

The quintet Majorana fermions is defined as follows:

$$\Sigma_R \equiv [\Sigma_1^{++}, \Sigma_1^+, \Sigma^0, \Sigma_2^-, \Sigma_2^{--}]_R^T, \quad (5)$$

where the upper indices for components represent the electric charges, while the lower indices distinguish components with the same electric charge. Then, their masses are simply given by

$$\begin{aligned} M_R [\tilde{\Sigma}_R^c \Sigma_R] = & M_R [\tilde{\Sigma}_{1R}^{++c} \Sigma_{2R}^{--} + \tilde{\Sigma}_{1R}^{+c} \Sigma_{2R}^- + \tilde{\Sigma}_{2R}^{-c} \Sigma_{1R}^+ \\ & + \tilde{\Sigma}_{2R}^{--c} \Sigma_{1R}^{++} + \tilde{\Sigma}_R^{0c} \Sigma_R^0]. \end{aligned} \quad (6)$$

Thus, $\Sigma_1^{\pm(\pm\pm)}$ and $\Sigma_2^{\pm(\pm\pm)}$ are combined to make singly (doubly) charged Dirac fermions, while Σ_R^0 remains as the Majorana fermions. Each of mass is almost degenerate.

B. Higgs potential

The scalar fields can be parametrized as

$$H = \begin{bmatrix} w^+ \\ \frac{v_H + h + iz}{\sqrt{2}} \end{bmatrix}, \quad \phi_4 = [\varphi^{++}, \varphi_2^+, \varphi^0, \varphi_1^-]^T, \quad (7)$$

where the upper indices of each component represents the electric charges, while the lower indices distinguish components with same electric charge, and $\varphi^0 \equiv \frac{v_4 + \varphi_R + i\varphi_I}{\sqrt{2}}$. Then, the renormalizable potential $\mathcal{V} = \mathcal{V}_2 + \mathcal{V}_4$ under our symmetries is given by

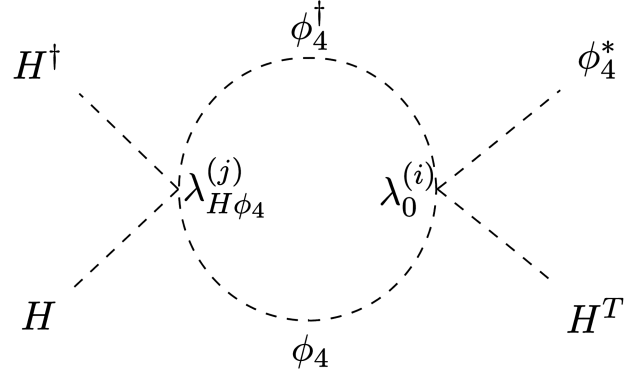


FIG. 1. One-loop diagram of the quartic term $\delta\lambda_{ij}$.

$$\mathcal{V}_2 = -\mu_H^2 H^\dagger H + \mu_4^2 \phi_4^\dagger \phi_4, \quad (8)$$

$$\mathcal{V}_4 = \lambda_0^{(i)} [\phi_4^\dagger H \phi_4^\dagger \phi_4]_i + \lambda_{H\phi_4}^{(i)} [H^\dagger H \phi_4^\dagger \phi_4]_i + \text{c.c.} \quad (9)$$

$$+ \lambda_H |H|^4 + \lambda_{\phi_4}^{(i)} [\phi_4^\dagger \phi_4 \phi_4^\dagger \phi_4]_i, \quad (10)$$

where $i = 1, 2$, $\tilde{\Phi} \equiv i\tau_2 \Phi^*$, and we consider $\mathcal{V}_2$ almost corresponds to mass eigenvalues for each component. Note here that we assume $0 < \mu_H^2$ and $0 < \mu_4^2$. The positivity of both $0 < \mu_H^2$ and $0 < \mu_4^2$ is crucial for realizing the radiative origin of v_4 . In the absence of the loop-induced quartic interaction $[H^\dagger \phi_4^* H^T H]_{ij}$, the quartet scalar does not develop a VEV at tree level. Therefore, the electroweak symmetry breaking in the ϕ_4 sector is triggered solely by the radiatively generated interaction shown in Fig. 1. Here, we stress to mention that $\delta\lambda_{ij} [H^\dagger \phi_4^* H^T H]_{ij}$ term is generated at one-loop level through $\lambda_0^{(i)} [\phi_4^\dagger H \phi_4^\dagger \phi_4]_i$ and $\lambda_{H\phi_4}^{(i)} [H^\dagger H \phi_4^\dagger \phi_4]_i$ in Eq. (9) as shown in Fig. 1. Clearly, this term dynamically violates FFR and $\delta\lambda$ is given by

$$\delta\lambda_{ij} \approx -\frac{\lambda_0^{(i)} \lambda_{H\phi_4}^{(j)}}{(4\pi)^2} \left[\frac{2 - 2r + (1+r) \ln(r)}{1-r} \right], \quad (11)$$

where $r \equiv m_H^2 / \Lambda^2$, Λ being our cutoff scale which we discussed in introduction. Then, $v_H \approx \mu_H / \sqrt{\lambda_H}$, and v_4 is approximately given by [38]

$$v_4 \sim v_H^3 \sum_{i,j=1}^2 \frac{\delta\lambda_{ij}}{2\mu_4^2 + (\lambda_{H\phi_4}^{(1)} + \lambda_{H\phi_4}^{(2)}) v_H^2} \quad (12)$$

$$\begin{aligned} = & -\frac{v_H^3}{(4\pi)^2} \left[\frac{2 - 2r + (1+r) \ln(r)}{1-r} \right] \\ & \times \sum_{i,j=1}^2 \frac{\lambda_0^{(i)} \lambda_{H\phi_4}^{(j)}}{2\mu_4^2 + (\lambda_{H\phi_4}^{(1)} + \lambda_{H\phi_4}^{(2)}) v_H^2} \end{aligned} \quad (13)$$

$$\simeq -\frac{v_H}{2(4\pi)^2\mu_4^2} \left[\frac{2-2r+(1+r)\ln(r)}{1-r} \right] \sum_{i,j=1}^2 \lambda_0^{(i)} \lambda_{H\phi_4}^{(j)}, \quad (14)$$

where $\lambda_{H\phi_4}^{(1)} + \lambda_{H\phi_4}^{(2)} \ll \mu_4^2/v_H^2$ in the last line. In the parameter region considered in this work, we require

$$0 < 2\mu_4^2 + (\lambda_{H\phi_4}^{(1)} + \lambda_{H\phi_4}^{(2)})v_H^2, \quad (15)$$

which guarantees the stability of v_4 direction and leads to the perturbative solution given in Eq. (12). Since $v_4 \ll v_H$, the loop-induced vacuum corresponds to a local minimum. A complete analysis of the global vacuum structure involving all quartic couplings is beyond the scope of the present work. Now we need to know the concrete energy scale of Λ ($r \equiv m_H^2/\Lambda^2$) in order to estimate the value of v_4 , and we have to evaluate the renormalization group evolution specifically on g_2 before discussing the neutrino mass matrix. Here, we briefly comment on the dependence of v_4 on the cutoff scale. For instance, when the cutoff is rescaled as $\Lambda \rightarrow 2\Lambda$, with, e.g., $\Lambda = 100$ TeV, the ratio $v_4(2\Lambda)/v_4(\Lambda)$ is approximately 1.22. Therefore, the sensitivity of v_4 to the cutoff scale is negligibly small.

C. Beta function of g_2

Here we discuss running of the $SU(2)_L$ gauge coupling g_2 in the presence of new particles Σ_R and ϕ_4 .³ The new beta function of g_2 for three $SU(2)_L$ quintet fermions (Σ_R) and a quartet boson (Φ_4) is given by

$$\Delta b_g^{\Sigma_R} = 3 \times \frac{20}{3}, \quad \Delta b_g^{\Phi_4} = \frac{5}{3}. \quad (16)$$

Then, the energy evolution of the gauge coupling g_2 as [39]

$$\begin{aligned} \frac{1}{g_2^2(\mu)} &= \frac{1}{g_2^2(m_{\text{in}})} - \frac{b_g^{\text{SM}}}{(4\pi)^2} \ln \left[\frac{\mu^2}{m_{\text{in}}^2} \right] \\ &\quad - \theta(\mu - m_{\text{th}}) \frac{\Delta b_g^{\Sigma_R}}{(4\pi)^2} \ln \left[\frac{\mu^2}{m_{\text{th}}^2} \right] \\ &\quad - \theta(\mu - m_{\text{th}}) \frac{\Delta b_g^{\Phi_4}}{(4\pi)^2} \ln \left[\frac{\mu^2}{m_{\text{th}}^2} \right], \end{aligned} \quad (17)$$

where μ is a reference energy, $b_g^{\text{SM}} = -19/6$, and we suppose to be $m_{\text{in}} (= m_Z) < m_{\text{th}} = 1000$ GeV, being, respectively, threshold masses of Σ_R and ϕ_4 for m_{th} . The resultant energy flow of $g_2(\mu)$ is given by the Fig. 2, where a dashed (dotted) horizontal line represents $g_2 = \sqrt{4\pi}(1)$. This figure shows that g_2 is relevant up to the mass scale $\mu \simeq 2.56 \times 10^7 (5.05 \times 10^5)$ GeV when g_2 should be within

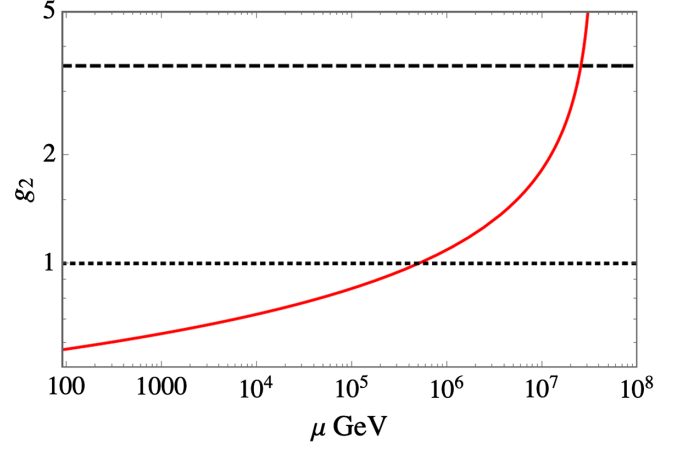


FIG. 2. The running of g_2 in terms of a reference energy of μ , where dashed (dotted) horizontal line represents $g_2 = \sqrt{4\pi}(1)$.

$\sqrt{4\pi}(1)$. Thus, we identify this upper mass scale of μ to be our cutoff scale Λ .

Note on RGE: Before proceeding with our discussion, it is important to address certain issues related to the renormalization group equations (RGEs) [40–42]. When a large number of $SU(2)_L$ multiplets are introduced in a model, one might naively expect that the cutoff scale is determined solely by the divergence of the gauge coupling g_2 , since it typically runs most rapidly toward strong coupling. However, this is not always the case. In particular, the self-quartic coupling of the multiplet boson (ϕ_4 in our case), denoted $\lambda_{\phi_4}^{(i)}$ ($i = 1, 2$), may diverge at an energy scale lower than or comparable to the divergence scale of g_2 . To ensure that the physical cutoff Λ is uniquely and consistently determined by the $SU(2)_L$ gauge sector, the initial values of the scalar quartic couplings at the electro-weak scale must be sufficiently suppressed, e.g., $\lambda_{\phi_4}^{(i)}(\mu)/g_2^2(\mu) \ll 1$ ($i = 1, 2$). This choice prevents the scalar sector from reaching a Landau pole prematurely, allowing the theory to remain under perturbative control until it encounters the gauge-induced singularity. Moreover, the neutrino Yukawa coupling y_ν can also influence the RGE flow of λ_4 if y_ν is taken too large. We follow the requirement $y_\nu \lesssim 0.2$ to minimize the Yukawa contribution to the RGEs. In this case, the contribution of y_ν to the running of λ_4 remains below 10% of the gauge contribution from g_2 , and the system remains under control. Before closing this sub section, let us comment on the stability of the positive mass parameter μ_4^2 under the RGE evolution. Since the neutrino Yukawa couplings are typically of order 10^{-3} (as can be seen later) in our framework, their contribution to the running of μ_4^2 is expected to be highly suppressed. Furthermore, the dominant one-loop corrections are proportional to μ_4^2 itself, implying that a positive initial value of μ_4^2 is not expected to be driven negative over the energy range of interest.

³Note here that $U(1)_Y$ gauge coupling g_Y runs far beyond the Planck scale without divergence.

Therefore, the radiative generation mechanism of v_4 remains consistent within the parameter region considered in this work. A full analysis of the RGE behaviors for all couplings lies beyond the scope of our work, but the above estimates suffice to determine the cutoff scale relevant to our discussion.

D. Neutrino mass matrix

The neutrino mass matrix is given through $(y_\nu)_{ij}[\overline{L_{Li}}(i\tau_2)\phi_4^*\Sigma_{Rj}]$ and $M_R[\overline{\Sigma_R^C}\Sigma_R]$ with the seesaw mechanism. The relevant component is found as

$$-\mathcal{L} \supset \frac{(y_\nu)_{ia}}{\sqrt{2}} \bar{\nu}_{Li} \Sigma_{Ra}^0 \phi^{0*} + M_{Ra} \overline{\Sigma_{Ra}^C} \Sigma_{Ra} + \text{H.c.} \quad (18)$$

After spontaneous symmetry breaking of ϕ_4 , the active neutrino mass matrix m_ν is given by

$$(m_\nu)_{ij} = v_4^2 \sum_{a=1}^3 (y_\nu)_{ia} M_{Ra}^{-1} (y_\nu^T)_{aj}. \quad (19)$$

Then, the mass matrix m_ν is diagonalized by an observed unitary mixing matrix V [43] as $D_\nu (\equiv \text{diag}[D_1, D_2, D_3]) = V^\dagger m_\nu V^*$, where we adopt V for the standard parametrization.

$$V = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha/2} & 0 \\ 0 & 0 & e^{i\beta/2} \end{bmatrix}, \quad (20)$$

where δ is Dirac CP phase, and α, β are Majorana phases. Two mass squared differences, three mixing angles, and δ are experimentally obtained in Nufit 6.1 [44]. Then, we can rewrite the Yukawa coupling y_ν in terms of observed values and some of the free parameters, so-called Casas-Ibarra parametrization [45], as follows:

$$y_\nu = \frac{1}{v_4} V \sqrt{D_\nu} O \sqrt{M_R}, \quad (21)$$

where O is an arbitrary complex orthogonal matrix with three degrees of freedom; $OO^T = O^T O = 1_{3 \times 3}$. Here we estimate v_4 . When $\Lambda \sim 2.56 \times 10^7 (5.05 \times 10^5)$ GeV, v_4 is given by

$$\Lambda \sim 2.56 \times 10^7 \text{ GeV: } v_4 \sim 11.23 \frac{v_H}{(4\pi)^2} \frac{v_H^2}{\mu_4^2} \sum_{i,j=1}^2 \lambda_0^{(i)} \lambda_{H\phi_4}^{(j)}, \quad (22)$$

$$\Lambda \sim 5.05 \times 10^5 \text{ GeV: } v_4 \sim 7.30 \frac{v_H}{(4\pi)^2} \frac{v_H^2}{\mu_4^2} \sum_{i,j=1}^2 \lambda_0^{(i)} \lambda_{H\phi_4}^{(j)}. \quad (23)$$

Furthermore, if $\sum_{i,j=1}^2 \lambda_0^{(i)} \lambda_{H\phi_4}^{(j)} \sim 0.1$ and $\mu_4 \sim 1000$ GeV, one obtains the following v_4 :

$$\Lambda \sim 2.56 \times 10^7 \text{ GeV: } v_4 \sim 0.1058 \text{ GeV}, \quad (24)$$

$$\Lambda \sim 5.05 \times 10^5 \text{ GeV: } v_4 \sim 0.0688 \text{ GeV}. \quad (25)$$

Then, y_ν can be estimated under the assumption $|V| \approx |O| = 1$ and $D_\nu \sim 100$ meV, $M_R \sim 1000$ GeV, and we obtain the following coupling:

$$\Lambda \sim 2.56 \times 10^7 \text{ GeV: } y_\nu \sim 3 \times 10^{-3}, \quad (26)$$

$$\Lambda \sim 5.05 \times 10^5 \text{ GeV: } y_\nu \sim 4.6 \times 10^{-3}. \quad (27)$$

The typical seesaw model requires $y_\nu \sim 10^{-6}$ under the same condition; our scenario is relaxed by 3 orders of magnitude!

III. CONCLUSIONS AND DISCUSSIONS

In this paper, we have proposed our framework as an effective field theory valid below the cutoff scale Λ to explain the tiny scale of neutrino masses by integrating noninvertible symmetries with the dynamical determination of the cutoff scale. Our model introduces three families of $SU(2)_L$ quintet fermions (Σ_R) and a quartet scalar (ϕ_4), where neutrino masses are generated via a seesaw mechanism mediated by the suppressed vacuum expectation value v_4 of the quartet. The core achievement of this work is the resolution of the arbitrariness of the cutoff scale Λ in loop-induced VEV models. We have demonstrated that the introduction of higher $SU(2)_L$ multiplets alters the renormalization evolution of the $SU(2)_L$ gauge coupling g_2 , causing it to encounter a Landau pole at high energies. By identifying the scale where g_2 reaches the strong coupling regime as the physical cutoff Λ , we eliminate the need to introduce this scale by hand. Our analysis shows that Λ is naturally determined to be in the range of approximately 10^5 to 10^7 GeV. Quantitatively, this framework successfully explains the neutrino mass hierarchy

without extreme parameter tuning. Under the Fibonacci fusion rule, the v_4 is induced at the one-loop level. For a cutoff scale $\Lambda = 10^5\text{--}10^7$ GeV, we obtained $v_4 = 0.07\text{--}0.1$ GeV. Consequently, the required neutrino Yukawa couplings are on the order of 10^{-3} , which is significantly larger and thus more natural than the 10^{-6} to 10^{-10} typically required in the standard seesaw or Type-II models. In conclusion, our model provides a structural explanation for the smallness of neutrino masses by linking the internal dynamics of gauge coupling evolution to the generation of loop-induced scales. This approach maintains a relatively simple particle content without requiring additional gauge bosons for symmetry breaking.

Before closing our paper, it is worthwhile to mention a brief comment on collider phenomenology. The model predicts exotic charged states originating from the $SU(2)_L$ quintet fermions Σ_R and the quartet scalar ϕ_4 . Through the neutrino Yukawa interaction, the quintet fermions can decay into a lepton and a quartet scalar, for example,

$$\Sigma^{++} \rightarrow \ell^+(\varphi_1^*)^+, \quad \Sigma^+ \rightarrow \ell^+(\varphi^0)^*, \quad \Sigma^+ \rightarrow \nu(\varphi_1^*)^+. \quad (28)$$

The quartet scalars subsequently decay into electroweak gauge bosons through their small vacuum expectation value. Consequently, pair production of the quintet fermions and quartet scalars at hadron colliders may lead to distinctive multilepton signatures, including same-sign dilepton events accompanied by multiple gauge bosons. A dedicated collider study is beyond the scope of the present work and will be presented elsewhere.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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