

Constrain the $\chi_{cJ} \rightarrow D^{(*)}\bar{D}^{(*)}$ effective couplings via the $X(3872) \rightarrow \pi^0\chi_{cJ}$ decays

Zhao-Sai Jia,^{1,2} Gang Li^{1,*} and Zhen-Hua Zhang^{3,†}

¹College of Physics and Engineering, Qufu Normal University, Qufu 273165, China

²CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

³Center for High Energy Physics, Peking University, Beijing 100871, China



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Hidden-charm decays provide irreplaceable platforms for probing the structures of charmoniumlike states, such as $X(3872)$, $Y(4260)$, and $Z_c(3900)$, and their heavy-quark-symmetry partners. In the hadronic molecular scenario, these hidden-charm decays are dominated by intermediate meson loops (IMLs), and the couplings of $\chi_{cJ} \rightarrow D^{(*)}\bar{D}^{(*)}$ are essential building blocks of the amplitudes for the pionic and radiative transitions of the charmoniumlike states to the χ_{cJ} and h_c states, e.g., $X(3872) \rightarrow \pi^0\chi_{cJ}, \pi\pi\chi_{cJ}, \gamma\chi_{cJ}$, and $Y(4260) \rightarrow \pi^0h_c, \eta h_c$. These couplings cannot be extracted from the partial decay widths of the χ_{cJ} directly and only have estimated values from the vector meson dominance (VMD) model. Utilizing the recent precise determination of the pole position and the isospin breaking properties of the $X(3872)$, we give an estimation on the upper bounds of the absolute values of the $\chi_{cJ} \rightarrow D^{(*)}\bar{D}^{(*)}$ couplings. Our results show that the VMD model may overestimate the $\chi_{cJ} \rightarrow D^{(*)}\bar{D}^{(*)}$ couplings considering the $X(3872)$ as a $D\bar{D}^*$ hadronic molecule with a binding energy about tens of keV. These upper limits can be used and tested in other hidden-charm transitions of the charmoniumlike states to the χ_{cJ} and h_c .

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I. INTRODUCTION

Over the past two decades, numerous candidates of exotic hadrons beyond the meson and baryon configurations in the conventional quark model have been observed in the charmonium energy region. These states are referred to as charmoniumlike states or XYZ states. The most well-known charmoniumlike states are, for example, $X(3872)$, $Y(4260)$, and $Z_c(3900)$. Extensive efforts, both experimental and theoretical, have been made to explore the internal structures of the XYZ states (for recent reviews, see Refs. [1–11]). In these studies, the XYZ states are interpreted as conventional charmonium states, compact tetraquarks, kinematic singularities, and hadronic molecules. Decays of XYZ states to χ_{cJ} and h_c states with the emission of pions or photons have been widely proposed and studied as crucial probes to distinguish these different interpretations [12–26]. For different configurations of the charmoniumlike states, these decays can involve different

mechanisms, resulting in significantly different decay widths. In the hadronic molecular scenario, the hidden-charm transitions of the XYZ states proceed predominantly through intermediate meson loops (IMLs) of open-charm particles [3,27]. In this framework, the couplings of the final-state charmonium to charmed meson pairs [g_1 for $\chi_{cJ}/h_c \rightarrow D^{(*)}\bar{D}^{(*)}$] are essential components of the decay amplitude. However, the coupling g_1 cannot be directly extracted from their partial widths and only have an estimated value from the vector meson dominance (VMD) model [28,29], $g_1^2 \approx m_{\chi_{c0}}/(6f_{\chi_{c0}}^2)$, where $m_{\chi_{c0}}$ is the mass of χ_{c0} , and the value of the χ_{c0} decay constant $f_{\chi_{c0}}$ is calculated to be about 300–500 MeV in the QCD sum rules [30–32]. Such estimation could bring large uncertainties in the partial decay widths of the XYZ states [13,15]. Recently, the properties of the $X(3872)$ states, including its pole positions, compositeness, and isospin breaking effects, have been determined precisely in Refs. [33,34] from the newly updated experimental data from the BESIII [35] and LHCb [36,37] Collaborations. This offers great opportunities to put limitations on the g_1^2 from the $X(3872) \rightarrow \pi^0\chi_{cJ}$ isospin-breaking decays.

The $X(3872)$, also known as $\chi_{c1}(3872)$ [38], was first discovered in 2003 by the Belle Collaboration [39] in the $\pi^+\pi^-J/\psi$ invariant mass spectrum from $B^\pm \rightarrow (X(3872) \rightarrow \pi^+\pi^-J/\psi)K^\pm$ produced in the e^+e^- collisions, with a mass

*Contact author: gli@qfnu.edu.cn

†Contact author: zhhzhang@pku.edu.cn

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of $3872.0 \pm 0.6 \pm 0.5$ MeV, a width less than 2.3 MeV, and quantum numbers $I^G(J^{PC}) = 0^+(1^{++})$ [38,40–43]. The most striking characteristic of the $X(3872)$ is that its mass coincides exactly with the $D^0\bar{D}^{*0}$ threshold, with a difference (hereinafter called binding energy) $BE_n = m_{D^0} + m_{D^{*0}} - m_{X(3872)} = (0.01 \pm 0.14)$ MeV [44] and (0.12 ± 0.13) MeV [36], where $m_{D^0} = (1864.84 \pm 0.05)$ MeV, $m_{D^{*0}} = (2006.85 \pm 0.15)$ MeV [45]. The newly updated study [34] determined the pole position of the $X(3872)$ to be $E_X = -53^{+9}_{-24} - i34^{+2}_{-12}$ keV relative to the $D^0\bar{D}^{*0}$ threshold. Despite the closeness to the threshold, the $X(3872)$ exhibits strong coupling to the $D^0\bar{D}^{*0}$ channel, as evidenced by the large branching fraction to the $D^0\bar{D}^{*0}$ and $D^0\bar{D}^{*0}\pi^0$ channels [38]. Because of these properties, the $X(3872)$ is widely hypothesized to be a $D\bar{D}^*$ hadronic molecule. The molecular component of the $X(3872)$ can be quantified by its compositeness (see Ref. [46] for a recent review). In a newly updated detailed study [34], the compositeness of the $X(3872)$ is determined to be $\tilde{X}_A > 0.99$, strongly support the dominance of the molecular component.

Another important feature of the $X(3872)$ is the significant isospin-breaking in its decays, such as $X(3872) \rightarrow \pi^+\pi^-J/\psi$, $X(3872) \rightarrow \pi^0\chi_{cJ}$, and $X(3872) \rightarrow \pi^0\pi^+\pi^-$ [12,13,15–20,33,37,47–61]. The isospin-breaking decay $X(3872) \rightarrow \pi^+\pi^-J/\psi$, which proceeds predominantly via the $\rho^0 J/\psi$ intermediate state, has a branching ratio comparable to that of the isospin-conserving decay $X(3872) \rightarrow \omega J/\psi$,

$$\frac{\text{Br}(X(3872) \rightarrow \omega J/\psi)}{\text{Br}(X(3872) \rightarrow \pi^+\pi^-J/\psi)} = 1.1 \pm 0.4. \quad (1)$$

Given the limited phase space for $X(3872) \rightarrow \omega J/\psi$ decay, the latest result from the LHCb Collaboration for the amplitude ratio R_X of $X(3872) \rightarrow \rho^0 J/\psi$ and $X(3872) \rightarrow \omega J/\psi$ [37] is

$$R_X = \left| \frac{\mathcal{M}(X(3872) \rightarrow \rho^0 J/\psi)}{\mathcal{M}(X(3872) \rightarrow \omega J/\psi)} \right| = 0.29 \pm 0.04, \quad (2)$$

which is about 1 order of magnitude larger than that of conventional charmonium. The value of R_X is recently determined more precisely to be $R_X = 0.28 \pm 0.04$ [34] and $R_X = 0.26 \pm 0.03$ [12] using dispersive formalisms with and without the contribution of isovector W_{c1}^0 [62]. Such a significant isospin-breaking effect may originate from the mass discrepancy between charged and neutral charm mesons. Studies [63,64] have highlighted the necessity of incorporating charged D mesons in the calculations of decays $X(3872) \rightarrow \gamma J/\psi$, $\pi^+\pi^-J/\psi$, and $\pi^+\pi^-\pi^0 J/\psi$.

Prior to the experimental study from the BESIII [16] and Belle [17] Collaborations, the $X(3872) \rightarrow \pi^0\chi_{cJ}$ decay had been proposed in Refs. [12,13,15] assuming the $X(3872)$

being a charmonium or molecular state. Subsequently, this decay mode was further investigated in various theoretical frameworks in, e.g., Refs. [18–20,22]. In Ref. [13], the $X(3872)$ was interpreted as a loosely bound state of neutral charmed mesons $D^0\bar{D}^{*0}$, and its decay into $\pi^0\chi_{cJ}$ mainly proceeds via a neutral charmed meson loop. The studies in Ref. [15] pointed out that the D^+D^{*-} components in the $X(3872)$ and thus, the charged meson loop also plays an important role in these isospin-breaking decays; otherwise, the partial width of $X(3872) \rightarrow \pi^0\chi_{cJ}$ would exceed the total width of the $X(3872)$ [38]. Therefore, the partial width of $X(3872) \rightarrow \pi^0\chi_{cJ}$ are sensitive to the relative amount of the $D^0\bar{D}^{*0}$ and D^+D^{*-} components in the $X(3872)$ wave function, which can be quantified by the mixing angle θ [65],

$$|X(3872)\rangle = \frac{1}{\sqrt{2}} \cos \theta (|D^0\bar{D}^{*0}\rangle + |D^{*0}\bar{D}^0\rangle) + \frac{1}{\sqrt{2}} \sin \theta (|D^+D^{*-}\rangle + |D^-D^{*+}\rangle), \quad (3)$$

where the charge conjugation conventions $D^{*C} \xrightarrow{C} \bar{D}^*$ and $D^C \xrightarrow{C} \bar{D}$ are implemented. The mixing angle θ can be determined from the isospin-breaking decay properties of the $X(3872)$ [62]. Due to the lack of precise experimental data, the mixing angle was estimated to be $\theta = 0.37 \pm 0.04$ through the maximum cancellation between the charged and neutral meson loops in Ref. [15]. The partial widths for different mixing angles of the isospin-breaking decays $X(3872) \rightarrow \pi^+\pi^-J/\psi$ and $X(3872) \rightarrow \pi^0\chi_{cJ}$ were also investigated in Ref. [19]. All of these works utilized the VMD model-estimated value of the $\chi_{cJ} \rightarrow D^{(*)}\bar{D}^{(*)}$ coupling g_1 in the IMLs, which brings further uncertainties to the final results. With the input from the precisely determined isospin-breaking amplitude ratio R_X , the mixing angle θ can be settled specifically, and the upper limit of the branching ratio of the $X(3872) \rightarrow \pi^0\chi_{cJ}$ can be used to put an upper bound on the g_1^2 .

In this work, we take the decay $X(3872) \rightarrow \pi^0\chi_{c0}(1P)$ serves as a pivotal probe to investigate the coupling between χ_{cJ} and charmed mesons. We first utilize the R_X extracted from the LHCb and BESIII data [34] to determine the proportion of neutral and charged constituents in $X(3872)$, and then put an upper limit on g_1^2 through the branching ratio of the $X(3872) \rightarrow \pi^0\chi_{c0}(1P)$ [38] decay. Finally, for a consistency check, the upper limit of g_1^2 determined from the $X(3872) \rightarrow \pi^0\chi_{c0}(1P)$ decay is applied to the $X(3872) \rightarrow \pi^0\chi_{c1,2}(1P)$ decays to give the upper bounds of their partial widths.

The structure of the paper is as follows. In Sec. II, we introduce the effective Lagrangians and Feynman diagrams for $X(3872) \rightarrow \pi^0\chi_{cJ}$. The numerical results of the upper bounds of g_1^2 and the $X(3872) \rightarrow \pi^0\chi_{c1,2}(1P)$ branching

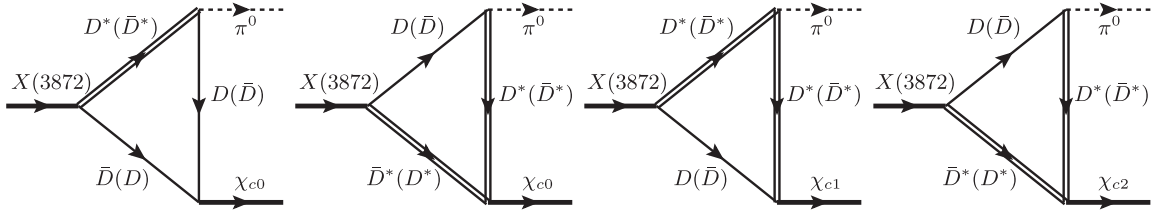


FIG. 1. Feynman diagrams for $X(3872) \rightarrow \pi^0 \chi_{cJ}$. The double lines represent vector mesons (D^* , $\bar{D}^*$), the single solid lines represent scalar mesons (D , $\bar{D}$), and the dashed lines represent the scalar meson π^0 .

ratios are presented in Sec. III, and a brief summary is given in Sec. IV.

II. FRAMEWORK

The Feynman diagrams for $X(3872) \rightarrow \pi^0 \chi_{cJ}(1P)$ are shown in Fig. 1. We assume the $X(3872)$ is an S -wave molecular state with $J^{PC} = 1^{++}$, represented as the superposition $D^0 \bar{D}^{*0} + \text{c.c}$ and $D^- \bar{D}^{*-} + \text{c.c}$ given in Eq. (3). Correspondingly, the effective Lagrangian describing the coupling of the $X(3872)$ to $D^* \bar{D}$ can be written as

$$\mathcal{L}_{XDD^*} = \frac{g_n}{\sqrt{2}} X^{i\dagger} (D^{*0i} \bar{D}^0 + D^0 \bar{D}^{*0i}) + \frac{g_c}{\sqrt{2}} X^{i\dagger} (D^{*+i} D^- + D^+ D^{*-i}) + \text{H.c.} \quad (4)$$

The coupling constants of the $X(3872)$ to neutral and charged charmed mesons, g_n and g_c , can be extracted from the residues of the $D^0 \bar{D}^{*0} - D^+ \bar{D}^{*-}$ coupled-channel scattering T matrix at the $X(3872)$ pole position [66] as

$$g_n = 4\sqrt{\frac{\pi\gamma_n}{\mu_n^2}} \cos\theta, \quad g_c = 4\sqrt{\frac{\pi\gamma_c}{\mu_c^2}} \sin\theta, \quad (5)$$

where $\gamma_n = \sqrt{2\mu_n BE_n}$, $\gamma_c = \sqrt{2\mu_c BE_c}$ with $BE_n = m_{D^{*0}} + m_{D^0} - m_{X(3872)}$, $BE_c = m_{D^{*-}} + m_{D^-} - m_{X(3872)} = \Delta + BE_n$ being the binding energies of the $X(3872)$ relative to the thresholds of the neutral and charged channels, respectively. Here, $\Delta = m_{D^{*+}} + m_{D^-} - m_{D^{*0}} - m_{D^0}$ is the gap between the thresholds of $D^0 \bar{D}^{*0}$ and $D^+ \bar{D}^{*-}$, and μ_n and μ_c are the reduced masses of $D^0 \bar{D}^{*0}$ and $D^+ \bar{D}^{*-}$, respectively. These coupling constants can be related to the amplitude ratio R_X for $X(3872) \rightarrow \rho^0 J/\psi$ and $X(3872) \rightarrow \omega J/\psi$ by [37,62],

$$R_X = \left| \frac{1 - R_{c/n}}{1 + R_{c/n}} \right|, \quad R_{c/n} \equiv \frac{g_c}{g_n} = \left(\frac{\mu_n}{\mu_c} \right)^{\frac{3}{4}} \left(1 + \frac{\Delta}{BE_n} \right)^{\frac{1}{4}} \tan\theta. \quad (6)$$

Therefore, R_X can be used as an input to constrain the mixing angle θ . There are two solutions of $R_{c/n}$, $R_{c/n} = 0.56 \pm 0.05$, and $R_{c/n} = 1.78^{+0.16}_{-0.15}$. The later one means the coupling of the $X(3872)$ to $D^+ \bar{D}^{*-}$ is stronger

than to $D^0 \bar{D}^{*0}$. However, this scenario is not supported by experimental evidence, which indicates the opposite hierarchy. Therefore, the second solution should be excluded.

The leading-order effective Lagrangian for the $D^{(*)} D^* \pi$ coupling is given by the heavy hadron chiral perturbation theory (HH χ PT) as [13,15]

$$\mathcal{L}_{D^{(*)} D^* \pi} = -g \text{Tr}[H_a^\dagger H_b \vec{\sigma} \cdot \vec{A}_{ba}] + g \text{Tr}[\bar{H}_a^\dagger \vec{\sigma} \cdot \vec{A}_{ab} \bar{H}_b], \quad (7)$$

where $\vec{\sigma}$ denote the Pauli matrices and a is the light flavor index, the charmed mesons are given by the two-component notation [67] as $H_a = \vec{V}_a \cdot \vec{\sigma} + P_a$ with $P = (D^-, \bar{D}^0)$ and $V = (D^{*-}, \bar{D}^{*0})$ being the pseudoscalar and vector heavy mesons, respectively, and the field for the antimesons is $\bar{H}_a = -\vec{V}_a \cdot \vec{\sigma} + \bar{P}_a$ with $\bar{P} = (D^+, D^0)$ and $\bar{V} = (D^{*+}, D^{*0})$. The field $\vec{A}_{ab} = -\vec{\nabla} \Phi_{ab} / f_\pi + \dots$ is the axial current in the chiral perturbation theory (χ PT) and couples to the heavy mesons with the axial coupling $g = 0.54$ [15], where $f_\pi = 130$ MeV is the pion decay constant, and the Φ field contains the Goldstone bosons as components,

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 \end{pmatrix}. \quad (8)$$

The Lagrangian couples the χ_{cJ} to the $D^{(*)}$ mesons reads

$$\mathcal{L}_{\chi D^{(*)} D^{(*)}} = i \frac{g_1}{2} \text{Tr}[\chi^{i\dagger} H_a \sigma^i \bar{H}_a] + \text{H.c.}, \quad (9)$$

where the χ_{cJ} field is expressed as [13]

$$\chi^i = \sigma^j \chi^{ij} = \sigma^j \left(\chi_{c2}^{ij} + \frac{1}{\sqrt{2}} \epsilon^{ijk} \chi_{c1}^k + \frac{\delta^{ij}}{\sqrt{3}} \chi_{c0} \right) + h_c^i, \quad (10)$$

with the unknown coupling of the χ_{cJ} to charmed mesons g_1 . In Ref. [15], the coupling constant was estimated in the VMD model as $g_1^2 \approx m_{\chi_{c0}} / (6f_{\chi_{c0}}^2) = 1/(457 \text{ MeV})$ with $f_{\chi_{c0}} = 510$ MeV from the QCD sum rule calculation [30]. In this study, we will first determine the range of the mixing angle θ using the input value $R_X = 0.28(4)$ from Ref. [34] and then utilize the upper bounds $\Gamma(X(3872) \rightarrow \pi^0 \chi_{c0}(1P)) / \Gamma_{X(3872)} < 16\%$ [38] and $\Gamma_{X(3872)} \lesssim 131 \text{ keV}$ [15]

TABLE I. All the possible combinations of the intermediate charmed mesons for the diagrams in Fig. 1. The first particle in each square bracket denotes top left intermediate charmed meson in the corresponding diagram, and the other intermediate charmed mesons in the same diagram are listed in the square bracket in counterclockwise order along the loop.

$X(3872) \rightarrow \pi^0 \chi_{c0}$	$[D^{*0}, \bar{D}^0, D^0], [\bar{D}^{*0}, D^0, \bar{D}^0],$ $[D^{*+}, D^-, D^+], [D^{*-}, D^+, D^-],$ $[\bar{D}^0, D^{*0}, \bar{D}^{*0}], [D^0, \bar{D}^{*0}, D^{*0}],$ $[D^-, D^{*+}, D^{*-}], [D^+, D^{*-}, D^{*+}]$
$X(3872) \rightarrow \pi^0 \chi_{c1}$	$[D^{*0}, \bar{D}^0, D^{*0}], [\bar{D}^{*0}, D^0, \bar{D}^{*0}],$ $[D^{*+}, D^-, D^{*+}], [D^{*-}, D^+, D^{*-}]$
$X(3872) \rightarrow \pi^0 \chi_{c2}$	$[\bar{D}^0, D^{*0}, \bar{D}^{*0}], [D^0, \bar{D}^{*0}, D^{*0}],$ $[D^-, D^{*+}, D^{*-}], [D^+, D^{*-}, D^{*+}]$

to constrain the coupling constant square g_1^2 . The upper bound of $\Gamma_{X(3872)}$ in Ref. [15] was derived from the theoretical calculation of $\Gamma(X(3872) \rightarrow D^0 \bar{D}^0 \pi^0)$ within the molecular framework, combined with the experimentally measured lower bound of the $X(3872) \rightarrow D^0 \bar{D}^0 \pi^0$

branching ratio [38]. Based on the Lagrangians given above, the loop transition amplitudes in Fig. 1 can be expressed in a general form as follows:

$$\mathcal{M}(X(3872) \rightarrow \pi \chi_{cJ}) = \sqrt{m_{X(3872)} m_{\chi_{cJ}}} m_1 m_2 m_3 V_1 V_2 V_3 \times I[m_1, m_2, m_3], \quad (11)$$

where V_l ($l = 1, 2, 3$) represent the vertexes of the charmed mesons in the loop coupling to the initial $X(3872)$, final χ_{cJ} and π^0 , respectively, and the prefactor $\sqrt{m_{X(3872)} m_{\chi_{cJ}}} m_1 m_2 m_3$ accounts for the relativistic normalization. The $I[m_1, m_2, m_3]$ represents the scalar 3-point integral, where m_k ($k = 1, 2, 3$) represents the mass of k th particle of each combination of the intermediate states in Table I for different processes. The first particle in each square bracket denotes the top left intermediate charmed meson in the corresponding triangle diagram, and the other intermediate charmed mesons in the same diagram are listed in the square bracket in counterclockwise order along the loop. The scalar three-point loop integral $I[m_1, m_2, m_3]$ is ultraviolet (UV) convergent and can be worked out as [27]

$$I[m_1, m_2, m_3] = i \int \frac{d^4 l}{(2\pi)^4} \frac{1}{[l^2 - m_1^2 + i\epsilon][(p-l)^2 - m_2^2 + i\epsilon][(l-q)^2 - m_3^2 + i\epsilon]} \simeq \frac{\mu_{12}\mu_{23}}{16\pi m_1 m_2 m_3 \sqrt{a}} \left[\tan^{-1} \left(\frac{c_2 - c_1}{2\sqrt{ac_1}} \right) + \tan^{-1} \left(\frac{2a + c_1 - c_2}{2\sqrt{a(c_2 - a)}} \right) \right], \quad (12)$$

where p , l , and q are the four-momenta of the initial $X(3872)$, the top left intermediate charmed meson in the triangle diagram, and the final-state π^0 in the $X(3872)$ rest frame, respectively, $\mu_{ij} = m_i m_j / (m_i + m_j)$ are the reduced masses, and

$$a = \left(\frac{\mu_{23}}{m_3} \right)^2 \bar{q}^2, \quad c_1 = 2\mu_{12} b_{12}, \quad c_2 = 2\mu_{23} b_{23} + \frac{\mu_{23}}{m_3} \bar{q}^2, \quad (13)$$

with $b_{12} = m_1 + m_2 - M$, $b_{23} = m_2 + m_3 + q^0 - M$, and the magnitude of three-momentum of the final state in the initial state rest frame,

$$|\vec{q}| = \frac{\sqrt{(m_{X(3872)} - (m_{\chi_{cJ}} + m_\pi)^2)(m_{X(3872)} - (m_{\chi_{cJ}} - m_\pi)^2)}}{2m_{X(3872)}}. \quad (14)$$

All the combinations in each diagram should be summed to get the final amplitude, and $X(3872) \rightarrow \pi^0 \chi_{cJ}$ decay

widths are given by

$$\Gamma(X(3872) \rightarrow \pi^0 \chi_{cJ}) = \frac{1}{2j+1} \frac{|\vec{q}|}{8\pi m_{X(3872)}^2} \sum_{\text{spins}} |\mathcal{M}|^2, \quad (15)$$

where $j = 1$ is the spin of the initial $X(3872)$, and there is a sum over all polarizations of the final-state particles.

III. NUMERICAL RESULTS

In this section, we present the allowed ranges of the mixing angle θ and coupling constant square g_1^2 . The angle θ characterizes the proportion of neutral and charged constituents within the $X(3872)$, while g_1^2 describes the interaction strength between χ_{cJ} and charmed mesons. These quantities are determined with the binding energy BE_n of the $X(3872)$ varying from 2 to 180 keV.

Figure 2(a) illustrates the dependence of the mixing angle θ on BE_n over the range [2, 180] keV. The band shows the uncertainty propagated from R_X . The central value corresponds to $R_X = 0.28$, and the upper and lower bounds correspond to $R_X = 0.24$ and $R_X = 0.32$, respectively. For a certain binding energy BE_n , the error of θ is

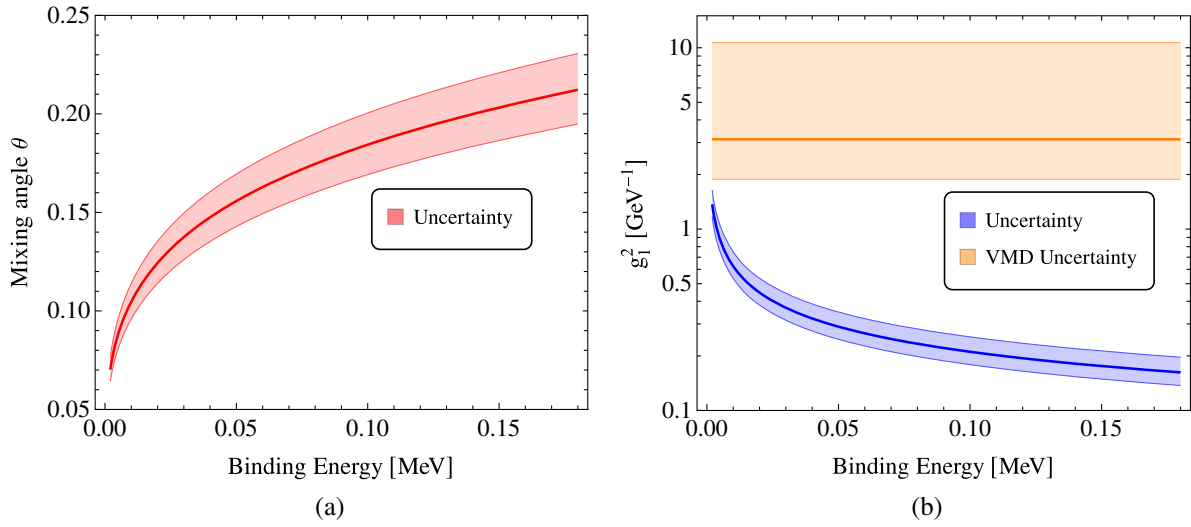


FIG. 2. The mixing angle θ describing the proportion of neutral and charged constituents in $X(3872)$ (a), and the upper limits of the coupling constant square g_1^2 of χ_{cJ} to charmed mesons (b) as a function of the binding energy BE_n . The red and blue bands show the uncertainties of θ and g_1^2 determined from the $X(3872)$ properties propagated from the uncertainty of R_X , and the orange band gives the uncertainty of g_1^2 from the VMD model propagated from the uncertainty of the χ_{c0} decay constant $f_{\chi_{c0}}$.

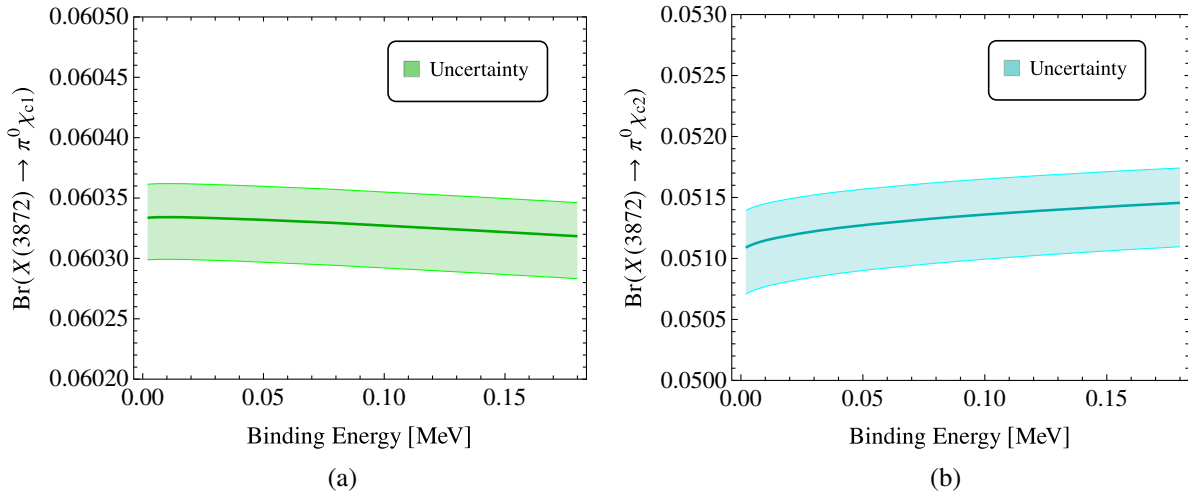


FIG. 3. Branching ratios of $X(3872) \rightarrow \pi^0 \chi_{c1}$ (a) and $X(3872) \rightarrow \pi^0 \chi_{c2}$ (b) as functions of the binding energy BE_n calculated by utilizing the g_1^2 determined from the $X(3872) \rightarrow \pi^0 \chi_{c0}$ decay. The uncertainties are propagated from that of R_X .

about $\mathcal{O}(0.1)$, much smaller than that in the previous effective field theory analysis [15,19], which is about $\mathcal{O}(1)$. As shown, θ increases with increasing BE_n . This trend implies a systematic rearrangement of the internal composition of $X(3872)$ as its binding energy changes. Nevertheless, the neutral constituent remains dominant across the explored range of $BE_n \in [2, 180]$ keV. Taking the central value of θ at $BE_n = 53$ keV as representative and incorporating uncertainties from both BE_n and R_X , we obtain the mixing angle,

$$\theta = 0.16^{+0.07}_{-0.09}. \quad (16)$$

whose upper and lower limits are determined at $R_X = 0.24$, $BE_n = 180$ keV, and $R_X = 0.32$, $BE_n = 2$ keV, respectively.

The variation of the upper limit of the coupling constant square g_1^2 determined from the upper bound of $\Gamma(X(3872) \rightarrow \pi^0 \chi_{c0})$ versus $BE_n \in [2, 180]$ keV is presented by the blue curve in Fig. 2(b). The band again shows the uncertainty propagated from R_X , with the central value of g_1^2 determined at $R_X = 0.28$. The upper and lower bounds of the band also correspond to $R_X = 0.24$ and $R_X = 0.32$, respectively. One can see that the g_1^2 decreases monotonically as BE_n increases. Taking the central value of g_1^2 to be that at $BE_n = 53$ keV and considering the uncertainties from BE_n and R_X , the upper

bound of g_1^2 derived from the $X(3872)$ partial width constraints is

$$g_1^2 \lesssim 0.28_{-0.14}^{+1.36} \text{ GeV}^{-1}, \quad (17)$$

where the upper and lower limits on the right-hand side are determined at $R_X = 0.24$, $BE_n = 2$ keV, and $R_X = 0.32$, $BE_n = 180$ keV, respectively.

The $g_1^2 \approx m_{\chi_{c0}}/(6f_{\chi_{c0}}^2)$ from the VMD model used in the previous effective field theory analysis [15,19] is shown by the orange line in Fig. 2(b), with $m_{\chi_{c0}} = 3414.71$ MeV [38]. The uncertainty shown by the orange band mainly comes from the uncertainty of $f_{\chi_{c0}}$ calculated from the QCD sum rules, which is also much larger than the uncertainty in our analysis for a certain binding energy BE_n . We have taken the average central value of $f_{\chi_{c0}} = 510 \pm 40$ MeV in Ref. [30] and $f_{\chi_{c0}} = 343 \pm 112$ MeV in Ref. [31] to evaluate the central value of g_1^2 . The VMD model estimation of the g_1^2 is

$$g_1^2(\text{VMD}) = 3.13_{-1.25}^{+7.54} \text{ GeV}^{-1}, \quad (18)$$

whose central value is 1 order of magnitude larger than that determined from the $X(3872)$ decay in Eq. (17). The lower limit of the g_1^2 from the VMD model is still larger than the upper limit of that from the $X(3872)$ decay. Therefore, the VMD model may over estimate the coupling of $\chi_{cJ} \rightarrow D^{(*)}\bar{D}^{(*)}$, at least for the $X(3872)$ being a $D\bar{D}^*$ bound state with its binding energy BE_n at tens of keV.

To check the validity of our result, we calculate the upper bounds of the branching ratios of the $X(3872) \rightarrow \pi^0\chi_{c1,2}$ decays using the g_1^2 determined from the $X(3872) \rightarrow \pi^0\chi_{c0}$ decay. These branching ratios are shown in Fig. 3 as functions of the $X(3872)$ binding energy BE_n . The upper bounds are $\text{Br}(X(3872) \rightarrow \pi^0\chi_{c1}) \lesssim 6\%$ and $\text{Br}(X(3872) \rightarrow \pi^0\chi_{c2}) \lesssim 5\%$, consistent with the experimental results [38].

IV. SUMMARY

In this work, we employed the amplitude ratio R_X of $X(3872) \rightarrow \rho^0 J/\psi$ and $X(3872) \rightarrow \omega J/\psi$ [37,62] as an input to determine the mixing angle θ of the $D^0\bar{D}^{*0}$ and D^+D^{*-} components in the $X(3872)$, where $X(3872)$ is assumed to be a $D\bar{D}^*$ hadronic molecule. For the $X(3872)$ binding energy $BE_n \in [2, 180]$ keV, we constrain $\theta = 0.16_{-0.09}^{+0.07}$, indicating a dominant $D^0\bar{D}^{*0}$ component in $X(3872)$. Further, we utilize the mixing angle and the upper limit of the branching fraction $\Gamma(X(3872) \rightarrow \pi^0\chi_{c0}(1P))/\Gamma_{X(3872)} < 16\%$ [38] with $\Gamma_{X(3872)} \lesssim 131$ keV [15], to constrain the upper bound of the coupling constant square g_1^2 for $\chi_{cJ} \rightarrow D^{(*)}\bar{D}^{(*)}$. The g_1^2 is determined to be $g_1^2 \lesssim 0.28_{-0.14}^{+1.36} \text{ GeV}^{-1}$, whose central value is about 1 order-of-magnitude smaller than that from the VMD model estimation.

To check the reliability of our result about g_1^2 , we compute the partial decay widths of $X(3872) \rightarrow \pi^0\chi_{c1,2}$ using the g_1^2 determined from the $X(3872) \rightarrow \pi^0\chi_{c0}$ decay. The results are consistent with the experimental measurement [38]. Our results of g_1^2 can be further used and examined in other hidden-charm decay processes of the XYZ states, e.g., $X(3872) \rightarrow \pi\pi\chi_{cJ}$, $Y(4260) \rightarrow \pi h_c$, and $Z_c(3900)/Z_c(4020) \rightarrow \pi h_c$.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [30,31,37].

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