

$SL(2N, C)$ Yang-Mills theories: Direct internal forces and emerging gravityJ. L. Chkareuli *Institute of Theoretical Physics, Ilia State University, 0179 Tbilisi, Georgia
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A four-dimensional gauge-gravity unification based on local $SL(2N, C)$ symmetry is developed in a universal Yang-Mills-type setting, which, however, appears dynamically consistent only in the symmetry-broken phase. In the exact symmetry limit, the theory may only be formulated in a premetric framework, where the accompanying tetrad multiplets, though promoted to dynamical fields, do not yet satisfy the conventional invertibility conditions. An ordinary Einstein-Cartan spacetime geometry emerges only in the broken postsoldering phase, in which the $SL(2N, C)$ tetrad multiplets are treated as constrained dynamical fields selecting a neutral internal symmetry branch. This realizes the breaking $SL(2N, C) \rightarrow SL(2, C) \times SU(N)$, thereby lifting all noncompact internal directions, while the surviving neutral tetrad is, as usual, associated with the gravitational field. A special ghost-free curvature-squared Lagrangian provides a consistent quadratic sector for the spin connection, propagating only admissible connection modes: the massless $SU(N)$ vector fields together with massive axial-vector and pseudoscalar multiplets. The Einstein-Cartan linear curvature term is argued to arise radiatively from fermion loops, thereby relating the gravitational scale to the same $SL(2N, C)$ -covariant matter sector that defines the unified gauge coupling. Finally, the matter sector points to a deeper elementarity of $SL(2N, C)$ spinors, identified with preon constituents whose bound states form the observed quarks and leptons. Anomaly matching between preons and composites singles out $N = 8$. The chain $SL(16, C) \rightarrow SL(2, C) \times SU(8)$ then naturally yields three composite quark-lepton families, while filtering out extraneous heavy states.

DOI: [10.1103/4v84-pz6v](https://doi.org/10.1103/4v84-pz6v)**I. INTRODUCTION**

It has long been recognized [1,2] that gravity can be cast as a gauge theory in close analogy with the gauge description of electromagnetic, weak, and strong interactions. In this language, the spin connection associated with local Lorentz transformations plays the role of a gauge field, much as Yang-Mills bosons do for internal symmetry groups [3]. Once this parallel is taken seriously, it is natural to ask whether gravitational and internal gauge symmetries can be embedded into a single noncompact group, so that all interactions are described within a common gauge-theoretic framework. Such possibilities have been discussed in various forms [4–10]. However, despite their attractive algebraic structure, these schemes face well-known obstacles, which we consider here in the unified $SL(2N, C)$ symmetry case.

For the gravitational sector of such unification, the relevant local symmetry is $SL(2, C)$, the double cover of the proper orthochronous Lorentz group, which furnishes

the appropriate spinorial representation for fermions in four-dimensional spacetime. If one assumes that fundamental spinor fields are also responsible for organizing internal quantum numbers, it is natural to extend $SL(2, C)$ to a larger group $SL(2N, C)$, in which $SU(N)$ appears as an internal symmetry subgroup. In what follows, we refer to such $SL(2N, C)$ -based constructions as hyperunified theories (HUTs), since they combine $SL(2, C)$ gauge gravity and $SU(N)$ grand unification within a single local group. The internal $SU(N)$ factor will be called hyperflavor, in order to distinguish it from the Lorentzian $SL(2, C)$ sector, which remains neutral under internal charges. The full $SL(2N, C)$ gauge multiplet contains vector, axial-vector, and Lorentz-tensor connection fields, which provide a possible algebraic framework for combining Standard Model (SM) interactions with gravity, provided the known dynamical obstacles are resolved.

First, the unavoidable axial-vector and tensor partners of the usual vector fields have no experimental support and therefore must either decouple or lie at very high energies. Moreover, while internal forces are naturally described by Yang-Mills kinetic terms quadratic in the field strength, gravity in its Einstein-Cartan form is linear in curvature. A naive unified Lagrangian therefore typically contains both linear and quadratic curvature terms with different

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couplings. This tends to weaken the idea of a fully unified gauge sector and may generically lead to multiple tensor “gravitonlike” modes carrying $SU(N)$ quantum numbers, together with ghost or tachyon modes in standard $R + R^2$ constructions.

A further difficulty is that the matter sector of a hyperunified $SL(2N, C)$ theory is strongly constrained. Higher nonfundamental representations of $SL(2N, C)$ generally mix fields of different spins inside a single multiplet and therefore are not suitable for describing purely fermionic states such as quarks and leptons. Conversely, the fundamental $2N$ -dimensional representation contains only spin- $1/2$ fields, but its internal $SU(N)$ quantum numbers do not match the observed pattern of the Standard Model charges. Thus, $SL(2N, C)$ cannot act directly on quarks and leptons without introducing a large number of unobserved higher-spin or exotic-charged states. This strongly suggests that quarks and leptons may not be truly elementary in the $SL(2N, C)$ sense, but instead emerge as composites of more fundamental preons. If these preons are chiral spin- $1/2$ fermions transforming in the fundamental representation of $SL(2N, C)$, the group becomes a natural candidate for organizing both gravitational and internal dynamics at very short distances.

Finally, a conceptual issue concerns the Coleman-Mandula theorem [11], which severely restricts nontrivial unifications of spacetime and internal symmetries within a simple Lie algebra. However, the $SL(2N, C)$ framework developed here does not propose such a direct unification at the level of an ordinary S -matrix theory on a fixed metric background. In the exact symmetry limit, the theory is formulated on a premetric differentiable manifold, while the usual Einstein-Cartan spacetime geometry and internal symmetries emerge only in the spontaneously broken phase. This phase lifts the noncompact directions through tetrad dynamics, leaving the residual low-energy symmetry in the standard product form $SL(2, C) \times SU(N)$.

The central idea in this work is that the tetrad can simultaneously serve several distinct roles. First, it is, as usual, the soldering tool that connects the rigid spinorial tangent space with the flexible spacetime manifold and thereby determines the true metric structure of spacetime. Second, the tetrad is treated as a genuine dynamical field. Its kinetic sector, which generally contains ghostlike components, is chosen to be the teleparallel-gravity-type (TGR) torsion invariant [12], thereby giving the healthy Fierz-Pauli kinetic structure for the neutral graviton and simultaneously supplying algebraic lifting terms for the broken connection components. Third, a nonlinear σ -model-type constraint [13] fixes a nonzero invariant soldering norm of the tetrad multiplets, while their hyperflavor orientation is selected dynamically by the Coleman-Weinberg effective potential. The resulting hyperflavor-blind vacuum spontaneously breaks $SL(2N, C)$ down to $SL(2, C) \times SU(N)$, leaving the neutral gravitational tetrad and the $SU(N)$ vector fields

light, while the axial-vector and Lorentz-tensor submultiplets become heavy. Finally, in the presence of suitable fermion multiplets, the same tetrad and tensor fields can radiatively generate an Einstein-Cartan (EC) linear curvature term, even if such a term is absent at the tree level.

To make the quadratic-curvature sector consistent, we adopt a particular ghost-free combination (GFC) of curvature-squared terms originally identified by Neville [14,15], adapted here to the $SL(2N, C)$ symmetry case. We show that this structure arises naturally from an $SL(2N, C)$ -invariant quadratic Lagrangian once the tetrad is restricted to be neutral in hyperflavor space. With this choice, the field-strength-squared Lagrangian becomes fully unified: a single gauge coupling governs the dynamics of the vector, axial-vector, and tensor submultiplets.

Matter fields in the $SL(2N, C)$ framework are then reconsidered from a composite perspective. We argue that a preon model with chiral fermions in fundamental $SL(2N, C)$ representations, together with metacolor confinement and anomaly matching, picks out $SL(16, C)$ with an $SU(8)$ hyperflavor subgroup as a particularly compelling case. In this scenario, three chiral quark-lepton families arise as composites of preons and fit naturally within a single $SU(8)$ -based structure.

Compared with the earlier formulations of $SL(2N, C)$ hyperunification [5,7,9], and particularly with the recent ones [10], the present work focuses on the dynamical origin of the broken postsoldering phase. In the exact symmetry limit, the theory may only be formulated in a premetric framework, where the tetrad multiplets, though promoted to dynamical fields, do not yet satisfy the conventional invertibility conditions. An ordinary EC spacetime geometry emerges only in the broken postsoldering phase, in which the $SL(2N, C)$ tetrad multiplets are treated as constrained dynamical fields selecting the neutral branch. The resulting TGR tetrad kinetic structure both gives the healthy massless graviton and lifts the broken noncompact directions, while the GFC curvature-squared structure projects the spin-connection sector onto its ghost-free propagating particle content. Another distinctive point is that, despite the formal presence of the linear Einstein-Cartan term in the theory, this term may be induced radiatively from the same $SL(2N, C)$ -covariant matter couplings, so that the gravitational scale need not be introduced as an independent bare parameter, but may emerge universally from the unifying Yang-Mills setting. Finally, the preon sector, already motivated in earlier work [10], completes here an important matter-side interpretation of the present dynamical construction: the true carriers of the full hyperunified symmetry are not the observed quarks and leptons themselves, but subelementary preon spinors whose bound states form the observed chiral families.

The paper is organized as follows. Section II reviews the construction of $SL(2N, C)$ gauge theories, beginning with neutral spinors and $SL(2, C)$ symmetry and then extending

to charged spinors and total $SL(2N, C)$ framework. Section III is devoted to the role of tetrads in the theory being crucial for the compatibility of spacetime geometry and internal symmetries. Accordingly, the $SL(2N, C)$ theory is first formulated in a premetric phase and then acquires EC spacetime in the broken postsoldering phase with the residual $SL(2N, C) \rightarrow SL(2, C) \times SU(N)$ symmetry. Section IV develops the quadratic-strength-tensor Lagrangians and identifies a ghost-free choice that treats all gauge submultiplets on an equal footing. Section V explains how an EC gravity term is induced at one loop by fermion bubbles involving tetrads and tensor fields. Section VI reviews the full $SL(2N, C)$ hyperunification scenario in the spontaneously broken phase. Section VII turns to the matter sector and shows how preon-based $SL(16, C)$ hyperunification can accommodate three composite quark-lepton families. Section VIII summarizes the main results and outlines directions for further work.

II. $SL(2N, C)$ GAUGE THEORIES

Assuming that spinor fields provide the fundamental building blocks of both spacetime and internal structure leads naturally from the $SL(2, C)$ gauge group of gravity [4] to an enlarged local symmetry $SL(2N, C)$, where N measures the size of an internal $SU(N)$ subgroup. This $SU(N)$ is interpreted as a hyperflavor symmetry intended to organize the internal quantum numbers of the quark-lepton sector, including color, weak isospin, and family labels. In what follows, we outline how such a hyperflavor symmetry arises and how it feeds into hyperunification.

A. Neutral spinors: $SL(2, C)$ symmetry

We start with the $SL(2, C)$ gauge-symmetry case for a neutral spinor. Consider a Dirac field Ψ that transforms in the fundamental spinor representation of $SL(2, C)$, acting in the tangent (spinor) bundle at each spacetime point [4],

$$\Psi \rightarrow \Omega \Psi, \quad \Omega = \exp \left\{ \frac{i}{4} \theta_{ab} \gamma^{ab} \right\}, \quad (1)$$

where the matrix Ω satisfies the pseudounitariness condition

$$\Omega^{-1} = \gamma_0 \Omega^\dagger \gamma_0.$$

To preserve the invariance of the free kinetic term $i\bar{\Psi}\gamma^\mu\partial_\mu\Psi$ under local $SL(2, C)$ transformations, the constant matrices γ^μ must be replaced by tetrad-valued matrices e^μ that transform as

$$e^\mu \rightarrow \Omega e^\mu \Omega^{-1} \quad (2)$$

and are related to the usual tetrad fields e^a_μ by

$$e^\mu = e^\mu_a \gamma^a, \quad e_\mu = e^a_\mu \gamma_a.$$

Infinitesimally, this implies for the tetrad components

$$\delta e^{\mu c} = \frac{1}{2} \theta_{ab} (e^{\mu a} \eta^{bc} - e^{\mu b} \eta^{ac}),$$

while the usual invertibility conditions

$$e^a_\mu e^\nu_a = \delta^\nu_\mu, \quad e^a_\mu e^\mu_b = \delta^a_b \quad (3)$$

ensure that these tetrads define a nondegenerate metric,

$$g_{\mu\nu} = \frac{1}{4} \text{Tr}(e_\mu e_\nu) = e^a_\mu e^b_\nu \eta_{ab},$$

$$g^{\mu\nu} = \frac{1}{4} \text{Tr}(e^\mu e^\nu) = e^\mu_a e^\nu_b \eta^{ab}. \quad (4)$$

Promoting $SL(2, C)$ to a local symmetry, $\theta_{ab} \rightarrow \theta_{ab}(x)$, requires introducing a spin-connection 1-form I_μ transforming as

$$I_\mu \rightarrow \Omega I_\mu \Omega^{-1} - \frac{1}{ig} (\partial_\mu \Omega) \Omega^{-1}$$

and defining the covariant derivative

$$\partial_\mu \Psi \rightarrow D_\mu \Psi = \partial_\mu \Psi + ig I_\mu \Psi, \quad (5)$$

with gauge coupling g . The spin connection can be expanded in terms of Dirac generators as

$$I_\mu = \frac{1}{4} T^{[ab]}_\mu \gamma_{ab}, \quad (6)$$

where $T^{[ab]}_\mu$ transforms as

$$\delta T^{[ab]}_\mu = \frac{1}{2} \theta_{[cd]} [(T^{[ac]}_\mu \eta^{bd} - T^{[ad]}_\mu \eta^{bc}) - (T^{[bc]}_\mu \eta^{ad} - T^{[bd]}_\mu \eta^{ac})] - \frac{1}{g} \partial_\mu \theta^{[ab]},$$

i.e., as a Lorentz-antisymmetric tensor in the local frame and a spacetime 1-form.

When squared with the appropriate tetrad contractions, this field strength

$$T^{[ab]}_{\mu\nu} = \partial_{[\mu} T^{[ab]}_{\nu]} + g \eta_{cd} T^{[ac]}_{[\mu} T^{[bd]}_{\nu]}$$

leads to an $SL(2, C)$ -invariant Yang-Mills-type Lagrangian. However, in the presence of a tetrad, an additional invariant term linear in $T^{[ab]}_{\mu\nu}$, can also be written as

$$e\mathcal{L}_G^{(1)} = \frac{1}{2\kappa} e^\mu_{[a} e^\nu_{b]} T^{[ab]}_{\mu\nu}, \quad e \equiv [-\det \text{Tr}(e^\mu e^\nu)/4]^{-1/2}, \quad (7)$$

where κ is a dimensionful coefficient. Identifying κ with $8\pi/M_P^2$ yields the EC action in Palatini formulation [4,16]. Varying with respect to $T_\mu^{[ab]}$ expresses it algebraically in terms of the tetrad and its derivatives, and one recovers the usual Einstein action plus torsion-induced four-fermion terms [16]. The factor e ensures invariance under general four-dimensional diffeomorphisms as well, while the $SL(2, C)$ gauge group solely acts in the tangent spinorial space [4].

The coupling of the fermion to I_μ through (5) and (6) produces an additional interaction term

$$e\mathcal{L}_M = -\frac{1}{2}ge^{abcd}T_{\mu[ab]}e_c^\mu\bar{\Psi}\gamma_d\gamma_5\Psi,$$

which, after eliminating torsion, leads to an effective four-fermion contact term

$$\kappa(\bar{\Psi}\gamma_d\gamma_5\Psi)(\bar{\Psi}\gamma^d\gamma_5\Psi)$$

characteristic of EC gravity [2].

We show below, in Sec. V, that in the $SL(2N, C)$ hyperunified theory, whose fundamental bosonic sector is governed by a universal quadratic-strength Lagrangian with a single gauge coupling, the EC term need not be introduced as an independent tree-level input: it can instead be generated radiatively by the corresponding fermion-loop diagrams.

B. Charged spinors: $SL(2N, C)$ hyperunification

We now pass to the full $SL(2N, C)$ group, which contains the spacetime symmetry $SL(2, C)$ and an internal hyperflavor $SU(N)$ among its primary subgroups. The $(8N^2 - 2)$ generators of $SL(2N, C)$ can be written as tensor products of Dirac matrices with $SU(N)$ generators. A convenient parametrization [5] of a general transformation acting on fermions is

$$\Omega = \exp\left\{\frac{i}{2}\left[(\theta^k + \theta_5^k\gamma_5)\lambda^k + \frac{1}{2}\theta_{ab}^K\gamma^{ab}\lambda^K\right]\right\} (K = 0, k), \quad (8)$$

where λ^k ($k = 1, \dots, N^2 - 1$) are the standard $SU(N)$ Gell-Mann matrices and $\lambda^0 = \sqrt{2/N}\mathbf{1}_N$ denotes the normalized hyperflavor-singlet matrix. The parameters θ^k , θ_5^k , and θ_{ab}^K may be global or local.¹ Uppercase indices I, J, K refer to

¹The γ and λ matrices obey

$$\begin{aligned} \gamma^{ab} &= i[\gamma^a, \gamma^b]/2, & \{\gamma^a, \gamma^b\} &= 2\eta^{ab}\mathbf{1}, \\ [\lambda^k, \lambda^l] &= 2if^{klm}\lambda^m, & \{\lambda^k, \lambda^l\} &= \frac{4}{N}\delta^{kl}\hat{1} + 2d^{klm}\lambda^m. \end{aligned}$$

the singlet-plus-adjoint $U(1) \times SU(N)$ -type basis, while lowercase indices i, j, k denote the purely $SU(N)$ adjoint part.

Equation (8) describes the action of the $SL(2N, C)$ symmetry on the fermion multiplet Ψ , namely, on Dirac spinors carrying an additional hyperflavor $SU(N)$ index, so that Ψ lives in the space $\mathbb{C}^4 \otimes \mathbb{C}^N$. Equivalently, in the chiral basis for γ matrices, this symmetry acts separately on the left- and right-handed Weyl components, Ψ_L and Ψ_R , through the transformation matrices

$$\begin{aligned} \Omega_L &= \exp\left\{\frac{i}{2}\left[(\theta^k - \theta_5^k)\lambda^k + \frac{1}{2}\theta_{ab}^K\Sigma^{ab}\lambda^K\right]\right\}, \\ \Omega_R &= \exp\left\{\frac{i}{2}\left[(\theta^k + \theta_5^k)\lambda^k + \frac{1}{2}\theta_{ab}^K\bar{\Sigma}^{ab}\lambda^K\right]\right\}, \end{aligned} \quad (9)$$

respectively. Here Σ^{ab} and $\bar{\Sigma}^{ab}$ are given by

$$\Sigma^{ab} = \frac{i}{2}(\sigma^a\bar{\sigma}^b - \sigma^b\bar{\sigma}^a), \quad \bar{\Sigma}^{ab} = \frac{i}{2}(\bar{\sigma}^a\sigma^b - \bar{\sigma}^b\sigma^a),$$

where the two-dimensional matrices σ^a and $\bar{\sigma}^a$ are expressed in terms of the unit and Pauli matrices as

$$\sigma^a = (\hat{1}, \boldsymbol{\sigma}), \quad \bar{\sigma}^a = (\hat{1}, -\boldsymbol{\sigma}).$$

Accordingly, the action of the $SL(2N, C)$ gauge multiplet on both left- and right-handed Weyl sectors is explicitly specified.

The tetrad multiplet in the full $SL(2N, C)$ theory may, in general, contain both vector and axial-vector Dirac components,

$$e_\mu = (e_\mu^{aK}\gamma_a + e_{\mu 5}^{aK}\gamma_a\gamma_5)\lambda^K, \quad K = 0, 1, \dots, N^2 - 1 \quad (10)$$

and transforms as in (2), where Ω is now defined by (8). One can simplify this structure and eliminate the axial-vector components by introducing an auxiliary scalar multiplet,

$$S = \exp\{i[(s^k + p^k\gamma_5)\lambda^k + T_{[ab]}\gamma^{ab}\lambda^K/2]\}$$

that transforms as $S \rightarrow \Omega S$. By requiring the invariant combination $S^{-1}e_\mu S$ to have zero projection onto the axial-vector directions, we impose the constraint

$$\text{Tr}[\gamma_a\gamma_5\lambda^K(S^{-1}e_\mu S)] = 0$$

with the trace taken over both Dirac and hyperflavor indices. Subsequently, selecting $S = 1$ within the gauge orbit of the pair (e_μ, S) yields the unitary gauge representative of the covariantly constrained theory, in which the

tetrad reduces to a purely vectorial form [5]. Finally, we adopt the tetrad form

$$e_\mu = e_\mu^{aK} \gamma_a \lambda^K, \quad e = e_a^{\mu K} \gamma^a \lambda^K \quad (11)$$

so that the neutral tetrad components in e_μ and e^μ are identified with the pure $SL(2, C)$ tetrads, $e_\mu^{a0} = e_\mu^a$ and $e_a^{\mu 0} = e_a^\mu$.

As in the previous $SL(2, C)$ case, one may formally write the tetrad invertibility condition

$$\frac{1}{32} \text{Tr}(e_\mu e^\nu) = \delta_\mu^\nu, \quad e_\mu^{aK} e_a^{\nu K} = \delta_\mu^\nu, \quad e_\mu^{aK} e_b^{\mu K} = \delta_b^a$$

and corresponding metric tensors

$$\begin{aligned} \frac{1}{8} \text{Tr}(e_\mu e_\nu) &= e_\mu^{aK} e_{\nu a}^K = g_{\mu\nu}, \\ \frac{1}{8} \text{Tr}(e^\mu e^\nu) &= e_a^{\mu K} e^{\nu a K} = g^{\mu\nu}, \end{aligned}$$

where we have used the trace conditions for γ and λ matrices.

Localizing the transformation (8) introduces a gauge field I_μ transforming as

$$I_\mu \rightarrow \Omega I_\mu \Omega^{-1} - \frac{1}{ig} (\partial_\mu \Omega) \Omega^{-1} \quad (12)$$

with field strength

$$I_{\mu\nu} = \partial_{[\mu} I_{\nu]} + ig [I_\mu, I_\nu] \quad (13)$$

and fermionic-covariant derivative

$$\partial_\mu \Psi \rightarrow D_\mu \Psi = \partial_\mu \Psi + ig I_\mu \Psi.$$

The gauge connection I_μ can be decomposed into vector, axial, and tensor parts

$$\begin{aligned} I_\mu &= V_\mu + A_\mu + T_\mu \\ &= \frac{1}{2} (V_\mu^k + A_\mu^k \gamma_5) \lambda^k + \frac{1}{4} T_\mu^{[ab]K} \gamma_{ab} \lambda^K (K=0, k), \end{aligned} \quad (14)$$

where each submultiplet carries the appropriate spacetime, Lorentz, and internal indices. The corresponding contribution to the field strength reads as

apart from some unessential V - T and A - T mixed terms.² Meanwhile, the generic gauge-invariant matter coupling can be written in terms of e^μ and the submultiplets as

$$e \mathcal{L}_M = -\frac{g}{2} \bar{\Psi} \left\{ e^\mu, \left[\frac{1}{2} (V_\mu^k + A_\mu^k \gamma_5) \lambda^k + \frac{1}{4} T_\mu^{[ab]K} \gamma_{ab} \lambda^K \right] \right\} \Psi. \quad (16)$$

Thus, all three types of gauge fields couple with the same universal coupling g in the underlying $SL(2N, C)$ theory,

²The general field strength (13) contains, in addition to the sectoral vector/axial-vector and tensor contributions displayed in (15), the mixed V - T and A - T commutator terms

$$\Delta I_{\mu\nu} = -(g/4) f^{ijk} (V^i + A^i \gamma_5)_{[\mu} T_{\nu]}^{[ab]j} \gamma_{ab} \lambda^k.$$

These terms describe the gauge interactions of the vector and axial-vector submultiplets with the hyperflavored tensor fields. They introduce no independent coupling and no new kinetic structure; after tetrad condensation (presented below) they involve tensor fields belonging to the heavy sector. For this reason, they are suppressed in (15), where only the sectoral contributions relevant for constructing the independent quadratic-strength Lagrangians are displayed.

which typically includes a standard quadratic field-strength term for each submultiplet, while a possible linear curvature term, analogous to (7), may be induced radiatively (Sec. V). The key question is how the observed low-energy spectrum can retain only the $SU(N)$ vector fields and a single graviton, while the axial-vector and tensor states are decoupled.

III. TETRADS IN $SL(2N, C)$ THEORY

The full $SL(2N, C)$ gauge theory contains many more degrees of freedom than are needed to describe known low-energy physics. Our strategy is to show that a dynamical tetrad, with its norm fixed by the length constraint and its hyperflavor orientation selected by quantum effects, can spontaneously break $SL(2N, C)$ and remove the unwanted noncompact directions. In this picture, the tetrad not only defines the spacetime geometry but also selects the internal symmetry and filters the spin-connection multiplets that remain light. The low-energy theory then exhibits only $SL(2, C) \times SU(N)$ symmetry with massless $SU(N)$ vectors, while hyperflavored axial-vector and tensor fields are lifted to high masses. As will be seen below, a crucial simplification occurs once tetrads are constrained to be

neutral in hyperflavor space. In that case, only the V_μ^k and the singlet tetrad e_μ^{a0} remain relevant for low-energy physics: the former mediate internal forces, while the latter underlies the induced EC gravity.

A. Gravity-inspired geometry

The general tetrad ansatz (10), even after elimination of the axial-vector part, still includes the hyperflavor components as well (11). However, the invertibility conditions suitable for coupling to a four-dimensional metric severely restrict this structure. Let us assume that these conditions take the simple multiplicative form

$$e_\mu^{aK} e_a^{\nu K'} = \delta_\mu^\nu n^{KK'}, \quad e_\mu^{aK} e_b^{\mu K'} = \delta_b^a n^{KK'} \quad (17)$$

with some matrix n , whose form is to be determined. In the pure gravity limit, one has constraints

$$K = K' = 0; n^{00} = 1, \quad n^{kk} = 0, \quad (18)$$

which exclude a conventional $\delta^{KK'}$ form for the matrix n .

Multiplying (17) by $e_\rho^{bK''}$ and $e_a^{\rho K''}$ and using basic algebraic identities one finds

$$e_\mu^{aK} = n^{K0} e_\mu^{a0}, \quad e_a^{\mu K'} = n^{K'0} e_a^{\mu 0},$$

which leads to the equality

$$e_\mu^{aK} e_a^{\mu K'} / 4 = n^{K0} n^{K'0}.$$

Assuming that $n^{KK'}$ is a constant numerical matrix, one easily finds that the only consistent solution compatible with the conditions (17), (18)

$$n^{KK'} = \delta^{K0} \delta^{K'0},$$

i.e., the standard invertibility relations hold only for the $K = 0$ component. This implies that the general tetrad must effectively lie in the $SL(2, C)$ subgroup,

$$e_\mu^{aK} = e_\mu^a \delta^{K0}, \quad e_a^{\mu K} = e_a^\mu \delta^{K0} \quad (19)$$

if it is to define a conventional spacetime metric. Thus, within the simple factorized invertibility ansatz (17), the ordinary tetrad relations select the hyperflavor-singlet direction. The neutral tetrad branch (19) should therefore be viewed not as an arbitrary truncation, but as the branch on which the usual EC spacetime geometry is recovered.

One can enforce this by imposing (19) as a background constraint which explicitly reduces the starting $SL(2N, C)$ to $SL(2, C) \times SU(N)$. Alternatively, when tetrads are treated as dynamical fields, the neutral tetrad may emerge via the spontaneous breaking of $SL(2N, C)$, as we see below.

B. Premetric phase

In general, the main difficulty is how to formulate the theory in a manifestly $SL(2N, C)$ -invariant way, since, as we have seen, a general tetrad multiplet e_μ^{aK} does not satisfy the ordinary tetrad invertibility condition in the unbroken phase. The compound index (a, K) contains more components than the spacetime index μ , so the full tetrad multiplet cannot be treated as an ordinary square tetrad with a standard inverse.

A useful way to formulate the theory before soldering is not to require the matrix tetrad multiplet to possess an ordinary inverse. In the unbroken $SL(2N, C)$ phase, one introduces two independent tetrad-type fields, the “lower” and “upper” tetrads,

$$e_\mu = e_\mu^{aK} \gamma_a \lambda_K, \quad E^\mu = E_a^{\mu K} \gamma^a \lambda_K, \quad K = 0, k, \quad (20)$$

respectively, transforming in the adjoint representation of the local $SL(2N, C)$ group

$$e_\mu \rightarrow \Omega e_\mu \Omega^{-1}, \quad E^\mu \rightarrow \Omega E^\mu \Omega^{-1}.$$

In that invariant phase, the theory does not yet possess an ordinary metric geometry. Instead, the lower and upper tetrad multiplets define two independent invariant bilinears

$$g_{\mu\nu} = e_\mu^{aK} e_\nu^{bK} \eta_{ab}, \quad G^{\mu\nu} = E_a^\mu E_b^\nu \eta^{ab},$$

which are not yet a metric and its inverse. These tensors can be used to form invariant contractions of vectors and covectors, but in the unbroken phase they are not inverse to one another. Thus, the symmetric phase is best understood as a presoldering bipremetric spacetime, with no canonical operation of raising and lowering indices and no ordinary EC metric structure. Ordinary spacetime geometry appears only after the transition to the dynamically selected soldered neutral branch. On that branch, the tetrads reduce to the ordinary $SL(2, C)$ tetrad and inverse tetrad, and the usual metric relation, $g_{\mu\nu} G^{\nu\rho} = \delta_\mu^\rho$, emerges as a property of the broken phase rather than as an assumption of the unbroken presoldering theory.

The same lower-upper presoldering formulation may be made also in the gauge sector. The fields are still defined on a differentiable manifold with local coordinates, ordinary partial derivatives, and local $SL(2N, C)$ gauge transformations. What is absent in this phase is a prior physical metric which would identify covariant and contravariant world indices.

Accordingly, besides the lower tetrad-type field e_μ and the upper tetrad-type field E^μ , one may introduce lower and upper gauge connections, I_μ and I^μ , and the corresponding covariant derivatives

$$D_\mu = \partial_\mu + ig I_\mu, \quad D^\mu = \partial^\mu + ig I^\mu.$$

Here ∂^μ is not obtained from ∂_μ by raising an index with a background Minkowski metric. It is the contravariant derivative operator of the premetric formulation and becomes $G^{\mu\nu}\partial_\nu$ only in the soldered phase.

The I_μ and I^μ gauge multiplets and their strength tensors $I_{\mu\nu}$ and $I^{\mu\nu}$ are appropriately transformed under local $SL(2N, C)$ transformations (8), (12), thus leading to the direct invariant lower-upper pairing

$$i_0 = \text{Tr}(I_{\mu\nu}I^{\mu\nu}). \quad (21)$$

The two premetrics $g_{\mu\nu}$ and $G^{\mu\nu}$ also allow the lower- and upper-sector self-contractions

$$i_1 = \text{Tr}(I_{\mu\nu}I_{\rho\sigma})G^{\mu\rho}G^{\nu\sigma}, \quad i_2 = \text{Tr}(I^{\mu\nu}I^{\rho\sigma})g_{\mu\rho}g_{\nu\sigma}. \quad (22)$$

These are independent invariants in the presoldering phase, and if one imposes the lower-upper exchange symmetry

$$e_\mu \leftrightarrow E^\mu, \quad g_{\mu\nu} \leftrightarrow G^{\mu\nu}, \quad I_\mu \leftrightarrow I^\mu,$$

then the two self-contractions must enter in the action with equal coefficients. Thus, the most general quadratic gauge sector may contain the direct invariant and a symmetric pair of contracted invariants.

The fermion sector can be written in the same lower-upper symmetric form. For a spinor multiplet Ψ ,

$$D_\mu\Psi = (\partial_\mu + igI_\mu)\Psi, \quad D^\mu\Psi = (\partial^\mu + igI^\mu)\Psi.$$

The corresponding presoldering Dirac term may be written as

$$\mathcal{L}_\Psi = \frac{i}{4}\mathcal{E}\bar{\Psi}(E^\mu\overleftrightarrow{D}_\mu + e_\mu\overleftrightarrow{D}^\mu)\Psi. \quad (23)$$

Both pieces are $SL(2N, C)$ invariant: e_μ and E^μ transform adjointly, while Ψ transforms in the fundamental representation. The factor $\mathcal{E}$ is the premetric volume density, reducing to the usual tetrad determinant after soldering.

This formulation does not imply two independent gauge multiplets in the low-energy theory. The soldering transition identifies the lower and upper sectors. On the neutral tetrad branch (see below),

$$g_{\mu\rho}G^{\rho\nu} = \delta_\mu^\nu, \quad E^\mu = G^{\mu\nu}e_\nu,$$

and the upper connection is identified with the raised lower connection,

$$I^\mu = G^{\mu\nu}I_\nu, \quad D^\mu = G^{\mu\nu}D_\nu.$$

Consequently,

$$I^{\mu\nu} = G^{\mu\rho}G^{\nu\sigma}I_{\rho\sigma},$$

and all three quadratic gauge invariants $i_{0,1,2}$ in (21) and (22) reduce to the same ordinary Yang-Mills kinetic structure, differing only by an overall normalization of the effective gauge coupling. Likewise, the symmetric fermion term (23) reduces to the usual tetrad Dirac action,

$$e\mathcal{L}_\Psi \rightarrow \frac{i}{2}\bar{\Psi}e^\mu\overleftrightarrow{D}_\mu\Psi, \quad e \equiv [-\det\text{Tr}(e^\mu e^\nu)/4]^{-1/2}$$

up to the conventional normalization. Thus, the premetric theory is a local $SL(2N, C)$ -invariant premetric gauge theory, while the ordinary metric Yang-Mills and fermion Lagrangians emerge only on the dynamically selected hyperflavor-blind soldered branch.³

C. Toward tetrad condensation

The presoldering $SL(2N, C)$ -invariant theory contains two independent tetrad-type fields (20). The soldered phase leading to ordinary EC spacetime geometry is described by a σ -model-type constraint, which in the simplest fixed-norm form can be presented as

$$\frac{1}{32}\text{Tr}(e_\mu E^\mu) = \frac{1}{4}e_\mu^{aK}E_{aK}^\mu = 1. \quad (24)$$

The tetrad multiplets themselves play the role of the soldering order parameter, while the constraint (24) fixes only its invariant magnitude and thus defines the nonlinear σ -model surface of the broken phase. In the present effective description, the dynamics that produces the non-zero soldering norm is not explicitly modeled; the constraint (24) may be viewed as the fixed-norm limit of a more general tetrad potential. It does not select a definite $SU(N)$ hyperflavor direction. This orientation is instead selected dynamically by the Coleman-Weinberg effective potential considered below, whose preferred vacuum is the hyperflavor-blind branch. On this branch, we write

$$e_\mu^{aK} = e_\mu^a\delta^{K0} + e_\mu^{*aK}, \quad E_a^{\mu K} = e_a^\mu\delta^{K0} + E_a^{*\mu K}, \quad (25)$$

where, to maintain correspondence with the pure gravity case, we take

$$e_\mu^{*a0} = 0, \quad E_a^{*\mu 0} = 0$$

³We recently became aware that a certain premetric approach to gauge-gravity unification was discussed earlier in the $SO(13, 1)$ symmetry framework [8], which contains the local Lorentz generators and the $SO(10)$ grand unified theory (GUT) generators as commuting subalgebras. In that approach, the soldering 1-form plays the role of an order parameter, while the unified-symmetry limit is associated with the vanishing of an ordinary metric. In contrast, the present construction uses the general $SL(2N, C)$ hyperunification framework, with independent lower and upper premetrics, and the subsequent broken $SL(2N, C) \rightarrow SL(2, C) \times SU(N)$ postsoldering phase.

(the asterisk will denote hyperflavor-carrying quantities throughout). Once the hyperflavor-blind branch (25) is selected dynamically, the vacuum realizes the spontaneous symmetry breaking of $SL(2N, C)$ down to $SL(2, C) \times SU(N)$. The neutral tetrad components e_μ^a and e_a^μ , satisfying the standard invertibility relations (3), then define the soldered spacetime geometry, while the flavored components e_μ^{*ak} and $E_a^{*\mu k}$ contain the modes associated with the broken noncompact directions. Absorption of these modes induces masses for the axial-vector and tensor-field submultiplets, related to the $7(N^2 - 1)$ broken coset generators beyond the residual $SL(2, C) \times SU(N)$ symmetry.

With the parametrization (25), the constraint (24) gives

$$\frac{1}{4} e_\mu^{aK} E_a^{\mu K} = 1 + e_\mu^{*ak} E_a^{*\mu k} / 4 \rightarrow e_\mu^{*ak} E_a^{*\mu k} = 0. \quad (26)$$

Thus, the minimal scalar constraint removes only the total mixed trace of the flavored sector. Meanwhile, the ordinary invertibility conditions

$$\begin{aligned} e_\mu^{aK} E_a^{\mu K} &= \delta_\mu^\nu + e_\mu^{*ak} E_a^{*\nu k}, \\ e_\mu^{aK} E_b^{\mu K} &= \delta_b^a + e_\mu^{*ak} E_b^{*\mu k} \end{aligned}$$

hold only up to quadratic flavored corrections. Accordingly, the lower and upper premetrics are

$$\begin{aligned} g_{\mu\nu} &= e_\mu^{aK} e_\nu^{bK} \eta_{ab} = g_{\mu\nu}^0 + g_{\mu\nu}^*, \\ G^{\mu\nu} &= E_a^{\mu K} E_b^{\nu K} \eta^{ab} = g_0^{\mu\nu} + G_*^{\mu\nu}, \end{aligned}$$

where $g_{\mu\nu}^0$ and $g_0^{\mu\nu}$ are the standard $SL(2, C)$ metric (4), while

$$g_{\mu\nu}^* = e_\mu^{*ak} e_\nu^{*bk} \eta_{ab}, \quad G_*^{\mu\nu} = E_a^{*\mu k} E_b^{*\nu k} \eta^{ab}.$$

Thus, the product of full metrics

$$\begin{aligned} g_{\mu\rho} G^{\nu\rho} &= \delta_\mu^\nu + g_{\mu\rho}^0 G_*^{\nu\rho} + g_{\mu\rho}^* g_0^{\nu\rho} + g_{\mu\rho}^* G_*^{\nu\rho} \\ &= \delta_\mu^\nu + O(e^* e^*, E^* E^*) \end{aligned}$$

can significantly deviate from an exact EC geometry unless the flavored metric corrections are removed or dynamically suppressed.

By itself, the $SL(2N, C)$ -invariant constraint surface (24) possesses a much larger accidental symmetry leaving many flat directions. To display it, introduce the collective index

$$\begin{aligned} Q &= (\mu, a, K), \quad K = 0, 1, \dots, N^2 - 1, \\ Q &= 1, \dots, 16N^2 \end{aligned}$$

and define $e_Q \equiv e_\mu^{aK}$ and $E_Q \equiv E_a^{\mu K}$. The two premetric tetrads may then be assembled into a single complex vector

$$\Phi^A = \begin{pmatrix} e_Q \\ E_Q \end{pmatrix} = \begin{pmatrix} e_\mu^{aK} \\ E_a^{\mu K} \end{pmatrix}, \quad A = 1, \dots, 32N^2.$$

With the off-diagonal bilinear form

$$\mathcal{J} = \begin{pmatrix} 0 & \mathbf{1}_{16N^2} \\ \mathbf{1}_{16N^2} & 0 \end{pmatrix},$$

the singlet constraint becomes

$$\frac{1}{8} \Phi^T \mathcal{J} \Phi = 1.$$

Therefore, the isolated constraint possesses the accidental complex orthogonal symmetry

$$SO(32N^2, C), \quad O^T \mathcal{J} O = \mathcal{J}, \quad (27)$$

where O is a general global rotation/boost in the full $32N^2$ -dimensional complex tetrad space, mixing e_Q and E_Q , while leaving the singlet trace invariant. This symmetry appears because the constraint (24) does not resolve the separate geometric roles of the spacetime index μ , the Lorentz index a , and the hyperflavor index K ; it only sees the combined vector Φ . In this representation, the total premetric tetrads e_μ^{aK} and $E_a^{\mu K}$ are the two isotropic halves of the fundamental vector representation of $SO(32N^2, C)$.

However, in the gravitational soldered phase we considered above in (25), the neutral tetrads e_μ^{a0} and $E_a^{\mu 0}$ are not Goldstone fields of the symmetry breaking. They are retained as the real neutral Higgs sector that defines the postsoldering EC spacetime geometry, while the residual local Lorentz group $SL(2, C)$ remains unbroken. Therefore, after the neutral tetrad sector saturates the constraint in (26), the relevant accidental symmetry for the remaining flat directions is not the formal full symmetry (27), but the flavored-sector symmetry $SO[32(N^2 - 1), C]$, which acts on the combined flavored tetrad vector $\Phi^* = (e_\mu^{*ak}, E_a^{*\mu k})^T$. The constraint (24) reduces to the condition $e_\mu^{*ak} E_a^{*\mu k} = 0$ on the charged tangent space, which is invariant under this accidental symmetry. Thus, the charged accidental zero-mode spectrum contains $32(N^2 - 1)$ independent flavored tetrad modes, whose number exactly corresponds to the number of coset flat directions in the symmetry breaking

$$SO[32(N^2 - 1), C] \longrightarrow SO[32(N^2 - 1) - 1, C]. \quad (28)$$

This large accidental zero-mode spectrum must be distinguished from the true gauge Goldstone spectrum of the underlying local theory, since this counting refers to the accidental flat directions of the isolated soldering constraint (24), not to the $7(N^2 - 1)$ Goldstone modes of the local $SL(2N, C)$ gauge symmetry. They are removed in

unitary gauge and become the longitudinal components of the massive axial-vector and tensor-field spin connections. The remaining $25(N^2 - 1)$ charged zero modes are pseudo-Goldstone modes emerging in the accidental symmetry breaking (28). Since the full theory possesses only the smaller local $SL(2N, C)$ symmetry, these modes are not protected and are expected to acquire large masses from the remaining dynamics.

The flavored tetrads e_μ^{*ak} and $E_a^{*\mu k}$ are generically massless fields. However, once the neutral soldering branch has been selected, the cotetrad fluctuation $E_a^{*\mu k}$ can be converted to $E_\mu^{*ak} = g_{\mu\nu}^0 \eta^{ab} E_b^{*\nu k}$, and the pseudo-Goldstone lifting appears as an ordinary quadratic mass matrix for the pair $(e_\mu^{*ak}, E_\mu^{*ak})$, summed over the adjoint index k . In postsoldering phase, one may then impose, for simplicity, the extra $16(N^2 - 1)$ conditions, $e_\mu^{*ak} = E_\mu^{*ak}$, leaving in the theory a single tetrad multiplet e_μ^{aK} with the flavored e_μ^{*ak} components. In the low-energy neutral branch, they may therefore be consistently neglected only after they are properly lifted. Thus, the neutral tetrad presentation (19) should not be interpreted as an explicit breaking of the underlying $SL(2N, C)$ -invariant formulation. It is the effective unitary-gauge representative of the soldered branch after the gauge Goldstone modes have been absorbed and the remaining hyperflavor-carrying tetrad modes have been lifted. Therefore, both the presoldering covariant formulation and the broken postsoldering phase can be described within an otherwise $SL(2N, C)$ -invariant theory while using the neutral tetrad branch for the low-energy EC spacetime geometry. The full tetrad multiplet reduces to the neutral $SL(2, C)$ tetrad sector, and all departures from it are either Goldstone modes eaten by the massive spin connections or residual heavy flavored tetrads outside the leading low-energy geometry. We will proceed with a neutral tetrad for the rest of the analysis, rather than considering the general tetrad case first, subsequently substituting its vacuum-inspired form (19) into the relevant equations.

D. Dynamical tetrads: Lifting noncompact directions

We now promote the tetrad to a dynamical field. However, the naive quadratic term

$$(D_{[\mu} e_{\nu]}^{aK})(D^{[\mu} e^{\nu]}_{aK}) \quad (29)$$

is not appropriate as a fundamental tetrad kinetic term. A generic quadratic tetrad-derivative form of this type propagates wrong-sign tetrad components. The healthy gravitational kinetic term is instead provided by the teleparallel equivalent of general relativity [12]. In ordinary TGR, the tetrad is used as the fundamental gravitational variable and the Einstein-Hilbert scalar is replaced, up to a total derivative, by a special quadratic combination of the torsion tensor.

In this case, the tetrad torsion is

$$C_{\mu\nu}^a = \partial_\mu e_\nu^a - \partial_\nu e_\mu^a,$$

or, in local Lorentz components,

$$C_{abc} = e_b^\mu e_c^\nu C_{\mu\nu a}, \quad C_{abc} = -C_{acb}, \quad C_a = C^b_{ba}.$$

So, the TGR torsion scalar is

$$\mathbb{T}(C) = \frac{1}{4} C_{abc} C^{abc} + \frac{1}{2} C_{abc} C^{cab} - C_a C^a.$$

Thus, the healthy tetrad kinetic term is chosen as

$$e\mathcal{L}_\mathbb{T} = -M^2 \mathbb{T}(C), \quad (30)$$

where M is the mass scale supplied by the TGR sector. Since the tetrad is dimensionless, this is the only dimensionful parameter appearing in the tetrad kinetic term.

The sign in (30) is fixed by the requirement that the quadratic expansion around a nondegenerate tetrad background gives the correct Fierz-Pauli kinetic term for the massless graviton. Writing

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad h = \eta^{\mu\nu} h_{\mu\nu},$$

one obtains, up to the overall normalization M^2 ,

$$e\mathcal{L}_\mathbb{T}(h) = \frac{M^2}{8} [\partial_\rho h_{\mu\nu} \partial^\rho h^{\mu\nu} - 2\partial_\mu h^{\mu\nu} \partial^\rho h_{\rho\nu} + 2\partial_\mu h^{\mu\nu} \partial_\nu h - \partial_\rho h \partial^\rho h]. \quad (31)$$

This is the standard Fierz-Pauli kinetic structure for a conventional massless spin-two field. So, the tetrad kinetic term, being now the special TGR combination, propagates the healthy graviton rather than the ghost modes present in a generic quadratic tetrad-derivative term (29).

In the present $SL(2N, C)$ theory, we keep the physical tetrad neutral or hyperflavor blind (19). We do not specialize e_μ^a to a flat Minkowski background at this stage. It is the dynamical EC or TGR tetrad defining the spacetime metric. The $SL(2N, C)$ -covariant torsion is obtained by replacing the ordinary derivative by the full covariant derivative,

$$C_{\mu\nu} = D_\mu e_\nu - D_\nu e_\mu,$$

where

$$D_\mu e_\nu = \partial_\mu e_\nu + ig[I_\mu, e_\nu],$$

and I_μ is the full spin-connection multiplet (14). Thus,

$$C_{\mu\nu} = \partial_\mu e_\nu - \partial_\nu e_\mu + ig[I_\mu, e_\nu] - ig[I_\nu, e_\mu]. \quad (32)$$

For an axial-free tetrad $e_\mu = e_\mu^{aK} \gamma_a \lambda_K$, the covariant torsion has the explicit Dirac decomposition

$$C_{\mu\nu} = C_{\mu\nu}^{aK} \gamma_a \lambda_K + C_{\mu\nu 5}^{aK} \gamma_a \gamma_5 \lambda_K.$$

There is no independent $\gamma_{ab} \lambda_K$ component in $C_{\mu\nu}$. The reason is that the connection I_μ is Dirac even, whereas the tetrad is Dirac odd; hence the commutator $[I_\mu, e_\nu]$ is again Dirac odd. The tensor gauge fields $T_\mu^{[ab]K}$ are therefore not absent; rather, they enter the vector and axial-vector coefficients $C_{\mu\nu}^{aK}$ and $C_{\mu\nu 5}^{aK}$, but they do not generate a separate Dirac-tensor torsion component proportional to $\gamma_{ab} \lambda_K$.

The $SL(2N, C)$ TGR Lagrangian has the same formal structure as (30), but now the torsion components entering $\mathbb{T}(C)$ are built from the flavored covariant object $C_{\mu\nu}$. In local indices,

$$C_{abc}^K = e_b^\mu e_c^\nu C_{\mu\nu a}^K, \quad C_a^K = C^{bK}_{ba},$$

and the similar TGR combination is used,

$$\mathbb{T}(C) = \frac{1}{4} C_{abc}^K C^{abcK} + \frac{1}{2} C_{abc}^K C^{cabK} - C_a^K C^{aK}. \quad (33)$$

The normalization is chosen so that, for the neutral $K = 0$ tetrad sector, the action reduces to the ordinary TGR action (30).

The hyperflavor-blind vacuum branch is represented by the tetrad field configuration

$$\bar{e}_\mu = e_\mu^a \gamma_a \lambda^0, \quad \lambda^0 = \sqrt{2/N} \mathbf{1}_N, \quad (34)$$

providing the true vacuum in the theory.⁴ Since λ^0 is proportional to the identity in hyperflavor space, the vector generators commute with the tetrad field configuration on this vacuum branch,

$$[\lambda^k, \bar{e}_\mu] = 0.$$

Consequently, the vector gauge fields V_μ^k do not enter the algebraic part of the covariant torsion,

$$ig[V_\mu^k \lambda_k, \bar{e}_\nu] = 0.$$

The TGR term therefore gives no mass to the vector multiplet,

⁴Here “vacuum” refers only to the alignment of the tetrad multiplet in the neutral hyperflavor direction. The surviving field $e_\mu^a(x)$ remains the dynamical gravitational tetrad and is not fixed to a constant Minkowski value. The conventional flat-background choice $\langle e_\mu^a \rangle \sim \delta_\mu^a$ is a further specialization used only for a perturbative expansion around Minkowski spacetime.

$$m_V^2 = 0.$$

Thus, the $SU(N)$ vector gauge bosons remain massless at this stage.

The situation is different for the axial-vector generators. Their contribution to the tetrad-covariant derivative follows from

$$ig \left[\frac{1}{2} A_\mu^k \gamma_5 \lambda_k, e_\nu^a \gamma_a \lambda^0 \right].$$

Using the standard group condition for γ and λ matrices, one comes to

$$D_\mu e_\nu \Big|_A = -ig \sqrt{2/N} e_\nu^a A_\mu^k \gamma_a \gamma_5 \lambda_k,$$

and hence

$$C_{\mu\nu} \Big|_A = -ig \sqrt{2/N} (A_\mu^k e_\nu^a - A_\nu^k e_\mu^a) \gamma_a \gamma_5 \lambda_k.$$

Thus, the axial-vector fields enter the TGR torsion algebraically. Substitution into the TGR scalar produces a mass term of the form $m_A^2 \sim g^2 M^2$, once one uses the tetrad invertibility condition (3). Note that the exact numerical coefficient depends only on the normalization of the $SL(2N, C)$ generators: for the conventional normalization taken in (14), $m_A^2 = (36/N) g^2 M^2$.

The tensor generators also do not commute with the tetrad. Their contribution follows from

$$ig \left[\frac{1}{4} T_\mu^{[ab]K} \gamma_{ab} \lambda_K, e_\nu^c \gamma_c \lambda^0 \right].$$

Using the Lorentz algebra relation

$$[\gamma_{ab}, \gamma_c] = 2(\eta_{bc} \gamma_a - \eta_{ac} \gamma_b),$$

one finds that the tensor connection contributes to the vector torsion components as

$$C_{abc}^K(T) = g \sqrt{2/N} (T_{b[ac]}^K - T_{c[ab]}^K), \quad (35)$$

up to the same generator-normalization convention. Therefore, the TGR term [(30), (33)] with these torsion components explicitly produces the tensor-field mass block with a general factor $(2/N) g^2 M^2$, which gives mass of order gM to the tensor spin-connection multiplet. The detailed decomposition of T_{abc}^K into its irreducible Lorentz blocks will be discussed later. At this stage, it is enough to note that the tensor fields are lifted by the same TGR structure that gives the healthy kinetic term for the tetrad.

Thus, the TGR tetrad sector gives the desired pattern,

$$\begin{aligned} V_\mu^k: \quad & m_V^2 = 0, \\ A_\mu^k: \quad & m_A^2 \sim g^2 M^2, \\ T_\mu^{[ab]K}: \quad & m_T^2 \sim g^2 M^2. \end{aligned} \quad (36)$$

This is precisely the pattern expected from the hyperflavor-blind tetrad. The unbroken gauge sector is the vector $SU(N)$, while the noncompact axial-vector and tensor directions are lifted. In group-theoretic terms, the hyperflavor-blind tetrad selects the residual structure, $SL(2, C) \times SU(N)$, with the Lorentz sector realized through the physical tetrad and the $SU(N)$ vector bosons remaining massless.

It is important that the same TGR sign is responsible for both effects. It gives the correct kinetic term for the dimensionless tetrad and hence for the emergent graviton, while also giving the algebraic mass terms for the noncompact spin-connection fields. In particular, as will be shown below, this sign is also compatible with the healthy propagation of the longitudinal pseudoscalar mode contained in the tensor multiplet. Thus, the TGR replacement of the naive kinetic terms (29) is not merely a cosmetic change; it is required both for the absence of tetrad ghosts and for the correct lifting of the noncompact $SL(2N, C)$ directions.

Consequently, the theory contains the characteristic scale M , introduced by the TGR tetrad sector. Above the scale gM , the full spin-connection multiplet propagates according to the Yang-Mills-type curvature terms, which we consider in the next section. Below this scale, the heavy axial-vector and tensor connections can be integrated out. Their equations of motion enforce the covariant tetrad compatibility condition in the noncompact directions,

$$D_{[\mu} e_{\nu]} = 0,$$

up to corrections suppressed by powers of $p^2/(g^2 M^2)$. The vector gauge fields V_μ^k do not participate in this algebraic compatibility condition for the neutral tetrad, because they commute with λ^0 and hence with $\bar{e}_\mu$. They remain as massless $SU(N)$ gauge fields with their own Yang-Mills dynamics. Thus, at low energies, integrating out the heavy axial-vector and tensor connections leaves ordinary TGR, or equivalently, Einstein gravity, for the neutral tetrad, together with the massless $SU(N)$ vector sector and higher-derivative corrections generated by the heavy fields.

The $SL(2, C)$ EC term (7)) and the TGR tetrad-torsion term (30) are both dimension-two gravitational operators, but they play different roles in the present construction. The EC term fixes the observed Planck scale and dominates the infrared graviton dynamics when $M \ll M_P$. The TGR term is introduced instead of the naive tetrad kinetic term (29) because its specific torsion-scalar combination provides a ghost-free kinetic term for the dynamical tetrad. Expanded

around a nondegenerate background, it gives the standard Fierz-Pauli kinetic structure (31) for the massless graviton rather than the ghost modes generated by a generic quadratic tetrad-derivative term. At the same time, when this TGR torsion is built with the full $SL(2N, C)$ -covariant derivative, it provides algebraic lifting of the noncompact axial-vector and tensor spin connections. Thus, M is the scale of tetrad-induced $SL(2N, C)$ breaking and of the heavy connection masses, while M_P continues to control Newton's constant.

The constraint (24) fixes only the overall length of the unified tetrad, leaving a continuous set of possible orientations in hyperflavor space. Moreover, in the generic parametrization (25), part of the flavored tetrad modes are absorbed by the broken tensor and axial-vector gauge fields, while the remaining modes stay as physical fields and must be given large masses. Both issues are resolved once quantum fluctuations of the gauge fields are included: they generate a Coleman-Weinberg potential [17] that selects the hyperflavor-blind vacuum and lifts the hyperflavored symmetric modes.

IV. LAGRANGIANS

Having formulated the $SL(2N, C)$ -invariant presoldering theory, we now turn to the gauge-field Lagrangians in the postsoldering phase. Once the neutral tetrad branch is selected, the two premetrics become inverse to one another, and the upper gauge connection is identified with the raised lower connection. Consequently the presoldering gauge invariants (21), (22) reduce to the usual Yang-Mills-type quadratic structures.

In general, an $SL(2N, C)$ -invariant action may contain both linear and quadratic terms in the field strengths, each with its own coupling. For our purposes, we focus on the quadratic sector and require it to be governed by a single universal gauge coupling, common to vector, axial, and tensor parts of the connection (14). The corresponding linear curvature term is the Einstein-Cartan-type action (7). In the present approach, it may be viewed as an emergent contribution generated primarily by fermion loops. In this way, one can maintain hyperunification in the quadratic sector while letting gravity arise from quantum effects, as will be seen in the next section. The present section is therefore devoted mainly to the quadratic-strength-tensor Lagrangians, which determine the propagating gauge-field content of the postsoldering theory.

We first consider the vector and axial-vector components of the connection and then turn to the full gauge multiplet, where the tensor connection requires the ghost-free curvature-squared combination.

A. Vector and axial-vector fields

We first examine the part of the postsoldering Lagrangian associated with the $SU(N)$ vector and axial-vector fields,

which are the basic spin-one carriers of hyperflavor. From the quadratic-strength term built out of (15), one finds

$$\begin{aligned} e\mathcal{L}_H^{VA} &= -\frac{1}{2}\text{Tr}[(V_{\mu\nu}^k\lambda^k + \gamma_5 A_{\mu\nu}^k\lambda^k)^2/4] \\ &= -\frac{1}{4}(V_{\mu\nu}^k)^2 - \frac{1}{4}(A_{\mu\nu}^k)^2 \\ &= -\frac{1}{4}[\partial_{[\mu}V_{\nu]}^k - gf^{ijk}(V_{\mu}^iV_{\nu}^j + A_{\mu}^iA_{\nu}^j)]^2 \\ &\quad - \frac{1}{4}[\partial_{[\mu}A_{\nu]}^k - gf^{ijk}(V_{\mu}^iA_{\nu}^j)]^2, \end{aligned} \quad (37)$$

where $V_{\mu\nu}^k$ and $A_{\mu\nu}^k$ are the corresponding field strengths. The vector fields thus have the standard $SU(N)$ gauge structure. The axial-vector fields, however, are not gauge bosons of an independent surviving axial gauge symmetry. They transform as adjoint fields under the residual compact $SU(N)$, with interactions fixed by the original $SL(2N, C)$ curvature. The terms involving A_{μ}^k reflect the noncompact part of the hyperunified algebra. After tetrad condensation, these axial-vector fields acquire mass (36) from the tetrad kinetic sector and decouple at energies well below the scale gM .

In the matter sector, the vector fields couple to ordinary fermions through

$$e\mathcal{L}_{HM}^V = -\frac{g}{2}V_{\mu}^i\bar{\Psi}e_{aI}^{\mu}\gamma^a\lambda^i\Psi,$$

whereas the axial-vector couplings vanish for a neutral tetrad, so A_{μ}^k have no direct interactions with fermionic matter on the neutral branch. They can still participate in loop processes through their self-interactions and through their interactions with the vector fields. Present experimental data [18] do not forbid heavy states with such quantum numbers in the multi-TeV range. Once they are integrated out, the vector sector recovers gauge invariance at lower energies. Thus, the postsoldering vector sector contains massless $SU(N)$ gauge bosons V_{μ}^k , while the axial-vector fields A_{μ}^k are heavy adjoint states associated with the lifted noncompact directions.

B. Total gauge multiplet

As discussed earlier, the quadratic part of a consistent postsoldering $SL(2N, C)$ theory must contain strength-tensor invariants for the vector, axial-vector, and tensor components of the total connection. For the vector and axial-vector sectors, the Yang-Mills form is sufficient. The tensor sector is more restrictive: a generic curvature-squared spin-connection term propagates ghosts. Among the possible combinations, the GFC, identified for the first time in Refs. [14,15], to the best of our knowledge, stands out as the appropriate choice. In the pure $SL(2, C)$ case, this Lagrangian reads

$$e\mathcal{L}_G^{(2)} = \lambda T_{[ab]}(T^{abcd} - 4T^{acbd} + T^{cdab}),$$

where $T^{abcd} = T_{\mu\nu}^{[ab]}e^{\mu c}e^{\nu d}$ and $T_{\mu\nu}^{[ab]}$ is the curvature of the spin connection. Writing everything in terms of the tensor fields and tetrads gives

$$\begin{aligned} e\mathcal{L}_G^{(2)} &= \lambda(T_{[ab]}^{\mu\nu}T_{\mu\nu}^{[ab]} - 4T_{[ab]}^{\mu\nu}T_{\rho\nu}^{[ac]}e_{\mu c}e^{\rho b} \\ &\quad + T_{[ab]}^{\mu\nu}T_{\rho\sigma}^{[cd]}e_{\mu c}e_{\nu d}e^{\rho a}e^{\sigma b}), \end{aligned} \quad (38)$$

where λ is a constant. In this model, the spin connection becomes a propagating field, but not as an ordinary Yang-Mills gauge field. The special relative coefficients in the GFC eliminate the dangerous ghost modes and isolate the healthy spin-connection excitation identified as the massive pseudoscalar state. In the present hyperunified theory, the actual propagating tensor-sector mode will be determined below after the TGR mass lifting terms are included.

We now embed this structure into the postsoldering hyperunified $SL(2N, C)$ theory by writing a general quadratic Lagrangian built from the total curvature and the neutral tetrad background

$$e\mathcal{L}_H^{(2)} = -\lambda_I\text{Tr}(aI^{\mu\nu}I_{\mu\nu} + bI^{\mu\nu}I_{\rho\nu}e_{\mu}e^{\rho} + cI^{\mu\nu}I_{\rho\sigma}e_{\mu}e_{\nu}e^{\rho}e^{\sigma}), \quad (39)$$

with constants a, b, c to be fixed by the requirement that the $SL(2, C)$ limit of (39) reproduces (38). It is important to distinguish the origin of the ghost-free kinetic projector from the origin of the algebraic lifting. The special GFC coefficients (1, -4, 1) are fixed solely by the Lorentz spin-connection kinetic problem and therefore apply independently to each hyperflavor component $T_{\mu}^{[ab]K}$. In the pure $SL(2, C)$ case, the degeneracy of this projector is lifted by the linear Einstein-Cartan term, whereas in the flavored $SL(2N, C)$ sectors no corresponding EC term is available; there, the same degeneracy is lifted instead by the TGR-induced tensor mass term. Thus, the GFC structure is universal, while the lifting mechanism is sector dependent.

Using neutral tetrads satisfying (19) and the trace properties of Dirac and $SU(N)$ matrices, one finds for the vector and axial-vector contributions

$$\begin{aligned} \text{Tr}(I^{(W)\mu\nu}I_{\mu\nu}^{(W)}) &= (W_{\mu\nu}^k W^{\mu\nu k})/2, \\ \text{Tr}(I^{(W)\mu\nu}I_{\rho\nu}^{(W)}e_{\mu}e^{\rho}) &= 2(W_{\mu\nu}^k W^{\mu\nu k}), \\ \text{Tr}(I^{(W)\mu\nu}I_{\rho\sigma}^{(W)}e_{\mu}e_{\nu}e^{\rho}e^{\sigma}) &= 4(W_{\mu\nu}^k W^{\mu\nu k})(W \equiv V, A) \end{aligned}$$

and for the tensor submultiplet

$$\begin{aligned}\text{Tr}(I^{(T)\mu\nu} I_{\mu\nu}^{(T)}) &= T_{[ab]}^{\mu\nu K} T_{\mu\nu}^{[ab]K}, \\ \text{Tr}(I^{(T)\mu\nu} I_{\rho\nu}^{(T)} e_\mu e^\rho) &= 3T_{[ab]}^{\mu\nu K} T_{\mu\nu}^{[ab]K} + 4(T_{[ab]}^{\mu\nu K} T_{\rho\nu}^{[ac]K} e_{\mu c} e^{\rho b}), \\ \text{Tr}(I^{(T)\mu\nu} I_{\rho\sigma}^{(T)} e_\mu e_\nu e^\rho e^\sigma) &= 2T_{[ab]}^{\mu\nu K} T_{\mu\nu}^{[ab]K} + 8(T_{[ab]}^{\mu\nu K} T_{\rho\sigma}^{[cd]K} e_{\mu c} e_{\nu d} e^{\rho a} e^{\sigma b}).\end{aligned}$$

Combining these pieces, the quadratic Lagrangian involves

$$(a/2 + 2b + 4c)W^2 + (a + 3b + 2c)TT + 4bTTee + 8cTTeeee, \quad (40)$$

where W^2 denotes the vector/axial-vector term and the symbols TT , etc. represent the tensor structures in (38).

To reproduce the pure gravity GFC combination (38), one must impose

$$a + 3b + 2c = 1, \quad b = -1, \quad c = 1/8 \rightarrow a = 15/4,$$

which fixes a, b, c and yields a coefficient $(3/8)$ for W^2 in (40). With these values, the hyperunified quadratic Lagrangian becomes

$$e\mathcal{L}_H^{(2)} = -\frac{1}{4}(V_{\mu\nu}^k V^{\mu\nu k} + A_{\mu\nu}^k A^{\mu\nu k}) - \frac{1}{4} \left(T_{[ab]}^{\mu\nu K} T_{\mu\nu}^{[ab]K} - 4T_{[ab]}^{\mu\nu K} T_{\rho\nu}^{[ac]K} e_{\mu c} e^{\rho b} + T_{[ab]}^{\mu\nu K} T_{\rho\sigma}^{[cd]K} e_{\mu c} e_{\nu d} e^{\rho a} e^{\sigma b} \right) \quad (41)$$

after choosing $\lambda_I = -2/3$, so that the vector and axial sectors, given above in (37), have canonical normalization. A further rescaling of tensor fields and of the coupling

$$T_{[ab]}^{\mu\nu K} \rightarrow \sqrt{8/3} T_{[ab]}^{\mu\nu K}, \quad g \rightarrow \sqrt{3/8} g$$

puts the tensor sector in canonical form as well. The result is a unified quadratic postsoldering Lagrangian in which the vector, axial-vector, and tensor curvatures are normalized consistently, and a single gauge coupling g controls the interactions of all components of the $SL(2N, C)$ connection.

C. Propagating fields

Whereas the massless vector and massive axial-vector submultiplets of the total gauge multiplet I_μ (14) freely propagate, the tensor field submultiplet may, in general, contain the ghostly kinetic terms. In this connection, let us now examine which states are actually propagated by the tensor part of the GFC term in the Lagrangian (41) and how they are compatible with the lifting of noncompact directions mentioned above.

For this purpose, it is sufficient to work in the quadratic approximation around the neutral tetrad background and to keep the bilinear derivative part of the tensor curvature. We write

$$T_{\mu\nu}^{[ab]K} = \partial_\mu T_\nu^{[ab]K} - \partial_\nu T_\mu^{[ab]K}$$

and introduce local Lorentz indices,

$$T_{abc}^K = e_a^\mu T_{\mu bc}^K, \quad T_{abc}^K = -T_{acb}^K.$$

For every hyperflavor index $K = 0, 1, \dots, N^2 - 1$, this can be decomposed into irreducible Lorentz pieces as

$$T_{abc}^K = q_{abc}^K + \frac{1}{3}(\eta_{ab} v_c^K - \eta_{ac} v_b^K) + \frac{1}{6} \epsilon_{abcd} B^{dK}.$$

Here

$$v_a^K = T_{ba}^{bK}, \quad B^{aK} = \epsilon^{abcd} T_{bcd}^K,$$

while the remaining tensor q_{abc}^K satisfies

$$q_{ba}^{bK} = 0, \quad \epsilon^{abcd} q_{bcd}^K = 0.$$

The GFC in the Lagrangian (41) in local Lorentz indices looks as

$$\mathcal{N}_T^K = T_{abcd}^K T^{abcdK} - 4T_{abcd}^K T^{acbdK} + T_{abcd}^K T^{cdabK}. \quad (42)$$

Substituting the axial irreducible component

$$T_{abc}^K \Big|_B = \frac{1}{6} \epsilon_{abcd} B^{dK}$$

into the GFC bracket (42), one obtains

$$e\mathcal{L}_T^{(2)} \Big|_B = \frac{1}{4} (\partial_a B^{aK})^2.$$

Thus, the GFC term does not give a Maxwell-type kinetic term for the full pseudovector B_a^K . It gives only a kinetic

structure for its divergence. The transverse components of B_a^K , as well as the q_{abc}^K and v_a^K irreducible components, do not propagate from the GFC bracket alone. The possible propagating tensor-sector state is therefore the longitudinal pseudoscalar combination

$$\pi^K \sim \partial_a B^{aK}.$$

This conclusion must be combined with the algebraic mass lifting terms generated by the TGR tetrad sector considered in Sec. III D. In the flavor-blind tetrad background (19) and with the standard normalization of the $SL(2N, C)$ generators taken in (14), the tensor contribution to the covariant torsion is given in (35). For the axial irreducible tensor component, this gives

$$C_{abc}^K \Big|_B = -\frac{g}{3} \sqrt{\frac{2}{N}} \epsilon_{abcd} B^{dK}.$$

Because of its total antisymmetry, the trace vector $C_a^K \equiv C_{ba}^{bK}$ vanishes, so the last term in TGR torsion scalar $\mathbb{T}(C)$ does not contribute to the B_a^K sector. As a result, one obtains

$$C_{abc}^K C^{abcK} = -\frac{4}{3N} g^2 B_a^K B^{aK}$$

and eventually

$$\mathbb{T}(C) \Big|_B = \frac{1}{3N} g^2 B_a^K B^{aK}.$$

Combining the GFC derivative term with the TGR lifting term gives

$$e\mathcal{L}_B^{(2)} = \frac{1}{4} (\partial_a B^{aK})^2 + \frac{1}{2} C_B B_a^K B^{aK}, \quad C_B = -\frac{2}{3N} g^2 M^2. \quad (43)$$

The sign of the algebraic term would be problematic if B_a^K were an ordinary propagating pseudovector. However, the GFC bracket does not propagate B_a^K as a full vector field. Only its longitudinal pseudoscalar component, $\pi^K = \partial_a B^{aK}$, is dynamical. Varying (43) with respect to B_a^K and taking the divergence gives

$$(\square - 2C_B) \pi^K = 0,$$

which can equivalently be represented by the effective Lagrangian

$$e\mathcal{L}_{\text{eff}}^{(\pi)} = \frac{1}{2} (\partial_a \pi_c^K) (\partial^a \pi_c^K) - \frac{1}{2} m_\pi^2 \pi_c^K \pi_c^K, \\ \pi_c^K = \sqrt{3N/2} \pi^K / gM$$

for the canonically normalized pseudoscalar fields.

Remarkably, the sign fixed by the ordinary TGR sector in (30), which provides the correct kinetic term for the tetrad and eventually for the graviton, is precisely the sign required for the longitudinal pseudoscalar selected by the GFC kinetic structure to be ghost-free and nontachyonic. The transverse components of B_a^K , together with v_a^K and q_{abc}^K , remain nonpropagating auxiliary fields at the quadratic level. Hence, the tensor connection does not propagate as a full tensor gauge multiplet; it propagates one pseudoscalar state π^K for each hyperflavor index K .

Thus, at the quadratic level, the propagating physical sector contained in (41), supplemented by the TGR lifting masses, may be written as

$$e\mathcal{L}_{H\text{phys}}^{(2)} = -\frac{1}{4} V_{\mu\nu}^k V^{\mu\nu k} - \frac{1}{4} A_{\mu\nu}^k A^{\mu\nu k} + \frac{1}{2} m_A^2 A_\mu^k A^{\mu k} \\ + \frac{1}{2} (\partial_a \pi_c^K) (\partial^a \pi_c^K) - \frac{1}{2} m_\pi^2 \pi_c^K \pi_c^K, \quad (44)$$

where the corresponding mass terms have also been included

$$m_A^2 = \frac{36}{N} g^2 M^2, \quad m_\pi^2 = \frac{4}{3N} g^2 M^2.$$

Finally, let us extract the coupling of the physical pseudoscalar π_c^K to fermions. Using the axial irreducible component

$$T_{abc}^K \Big|_B = \frac{1}{6} \epsilon_{abcd} B^{dK},$$

in the relevant tensor part of the gauge-field fermion coupling in (16) one comes to the axial-current coupling

$$e\mathcal{L}_{B\Psi} = \frac{g}{4} \sqrt{\frac{2}{N}} B_a^K J_5^{aK}, \quad J_5^{aK} \equiv \bar{\Psi} \gamma^a \gamma_5 \lambda_K \Psi,$$

up to the overall sign fixed by the convention for ϵ_{abcd} and γ_5 . Eliminating the auxiliary pseudovector B_a^K through its algebraic equation of motion gives, to first order in the fermion current,

$$e\mathcal{L}_{\pi\Psi} = -\frac{\sqrt{3}}{4M} (\partial_a \pi_c^K) \bar{\Psi} \gamma^a \gamma_5 \lambda_K \Psi. \quad (45)$$

Thus, the physical pseudoscalar mode contained in the tensor connection couples only to the nonconserved part of the hyperflavor axial fermion current, with a strength suppressed by the TGR scale M . The absence of the gauge coupling constant g in (45) is natural: π_c^K is the canonically normalized longitudinal mode of the heavy tensor connection, so its derivative coupling is solely controlled by the symmetry-breaking scale M .

D. Hyperflavor-blind vacuum

In the fixed-norm formulation, the length constraint (24) may be imposed via a Lagrange multiplier. It fixes the norm of the tetrad but leaves its internal orientation undetermined. A convenient parametrization of the vacuum is (see footnote 4)

$$\bar{e}_\mu = v_\mu \otimes H, \quad v_\mu = e_\mu^a \gamma_a, \quad H^\dagger = H, \quad (46)$$

so that the hyperflavor dependence resides entirely in the Hermitian matrix H . The choice $H \propto \mathbf{1}_N$ realizes a hyperflavor-blind vacuum (HBV) that preserves the residual $SU(N)$ and evidently satisfies (24). It is, however, necessary to show that quantum corrections select this orientation as the true minimum.

The one-loop Coleman-Weinberg (CW) potential for the vector fields in a background $\bar{e}$ is

$$U_V(e) = \frac{3}{64\pi^2} \sum_n m_n^4(e) \left(\ln \frac{m_n^2(e)}{\mu^2} - \frac{5}{6} \right),$$

where m_n^2 are eigenvalues of the vector mass matrix $M_V^2(e)$ and μ is the renormalization scale. For $SU(N)$ generators $G_V = \lambda_k$ one finds

$$\begin{aligned} (M_V^2)_{kl}(e) &= -g^2 \text{Tr}([\lambda_k, \bar{e}_\mu][\lambda_l, \bar{e}^\mu]) \\ &= -4g^2 M^2 \text{Tr}([\lambda_k, H][\lambda_l, H]), \end{aligned}$$

where we used

$$[\lambda_k, \bar{e}_\mu] = v_\mu \otimes [\lambda_k, H].$$

If H is proportional to the identity, all commutators vanish, the vector mass eigenvalues are zero, and the vector contribution to the CW potential vanishes after the standard subtraction. For a nonsinglet orientation of H , some vector fields acquire nonzero masses and generate a nontrivial positive contribution to the curvature of the effective potential around the hyperflavor-blind point. Thus, the vector sector selects the hyperflavor-blind direction as the preferred vacuum orientation.

The axial and tensor contributions to the CW potential, $U_A(e)$ and $U_T(e)$, do not change the fact that the hyperflavor-blind direction is selected by the compact vector sector. Near the HBV, their leading masses are set by the scale gM and are largely controlled by the Dirac structure of the corresponding generators, thus contributing only an additive constant at $H \propto \mathbf{1}_N$. Nevertheless, their dependence on tetrad fluctuations gives a nonzero curvature of the effective potential in the hyperflavor-adjoint directions and thereby generates masses for the hyperflavor-carrying tetrad modes.

In the postsoldering phase, the lower and upper tetrad-type fields are no longer independent premetric variables.

The flavored tetrads, e_μ^{*ak} and $E_a^{*\mu k}$, as well as the two premetrics $g_{\mu\nu}$ and $G^{\mu\nu}$, are related as Lorentz-conjugated descriptions of the same metric geometry. Thus, the independent hyperflavor-adjoint tetrad fluctuations reduce to $16(N^2 - 1)$ components. At the HBV these decompose, in local Lorentz indices, into antisymmetric and symmetric parts

$$e_\mu^{*ak} = \frac{1}{2} e_{\mu b} (e^{*[ab]k} + e^{*\{ab\}k}), \quad (47)$$

where $e_{\mu b}$ represents the corresponding neutral tetrad on the branch (46).

The antisymmetric $6(N^2 - 1)$ modes in (47), being Lorentz-rotation-type tetrad components, are absorbed by the tensor gauge fields $T_\mu^{[ab]K}$. Among the symmetric $10(N^2 - 1)$ modes, the $(N^2 - 1)$ adjoint traces $e_\mu^{*\mu k}$ become longitudinal for the axial vectors A_μ^k . This leaves $9(N^2 - 1)$ independent symmetric traceless modes in the $SU(N)$ adjoint, apart from the neutral tetrad e_μ^a in the singlet sector. Its six antisymmetric components analogously provide masses for the $SL(2, C)$ spin connections, while its ten symmetric components contain the ordinary metric perturbation. The role of the TGR combination in (30) is precisely to give these symmetric neutral components the standard massless spin-two kinetic structure, rather than a generic ghostful tetrad kinetic term. Consequently, one obtains the Fierz-Pauli kinetic operator (31) for the metric variations $h_{\mu\nu}$. Four components are gauge redundancies of diffeomorphism invariance, while the components h_{00} and h_{0i} enter without independent time-derivative kinetic terms and act as constraint variables. Their equations remove the nonpropagating scalar and vector parts of $h_{\mu\nu}$, so the neutral TGR tetrad sector propagates only the two transverse-traceless graviton polarizations.

Near the HBV, the axial and tensor loops generate a positive-definite CW mass matrix along the hyperflavor-adjoint symmetric directions. Schematically,

$$\begin{aligned} M_{A,T}^2(e) &\propto g^2 \text{Tr}(i[G_{A,T}, e_\mu]i[G_{A,T}, e^\mu]), \\ G_A &= \gamma_5 \lambda^k, \quad G_T = \gamma_{ab} \lambda^K \end{aligned}$$

and at quadratic order this gives

$$\Delta \mathcal{L}_{\text{CW}}^{(2)} = -\frac{1}{2} e_\mu^{*ak} (\mathcal{M}^2)_{kl} e_a^{*\mu l}, \quad (\mathcal{M}^2)_{kl} \sim \frac{g^4}{16\pi^2} M^2$$

for the $9(N^2 - 1)$ flavored symmetric modes. This CW lifting applies only to the flavored tetrad components e_μ^{*ak} , while the neutral gravitational tetrad remains massless. Its masslessness is protected by the residual spacetime gauge symmetries: the TGR sector gives the usual Fierz-Pauli kinetic operator for the metric perturbation, but no graviton mass term. Thus, as required, the gravitational sector is

described entirely by the flavor-singlet tetrad (19), while the flavored tetrad components are either absorbed by the tensor and axial-vector gauge fields or acquire large CW masses and thus decouple.

Summarizing this stage: (i) the dynamical tetrad selects the hyperflavor-blind branch of the theory, thereby breaking the noncompact directions of $SL(2N, C)$ while preserving the compact hyperflavor group $SU(N)$; (ii) on this branch, the tetrad-condensation mechanism filters the original unified multiplet: the compact vector gauge fields remain massless, whereas the noncompact axial-vector and tensor directions, together with the nonsinglet tetrad components, decouple from the low-energy sector; (iii) in general, the theory may contain two independent dimensional scales, M_1 and M_2 , associated with the independent linear and quadratic curvature structures. At the fundamental level, these scales are arbitrary parameters, although one or both of them may be related to the Planck scale; and (iv) this arbitrariness naturally motivates an emergent-gravity scenario, in which the low-energy Einstein-Cartan operator (7) is not necessarily fundamental, but can be generated radiatively. This possibility will be discussed in detail in the next section.

V. EMERGING GRAVITY

A. Preliminaries

We now turn to the emergence of the EC term from radiative corrections. The hyperunified $SL(2N, C)$ gauge theory described above is formulated in a Yang-Mills-like fashion, with all field strengths entering quadratically and tetrads treated as constrained dynamical fields. The nonlinear constraint (24) fixes the soldering norm, while the dynamically selected hyperflavor-blind branch (34) corresponds to the neutral tetrad component (19) and realizes the spontaneous breaking $SL(2N, C) \rightarrow SL(2, C) \times SU(N)$. After these steps, the low-energy gauge multiplet consists of the $SU(N)$ vectors and the flavor-singlet tensor connection $T_\mu^{[ab]}$.

At this stage, the Lagrangian need not contain a tree-level EC term linear in the tensor curvature $R[T]$, even though such a term is allowed by symmetry. Instead, we will show that this term is naturally generated at one loop by fermion bubbles that involve both the tensor connection and the tetrad. Two distinct but complementary routes may be considered, depending on the masses of the fermions contributing to the loops. In a “threshold” scenario, heavy vectorlike fermions of mass m_ψ induce a finite, regulator-independent EC term proportional to $N_f m_\psi^2$ (for N_f species), reminiscent of Sakharov’s induced gravity [19] and related effective field theory analyses [20–22], although such a mechanism has not yet been applied to gravity-unified theories. In a “universal cutoff” scenario, if gravity or some UV completion provides a physical cutoff $\Lambda \sim M_P$ for the relevant loop integrals, even light fermions

induce an EC term of order $N_f \Lambda^2$; this is regulator dependent but fixes the correct overall scale. In both pictures, the EC term becomes radiative in origin, expressed in terms of the underlying $SL(2N, C)$ parameters while maintaining the unified structure of the quadratic sector.

B. Radiative corrections

After tetrad condensation, the quadratic curvature sector is governed by the ghost-free Lagrangian $\mathcal{L}_G^{(2)}$ (38) embedded into the $SL(2N, C)$ framework. A separate linear curvature term of the EC form for the surviving flavor-singlet tensor-field submultiplet

$$e\mathcal{L}_{\text{EC}} = \frac{1}{2\kappa} e_{[a}{}^\mu e_{b]}{}^\nu R_{\mu\nu}^{ab}[T] \quad (48)$$

is allowed by symmetry but need not appear at tree level. We show that this structure—with $R_{\mu\nu}^{ab}[T]$ denoting the curvature of the flavor-singlet tensor connection $T_\mu^{[ab]}$ —is generated radiatively by fermion loops in which both the tetrad and the tensor field couple to matter.

The relevant interaction vertices follow from the fermion kinetic term and the $SL(2N, C)$ -covariant derivative (16) when specialized to the neutral tensor field. The basic insertions in the fermion loops are

$$\begin{aligned} V_e &: ie^\mu{}_a \bar{\Psi} \gamma^a \overleftrightarrow{\partial}_\mu \Psi, \\ V_T &: \frac{ig}{4} e^\mu{}_c \bar{\Psi} \gamma^c \gamma_{ab} \Psi T_\mu^{[ab]}. \end{aligned}$$

Correspondingly, the vertex V_e brings an external momentum, while the vertex V_T insertion involves both an external tetrad leg and an external T leg with the coupling constant g .

The minimal one-loop origin of the operator $e \wedge e \wedge R[T]$ is then a pair of fermion-bubble diagrams, illustrated in Fig. 1. The first contains one pure tetrad insertion V_e and one mixed tensor-tetrad insertion $V_T - V_e$, producing a three-point function corresponding to $e \wedge e \wedge dT$. The second contains two mixed $V_T - V_e$ insertions, producing $e \wedge e \wedge (gT \wedge T)$. Taken together they reconstruct the full $e \wedge e \wedge R[T]$ structure with $R[T] = dT + gT \wedge T$.

The coefficient of (48) depends on the fermion content and on the UV completion. Two complementary scenarios

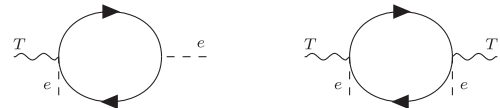


FIG. 1. Fermion-bubble diagrams generating the Einstein-Cartan term. Left: one pure tetrad insertion V_e and one mixed $V_T - V_e$ insertion generate $e \wedge e \wedge dT$. Right: two mixed $V_T - V_e$ insertions generate $e \wedge e \wedge (gT \wedge T)$. Their sum reconstructs $e \wedge e \wedge R[T]$.

are illustrative. In a threshold scenario, vectorlike fermions with masses $m_\psi \sim M_P$ generate a regulator-independent contribution

$$e\mathcal{L}_{\text{EC}}^{(t)} \sim \frac{N_f}{16\pi^2} m_\psi^2 e \wedge e \wedge R[T],$$

so that $M_P^2 \sim (N_f/16\pi^2)m_\psi^2$. In a universal cutoff scenario, quantum gravity itself provides a physical cutoff $\Lambda \sim M_P$, and even light (or massless) fermions induce

$$e\mathcal{L}_{\text{EC}}^{(c)} \sim \frac{N_f}{16\pi^2} \Lambda^2 e \wedge e \wedge R[T],$$

with the Planck scale determined by the UV boundary condition set by the UV completion. In both cases, the emergent EC term is expressed in terms of the parameters of the underlying $SL(2N, C)$ gauge theory and its fermion sector. The quadratic curvature terms are already present via the GFC-type Lagrangian, so the full low-energy gravitational sector is an EC-plus-curvature-squared theory with the correct sign and no ghosts.

Analogously, the TGR term is generated instead by torsion-square-type fermion-loop structures with insertions

$$\begin{aligned} V_e - V_e &\rightarrow (\partial e)^2, \\ V_e - V_T &\rightarrow (\partial e)(Te), \\ V_T - V_T &\rightarrow (Te)^2. \end{aligned}$$

Together these reconstruct the tensor-connection-covariant part of the TGR torsion

$$C_{\mu\nu}^a = C_{\mu\nu}^a(e) + C_{\mu\nu}^a(T)$$

and hence the corresponding torsion-scalar operator $T(C)$ in (30), (33). On the strict hyperflavor-blind branch, the direct axial fermion vertex vanishes. Therefore, the fermion determinant on that branch need not generate the axial-vector lifting part $C_{\mu\nu}^a(A)$ of the TGR torsion.

C. Concluding remarks

These considerations show that the linear curvature term allowed by $SL(2N, C)$ is generically a sum of a bare piece and an induced one-loop contribution. In a threshold scenario, superheavy vectorlike fermions near the Planck scale generate a finite EC term, which may dominate over any tree-level coefficient if the latter is tied to lower mass scales. The Planck mass may then be viewed as largely radiative in origin, controlled by the multiplicity and masses of heavy fermion states.

In the universal-cutoff picture, quantum gravity or some UV completion supplies a physical scale $\Lambda \sim M_P$ that regularizes the relevant loop integrals. The induced EC term from light or massless fermions then takes the form

$\Delta M^2 \propto N_f \Lambda^2$, so the observed Planck mass squared M_P^2 can be viewed as the sum $M^2 + \Delta M^2$, where M^2 is a bare parameter. Assuming M^2 is tied to much lower scales [e.g., the $SL(2N, C)$ breaking scale or the weak scale] makes the induced contribution dominant.

One may further entertain the idea that the $SL(2N, C)$ theory (including its SM sector) is classically scale invariant, with all large scales generated radiatively or via the UV completion. In such a setting the EC term is absent at tree level, so that the Planck scale and the effective EC coupling necessarily arise solely from such radiative corrections. This does not, by itself, solve the hierarchy problem, but it clarifies how a large gravitational scale may arise without introducing Planck-scale vacuum expectation values (VEVs) inside the $SL(2N, C)$ theory.

Finally, let us comment on the radiative stability of the ghost-free TGR and GFC frameworks used above. These structures should be regarded as the unitary kinetic projectors of the two dynamical sectors, not as accidental tunings among arbitrary quadratic invariants. The TGR torsion term selects the tetrad kinetic structure that propagates only the two massless spin-two graviton polarizations, eliminating the would-be ghostlike scalar, vector, and antisymmetric tetrad components. The curvature-square GFC plays the analogous role for the spin connection: its special structure projects out the unwanted ghostlike connection components. In a unitary quantum theory, radiative corrections may renormalize the overall coefficients of these admissible projectors and may also generate higher-dimensional operators suppressed by the ultraviolet scale, but they cannot reintroduce uncanceled negative-norm poles into the physical spectrum. Any same-order deformation away from the TGR or GFC surfaces that would produce such poles must therefore be absent, canceled by the full set of quantum contributions or fixed by the unitarity-preserving renormalization conditions. This is the sense in which the TGR and GFC frameworks are radiatively stable within the unitary effective theory.

VI. $SL(2N, C)$ BREAKING SCENARIO

A. Postsoldering phase

In general, the logic of the broken phase should not be understood as a truncation of the original $SL(2N, C)$ theory to an ordinary $SL(2, C)$ theory. Rather, we keep the full $SL(2N, C)$ -covariant structure, while the soldering branch selects the hyperflavor-blind direction (19). Thus, the physical spacetime metric is determined by the neutral tetrad e_μ^a , but the covariant derivatives acting on all multiplets remain the full $SL(2N, C)$ -covariant derivatives. In particular, the covariant derivative for a tetrad in (32) is written with the complete $SL(2N, C)$ connection. This is the standard Higgs-phase logic: the vacuum chooses a direction in group space, but the broken gauge fields still appear through the covariant derivative of that vacuum.

Consequently, although the tetrad is neutral in hyperflavor space, the full connection still probes the broken directions through commutators with the neutral tetrad.

This same principle is used in all sectors of the theory. First, in the tetrad sector, the TGR torsion scalar is interpreted as the ghost-free kinetic projector for the dynamical tetrad. On the hyperflavor-blind branch, it gives the healthy massless spin-two kinetic structure for the neutral tetrad fluctuations and removes the would-be ghostlike scalar, vector, and antisymmetric tetrad components. The essential requirement is therefore the TGR no-ghost structure for the physical neutral tetrad sector. The full $SL(2N, C)$ -covariant derivative may also contain lifting terms for the broken connection components, but the absence of a particular radiatively induced axial-vector contribution does not by itself reintroduce ghostlike tetrad modes.

Second, the spin-connection sector is treated analogously. The full $SL(2N, C)$ gauge multiplet is retained, but its Yang-Mills-type kinetic term must be chosen in the generalized GFC combination. This special curvature-square structure is the no-ghost kinetic projector for the spin connections, just as TGR is the no-ghost kinetic projector for the tetrad. Thus, the $SL(2N, C)$ gauge fields are not arbitrary higher-derivative connection modes; their healthy propagation is defined by the GFC-type structure.

Third, the same broken-phase logic applies to induced gravitational terms. The radiative corrections are to be computed from the full matter coupling, rather than from an artificially neutral-truncated set of interaction vertices. After the effective action is obtained, one evaluates it on the hyperflavor-blind soldering vacuum. In this way, the induced EC term, and likewise the induced TGR-type tetrad kinetic term, should be understood as broken-phase remnants of the full $SL(2N, C)$ -covariant matter theory. The EC term is generated by the familiar fermion-loop mechanism involving tetrad and spin-connection insertions, while the TGR term corresponds to the induced ghost-free kinetic operator for the neutral dynamical tetrad. The loop calculation need not generate every possible broken connection lifting term in the covariant torsion; what is essential is that the induced tetrad kinetic sector lies on the TGR no-ghost surface.

In this sense, the construction is internally consistent. The hyperflavor-blind tetrad vacuum provides the ordinary spacetime geometry, while the full $SL(2N, C)$ -covariant derivatives keep track of the broken gauge directions and their mass generation. The TGR and GFC structures then supply the two required ghost-free kinetic projectors: TGR for the dynamical tetrad and GFC for the spin connection. Radiative fermion loops can induce the low-energy EC and TGR gravitational operators in this broken phase, while the no-ghost structure fixes which quadratic combinations are physically admissible. Thus, one obtains a gravity-inspired hyperunified $SL(2N, C)$ theory without reducing the parent gauge-covariant framework to an ordinary neutral $SL(2, C)$ theory.

B. Interplay of two gravities

In the hyperflavor-blind broken phase, the tetrad selects the neutral direction (19) inside the $SL(2N, C)$ group space, but the theory is not reduced to an ordinary $SL(2, C)$ theory. The covariant derivatives acting on all multiplets still contain the full $SL(2N, C)$ connection I_μ . Thus, EC gravity and TGR gravity in this framework use the same neutral tetrad and the same full spin-connection multiplet, but they organize the gravitational dynamics differently.

The EC-type term is curvature based,

$$e\mathcal{L}_{\text{EC}} \sim M_{\text{EC}}^2 \text{Tr}(e^\mu e^\nu I_{\mu\nu}),$$

where $I_{\mu\nu}$ is the curvature of the full $SL(2N, C)$ connection, projected onto the neutral Lorentz-tetrad direction. Meanwhile, the TGR-type term is torsion based,

$$e\mathcal{L}_{\text{T}} \sim M_{\text{T}}^2 \text{Tr}\mathbb{T}(C)$$

as presented above in Sec. III D. Therefore, EC measures the curvature of the total connection, while TGR measures the covariant nonintegrability of the neutral tetrad under the same total connection. In the limit in which the extra broken connection components are heavy or decouple, both descriptions reduce to the same low-energy neutral metric gravity: the same massless spin-two graviton, the same Newtonian limit, light bending, redshift, and ordinary gravitational-wave propagation. Away from this low-energy neutral limit, they differ through their coupling to the full spin-connection multiplet.

The main difference relevant here appears in the mass terms of the gauge submultiplets, when the generalized GFC term supplies the quadratic kinetic operator for the full $SL(2N, C)$ spin-connection multiplet. Though both gravity operators, $e\mathcal{L}_{\text{EC}}$ and $e\mathcal{L}_{\text{T}}$, leave the vector submultiplets massless, they generate different mass patterns for the axial-vector and tensor submultiplets. The EC term, on the hyperflavor-blind tetrad branch, $e_\mu = e_\mu^a \gamma_a \lambda^0$, projects the full connection curvature onto the Lorentz-tensor part. Its quadratic expansion gives an algebraic mass term of the order M_{EC}^2 for the tensor spin-connection submultiplet $T_\mu^{[ab]K}$, while the axial-vector submultiplet A_μ^k does not enter the neutral EC projection at quadratic order and thereby is not lifted by the EC term itself. By contrast, the TGR term contains the full covariant torsion of the neutral tetrad. Since the broken tensor and axial-vector generators do not commute with the neutral tetrad term, the TGR torsion contains both tensor and axial-vector lifting pieces proportional to M_{T}^2 .

C. Achieving hyperunification

The existence of an $SL(2N, C)$ gauge structure by itself does not automatically imply a full dynamical unification of gravity with the other elementary interactions. The

symmetry organizes the Lorentz, tensor, axial-vector, and internal gauge fields into one $SL(2N, C)$ connection I_μ , whose universal Yang-Mills-type kinetic term must be chosen in the generalized GFC form. However, once tetrad multiplets are present, the same general $SL(2N, C)$ -covariant framework also permits independent gravitational invariants built from the tetrads and the full connection. Thus, besides the GFC gauge kinetic terms $\mathcal{L}_{\text{GFC}}$ presented in terms of the propagating fields in (44), one may also write independent EC- and TGR-type terms, $\mathcal{L}_{\text{EC}}$ and $\mathcal{L}_{\text{T}}$, with their own dimensionful coefficients, M_{EC}^2 and M_{T}^2 . In general, these terms are allowed by the starting $SL(2N, C)$ structure already in the premetric invariant phase and are then reduced on the hyperflavor-neutral tetrad branch discussed above. Therefore, if M_{EC}^2 and M_{T}^2 are arbitrary bare parameters, the theory is not yet a true dynamical unification of gravity with the other forces. It is a unification at the level of covariant derivatives: the spin-connection and internal symmetry objects are embedded into the same $SL(2N, C)$ gauge multiplet, but the gravitational strengths are still inserted independently through the separate mass-squared coefficients M_{EC}^2 and M_{T}^2 . In that case, the Planck scale is not explained by the unified gauge structure; it is put in by hand.

A genuine hyperunification interpretation requires an additional dynamical assumption. The fundamental bare EC and TGR terms should be absent, suppressed, or insignificant compared with the induced terms generated by the heavy matter sector. Then the dimensionful gravitational coefficients are not independent input parameters. Instead, they arise radiatively from fermion loops involving the same $SL(2N, C)$ -covariant matter couplings (16) that also define the unified gauge interaction. In this case, the induced EC and TGR coefficients inherit the same universal gauge coupling g that appears in the covariant derivative, in the $SL(2N, C)$ curvature, and in the generalized GFC term. Schematically, in the chosen normalization,

$$M_{\text{EC}}^2, M_{\text{T}}^2 \sim \frac{N_f}{16\pi^2} g^2 \mathbf{M}^2,$$

up to numerical coefficients, where $\mathbf{M}$ denotes the heavy fermion threshold or cutoff scale. The bare coefficients, $M_{\text{EC},0}^2$ and $M_{\text{T},0}^2$, may then be taken much smaller than the induced threshold contributions, so that the observed gravitational scales are generated by the same universal gauge-coupled matter sector rather than imposed independently. There is, however, an important distinction between the bare $M_{\text{EC},0}^2$ and $M_{\text{T},0}^2$ coefficients. $M_{\text{EC},0}^2$ merely shifts the effective EC scale and can be absorbed into the definition of the total induced gravitational constant. By contrast, $M_{\text{T},0}^2$ not only contributes to this constant, but also supplies a mass lifting term for the axial-vector submultiplet A_μ^k , which would remain massless in the absence of the corresponding TGR lifting.

This clarifies the meaning of hyperunification. The universal GFC term supplies the no-ghost kinetic structure for the full $SL(2N, C)$ gauge multiplet and contains the same gauge coupling g that controls the matter interactions. The EC and TGR terms are allowed by the same symmetry, but if their coefficients are arbitrary bare couplings, they weaken the claim of dynamical unification. The stronger hyperunification scenario is obtained only when these gravitational terms are radiatively induced from the same $SL(2N, C)$ -covariant fermion sector. Then the Planck-scale gravitational dynamics originate from the same high-energy gauge-coupled matter dynamics.

The ultraviolet interpretation should therefore be that the fundamental theory is a fully $SL(2N, C)$ -invariant premetric theory with massless gauge fields and no prior ordinary spacetime metric. The tetrad multiplets define the premetric soldering sector, while the full connection I_μ is governed by the generalized GFC kinetic term. In the infrared, the theory selects the hyperflavor-blind neutral tetrad branch and breaks to the ordinary spacetime phase with the $SL(2, C) \times SU(N)$ symmetry as the surviving gauge structure. The $SU(N)$ vector fields V_μ^k remain massless and the axial-vector multiplet A_μ^k becomes massive at the generic TGR scale through the covariant-torsion lifting term. Meanwhile, the tensor connection sector $T_\mu^{[ab]K}$ produces the massive pseudoscalar multiplet π^K of order the Planck scale. With the TGR and GFC chosen as the no-ghost kinetic projectors, and with positive induced mass-squared coefficients, the infrared broken-phase theory is ghost- and tachyon-free, as summarized in (44). The ordinary gravitational scale is not a fundamental independent constant, but an induced threshold effect of the same $SL(2N, C)$ -covariant matter sector. This is the sense in which the construction becomes a genuine gravity-inspired hyperunification rather than merely a representation-level embedding of gravity and internal gauge fields into a larger group. The most important point is that genuine gravitational unification emerges only in the broken postsoldering phase, in contrast to ordinary grand unification of internal forces, which has both an exact high-symmetry limit and a broken low-energy realization.

VII. FROM GAUGE TO MATTER SECTOR: TRUE HUT CARRIERS

A. Are quarks and leptons truly elementary?

So far we have concentrated on the gauge sector. We now address the matter sector and its relation to hyperunification. After tetrad condensation, the effective local symmetry is $SL(2, C) \times SU(N)$, while the full $SL(2N, C)$ algebra continues to organize multiplets of fields. On the one hand, this yields $SL(2, C)$ gauge gravity; on the other, it provides $SU(N)$ as the candidate grand unified group. However, because $SL(2N, C)$ multiplets carry both spinor and internal indices, such GUT constructions do not accommodate

ordinary quarks and leptons comfortably. The spin-1/2 fundamental representation of $SL(2N, C)$ has quantum numbers that do not match the observed pattern of Standard Model charges, while higher representations generally mix fields of different spins inside a single multiplet and therefore are not suitable for describing purely fermionic states.⁵

This can be seen already in the simplest case of $SU(5)$, which might be expected to arise from an $SL(10, C)$ hyperunification scheme. Some low-dimensional chiral multiplets, say left-handed ones transforming under Ω_L in (9), may then be written in $SU(5) \times SL(2, C)$ components as

$$\Psi_L^{ia}, 10 = (\bar{5}, 2)$$

and

$$\Psi_{L[ai,jb]} = \Psi_{L[ij]\{ab\}} + \Psi_{L\{ij\}[ab]}, \quad 45 = (10, 3) + (15, 1). \quad (49)$$

Here $i, j = 1, \dots, 5$ label $SU(5)$ indices and $a, b = 1, 2$ label $SL(2, C)$ spinor indices. In these decompositions, antisymmetry in the combined (ia) indices implies antisymmetry in i, j and symmetry in a, b , and conversely. One finds that the $\bar{5}$ representation appears naturally, but the usual fermionic decuplet of $SU(5)$ does not emerge from a purely antisymmetric $SL(10, C)$ representation: the tensor (49) instead corresponds to vector and scalar multiplets rather than to a fermionic decuplet. As a result, the standard $SU(5)$ GUT [23], together with many of its supersymmetric [24] or string-inspired extensions [25,26], cannot be straightforwardly realized in this hyperunified setting.

More generally, GUTs in which quarks and leptons are placed in purely antisymmetric representations [such as $SU(11)$ models [27]] are problematic: the spin content of the resulting states does not match that of observed fermions once the requirement of an underlying $SL(2N, C)$ structure is imposed. To maintain spin-1/2 assignments for ordinary quarks and leptons within an $SL(2N, C)$ framework, one would need more complicated multiplets involving both upper and lower indices, which tend to be extremely large and introduce many exotic states.

This strongly points toward a composite interpretation: quarks and leptons should not be identified with fundamental $SL(2N, C)$ spinors, but may instead emerge as bound states of more fundamental chiral preons. For such preons, the $SL(2N, C)$ representation theory becomes much more economical, with preons transforming in fundamental spinor representations and composites living in higher-rank

tensors. In the remainder of this section, we follow this line of thought and argue that an $SL(16, C)$ hyperunified model with $SU(8)$ hyperflavor provides a natural home for three generations of composite quarks and leptons [10].

B. Preons: metaflavors and metacolors

We adopt the preon framework developed in [28]. At very short distances, potentially near the Planck scale, we postulate the existence of $2N$ massless preons, N left-handed and N right-handed, described by independent Weyl spinors P_{iaL} and Q_{iaR} with $i = 1, \dots, N$ and $a = 1, 2$. They carry a common local metaflavor symmetry

$$G_{MF} = SL(2N, C),$$

acting in the fundamental representation on both P_{iaL} and Q_{iaR} . This is the same hyperunified group that organized the gauge and tetrad sectors above, now acting on the truly fundamental matter fields. At the fundamental level we take this metaflavor symmetry to be vectorlike, so that $SL(2N, C)$ acts identically on left- and right-handed preons; left-right symmetry is therefore exact in the preon sector. At large distances, however, the effective symmetry realized on composite states may be reduced to chiral subgroups.

In addition to metaflavor, preons transform under a chiral metacolor gauge group,

$$G_{MC} = SO(n)^L \times SO(n)^R,$$

in whose vector representation they reside. We denote the corresponding indices by α and α' for the left and right metacolor factors, respectively, and write P_{iaL}^α and $Q_{iaR}^{\alpha'}$ with $\alpha, \alpha' = 1, \dots, n$. The chiral metacolor interactions confine preons into bound states, among which quarks, leptons, and possible additional composites may appear with a characteristic confinement radius

$$R_{MC} \sim 1/\Lambda_{MC},$$

where Λ_{MC} is the scale at which metacolor becomes strong. By assumption, left and right metacolor sectors have comparable confinement scales due to the underlying L-R symmetry.

The choice of orthogonal metacolor has several advantages. Vector representations of $SO(n)$ are free of gauge anomalies, consistent with the preon assignment, and orthogonal groups allow composites in a variety of representations of the metaflavor group $SL(2N, C)$, including tensors with both upper and lower indices. Under the combined symmetry $SL(2N, C) \times SO(n)^L \times SO(n)^R$ the preons transform as

$$P_{iaL}^\alpha [N, (1/2, 0); (n, 1)], \quad Q_{iaR}^{\alpha'} [N, (0, 1/2); (1, n)],$$

⁵Thus, the matter-sector question is not merely how to embed known quarks and leptons into $SL(2N, C)$, but which truly elementary spinor fields can consistently carry the full hyperunified symmetry.

where we have indicated the transformation properties under metaflavor, Lorentz, and metacolor factors.

In the limit where metaflavor gauge interactions are formally turned off, the preon system exhibits an additional global chiral symmetry,

$$K(N) = SU(N)_L \times SU(N)_R, \quad (50)$$

acting independently on left- and right-handed preons. A bilinear condensate $\langle P_L Q_R \rangle$ that would generate large Dirac masses of order Λ_{MC} for composites is forbidden by this chiral symmetry. This is a necessary (though not sufficient) condition for the existence of massless composite fermions. In fact, massless composites are expected only when the preon chiral symmetry (50) survives at large distances, and the corresponding pattern is constrained by the 't Hooft anomaly matching (AM) condition [29]. Roughly speaking, the $SU(N)_{L,R}^3$ anomalies carried by preons must match those of the composite fermions in each chiral sector.

An important distinction arises between the vectorlike metaflavor symmetry and the chiral spectator symmetry (50). For the local $SL(2N, C)$, gauge anomalies cancel automatically between left- and right-handed preons and their composites since the theory is vectorlike. For the global chiral symmetry $SU(N)_{L,R}$, however, anomalies must be matched separately in the left and right sectors by suitable combinations of preons and massless composites. In practice, one can imagine gauging $SU(N)_{L,R}$ as a spectator symmetry and introducing additional spectator fermions as in [29]. The AM condition then constrains the number of preon species N and the representations in which the massless composite states transform.

C. Composites: $SL(16, C)$ and residual $SL(2, C) \times SU(8)$

The metaflavor dynamics is governed by the local vectorlike group $SL(2N, C)$, while the chiral symmetry $SU(N)_L \times SU(N)_R$ remains global. To analyze anomaly constraints, one promotes the chiral symmetry temporarily to a spectator gauge symmetry with auxiliary gauge fields, allowing one to compute the $SU(N)_L^3$ and $SU(N)_R^3$ anomalies produced by preons and composites. In this auxiliary theory, anomalies in the $SU(N)_L$ and $SU(N)_R$ sectors are canceled by suitable spectra of spectator fermions, but the AM condition enforces equality of the anomalies carried by preons and by composites in each sector separately.

In the $SL(2N, C)$ metaflavor theory, the cubic $SL(2N, C)^3$ anomalies cancel between left- and right-handed states for any N , since the theory is vectorlike. The nontrivial constraint comes instead from the chiral symmetry (50). Let $a(N)$ be the anomaly coefficient for $SU(N)$ in the fundamental representation [so $a(N) = \pm 1$ for left- and right-handed preons], and let $a(r)$ denote the anomaly coefficient for a given composite representation r .

If i_r counts the net number of massless composites in representation r (positive for left-handed, negative for right-handed states), the AM condition takes the form

$$na(N) = \sum_r i_r a(r), \quad (51)$$

where n is the number of preons of a given chirality. The coefficients $a(r)$ depend on N , so (51) can, in principle, fix N once a particular pattern of composite representations is specified.

To sharpen the constraints, we impose three assumptions: (i) all massless composites contributing to (51) are spin-1/2; (ii) only the simplest three-preon composites formed by metacolor interactions are present; (iii) they form a single irreducible representation of the hyperunified symmetry, which at low energies reduces to a single irreducible representation of $SL(2, C) \times SU(N)$. Assumption (i) means that higher-spin components of generic $SL(2N, C)$ multiplets must become massive and decouple, which, in turn, implies that the preon metaflavor symmetry is effectively reduced to $SL(2, C) \times SU(N)$ at large distances, consistent with the gauge-sector analysis of Secs. II–IV. Assumptions (ii) and (iii) significantly restrict the possible composite spectra.

Under these assumptions, only three-preon composites built from P_L (and analogously from Q_R) are considered. The possible $SU(N)$ representations for spin-1/2 composites are third-rank tensors of the form

$$\begin{aligned} &\Psi_{L,R}^{\{ijk\}}, & \Psi_{[ijk]L,R}, & \Psi_{\{[ij]k\}L,R}, \\ &\Psi_{\{jk\}L,R}^i, & \Psi_{[jk]L,R}^i, \end{aligned}$$

with symmetrization or antisymmetrization in the flavor indices $i, j, k = 1, \dots, N$. Evaluating the anomaly coefficients $a(r)$ for these representations and imposing (51) with $n = 3$, $a(N) = 1$, and $i_r = 1$ for a single composite representation $r = r_0$ yields

$$3 = a(r_0).$$

One finds that an integer solution for N exists only when r_0 is the mixed tensor $\Psi_{[jk]L}^i$ (and its right-handed analog) and, moreover, only for a specific value of N ,

$$3 = \frac{N^2}{2} - \frac{7N}{2} - 1, \quad N = 8, \quad (52)$$

which corresponds to a 216-dimensional representation for each chirality. Thus, among all candidate values, only $SU(8)_L \times SU(8)_R$ can be realized on massless three-preon composites consistent with anomaly matching, thereby

pointing uniquely, within these assumptions, to $SL(16, C)$ as the underlying metaflavor symmetry.⁶

The unique solution of the AM condition with three-preon composites thus picks out $N = 8$ and identifies

$$SL(16, C) \longrightarrow SL(2, C) \times SU(8)$$

as the relevant metaflavor pattern: the full $SL(16, C)$ is exact at the preon level, whereas the residual $SL(2, C) \times SU(8)$ is the symmetry realized on massless composites at large distances. The corresponding massless multiplets $\Psi_{[jk]aL}^i$ and $\Psi_{[jk]aR}^i$ fill the $216_{L,R}$ representations of $SU(8)$, with spinor index $a = 1, 2$.

Upon decomposing the $SU(8)$ multiplet under $SU(5) \times SU(3)$ one obtains (suppressing spinor indices)

$$216_{L,R} = [(\bar{5} + 10, \bar{3}) + (45, 1) + (5, 8 + 1) + (24, 3) + (1, 3) + (1, \bar{6})]_{L,R}, \quad (53)$$

where $(\bar{5} + 10, \bar{3})_L$ corresponds precisely to three generations of quarks and leptons forming a triplet of a family symmetry $SU(3)$ [30]. The same pattern appears for 216_R due to the underlying vectorlike $SL(16, C)$ symmetry. Even though preons remain massless, all composite states listed in (53) are, in principle, allowed to pair up and acquire heavy masses through Dirac terms.

D. L-R symmetry violation: Chiral quark-lepton families

To obtain a chiral low-energy spectrum containing three light quark-lepton families, one needs to break the preon-level L-R symmetry effectively, so that only the left-handed combination $(\bar{5} + 10, \bar{3})_L$ remains massless, while its right-handed counterpart acquires a large mass. Following [28], we assume that this breaking is triggered by interactions in the right-handed preon sector, such that the chiral symmetry realized on right-handed composites at large distances is reduced to $[SU(5) \times SU(3)]_R$, while the left-handed sector retains the full $SU(8)_L$.

The corresponding breaking pattern may be caused by a typical L-R symmetric polynomial potential for the

third-rank antisymmetric scalars $\Phi_{[pqr]}$ and $\Phi'_{[pqr]}$ composed from P and Q preons, respectively. This potential, after breaking of $SL(16, C)$ to $SL(2, C) \times SU(8)$, can be written in the form

$$U = -M_U^2(\Phi^2 + \Phi'^2) + h_1(\Phi^2\Phi^2 + \Phi'^2\Phi'^2) + h_2(\Phi^4 + \Phi'^4) + h_3(\Phi^2\Phi'^2), \quad (54)$$

which is presumably induced by the multipreon interactions at large distances [here the notations $\Phi^2 = \text{Tr}(\Phi^+\Phi)$, $\Phi'^2 = \text{Tr}(\Phi'^+\Phi')$, $\Phi^4 = \text{Tr}(\Phi^+\Phi\Phi^+\Phi)$, and $\Phi'^4 = \text{Tr}(\Phi'^+\Phi'\Phi'^+\Phi')$ were used]. Likewise, these interactions may also produce, among others, the invariant Yukawa-type couplings for the composite multiplets (53),

$$L_Y = \frac{1}{M_Y^2} \left(\bar{\Psi}_{Lr}^{[pq]} \hat{D} \Psi_{L[tu]}^s \Phi_{[pqs]} \Phi'^{[rtu]} + \bar{\Psi}_{Rr}^{[pq]} \hat{D} \Psi_{R[tu]}^s \Phi'_{[pqs]} \Phi'^{[rtu]} \right) \quad (55)$$

($p, q, r, s, t, u = 1, \dots, 8$) being initially L-R symmetric as well. The mass parameter M_Y is some UV cutoff which in our case can be ultimately related to the preon confinement energy scale Λ_{MC} .

As in the known left-right models [31], in the potential (54) for a natural range of the parameters, particularly for $h_3 > 2(h_1 + h_2)$, and for properly chosen higher-dimensional coupling constants, the scalars $\Phi_{[ijk]}$ and $\Phi'_{[ijk]}$ may develop the totally antisymmetric VEV configuration

$$\langle \Phi_{[pqr]} \rangle = 0, \quad \langle \Phi'^{[pqr]} \rangle = \delta_{\bar{p}}^p \delta_{\bar{q}}^q \delta_{\bar{r}}^r \epsilon^{\bar{p}\bar{q}\bar{r}} M_{LR} \quad (\bar{p}, \bar{q}, \bar{r} = 1, 2, 3), \quad (56)$$

which leads at the scale M_{LR} to the spontaneous breakdown of the starting $SU(8)$ down to the $SU(5) \times SU(3)$ symmetry group, which we may identify as the conventional $SU(5)$ grand unification times the $SU(3)_F$ family symmetry. Moreover, due to asymmetric VEV (56), the Yukawa couplings (55) certainly lead to the chiral symmetry breaking for the right-handed composites, $SU(8)_R \rightarrow [SU(5) \times SU(3)]_R$, leaving intact the $SU(8)_L$ symmetry for the left-handed ones. In terms of the spectator gauge chiral symmetry, this means that all nondiagonal gauge bosons related to the broken generators of the coset $SU(8)_R/[SU(5) \times SU(3)]_R$ acquire at large distances the scale M_{LR} order masses. This means that, though the massless right-handed preons still possess the $SU(8)_R$ symmetry, the masslessness of their composites at large distances is now controlled solely by the remaining $[SU(5) \times SU(3)]_R$ part. Indeed, the right-handed composites no longer fill the complete 216_R multiplet but instead satisfy a modified anomaly matching condition based on

⁶Remarkably, due to the direct screening effect of the orthogonal metacolor, the AM condition (52) can be extended to the $SO(n)^L \times SO(n)^R$ metacolor symmetry case by supplementing the above three-preon representation $\Psi_{[jk]L}^i$ with p composite fundamental one-preon multiplets Ψ_{iL} , appropriately screened by metagluons. Thus, the new AM condition acquires the form

$$n = \frac{N^2}{2} - \frac{7N}{2} - 1 + p$$

whose solutions exist only if $n - p = 3$, so the same eightfold chiral symmetry $SU(8)_L \times SU(8)_R$ is selected even in the general case [28].

$[SU(5) \times SU(3)]_R$. One finds that the right-handed spectrum consistent with these anomalies excludes the multiplet $(5 + 10, 3)_R$, leaving only the submultiplet combination

$$[(45, 1) + (5, 8 + 1) + (5 + \bar{5}, 3) + (1, \bar{6})]_R, \quad (57)$$

which satisfies the $SU(5)^3$ and $SU(3)^3$ anomaly matching conditions with the right-handed Q preons. When the left-handed multiplet (53) and the reduced right-handed multiplet (57) are combined, all components except $(\bar{5} + 10, \bar{3})_L$ can pair up and become heavy, leaving only

$$(\bar{5} + 10, \bar{3})_L + (24 + 1, 3)_L + (5 + \bar{5}, 3)_R$$

as the anomaly-free light spectrum, with $(24 + 1, 3)_L$ and $(5 + \bar{5}, 3)_R$ expected to be lifted to higher scales through the family-symmetry interactions. Thus, three chiral families of quarks and leptons naturally remain light, transforming as $(\bar{5} + 10, \bar{3})_L$.

Once L-R symmetry is broken in this way, the initially vectorlike metaflavor symmetry $SU(8) \times SL(2, C)$ effectively reduces in the composite sector to a chiral subgroup $[SU(5) \times SU(3)_F] \times SL(2, C)$, where $SU(3)_F$ is a family symmetry acting on the three quark-lepton generations [30]. Further symmetry breaking is then driven by scalar multiplets (which may themselves be composite) that break $SU(5)$ and $SU(3)_F$ to the Standard Model, generating the observed pattern of gauge bosons and fermion masses and mixings.

E. Down to the Standard Model

A conventional scenario for further breaking of the $SU(5) \times SU(3)_F$ symmetry—identified here as the GUT and family symmetry—introduces a suitable set of scalar fields that may reduce this theory down to the Standard Model. For this, one takes the $SL(16, C)$ adjoint scalar multiplets of the type

$$\begin{aligned} & \frac{1}{M} \left[\Psi_{[qa,rb]L}^{pc} C \Psi_{[te,uf]L}^{sd} \right] H^{\{[qa,rb],[te,uf]\}} (a_U \xi_{[pc,sd]} + b_U \chi_{\{pc,sd\}}), \\ & \frac{1}{M} \left[\Psi_{[qa,rb]L}^{pc} C \Psi_{[pc,te]L}^{sd} \right] H^{\{[qa,rb],[te,uf]\}} (a_D \xi_{[sd,uf]} + b_D \chi_{\{sd,uf\}}) \end{aligned} \quad (59)$$

with distinct index contraction for the up quarks, down quarks, and leptons, respectively. The mass M represents an effective scale in the theory that, in the composite model of quarks and leptons, can be linked to their compositeness scale Λ_{MC} , while $a_{U,D}$ and $b_{U,D}$ are dimensionless constants of the order of 1.

Actually, these couplings contain two types of scalar-containing multiplets with the following $SU(8) \times SL(2, C)$

$$\Sigma = (\sigma^k + i\sigma_5^k \gamma_5) \lambda^k + \sigma_{ab}^K \gamma^{ab} \lambda^K / 2 \quad (58)$$

($k = 1, \dots, 64$; $K = 0, k$), which transform as

$$\Sigma \rightarrow \Omega \Sigma \Omega^{-1}.$$

In general, Σ contains not only scalar components but also pseudoscalar and spinorial tensor components in (58), which can be filtered out by the covariant tetrad-constraint mechanism. Imposing on Σ the constraint

$$\Sigma = e_\rho \Sigma e^\rho / 4,$$

one finds, after tetrad condensation, that Σ is effectively “sandwiched” by the neutral tetrads (19). As a result, only the pure scalar components remain in the $SU(5)$ symmetry-breaking multiplet, $\Sigma = \sigma^k \lambda^k$, thereby enabling the breaking of the $SU(5)$ GUT down to the Standard Model, $SU(5) \rightarrow SU(3)_c \times SU(2)_W \times U(1)_Y$, caused by the conventional potential [31].

The final breaking of the SM and of the accompanying family symmetry $SU(3)_F$ to $SU(3)_c \times U(1)_{em}$ occurs through the extra multiplets with assignments determined by representations chosen for quarks and leptons. They are $H^{[pa,qb,rc,sd]}$, and $\xi_{[pa,qb]}$ and $\chi_{\{pa,qb\}}$ of $SL(16, C)$, which contain an even number of antisymmetrized and symmetrized $SU(8)$ and $SL(2, C)$ indices ($p, q, r, s, t, u = 1, \dots, 8$; $a, b, c, d, e, f = 1, 2$). These multiplets include, among other components, genuine scalar components that develop the corresponding VEVs giving masses to the weak bosons, as well as to the family bosons of the $SU(3)_F$. They also generate masses for quarks and leptons located in the left-handed fermion multiplet (53) through the $SL(16, C)$ -invariant Yukawa couplings. Particularly, for the submultiplet $(\bar{5} + 10, \bar{3})_L$ one has, as in a conventional $SU(5)$ GUT, the two independent couplings

components—the H multiplet with the scalar components

$$H^{[pqrs]\{[ab],[cd]\}}(70, 1) \quad (60)$$

and the ξ and χ multiplets, whose scalar components look as

$$\xi_{[pq][ab]}(28, 1), \quad \chi_{\{pq\}[ab]}(36, 1). \quad (61)$$

Decomposing them into the components of the final $SU(5) \times SU(3)_F$ symmetry, one finds the full set of scalars

$$\begin{aligned} 70 &= (5, 1) + (\bar{5}, 1) + (10, \bar{3}) + (\bar{10}, 3), \\ 28 &= (5, 3) + (10, 1) + (1, \bar{3}), \\ 36 &= (5, 3) + (15, 1) + (1, 6) \end{aligned}$$

containing the $SU(5)$ quintets $(5, 1)$ and $(\bar{5}, 1)$ to break the Standard Model at the electroweak scale M_{SM} , and the $SU(3)_F$ triplet and sextet, $(1, \bar{3})$ and $(1, 6)$, to properly break the family symmetry at some larger scale M_F . One may refer to the scalars (60) and (61) as the corresponding “vertical” and “horizontal” scalars, which provide the simplest form of the above Yukawa couplings. Acting in pairs, they presumably determine the masses and mixings of all quarks and leptons. Finally, and importantly, in the model under consideration, these scalars may themselves be composite states formed from the same preons as quarks and leptons [28].

After the $SU(8)$ symmetry breaking, the Yukawa couplings (59) acquire the transparent $SU(5) \times SU(3)_F$ -invariant form (all metaflavor indices are omitted)

$$\begin{aligned} &[(10, \bar{3})_L C(10, \bar{3})_L](5, 1)_U [a_U(1, \bar{3}) + b_U(1, 6)]/\mathcal{M}, \\ &[(\bar{5}, \bar{3})_L C(10, \bar{3})_L](\bar{5}, 1)_D [a_D(1, \bar{3}) + b_D(1, 6)]/\mathcal{M}, \end{aligned} \quad (62)$$

where we only include those components of the vertical scalar H and horizontal scalars ξ and χ which develop the VEVs. Just the horizontal scalar VEVs determine through the Yukawa couplings (62) the mass matrices for quarks and leptons

$$\begin{aligned} \hat{m}_{\bar{p}q}^U &= \langle 5, 1 \rangle_U [a_U \langle 1, \bar{3} \rangle_{[\bar{p}q]} + b_U \langle 1, 6 \rangle_{\{\bar{p}q\}}]/\mathcal{M}, \\ \hat{m}_{\bar{p}q}^D &= \langle \bar{5}, 1 \rangle_D [a_D \langle 1, \bar{3} \rangle_{[\bar{p}q]} + b_D \langle 1, 6 \rangle_{\{\bar{p}q\}}]/\mathcal{M}, \end{aligned}$$

where the angle brackets denote the corresponding VEVs, while $\bar{p}, q = 1, 2, 3$ stand for family indices. The matrices $\hat{m}_{\bar{p}q}^U$ and $\hat{m}_{\bar{p}q}^D$ are defined at the grand unified scale and must then be extrapolated down to the physical mass range of quarks and leptons. Depending on which components of the symmetric and asymmetric VEVs develop, different texture-zero patterns arise for the mass matrices. The strong hierarchies of the quark-lepton masses and mixings may then be explained by softer hierarchies among the breaking directions of the $SU(3)_F$ family symmetry, whose scale M_F is assumed to lie close to the effective scale M .

The composite nature of quarks and leptons introduces, in addition to the Planck scale M_P and the $SL(16, C)$ breaking scale M (determined by the tetrad condensate), a new fundamental scale: the preon confinement scale Λ_{MC} . Thus, the same $SL(16, C)$ metaflavor structure organizes both the gauge-gravity sector and the preonic origin of the observed quarks and leptons, while their composition is governed by the independent metacolor dynamics at the scale Λ_{MC} . The phenomenology depends on the relative

hierarchy among these three scales. If M , Λ_{MC} , and M_P are all near one another, the framework mainly provides a structural reinterpretation of known physics. If M and Λ_{MC} are considerably below M_P , a rich spectrum of preonic and hyperunified states may become accessible, offering possible experimental windows into the underlying $SL(16, C)$ hyperunification.

VIII. SUMMARY AND OUTLOOK

We have examined a class of hyperunified gauge theories based on the noncompact group $SL(2N, C)$ that aim to unify gravity with internal interactions. Gauging neutral spinors leads to $SL(2, C)$ as the natural local symmetry associated with spacetime spin structure, while gauging charged spinors extends the symmetry to $SL(2N, C)$, with an internal $SU(N)$ hyperflavor subgroup. Within this framework the accompanying tetrads play a central role: once treated as dynamical fields, the nonlinear length constraint fixes their soldering norm, while quantum effects select the hyperflavor-blind vacuum orientation; the resulting configuration filters out the noncompact directions in the gauge algebra and supports a radiative origin of Einstein-Cartan gravity.

The low-energy symmetry that remains after tetrad condensation through the hyperflavor-blind vacuum is $SL(2, C) \times SU(N)$, realized on a spectrum in which only the neutral tetrad, identified with the graviton, and the $SU(N)$ vector field multiplet are massless. Starting from a unified quadratic-strength Lagrangian with a single gauge coupling, we argued that the EC term can be induced by fermion loops involving the flavor-singlet tensor connection and the tetrad, with either heavy vectorlike fermions near the Planck scale or a universal UV cutoff providing the required scale. At the same time, the TGR and GFC structures, described in Secs. III and IV, play the role of no-ghost kinetic projectors: TGR selects the healthy massless spin-two dynamics of the neutral tetrad, while GFC ensures the admissible propagation of the spin-connection sector.

A distinctive feature of the whole construction is that gravitational unification does not appear as an ordinary exact high-symmetry phase followed by conventional symmetry breaking, as in internal GUTs. The full $SL(2N, C)$ covariance is present in the premetric formulation, but the physical meaning of gravity emerges only after tetrad soldering selects the neutral spacetime branch. Thus, the theory is not merely a representation-level embedding of Lorentz and internal generators into a larger algebra; it becomes a dynamical hyperunification only when the gravitational scales are generated by the same gauge-coupled matter sector rather than inserted as independent constants.

On the matter side, we saw that attempting to assign elementary quarks and leptons to $SL(2N, C)$ multiplets usually leads to large representations with unwanted spin content. This motivates a composite viewpoint: the truly

elementary spinor fields are not the observed quarks and leptons themselves, but preons in fundamental $SL(2N, C)$ representations, while quarks and leptons arise as their bound states. Applying the 't Hooft anomaly matching condition to the global chiral symmetry $SU(N)_L \times SU(N)_R$ of preons, while assuming that only three-preon spin-1/2 composites remain massless and fit into a single representation, singles out $SU(8)_L \times SU(8)_R$ at large distances and thus $SL(16, C)$ as the hyperunified metaflavor group. The residual symmetry $SL(2, C) \times SU(8)$ naturally yields three families of composite quarks and leptons, organized into an $SU(5) \times SU(3)_F$ structure after L-R breaking in the composite sector.

Remarkably, compositeness appears here to be dictated by the structure of the theory itself, rather than introduced solely on phenomenological grounds such as the existence of identical quark-lepton families and their mass and mixing hierarchies, as is usually the case. In the present context, applying the fundamental $SL(2N, C)$ symmetry directly to the observed quarks and leptons would be somewhat analogous to imagining the Standard Model, or its conventional GUT extensions, being applied directly to hadrons in order to describe their classification and interactions. Such an identification would obscure the underlying constituent structure and could therefore be fundamentally misleading.

The proposed hyperunification mechanism is purely four dimensional and gauge based; it does not rely on extra dimensions [32–34] or string excitations [35]. The tetrad, which is required in any case to couple spinors to gravity, is used as the field that condenses; its dynamically selected

hyperflavor-blind orientation fixes the vacuum and lifts noncompact directions with a clear understanding of which gauge components have been decoupled. In this way, the gauge-gravity construction and the preon construction meet at the level of the same $SL(16, C)$ metaflavor symmetry: the former uses it to organize the unified connection and tetrad sector, while the latter uses it to classify the truly elementary preons whose metacolor bound states become the observed quarks and leptons.

Further work should complete the preon model for $N = 8$, including the full set of composites. If the tetrad-condensation scale M and the preon confinement scale Λ_{MC} lie well below M_P , remnants of the filtered hyperunified structure could leave characteristic signatures in the spectrum of heavy bosonic and fermionic composites. A related possibility is that all $SL(2N, C)$ gauge fields themselves may be composite, built from preon bilinears with only global $SL(2N, C)$ imposed at the fundamental level. This approach, long recognized as a viable alternative to conventional gauge theories and gravity [36–42], has not yet been explored in the context of noncompact unified symmetries. Extending the present work to such a fully composite setting remains an interesting direction for future study.

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- [1] R. Utiyama, *Phys. Rev.* **101**, 1597 (1956).
 - [2] T. W. Kibble, *J. Math. Phys. (N.Y.)* **2**, 212 (1960).
 - [3] C. N. Yang and R. L. Mills, *Phys. Rev.* **96**, 191 (1954).
 - [4] C. J. Isham, A. Salam, and J. Strathdee, *Lett. Nuovo Cimento* **5**, 969 (1972).
 - [5] C. J. Isham, A. Salam, and J. Strathdee, *Phys. Rev. D* **8**, 2600 (1973); **9**, 1702 (1974).
 - [6] Y. M. Cho and P. G. O. Freund, *Phys. Rev. D* **12**, 1711 (1975); Y. M. Cho, *Phys. Rev. D* **14**, 3335 (1976).
 - [7] J. C. Huang and P. W. Dennis, *Phys. Rev. D* **15**, 983 (1977); **24**, 3125 (1981).
 - [8] R. Percacci, *Phys. Lett.* **144B**, 37 (1984); F. Nesti and R. Percacci, *J. Phys. A* **41**, 075405 (2008); *Phys. Rev. D* **81**, 025010 (2010).
 - [9] A. H. Chamseddine, *Phys. Rev. D* **70**, 084006 (2004).
 - [10] J. L. Chkareuli, *Phys. Lett. B* **834**, 137417 (2022); *Eur. Phys. J. C* **84**, 1212 (2024); *Nucl. Phys.* **B1020**, 117150 (2025).
 - [11] S. Coleman and J. Mandula, *Phys. Rev.* **159**, 1251 (1967).
 - [12] K. Hayashi and T. Shirafuji, *Phys. Rev. D* **19**, 3524 (1979); R. Aldrovandi and J. G. Pereira, *Teleparallel Gravity: An Introduction, Fundamental Theories of Physics* (Springer, New York, 2013).
 - [13] Steven Weinberg, *The Quantum Theory of Fields, Volume II: Modern Applications* (Cambridge University Press, Cambridge, England, 1996).
 - [14] D. E. Neville, *Phys. Rev. D* **18**, 3535 (1978).
 - [15] E. Sezgin and P. van Nieuwenhuizen, *Phys. Rev. D* **21**, 3269 (1980).
 - [16] F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, *Rev. Mod. Phys.* **48**, 393 (1976).
 - [17] S. Coleman and E. Weinberg, *Phys. Rev. D* **7**, 1888 (1973).
 - [18] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
 - [19] A. D. Sakharov, *Sov. Phys. Dokl.* **12**, 1040 (1968).
 - [20] S. L. Adler, *Rev. Mod. Phys.* **54**, 729 (1982).
 - [21] A. Zee, *Phys. Rev. D* **23**, 858 (1981).
 - [22] J. F. Donoghue, *Phys. Rev. D* **50**, 3874 (1994).

- [23] H. Georgi and S. L. Glashow, *Phys. Rev. Lett.* **32**, 438 (1974).
- [24] S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193**, 150 (1981).
- [25] J. Hisano, H. Murayama, and T. Yanagida, *Nucl. Phys. B* **402**, 46 (1993).
- [26] J. L. Chkareuli and I. G. Gogoladze, *Phys. Rev. D* **58**, 055011 (1998).
- [27] H. Georgi, *Nucl. Phys.* **B156**, 126 (1979).
- [28] J. L. Chkareuli, *Nucl. Phys.* **B941**, 425 (2019); **B1020**, 117150 (2025).
- [29] G. 't Hooft, in *Recent Developments in Gauge Theories*, edited by G. 't Hooft *et al.* (Plenum, New York, 1980).
- [30] J. L. Chkareuli, *JETP Lett.* **32**, 671 (1980); Z. G. Berezhiani and J. L. Chkareuli, *Sov. J. Nucl. Phys.* **37**, 618 (1983); J. L. Chkareuli, C. D. Froggatt, and H. B. Nielsen, *Nucl. Phys.* **B626**, 307 (2002).
- [31] R. N. Mohapatra, *Unification and Supersymmetry* (Springer-Verlag, New York, 2003).
- [32] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *Phys. Lett. B* **436**, 257 (1998).
- [33] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).
- [34] K. Krasnov and R. Percacci, *Classical Quantum Gravity* **35**, 143001 (2018).
- [35] K. Becker, M. Becker, and J. H. Schwarz, *String Theory and M-Theory: A Modern Introduction* (Cambridge University Press, Cambridge, England, 2007).
- [36] J. D. Bjorken, *Ann. Phys. (N.Y.)* **24**, 174 (1963).
- [37] P. R. Phillips, *Phys. Rev.* **146**, 966 (1966).
- [38] T. Eguchi, *Phys. Rev. D* **14**, 2755 (1976).
- [39] H. Terazawa, Y. Chikashige, and K. Akama, *Phys. Rev. D* **15**, 480 (1977).
- [40] M. Suzuki, *Phys. Rev. D* **37**, 210 (1988); **82**, 045026 (2010).
- [41] J. L. Chkareuli, C. D. Froggatt, and H. B. Nielsen, *Phys. Rev. Lett.* **87**, 091601 (2001).
- [42] J. L. Chkareuli, *Phys. Lett. B* **817**, 136281 (2021).