

High-frequency gravitational waves from a first-order phase transitions

Wen-Yuan Ai^{*}

State Key Laboratory of Dark Matter Physics, Tsung-Dao Lee Institute and School of Physics and Astronomy, *Shanghai Jiao Tong University*, Shanghai 201210, China
and Key Laboratory for Particle Astrophysics and Cosmology (MOE) and Shanghai Key Laboratory for Particle Physics and Cosmology, *Shanghai Jiao Tong University*,
Shanghai 201210, China and Marietta Blau Institute for Particle Physics,
Austrian Academy of Sciences, Dominikanerbastei 16, 1010 Vienna, Austria



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First-order phase transitions in the early Universe are a well-motivated source of gravitational waves (GWs). In this paper, we identify a previously overlooked GW production mechanism: *gravitational transition radiation*, arising from graviton emission by particles whose mass changes as they pass through expanding bubble walls. Unlike conventional sources, such as bubble collisions or sound waves, this mechanism operates at the microscopic scale set by the Lorentz-contracted wall thickness, leading to GW emission at significantly higher frequencies. The resulting spectrum features a distinctive shape with a peak frequency redshifting to $f_{\text{peak}} \sim T_0 \sim 10^{10}$ Hz, where T_0 is the current temperature of the Universe. This mechanism is generic and is expected to operate similarly for domain walls and other relativistic interfaces.

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I. INTRODUCTION

The detection of gravitational waves (GWs) by LIGO/Virgo collaboration in 2015 [1] has led us to the era of GW astronomy. Future GW observations hold great promise for probing fundamental questions in high-energy physics and for discovering new physics beyond the standard model of particle physics and cosmology. It is, therefore, crucial to theoretically identify all possible sources of GW production. While conventional sources typically involve abrupt, macroscopic classical processes—such as compact object mergers, the collision and coalescence of cosmological defects, or primordial plasma turbulence—GWs can also be generated by microscopic particle processes. These are often described in terms of graviton production. Notable examples include graviton production from the Sun [2,3], freeze-in production of GWs from the primordial thermal plasma [4–10], gravitational bremsstrahlung during the cosmic reheating [11–18], leptogenesis [19–21], or dark matter freeze-in production [22], and graviton emission from evaporating primordial black holes [23–30]. In this article, for the first time, we identify a new mechanism for GW production, arising from a novel phenomenon that we term *gravitational transition radiation* (GTR).

Transition radiation is usually understood as an electromagnetic phenomenon in which a charged particle emits light when crossing an interface that modifies the photon dispersion relation. Radiation can also be produced if the particle itself experiences a sudden change in properties, such as its mass or charge. In cosmology, such conditions naturally arise near phase boundaries, for example, bubble walls or domain walls. The emission is significantly enhanced when the boundary moves relativistically. Analogously, a sudden change in a particle's mass can induce the emission of gravitons. In this work, we focus on bubble walls from a first-order phase transition (FOPT) as a concrete realization of this phenomenon and study its associated GW background. However, the mechanism is expected to apply more broadly to other relativistic interfaces, such as domain walls.

On the other hand, GWs generated by FOPTs in the early Universe offer a powerful probe of new physics [31–37]. So far, significant attention has been devoted to GWs sourced by bulk hydrodynamic effects, such as sound waves [38,39] and turbulence [34,40–42], as well as scalar field bubble collisions [32–34,43–45]. These processes operate at macroscopic scales, with typical GW frequencies tied to the inverse size of the bubbles at collision. In this work, we demonstrate that GTR generates a qualitatively distinct GW signal during the bubble expansion phase of the transition, characterized by a frequency and spectral shape that are dramatically different from those of conventional sources.

II. GRAVITATIONAL TRANSITION RADIATION

Consider a scalar field Φ , which may be either fundamental or composite, serving as the order parameter for a

^{*}Contact author: wenyuan.ai@oeaw.ac.at

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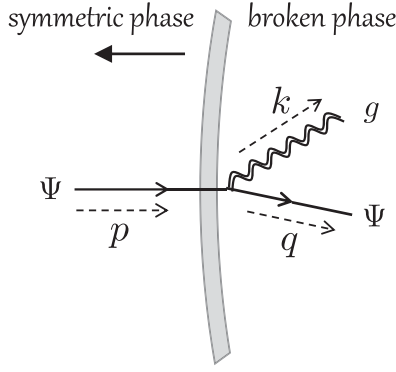


FIG. 1. GTR: a particle hitting an expanding wall emits a graviton due to the change in its mass.

FOPT at temperature T . This phase transition breaks a certain symmetry and proceeds via the nucleation of bubbles, which separate regions of the broken phase inside the bubble from the surrounding symmetric phase outside. After nucleation, these bubbles will expand until collision. The process of GW production we shall study occurs during the bubble expansion. The bubble field configuration is described by the vacuum expectation value (VEV) of Φ , $\varphi \equiv \langle \Phi \rangle$. Without loss of generality, consider a bubble wall expanding in the negative z -direction with a Lorentz factor $\gamma_w \equiv 1/\sqrt{1-v_w^2}$, where v_w is the wall velocity. As the bubble wall expands, particles outside of the bubble impinge on the wall and enter the bubble. A particle Ψ that has a φ -dependent mass, which can be a scalar, fermion, or vector boson, can emit a graviton as it passes through the wall due to the change in its mass, leading to the GTR process $\Psi \rightarrow \Psi + g$, as illustrated in Fig. 1.

We write the four-momenta for this process in the rest frame of the wall as

$$\text{incoming } \Psi: \quad p = (p^0, \mathbf{p}_\perp, p^z), \quad (1a)$$

$$\text{outgoing } \Psi: \quad q = (q^0, \mathbf{q}_\perp, q^z), \quad (1b)$$

$$\text{graviton } g: \quad k = (k^0, \mathbf{k}_\perp, k^z), \quad (1c)$$

where a subscript $\perp$ denotes the direction parallel to the wall and on-shell conditions are understood: $p^0 = \sqrt{\mathbf{p}_\perp^2 + (p^z)^2 + m_\Psi^2(z)}$ (same for q), $k^0 = |\mathbf{k}|$. Since gravitons remain massless in the presence of the wall, one can use $\mathbf{k}$ to label a free graviton. However, for Ψ , the presence of the wall spontaneously breaks the z -translation invariance so that $p^0, \mathbf{p}_\perp$ (similarly for q) are conserved quantities for a one-particle state of Ψ but p^z is z -dependent. Although one can use $p^0, \mathbf{p}_\perp$ to label one-particle states of Ψ , it is more convenient to use $\mathbf{p}_s \equiv (\mathbf{p}, p_s^z)$ and $\mathbf{p}_b \equiv (\mathbf{p}, p_b^z)$ to label the incoming and outgoing Ψ particles, respectively. Here, the subscripts “s” and

“b” denote quantities in the symmetric and broken phases, respectively. For example, $p_{s/b}^z \equiv \sqrt{(p^0)^2 - \mathbf{p}_\perp^2 - m_{\Psi,s/b}^2}$ is the z -momentum in the symmetric/broken phase. For simplicity, we take $m_{\Psi,s} = 0$ below. The generalization to the case of $m_{\Psi,s} \neq 0$ is straightforward.

In the wall’s rest frame, the flux of Ψ particles impinging on the wall reads

$$J_\Psi^{(\text{wall})} = \int \frac{d^3\mathbf{p}_s}{(2\pi)^3} \frac{p_s^z}{p^0} f_{\Psi;s}^{(\text{wall})}(\mathbf{p}_s, T), \quad (2)$$

where $f_{\Psi;s}^{(\text{wall})}(\mathbf{p}_s, T) = 1/[\exp(\gamma_w(p^0 - v_w p^z)/T) - 1]$ is the thermal-equilibrium distribution function of Ψ particles in the symmetric phase in the wall frame. The graviton energy density in the *plasma* frame at the time of generation is given by [46] (see Appendix A)

$$\begin{aligned} \rho_{\text{GW}}^{\text{gen}} &= \frac{1}{v_w \gamma_w} \int \frac{d^3\mathbf{p}_s}{(2\pi)^3} \frac{p_s^z}{p^0} \times \frac{1}{2p_s^z} \int \frac{d^3\mathbf{q}_b}{(2\pi)^3} \frac{1}{2q^0} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{2k^0} \\ &\times \tilde{k}^0 |\mathcal{M}|^2 (2\pi)^3 \delta(p^0 - q^0 - k^0) \delta^2(\mathbf{p}_\perp - \mathbf{q}_\perp - \mathbf{k}_\perp) \\ &\times f_{\Psi;s}^{(\text{wall})}(\mathbf{p}_s, T) \left[1 \pm f_{\Psi;b}^{(\text{wall})}(\mathbf{q}_b, T) \right], \end{aligned} \quad (3)$$

where $\tilde{k}^0 \equiv \gamma_w(k^0 - v_w k^z) \approx k^0/2\gamma_w$ is the graviton energy in the *plasma* frame, and $\mathcal{M}$ is the invariant transition amplitude for the process $\Psi(p_s) \rightarrow \Psi(q_b) + g(k)$.¹ The sign in $\pm$ depends on whether Ψ is bosonic or fermionic.

Above, we have approximated $1 + f_g^{(\text{wall})}(\mathbf{k}) \approx 1$ on the right-hand side, as $f_g^{(\text{wall})}$ is expected to be small at production. [Another argument, as used for gauge boson emission in [47], is that the produced graviton can also be absorbed by an incoming Ψ particle. The net effect of including both emission and absorption processes is, approximately, to replace the factor $1 + f_g^{(\text{wall})}$ by unity.]

III. INVARIANT TRANSITION AMPLITUDE FOR GTR

Because the bubble wall spontaneously breaks the z -translation invariance, the transition amplitude $\mathcal{M}$ cannot be computed in the conventional way. One must first construct the mode functions for particles with a z -dependent mass. Fortunately, the full procedure has been worked out by Bödeker and Moore [47]. We provide a comprehensive review of the Bödeker-Moore method in Appendix A. The general form of $\mathcal{M}$ reads

¹Note that here $\mathcal{M}$ is defined from the S-matrix element with a three-dimensional Dirac delta function factored out. As a result, it carries different mass dimensions compared to the conventional definition. For example, in a $1 \rightarrow 2$ process, $\mathcal{M}$ is dimensionless, whereas the conventional invariant transition amplitude has mass dimension 1.

$$\mathcal{M} = - \left[2ip^0 \left(\frac{V_b}{A_b} - \frac{V_s}{A_s} \right) \right], \quad (4)$$

where $A_{s/b}$ depend on the kinematics and $V_{s/b}$ depend on the specific vertex under consideration. For our case,

$$A_s = -\mathbf{p}_\perp^2 + \frac{x\mathbf{q}_\perp^2 + (1-x)\mathbf{k}_\perp^2}{x(1-x)}, \quad (5a)$$

$$A_b = -(\mathbf{p}_\perp^2 + m_{\Psi,b}^2) + \frac{x(\mathbf{q}_\perp^2 + m_{\Psi,b}^2) + (1-x)\mathbf{k}_\perp^2}{x(1-x)}, \quad (5b)$$

where $x = k^0/p^0$.

To get $V_{s/b}$, we consider the minimal coupling between a field Ψ and the graviton $h_{\mu\nu}$,

$$-\mathcal{L} \supset \frac{1}{m_{\text{Pl}}} h_{\mu\nu} T_{(\Psi)}^{\mu\nu} \equiv \frac{1}{2} \kappa h_{\mu\nu} T_{(\Psi)}^{\mu\nu}, \quad (6)$$

where $T_{(\Psi)}^{\mu\nu}$ is the energy-momentum tensor of the Ψ field and m_{Pl} is the reduced Planck mass. In Appendix B, we review the Feynman rules for the coupling between a graviton and a scalar, fermion, and vector particle. Here, we take a scalar particle $\Psi = \chi$ as an example. Up to the quadratic term in χ we have

$$T_{(\chi)}^{\mu\nu} = (\partial^\mu \chi)(\partial^\nu \chi) - \eta^{\mu\nu} \left[\frac{1}{2} (\partial_\rho \chi) \partial^\rho \chi - \frac{1}{2} m_\chi^2(z) \chi^2 \right],$$

where $m_\chi^2(z) = \lambda \varphi^2(z)/2$. The vertex (6) then gives the following V function:

$$V(z) = \frac{\kappa}{2} \epsilon_{\mu\nu}^*(k) \{ p^\mu q^\nu + p^\nu q^\mu - \eta^{\mu\nu} [p \cdot q - m_\chi^2(z)] \},$$

where $\epsilon_{\mu\nu}$ is the graviton polarization tensor. Then, $V_{s/b} \equiv V(z = \pm\infty)$ corresponds to taking $m_\chi(z)$ in $V(z)$ as the mass in the symmetric/broken phase. Substituting the obtained $|\mathcal{M}|^2$ into Eq. (3), we are prepared to compute the GW energy density at production.

IV. GRAVITATIONAL WAVE POWER SPECTRUM

Once gravitons are produced at the phase transition temperature T , they redshift as radiation. Assuming no entropy generation after the phase transition,² we then have

²This also assumes that there is no exceptionally strong supercooling. Otherwise, the produced GWs would be significantly diluted by the reheating via percolation of bubbles, leading to an additional factor $(g_{\star,s}(T)/g_{\star,s}(T_{\text{RH}}))^{4/3} (T/T_{\text{RH}})^4 \approx (g_{\star,s}(T)/g_{\star,s}(T_{\text{RH}}))^{4/3} (1 + \alpha_n)^{-1}$, where T_{RH} is the reheating temperature and α_n is the phase transition strength, in the power spectrum. The relation between k^0 and the frequency observed today will be modified to $\tilde{k}^0 = (g_{\star,s}(T)/g_{\star,s}(T_0))^{1/3} (T_{\text{RH}}/T_0) 2\pi f$.

$$\rho_{\text{GW}}^{\text{today}} = \rho_{\text{GW}}^{\text{gen}} \left(\frac{g_{\star,s}(T_0)}{g_{\star,s}(T)} \right)^{4/3} \left(\frac{T_0}{T} \right)^4, \quad (7)$$

where T_0 is the temperature today, and $g_{\star,s}$ is the number of entropic degrees of freedom. On the other hand, the frequency measured today is related to the graviton energy at production via $\tilde{k}^0 = [a_0/a(T)] 2\pi f$. Substituting the above equations into $\Omega_{\text{GW}} = 1/\rho_c d\rho_{\text{GW}}^{\text{today}}(f)/d \ln f$, where ρ_c is the critical energy density, we get

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \left(\frac{g_{\star,s}(T_0)}{g_{\star,s}(T)} \right)^{4/3} \left(\frac{T_0}{T} \right)^4 \times \frac{d\rho_{\text{GW}}^{\text{gen}}(\tilde{k}^0)}{d \ln \tilde{k}^0} \bigg|_{\tilde{k}^0 = \left(\frac{g_{\star,s}(T)}{g_{\star,s}(T_0)} \right)^{1/3} \left(\frac{T}{T_0} \right) 2\pi f}. \quad (8)$$

For simplicity, we still consider GTR from a scalar particle. The generalization to fermions and vector bosons is straightforward. After computing $d\rho_{\text{GW}}^{\text{gen}}(\tilde{k}^0)/d \ln \tilde{k}^0$ (see Appendix C), we find, for $\gamma_w T \gg m_{\chi,b}$,

$$\Omega_{\text{GW}}(f) h^2 \simeq 3.6 \times 10^{-10} \left(\frac{100}{g_{\star,s}(T)} \right)^{4/3} \left(\frac{m_{\chi,b}}{m_{\text{Pl}}} \right)^2 \times I \left(\frac{f}{T_0}; g_{\star,s}(T) \right). \quad (9)$$

We note that the factor of $(m_{\chi,b}/m_{\text{Pl}})^2$ is a universal feature for a single graviton emission or absorption [2,48]. The shape of the power spectrum is described by the dimensionless integral I . This integral actually has additional *weak* dependences on γ_w and $m_{\chi,b}$. Varying these parameters while keeping f/T_0 fixed would slightly alter the shape of I , but the results remain of the same order of magnitude. For simplicity, we therefore neglect these weak dependencies. Taking an example, $g_{\star,s}(T) = 100$, $m_{\chi,b} = T$ and $\gamma_w = 20$, we show it in Fig. 2. We observe that there is an upper bound for the allowed frequency at $f_{\text{max}} \approx 0.34 T_0 \approx 1.9 \times 10^{10}$ Hz and a peak at $f_{\text{peak}} \approx 0.176 T_0 \approx 1 \times 10^{10}$ Hz. These features can be understood from the kinematics. First, the energy of the outgoing particle χ in the broken phase must be larger than its mass, giving $(3\gamma_w T/2 - k^0) \geq m_{\chi,b}$, where $3\gamma_w T/2$ is the averaged energy of the incoming χ particle. This leads to the constraint $k^0 \leq k_{\text{max}}^0 \approx 3\gamma_w T/2$, which gives the maximal frequency. Second, the peak frequency occurs when both the z -momentum of the graviton and the outgoing χ particle can go to zero, such that the integrand in the I integral is resonantly enhanced. This can also be seen from Eq. (3). In view of that there is the Dirac delta function for the energy conservation, it is convenient to make a change of integral variables: $d\tilde{q}_b^z/q^0 = dq^0/q_b^z$, $dk^z/k^0 = dk^0/k^z$. Then, the integral is resonantly enhanced at $q_b^z = k^z = 0$, while when only one of them is zero, we do not observe a local peak

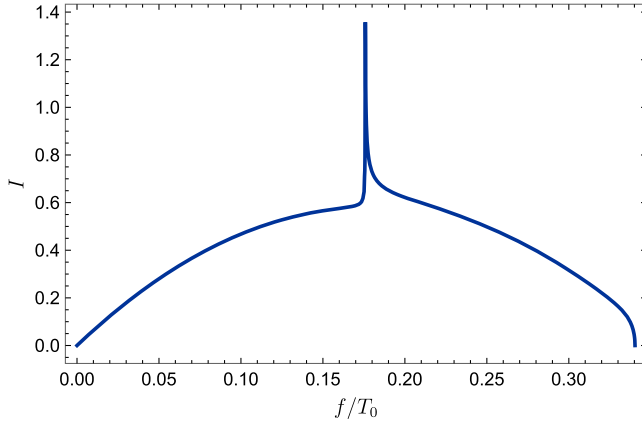


FIG. 2. The shape of the power spectrum for $g_{\star,s}(T) = 100$. The peak frequency today is at $f_{\text{peak}} \approx 0.176T_0$.

showing up. Taking $q_b^z = k^z = 0$ in the on-shell conditions, $(k^0)^2 = (k^z)^2 + |\mathbf{k}_\perp|^2$ and $(p_s^0 - k^0)^2 = (q_b^z)^2 + |\mathbf{k}_\perp|^2 + m_{\chi,b}^2$, where we have taken $\mathbf{q}_\perp = -\mathbf{k}_\perp$ assuming that in the plasma the average $\mathbf{p}_\perp$ is zero, and we obtain $k^0 = \sqrt{(3\gamma_w T/2 - k^0)^2 - m_{\chi,b}^2}$. This gives $k_{\text{peak}}^0 \approx 0.75\gamma_w T$ for $\gamma_w T \gg m_{\chi,b}$. These particular values, k_{max}^0 and k_{peak}^0 , are related to the maximal and peak frequencies via the relation $\frac{k^0}{2\gamma_w} \approx \tilde{k}^0 = \left(\frac{g_{\star,s}(T)}{g_{\star,s}(T_0)}\right)^{\frac{1}{3}} \left(\frac{T}{T_0}\right) 2\pi f$.

It is interesting to note that the GW power spectrum is not explicitly sensitive to γ_w . However, for most of the χ particles to enter the bubble, we require $\gamma_w T \gg m_{\chi,b}$. For $f \ll f_{\text{peak}}$, the I function can be fitted with a linear function. Taking $g_{\star,s}(T) = 100$, we find, for $\gamma_w T \gg m_{\chi,b}$,

$$\Omega_{\text{GW}}^{\text{IR}}(f)h^2 \simeq 4 \times 10^{-17} \left(\frac{m_{\chi,b}}{m_{\text{pl}}}\right)^2 \left(\frac{f}{\text{kHz}}\right). \quad (10)$$

Such a linear spectrum is expected to break down in the infrared regime, likely when $\tilde{k}^0(f) \sim H(T)$ with H being the Hubble rate at production.

In Fig. 3, we compare the GTR GWs, represented by the brown curve, with the cosmic gravitational microwave background (CGMB) [4–6], represented by the orange curve, and the conventional FOPT GW signals, represented by the red curve, with the following benchmark parameters:

$$T = 10^{14} \text{ GeV}, \quad \gamma_w = 500, \quad m_{\chi,b} = 50T, \\ T_{\text{max}} = m_{\text{pl}}, \quad \beta/H = 20, \quad \alpha_n = 1, \quad v_w \rightarrow 1. \quad (11)$$

Above, T_{max} is the maximal temperature ever reached by the Universe, and β/H is the dimensionless inverse duration parameter for the phase transition. Also shown are various current experimental bounds (CAST, LIGO,

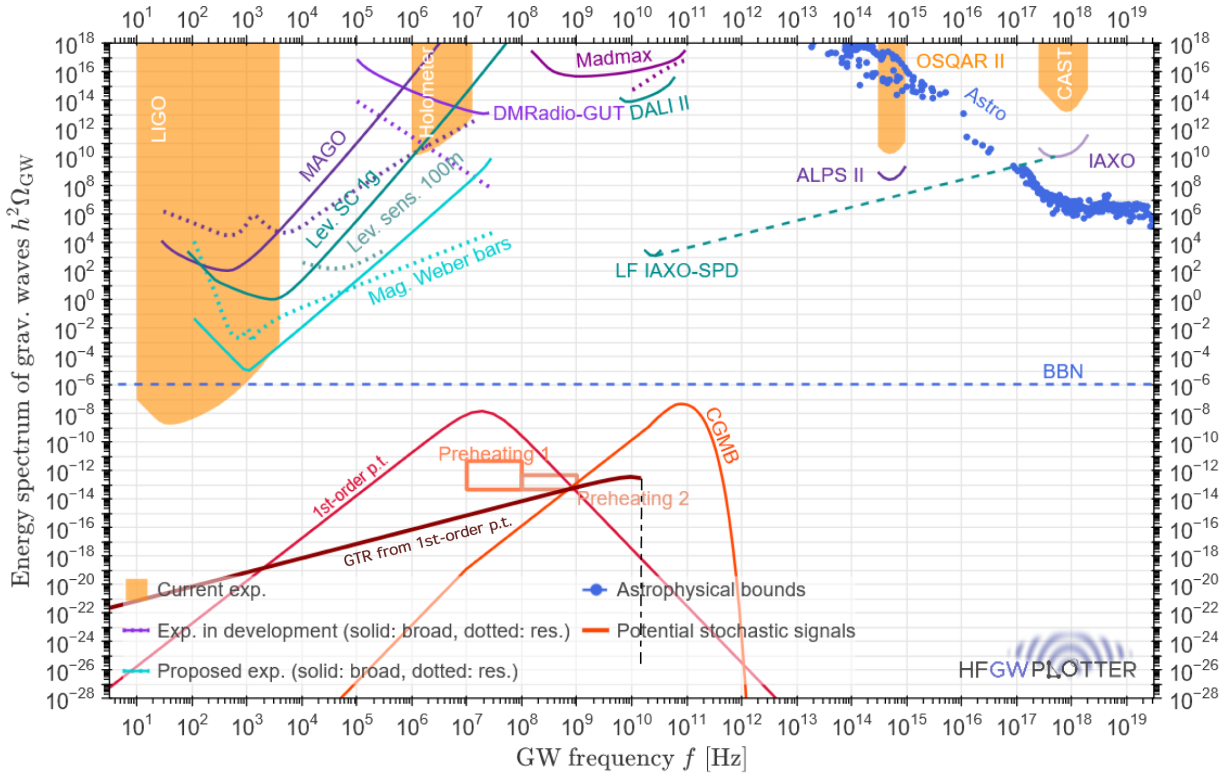


FIG. 3. Comparison of the GWs studied in this work (brown) with CGMB (orange) and those from conventional FOPT sources (red), using the benchmark parameters specified in Eq. (11). In computing the GW power spectrum, we have multiplied Eq. (9) by a factor of 10 to account for multiple degrees of freedom. The figure is generated with the high-frequency GW plotter [49].

OSQAR II, and Holometer), astrophysical bounds (blue dots), and projected sensitivities of ongoing and proposed experiments. The blue dashed line denotes the big bang nucleosynthesis (BBN) bound, while two light-red boxes are envelopes of potential signals from preheating. For more details, see Refs. [49,50]. Owing to limitations of the interpolated function on a large logarithmic scale, we have manually added a dashed gray line to illustrate the rapid falloff of the GTR GW power spectrum near the maximal frequency. The plot appears to indicate that, like all other cosmological sources of high-frequency GWs, the new signals remain beyond current experimental reach.³ Note, however, that this plot is conservative, and new designs for high-frequency GW detection are actively being developed.

In Fig. 3, we see that the new GW signals are relatively suppressed compared with the conventional FOPT GW signals. This may be understood from a quick order-of-magnitude analysis. The peak GW power spectrum from GTR of a scalar particle is given by $\Omega_{\text{GW,peak}}^{(\text{GTR})} \sim 10^{-10} (m_{\chi,b}/m_{\text{pl}})^2 \sim 10^{-13}$. Since we are considering fast bubble walls $v_w \rightarrow 1$, we assume the conventional FOPT GWs are dominated by bubble collisions, which gives $\Omega_{\text{GW,peak}}^{(\text{bubble})} \sim 10^{-5} (H/\beta)^2 \sim 10^{-8}$, where we have used the value of $\beta/H = 20$ given in Eq. (11).

V. DISCUSSION

Observable high-frequency GWs are particularly compelling because they are typically tied to new physics beyond the standard model, with perhaps the sole exception being the CGMB if produced minimally from the standard model plasma. For a summary of the various known sources, see Ref. [50]. Interest in this area has grown substantially in recent years. There are already interesting proposals and theoretical studies for high-frequency GW detection, including interferometers [51–55], microwave and optical cavities [56–73], optically levitated sensors [74], mechanical resonators [75–79], superconducting rings [80], detectors exploiting the inverse Gertsenshtein effect [81–83], frequency modulation of photons by GWs [84], and schemes based on the excitation of collective magnon modes [85]. For this reason, it is crucial to identify potential sources of high-frequency GWs.

In this work, we have identified a new source of high-frequency GW production from FOPTs, introducing the GTR mechanism. This mechanism generates a power spectrum with features qualitatively distinct from those of conventional sources. Although detecting such signals—along with other high-frequency GWs—remains challenging in the near future, our work fills an important gap in the understanding of microscopic GW sources and highlights

the rich phenomenology associated with relativistic interfaces in the early Universe. In this paper, we have illustrated the mechanism in its simplest realization, where a scalar particle emits a graviton through minimal gravitational coupling. Exploring how an additional structure—such as nonminimal couplings to gravity—could imprint new spectral features is a natural next step. GTR from a vector boson may give a different, potentially larger, GW power spectrum. Another immediate application of our work is to investigate new GW signals sourced by GTR from particles crossing axion domain walls. In this case, the production can last for many Hubble times and hence a larger GW power spectrum is expected. Further, the GTR mechanism may also be applied to astrophysical processes, e.g., a FOPT inside a neutron star [86], where the resulting GWs do not need to respect the BBN upper bound. Our results thus open a new direction for theoretical and phenomenological investigation, and meanwhile, provide a target for future experimental efforts.

Note added. Shortly after our paper was posted on arXiv, a related study [87] appeared, which examines the same process.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [49].

APPENDIX A: BÖDEKER-MOORE METHOD

In this appendix, we review the Bödeker-Moore method [47] for computing the transition amplitudes in the presence of a planar wall. This method has been used in, e.g., Refs. [88–90]. Let us consider the following general interaction:

$$-\mathcal{L} \supset v(z) \chi_1 \chi_2 \chi_3, \quad (\text{A1})$$

that can describe the 1-to-2 process $\chi_1(p) \rightarrow \chi_2(q) \chi_3(k)$. Here, we have followed the same notation in Eq. (1) for the kinematics, with the correspondence $\chi_1(p) \leftrightarrow \Psi(p)$,

³From the plot, we can see that any high-frequency GWs from cosmological sources that do not violate the BBN bound are beyond the current experimental reach.

$\chi_2(q) \leftrightarrow \Psi(q)$, $\chi_3(k) \leftrightarrow g(k)$. Note that the fields and the factor $v(z)$ may carry additional indices reflecting the inner structure in the fields. The factor of $v(z)$ may or may not depend on z , depending on whether the VEV φ appears in the vertex. For example, one may have an interaction $\lambda\varphi(z)\phi\chi^2$, where ϕ is the fluctuation field of Φ upon the background φ [i.e., $\Phi = \varphi(z) + \phi$], and χ is another scalar field, then $v(z) = \lambda\varphi(z)$. One can also have a Yukawa interaction $y\bar{\psi}\phi\psi$ [$v(z) = y$] or fermion-vector-boson interaction $g_A\bar{\psi}\gamma^\mu A_\mu\psi$ [$v(z) = g_A\gamma^\mu$], where γ^μ are the Dirac gamma matrices.

Even though the VEV φ does not appear in the vertex, i.e., $v(z)$ does not depend on z , the z -translation invariance can still be broken by the φ -dependent mass terms. As explained in the main text, we shall use $\mathbf{p}_s$ to denote the one-particle states χ_1 , and $\mathbf{q}_b/\mathbf{k}_b$ to denote the one-particle states of χ_2/χ_3 . (If χ_3 is to be identified as the graviton, then there is no need to use $\mathbf{k}_b$ since gravitons do not have a φ -dependent mass.) Taking a scalar field as an example, the free χ_1 field (with the φ -dependent mass term taken into account) has the following decomposition:

$$\chi_1^{\text{(free)}}(x) = \int \frac{d^3\mathbf{p}_s}{(2\pi)^3 2p^0} (\hat{a}_{\mathbf{p}_s} \zeta_{\mathbf{p}_s}(z) e^{-i(p^0 t - \mathbf{p}_\perp \cdot \mathbf{x}_\perp)} + \hat{a}_{\mathbf{p}_s}^\dagger \zeta_{\mathbf{p}_s}^*(z) e^{i(p^0 t - \mathbf{p}_\perp \cdot \mathbf{x}_\perp)}), \quad (\text{A2})$$

where again p^0 should be understood as the on-shell energy. The momentum labels are also used to distinguish the particle species to simplify the notation. Therefore, $p^0 = \sqrt{\mathbf{p}_\perp^2 + (p^z)^2 + m_{\chi_1}^2(z)} = \sqrt{\mathbf{p}_\perp^2 + (p_s^z)^2 + m_{\chi_{1,s}}^2}$, where $m_{\chi_{1,s}}$ is the mass of the χ_1 particles in the symmetric phase. Since we consider a process with χ_1 as an incoming particle, the χ_1 distribution function in the symmetric phase $f_{\chi_{1,s}}^{\text{(wall)}}(\mathbf{p}_s, T)$ always appears in the expressions of interesting physical quantities. This makes using $\mathbf{p}_s$ to label χ_1 particles convenient. The equation of motion for $\zeta_{\mathbf{p}_s}$ is obtained by substituting Eq. (A2) into the free equation of motion for χ_1 . For example, if χ_1 is a scalar field, then we have

$$[-(p^0)^2 + \mathbf{p}_\perp^2 - \partial_z^2 + m_{\chi_1}^2(z)]\zeta_{\mathbf{p}_s}(z) = 0. \quad (\text{A3})$$

We require the normalization for $\zeta_{\mathbf{p}_s}$,

$$\int dz \zeta_{\mathbf{p}_s}^*(z) \zeta_{\mathbf{p}'_s}(z) = \delta(p_s^z - p'^z_s). \quad (\text{A4})$$

This gives the commutation relations

$$\begin{aligned} [\hat{a}_{\mathbf{p}_s}, \hat{a}_{\mathbf{p}'_s}] &= 0, & [\hat{a}_{\mathbf{p}_s}^\dagger, \hat{a}_{\mathbf{p}'_s}^\dagger] &= 0, \\ [\hat{a}_{\mathbf{p}_s}, \hat{a}_{\mathbf{p}'_s}^\dagger] &= 2p^0(2\pi)^3 \delta^3(\mathbf{p}_s - \mathbf{p}'_s), \end{aligned} \quad (\text{A5})$$

and

$$\begin{aligned} \langle \mathbf{p}_s | \mathbf{p}'_s \rangle &= 2p^0(2\pi)^3 \delta^3(\mathbf{p}_s - \mathbf{p}'_s) \\ &= 2p_s^z(2\pi)^3 \delta(p^0 - p'^0) \delta^2(\mathbf{p}_\perp - \mathbf{p}'_\perp). \end{aligned} \quad (\text{A6})$$

For $\chi_{2/3}$ free particle states, we have similar expressions but with $\mathbf{p}_s$ replaced by $\mathbf{q}_b/\mathbf{k}_b$.

To study the process $\chi_1(p) \rightarrow \chi_2(q)\chi_3(k)$, we look into the following matrix element:

$$\begin{aligned} \mathcal{A}_{\chi_1(p) \rightarrow \chi_2(q)\chi_3(k)} &= \langle \mathbf{q}_b \mathbf{k}_b | (-i) \int dt H_{\text{int}} | \mathbf{p}_s \rangle \\ &= -i \langle \mathbf{q}_b \mathbf{k}_b | \int d^4x v(z) \chi_1(x) \chi_2(x) \chi_3(x) | \mathbf{p}_s \rangle. \end{aligned} \quad (\text{A7})$$

The differential probability is then given by

$$\begin{aligned} d\mathbb{P}_{\chi_1 \rightarrow \chi_2 \chi_3}(\mathbf{p}_s) &= \frac{1}{\langle \mathbf{p}_s | \mathbf{p}_s \rangle} \int \frac{d^3\mathbf{q}_b}{(2\pi)^3 2q^0} \left[1 \pm f_{\chi_2;b}^{\text{(wall)}}(\mathbf{q}_b, T) \right] \\ &\quad \times \int \frac{d^3\mathbf{k}_b}{(2\pi)^3 2k^0} \left[1 \pm f_{\chi_3;b}^{\text{(wall)}}(\mathbf{k}_b, T) \right] \\ &\quad \times |\mathcal{A}_{\chi_1(p) \rightarrow \chi_2(q)\chi_3(k)}|^2. \end{aligned} \quad (\text{A8})$$

Above, we have inserted the normalization factor $\langle \mathbf{p}_s | \mathbf{p}_s \rangle$. This is a formal procedure. A more rigorous, but slightly more complicated, way is to construct a wave packet state for the incoming χ_1 particle. Substituting the decomposition of the free fields into Eqs. (A8) and (A7), we obtain

$$\begin{aligned} d\mathbb{P}_{\chi_1 \rightarrow \chi_2 \chi_3}(\mathbf{p}_s) &= \frac{1}{2p_s^z} \int \frac{d^3\mathbf{q}_b}{(2\pi)^3 2q^0} \left[1 \pm f_{\chi_2;b}^{\text{(wall)}}(\mathbf{q}_b, T) \right] \\ &\quad \times \int \frac{d^3\mathbf{k}_b}{(2\pi)^3 2k^0} \left[1 \pm f_{\chi_3;b}^{\text{(wall)}}(\mathbf{k}_b, T) \right] (2\pi)^3 \\ &\quad \times \delta(p^0 - q^0 - k^0) \delta^2(\mathbf{p}_\perp - \mathbf{q}_\perp - \mathbf{k}_\perp) \\ &\quad \times |\mathcal{M}_{\chi_1(p) \rightarrow \chi_2(q)\chi_3(k)}|^2, \end{aligned} \quad (\text{A9})$$

where

$$\begin{aligned} i\mathcal{M}_{\chi_1(p) \rightarrow \chi_2(q)\chi_3(k)} &= (-i) \int dz V(z; \mathbf{p}_s, \mathbf{q}_b, \mathbf{k}_b) \\ &\quad \times \zeta_{\mathbf{p}_s}(z) \zeta_{\mathbf{q}_b}^*(z) \zeta_{\mathbf{k}_b}^*(z). \end{aligned} \quad (\text{A10})$$

To be general, we have indicated the possible dependence of $V(z; \mathbf{p}_s, \mathbf{q}_b, \mathbf{k}_b)$ on the momenta. Occasionally, we will suppress the momenta variables. $V(z)$ can differ from $v(z)$ by the contraction in the inner structure of the fields, e.g., contraction of polarizations. Comparing Eq. (A10) with Eq. (3.8) of Ref. [47], one may recognize that our $V(z)$ differs from the $V(z)$ in [47] by a minus sign. Also note that a total minus sign in $\mathcal{M}$ will not affect physical results anyway.

A quantity of interest could be the number density of the produced χ_3 particles, i.e., gravitons in the case of gravitational transition radiation. To derive it, we consider a duration Δt in the wall frame. In this frame, the flux of χ_1 particles impinging on the wall is

$$J_{\chi_1}^{(\text{wall})} = \int \frac{d^3 \mathbf{p}_s}{(2\pi)^3} \frac{p_s^z}{p^0} f_{\chi_1;s}^{(\text{wall})}(\mathbf{p}_s, T). \quad (\text{A11})$$

The total number of χ_3 particles produced in this time then reads

$$N_{\chi_3} = \text{wall area} \times \Delta t \int \frac{d^3 \mathbf{p}_s}{(2\pi)^3} \frac{p_s^z}{p^0} d\mathbb{P}_{\chi_1 \rightarrow \chi_2 \chi_3}(\mathbf{p}_s) \times f_{\chi_1;s}^{(\text{wall})}(\mathbf{p}_s, T). \quad (\text{A12})$$

In the plasma frame, the volume swept by the wall is

$$\Delta V^{(\text{plasma})} = \text{wall area} \times v_w \times \gamma_w \Delta t. \quad (\text{A13})$$

Dividing N_{χ_3} by $\Delta V^{(\text{plasma})}$ then gives the density of produced χ_3 particles in the *plasma frame*,

$$n_{\chi_3} = \frac{1}{v_w \gamma_w} \int \frac{d^3 \mathbf{p}_s}{(2\pi)^3} \frac{p_s^z}{p^0} d\mathbb{P}_{\chi_1 \rightarrow \chi_2 \chi_3}(\mathbf{p}_s) \times f_{\chi_1;s}^{(\text{wall})}(\mathbf{p}_s, T). \quad (\text{A14})$$

The energy density of χ_3 particles can be obtained by inserting a factor of $\tilde{k}^0$, i.e., the graviton energy in the *plasma frame*, inside the integrals in $d\mathbb{P}_{\chi_1 \rightarrow \chi_2 \chi_3}(\mathbf{p}_s)$, Eq. (A9). This leads to Eq. (3).

For relativistic bubble walls ($\gamma_w \gg 1$), and assuming a thermal distribution of χ_1 particles in the symmetric phase

$$f_{\chi_1;s}^{(\text{wall})}(\mathbf{p}_s, T) = \frac{1}{e^{\frac{\gamma_w(p^0 - v_w p_s^z)}{T}} - 1}, \quad (\text{A15})$$

one has that the distribution sharply peaks at $p^z = \gamma_w |\mathbf{p}_\perp| \sim \gamma_w T$ [91]. Then, we are in the Wentzel–Kramers–Brillouin regime

$$p^0, q^0, k^0 \sim \gamma_w T \gg \frac{1}{L_w} \sim T, \quad (\text{A16})$$

where L_w is the bubble wall width in the wall frame (the proper wall width). We have

$$\zeta_{\mathbf{p}_s}(z) \simeq \sqrt{\frac{p_s^z}{p^z(z)}} e^{i \int_0^z dz' p^z(z')}, \quad (\text{A17a})$$

$$\zeta_{\mathbf{q}_b}(z) \simeq \sqrt{\frac{q_b^z}{q^z(z)}} e^{i \int_0^z dz' q^z(z')} \text{ (similarly for } \mathbf{k}_b). \quad (\text{A17b})$$

The above forms satisfy $\zeta_{\mathbf{p}_s}(z) \rightarrow e^{ip_s^z z}$ for $z \rightarrow -\infty$ and $\zeta_{\mathbf{q}_b}(z) \rightarrow e^{iq_b^z z}$ for $z \rightarrow \infty$.

Using the expansion

$$p^z(z') = \sqrt{(p^0)^2 - \mathbf{p}_\perp^2 - m_{\chi_1}^2(z')} \approx p^0 - \frac{1}{2} \frac{\mathbf{p}_\perp^2 + m_{\chi_1}^2(z')}{p^0}, \quad (\text{A18})$$

and similarly for $q^z(z')$ and $k^z(z')$, we have

$$\zeta_{\mathbf{p}_s}(z) \zeta_{\mathbf{q}_b}^*(z) \zeta_{\mathbf{k}_b}^*(z) \approx e^{i \int_0^z dz' A(z')}, \quad (\text{A19})$$

where

$$A(z') = -(\mathbf{p}_\perp^2 + m_{\chi_1}^2(z')) + \frac{m_{\chi_2}^2(z') + \mathbf{q}_\perp^2}{1-x} + \frac{m_{\chi_3}^2(z') + \mathbf{k}_\perp^2}{x}. \quad (\text{A20})$$

Above, we have defined $x = k^0/p^0$, $(1-x) = q^0/p^0$ and have used the condition of energy conservation. When doing the expansion (A18) for $q^z(z')$ and $k^z(z')$, we have assumed

$$\begin{aligned} (p^0)^2(1-x)^2 &\gg \mathbf{q}_\perp^2 + m_{\chi_2}^2 \sim T^2, \\ (p^0)^2 x^2 &\gg \mathbf{k}_\perp^2 + m_{\chi_3}^2 \sim T^2 \\ \Rightarrow 1-x &\gg \frac{T}{p^0} \sim \frac{1}{\gamma_w} \text{ \& } x \gg \frac{1}{\gamma_w}. \end{aligned} \quad (\text{A21})$$

As long as γ_w is large enough, the phase space violating the above conditions is negligibly small.

The phase in Eq. (A19) is small for $|z| < L_w$. So, we can approximate

$$\int dz' A(z') \approx \Theta(-z) A_s z + \Theta(z) A_b z, \quad (\text{A22})$$

where $A_{s,b}$ is the function $A(z')$ evaluated in the symmetric/broken phase. Similarly, $V(z) \approx \Theta(-z) V_s + \Theta(z) V_b$. Substituting these equations into Eq. (A10), one obtains

$$i\mathcal{M} = -i \left[2ip^0 \left(\frac{V_b}{A_b} - \frac{V_s}{A_s} \right) \right]. \quad (\text{A23})$$

Once we know $\mathcal{M}$, we can substitute it into Eq. (A14) or (3) to compute quantities of interest.

Gravitational transition radiation. For gravitational transition radiation, we do the following replacement:

$$\chi_1(p) \rightarrow \Psi(p), \quad \chi_2(q) \rightarrow \Psi(q), \quad \chi_3(k) \rightarrow g(k). \quad (\text{A24})$$

For simplicity, we assume $m_{\Psi,s} = 0$. Then, we have

$$A_s = \frac{-x(1-x)\mathbf{p}_\perp^2 + x\mathbf{q}_\perp^2 + (1-x)\mathbf{k}_\perp^2}{x(1-x)}, \quad (\text{A25a})$$

$$A_b = \frac{-x(1-x)(\mathbf{p}_\perp^2 + m_{\Psi,b}^2) + x(\mathbf{q}_\perp^2 + m_{\Psi,b}^2) + (1-x)\mathbf{k}_\perp^2}{x(1-x)}. \quad (\text{A25b})$$

In the next appendix, we will discuss the function $V(z)$, and therefore $V_{s/b}$, for gravitational transition radiation.

APPENDIX B: MINIMAL COUPLING BETWEEN Ψ AND GRAVITON AND FEYNMAN RULES

1. Gravitational transition radiation from a scalar particle

Consider the following Lagrangian for a scalar field:

$$\mathcal{L}_s/\sqrt{-g} = \frac{1}{2}g^{\mu\nu}(\partial_\mu\chi)\partial_\nu\chi - \frac{1}{2}m_\chi^2(z)\chi^2, \quad (\text{B1})$$

where $m_\chi^2(z) = \lambda\varphi^2(z)/2$. Doing the expansion $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, where $\kappa = 2/M_{\text{Pl}}$ with M_{Pl} being the reduced Planck mass, we have $\mathcal{L}_s = \tilde{\mathcal{L}}_s + \delta\mathcal{L}_{gs} + \mathcal{O}(\kappa^2)$, where $\tilde{\mathcal{L}}_s = \mathcal{L}_s|_{g_{\mu\nu}=\eta_{\mu\nu}}$ and

$$-\delta\mathcal{L}_{gs} = \frac{\kappa}{2}h_{\mu\nu}\left\{(\partial^\mu\chi)(\partial^\nu\chi) - \eta^{\mu\nu}\left[\frac{1}{2}(\partial_\rho\chi)\partial^\rho\chi - \frac{1}{2}m_\chi^2(z)\chi^2\right]\right\}. \quad (\text{B2})$$

This gives the following V function:

$$V^{(gs)}(z; p, q, k) \approx \frac{\kappa}{2}\epsilon_{\mu\nu}^*(k)\{p^\mu q^\nu + p^\nu q^\mu - \eta^{\mu\nu}[p \cdot q - m_\chi^2(z)]\}, \quad (\text{B3})$$

where $\epsilon_{\mu\nu}(k)$ is the graviton polarization tensor. Above,

$$p = \left(p^0, \mathbf{p}_\perp, \sqrt{(p^0)^2 - \mathbf{p}_\perp^2 - m_\chi^2(z)}\right), \quad (\text{B4a})$$

$$q = \left(q^0, \mathbf{q}_\perp, \sqrt{(q^0)^2 - \mathbf{q}_\perp^2 - m_\chi^2(z)}\right), \quad (\text{B4b})$$

and it is understood that p is an incoming four-momentum and q is an outgoing four-momentum. Note that in the transverse-traceless gauge, the term proportional to $\eta^{\mu\nu}$ in Eq. (B3) would vanish due to $\epsilon_{\mu\nu}\eta^{\mu\nu} = 0$. We have checked that removing that term indeed does not change the numerical results given below. The nonvanishing $|\mathcal{M}|^2$ for GTR is, therefore, due to $A_s \neq A_b$.

Gravitons remain massless in the presence of the bubble wall. So, we can use the standard polarization sum rule for $\epsilon_{\mu\nu}$ [92],

$$\sum_{\lambda=\pm} \epsilon_{\mu\nu}^{\lambda*}(k)\epsilon_{\alpha\beta}^\lambda(k) = \frac{1}{2}(\hat{\eta}_{\mu\alpha}\hat{\eta}_{\nu\beta} + \hat{\eta}_{\mu\beta}\hat{\eta}_{\nu\alpha} - \hat{\eta}_{\mu\nu}\hat{\eta}_{\alpha\beta}), \quad (\text{B5})$$

where

$$\hat{\eta}_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu \bar{k}_\nu + \bar{k}_\mu k_\nu}{k \cdot \bar{k}}, \quad (\text{B6})$$

with $\bar{k} = (k^0, -\mathbf{k})$. Note that in the presence of the bubble wall, the gravitational Ward identity is violated and, therefore, one cannot neglect the second term in Eq. (B6) in summing the polarizations of the graviton.

2. Gravitational transition radiation from a fermion

To derive the interaction between a graviton and a Dirac fermion, one needs to generalize the conventional Dirac Lagrangian in flat spacetime to curved spacetime. For this purpose, one needs to define the Dirac matrices in curved spacetime. This can be done by introducing the vierbein fields [93]. The vector space at any spacetime point is identical to a Minkowski spacetime. Therefore, at any spacetime point, one can introduce an orthonormal basis $\{e^m(x)\}$ (with $m = 1, \dots, 4$), where the index is lowered and raised by the Minkowski metric η_{mn} (η^{mn}). The vierbein fields are then defined as

$$e_\mu^m(x) \equiv \frac{\partial e^m(x)}{\partial x^\mu}. \quad (\text{B7})$$

Then,

$$g_{\mu\nu}(x) = \eta_{mn}e_\mu^m(x)e_\nu^n(x). \quad (\text{B8})$$

The indices μ, ν are raised and lowered by the curved spacetime metric g . For example, $e_\nu^\nu(x) = \eta_{mn}g^{\mu\nu}(x)e_\mu^m(x)$, which is the inverse of $e_\mu^n(x)$. The Dirac matrices in curved spacetime are then defined as $\gamma^\mu(x) = e_\mu^m(x)\gamma^m$. Furthermore, in curved spacetime one has to introduce the covariant derivative on fermionic fields,

$$\nabla_\mu = \partial_\mu + i\omega_\mu, \quad (\text{B9})$$

where

$$\omega_\mu(x) = \frac{1}{4}\sigma^{mn}\left[e_\mu^\nu(\partial_\mu e_{n\nu} - \partial_\nu e_{n\mu}) + \frac{1}{2}e_\mu^\rho e_n^\sigma(\partial_\sigma e_{\rho\mu} - \partial_\rho e_{\sigma\mu})e_\mu^l - (m \leftrightarrow n)\right], \quad (\text{B10})$$

with $\sigma^{mn} = i[\gamma^m, \gamma^n]/2$.

Then, the Dirac Lagrangian in curved spacetime is

$$\mathcal{L}_{\text{Dirac}}/\sqrt{-g} = \frac{1}{2}\bar{\psi}\left[i\gamma^\mu\vec{\nabla}_\mu - i\gamma^\mu\vec{\nabla}_\mu - 2m_\psi(z)\right]\psi, \quad (\text{B11})$$

where $\tilde{\psi}\tilde{\nabla}_\mu = \partial_\mu\tilde{\psi} - i\tilde{\psi}\omega_\mu$, and we have considered a background field-dependent Dirac mass term $m_\psi(z) = y\varphi(z)$. Expanding the vierbein around the flat spacetime background gives [93]

$$e_\mu^m = \delta_\mu^m + \frac{\kappa}{2}h_\mu^m + \mathcal{O}(\kappa^2). \quad (\text{B12})$$

With all these prepared, we can expand the Dirac Lagrangian around the Minkowski spacetime and obtain the graviton-fermion interaction,

$$\begin{aligned} -\delta\mathcal{L}_{gf} = & \frac{\kappa}{2}h_{\mu\nu}\left\{\frac{i}{4}\tilde{\psi}\left[\gamma^\mu\overrightarrow{\partial}^\nu - \gamma^\mu\overleftarrow{\partial}^\nu\right]\psi\right. \\ & \left.-\frac{1}{4}\eta^{\mu\nu}\tilde{\psi}\left[i\gamma^\rho\overrightarrow{\partial}_\rho - i\gamma^\rho\overleftarrow{\partial}_\rho - 2m_\psi(z)\right]\psi + (\mu \leftrightarrow \nu)\right\}. \end{aligned} \quad (\text{B13})$$

It is easy to read off the V function from the above equation,

$$\begin{aligned} V^{(gf)}(z; p, q, k) \approx & \frac{\kappa}{8}\epsilon_{\mu\nu}^*(k)\bar{u}^{s'}(q)[(p+q)^\mu\gamma^\nu + (p+q)^\nu\gamma^\mu \\ & - 2\eta^{\mu\nu}(\not{p} + \not{q} - 2m_\psi(z))]u^s(p). \end{aligned} \quad (\text{B14})$$

Again, p and q are incoming and outgoing four-momenta, respectively, defined in Eq. (B4). For antifermions, one needs to replace the spinors $u, \bar{u}$ with $v, \bar{v}$, respectively. As commented above, the term with $\eta^{\mu\nu}$ does not contribute in the transverse-traceless gauge. Also note that the spinors are z -dependent and hence one should distinguish the spinors in the symmetric phase from those in the broken phase. For a related discussion, see [94].

3. Gravitational transition radiation from a vector boson

We consider

$$\mathcal{L}_A/\sqrt{-g} = -\frac{1}{4}(F_{\alpha\beta}F^{\alpha\beta})|_g + \frac{1}{2}m_A^2(z)g^{\alpha\beta}A_\alpha A_\beta, \quad (\text{B15})$$

where $F_{\alpha\beta}|_g = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$ and $m_A^2(z) = g_A^2\varphi^2(z)$. This gives the following graviton-vector-boson interaction:

$$-\delta\mathcal{L}_{gA} = \frac{\kappa}{2}h_{\mu\nu}\{-F^{\mu\alpha}F_\alpha^\nu + m_A^2(z)A^\mu A^\nu - \eta^{\mu\nu}\mathcal{L}_A|_{g=\eta}\}, \quad (\text{B16})$$

where $F_{\mu\nu}$ without $|_g$ is the field strength in flat spacetime. This gives

$$\begin{aligned} V^{(gA)}(z; p, q, k) \approx & \frac{\kappa}{2}\epsilon_\beta^*(q)\epsilon_{\mu\alpha}^*(k)[\eta^{\mu\nu}\eta^{\alpha\beta}(p \cdot q - m_A^2(z)) \\ & - \eta^{\mu\nu}p^\beta q^\alpha + \eta^{\nu\alpha}p^\beta q^\mu - \eta^{\alpha\beta}p^\nu q^\mu \\ & + \eta^{\mu\beta}p^\nu q^\alpha - \eta^{\nu\alpha}\eta^{\mu\beta}(p \cdot q - m_A^2(z)) \\ & + \eta^{\nu\beta}p^\mu q^\alpha - \eta^{\alpha\beta}p^\mu q^\nu + \eta^{\mu\alpha}p^\beta q^\nu \\ & - \eta^{\nu\beta}\eta^{\mu\alpha}(p \cdot q - m_A^2(z))]\epsilon_\alpha(p). \end{aligned} \quad (\text{B17})$$

APPENDIX C: SIMPLIFY THE INTEGRAL FOR THE PRODUCED GRAVITATIONAL WAVE ENERGY DENSITY

It is difficult to do the integral (3) analytically, and so we will rely on numerics. To this end, we simplify the integral (3) first. From the V functions, Eqs. (B3), (B14), (B17), and $A_{s/b}$ in Eq. (A25), we can obtain $M(p, q, k)$. Since p, q , and k satisfy the on-shell conditions, we can always express $\mathcal{M}$ in terms of $\mathbf{p}_s, \mathbf{q}_b$, and $\mathbf{k}$. Below, it is more convenient to write $\rho_{\text{GW}}^{\text{gen}}$ as a function of k^0 instead of $\tilde{k}^0$ (recall $\tilde{k}^0 \approx k^0/2\gamma_w$). Taking $[1 \pm f_{\Psi;b}^{(\text{wall})}(\mathbf{q}_b, T)] \approx 1$ and integrating over $\mathbf{q}_b$, we obtain

$$\begin{aligned} \frac{d\rho_{\text{GW}}^{\text{gen}}(k^0)}{d \ln k^0} = & \frac{1}{8\pi v_w \gamma_w^2} \int \frac{d^3\mathbf{p}_s}{(2\pi)^3 2p^0} f_{\Psi;s}^{(\text{wall})}(\mathbf{p}_s, T) \\ & \times \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \frac{(k^0)^2}{\sqrt{(k^0)^2 - \mathbf{k}_\perp^2}} \frac{1}{2\sqrt{H(\mathbf{p}_s, \mathbf{k})}} \\ & \times |\mathcal{M}(\mathbf{p}_s, \mathbf{k}_\perp, k^0)|^2, \end{aligned} \quad (\text{C1})$$

where

$$H(\mathbf{p}_s, \mathbf{k}) = (p^0 - k^0)^2 - (\mathbf{p}_\perp - \mathbf{k}_\perp)^2 - m_{\Psi;b}^2. \quad (\text{C2})$$

$\mathcal{M}(\mathbf{p}_s, \mathbf{k})$ is obtained by taking $\mathbf{q}_\perp = \mathbf{p}_\perp - \mathbf{k}_\perp$ and $q_b^z = \sqrt{H(\mathbf{p}_s, \mathbf{k})}$ in the original expression. Above, we have used $dk^z = (k^0/k^z)dk^0$. In principle, we have two solutions for k^z that satisfy the energy conservation condition, differing from each other by a sign. Given that the incoming particle has a large positive z -momentum, we expect that the scattering amplitude receives a dominant contribution with positive k^z . Kinematics requires $p^0 > k^0$ and $H(\mathbf{p}_s, \mathbf{k}) > 0$.

We are interested in the case $\gamma_w \gg 1$ (and so $v_w \rightarrow 1$), since for $\gamma_w \lesssim 1$ the gravitational waves produced from bubble expansion are expected to be subdominant compared with conventional sources. In this case,

$$\begin{aligned} f_{\Psi;s}^{(\text{wall})}(\mathbf{p}_s, T) \approx & e^{-\frac{\gamma_w \left(\sqrt{|\mathbf{p}_\perp|^2 + (p_s^z)^2} - v_w p_s^z \right)}{T}} \\ \approx & e^{-\frac{1}{2T} \left(\frac{\gamma_w \mathbf{p}_\perp^2}{p_s^z} + \frac{p_s^z}{\gamma_w} \right)}, \end{aligned} \quad (\text{C3})$$

where we have used

$$\sqrt{|\mathbf{p}_\perp|^2 + (p_s^z)^2} \approx p_s^z + \frac{1}{2} \frac{|\mathbf{p}_\perp|^2}{p_s^z}, \quad v_w \approx 1 - \frac{1}{2\gamma_w^2}. \quad (\text{C4})$$

$f_{\Psi;s}^{(\text{wall})}$, as a function of p_s^z , is strongly peaked at $p_s^z = \gamma_w |\mathbf{p}_\perp|$. Following Ref. [91], we can do the integral over p_s^z using the method of steepest descent. Denote

$$h(p_s^z) \equiv -\frac{1}{2T} \left(\frac{\gamma_w |\mathbf{p}_\perp|^2}{p_s^z} + \frac{p_s^z}{\gamma_w} \right). \quad (\text{C5})$$

This function is minimal at $p_s^z = \gamma_w |\mathbf{p}_\perp|$. Since it appears in the exponent, we can do the Taylor expansion, obtaining

$$h(p_s^z) \approx -\frac{|\mathbf{p}_\perp|}{T} - \frac{1}{2\gamma_w^2 |\mathbf{p}_\perp| T} (p_s^z - \gamma_w |\mathbf{p}_\perp|)^2. \quad (\text{C6})$$

When we do the integral over p_s^z , we can simply do the Gaussian integral and let $p_s^z = \gamma_w |\mathbf{p}_\perp|$ (and hence $p^0 \approx \gamma_w |\mathbf{p}_\perp|$) in all other parts. We finally obtain

$$\begin{aligned} \frac{d\rho_{\text{GW}}^{\text{gen}}(k^0)}{d\ln k^0} &\approx \frac{\sqrt{2\pi T}}{512\pi^5 \gamma_w^2} \int d|\mathbf{p}_\perp| |\mathbf{p}_\perp|^{1/2} e^{-\frac{|\mathbf{p}_\perp|}{T}} \int d|\mathbf{k}_\perp| |\mathbf{k}_\perp| \\ &\times \int_0^{2\pi} d\theta \frac{(k^0)^2}{\sqrt{(k^0)^2 - |\mathbf{k}_\perp|^2}} \frac{|\mathcal{M}(\mathbf{p}_s, \mathbf{k})|_{p_s^z=\gamma_w |\mathbf{p}_\perp|}^2}{\sqrt{H(\mathbf{p}_s, \mathbf{k})|_{p_s^z=\gamma_w |\mathbf{p}_\perp|}}}, \end{aligned} \quad (\text{C7})$$

where $\theta = \arccos(\frac{\mathbf{p}_\perp \cdot \mathbf{k}_\perp}{|\mathbf{p}_\perp| |\mathbf{k}_\perp|})$.

To simplify the integral further, we notice that the integral over $|\mathbf{p}_\perp|$ receives the dominant contribution from small $|\mathbf{p}_\perp|/T$ due to the exponential function $\exp(-|\mathbf{p}_\perp|/T)$ in the integrand. On the other hand, the factor $|\mathbf{p}_\perp|^{1/2}$ and the fact that $|\mathcal{M}|_{p_s^z=\gamma_w |\mathbf{p}_\perp|}^2$ is regular at $|\mathbf{p}_\perp| = 0$ indicate that the integral receives negligible contribution as $|\mathbf{p}_\perp| \rightarrow 0$. Combining these two observations, one may effectively replace $|\mathbf{p}_\perp|$ inside the $\mathbf{k}$ integral by the following averaged value:

$$\langle |\mathbf{p}_\perp| \rangle = \frac{\int d|\mathbf{p}_\perp| |\mathbf{p}_\perp|^{3/2} e^{-\frac{|\mathbf{p}_\perp|}{T}}}{\int d|\mathbf{p}_\perp| |\mathbf{p}_\perp|^{1/2} e^{-\frac{|\mathbf{p}_\perp|}{T}}} = \frac{3T}{2}. \quad (\text{C8})$$

Mathematically, this amounts to doing a cumulant expansion and taking the leading-order result [95]. Under such an average and taking into account energy and transverse

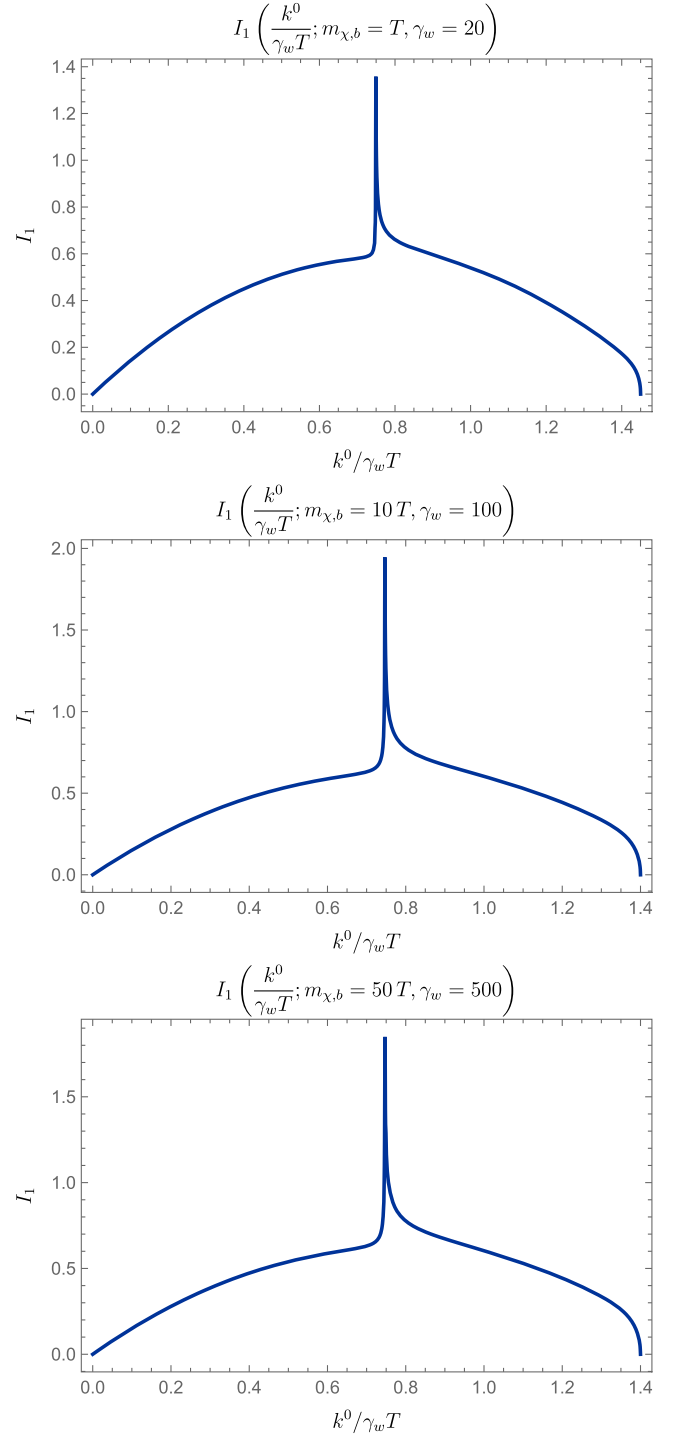


FIG. 4. The function I_1 in Eq. (C15) for a few values of $\{m_{\chi,b}, \gamma_w\}$. The additional dependence on these two parameters, aside from the dependence on the combination $k^0/\gamma_w T$, is weak. The GW power spectrum has the same shape as I_1 .

momentum conservation, we now have

$$p_s = \left(\frac{3\gamma_w T}{2}, 0, 0, \frac{3\gamma_w T}{2} \right), \quad (\text{C9a})$$

$$p_b = \left(\frac{3\gamma_w T}{2}, 0, 0, \sqrt{\frac{9\gamma_w^2 T^2}{4} - m_{\Psi,b}^2} \right), \quad (\text{C9b})$$

$$k = \left(k^0, \mathbf{k}_\perp, \sqrt{(k^0)^2 - |\mathbf{k}_\perp|^2} \right), \quad (\text{C9c})$$

$$q_s = \left(\frac{3\gamma_w T}{2} - k^0, -\mathbf{k}_\perp, \sqrt{\left(\frac{3\gamma_w T}{2} - k^0 \right)^2 - |\mathbf{k}_\perp|^2} \right), \quad (\text{C9d})$$

$$q_b = \left(\frac{3\gamma_w T}{2} - k^0, -\mathbf{k}_\perp, \sqrt{\left(\frac{3\gamma_w T}{2} - k^0 \right)^2 - |\mathbf{k}_\perp|^2 - m_{\Psi,b}^2} \right). \quad (\text{C9e})$$

Note that, although the averaged $|\mathbf{p}_\perp|$ is nonzero, we have taken the averaged $\mathbf{p}_\perp$ to be zero due to the isotropy in the plane parallel to the wall. This approximation of taking $\mathbf{p}_\perp = 0$ in computing $\mathcal{M}$ has been used in the literature, see, e.g., Refs. [47,89].

Now $|\mathcal{M}|^2(k^0, |\mathbf{k}_\perp|)$ becomes a function of k^0 and $|\mathbf{k}_\perp|$, and we can do the integrals over $|\mathbf{p}_\perp|$ and θ in Eq. (C7) trivially. We obtain

$$\frac{d\rho_{\text{GW}}^{\text{gen}}(k^0)}{d \ln k^0} \approx \frac{\sqrt{2} T^2}{512 \pi^3 \gamma_w^2} \int d|\mathbf{k}_\perp| |\mathbf{k}_\perp| \frac{(k^0)^2}{\sqrt{(k^0)^2 - |\mathbf{k}_\perp|^2}} \times \frac{|\mathcal{M}(k^0, |\mathbf{k}_\perp|)|^2}{\sqrt{G(k^0, |\mathbf{k}_\perp|)}}, \quad (\text{C10})$$

where

$$G(k^0, |\mathbf{k}_\perp|) = \left(\frac{3\gamma_w T}{2} - k^0 \right)^2 - |\mathbf{k}_\perp|^2 - m_{\Psi,b}^2. \quad (\text{C11})$$

The kinematic constraint gives

$$k^0 \leq \frac{3\gamma_w T}{2} - m_{\Psi,b}, \quad (\text{C12a})$$

$$0 \leq |\mathbf{k}_\perp| \leq \min \left\{ k^0, \sqrt{\left(\frac{3\gamma_w T}{2} - k^0 \right)^2 - m_{\Psi,b}^2} \right\}. \quad (\text{C12b})$$

Example: Gravitational transition radiation from a scalar particle χ . Taking

$$x_1 = \frac{k^0}{T}, \quad x_2 = \frac{|\mathbf{k}_\perp|}{T}, \quad x_3 = \frac{m_{\chi,b}}{T}, \quad (\text{C13})$$

we can write $|\mathcal{M}|^2$ as

$$|\mathcal{M}|^2 = \kappa^2 T^2 f(x_1, x_2, x_3, \gamma_w), \quad (\text{C14})$$

where f is a dimensionless function obtained numerically. The final result can be fitted as

$$\frac{d\rho_{\text{GW}}^{\text{gen}}(k^0)}{d \ln k^0} \approx 3.610^{-4} \left(\frac{m_{\chi,b}}{m_{\text{Pl}}} \right)^2 T^4 \times I_1 \left(\frac{k^0}{\gamma_w T}; m_{\chi,b}, \gamma_w \right), \quad (\text{C15})$$

where the dependence on $m_{\chi,b}$ and γ_w (with $k^0/\gamma_w T$ fixed) in the function I_1 is weak. We show and compare the results in Fig. 4 for a few values of $\{m_{\chi,b}, \gamma_w\}$.

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