

Beautiful and charming energy correlatorsEvan Craft,^{1,*} Kyle Lee^{2,3,†}, Bianka Meçaj,^{1,‡} and Ian Mould^{1,§}¹*Department of Physics, Yale University, New Haven, Connecticut 06511, USA*²*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*³*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

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Understanding the detailed structure of energy flow within jets, a field known as jet substructure, plays a central role in searches for new physics, and precision studies of QCD. Many applications of jet substructure require an understanding of jets initiated by heavy quarks, whose description has lagged behind remarkable recent progress for massless jets. In this Letter, we initiate a study of correlation functions of energy flow operators on beauty and charm jets to illuminate the effects of the intrinsic mass of the elementary particles of QCD. We present a factorization theorem incorporating the mass of heavy quarks, and show that the heavy quark jet functions for energy correlators have a simple structure in perturbation theory. Our results achieve the very first full next-to-leading-logarithmic calculation of the heavy quark jet substructure observable at the LHC. Using this framework, we study the behavior of the correlators, and show that they exhibit a clear transition from a massless scaling regime, at precisely the scale of the heavy quark mass. This manifests the long-sought-after dead-cone effect and illustrates fundamental effects from the intrinsic mass of beauty and charm quarks in a perturbative regime, before they are confined inside hadrons. Our theoretical framework for studying energy correlators using heavy jets has many exciting applications for improving the description of mass effects in next generation parton shower event generators, probing the QGP, and studying heavy flavor fragmentation functions.

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Introduction. Jet substructure is playing an increasingly central role in collider physics [1–3], with applications ranging from innovative searches for new physics, to unravelling the nature of the quark-gluon plasma (QGP) [4–8]. The progressively subtle features of the radiation pattern being exploited have motivated significant progress in the description of the perturbative substructure of jets, including the introduction of new theoretically motivated observables [9,10] and new theoretical techniques [11–15].

Despite this tremendous progress, many physical applications require an understanding of the substructure of jets initiated by massive (beauty or charm) jets, the most famous examples being the search for $H \rightarrow b\bar{b}$ [16], and recent searches for $H \rightarrow c\bar{c}$, for which the strongest constraints come from jet substructure [17,18]. In the

context of QCD, heavy quarks provide direct access to mass effects of quarks before their confinement into hadrons, and provide interesting probes of the QGP [19–23]. For a wealth of other discussions of the importance of heavy quarks, see, e.g., studies of heavy quark production [24–48], fragmentation [49–55], massive event shapes [56–59], jet substructure [6,60–69] and experimental measurements [70–76].

The theoretical description of the substructure of heavy quark jets is made difficult by the presence of the additional scale, and has seen much less progress than its massless counterpart. This is true both for perturbative calculations, as well as for parton shower simulations where large discrepancies with data are often observed [73–75]. While many beautiful measurements of event shapes on b-quark jets were made at stanford linear collider/large electron–positron collider [70–72], a particular complexity associated with the high energies of the LHC is that it requires a description of the transition from a regime where the quark behaves as massless to one where it behaves as massive. Motivated both by the wealth of physical applications, and by the development of a next generation of parton showers [77–85] that must incorporate heavy quark effects, we believe that the study of heavy quark jets requires renewed attention and a new approach.

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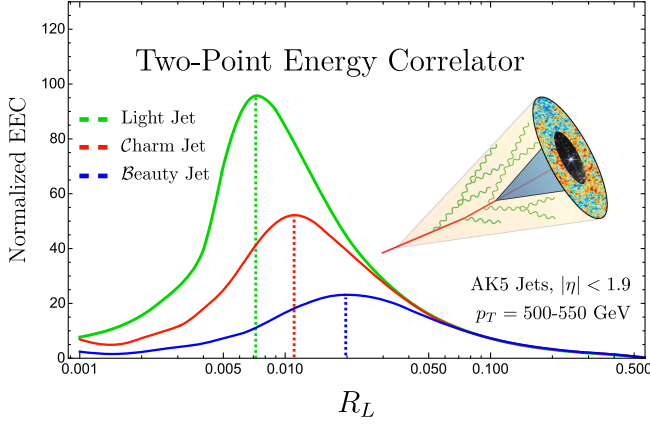


FIG. 1. EEC measured inside high- p_T jets as a function of the correlation angle, denoted as R_L . The radiation pattern in the jet is modified by the presence of heavy quarks masses, suppressing small angle radiation and giving rise to a “dead cone”.

In this Letter we initiate a study of heavy quark jets using correlation functions [86–89] $\langle \Psi | \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \cdots \mathcal{E}(\vec{n}_k) | \Psi \rangle$ of energy flow operators [11,90–92], $\mathcal{E}(\vec{n})$, allowing us to apply recent theoretical progress in this area¹ to study the effects of intrinsic quark masses within jets. We present a factorization formula that characterizes the universal behavior of energy flow operator correlators, referred to as energy-energy correlators (EEC), in the small angle limit, and perturbatively compute the effects of the heavy quark mass, unveiling their simple structure. We show that the heavy quark mass imprints itself as a characteristic angular scale in the energy correlator, see Fig. 1, allowing us direct access to the “dead-cone” effect [114] recently measured by the ALICE collaboration [115–117], but in an observable that can be systematically computed in perturbation theory. This extends the list of physical systems to which the energy correlators have recently been applied to derive new insights, ranging from top quark decays [118], to nucleon structure [119], and massless jets in the QGP [120].

We introduce a factorization formula that outlines the universal behavior of energy flow operator correlators, referred to as energy-energy correlators (EEC), in the small angle limit. Our perturbative computation reveals the impact of heavy quark mass on these correlators, demonstrating a straightforward structure in their behavior.

Factorization theorem. Since our interest is on the behavior of energy correlators *inside* jets, we are naturally led to study their universal behavior at small angles, which can be described by universal factorization theorems. Here we present the factorization theorem for a general class of scaling observables derived from the N -point correlation

¹For a selection, see, e.g., [9,11–15,93–113] and in particular the development of the light-ray operator product expansion [11,12,107] and celestial blocks [12,14,15,107].

functions by integrating out all the information about the shape while keeping the length of the longest side, R_L , fixed, which are referred to as projected correlators, and will be denoted ENC [9]. In the small angle limit, the cumulant of the projected N -point correlators $\Sigma^{[N]}(R_L, p_T^2, m_Q, \mu)$ factorizes into a hard function $\vec{H}(x, p_T^2, \mu)$, which describes the production of the collinear source, and the energy correlator jet function, $\vec{J}^{[N]}(R_L, x, \mu, m_Q)$, which describes the R_L dependence of the observable

$$\Sigma^{[N]}(R_L, p_T^2, m_Q, \mu) = \int_0^1 dx x^N \vec{J}^{[N]}(R_L, x, m_Q, \mu) \cdot \vec{H}(x, p_T^2, \mu). \quad (1)$$

This factorization was first derived in the context of e^+e^- [102] and was recently extended to hadron colliders [121]. It should be viewed as an extension of the classic factorization theorems [122–128] for massive quark fragmentation [49–54] to the case of jet substructure observables. Note that massive jets may introduce additional subtleties [129–132] in the factorization theorem beyond the next-to-leading-logarithmic (NLL) accuracy we work in this paper. In the case of hadron colliders, $\vec{H}(x, p_T^2, \mu)$ also includes the parton distribution functions of the incoming beams as well as the matching coefficients incorporating the details of the jet algorithm. The hard function in proton-proton was recently computed at next-to-next-to-leading order [24], which should allow us to extend the perturbative accuracy of our calculation. The corresponding hard scale of the hadron colliders is associated with the transverse momentum of the jet, $\mu_H \sim p_T$. When jet algorithms such as the anti- k_T jet algorithm [133–136] are used, they also introduce a jet scale associated with the jet radius as $\mu_J \sim p_T R$. Compared to what is presented in literature, we now also include the heavy quark mass in our factorization framework. See also [126,128]. As long as $\mu_J \gg m_Q$, the heavy quark mass dependence factorizes completely into the heavy flavor jet function. The μ scale given in Eq. (1) represents this factorization of the jet production scale μ_J from the EEC measurement scale $\mu_{\text{EEC}} \sim p_T R_L + M_Q$. In the region where the factorization is valid, the hard function and the jet function are vectors in flavor space such that $\vec{H} \equiv \{H_g, H_q, H_Q = H_{\bar{q}}\}$ and $\vec{J}^{[N]} \equiv \{J_g^{[N]}, J_q^{[N]}, J_Q^{[N]}\}$, where g, q and Q stand for gluon, light quark and massive quark labels, respectively.

Since factorization theorems involving massive quarks are much less well tested than their massless counterparts, to verify the factorization theorem in Eq. (1), we have computed the full angle EEC for massive quarks in $e^+e^- \rightarrow Q\bar{Q}g$, expanded it in the small angle limit, and used it to separately extract the heavy quark EEC jet function. We find exact agreement with the direct calculation of the jet function from its operator definition in the factorization theorem, which is presented in Eq. (3), providing a highly non-trivial check on the factorization

theorem. This also highlights the perturbative tractability of the energy correlators on massive partons, also highlighted by the perturbative calculations beyond the leading order for massless partons [95,98–100], and is deserving of further study.

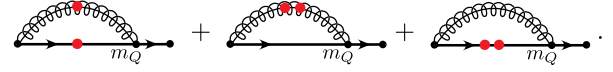
The heavy quark EEC jet function. The cumulant N -point heavy quark energy correlator jet function is defined as [102]

$$J_Q^{[N]}(R_L, m_Q) = \sum_X \sum_{i_1, i_2, \dots, i_N \in X} \langle 0 | \tilde{\chi}_n | X \rangle \frac{E_{i_1} E_{i_2} \cdots E_{i_N}}{p_T^N} \Theta(\max\{\theta_{ij}\} < R_L) \langle X | \chi_n | 0 \rangle, \quad (2)$$

where χ_n is the gauge invariant collinear field in soft-collinear effective theory (SCET) for the massive quark field [137–141]. We choose to retain only the maximum angle information here, which corresponds to the N -point projected energy correlator discussed below. As the UV physics is independent of the heavy quark mass, the UV poles of the heavy quark jet functions are identical to those of the light quark jet functions.

The light quark energy correlator jet function was computed in [102] by integrating the Altarelli-Parisi splitting functions [142] over the massless collinear phase

space. In the case of a massive quark, the phase space and the matrix elements are nontrivially modified due to the presence of the mass. Representative one-loop diagrams are



We have computed the jet function both using the massive splitting functions [143], as well as massive [144] SCET [137–141], finding agreement.

An interesting feature of the massive jet function calculation is that the heavy quark mass renders the virtual diagrams, as well as diagrams where both detectors lie on the same particle, not scaleless, and sensitive to the quark mass. With the mass regulating the IR poles, the dimensional regularization parameter ϵ only regulates the UV poles, and there is no ambiguity with respect to the nature of different poles. The IR sensitivity in the poles cancels in the sum of the diagrams as expected and the remaining UV divergence corresponds to the LO twist-2 spin- $N+1$ anomalous dimension as in the massless case [102]. This consistency is also required for the consistency of the factorization theorem given in Eq. (1). Suppressing the contact terms for simplicity, for $R_L \neq 0$, the next-to-leading order (NLO) heavy quark jet function is given by

$$J_Q^{[N]}(R_L, m_Q)|_{R_L \neq 0} = \frac{\alpha_s C_F}{4\pi} \int dx \frac{2(1 - (1-x)^N - x^N) [2x^3 + (1+x^2)(x+\delta)(x+\bar{\delta}) \ln \frac{\delta\bar{\delta}}{(x+\delta)(x+\bar{\delta})}]}{(-1+x)(x+\delta)(x+\bar{\delta})}, \quad (3)$$

where $\delta = \frac{im_Q}{p_T R_L}$. This integral can be done as a function of N in terms of hypergeometric functions, however, the particular result is not particularly enlightening. For integer N , the results are expressed in terms of the transcendental functions of weight 1 (i.e., logarithms) with alphabet $\{\delta, \bar{\delta}, 1+\delta, 1+\bar{\delta}\}$, and explicit results for up to $N=6$, as well as analogous

results for the mass-dependent part of the gluon jet function, are presented in Supplemental Material [145]. It would be interesting to understand if this simple structure for the heavy quark jet function persists at higher loop order.

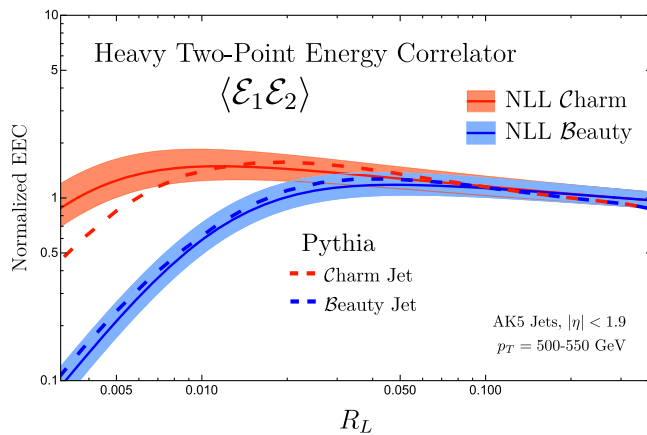


FIG. 2. EEC for beauty and charm jets illustrating a UV scaling behavior at large angles, and a mass dependent suppression at small angles.

The two-point energy correlator. The lowest order correlation function that exhibits a nontrivial dependence on the angular distance is the two-point energy correlator. This corresponds to taking $N=2$ in Eq. (1). As the angular scale R_L is associated with the transverse momentum exchange $\sim p_T R_L + m_Q$ between the two point, we expect to observe distinct regimes as the angular scale is varied. In Fig. 2, we compare our analytic calculation of the two-point energy correlators for beauty and charm quark jets with results from the parton shower Monte Carlo Pythia 8.2 [146].² We have verified that Vincia [147–149] gives compatible results. In Pythia we select $gg \rightarrow Q\bar{Q}$ and $q\bar{q} \rightarrow Q\bar{Q}$ processes, where Q is beauty or charm or light flavor quarks. In order to carry out a corresponding theoretical calculation, we choose $H_q = H_g = 0$ in Eq. (1), whereas

²Note that this figure contains information identical to Fig. 1, but is plotted as log-log scales to emphasize the scaling behaviors.

this may experimentally correspond to using experimental tagging techniques to select the heavy flavor jets [73–75]. Our analytic calculation is performed at NLL (single log counting), using the ingredients developed in [121], including the NLO hard functions [150–155], the NLO fragmenting jet functions [156,157], and the NLO energy correlator jet constants [102]. We estimate theoretical uncertainties by varying all ingredients around their canonical scales by a factor of 2 and taking their envelope. This is the first calculation of a heavy quark jet substructure observable at the LHC at this perturbative accuracy.

At scales larger than the heavy quark masses, we observe that the correlators exhibit a scaling behavior identical to that for massless quarks (we will verify this quantitatively in the next section).³ As $R_L \rightarrow m_Q/p_T$, we observe an onset of sensitivity to the heavy quark mass scale. A similar behavior was observed for light quark jets, but with the turnover at a scale of $R_L \rightarrow \Lambda_{\text{QCD}}/p_T$ [119,159]. An interesting feature of heavy quark jets is that this turnover is described by perturbation theory, in particular, the heavy quark EEC jet function in Eq. (3). Using a profile function to match between the resummation in the scaling and the heavy quark mass regions, we find good agreement with parton shower simulations, particularly for b -quark jets, where the turnover is more in the perturbative region. We believe this turnover region is particularly interesting for improving the description of heavy quark dynamics in parton shower programs. For instance, despite qualitatively exhibiting a turnover as well, c -quark jets display some discrepancies in comparison to the Monte Carlo simulations, potentially owing to the separation of perturbative and nonperturbative dynamics in Pythia. To our knowledge, ours is the first infrared safe heavy flavor observable (only sensitive to collinear dynamics) that exhibits such strong sensitivity to the intrinsic mass effects. We will now study the UV (large angle) and IR (small angle) regimes in more detail.

Projected correlators and anomalous dimensions. A particularly interesting feature of the high LHC energies is that we are able to clearly see the transition between a regime where the heavy quark behaves as massless to a regime where the mass dominates. To make this quantitative, we can study ratio of projected correlators. This corresponds to taking the ratio of $N > 2$ and $N = 2$ point correlators calculated using Eq. (1). These ratios were designed to remove IR effects and isolate UV scaling associated with twist-2 anomalous dimensions [9]. Although this was originally intended to eliminate nonperturbative effects at the scale Λ_{QCD} , we find that it also cancels the heavy quark effects, which are also IR. In Fig. 3 we show the projected correlator ratios up to six points for massive quark, which exhibit a clear scaling behavior.

³As the anomalous dimension is positive and directly proportional to α_s , the scaling exponent diminishes as p_T increases. Recent data affirm this observation [158].

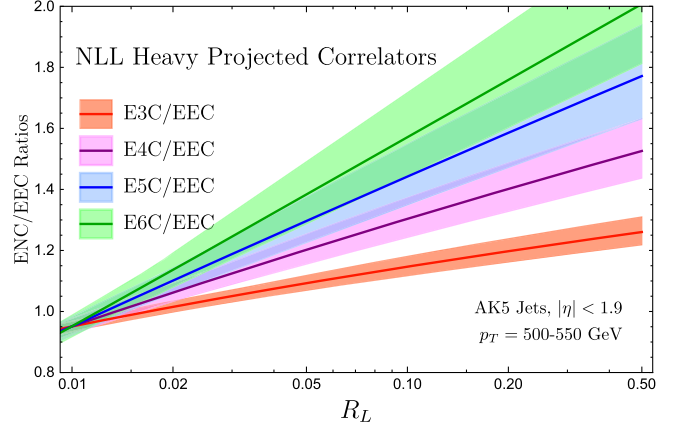


FIG. 3. Ratios of the multipoint projected correlators resolve the structure of the UV scaling dimensions of QCD, and are independent of the heavy quark masses.

The uncertainty bands show the envelope obtained by varying both the hard and jet scales up and down by a factor of two, with numerator and denominator uncertainties treated as fully correlated. Since both inherit their anomalous scaling from twist-two operators, differing only in spin and accessed through different moments of the same QCD splitting functions, this correlated variation is a natural starting point. We emphasize, however, that for ratio observables a quantitative treatment of the correlation between numerator and denominator uncertainties is particularly important and deserves careful study [160]. The difference compared to [121] only arises from making the choice $H_q = H_g = 0$ in Eq. (1) as discussed, and ratios are otherwise identical regardless of whether one measures them on a charm or beauty or massless jet in the angular region away from mass scales $R_L \gg m_Q/p_T$. While this is expected since the anomalous dimensions are a UV effect, we find it remarkable that we can isolate the scaling behavior from the mass effects in a clean manner.

The dead-cone effect. In the other extreme, we can isolate infrared effects by taking the ratio of the two-point correlator on massive and massless jets. Since the scaling behavior is the same in both cases, this ratio should be calculable in fixed order perturbation theory, and be reflective of the infrared dynamics of the quark mass. This ratio is shown in Fig. 4, where we find excellent agreement between our fixed order calculation and the Pythia parton shower, illustrating that the transition region is under perturbative control.

The apparent reduction of the uncertainty band in the small-angle region is mainly because the numerator becomes significantly smaller than the denominator in this area. The ratio of the numerator, between the central scale and its upper or lower variation, remains relatively flat over the full angular range plotted. Here, we allow the numerator and denominator scales to vary independently, yielding a more conservative estimate of the perturbative uncertainty. In the angular region near $p_T R_L \sim \Lambda_{\text{QCD}}$ (indicated in

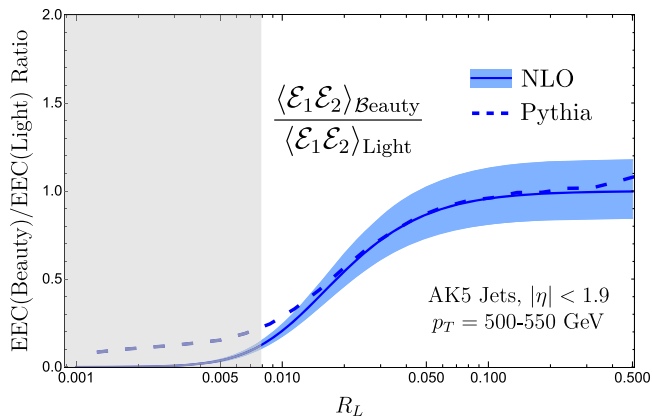


FIG. 4. Ratios of heavy/light correlators reveal a suppression of the small-angle gluon radiation at the angular scale of the heavy quark mass. This provides a calculable observable sensitive to the dead-cone effect.

gray), the distribution enters a fully nonperturbative regime and is described by dihadron fragmentation functions as noted in [161]. Therefore, predictions in this region must be interpreted with caution.

This shape of the ratio in Fig. 4 is a clean manifestation of what is commonly referred to as the “dead-cone” effect, which is a fundamental feature of all gauge field theories and is a direct signature of the intrinsic mass before confinement [114]. The dead-cone effect was recently observed experimentally [117] using an observable based on a de-clustering procedure [115]. Here we have shown how it can be seen in the energy correlators, where the description of the transition region can be computed systematically in perturbation theory. We look forward to real data observation of the dead-cone effect using energy correlators, and comparison to higher order calculations of heavy quarks in the near future.

Conclusions. In this Letter we have initiated a study of heavy quark jets using energy correlators. While the study of heavy quarks has long played an important role in QCD for understanding the mass effects of elementary particles, much less attention has been devoted to their study in jet substructure, primarily due to the prohibitive complexity associated with the additional mass scale.

We derived a factorization theorem for projected energy correlators on heavy quark jets, and showed that the jet functions, which encode the heavy quark mass effects can be computed analytically, and take a simple form. We demonstrated that the scaling of heavy energy correlators, like that of light energy correlators, is also governed by the twist-2 anomalous dimensions at large angles, but shows a turnover at angles of order the heavy quark mass. This provides an observable that is simultaneously experimentally measurable, and can be computed systematically in perturbation theory, that manifests the long-sought-after dead-cone effect [19,114–117,162,163].

We believe that a precision understanding of the radiation pattern of jets initiated by massive quarks, as provided by the energy correlators, has many important applications. First, they can be used to improve the description of heavy quarks within next generation parton showers [77–85], which play a crucial role in many LHC searches, such as $H \rightarrow c\bar{c}$. For this purpose it will also be important to extend our results to shape dependent multipoint correlators [10,101]. Second, heavy quark jets are a powerful probe of the quark-gluon plasma, and our calculation provides a first step toward understanding how they are modified by interactions with the medium. Finally, it would be interesting to apply our approach to other heavy quark systems, such as quarkonia to help in understanding the fragmentation mechanisms of heavy flavor hadrons and in resolving outstanding puzzles of heavy quarkonium production [23,50–54,128,164–169]. We hope that the beauty and charm of the energy correlators will open many new avenues to study heavy particles in QCD.

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- [1] A. J. Larkoski, I. Moult, and B. Nachman, *Phys. Rep.* **841**, 1 (2020).
- [2] R. Kogler *et al.*, *Rev. Mod. Phys.* **91**, 045003 (2019).
- [3] S. Marzani, G. Soyez, and M. Spannowsky, *Looking Inside Jets: An Introduction to Jet Substructure and Boosted-Object Phenomenology* (Springer, New York, 2019), Vol. 958.

- [4] M. Connors, C. Nattrass, R. Reed, and S. Salur, *Rev. Mod. Phys.* **90**, 025005 (2018).
- [5] W. Busza, K. Rajagopal, and W. van der Schee, *Annu. Rev. Nucl. Part. Sci.* **68**, 339 (2018).
- [6] H. A. Andrews *et al.*, *J. Phys. G* **47**, 065102 (2020).

- [7] L. Cunqueiro and A. M. Sickles, *Prog. Part. Nucl. Phys.* **124**, 103940 (2022).
- [8] L. Apolinário, Y.-J. Lee, and M. Winn, *Prog. Part. Nucl. Phys.* **127**, 103990 (2022).
- [9] H. Chen, I. Moul, X. Zhang, and H. X. Zhu, *Phys. Rev. D* **102**, 054012 (2020).
- [10] H. Chen, I. Moul, J. Thaler, and H. X. Zhu, *J. High Energy Phys.* **07** (2022) 146.
- [11] D. M. Hofman and J. Maldacena, *J. High Energy Phys.* **05** (2008) 012.
- [12] M. Kologlu, P. Kravchuk, D. Simmons-Duffin, and A. Zhiboedov, *J. High Energy Phys.* **01** (2021) 128.
- [13] H. Chen, I. Moul, and H. X. Zhu, *J. High Energy Phys.* **08** (2022) 233.
- [14] H. Chen, I. Moul, J. Sandor, and H. X. Zhu, *J. High Energy Phys.* **09** (2022) 199.
- [15] C.-H. Chang and D. Simmons-Duffin, *J. High Energy Phys.* **02** (2023) 126.
- [16] J. M. Butterworth, A. R. Davison, M. Rubin, and G. P. Salam, *Phys. Rev. Lett.* **100**, 242001 (2008).
- [17] H. Qu and L. Gouskos, *Phys. Rev. D* **101**, 056019 (2020).
- [18] A. Tumasyan *et al.* (CMS Collaboration), *Phys. Rev. Lett.* **131**, 061801 (2023).
- [19] N. Armesto, C. A. Salgado, and U. A. Wiedemann, *Phys. Rev. D* **69**, 114003 (2004).
- [20] Y. L. Dokshitzer and D. E. Kharzeev, *Phys. Lett. B* **519**, 199 (2001).
- [21] A. Adare *et al.* (PHENIX Collaboration), *Phys. Rev. Lett.* **98**, 172301 (2007).
- [22] G. D. Moore and D. Teaney, *Phys. Rev. C* **71**, 064904 (2005).
- [23] A. Andronic *et al.*, *Eur. Phys. J. C* **76**, 107 (2016).
- [24] M. Czakon, T. Generet, A. Mitov, and R. Poncelet, *J. High Energy Phys.* **10** (2021) 216.
- [25] M. Czakon and A. Mitov, *Comput. Phys. Commun.* **185**, 2930 (2014).
- [26] M. Czakon, P. Fiedler, and A. Mitov, *Phys. Rev. Lett.* **110**, 252004 (2013).
- [27] M. Czakon and A. Mitov, *J. High Energy Phys.* **01** (2013) 080.
- [28] P. Bärnreuther, M. Czakon, and A. Mitov, *Phys. Rev. Lett.* **109**, 132001 (2012).
- [29] M. Czakon and A. Mitov, *J. High Energy Phys.* **12** (2012) 054.
- [30] M. Czakon, A. Mitov, and S. Moch, *Phys. Lett. B* **651**, 147 (2007).
- [31] M. Czakon and A. Mitov, *Nucl. Phys.* **B824**, 111 (2010).
- [32] A. Mitov and S. Moch, *J. High Energy Phys.* **05** (2007) 001.
- [33] S. Catani, S. Devoto, M. Grazzini, S. Kallweit, and J. Mazzitelli, *J. High Energy Phys.* **03** (2021) 029.
- [34] P. Nason, S. Dawson, and R. K. Ellis, *Nucl. Phys.* **B327**, 49 (1989); **B335**, 260(E) (1990).
- [35] P. Nason, S. Dawson, and R. K. Ellis, *Nucl. Phys.* **B303**, 607 (1988).
- [36] M. L. Mangano, P. Nason, and G. Ridolfi, *Nucl. Phys.* **B373**, 295 (1992).
- [37] S. Frixione, M. L. Mangano, P. Nason, and G. Ridolfi, *Nucl. Phys.* **B431**, 453 (1994).
- [38] S. Frixione, M. L. Mangano, P. Nason, and G. Ridolfi, *Adv. Ser. Dir. High Energy Phys.* **15**, 609 (1998).
- [39] M. Cacciari, S. Frixione, N. Houdeau, M. L. Mangano, P. Nason, and G. Ridolfi, *J. High Energy Phys.* **10** (2012) 137.
- [40] M. Cacciari, P. Nason, and R. Vogt, *Phys. Rev. Lett.* **95**, 122001 (2005).
- [41] R. K. Ellis and P. Nason, *Nucl. Phys.* **B312**, 551 (1989).
- [42] Z.-B. Kang, F. Ringer, and I. Vitev, *J. High Energy Phys.* **03** (2017) 146.
- [43] L. Chen, Heavy quark-pair production in electron positron collisions at next-to-next-to-leading order QCD, Ph.D. thesis, RWTH Aachen University, 2017.
- [44] X. Chen, X. Guan, C.-Q. He, X. Liu, and Y.-Q. Ma, *Phys. Rev. Lett.* **132**, 101901 (2024).
- [45] S. Catani, M. Ciafaloni, and F. Hautmann, *Nucl. Phys.* **B366**, 135 (1991).
- [46] E. L. Berger and H. Contopanagos, *Phys. Rev. D* **54**, 3085 (1996).
- [47] S. Catani, M. L. Mangano, P. Nason, and L. Trentadue, *Nucl. Phys.* **B478**, 273 (1996).
- [48] N. Kidonakis and G. F. Sterman, *Nucl. Phys.* **B505**, 321 (1997).
- [49] B. Mele and P. Nason, *Nucl. Phys.* **B361**, 626 (1991); **B921**, 841(E) (2017).
- [50] B. Mele and P. Nason, *Phys. Lett. B* **245**, 635 (1990).
- [51] K. Melnikov and A. Mitov, *Phys. Rev. D* **70**, 034027 (2004).
- [52] A. Mitov, *eConf C050318*, 0608 (2005).
- [53] A. Mitov, *Phys. Rev. D* **71**, 054021 (2005).
- [54] M. Fickinger, S. Fleming, C. Kim, and E. Mereghetti, *J. High Energy Phys.* **11** (2016) 095.
- [55] M. Cacciari and S. Catani, *Nucl. Phys.* **B617**, 253 (2001).
- [56] C. Lepenik and V. Mateu, *J. High Energy Phys.* **03** (2020) 024.
- [57] A. Bris, V. Mateu, and M. Preisser, *J. High Energy Phys.* **09** (2020) 132.
- [58] P. Achar *et al.* (L3 Collaboration), *Phys. Rep.* **399**, 71 (2004).
- [59] P. Achar *et al.* (L3 Collaboration), *PMC Phys. A* **2**, 6 (2008).
- [60] R. Bain, L. Dai, A. Hornig, A. K. Leibovich, Y. Makris, and T. Mehen, *J. High Energy Phys.* **06** (2016) 121.
- [61] Y. Mehtar-Tani and K. Tywoniuk, *J. High Energy Phys.* **04** (2017) 125.
- [62] B. Dehnadi, Heavy quark mass determinations with sum rules and jets, Ph.D. thesis, Wien U., Vienna U., 2016.
- [63] Z.-B. Kang, F. Ringer, and I. Vitev, *Phys. Lett. B* **769**, 242 (2017).
- [64] J. Casallerrey-Solana, G. Milhano, D. Pablos, and K. Rajagopal, *J. High Energy Phys.* **01** (2020) 044.
- [65] C. Lee, P. Shrivastava, and V. Vaidya, *J. High Energy Phys.* **09** (2019) 045.
- [66] Y.-T. Chien and M. D. Schwartz, *J. High Energy Phys.* **08** (2010) 058.
- [67] Y. Makris and V. Vaidya, *J. High Energy Phys.* **10** (2018) 019.
- [68] H. T. Li and I. Vitev, *J. High Energy Phys.* **07** (2019) 148.
- [69] L. Dai, C. Kima, C. Kim, and A. K. Leibovich, *J. High Energy Phys.* **09** (2018) 109; **12** (2024) 003(E).

- [70] K. Abe *et al.* (SLD Collaboration), *Phys. Lett. B* **386**, 475 (1996).
- [71] K. Abe *et al.* (SLD Collaboration), *Phys. Lett. B* **578**, 45 (2004).
- [72] J. Abdallah *et al.* (DELPHI Collaboration), *Eur. Phys. J. C* **46**, 569 (2006).
- [73] S. Acharya *et al.* (ALICE Collaboration), *J. High Energy Phys.* **08** (2019) 133.
- [74] A. M. Sirunyan *et al.* (CMS Collaboration), *J. High Energy Phys.* **05** (2021) 054.
- [75] G. Aad *et al.* (ATLAS Collaboration), *J. High Energy Phys.* **12** (2021) 131.
- [76] G. Aad *et al.* (ATLAS Collaboration), *Phys. Rev. D* **106**, 032008 (2022).
- [77] H. T. Li and P. Skands, *Phys. Lett. B* **771**, 59 (2017).
- [78] S. Höche, F. Krauss, and S. Prestel, *J. High Energy Phys.* **10** (2017) 093.
- [79] S. Höche and S. Prestel, *Phys. Rev. D* **96**, 074017 (2017).
- [80] F. Dulat, S. Höche, and S. Prestel, *Phys. Rev. D* **98**, 074013 (2018).
- [81] L. Gellersen, S. Höche, and S. Prestel, *Phys. Rev. D* **105**, 114012 (2022).
- [82] K. Hamilton, R. Medves, G. P. Salam, L. Scyboz, and G. Soyez, *J. High Energy Phys.* **03** (2021) 041.
- [83] M. Dasgupta, F. A. Dreyer, K. Hamilton, P. F. Monni, G. P. Salam, and G. Soyez, *Phys. Rev. Lett.* **125**, 052002 (2020).
- [84] K. Hamilton, A. Karlberg, G. P. Salam, L. Scyboz, and R. Verheyen, *J. High Energy Phys.* **03** (2022) 193.
- [85] A. Karlberg, G. P. Salam, L. Scyboz, and R. Verheyen, *Eur. Phys. J. C* **81**, 681 (2021).
- [86] C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, *Phys. Lett.* **85B**, 297 (1979).
- [87] C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, *Phys. Rev. D* **19**, 2018 (1979).
- [88] C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, *Phys. Rev. Lett.* **41**, 1585 (1978).
- [89] C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, *Phys. Rev. D* **17**, 2298 (1978).
- [90] A. V. Belitsky, G. P. Korchemsky, and G. F. Sterman, *Phys. Lett. B* **515**, 297 (2001).
- [91] G. P. Korchemsky and G. F. Sterman, *Nucl. Phys.* **B555**, 335 (1999).
- [92] G. P. Korchemsky, G. Oderda, and G. F. Sterman, *AIP Conf. Proc.* **407**, 988 (1997).
- [93] A. V. Belitsky, S. Hohenegger, G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, *Nucl. Phys.* **B884**, 305 (2014).
- [94] A. V. Belitsky, S. Hohenegger, G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, *Nucl. Phys.* **B884**, 206 (2014).
- [95] A. V. Belitsky, S. Hohenegger, G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, *Phys. Rev. Lett.* **112**, 071601 (2014).
- [96] G. P. Korchemsky and E. Sokatchev, *J. High Energy Phys.* **12** (2015) 133.
- [97] A. V. Belitsky, S. Hohenegger, G. P. Korchemsky, and E. Sokatchev, *Nucl. Phys.* **B904**, 176 (2016).
- [98] L. J. Dixon, M.-X. Luo, V. Shtabovenko, T.-Z. Yang, and H. X. Zhu, *Phys. Rev. Lett.* **120**, 102001 (2018).
- [99] M.-X. Luo, V. Shtabovenko, T.-Z. Yang, and H. X. Zhu, *J. High Energy Phys.* **06** (2019) 037.
- [100] J. M. Henn, E. Sokatchev, K. Yan, and A. Zhiboedov, *Phys. Rev. D* **100**, 036010 (2019).
- [101] H. Chen, M.-X. Luo, I. Moul, T.-Z. Yang, X. Zhang, and H. X. Zhu, *J. High Energy Phys.* **08** (2020) 028.
- [102] L. J. Dixon, I. Moul, and H. X. Zhu, *Phys. Rev. D* **100**, 014009 (2019).
- [103] G. P. Korchemsky, *J. High Energy Phys.* **01** (2020) 008.
- [104] D. Chicherin, J. M. Henn, E. Sokatchev, and K. Yan, *J. High Energy Phys.* **02** (2021) 053.
- [105] P. Kravchuk and D. Simmons-Duffin, *J. High Energy Phys.* **11** (2018) 102.
- [106] M. Kologlu, P. Kravchuk, D. Simmons-Duffin, and A. Zhiboedov, *J. High Energy Phys.* **11** (2020) 096.
- [107] C.-H. Chang, M. Kologlu, P. Kravchuk, D. Simmons-Duffin, and A. Zhiboedov, *J. High Energy Phys.* **05** (2022) 059.
- [108] H. Chen, T.-Z. Yang, H. X. Zhu, and Y. J. Zhu, *Chin. Phys. C* **45**, 043101 (2021).
- [109] H. Chen, I. Moul, and H. X. Zhu, *Phys. Rev. Lett.* **126**, 112003 (2021).
- [110] G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, *J. High Energy Phys.* **08** (2022) 188.
- [111] G. P. Korchemsky and A. Zhiboedov, *J. High Energy Phys.* **02** (2022) 140.
- [112] K. Yan and X. Zhang, *Phys. Rev. Lett.* **129**, 021602 (2022).
- [113] T.-Z. Yang and X. Zhang, *J. High Energy Phys.* **09** (2022) 006.
- [114] Y. L. Dokshitzer, V. A. Khoze, and S. I. Troian, *J. Phys. G* **17**, 1602 (1991).
- [115] L. Cunqueiro and M. Płoskoń, *Phys. Rev. D* **99**, 074027 (2019).
- [116] N. Zardoshti (ALICE Collaboration), *Nucl. Phys.* **A1005**, 121905 (2021).
- [117] S. Acharya *et al.* (ALICE Collaboration), *Nature (London)* **605**, 440 (2022); **607**, E22 (2022).
- [118] J. Holguin, I. Moul, A. Pathak, and M. Procura, *Phys. Rev. D* **107**, 114002 (2023).
- [119] X. Liu and H. X. Zhu, *Phys. Rev. Lett.* **130**, 091901 (2023).
- [120] C. Andres, F. Dominguez, R. Kunawalkam Elayavalli, J. Holguin, C. Marquet, and I. Moul, *Phys. Rev. Lett.* **130**, 262301 (2023).
- [121] K. Lee, B. Meçaj, and I. Moul, *Phys. Rev. D* **111**, L011502 (2025).
- [122] J. C. Collins and G. F. Sterman, *Nucl. Phys.* **B185**, 172 (1981).
- [123] G. T. Bodwin, *Phys. Rev. D* **31**, 2616 (1985); **34**, 3932(E) (1986).
- [124] J. C. Collins, D. E. Soper, and G. F. Sterman, *Nucl. Phys.* **B261**, 104 (1985).
- [125] J. C. Collins, D. E. Soper, and G. F. Sterman, *Nucl. Phys.* **B308**, 833 (1988).
- [126] J. C. Collins, D. E. Soper, and G. F. Sterman, *Adv. Ser. Dir. High Energy Phys.* **5**, 1 (1989).
- [127] J. Collins, *Foundations of Perturbative QCD* (Cambridge University Press, Cambridge, England, 2011), Vol. 32.
- [128] G. C. Nayak, J.-W. Qiu, and G. F. Sterman, *Phys. Rev. D* **72**, 114012 (2005).

- [129] A. Banfi, G. P. Salam, and G. Zanderighi, *Eur. Phys. J. C* **47**, 113 (2006).
- [130] A. Banfi, G. P. Salam, and G. Zanderighi, *J. High Energy Phys.* **07** (2007) 026.
- [131] R. Gauld, A. Huss, and G. Stagnitto, *Phys. Rev. Lett.* **130**, 161901 (2023); **132**, 159901(E) (2024).
- [132] S. Caletti, A. J. Larkoski, S. Marzani, and D. Reichelt, *J. High Energy Phys.* **10** (2022) 158.
- [133] M. Cacciari and G. P. Salam, *Phys. Lett. B* **641**, 57 (2006).
- [134] G. P. Salam and G. Soyez, *J. High Energy Phys.* **05** (2007) 086.
- [135] M. Cacciari, G. P. Salam, and G. Soyez, *J. High Energy Phys.* **04** (2008) 063.
- [136] M. Cacciari, G. P. Salam, and G. Soyez, *Eur. Phys. J. C* **72**, 1896 (2012).
- [137] C. W. Bauer, S. Fleming, and M. E. Luke, *Phys. Rev. D* **63**, 014006 (2000).
- [138] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, *Phys. Rev. D* **63**, 114020 (2001).
- [139] C. W. Bauer and I. W. Stewart, *Phys. Lett. B* **516**, 134 (2001).
- [140] C. W. Bauer, D. Pirjol, and I. W. Stewart, *Phys. Rev. D* **65**, 054022 (2002).
- [141] C. W. Bauer, S. Fleming, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, *Phys. Rev. D* **66**, 014017 (2002).
- [142] G. Altarelli and G. Parisi, *Nucl. Phys.* **B126**, 298 (1977).
- [143] S. Catani, S. Dittmaier, and Z. Trocsanyi, *Phys. Lett. B* **500**, 149 (2001).
- [144] A. K. Leibovich, Z. Ligeti, and M. B. Wise, *Phys. Lett. B* **564**, 231 (2003).
- [145] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/4plr-bmv6> for fixed-order NLO computations of the heavy-quark jet function for projected (N)-point energy correlators up to ($N = 6$), for both quark and gluon channels.
- [146] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen, and P. Z. Skands, *Comput. Phys. Commun.* **191**, 159 (2015).
- [147] W. T. Giele, D. A. Kosower, and P. Z. Skands, *Phys. Rev. D* **78**, 014026 (2008).
- [148] A. Gehrmann-De Ridder, M. Ritzmann, and P. Z. Skands, *Phys. Rev. D* **85**, 014013 (2012).
- [149] N. Fischer, S. Prestel, M. Ritzmann, and P. Skands, *Eur. Phys. J. C* **76**, 589 (2016).
- [150] F. Aversa, M. Greco, P. Chiappetta, and J. P. Guillet, *Phys. Lett. B* **211**, 465 (1988).
- [151] F. Aversa, P. Chiappetta, M. Greco, and J. P. Guillet, *Phys. Lett. B* **210**, 225 (1988).
- [152] F. Aversa, P. Chiappetta, M. Greco, and J. P. Guillet, *Nucl. Phys.* **B327**, 105 (1989).
- [153] F. Aversa, M. Greco, P. Chiappetta, and J. P. Guillet, *Z. Phys. C* **46**, 253 (1990).
- [154] F. Aversa, M. Greco, P. Chiappetta, and J. P. Guillet, *Phys. Rev. Lett.* **65**, 401 (1990).
- [155] B. Jager, M. Stratmann, and W. Vogelsang, *Phys. Rev. D* **70**, 034010 (2004).
- [156] Z.-B. Kang, F. Ringer, and I. Vitev, *J. High Energy Phys.* **10** (2016) 125.
- [157] Z.-B. Kang, F. Ringer, and I. Vitev, *J. High Energy Phys.* **11** (2016) 155.
- [158] A. Hayrapetyan *et al.* (CMS Collaboration), *Phys. Rev. Lett.* **133**, 071903 (2024).
- [159] P. T. Komiske, I. Moulton, J. Thaler, and H. X. Zhu, *Phys. Rev. Lett.* **130**, 051901 (2023).
- [160] F. J. Tackmann, *J. High Energy Phys.* **08** (2025) 098.
- [161] K. Lee and I. W. Stewart, *Phys. Rev. Lett.* **136**, 081902 (2026).
- [162] R. Thomas, B. Kampfer, and G. Soff, *Acta Phys. Hung. A* **22**, 83 (2005).
- [163] F. Maltoni, M. Selvaggi, and J. Thaler, *Phys. Rev. D* **94**, 054015 (2016).
- [164] B.-W. Zhang, E.-k. Wang, and X.-N. Wang, *Nucl. Phys.* **A757**, 493 (2005).
- [165] N. Brambilla *et al.*, *Eur. Phys. J. C* **71**, 1534 (2011).
- [166] N. Brambilla *et al.* (Quarkonium Working Group), [10.5170/CERN-2005-005](https://arxiv.org/abs/10.5170/CERN-2005-005) (2004).
- [167] G. T. Bodwin, E. Braaten, and G. P. Lepage, *Phys. Rev. D* **51**, 1125 (1995); **55**, 5853(E) (1997).
- [168] R. Bain, L. Dai, A. Leibovich, Y. Makris, and T. Mehen, *Phys. Rev. Lett.* **119**, 032002 (2017).
- [169] Z.-B. Kang, J.-W. Qiu, F. Ringer, H. Xing, and H. Zhang, *Phys. Rev. Lett.* **119**, 032001 (2017).