

Alternative to axions

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The $U(1)$ CP -violating phase which arises below chiral symmetry breaking in a non-Abelian gauge theory is “secretly” the magnetic dual of the flux of a $U(1)$ 4-form gauge field. Thus, the discharge of this 4-form flux by Schwinger production of charged membranes reduces the total CP -violating phase toward zero. This can safely restore CP symmetry to within the observational limits, $\theta_{CP} \lesssim 10^{-10}$, while also satisfying all limits from cosmology, when the charge and the tension of the gauge field sources are in the $\lesssim \text{keV}$ range.

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The observed smallness of the strong CP -violating phase θ_{CP} is a naturalness problem of the Standard Model. It is a puzzle about why the phase of the quark mass matrix is so precisely aligned with the *a priori* arbitrary vacuum angle to cancel it with the precision of $\lesssim 10^{-10}$. The problem could be solved dynamically, if θ_{CP} receives a contribution from an axion, which then remains light enough to relax total θ_{CP} to a tiny value after QCD chiral symmetry breaking [1–3]. Realistic models could be constructed, starting with Refs. [4–7].

So far, there is no direct experimental evidence for the axion, while limits on axion’s features abound. It seems warranted to consider alternatives. Here, we propose that θ_{CP} is relaxed discretely, instead of smoothly, since it includes a flux of a 4-form (also known as “top form,” since its rank equals the spacetime dimension) sourced by membranes, whose Schwinger discharge [8] rapidly relaxes θ_{CP} right around the QCD scale. Our mechanism triggers a cascade of percolating bubbles near the QCD scale, relaxing θ_{CP} cosmologically, explicitly realizing some musings in Ref. [9]. This restores CP to within the observational limits and conforms with standard cosmology since it ends well before big bang nucleosynthesis (BBN).

The presence of top forms in strongly coupled gauge theory below confinement/chiral symmetry breaking has been known for a long time [10–21] (see Ref. [22] for a review). A key insight was that θ_{CP} is a magnetic dual of a top form, mimicking the $U(1)$ vector gauge theory in $1+1$ dimensions discovered in Refs. [23,24]. In Ref. [10], the

effective action of this top form was extracted from the nonperturbative contributions to the correlators of the duals of gauge theory anomaly terms. Below chiral symmetry breaking, they have nonvanishing values in the new ground state of the theory, and hence a dynamical top form effective action emerges. Above chiral symmetry breaking, these nonperturbative contributions vanish, and the top form remains a total derivative; i.e., it is “secret” [10].

The top form field equations show that its flux is degenerate with the total CP -violating phase θ_{CP} : vanishing of one implies vanishing of the other. In this context, the axion solution of the strong CP problem is the Higgsing of the top form gauge theory [11,17–21]. Even without the axion, assuming a contribution to θ_{CP} is a magnetic dual of a top form, Dvali proposed a model with an enhanced density of terminal states around $\theta_{CP} = 0$, making this region an attractor where the total θ_{CP} can relax by a discharge of membranes charged under this 4-form [20]. This can sidestep the concerns that nucleation rates of membranes “native” to QCD are too slow [15,25] (counterexamples to these ideas are offered in Refs. [26,27]).

Another way to sidestep the question of whether nucleation rates of QCD membranes are fast enough to discharge θ_{CP} , which we pursue here, is to invoke a new sector whose membranes have *smaller* charge and tension, which are weakly coupled to QCD. To this end, we use a replica of Lüscher’s nonperturbative top form in QCD but add membranes with tensions and charges in the keV range. The new 4-form is exactly degenerate away from the membranes with Lüscher’s top form, and so its discharge by membrane nucleation also reduces the strong CP -violating phase θ_{CP} . This occurs near the QCD scale, when the background universe is radiation dominated and expanding slowly. Hence, the bubbles of true vacuum bounded by membranes will expand at the speed of light, colliding with each other and easily percolating [28–30]. The colliding membranes decay into the strongly coupled

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QCD excitations. The vacuum with a tiny θ_{CP} is achieved quickly over large regions of the Universe. Working in the limit $M_{\text{Pl}} \rightarrow \infty$, we ignore any interplay of the strong CP problem with the cosmological constant, fine tuning the latter away. We also set aside questions of UV completion of the mechanism at this time.

Conceptually, our mechanism is actually very simple. As is well known, the strong CP violation is encoded by an arbitrary $U(1)$ phase of the theory, which takes any value on S^1 . The theory has an intrinsic higher rank $U(1)$ gauge field in the strong coupling regime, revealed by computing nonperturbative corrections below chiral symmetry breaking, by, e.g., evaluating the partition function and the correlators of the anomaly operator $q(x) = \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu\lambda\sigma}$ [10]. Above chiral symmetry breaking, these correlators are zero; below, they do not vanish as shown by Lüscher. The operator q is the dual of the emergent 4-form field strength $F_{\mu\nu\lambda\sigma}$. Its flux is dual to the total CP -violating phase [10,15,20].

When we kinetically mix this 4-form to an additional external 4-form gauge field with a sufficiently light charge, the flux will discharge by quantum nucleations of charges, as shown by Schwinger [8]. This can be fast enough to restore CP before BBN, and so all of tested cosmology can remain unaffected; the mechanism can pass both the particle physics and cosmological tests at the same time. The charge carriers are not point particles but membranes; this is merely a technical nuisance, which is understood well enough by now. In fact, on spherically symmetric backgrounds, when we integrate out the angular coordinates, the resulting picture is identical to the discharge of a uniform electric field inside a parallel plate capacitor. The top form sector which relaxes CP is a generalization to three spatial dimensions of a Maxwell $U(1)$ gauge theory in one dimension of space. One can see this by noting that for well-separated spherical membranes one can integrate angular coordinates and reduce the theory to $1+1$ dimensions. The membranes, being codimension-1 objects, correspond to points on a line. The discharges are but particle production in a background electric field, akin to how a parallel plate capacitor discharges by pair production.

Let us first formulate the strong CP problem using top forms. The perturbative definition of the gauge sector of QCD must be extended by an extra topological term, which follows because of the nontrivial topology of $SU(3)$,

$$\mathcal{L}_{\text{QCD}} \ni \frac{g^2}{64\pi^2} \theta \epsilon^{\mu\nu\lambda\sigma} \sum_a G_{\mu\nu}^a G_{\lambda\sigma}^a, \quad (1)$$

with a canonically normalized gauge field strength $G^a \in SU(3)$. We sum over colors, with θ a “vacuum angle,” $\theta \in [0, 2\pi]$. Aside from the $1/g^2$ factor in the gauge kinetic term, we follow the normalizations in Ref. [10]. This term

ensures invariance of the theory under large gauge transformations; θ is *a priori* arbitrary; loosely, a nonzero θ implies CP violation.

However, with unbroken chiral symmetry, all values of θ are degenerate. To see it, we can rewrite (1) using $\frac{g^2}{32} \epsilon^{\mu\nu\lambda\sigma} \sum_a G_{\mu\nu}^a G_{\lambda\sigma}^a = \partial_\mu K^\mu$, where K_μ is the Chern-Simons current. Hodge-dualizing, $K_\mu = \epsilon_{\mu\nu\lambda\sigma} A^{\nu\lambda\sigma}/6$, brings about the first apparition of QCD’s “secret” top form [10]: $\partial_\mu K^\mu = \frac{1}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu A_{\nu\lambda\sigma} = \frac{1}{24} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu\lambda\sigma}$ with $F_{\mu\nu\lambda\sigma} = 4\partial_{[\mu} A_{\nu\lambda\sigma]}$. This yields $F_{\mu\nu\lambda\sigma} = \frac{3}{4} g^2 \sum_a G_{\mu\nu}^a G_{\lambda\sigma}^a$, where $[\dots]$ denotes antisymmetrization. So, the θ term in the perturbative $\mathcal{L}_{\text{QCD}}$ is $\frac{1}{48\pi^2} \theta \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu\lambda\sigma}$, which is a total derivative.

With massive quarks, whose mass matrix is $\mathcal{M}$, an additional contribution comes from $\text{Arg det } \mathcal{M}$. In the fermion basis where $\det \mathcal{M}$ is real, the total CP -violating phase is $\theta \rightarrow \theta_{\text{CP}} = \hat{\theta} = \theta + \text{Arg det } \mathcal{M}$. Even so, different θ_{CP} ’s are still degenerate above the QCD scale, before the instanton effects kick in. Further, these superselection sectors disappear if even one of the quarks were massless, since then the determinant of $\mathcal{M}$ would have been zero, and so $\text{Arg det } \mathcal{M}$ would have been completely arbitrary. If so, we could cancel any value of $\hat{\theta}$ by picking at will an appropriate value of $\text{Arg det } \mathcal{M}$.

Now, we turn to the top form $F^{\mu\nu\lambda\sigma}$ below chiral symmetry breaking. Following Ref. [10], the operator expectation values are

$$\begin{aligned} \langle \mathcal{O} \rangle_{\hat{\theta}} &= \frac{1}{\mathcal{Z}[\hat{\theta}]} \int [\mathcal{D}\mathcal{B}] \mathcal{O} e^{iS + i\hat{\theta} \int q(x)}, \\ \mathcal{Z}[\hat{\theta}] &= \int [\mathcal{D}\mathcal{B}] e^{iS + i\hat{\theta} \int q(x)}, \end{aligned} \quad (2)$$

where $q(x) = \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\lambda\sigma} \sum_a G_{\mu\nu}^a(x) G_{\lambda\sigma}^a(x) = \frac{1}{48\pi^2} \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu\lambda\sigma}$, as explained above. In (2), we decoupled the fermions for simplicity, by assuming they are massive, which absorbs the mass matrix $U(1)$ phase into $\hat{\theta}$ and leaves us with a path integral over the gauge fields $B_\mu^a = \{\mathcal{B}\}$. Although the quark mass scale is comparable to the chiral symmetry breaking scale in real QCD, this simplification captures the key features of the dynamics well. The Euclidean partition function $\mathcal{Z}[\hat{\theta}]$ has minima at $\hat{\theta} = 0 + 2n\pi$ [31], where CP is restored, and so it is an even function of $\hat{\theta}$ [31].

As in Ref. [10], we are interested in the correlators of $q(x)$. Below chiral symmetry breaking, they can be computed using instanton dilute gas approximation. First off, $\langle q \rangle_{\hat{\theta}}$ is not vanishing unless $\hat{\theta} = 0$,

$$\langle q \rangle_{\hat{\theta}} = \frac{1}{\mathcal{Z}[\hat{\theta}]} \int [\mathcal{D}\mathcal{B}] q(x) e^{iS + i\hat{\theta} \int q(x)} = iH(\hat{\theta}), \quad (3)$$

where $H(-\hat{\theta}) = -H(\hat{\theta})$. For $SU(2)$ in a particular $\hat{\theta}$ state, Lüscher found this function to be $H(\hat{\theta}) = 0.078(\mu_0 \frac{8\pi^2}{g^2})^4 (\mu_0 \rho_c)^{10/3} e^{-8\pi^2/g^2(\mu_0)} \sin(\hat{\theta})$, where μ_0 is a

dimensional normalization parameter, $g(\mu_0)$ is the running coupling, and ρ_c is the IR cutoff for the instanton size. A qualitatively similar result also holds for CP^{n-1} nonlinear σ -models in $1+1$ dimensions. Similar features are also found in more realistic setups in [14–16]. Note that, because the vacuum angle $\hat{\theta}$ is removable by chiral transformations *above complete breaking of chiral symmetry*, where at least one fermion remains massless, this expectation value must be zero at scales above Λ_{QCD} . Thus, since as noted above $q(x) = \frac{1}{48\pi^2} \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu\lambda\sigma}$, below chiral symmetry breaking, the secret top form field strength is not zero when $\hat{\theta}$ is not zero since $q \neq 0$: inverting the Hodge dual, and transforming to Lorentzian signature variables,

$$F_{\mu\nu\lambda\sigma} = -2\pi^2 i \epsilon_{\mu\nu\lambda\sigma} q = 2\pi^2 \epsilon_{\mu\nu\lambda\sigma} H(\hat{\theta}). \quad (4)$$

Hence, $F_{\mu\nu\lambda\sigma} \neq 0$ is a faithful diagnostic of CP violation parametrized by $\hat{\theta}$. Further, restoring CP is equivalent to selecting a state in which the top form field strength is zero.

There are also additional $F^{\mu\nu\lambda\sigma}$ -dependent corrections [10]. They are duals to the higher-order correlators $\langle q(x_1)q(x_2)\dots q(x_n) \rangle_{\hat{\theta}}$, which are nonzero below chiral symmetry breaking/confinement scale, while vanishing when chiral symmetry is restored. For our purposes, it suffices to keep only quadratic terms and ignore higher powers. To extract these terms from the partition function in the IR, following [10,14–16], we can define a gauge field $\mathcal{Y}_\mu = \epsilon_{\mu\nu\lambda\sigma} A^{\nu\lambda\sigma}/6$ such that $q = \frac{1}{2\pi^2} \partial_\mu \mathcal{Y}^\mu$, compute the 1-particle irreducible correlator $\langle q(x)q(y) \rangle_{\hat{\theta}}$ and, using Lorentz symmetry, extract from it the correlator for $\langle \mathcal{Y}_\mu \mathcal{Y}_\nu \rangle_{\hat{\theta}}$. After some straightforward albeit tedious calculations, using the defining relation of $\mathcal{Y}$ in terms of A above, we finally find that in Lorenz gauge $\partial_\mu A^{\mu\lambda} = 0$ the two-point function for the top form field potential is

$$\begin{aligned} & \frac{1}{(3!)^2} \epsilon_{\mu\alpha\beta\gamma} \epsilon_{\nu\sigma\rho\delta} \langle A^{\alpha\beta\gamma}(x) A^{\sigma\rho\delta}(y) \rangle_{\hat{\theta}} \\ &= 4\pi^4 \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-y)} \frac{p_\mu p_\nu}{p^4} \mathcal{X}, \end{aligned} \quad (5)$$

where $\mathcal{X} = -i \int d^4 x \langle T(q(x)q(0)) \rangle_{\hat{\theta}} = \frac{d}{d\hat{\theta}} H(\hat{\theta}) \simeq (\Lambda_{\text{QCD}})^4$ is the topological susceptibility of the theory. The relationship $\mathcal{X} = \frac{d}{d\hat{\theta}} H(\hat{\theta})$ for momentum space correlator in Eq. (5) follows directly from $\int d^4 y \langle q(x)q(y) \rangle_{\hat{\theta}} = \frac{1}{i} \frac{d}{d\hat{\theta}} \langle q \rangle_{\hat{\theta}}$ and Eq. (3), after Fourier transforming to momentum space and using Lorentz invariance of the ground state; it shows that the nonperturbatively induced powers of $F_{\mu\nu\lambda\sigma}$ have correlated coefficients. Hence, well below Λ_{QCD} , the effective theory can be thought of as an operator series including terms $\propto (F_{\mu\nu\lambda\sigma})^n$.

From Eq. (5), we confirm that (1) above the complete chiral symmetry breaking scale, when at least

one quark is massless, $H = \mathcal{X} = 0$, the QCD top form is secret (the term linear in flux is physically irrelevant since we can pick $\hat{\theta}$ to be zero by a chiral gauge transformation, and the higher power correlators, starting with the two-point function $\langle AA \rangle_{\hat{\theta}}$, do not appear in the effective action) and (2) below chiral symmetry breaking, when $\mathcal{X} \neq 0$, and so H is a nontrivial function of $\hat{\theta}$, the effective theory must include terms $\propto (F_{\mu\nu\lambda\sigma})^n$, starting with

$$\mathcal{L}_{\text{QCD}} \ni \frac{1}{48\pi^2} \hat{\theta} \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu\lambda\sigma} - \frac{1}{4\pi^2 \cdot 4! \pi^2 \mathcal{X}} F_{\mu\nu\lambda\sigma}^2, \quad (6)$$

up to gauge-fixing terms, that reproduces the correlator in Eq. (5). Recall, that classically the 4-form field strength does not include new local degrees of freedom. For a purely quadratic theory, $\epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu\lambda\sigma} = \text{const}$ by equations of motion, which is the hidden constant elucidated in Ref. [12].

It is convenient to replace the field strength $F_{\mu\nu\lambda\sigma}$ with its magnetic dual $\propto F_{\mu\nu\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma}/4!$. This is the field theoretic realization of the canonical transformation exchanging momenta and coordinates. The procedure is discussed at length in, e.g., Refs. [12,19,32–38]. Here, we will merely summarize the main steps. We start with the action based on the Lagrangian (6). Recalling that $F_{\mu\nu\lambda\sigma} = 4\partial_{[\mu} A_{\nu\lambda\sigma]}$, we rewrite this action in the first-order formalism, employing a Lagrange multiplier F :

$$\begin{aligned} S = \int d^4 x \left(-\frac{1}{4\pi^2 \times 4! \pi^2 \mathcal{X}} F_{\mu\nu\lambda\sigma}^2 + \frac{\hat{\theta}}{2 \times 4! \pi^2} \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu\lambda\sigma} \right. \\ \left. + \frac{F}{4\pi^2 \times 4! \pi^2 \mathcal{X}} \epsilon_{\mu\nu\lambda\sigma} (F^{\mu\nu\lambda\sigma} - 4\partial_\mu A_{\nu\lambda\sigma}) \right). \end{aligned} \quad (7)$$

Next, we complete the square for $F_{\mu\nu\lambda\sigma}$, by defining $\tilde{F}_{\mu\nu\lambda\sigma} = F_{\mu\nu\lambda\sigma} - (F + 2\pi^2 \mathcal{X} \hat{\theta}) \epsilon_{\mu\nu\lambda\sigma}$, and integrate $\tilde{F}_{\mu\nu\lambda\sigma}$ out. This only yields a Gaussian factor in the partition function since the Jacobian is unity. The precise details of this calculation are given in Refs. [33–38], and we do not repeat them here. After integrating out $\tilde{F}_{\mu\nu\lambda\sigma}$, the remaining terms are

$$S = \int d^4 x \left(-\frac{\mathcal{X}}{2} \left(\hat{\theta} + \frac{F}{2\pi^2 \mathcal{X}} \right)^2 - \frac{F}{24\pi^4 \mathcal{X}} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu A_{\nu\lambda\sigma} \right). \quad (8)$$

The variation with respect to $A_{\nu\lambda\sigma}$ yields field equation for F : $\partial_\mu F = 0$. Thus, the local solutions for the field F are just constants. The quadratic term

$$V = \frac{\mathcal{X}}{2} \left(\hat{\theta} + \frac{F}{2\pi^2 \mathcal{X}} \right)^2 \quad (9)$$

is the contribution to the vacuum energy when CP is violated, calculated in, e.g., Refs. [14,16]. This shows that

the total CP -violating phase actually is $\theta_{\mathcal{CP}} = \hat{\theta} + \frac{F}{2\pi^2\sqrt{\mathcal{X}}}$. It sources the field $F_{\mu\nu\lambda\sigma} = 4\partial_{[\mu}A_{\nu\lambda\sigma]}$, as can be seen varying (8) with respect to F ; after some manipulation [22], $F_{\mu\nu\lambda\sigma} = 2\pi^2\mathcal{X}\theta_{\mathcal{CP}}\epsilon_{\mu\nu\lambda\sigma}$. So, $F_{\mu\nu\lambda\sigma}$ probes the presence of total CP violation: if $\theta_{\mathcal{CP}}$ vanishes, so does $F_{\mu\nu\lambda\sigma}$ (but not the dual F ; this feature is a well-known property of Legendre transforms). Next, although F is locally constant, because it is a magnetic dual of $F_{\mu\nu\lambda\sigma}$, it can vary from region to region when there are charges sourcing it [10,14,16,32,39]. The mechanics of quantum discharges is discussed at length in, e.g., Refs. [40,41].

The discharges might be very slow [15,25], but there are examples which suggest otherwise [26,27]. We sidestep this and add one more copy of the QCD top form sector. We also add membranes charged under it, with tension $\mathcal{T}$ and charge $\mathcal{Q}$, treating them as the input parameters of the theory. As we are interested in the proof of principle, we pick the scales $\mathcal{T}$, $\mathcal{Q}$ such that the relaxation processes are able to resolve the strong CP problem. It turns out that a tension and charge $\sim \text{keV}$ do the job.

To include our new top form, we imitate the axion couplings [2,3] (see Ref. [22] for details),

$$S_{F+\mathcal{H}} = \int d^4x \left(-\frac{1}{2} \left(\mathcal{H} + \sqrt{\mathcal{X}}\hat{\theta} + \frac{F}{2\pi^2\sqrt{\mathcal{X}}} \right)^2 + \frac{\epsilon^{\mu\nu\lambda\sigma}}{24\pi^4\mathcal{X}} \partial_\mu(F)A_{\nu\lambda\sigma} + \frac{\epsilon^{\mu\nu\lambda\sigma}}{6} \partial_\mu(\mathcal{H})C_{\nu\lambda\sigma} \right) - \mathcal{T} \int d^3\xi \sqrt{\left| \det \left(\eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \right) \right|} - \frac{\mathcal{Q}}{6} \int d^3\xi C_{\mu\nu\lambda} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \frac{\partial x^\lambda}{\partial \xi^c} \epsilon^{abc}, \quad (10)$$

where $\mathcal{H}$ is the dual of the new top form.¹ Varying (10) yields $\frac{1}{2\pi^2\sqrt{\mathcal{X}}}F_{\mu\nu\lambda\sigma} = \mathcal{H}_{\mu\nu\lambda\sigma} = (\mathcal{H} + \sqrt{\mathcal{X}}\hat{\theta} + \frac{F}{2\pi^2\sqrt{\mathcal{X}}})\epsilon_{\mu\nu\lambda\sigma}$. As before, away from the charges, both fluxes F and $\mathcal{H}$ are constant. Once charges are included, the flux $\mathcal{H}$ can change discretely. Note that the CP -violating phase now is $\theta_{\mathcal{CP}} = \frac{\mathcal{H}}{\sqrt{\mathcal{X}}} + \hat{\theta} + \frac{F}{2\pi^2\sqrt{\mathcal{X}}}$. The 3-form $\mathcal{C}$ comes in as a Lagrange multiplier enforcing $\partial_\mu \mathcal{H} = 0$. It yields $\mathcal{H}_{\mu\nu\lambda\sigma} = 4\partial_{[\mu}C_{\nu\lambda\sigma]}$ on shell. Note that if we integrate out the angular variables on S^2 and fix F Eq. (10) is just the standard Maxwell theory in one time and one spatial dimension, written in terms of the dual magnetic field.

To determine the quantum nucleation rates, we Euclideanize this action, working in the decoupling limit of gravity $M_{\text{Pl}} \rightarrow \infty$ [42]. Following Refs. [42–44] and [33,34], the Wick-rotated Euclidean action defined by $iS \rightarrow -S_E$ is

¹The bilinears $\propto \epsilon^{\mu\nu\lambda\sigma}$ were integrated by parts to covariantize the top forms on the membranes [33,34].

$$S_E = \int d^4x \left(\frac{1}{2} \left(\mathcal{H} + \sqrt{\mathcal{X}}\hat{\theta} + \frac{F}{2\pi^2\sqrt{\mathcal{X}}} \right)^2 + \frac{\epsilon^{\mu\nu\lambda\sigma}}{24\pi^4\mathcal{X}} \partial_\mu(F)A_{\nu\lambda\sigma} + \frac{\epsilon^{\mu\nu\lambda\sigma}}{6} \partial_\mu(\mathcal{H})C_{\nu\lambda\sigma} \right) + \mathcal{T} \int d^3\xi \sqrt{\gamma_C} - \frac{\mathcal{Q}}{6} \int d^3\xi C_{\mu\nu\lambda} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} \frac{\partial x^\lambda}{\partial \xi^\gamma} \epsilon^{\alpha\beta\gamma}. \quad (11)$$

When $M_{\text{Pl}} \rightarrow \infty$, the only available nucleation channel is the simplest bounce solution relating the backgrounds with nonnegative vacuum energy $V = \frac{1}{2}(\mathcal{H} + \sqrt{\mathcal{X}}\hat{\theta} + \frac{F}{2\pi^2\sqrt{\mathcal{X}}})^2$, where $\mathcal{H}_{\text{in,out}}$ differ by a unit of charge, $\mathcal{H}_{\text{out}} - \mathcal{H}_{\text{in}} = \mathcal{Q}$. Although $\mathcal{H}$ is quantized, $\mathcal{H} = \mathcal{N}\mathcal{Q}$, it is degenerate with $\sqrt{\mathcal{X}}\hat{\theta}$ and $\frac{F}{2\pi^2\sqrt{\mathcal{X}}}$, which can take arbitrary values when there are no restrictions on the overall phase of the quark mass matrix, and so we ignore the $\mathcal{H}$ quantization.

A membrane which nucleates must also satisfy energy conservation: the energy difference in the interior must equal the rest energy set by the tension [42]. Subtracting the backgrounds with and without a membrane yields the membrane action $S_{\text{membrane}} = 2\pi^2 r_0^3 \mathcal{T} - \frac{1}{2} \pi^2 r_0^4 \Delta V$, that follows since the volume integrals are $V_{S^4} = \pi^2 r_0^4/2$ and $V_{S^3} = 2\pi^2 r_0^3$. The bounce is the minimum with respect to r_0 which yields $r_0 = 3\mathcal{T}/\Delta V$. The actual bounce action is [42]

$$B = \frac{27\pi^2}{2} \frac{\mathcal{T}^4}{(\Delta V)^3}. \quad (12)$$

The bubble nucleation rate per unit time per unit volume is $\Gamma = Ae^{-B}$ [42,43]. The prefactor A for membrane production in flat space was found by Garriga in Ref. [44], and so

$$\Gamma \simeq 9 \frac{\mathcal{T}^4}{(\Delta V)^2} \exp \left(-\frac{27\pi^2}{2} \frac{\mathcal{T}^4}{(\Delta V)^3} \right). \quad (13)$$

Inside a membrane, the potential difference is $\Delta V \simeq \mathcal{Q}(\mathcal{H} + \sqrt{\mathcal{X}}\hat{\theta} + \frac{F}{2\pi^2\sqrt{\mathcal{X}}})$, which receives $\mathcal{O}(1)$ corrections when the QCD top form is near zero. The QCD top form also decreases by $\Delta F_{\mu\nu\lambda\sigma} = 2\pi^2\sqrt{\mathcal{X}}\mathcal{Q}\epsilon_{\mu\nu\lambda\sigma}$, since both it and the potential are set by $\theta_{\mathcal{CP}}$. Since the membrane tension is strictly positive, discharges decrease the potential and $\theta_{\mathcal{CP}}$ until $|F_{\mu\nu\lambda\sigma}| < 2\pi^2\sqrt{\mathcal{X}}\mathcal{Q}$: since $\mathcal{T} > 0$, and $r_0 = 3\mathcal{T}/\Delta V$, when $\Delta V < 0$ we would need $r_0 < 0$, which is clearly unphysical.

With a single membrane, the requirement $\theta_{\mathcal{CP}} \lesssim 10^{-10}$ and the bound $|F_{\mu\nu\lambda\sigma}| < 2\pi^2\sqrt{\mathcal{X}}\mathcal{Q}$ impose $\mathcal{Q}/\sqrt{\mathcal{X}} \lesssim 10^{-10}$. Using $\mathcal{X}^{1/4} \simeq \text{few} \times 100 \text{ MeV}$, we find $\mathcal{Q} \lesssim 10^{-11} \text{ GeV}^2 \simeq (3 \text{ keV})^2$. Further, the discharge mechanism should be faster than the cosmic dilution close to the QCD scale, $\Gamma \lesssim H_{\text{QCD}}^4$ [28–30]. Since $H_{\text{QCD}} \simeq (100 \text{ MeV})^2/M_{\text{Pl}} \simeq 10^{-20} \text{ GeV}$, we require

$$10^{81} \frac{T^4}{(\Delta V)^2 (\text{GeV})^4} \gtrsim \exp\left(\frac{27\pi^2}{2} \frac{T^4}{(\Delta V)^3}\right). \quad (14)$$

The nucleations are the slowest close to the terminal state [22], where ΔV is the smallest; there, $\Delta V = (Q)^2 \lesssim 10^{-22} \text{ GeV}^4$. The decay rate will be fast enough, $\Gamma > H_{\text{QCD}}^4$, if $T^{1/3} \lesssim 3 \text{ keV}$. Note that, since the decay rate depends on the tension and charge exponentially, even if the actual value of the strong CP phase θ_{CP} is a few orders of magnitude smaller than 10^{-10} , which could be tested by future experiments [45,46], our mechanism would remain operational. For example, if θ_{CP} were as small as 10^{-13} , the relaxation of $\theta_{\text{CP}} \sim \mathcal{O}(1)$ to 10^{-13} would still occur if the charge and tension were $\sim \mathcal{O}(100) \text{ eV}$ (see Ref. [22] for details).

We can now confirm that neglecting gravity is justified. Since $H_{\text{QCD}} \simeq \sqrt{\chi}/M_{\text{Pl}}$, the radius of a membrane at nucleation is $r_0 H_{\text{QCD}} \simeq \frac{T}{\Delta V} \frac{\sqrt{\chi}}{M_{\text{Pl}}} \simeq \frac{T}{\sqrt{\chi} M_{\text{Pl}}} \frac{1}{\theta_{\text{CP}} \Delta \theta_{\text{CP}}}$. For transitions close to the final state, with $\theta_{\text{CP}} \lesssim 10^{-10}$, this means $r_0 H_{\text{QCD}} \lesssim 10^{-12}$. Initially, for $\theta_{\text{CP}} \sim 1$, the bubbles are even much smaller, with $r_0 H_{\text{QCD}} \lesssim 10^{-22}$. Hence, our bubbles are very small compared to the background curvature scale $1/H_{\text{QCD}} \simeq M_{\text{Pl}}/\sqrt{\chi} \simeq 10^{11} (\text{eV})^{-1} \simeq 100 \text{ km}$, and we can always pick a large freely falling local frame, smaller than $1/H_{\text{QCD}}$ and much larger than the size of a nucleating bubble.

Now, we turn to the cosmology of CP restoration. We start with the hot universe after inflation, evolving as a radiation-dominated, spatially flat, isotropic and homogeneous Friedmann-Robertson-Walker cosmology, with temperature T well above $\Lambda_{\text{QCD}} \sim \text{GeV}$. As long as $T \gg \Lambda_{\text{QCD}}$, the QCD chiral symmetry will not be broken, and so, initially, all the θ_{CP} states will be degenerate, due to massless quarks. There will be no QCD-related CP violation, but as the universe cools, electroweak symmetry breaking at $T \sim \text{TeV}$ will fix the quark mass matrix phase contribution to θ_{CP} , initiating chiral symmetry breaking, and subsequent contributions to chiral symmetry breaking will come when the instanton effects arise, which will lift the degeneracy of θ_{CP} . Concurrently, the discharges of θ_{CP} induced by discharges of $\mathcal{H}$ will remain dormant until $H \sim H_{\text{QCD}}$ because the vacua with different θ_{CP} were initially degenerate, and the QCD top form and $\mathcal{H}$ are decoupled before the QCD scale. The instanton effects kick in around the QCD scale, explicitly lifting the degeneracy of different θ_{CP} states. The domains with a value of θ_{CP} develop vacuum energy $V = \frac{1}{2} \chi \theta_{\text{CP}}^2 = \frac{1}{2} (\mathcal{H} + \sqrt{\chi} \hat{\theta} + \frac{F}{2\pi^2 \sqrt{\chi}})^2$. Those regions are unstable to rapid nucleation of small bubbles surrounded by membranes with tension $T^{1/3} \sim 3 \text{ keV}$ and charge $\sqrt{Q} \sim 3 \text{ keV}$. They will decay toward the CP -invariant vacuum $\theta_{\text{CP}} = 0$ by discharging $\mathcal{H}$ very prolifically, thanks to (14). This will continue until $|F_{\mu\nu\lambda\sigma}| < 2\pi^2 \sqrt{\chi} Q$. Once $|F_{\mu\nu\lambda\sigma}|$ is so small, the net CP -violating phase will be reduced to

$$\theta_{\text{CP}} \lesssim \frac{Q}{\sqrt{\chi}} \leq 10^{-10}, \quad (15)$$

and the nucleations will cease.

The charges and tensions in the range of keV are small enough to ensure that the decay rate (13) is fast compared to the age of the Universe near and below the QCD scale, as seen in (14). The nucleations rapidly minimize CP -breaking effects throughout the Universe.² This dynamics is very similar to how the axion relaxes θ_{CP} . The main difference is that in our case the relaxation occurs in small discrete steps instead of continuously. However, since this is happening during radiation domination, the minimal θ_{CP} regions will percolate. Their membrane mantles will expand at the speed of light and will collide with each other in a time shorter than the Hubble time since the nucleation is so prolific (14).

Upon a collision, the membranes would burn out by production of strongly interacting particles, i.e., predominantly pions. To infer this, using (10), we see that below the chiral symmetry breaking we have

$$\begin{aligned} \int d^4x \mathcal{L}_{F+\mathcal{H}} &\ni \frac{1}{2\pi^2 \sqrt{\chi}} \int d^4x \mathcal{H} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu\lambda\sigma} \\ &\sim \frac{12}{\pi^2 \sqrt{\chi}} \int d^4x \mathcal{H} \partial_\mu K^\mu, \end{aligned} \quad (16)$$

where in the last step we used the perturbative definition of the top form. After a chiral transformation the Chern-Simons current will shift, on shell, to $\partial K \rightarrow \partial K + f_\pi \partial^2 \mathcal{P}$, with $\mathcal{P}$ denoting the pion field. Substituting into (16), integrating by parts and recalling that $\partial \mathcal{H} = 0$ away from membranes, and $\Delta \mathcal{H} = Q$ on a membrane,

$$\frac{12f_\pi}{\pi^2 \sqrt{\chi}} \int d^4x \mathcal{H} \partial^2 \mathcal{P} \sim 10^{-11} f_\pi \int_{R \times S^2} d^3 \xi n \cdot \partial \mathcal{P}, \quad (17)$$

where n is the outward normal to the membrane, and we substituted the membrane charge value. At nucleation, the membranes are very small, $r_0 H_{\text{QCD}} \sim 10^{-22} - 10^{-12}$, moving out at the speed of light. By the time they reach the size of the horizon at the QCD scale, they will have propagated through the background Friedmann-Robertson-Walker for a time $\sim 1/H_{\text{QCD}}$, which is at least a factor of 10^{12} larger than their initial size. The emission rate accumulates over a Hubble time, and the energy transfer of the membrane walls can be completed, compensating the suppression of 10^{-11} in Eq. (17).

²Interestingly, the discharge of θ_{CP} toward zero also reduces the vacuum energy contribution from the QCD confinement phase transition. To study this in more detail, one must go beyond the decoupling limit of gravity adopted in this paper, and so here we ignore it for the time being.

To summarize, in this work, we presented a mechanism of cosmological discharge of strong CP -violating phase by rapid membrane production. These processes start right after chiral symmetry breaking. They look like fast “evaporation” of false vacua. When the membrane tension and charge are in the keV range, the discharges are fast enough to complete the relaxation of θ_{CP} , and so approximately restore CP , well before BBN. As a consequence, the resulting cosmology passes the observational bounds. A small membrane charge ensures that the discharge will be refined enough to yield the terminal θ_{CP} below 10^{-10} , and a small membrane tension then makes the discharges fast, achieving $\theta_{CP} \lesssim 10^{-10}$ quickly.

The mechanism we proposed here must begin right around chiral symmetry breaking, discharging θ_{CP} as soon as θ_{CP} degeneracy is lifted. Many individual discharge processes are needed. Hence, to complete the transition to terminal θ_{CP} , we need many bubbles to form, which complicates the description of the process. Another question concerns the origin of the scale of charges and tensions. Small charges might appear if there is kinetic mixing, imitating how the millicharge particles arise in field theory [47]. Alternatively, they might come in models with UV completions with large extra dimensions, where four-dimensional charges include a factor of compact internal space volume [48].

Small tensions could come from wrapping p -branes on shrinking cycles in internal dimensions [48]. Since their

scale is picked to make transitions fast, an alternative approach might be faster false vacua decay rates, which could be done in a variety of ways [49–54]. Another possibility might also be using heavier axions, which should sit in their potential minima at chiral symmetry breaking, being effectively rendered immobile by their large mass and so decoupled, but where potentials have a large number of vacua which could have enhanced tunneling rates [55]. In any case, an alternative mechanism to relax θ_{CP} warrants a closer look. It has been known for some time that in theories with strong coupling in the IR topological defects may play a role of fundamental excitations. We advocate that it is interesting to explore how their dynamics could help with understanding deeper problems of fundamental physics of pointlike particles.

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