Investigating charmed hybrid baryons via QCD sum rules

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We investigate charmed hybrid baryons using the QCD sum rule method within the framework of heavy quark effective theory. We construct 28 interpolating currents for charmed hybrid baryons, seven of which are employed in QCD sum rule analyses of 19 states with quark-gluon configurations $qqcg$, $qscg$, and $sscg$ ($q = u/d$). The masses of the lowest-lying charmed hybrid baryons in the $SU(3)$ flavor $\mathbf{6}_F$ representation are calculated to be $M_{\Sigma_{cg}(1/2^+)} = 3.36^{+0.27}_{-0.26}$ GeV, $M_{\Xi'_{cg}(1/2^+)} = 3.59 \pm 0.20$ GeV, and $M_{\Omega_{cg}(1/2^+)} = 3.82 \pm 0.21$ GeV. We propose that future experiments search for these states via their P-wave decay channels $ND^{(*)}$, $\Lambda D^{(*)}$, and $\Xi D^{(*)}$, respectively. Such investigations would provide valuable insight into the role of gluonic excitations in hadron structure.

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I. INTRODUCTION

Quantum chromodynamics (QCD) allows for the existence of color singlets, including conventional hadrons, multiquark states, glueballs, and hybrids. Exploring exotic states beyond conventional hadrons is one of the most intriguing areas of research in hadronic physics, as such studies enhance our understanding of nonperturbative QCD. Gluonic degrees of freedom play a crucial role in these exotic states. The excitation of the gluonic field, analogous to the role of valence quarks, manifests in glueballs, hybrid mesons, and hybrid baryons. While substantial research has been conducted on hybrid mesons, relatively few studies have focused on hybrid baryons. Consequently, investigating hybrid baryons beyond conventional hadron configurations remains a significant and challenging topic in hadronic physics, requiring the identification of distinctive characteristics to confirm their existence. A common approach involves predicting the mass spectra and decay widths of hybrid baryons and offering specific recommendations for experimental searches.

In 2022 the BESIII Collaboration observed the isoscalar state $\eta_1(1855)$ with the exotic quantum numbers $I^G J^{PC} = 0^+1^{-+}$ [1,2]. Additionally, several collaborations [3–7] have reported three isovector states—$\pi_1(1400)$, $\pi_1(1600)$, and $\pi_1(2015)$—all possessing the exotic quantum numbers $I^G J^{PC} = 1^-1^{-+}$. These states are regarded as strong candidates for hybrid mesons. In 2024, the BESIII Collaboration also reported the $X(2370)$ as the possible lightest pseudoscalar glueball [8,9]. These observations collectively support the potential existence of hybrid baryons, although no hybrid hadron has yet been conclusively identified in experiments. Ongoing efforts to search for hybrid baryons are underway at CLAS in Hall B of Jefferson Lab [10,11], with the first results expected in the coming years. The hybrid hadron system has also attracted considerable interest from theorists. These exotic hadrons have been studied using various theoretical frameworks, including the bag model [12–17], the lattice QCD [18–27], the large-N_c approach [28], the flux-tube model [29–34], the hybrid quark model [35–37], the constituent gluon model [38–40], and QCD sum rules [41–46], among others. Additional discussions on related developments can be found in Refs. [47–52], which provide insights into recent progress. Notably, most studies of hybrid baryons have concentrated on systems composed entirely of either light or heavy quarks. In contrast, our focus is on singly heavy hybrid baryons containing only one heavy quark. In such systems, the heavy quark effective theory can be effectively applied, providing a promising framework for advancing our understanding of nonperturbative QCD.

In this paper we systematically investigate charmed hybrid baryons using the QCD sum rule method within the framework of heavy quark effective theory. We

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construct 28 interpolating currents for charmed hybrid baryons, seven of which are employed to perform sum rule analyses for 19 states with quark-gluon configurations $qqcg$, $qscg$, and $sscg$ ($q = u/d$). We carry out comprehensive analyses of their mass spectra for quantum numbers $J^P = 1/2^+$, $3/2^+$, and $5/2^+$, and determine their decay constants, which are essential for understanding their decay behaviors in future studies. In particular, we calculate the masses of the lowest-lying charmed hybrid baryons within the $SU(3)$ flavor $\mathbf{6}_F$ representation to be

$$M_{\Sigma_{cg}(1/2^+)} = 3.36^{+0.27}_{-0.26}\ \text{GeV},$$
$$M_{\Xi'_{cg}(1/2^+)} = 3.59 \pm 0.20\ \text{GeV},$$
$$M_{\Omega_{cg}(1/2^+)} = 3.82 \pm 0.21\ \text{GeV}.$$

All these states carry the quantum numbers $J^P = 1/2^+$. We propose that future experimental studies search for them through their P-wave decay channels $ND^{(*)}$, $\Lambda D^{(*)}$, and $\Xi D^{(*)}$, respectively. Such investigations would be instrumental in deepening our understanding of gluonic field excitations within hadronic systems.

This paper is organized as follows. In Sec. II we systematically construct the interpolating currents for charmed hybrid baryons. These currents are employed in Sec. III to perform QCD sum rule analyses at leading order, and in Sec. IV to incorporate $\mathcal{O}(1/m_Q)$ corrections. A summary and discussion of our findings are presented in Sec. V.

II. CHARMED HYBRID BARYON CURRENTS

In this section we systematically construct the interpolating currents for charmed hybrid baryons, where the heavy quark symmetry plays a crucial role. In the charmed hybrid baryon system, the heavy quark serves as a static color source, and the system's properties are primarily determined by the light degrees of freedom. Let us focus on the ground-state Qq_1q_2g system, where Q represents the heavy *charm* quark, q_1 and q_2 represent the two valence light quarks (*up*, *down*, or *strange*), and g represents the valence gluon. In the heavy quark limit, the total angular momentum of the system reduces to

$$J = s_Q \otimes s_{q_1} \otimes s_{q_2} \otimes s_g = s_Q \otimes j_l, \tag{1}$$

where s_Q, s_{q_1}, s_{q_2}, and s_g denote the spins of the heavy quark Q, the light quarks q_1 and q_2, and the gluon g, respectively. Additionally, j_l denotes the total angular momentum of the light degrees of freedom, expressed as

$$j_l = s_{q_1} \otimes s_{q_2} \otimes s_g = j_{qq} \otimes s_g. \tag{2}$$

Here, $j_{qq} = s_{q_1} \otimes s_{q_2}$ denotes the combined spin of the two light quarks. These two light quarks satisfy the exchange antisymmetry requirement for identical particles:

(i) The flavor structure of the two light quarks can be either antisymmetric or symmetric. Accordingly, we use Λ_{cg} and Ξ_{cg} to denote the charmed hybrid baryons in the $SU(3)$ flavor $\bar{\mathbf{3}}_F$ representation, while Σ_{cg}, Ξ'_{cg}, and Ω_{cg} denote those in the $SU(3)$ flavor $\mathbf{6}_F$ representation.

(ii) The color structure of the two light quarks can be either antisymmetric ($\bar{\mathbf{3}}_C$) or symmetric ($\mathbf{6}_C$), while the overall color structure of the three quarks is always $\mathbf{8}_C$.

(iii) The spin structure of the two light quarks can be either antisymmetric ($j_{qq} = 0$) or symmetric ($j_{qq} = 1$).

The charmed hybrid baryon currents can generally be constructed using one charm quark field $h_v^a(x)$, two light quark fields $q_a(x)$ and $q_b(x)$, and one gluon field strength tensor $G^n_{\mu\nu}(x)$ [or one dual gluon field strength tensor $\tilde{G}^n_{\mu\nu}(x) = G^{n,\rho\sigma}(x) \times \epsilon_{\mu\nu\rho\sigma}/2$], with the color indices $a/b = 1\ldots3$ and $n = 1\ldots8$ as well as the Lorentz indices $\mu/\nu = 0\ldots3$. The (dual) gluon field is a color octet, so the combination of the three quark fields should also form a color octet, with two possible combinations:

$$\epsilon_{abe}[q^a q^b]\frac{\lambda^n_{ec}}{2} h_v^c \times G^n/\tilde{G}^n, \tag{3}$$

$$\epsilon_{aec}[q^a q^b + q^b q^a]\frac{\lambda^n_{eb}}{2} h_v^c \times G^n/\tilde{G}^n. \tag{4}$$

The light diquark in the former combination is a color antitriplet, while the one in the latter is a color sextet.

As summarized in Fig. 1, we identify a total of 28 charmed hybrid baryon currents, denoted as $J^{\alpha_1\cdots\alpha_{J-1/2}}_{F,C,j_{qq},j_l,J^P}/\tilde{J}^{\alpha_1\cdots\alpha_{J-1/2}}_{F,C,j_{qq},j_l,J^P}$, which can be categorized into eight multiplets, denoted as $[F, C, j_{qq}, j_l, G/\tilde{G}]$. Here, F and C represent the flavor and color representations of the two light quarks, while $J(\tilde{J})$ and $G(\tilde{G})$ indicate the presence of a gluon (or dual gluon) in the current. The explicit expressions of these currents are given as follows:

(i) The doublet $[\bar{3}_f, \bar{3}_c, 0, 1, G]$ contains two states with $J^P = 1/2^+$ and $3/2^+$. Its dual doublet $[\bar{3}_f, \bar{3}_c, 0, 1, \tilde{G}]$ contains two other states with $J^P = 1/2^-$ and $3/2^-$. Their corresponding currents are

$$J_{\bar{3}_f,\bar{3}_c,0,1,1/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_5 q^b]\sigma_t^{\mu\nu} g_s \frac{\lambda^n_{ec}}{2} G^n_{\mu\nu} h_v^c, \tag{5}$$

$$\tilde{J}_{\bar{3}_f,\bar{3}_c,0,1,1/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_5 q^b]\sigma_t^{\mu\nu} g_s \frac{\lambda^n_{ec}}{2} \tilde{G}^n_{\mu\nu} h_v^c, \tag{6}$$

$$J^\alpha_{\bar{3}_f,\bar{3}_c,0,1,3/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_5 q^b] \times \left(g_t^{\alpha\mu} - \frac{1}{3}\gamma_t^\alpha\gamma_t^\mu\right)\gamma_t^\nu\gamma_5 g_s \frac{\lambda^n_{ec}}{2} G^n_{\mu\nu} h_v^c, \tag{7}$$

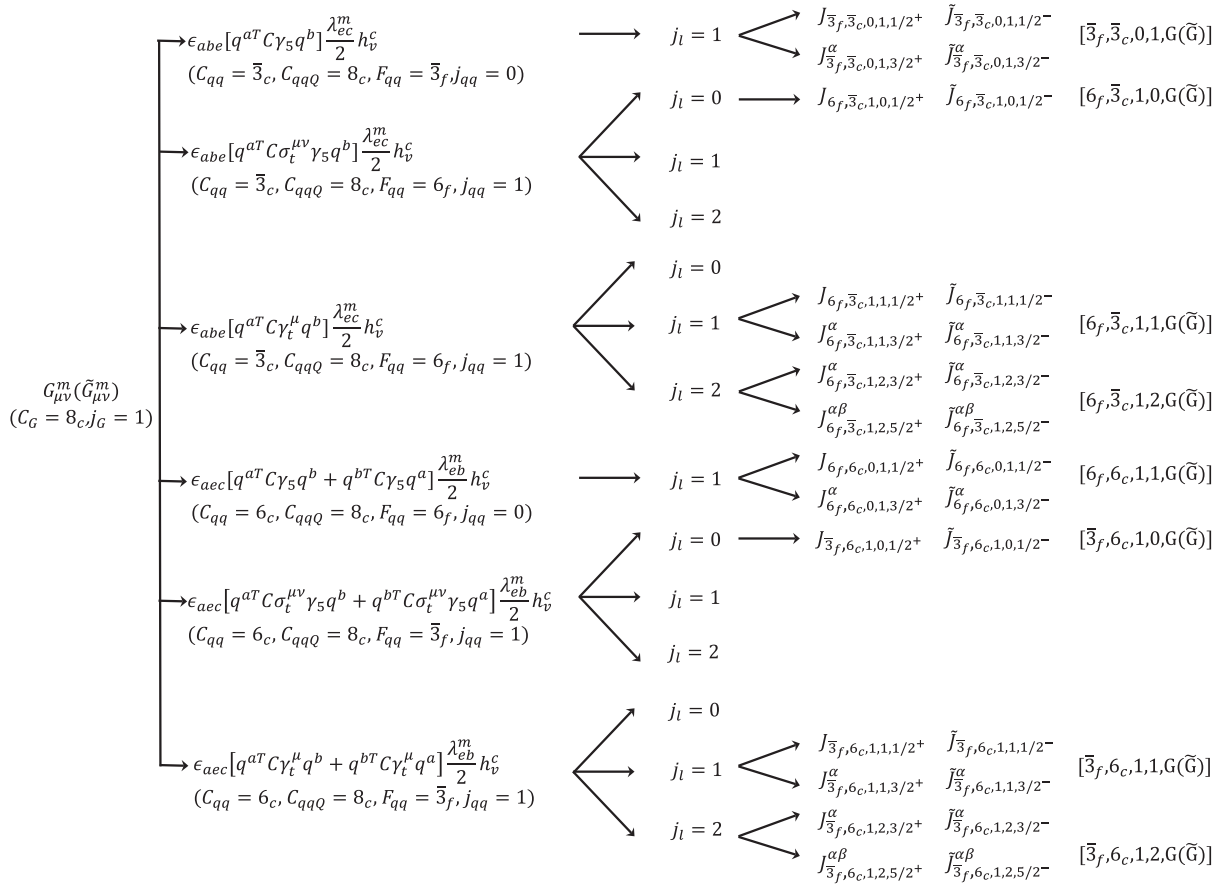


FIG. 1. Categorization of charmed hybrid baryon currents.

$$\tilde{J}^\alpha_{\bar{3}_f,\bar{3}_c,0,1,3/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_5 q^b] \times \left(g_t^{\alpha\mu} - \frac{1}{3}\gamma_t^\alpha\gamma_t^\mu\right)\gamma_t^\nu\gamma_5 g_s\frac{\lambda^n_{ec}}{2}\tilde{G}^n_{\mu\nu}h^c_v. \tag{8}$$

(ii) The singlet $[6_f, \bar{3}_c, 1, 0, G]$ contains one state with $J^P = 1/2^+$, and its dual singlet $[6_f, \bar{3}_c, 1, 0, \tilde{G}]$ contains another state with $J^P = 1/2^-$. Their corresponding currents are

$$J_{6_f,\bar{3}_c,1,0,1/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\sigma_t^{\mu\nu}\gamma_5 q^b]g_s\frac{\lambda^n_{ec}}{2}G^n_{\mu\nu}h^c_v, \tag{9}$$

$$\tilde{J}_{6_f,\bar{3}_c,1,0,1/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\sigma_t^{\mu\nu}\gamma_5 q^b]g_s\frac{\lambda^n_{ec}}{2}\tilde{G}^n_{\mu\nu}h^c_v. \tag{10}$$

(iii) The doublet $[6_f, \bar{3}_c, 1, 1, G]$ contains two states with $J^P = 1/2^+$ and $3/2^+$. Its dual doublet $[6_f, \bar{3}_c, 1, 1, \tilde{G}]$ contains two other states with $J^P = 1/2^-$ and $3/2^-$. Their corresponding currents are

$$J_{6_f,\bar{3}_c,1,1,1/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_t^\mu q^b]\gamma_t^\nu\gamma_5 g_s\frac{\lambda^n_{ec}}{2}G^n_{\mu\nu}h^c_v, \tag{11}$$

$$\tilde{J}_{6_f,\bar{3}_c,1,1,1/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_t^\mu q^b]\gamma_t^\nu\gamma_5 g_s\frac{\lambda^n_{ec}}{2}\tilde{G}^n_{\mu\nu}h^c_v, \tag{12}$$

$$J^\alpha_{6_f,\bar{3}_c,1,1,3/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_t^\mu q^b] \times \left(g_t^{\alpha\nu} - \frac{1}{3}\gamma_t^\alpha\gamma_t^\nu\right)g_s\frac{\lambda^n_{ec}}{2}G^n_{\mu\nu}h^c_v, \tag{13}$$

$$\tilde{J}^\alpha_{6_f,\bar{3}_c,1,1,3/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_t^\mu q^b] \times \left(g_t^{\alpha\nu} - \frac{1}{3}\gamma_t^\alpha\gamma_t^\nu\right)g_s\frac{\lambda^n_{ec}}{2}\tilde{G}^n_{\mu\nu}h^c_v. \tag{14}$$

(iv) The doublet $[6_f, \bar{3}_c, 1, 2, G]$ contains two states with $J^P = 3/2^+$ and $5/2^+$. Its dual doublet $[6_f, \bar{3}_c, 1, 2, \tilde{G}]$ contains two other states with $J^P = 3/2^-$ and $5/2^-$. Their corresponding currents are

$$J^\alpha_{6_f,\bar{3}_c,1,2,3/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_{t,\mu_1}q^b] \times \Gamma^{\alpha\mu_4,\mu_1\nu_2}_{J=2}\gamma_t^{\mu_4}\gamma_t^{\nu_1}g_s\frac{\lambda^n_{ec}}{2}G^n_{\nu_1\nu_2}h^c_v, \tag{15}$$

$$\tilde{J}^{\alpha}_{6_f,\bar{3}_c,1,2,3/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_{t,\mu_1}q^b] \times \Gamma^{\alpha\mu_4,\mu_1\nu_2}_{J=2}\gamma_t^{\mu_4}\gamma_t^{\nu_1}g_s\frac{\lambda^n_{ec}}{2}\tilde{G}^n_{\nu_1\nu_2}h^c_v, \quad (16)$$

$$J^{\alpha\beta}_{6_f,\bar{3}_c,1,2,5/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_{t,\mu_1}q^b] \times \Gamma^{\alpha\beta,\mu_1\nu_2}_{J=\frac{5}{2}}\gamma_t^{\nu_1}\gamma_5 g_s\frac{\lambda^n_{ec}}{2}G^n_{\nu_1\nu_2}h^c_v, \quad (17)$$

$$\tilde{J}^{\alpha\beta}_{6_f,\bar{3}_c,1,2,5/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_{t,\mu_1}q^b] \times \Gamma^{\alpha\beta,\mu_1\nu_2}_{J=\frac{5}{2}}\gamma_t^{\nu_1}\gamma_5 g_s\frac{\lambda^n_{ec}}{2}\tilde{G}^n_{\nu_1\nu_2}h^c_v. \quad (18)$$

(v) The doublet $[6_f, 6_c, 0, 1, G]$ contains two states with $J^P = 1/2^+$ and $3/2^+$. Its dual doublet $[6_f, 6_c, 0, 1, \tilde{G}]$ contains two other states with $J^P = 1/2^-$ and $3/2^-$. Their corresponding currents are

$$J_{6_f,6_c,0,1,1/2^+} = \epsilon_{aec}[q^{aT}\mathcal{C}\gamma_5 q^b + q^{bT}\mathcal{C}\gamma_5 q^a] \times \sigma_t^{\mu\nu}g_s\frac{\lambda^m_{eb}}{2}G^m_{\mu\nu}h^c_v, \quad (19)$$

$$\tilde{J}_{6_f,6_c,0,1,1/2^-} = \epsilon_{aec}[q^{aT}\mathcal{C}\gamma_5 q^b + q^{bT}\mathcal{C}\gamma_5 q^a] \times \sigma_t^{\mu\nu}g_s\frac{\lambda^m_{eb}}{2}\tilde{G}^m_{\mu\nu}h^c_v, \quad (20)$$

$$J^{\alpha}_{6_f,6_c,0,1,3/2^+} = \epsilon_{aec}[q^{aT}\mathcal{C}\gamma_5 q^b + q^{bT}\mathcal{C}\gamma_5 q^a] \times \left(g_t^{\alpha\mu} - \frac{1}{3}\gamma_t^{\alpha}\gamma_t^{\mu}\right)\gamma_t^{\nu}\gamma_5 g_s\frac{\lambda^m_{eb}}{2}G^m_{\mu\nu}h^c_v, \quad (21)$$

$$\tilde{J}^{\alpha}_{6_f,6_c,0,1,3/2^-} = \epsilon_{aec}[q^{aT}\mathcal{C}\gamma_5 q^b + q^{bT}\mathcal{C}\gamma_5 q^a] \times \left(g_t^{\alpha\mu} - \frac{1}{3}\gamma_t^{\alpha}\gamma_t^{\mu}\right)\gamma_t^{\nu}\gamma_5 g_s\frac{\lambda^m_{eb}}{2}\tilde{G}^m_{\mu\nu}h^c_v. \quad (22)$$

(vi) The singlet $[\bar{3}_f, 6_c, 1, 0, G]$ contains one state with $J^P = 1/2^+$, and its dual singlet $[\bar{3}_f, 6_c, 1, 0, \tilde{G}]$ contains another state with $J^P = 1/2^-$. Their corresponding currents are

$$J_{\bar{3}_f,6_c,1,0,1/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\sigma_t^{\mu\nu}\gamma_5 q^b + q^{bT}\mathcal{C}\sigma_t^{\mu\nu}\gamma_5 q^a] \times g_s\frac{\lambda^m_{ec}}{2}G^m_{\mu\nu}h^c_v, \quad (23)$$

$$\tilde{J}_{\bar{3}_f,6_c,1,0,1/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\sigma_t^{\mu\nu}\gamma_5 q^b + q^{bT}\mathcal{C}\sigma_t^{\mu\nu}\gamma_5 q^a] \times g_s\frac{\lambda^m_{ec}}{2}\tilde{G}^m_{\mu\nu}h^c_v. \quad (24)$$

(vii) The doublet $[\bar{3}_f, 6_c, 1, 1, G]$ contains two states with $J^P = 1/2^+$ and $3/2^+$. Its dual doublet $[\bar{3}_f, 6_c, 1, 1, \tilde{G}]$ contains two other states with $J^P = 1/2^-$ and $3/2^-$. Their corresponding currents are

$$J_{\bar{3}_f,6_c,1,1,1/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_t^{\mu}q^b + q^{bT}\mathcal{C}\gamma_t^{\mu}q^a] \times \gamma_t^{\nu}\gamma_5 g_s\frac{\lambda^m_{ec}}{2}G^m_{\mu\nu}h^c_v, \quad (25)$$

$$\tilde{J}_{\bar{3}_f,6_c,1,1,1/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_t^{\mu}q^b + q^{bT}\mathcal{C}\gamma_t^{\mu}q^a] \times \gamma_t^{\nu}\gamma_5 g_s\frac{\lambda^m_{ec}}{2}\tilde{G}^m_{\mu\nu}h^c_v, \quad (26)$$

$$J^{\alpha}_{\bar{3}_f,6_c,1,1,3/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_t^{\mu}q^b + q^{bT}\mathcal{C}\gamma_t^{\mu}q^a] \times \left(g_t^{\alpha\nu} - \frac{1}{3}\gamma_t^{\alpha}\gamma_t^{\nu}\right)g_s\frac{\lambda^m_{ec}}{2}G^m_{\mu\nu}h^c_v, \quad (27)$$

$$\tilde{J}^{\alpha}_{\bar{3}_f,6_c,1,1,3/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_t^{\mu}q^b + q^{bT}\mathcal{C}\gamma_t^{\mu}q^a] \times \left(g_t^{\alpha\nu} - \frac{1}{3}\gamma_t^{\alpha}\gamma_t^{\nu}\right)g_s\frac{\lambda^m_{ec}}{2}\tilde{G}^m_{\mu\nu}h^c_v. \quad (28)$$

(viii) The doublet $[\bar{3}_f, 6_c, 1, 2, G]$ contains two states with $J^P = 3/2^+$ and $5/2^+$. Its dual doublet $[\bar{3}_f, 6_c, 1, 2, \tilde{G}]$ contains two other states with $J^P = 3/2^-$ and $5/2^-$. Their corresponding currents are

$$J^{\alpha}_{\bar{3}_f,6_c,1,2,3/2^+} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_{t,\mu_1}q^b + q^{bT}\mathcal{C}\gamma_{t,\mu_1}q^a] \times \Gamma^{\alpha\mu_4,\mu_1\nu_2}_{J=2}\gamma_t^{\mu_4}\gamma_t^{\nu_1}g_s\frac{\lambda^m_{ec}}{2}G^m_{\nu_1\nu_2}h^c_v, \quad (29)$$

$$\tilde{J}^{\alpha}_{\bar{3}_f,6_c,1,2,3/2^-} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_{t,\mu_1}q^b + q^{bT}\mathcal{C}\gamma_{t,\mu_1}q^a] \times \Gamma^{\alpha\mu_4,\mu_1\nu_2}_{J=2}\gamma_t^{\mu_4}\gamma_t^{\nu_1}g_s\frac{\lambda^m_{ec}}{2}\tilde{G}^m_{\nu_1\nu_2}h^c_v, \quad (30)$$

$$J^{\alpha\beta}_{\bar{3}_f,6_c,1,2,5/2+} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_{t,\mu_1}q^b + q^{bT}\mathcal{C}\gamma_{t,\mu_1}q^a] \times \Gamma^{\alpha\beta,\mu_1\nu_2}_{J=\frac{5}{2}}\gamma_t^{\nu_1}\gamma_5 g_s\frac{\lambda^m_{ec}}{2}G^m_{\nu_1\nu_2}h^c_v, \quad (31)$$

$$\tilde{J}^{\alpha\beta}_{\bar{3}_f,6_c,1,2,5/2-} = \epsilon_{abe}[q^{aT}\mathcal{C}\gamma_{t,\mu_1}q^b + q^{bT}\mathcal{C}\gamma_{t,\mu_1}q^a] \times \Gamma^{\alpha\beta,\mu_1\nu_2}_{J=\frac{5}{2}}\gamma_t^{\nu_1}\gamma_5 g_s\frac{\lambda^m_{ec}}{2}\tilde{G}^m_{\nu_1\nu_2}h^c_v. \quad (32)$$

In the above expressions, $\gamma_t^{\mu} = \gamma^{\mu} - v\!\!\!/ v^{\mu}$, $g_t^{\mu\nu} = g^{\mu\nu} - v^{\mu}v^{\nu}$, $\sigma_t^{\mu\nu} = \sigma^{\mu\nu} - \sigma^{\mu\alpha}v_{\alpha}v^{\nu} - \sigma^{\alpha\nu}v_{\alpha}v^{\mu}$, and $G^n_{\mu\nu} = \partial_{\mu}A^n_{\nu} - \partial_{\nu}A^n_{\mu} + g_s f^{npq}A_{p,\mu}A_{q,\nu}$; $\Gamma^{\alpha\beta,\mu\nu}_{J=2}$ and $\Gamma^{\alpha\beta,\mu\nu}_{J=5/2}$ are the $J = 2$ and $J = 5/2$ projection operators:

$$\Gamma^{\alpha\beta,\mu\nu}_{J=2} = g_t^{\alpha\mu}g_t^{\beta\nu} + g_t^{\alpha\nu}g_t^{\beta\mu} - \frac{2}{3}g_t^{\alpha\beta}g_t^{\mu\nu}, \quad (33)$$

$$\Gamma^{\alpha\beta,\mu\nu}_{J=5/2} = g_t^{\alpha\mu} g_t^{\beta\nu} + g_t^{\alpha\nu} g_t^{\beta\mu} - \frac{2}{5} g_t^{\alpha\beta} g_t^{\mu\nu} - \frac{1}{5} g_t^{\alpha\mu} \gamma_t^\beta \gamma_t^\nu - \frac{1}{5} g_t^{\alpha\nu} \gamma_t^\beta \gamma_t^\mu - \frac{1}{5} g_t^{\beta\mu} \gamma_t^\alpha \gamma_t^\nu - \frac{1}{5} g_t^{\beta\nu} \gamma_t^\alpha \gamma_t^\mu. \tag{34}$$

III. RESULTS AT THE LEADING ORDER

In this section we apply the QCD sum rule method to investigate the charmed hybrid baryon currents given in Eqs. (5)–(18) within the framework of heavy quark effective theory. These seven currents form the four baryon multiplets $[\bar{3}_f, \bar{3}_c, 0, 1, G]$, $[6_f, \bar{3}_c, 1, 0, G]$, $[6_f, \bar{3}_c, 1, 1, G]$, and $[6_f, \bar{3}_c, 1, 2, G]$. The other currents given in Eqs. (19)–(32) will be investigated in our future studies. We shall find that the current $J_{6_f,\bar{3}_c,1,1,1/2^+}$, belonging to the doublet $[6_f, \bar{3}_c, 1, 1, G]$, couples to the lowest-lying charmed hybrid baryon, so we use this current as a representative example. The explicit forms of this current and its spin partner for the Σ_{cg} states are presented as follows ($q = up/down$):

$$J_{\Sigma_{cg},\bar{3}_c,1,1,1/2^+} = \epsilon_{abe}[q^{aT} C\gamma_t^\mu q^b]\gamma_t^\nu \gamma_5 g_s \frac{\lambda^m_{ec}}{2} G^m_{\mu\nu} h^c_v, \tag{35}$$

$$J^\alpha_{\Sigma_{cg},\bar{3}_c,1,1,3/2^+} = \epsilon_{abe}[q^{aT} C\gamma_t^\mu q^b]\left(g_t^{\alpha\nu} - \frac{1}{3}\gamma_t^\alpha \gamma_t^\nu\right) g_s \frac{\lambda^m_{ec}}{2} G^m_{\mu\nu} h^c_v. \tag{36}$$

The current $J_{\Sigma_{cg},\bar{3}_c,1,1,1/2^+}$ couples to both the charmed hybrid baryon with $J^P = 1/2^+$ and the one with $J^P = 1/2^-$ through

$$\langle 0|J_{\Sigma_{cg},\bar{3}_c,1,1,1/2^+}|\Sigma^+_{cg}\rangle = f_+ u, \tag{37}$$

$$\langle 0|J_{\Sigma_{cg},\bar{3}_c,1,1,1/2^+}|\Sigma^-_{cg}\rangle = f_- \gamma_5 u, \tag{38}$$

where $f_\pm$ are the decay constants and u is the Dirac spinor. Consequently, the two-point correlation function can be constructed as

$$\begin{aligned}\Pi_{\Sigma_{cg},\bar{3}_c,1,1,1/2^+}(\omega) &= i\int d^4x e^{ikx}\langle 0|T[J_{\Sigma_{cg},\bar{3}_c,1,1,1/2^+}(x)\bar{J}_{\Sigma_{cg},\bar{3}_c,1,1,1/2^+}(0)]|0\rangle \\ &= \frac{1+\not{v}}{2}\Pi_+(\omega) + \frac{1-\not{v}}{2}\Pi_-(\omega),\end{aligned} \tag{39}$$

where $\omega = v \cdot k$ is the external off-shell energy and $\Pi_\pm$ are, respectively, contributed by $\Sigma^\pm_{cg}$. We calculate Eq. (39) at the quark-gluon level using the operator product expansion (OPE) method to derive

$$\Pi_-(\omega) = 0, \tag{40}$$

suggesting that the current $J_{\Sigma_{cg},\bar{3}_c,1,1/2^+}$ fully couples to the positive-parity state Σ^+_{cg}. This allows us to simplify Eq. (39) at the hadron level as

$$\Pi_+(\omega) = \frac{f_+^2}{\bar{\Lambda}_+ - \omega} + \text{higher states}, \tag{41}$$

with $\bar{\Lambda}_+$ the sum rule result at the leading order. After performing the Borel transformation at both the hadron and the quark-gluon levels, we obtain

$$\begin{aligned}\Pi_+(\omega_c, T) &= f_+^2 e^{-\bar{\Lambda}_+/T} \\ &= \int_0^{\omega_c} e^{-\omega/T} d\omega \times \left(\frac{\alpha_s}{1890\pi^5}\omega^9 + \frac{\langle g_s^2 GG\rangle}{240\pi^4}\omega^9 - \frac{\alpha_s\langle g_s^2 GG\rangle}{1440\pi^5}\omega^5 - \frac{\langle g_s^3 G^3\rangle}{192\pi^4}\omega^3 + \frac{8\alpha_s\langle\bar{q}q\rangle^2}{9\pi}\omega^3 \right. \\ &\quad \left. + \frac{\langle g_s^2 GG\rangle^2}{4608\pi^4}\omega - \frac{41\alpha_s\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{48\pi}\omega\right) + \frac{\langle\bar{q}q\rangle^2\langle g_s^2 GG\rangle}{72} - \frac{\langle\bar{q}q\rangle\langle g_s^2 GG\rangle\langle g_s\bar{q}\sigma Gq\rangle}{576T^2} \\ &\quad - \frac{\langle\bar{q}q\rangle^2\langle g_s^3 G^3\rangle}{576T^2} + \frac{25\alpha_s\langle\bar{q}\sigma Gq\rangle^2}{768\pi},\end{aligned} \tag{42}$$

from which we further derive

$$\bar{\Lambda}_+(\omega_c, T) = \frac{1}{\Pi_+(\omega_c, T)} \times \frac{\partial\Pi_+(\omega_c, T)}{\partial(-1/T)}, \tag{43}$$

$$f_+^2(\omega_c, T) = \Pi_+(\omega_c, T) \times e^{\bar{\Lambda}_+(\omega_c,T)/T}. \tag{44}$$

In the above expressions, we have evaluated the two-point correlation function up to the12th dimension, including the quark condensate $\langle\bar{q}q\rangle$, the two-gluon condensate $\langle g_s^2 GG\rangle$, the three-gluon condensate $\langle g_s^3 G^3\rangle$, the quark-gluon mixed condensate $\langle g_s\bar{q}\sigma Gq\rangle$, and their combinations. The sum rule equations for other currents belonging to the multiplets $[\bar{3}_f, \bar{3}_c, 0, 1, G]$, $[6_f, \bar{3}_c, 1, 0, G]$, $[6_f, \bar{3}_c, 1, 1, G]$, and $[6_f, \bar{3}_c, 1, 2, G]$ are provided in the supplemental *Mathematica* file "OPE. nb."

To perform numerical analyses, we use the following values for various quark and gluon condensates [53–61]:

$$\begin{aligned}
\langle\bar{q}q\rangle &= -(0.240 \pm 0.010)^3 \text{ GeV}, \\
\langle\bar{s}s\rangle &= (0.8 \pm 0.1) \times \langle\bar{q}q\rangle, \\
\langle\alpha_s GG\rangle &= (6.35 \pm 0.35) \times 10^{-2} \text{ GeV}^4, \\
\langle g_s^3 G^3\rangle &= (8.2 \pm 1.0) \times \langle\alpha_s GG\rangle \text{ GeV}^2, \\
\langle g_s\bar{q}\sigma Gq\rangle &= (0.8 \pm 0.2) \times \langle\bar{q}q\rangle \text{ GeV}^2, \\
\langle g_s\bar{s}\sigma Gs\rangle &= (0.8 \pm 0.2) \times \langle\bar{s}s\rangle \text{ GeV}^2. \qquad (45)
\end{aligned}$$

Additionally, we use the following values for the running masses of the charm and strange quarks at the renormalization scale of 2 GeV [62]:

$$\begin{aligned}
\alpha_s(\mu = M_\tau) &= 0.33, \\
\alpha_s(\mu = 2 \text{ GeV}) &= \frac{\alpha_s(M_\tau)}{1 + \frac{25\alpha_s(M_\tau)}{12\pi}\log(\frac{\mu^2}{M_\tau^2})} = 0.31, \\
\bar{m}_c(\mu = m_c) &= 1.23 \pm 0.09 \text{ GeV}, \\
m_c(\mu = 2 \text{ GeV}) &= \bar{m}_c\left(\frac{\alpha_s(\mu)}{\alpha_s(\bar{m}_c)}\right)^{12/25} = 1.10 \text{ GeV}, \\
m_s(\mu = 2 \text{ GeV}) &= 93^{+11}_{-5} \text{ MeV}. \qquad (46)
\end{aligned}$$

The sum rule equation listed in Eq. (42) contains two free parameters: the threshold value ω_c and the Borel mass T. Their appropriate working regions are determined based on three criteria: (a) sufficient convergence of the OPE, (b) a sufficiently large pole contribution, and (c) weak dependence of the sum rule results on these parameters.

To ensure sufficient convergence of the OPE, we require the g_s^4 terms to be less than 5%, the $D = 8$ terms to be less than 10%, and the $D = 6$ terms to be less than 20%:

$$\begin{aligned}
\text{CVG} &\equiv \left|\frac{\Pi_+^{g_s^n=4}(\infty, T)}{\Pi_+(\infty, T)}\right| \leq 5\%, \\
\text{CVG}' &\equiv \left|\frac{\Pi_+^{D=8}(\infty, T)}{\Pi_+(\infty, T)}\right| \leq 10\%, \\
\text{CVG}'' &\equiv \left|\frac{\Pi_+^{D=6}(\infty, T)}{\Pi_+(\infty, T)}\right| \leq 20\%. \qquad (47)
\end{aligned}$$

As shown in Fig. 2, the lower limit of the Borel mass is determined to be $T_{\min} = 0.37$ GeV.

To ensure a sufficiently large pole contribution, we require the pole contribution to exceed 40%:

$$\text{PC} \equiv \left|\frac{\Pi_+(\omega_0, T)}{\Pi_+(\infty, T)}\right| \geq 40\%. \qquad (48)$$

As shown in Fig. 2, the upper limit of the Borel mass is determined to be $T_{\max} = 0.40$ GeV, when setting $\omega_c = 2.95$ GeV.

Altogether, we obtain the Borel window 0.37 GeV $\leq T \leq$ 0.40 GeV for $\omega_c = 2.95$ GeV. By varying ω_c, we find

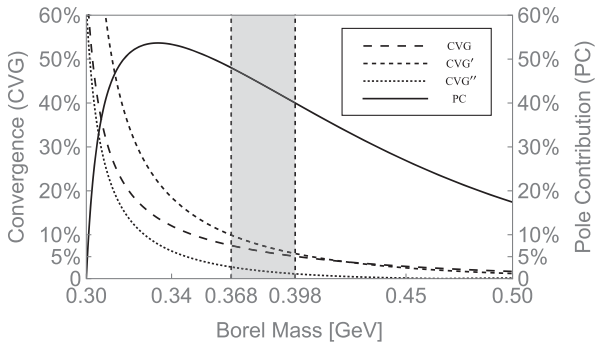


FIG. 2. CVG$^{(\prime,\prime\prime)}$ and PC as functions of the Borel mass T, with the threshold value set to be $\omega_c = 2.95$ GeV. These curves are derived using the current $J_{\Sigma_{cg}^+,\bar{3}_c,1,1,1/2^+}$, which belongs to the doublet $[\Sigma_{cg}^+, \bar{3}_c, 1, 1, G]$.

that nonvanishing Borel windows exist for $\omega_c \geq \omega_c^{\min} = 2.7$ GeV. We choose ω_c to be approximately 10% larger and establish its working region as 2.7 GeV $\leq \omega_c \leq$ 3.2 GeV, where the numerical results are derived as

$$\bar{\Lambda}_+ = 2.46^{+0.15}_{-0.14} \text{ GeV}, \qquad (49)$$

$$f_+ = 0.079^{+0.015}_{-0.013} \text{ GeV}^4. \qquad (50)$$

Their variations are shown in Fig. 3 as functions of the Borel mass T, where the dependence is weak and remains within an acceptable range inside the Borel window 0.37 GeV $\leq T \leq$ 0.40 GeV.

IV. RESULTS AT THE $\mathcal{O}(1/m_Q)$ ORDER

In this section we extend the analysis to include $\mathcal{O}(1/m_Q)$ corrections by considering the following Lagrangian from the heavy quark effective theory:

$$\mathcal{L}_{\text{eff}} = \bar{h}_v^c iv \cdot D_t h_v^c + \frac{1}{2m_c}\mathcal{K} + \frac{1}{2m_c}\mathcal{S}, \qquad (51)$$

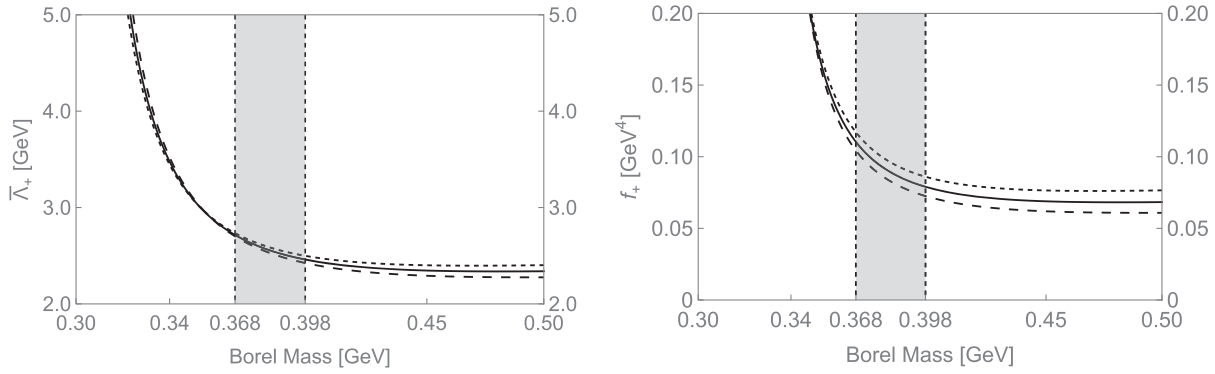
where $\mathcal{K}$ represents the operator of nonrelativistic kinetic energy and $\mathcal{S}$ represents the Pauli term describing the chromomagnetic interaction:

$$\mathcal{K} = \bar{h}_v^c (iD_t)^2 h_v^c, \qquad (52)$$

$$\mathcal{S} = \frac{g_s}{2} C_{\text{mag}}(m_c/\mu)\bar{h}_v^c \sigma_{\mu\nu} G^{\mu\nu} h_v^c, \qquad (53)$$

with $C_{\text{mag}}(m_c/\mu) = [\alpha_s(m_c)/\alpha_s(\mu)]^{3/\beta_0}$ and $\beta_0 = 11 - 2n_f/3$.

Following Eq. (41), we can express the pole term up to the $\mathcal{O}(1/m_c)$ order as



(a) (b)

FIG. 3. Variations of (a) $\bar{\Lambda}_+$ and (b) f_+ as functions of the Borel mass T, where the long-dashed, solid, and short-dashed curves correspond to fixed threshold values $\omega_c = 2.85$ GeV, 2.95 GeV, and 3.05 GeV, respectively. These results are obtained using the current $J_{\Sigma_{cg}^+,\bar{3}c,1,1,1/2^+}$, which is associated with the doublet $[\Sigma_{cg}^+, \bar{3}_c, 1, 1, G]$.

$$\Pi_+(\omega) = \frac{(f_+ + \delta f_+)^2}{\bar{\Lambda}_+ + \delta M_+ - \omega} = \frac{f_+^2}{\bar{\Lambda}_+ - \omega} - \frac{\delta M_+ f_+^2}{(\bar{\Lambda}_+ - \omega)^2} + \frac{f_+ \delta f_+}{\bar{\Lambda}_+ - \omega}, \quad (54)$$

where δM_+ and δf_+ denote the corrections to the mass and decay constant, respectively. To calculate δM_+, we consider the following three-point correlation functions:

$$\begin{aligned}
&\delta_{\mathcal{O}}\Pi^{\alpha_1\cdots\alpha_{J-1/2},\beta_1\cdots\beta_{J-1/2}}_{F,C,j_{qq},j_l,J^P}(\omega,\omega') \\
&= i^2\int d^4x d^4y e^{ikx-ik'y}\langle 0|TJ^{\alpha_1\cdots\alpha_{J-1/2}}_{F,C,j_{qq},j_l,J^P}(x)\mathcal{O}\bar{J}^{\beta_1\cdots\beta_{J-1/2}}_{F,C,j_{qq},j_l,J^P}(y)|0\rangle \\
&= \mathbb{S}[g_t^{\alpha_1\beta_1}\cdots g_t^{\alpha_{J-1/2}\beta_{J-1/2}}] \\
&\quad\times\left(\frac{1+\not{v}}{2}\delta_{\mathcal{O}}\Pi^+_{F,C,j_{qq},j_l,J^P} + \frac{1-\not{v}}{2}\delta_{\mathcal{O}}\Pi^-_{F,C,j_{qq},j_l,J^P}\right), \quad (55)
\end{aligned}$$

where $\mathcal{O} = \mathcal{K}$ or $\mathcal{S}$, $k' = k + q$, $\omega = v\cdot k$, and $\omega' = v\cdot k'$.

Based on Eq. (51), we can express Eq. (55) at the hadron level as

$$\delta_{\mathcal{K}}\Pi_+ = \frac{f_+^2 K_+}{(\bar{\Lambda}_+ - \omega)(\bar{\Lambda}_+ - \omega')} + \frac{f_+^2 G_{\mathcal{K}}}{\bar{\Lambda}_+ - \omega} + \frac{f_+^2 G'_{\mathcal{K}}}{\bar{\Lambda}_+ - \omega'}, \quad (56)$$

$$\delta_{\mathcal{S}}\Pi_+ = \frac{d_M f_+^2 \Sigma_+}{(\bar{\Lambda}_+ - \omega)(\bar{\Lambda}_+ - \omega')} + \frac{d_M f_+^2 G_{\mathcal{S}}}{\bar{\Lambda}_+ - \omega} + \frac{d_M f_+^2 G'_{\mathcal{S}}}{\bar{\Lambda}_+ - \omega'}, \quad (57)$$

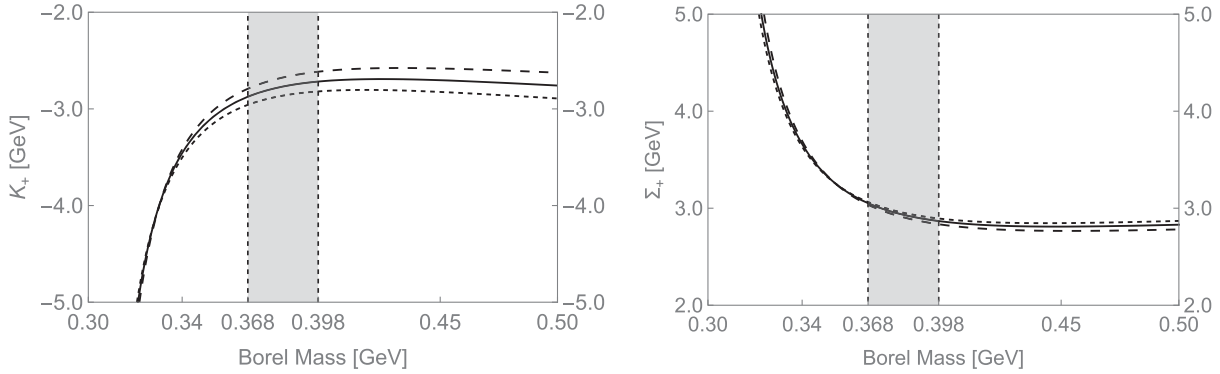
where K_+, Σ_+, and d_M are defined as

$$\begin{aligned}
K_+ &\equiv \langle\Sigma_{cg}^+|\bar{h}_v^c(iD_t)^2 h_v^c|\Sigma_{cg}^+\rangle, \\
d_M\Sigma_+ &\equiv \langle\Sigma_{cg}^+|\frac{g_s}{2}\bar{h}_v^c\sigma_{\mu\nu}G^{\mu\nu}h_v^c|\Sigma_{cg}^+\rangle, \\
d_M &\equiv d_{j,j_l}, \\
d_{j_l-1/2,j_l} &= 2j_l + 2, \\
d_{j_l+1/2,j_l} &= -2j_l. \quad (58)
\end{aligned}$$

Note that the term $\mathcal{S}$ causes a mass splitting within the same doublet, while it can also lead to a mixing of states with the same spin-parity quantum number. We have accounted for the former effect, but the latter effect is found in Ref. [63] to be negligible and is therefore not considered in the present study.

The correlation functions in Eq. (55) can also be evaluated at the quark-gluon level using the operator product expansion. After performing the double Borel transformation to convert ω and ω' into T_1 and T_2, and setting these two Borel parameters equal, we arrive at

$$\begin{aligned}
\delta_{\mathcal{K}}\Pi_+(\omega_c,T) &= f_+^2 K_+ e^{-\bar{\Lambda}_+/T} \\
&= \int_0^{\omega_c} e^{-\omega/T}d\omega\times\left(-\frac{\alpha_s}{4950\pi^5}\omega^{11} - \frac{11\langle g_s^2GG\rangle}{5040\pi^4}\omega^7 + \frac{13\alpha_s\langle g_s^2GG\rangle}{24192\pi^5}\omega^7 + \frac{\langle g_s^3G^3\rangle}{1920\pi^4}\omega^5 - \frac{8\alpha_s\langle\bar{q}q\rangle^2}{9\pi}\omega^5\right. \\
&\quad\left. + \frac{\langle g_s^2GG\rangle^2}{256\pi^4}\omega^3 - \frac{47\langle g_s^2GG\rangle^2}{110592\pi^4}\omega^3 + \frac{269\alpha_s\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{144\pi}\omega^3 - \frac{41\alpha_s\langle g_s\bar{q}\sigma Gq\rangle^2}{192\pi}\omega\right) \\
&\quad - \frac{5\langle\bar{q}q\rangle\langle g_s^2GG\rangle\langle g_s\bar{q}\sigma Gq\rangle}{512} - \frac{\langle\bar{q}q\rangle^2\langle g_s^3G^3\rangle}{288}, \quad (59)
\end{aligned}$$



(a) (b)

FIG. 4. Variations of (a) K_+ and (b) Σ_+ as functions of the Borel mass T, where the long-dashed, solid, and short-dashed curves correspond to fixed threshold values $\omega_c = 2.85$ GeV, 2.95 GeV, and 3.05 GeV, respectively. These results are obtained using the current $J_{\Sigma_{cg}^+,\bar{3}c,1,1,1/2^+}$, which is associated with the doublet $[\Sigma_{cg}^+, \bar{3}_c, 1, 1, G]$.

$$\begin{aligned}\delta_S\Pi_+(\omega_c,T) &= 4\times f_+^2\Sigma_+ e^{-\bar{\Lambda}_+/T}\\ &= \int_0^{\omega_c} e^{-\omega/T}d\omega\times\left(\frac{815\alpha_s\langle g_s^2GG\rangle}{24192\pi^5}\omega^7+\frac{\langle g_s^3G^3\rangle}{960\pi^4}\omega^5+\frac{251\langle g_s^2GG\rangle^2}{221184\pi^4}\omega^3-\frac{511\alpha_s\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{108\pi}\omega^3\right.\\ &\quad\left.+\frac{8779\alpha_s\langle g_s\bar{q}\sigma Gq\rangle^2}{4608\pi}\omega+\frac{9\alpha_s\langle\bar{q}q^2\rangle\langle g_s^2GG\rangle}{32\pi}\omega\right)-\frac{31\langle\bar{q}q\rangle\langle g_s^2GG\rangle\langle g_s\bar{q}\sigma Gq\rangle}{6912}+\frac{\langle\bar{q}q\rangle^2\langle g_s^3G^3\rangle}{288}\\ &\quad-\frac{9\alpha_s\langle\bar{q}q\rangle\langle g_s^2GG\rangle\langle g_s\bar{q}\sigma Gq\rangle}{256\pi}.\end{aligned}\tag{60}$$

The Borel mass dependence of these sum rule equations is shown in Fig. 4. Both K and Σ exhibit mild variation with T in the working region 2.7 GeV $\leq \omega_c \leq$ 3.2 GeV. From this region, we extract the following numerical results:

$$K_+ = -2.72^{+0.19}_{-0.20}\ \text{GeV}^2,\tag{61}$$

$$\Sigma_+ = 2.87\pm 0.25\ \text{GeV}^2.\tag{62}$$

These two values represent the sum rule results at the $\mathcal{O}(1/m_c)$ order, from which we derive

$$\delta M_+ = -\frac{1}{2m_c}(K_+ + d_M C_{\text{mag}}\Sigma_+),\tag{63}$$

and further extract the mass of the Σ_{cg}^+ state as

$$\begin{aligned}M_{\Sigma_{cg}(1/2^+)} &\equiv M_+ = m_c+\bar{\Lambda}_+ + \delta M_+\\ &= 1.10^{+0.08}_{-0.08}\ \text{GeV} + 2.46^{+0.15}_{-0.14}\ \text{GeV}\\ &\quad - 0.21^{+0.13}_{-0.12}\ \text{MeV}\\ &= 3.36^{+0.27}_{-0.26}\ \text{GeV},\end{aligned}\tag{64}$$

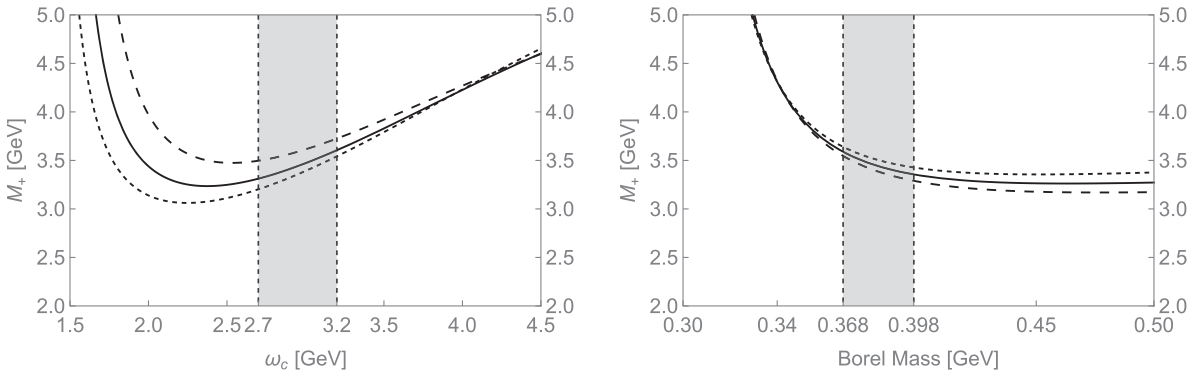
whose dependence on the threshold value ω_c and the Borel mass T is illustrated in Fig. 5. The uncertainties originate from variations in the threshold value ω_c, the Borel mass T, and the QCD parameters provided in Eqs. (45) and (46).

In addition, the mass splitting between the Σ_{cg} states of $J^P = 1/2^+$ and $J^P = 3/2^+$ is derived as

$$\Delta M_+ = M_{\Sigma_{cg}(3/2^+)} - M_{\Sigma_{cg}(1/2^+)} = 2.16^{+0.32}_{-0.29}\ \text{GeV}.\tag{65}$$

It is important to note that this mass splitting is rather large, an effect that should vanish in the heavy quark limit. As seen from Eq. (53), the term $\mathcal{S}$ is directly related to the mass splitting, and this term is significantly influenced by the gluon degrees of freedom. Therefore, the presence of a significant gluonic component in the charmed hybrid baryon might result in a substantial mass difference. Furthermore, as shown in Eqs. (61) and (62), both the K and Σ terms are $\mathcal{O}(1/m_Q)$ corrections, and their magnitudes are of the same order, indicating that the observed mass splitting is consistent with the $\mathcal{O}(1/m_Q)$ correction scale. The effects of these gluonic contributions will be addressed further in future studies to quantify their impact more rigorously.

Similarly, we apply the QCD sum rule method to study the other currents belonging to the multiplets $[\bar{3}_f, \bar{3}_c, 0, 1, G]$, $[6_f, \bar{3}_c, 1, 1, G]$, $[6_f, \bar{3}_c, 1, 2, G]$, and $[6_f, \bar{3}_c, 1, 0, G]$. The obtained results are summarized in Table I, where the current $J_{6_f,\bar{3}_c,1,1,1/2^+}$ belonging to the



(a) (b)

FIG. 5. Variations of M_+ as functions of (a) the threshold value ω_c and (b) the Borel mass T. In the left panel the long-dashed, solid, and short-dashed curves correspond to fixed Borel masses $T = 0.37$ GeV, 0.38 GeV, and 0.40 GeV, respectively. In the right panel the long-dashed, solid, and short-dashed curves correspond to fixed threshold values $\omega_c = 2.85$ GeV, 2.95 GeV, and 3.05 GeV, respectively. These results are obtained using the current $J_{\Sigma_{cg}^+,\bar{3}c,1,1,1/2^+}$, which is associated with the doublet $[\Sigma_{cg}^+, \bar{3}_c, 1, 1, G]$.

doublet $[6_f, \bar{3}_c, 1, 1, G]$ couples to the lowest-lying charmed hybrid baryon. A point of confusion arises because this current exhibits a symmetric flavor structure for the two light quarks, which contrasts with the flavor structure of the lowest-lying charmed baryon Λ_{cg}, but is consistent with the flavor structure of Σ_{cg}. To explain the possible reasons, we notice that there are two states with the flavor structure $\bar{3}_f$, but five with 6_f. Hence, if the mass of one of the five states with 6_f were to decrease, it would become possible for this state to become the lowest-lying one, leading to the situation where the lowest-lying charmed baryons are Σ_{cg} rather than Λ_{cg}. Indeed, one can observe in Table I that only one Σ_{cg} state is lower than the $\Lambda_{cg}(1/2^+)$ state belonging to the $[\bar{3}_f, \bar{3}_c, 0, 1, G]$ doublet, while all the others are higher than this $\Lambda_{cg}(1/2^+)$ state.

Additionally, we have examined the charmed hybrid baryons belonging to the other four multiplets —$[6_f, 6_c, 0, 1, G]$, $[\bar{3}_f, 6_c, 1, 0, G]$, $[\bar{3}_f, 6_c, 1, 1, G]$, and

TABLE I. Parameters of the charmed hybrid baryons belonging to the multiplets $[\bar{3}_f, \bar{3}_c, 0, 1, G]$, $[6_f, \bar{3}_c, 1, 0, G]$, $[6_f, \bar{3}_c, 1, 1, G]$, and $[6_f, \bar{3}_c, 1, 2, G]$, calculated using the QCD sum rule method within the framework of heavy quark effective theory.

Multiplets	ω_c (GeV)	Working region (GeV)	$\bar{\Lambda}$ (GeV)	State	Mass (GeV)	Difference (GeV)	Decay constant (GeV^4)
$[\bar{3}_f, \bar{3}_c, 0, 1, G]$	3.20	$0.400 \le T \le 0.410$	$2.72^{+0.17}_{-0.15}$	$\Lambda_{cg}(1/2^+)$	$4.74^{+0.23}_{-0.20}$	$0.62^{+0.12}_{-0.10}$	$0.155^{+0.031}_{-0.024}$
				$\Lambda_{cg}(3/2^+)$	$5.36^{+0.30}_{-0.26}$		$0.045^{+0.009}_{-0.007}$
	3.45	$0.409 \le T \le 0.437$	$2.76^{+0.11}_{-0.09}$	$\Xi_{cg}(1/2^+)$	$4.91^{+0.15}_{-0.14}$	$0.54^{+0.10}_{-0.08}$	$0.176^{+0.023}_{-0.019}$
				$\Xi_{cg}(3/2^+)$	$5.45^{+0.20}_{-0.17}$		$0.051^{+0.007}_{-0.005}$
$[6_f, \bar{3}_c, 1, 0, G]$	2.75	$0.394 \le T \le 0.396$	2.25 ± 0.12	$\Sigma_{cg}(1/2^+)$	$4.75^{+0.21}_{-0.20}$	⋯	$0.091^{+0.015}_{-0.013}$
	3.00	$0.373 \le T \le 0.420$	$2.30^{+0.14}_{-0.07}$	$\Xi'_{cg}(1/2^+)$	$4.88^{+0.20}_{-0.13}$	⋯	$0.104^{+0.021}_{-0.011}$
	3.25	$0.357 \le T \le 0.442$	$2.41^{+0.15}_{-0.08}$	$\Omega_{cg}(1/2^+)$	$5.09^{+0.21}_{-0.14}$	⋯	$0.183^{+0.041}_{-0.019}$
$[6_f, \bar{3}_c, 1, 1, G]$	2.95	$0.368 \le T \le 0.398$	$2.46^{+0.15}_{-0.14}$	$\Sigma_{cg}(1/2^+)$	$3.36^{+0.27}_{-0.26}$	$2.16^{+0.32}_{-0.29}$	$0.079^{+0.015}_{-0.013}$
				$\Sigma_{cg}(3/2^+)$	$5.51^{+0.29}_{-0.26}$		$0.046^{+0.009}_{-0.007}$
	3.20	$0.351 \le T \le 0.423$	2.50 ± 0.08	$\Xi'_{cg}(1/2^+)$	3.60 ± 0.20	$2.15^{+0.30}_{-0.27}$	0.090 ± 0.010
				$\Xi'_{cg}(3/2^+)$	$5.74^{+0.21}_{-0.19}$		0.052 ± 0.006
	3.45	$0.351 \le T \le 0.447$	2.62 ± 0.08	$\Omega_{cg}(1/2^+)$	3.82 ± 0.21	$2.14^{+0.30}_{-0.27}$	$0.158^{+0.018}_{-0.017}$
				$\Omega_{cg}(3/2^+)$	$5.96^{+0.22}_{-0.19}$		$0.091^{+0.010}_{-0.010}$
$[6_f, \bar{3}_c, 1, 2, G]$	3.50	$0.418 \le T \le 0.429$	$2.98^{+0.16}_{-0.14}$	$\Sigma_{cg}(3/2^+)$	$6.63^{+0.36}_{-0.30}$	$-1.39^{+0.21}_{-0.23}$	$0.364^{+0.063}_{-0.051}$
				$\Sigma_{cg}(5/2^+)$	$5.24^{+0.25}_{-0.22}$		$0.123^{+0.021}_{-0.017}$
	3.75	$0.428 \le T \le 0.457$	$3.04^{+0.10}_{-0.09}$	$\Xi'_{cg}(3/2^+)$	$6.76^{+0.28}_{-0.24}$	$-1.35^{+0.18}_{-0.20}$	$0.422^{+0.052}_{-0.043}$
				$\Xi'_{cg}(5/2^+)$	$5.42^{+0.17}_{-0.16}$		$0.143^{+0.018}_{-0.015}$
	4.00	$0.434 \le T \le 0.484$	3.16 ± 0.09	$\Omega_{cg}(3/2^+)$	$6.99^{+0.28}_{-0.24}$	$-1.31^{+0.18}_{-0.20}$	$0.740^{+0.080}_{-0.073}$
				$\Omega_{cg}(5/2^+)$	$5.69^{+0.17}_{-0.16}$		$0.250^{+0.027}_{-0.025}$

TABLE II. Possible decay channels of the charmed hybrid baryons belonging to the doublet $[6_f, \bar{3}_c, 1, 1, G]$.

States	Decay patterns
$\Sigma_{cg}(1/2^+)$	ND, ΣD_s, ND^*, ΣD_s^*, ΔD, $\Sigma^* D_s$
$\Xi'_{cg}(1/2^+)$	ΛD, ΞD_s, ΛD^*, ΞD_s^*, $\Xi^* D_s$
$\Omega_{cg}(1/2^+)$	ΞD, ΞD^*, $\Xi^* D$, ΩD_s, $\Xi^* D^*$

$[\bar{3}_f, 6_c, 1, 2, G]$—as well as those with the negative parity. We find that they have significantly larger masses, primarily because the relevant two-point correlation functions are negative and thus nonphysical when ω is not sufficiently large. These results will be discussed in detail in our future studies.

Before concluding this section, we would like to note that in this calculation, we have focused on the $\mathcal{O}(1/m_Q)$ corrections, particularly those related to the propagators in the QCD sum rules, as described above. However, the $\mathcal{O}(1/m_Q)$ corrections also affect the interpolating currents, known as the flow corrections, whose effects are not considered in the present study. Although the flow corrections are of the same order, their effects are generally smaller and mainly affect the fine details of the coupling constants and Dirac structures. Therefore, they are expected to have a relatively smaller effect compared to the propagator corrections [64,65]. For simplicity, we have neglected these corrections in the current analysis, but they will be systematically explored in future studies.

V. DISCUSSIONS AND CONCLUSION

In recent years substantial theoretical and experimental efforts have been devoted to the study of glueballs and hybrid mesons. However, their internal structures remain elusive, underscoring the necessity for continued investigations—particularly into the comparatively less-explored hybrid baryons. In this paper we employ the QCD sum rule method within the framework of heavy quark effective theory to investigate charmed hybrid baryons with various quantum numbers. We systematically construct 28 interpolating currents for these baryons and utilize seven of them to perform numerical analyses. These seven currents form the four baryon multiplets $[\bar{3}_f, \bar{3}_c, 0, 1, G]$, $[6_f, \bar{3}_c, 1, 0, G]$, $[6_f, \bar{3}_c, 1, 1, G]$, and $[6_f, \bar{3}_c, 1, 2, G]$. Altogether, we compute the masses of 19 states with quark-gluon configurations $qqcg$, $qscg$, and $sscg$ ($q = u/d$). The resulting mass predictions are summarized in Table I.

In particular, the charmed hybrid baryons belonging to the doublet $[6_f, \bar{3}c, 1, 1, G]$ comprise six states: $\Sigma_{cg}(1/2^+)$, $\Sigma_{cg}(3/2^+)$, $\Xi'_{cg}(1/2^+)$, $\Xi'_{cg}(3/2^+)$, $\Omega_{cg}(1/2^+)$, and $\Omega_{cg}(3/2^+)$. Among these, the three spin-1/2 states are predicted to be the lowest in mass:

$$\begin{aligned} M_{\Sigma_{cg}(1/2^+)} &= 3.36^{+0.27}_{-0.26}\ \text{GeV}, \\ M_{\Xi'_{cg}(1/2^+)} &= 3.59 \pm 0.20\ \text{GeV}, \\ M_{\Omega_{cg}(1/2^+)} &= 3.82 \pm 0.21\ \text{GeV}. \end{aligned} \tag{66}$$

It is important to note that all these states have positive parity. Recalling the case of hybrid mesons, the MIT bag model [66,67] suggested that the lightest supermultiplet states correspond to those with $J^{PC} = 1^{--}$, 0^{-+}, 1^{-+}, and 2^{-+}, all of which have negative parity, suggesting it is possible for the lightest hybrid baryons to have positive parity. However, we must acknowledge that the situation for hybrid baryons is quite complex due to the interplay between quark-gluon dynamics and the QCD vacuum structure, so the presence of gluonic excitations in hybrid baryons has the potential to lead to a different parity pattern compared to the naive constituent parton model, in which gluonic degrees of freedom are not explicitly considered. These hybrid states still involve many uncertainties, and thus, extensive further research is needed to fully understand their properties.

Charmed hybrid baryons can decay via the excitation of a color-octet $\bar{q}q$ or $\bar{s}s$ pair from the valence gluon, followed by the recombination of the remaining color-octet clusters —qqc, qsc, or ssc—with the excited $\bar{q}q$ or $\bar{s}s$ pair into color-singlet baryons and mesons. Representative decay channels for the charmed hybrid baryons belonging to the doublet $[6_f, \bar{3}_c, 1, 1, G]$ are listed in Table II, providing viable avenues for their experimental identification in future studies.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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