

Excited Ω hyperon in charmful Ω_b weak decays

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We investigate the sextet b -baryon decay processes $\Omega_b \rightarrow J/\psi \Omega^{(*)}$, where Ω^* represents the $1P$ -, $1D$ - and $2S$ -wave excited Ω hyperons in the spectroscopy. Using the constituent quark model, we obtain $\mathcal{B}(\Omega_b \rightarrow J/\psi \Omega) = 8.8 \times 10^{-4}$, which agrees with the previous studies to the order of magnitude. By identifying $\Omega(2102)$ as $\Omega(1^2P_{3/2-})$, $\mathcal{B}(\Omega_b \rightarrow J/\psi \Omega(2102)) = 1.1 \times 10^{-3}$ can be similarly significant. Additionally, $\Omega(1^2D_{5/2+})$ and $\Omega(1^4D_{3/2+}, 1^4D_{5/2+})$ states exhibit production rates of 0.5, and (0.6, 0.8), respectively, relative to their ground-state counterpart. Notably, our findings suggest that $\mathcal{B}(\Omega_b \rightarrow J/\psi \Omega(2^2S_{1/2+}, 2^4S_{3/2+}))$ are as large as $(4.5, 20) \times 10^{-4}$, making them accessible to experiments at LHCb.

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I. INTRODUCTION

In baryon spectroscopy, the formation of a colorless state from three quarks with three different colors is fundamentally linked to the principles of quantum chromodynamics (QCD) and hadron physics. While the combination of individual quark spins and orbital angular momenta is expected to produce a variety of light baryon states [1–3], only a subset of these states has been observed to date [4]. The so-called “missing excited baryon” problem highlights an incomplete theoretical understanding of baryon spectroscopy [3,5].

The Ω hyperon spectroscopy has been relatively underexplored. Nonetheless, Belle initiated a new era of discoveries with the identification of an excited hyperon, $\Omega(2102)$ [6], which was later reconfirmed through the process $\Omega_c^0 \rightarrow \pi^+ \Omega(2102)^- , \Omega(2102)^- \rightarrow \Xi^0 K^-$ [7]. Additionally, a heavier excited hyperon $\Omega(2109)^-$, observed via $e^+e^- \rightarrow \Omega(2109)^- \bar{\Omega}^+ + \text{c.c.}$, and reported by BESIII [8]. To investigate the nature of these new excited hyperons, significant theoretical attention has been devoted [9–26].

Notably, $\Omega(2102)$ is often preferred as an exotic molecule candidate [17–26] rather than as a conventional $1P$ -wave $\Omega(sss)$ state.

Clearly, the “missing excited baryon” problem persists in Ω hyperon spectroscopy, as all waves of Ω hyperons have been theoretically predicted [27–45], but only a few have been experimentally observed. These include $\Omega \equiv \Omega(1672)$ [46,47], $\Omega(2102, 2109)$ [6–8], and $\Omega(2250, 2380, 2470)$ [48–50]. Since further clarification and exploration are essential, we propose that recent measurements of the sextet b -baryon decay process $\Omega_b \rightarrow J/\psi \Omega$ [51–56] be extended to include $\Omega_b \rightarrow J/\psi \Omega^*$, where Ω^* denotes higher-wave excited Ω hyperons.

As LHCb continues to improve the statistics and precision [55,56], the absolute branching fractions of Ω_b^- decays may soon be measured with a similar accuracy to those of the antitriplet b -baryon decays [57–72]. Theoretical estimations are therefore needed, and a variety of theoretical tools are already available for such studies. These include the non-relativistic quark model [73–76], the covariant confined quark model [77–80], the covariant oscillator quark model [81], the relativistic three-quark model [82,83], the light-front quark model [84–87], the perturbative QCD approach [88,89], the generalized factorization approach [90–92], and SU(3) flavor analysis [93].

In this work, we employ the constituent quark model [94–100] to provide our estimations. As we will demonstrate, this model can be extended to study both the $\Omega_b \rightarrow J/\psi \Omega$ and $\Omega_b \rightarrow J/\psi \Omega^*$ decay processes, enabling a systematic analysis of the missing excited Ω states. This paper is organized as follows: In Sec. II, we apply the

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constituent quark model to two-body nonleptonic weak decays of $\Omega_b \rightarrow J/\psi \Omega^{(*)}$. In Sec. III, we present the numerical results. Finally, we provide our discussions and conclusion in Sec. IV.

II. FRAMEWORK

As shown in Fig. 1, the recently observed sextet b -baryon decay, $\Omega_b \rightarrow J/\psi \Omega$, proceeds exclusively via the internal W -boson emission diagram [101], where Ω is strait-forwardly formed through the $\Omega_b \rightarrow \Omega$ transition. The single decay topology, extended to $\Omega_b \rightarrow J/\psi \Omega^*$, also facilitates the investigation of the $1P$ -, $2S$ - and $1D$ -wave excited Ω^* states.

The quark-level effective Hamiltonian of $b \rightarrow c \bar{c} s$ weak transition induces the doubly charmful decay channels $\Omega_b \rightarrow J/\psi \Omega^{(*)}$, given by [102]

$$H_W = \frac{G_F}{\sqrt{2}} V_{bc} V_{cs}^* (C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2), \quad (1)$$

where G_F is the Fermi constant, C_1 and C_2 are the Wilson coefficients, and (V_{bc}, V_{cs}) are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Additionally, $\mathcal{O}_{1,2}$ are the current-current operators, which are written as

$$\begin{aligned} \mathcal{O}_1 &= \bar{\psi}_{\bar{s}\beta} \gamma_\mu (1 - \gamma_5) \psi_{c\beta} \bar{\psi}_{\bar{c}\alpha} \gamma^\mu (1 - \gamma_5) \psi_{b\alpha}, \\ \mathcal{O}_2 &= \bar{\psi}_{\bar{s}\alpha} \gamma_\mu (1 - \gamma_5) \psi_{c\beta} \bar{\psi}_{\bar{c}\beta} \gamma^\mu (1 - \gamma_5) \psi_{b\alpha}. \end{aligned} \quad (2)$$

Here, $\psi_{j\delta}$ is the j th quark field, where j can be s , c or b , and $\delta = (\alpha, \beta)$ is the color index. We define the convention for the quark and antiquark fields

$$\begin{aligned} \psi_c(x) &= \int \frac{dp}{(2\pi)^{3/2}} \left(\frac{m}{p^0} \right)^{1/2} \sum_s [u_s(p) b_{s,c}(p) e^{-ip \cdot x} \\ &\quad + v_s(p) d_{s,c}^\dagger(p) e^{ip \cdot x}], \\ \bar{\psi}_c(x) &= \int \frac{dp}{(2\pi)^{3/2}} \left(\frac{m}{p^0} \right)^{1/2} \sum_s [\bar{u}_s(p) b_{s,c}^\dagger(p) e^{ip \cdot x} \\ &\quad + \bar{v}_s(p) d_{s,c}(p) e^{-ip \cdot x}]. \end{aligned} \quad (3)$$

where c is color quantum number, and

$$\begin{aligned} u_s(p) &= \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \varphi_s \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m} \varphi_s \end{pmatrix}, \\ v_s(p) &= \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m} \chi_s \\ \chi_s \end{pmatrix}. \end{aligned} \quad (4)$$

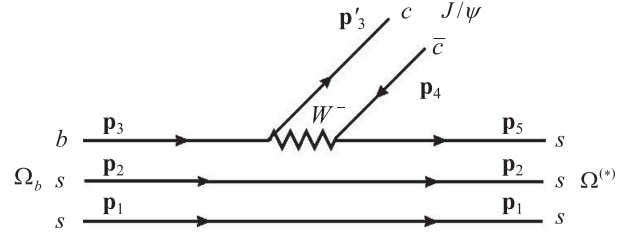


FIG. 1. Feynman diagram for the nonleptonic weak decay $\Omega_b \rightarrow J/\psi \Omega^{(*)}$.

Here, s represents spin labeling, so

$$\sum_s \varphi_s \varphi_s^\dagger = \sum_s \chi_s \chi_s^\dagger = 1. \quad (5)$$

The anticommutation relations of the creation and annihilation operators are given by

$$\begin{aligned} \{b_{s,c}(p), b_{s',c'}^\dagger(p')\} &= \{d_{s,c}(p), d_{s',c'}^\dagger(p')\} \\ &= \delta_{ss'} \delta_{cc'} \delta^3(p - p'). \end{aligned} \quad (6)$$

The normalization of spinor is

$$\bar{u}_s(p) u_{s'}(p) = -\bar{v}_s(p) v_{s'}(p) = \delta_{ss'}. \quad (7)$$

According to Refs. [103–105], $C_2 \langle J/\psi \Omega | \mathcal{O}_2 | \Omega_b \rangle$ corresponds to the internal W -boson emission diagram. On the other hand, $C_1 \langle J/\psi \Omega | \mathcal{O}_1 | \Omega_b \rangle$ is replaced by $C_1 / N_c^{\text{eff}} \langle J/\psi \Omega | \mathcal{O}_2 | \Omega_b \rangle$ in the generalized factorization, where N_c^{eff} is an effective color number that accounts for the nonfactorizable QCD corrections. In the large N_c^{eff} limit ($N_c^{\text{eff}} \rightarrow \infty$) [103–105], the contribution from $C_1 \mathcal{O}_1$ becomes negligible. Therefore, we only consider $C_2 \mathcal{O}_2$ in this work.

The constituent quark model separates H_W into the parity-conserving (PC) and parity-violating (PV) components [94]

$$H_W = H_W^{\text{PC}} + H_W^{\text{PV}}. \quad (8)$$

In the nonrelativistic approximation, the two components are given by [94–96]

$$\begin{aligned}
H_W^{PC} &\simeq \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb} C_2 \frac{\hat{\phi}_c \hat{\mathcal{O}}_f}{(2\pi)^3} \delta^3(\mathbf{p}_3 - \mathbf{p}_3' - \mathbf{p}_4 - \mathbf{p}_5) \left\{ \sigma_4 \cdot \left(\frac{\mathbf{p}_5}{2m_5} + \frac{\mathbf{p}_4}{2m_4} \right) + \sigma_3 \cdot \left(\frac{\mathbf{p}_3'}{2m_3'} + \frac{\mathbf{p}_3}{2m_3} \right) \right. \\
&\quad \left. - \left[\left(\frac{\mathbf{p}_3'}{2m_3'} + \frac{\mathbf{p}_3}{2m_3} \right) - i\sigma_3 \left(\frac{\mathbf{p}_3}{2m_3} - \frac{\mathbf{p}_3'}{2m_3'} \right) \right] \cdot \sigma_4 - \sigma_3 \cdot \left[\left(\frac{\mathbf{p}_5}{2m_5} + \frac{\mathbf{p}_4}{2m_4} \right) - i\sigma_4 \times \left(\frac{\mathbf{p}_4}{2m_4} - \frac{\mathbf{p}_5}{2m_5} \right) \right] \right\}, \\
H_W^{PV} &\simeq \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb} C_2 \frac{\hat{\phi}_c \hat{\mathcal{O}}^f}{(2\pi)^3} \delta^3(\mathbf{p}_3 - \mathbf{p}_3' - \mathbf{p}_4 - \mathbf{p}_5) (\sigma_3 \cdot \sigma_4 - 1),
\end{aligned} \tag{9}$$

where $\mathbf{p}_j$ and m_j , as assigned in Fig. 1, are the momentum and mass of the j th quark, respectively, and σ_j is the spin operator of the j th quark. Additionally, $\hat{\phi}_c$ is the color operator and $\hat{\mathcal{O}}_f$ is the flavor operator.

Explicitly, we present

$$\begin{aligned}
\hat{\mathcal{O}}_f &= \hat{b}_{3'}^\dagger(c) \hat{b}_4^\dagger(\bar{c}) \hat{b}_5^\dagger(s) \hat{b}_3(b), \\
\hat{\phi}_c &= \delta_{c_4 c_3'} \delta_{c_5 c_3},
\end{aligned} \tag{10}$$

where $\hat{b}^{(\dagger)}(q)$ annihilates (creates) the quark (q), and the $\delta_{c_5 c_3} (\delta_{c_4 c_3'})$ represents that the colors of $c_3(c_3')$ and $c_5(c_4)$ should be conserved.

For the color operator to act on the color wave functions of Ω_b , $\Omega^{(*)}$, and J/ψ , we define

$$\begin{aligned}
|\zeta_{\Omega_b}\rangle &= \frac{1}{\sqrt{6}} (RGB - RBG + GBR - GRB + BRG - BGR), \\
|\zeta_{\Omega^{(*)}}\rangle &= |\zeta_{\Omega_b}\rangle, \\
|\zeta_{J/\psi}\rangle &= \frac{1}{\sqrt{3}} (R\bar{R} + G\bar{G} + B\bar{B}).
\end{aligned} \tag{11}$$

From Eqs. (10) and (11), we obtain

$$\langle \zeta_{J/\psi} \zeta_{\Omega^{(*)}} | \hat{\phi}_c | \zeta_{\Omega_b} \rangle = \sqrt{3}. \tag{12}$$

In the constituent quark model, the decay amplitude is represented as

$$\mathcal{M}(\Omega_b \rightarrow J/\psi \Omega^{(*)}) = \mathcal{M}^{PC} + \mathcal{M}^{PV}, \tag{13}$$

where

$$\begin{aligned}
\mathcal{M}^{PC(PV)} &= \frac{1}{\sqrt{3}} \langle J/\psi(\mathbf{q}; 1, J_{J/\psi}^z) \Omega^{(*)}(\mathbf{P}_f; J_f, J_f^z) | \\
&\quad (1 + \hat{P}_{25} + \hat{P}_{15}) H_W^{PC(PV)} | \Omega_b(\mathbf{P}_i; J_i, J_i^z) \rangle.
\end{aligned} \tag{14}$$

Here, the factor $\frac{1}{\sqrt{3}}$ is the normalization coefficient and $\hat{P}_{ij}$ represents the permutation operator of quarks i and j . In the above equation, $J/\psi(\mathbf{q}; 1, J_{J/\psi}^z)$, $\Omega_b(\mathbf{P}_i; J_i, J_i^z)$, and $\Omega^{(*)}(\mathbf{P}_f; J_f, J_f^z)$ are wave functions, where $\mathbf{q}$ and $\mathbf{P}_{i(f)}$ are the total momentum, $J_{i(f)}$ are the total angular momentum, and $J_{i(f)}^z$ are the third component of the total angular momentum.

It is worth noting that the calculation of the amplitudes for $\Omega_b \rightarrow \Omega^{(*)} J/\psi$ decays should consider a question of symmetry. The flavor wave functions of Ω_b and Ω are $\varphi_{\Omega_b} = ssb$ and $\varphi_{\Omega} = sss$, respectively. In the $\Omega_b \rightarrow \Omega^{(*)} J/\psi$ nonleptonic decay process, the bottom quark of the Ω_b decays into an s quark, which can be any of the three s quarks in the Ω . Since the wave functions of the final state $\Omega^{(*)}$ baryon are fully antisymmetric under the exchange of any two quarks, so we obtain

$$\begin{aligned}
&\langle J/\psi(\mathbf{q}; 1, J_{J/\psi}^z) \Omega^{(*)}(\mathbf{P}_f; J_f, J_f^z) | H_W^{PC(PV)} | \Omega_b(\mathbf{P}_i; J_i, J_i^z) \rangle \\
&= \langle J/\psi(\mathbf{q}; 1, J_{J/\psi}^z) \Omega^{(*)}(\mathbf{P}_f; J_f, J_f^z) | \hat{P}_{25} H_W^{PC(PV)} | \Omega_b(\mathbf{P}_i; J_i, J_i^z) \rangle \\
&= \langle J/\psi(\mathbf{q}; 1, J_{J/\psi}^z) \Omega^{(*)}(\mathbf{P}_f; J_f, J_f^z) | \hat{P}_{15} H_W^{PC(PV)} | \Omega_b(\mathbf{P}_i; J_i, J_i^z) \rangle.
\end{aligned} \tag{15}$$

Therefore, one can obtain

$$\begin{aligned}
\mathcal{M}^{PC(PV)} &= \sqrt{3} \langle J/\psi(\mathbf{q}; 1, J_{J/\psi}^z) \Omega^{(*)}(\mathbf{P}_f; J_f, J_f^z) | \\
&\quad H_W^{PC(PV)} | \Omega_b(\mathbf{P}_i; J_i, J_i^z) \rangle.
\end{aligned} \tag{16}$$

Similar calculations have been applied in Ref. [99].

To calculate $\mathcal{M}^{PC(PV)}$ in Eq. (16), the details of the wave functions are required. The total wave function of a baryon

system should include four parts: a color wave function ζ , a flavor wave function ϕ , a spin wave function χ , and a spatial wave function ψ . The spin wave function of a baryon satisfies SU(2) symmetry, which can be expressed as symmetric ($\chi_{S_z}^S$), mixed antisymmetric ($\chi_{S_z}^{\rho}$), mixed symmetric ($\chi_{S_z}^{\lambda}$) spin wave functions. The detailed representations of the spin wave function is shown in Refs. [94,106,107].

TABLE I. The spin-flavor-space wave-functions of Ω baryons under $SU(6)$ quark model classification are listed below. We denote the baryon states as $|N_6, {}^{2S+1}N_3, N, L, J^P\rangle$ where N_6 stands for the irreducible representation of spin-flavor $SU(6)$ group, N_3 stands for the irreducible representation of flavor $SU(3)$ group and N, L, J^P as principal quantum number, total orbital angular momentum and spin-parity, respectively [9–11]. The ϕ, χ, ψ denote flavor, spin and spatial wave function, respectively. The Clebsch-Gorden coefficients of spin-orbital coupling and color wave function ζ_Ω have been omitted.

$n^{2S+1}L_{J^P}$	$ N_6, {}^{2S+1}N_3, N, L, J^P\rangle$	Wave function
$1^4S_{\frac{3}{2}^+}$	$ 56, {}^4 10, 0, 0, \frac{3}{2}^+\rangle$	$\phi^s \chi^s \psi_{000}^s$
$1^2P_{\frac{1}{2}^-}$	$ 70, {}^2 10, 1, 1, \frac{1}{2}^-\rangle$	$\frac{1}{\sqrt{2}}(\phi^s \chi^\rho \psi_{11L_z}^\rho + \phi^s \chi^\lambda \psi_{11L_z}^\lambda)$
$1^2P_{\frac{3}{2}^-}$	$ 70, {}^2 10, 1, 1, \frac{3}{2}^-\rangle$	$\frac{1}{\sqrt{2}}(\phi^s \chi^\rho \psi_{11L_z}^\rho + \phi^s \chi^\lambda \psi_{11L_z}^\lambda)$
$2^2S_{\frac{1}{2}^+}$	$ 70, {}^2 10, 2, 0, \frac{1}{2}^+\rangle$	$\frac{1}{\sqrt{2}}(\phi^s \chi^\rho \psi_{200}^\rho + \phi^s \chi^\lambda \psi_{200}^\lambda)$
$2^4S_{\frac{3}{2}^+}$	$ 56, {}^4 10, 2, 0, \frac{3}{2}^+\rangle$	$\phi^s \chi^s \psi_{200}^s$
$1^2D_{\frac{3}{2}^+}$	$ 70, {}^2 10, 2, 2, \frac{3}{2}^+\rangle$	$\frac{1}{\sqrt{2}}(\phi^s \chi^\rho \psi_{22L_z}^\rho + \phi^s \chi^\lambda \psi_{22L_z}^\lambda)$
$1^2D_{\frac{5}{2}^+}$	$ 70, {}^2 10, 2, 2, \frac{5}{2}^+\rangle$	$\frac{1}{\sqrt{2}}(\phi^s \chi^\rho \psi_{22L_z}^\rho + \phi^s \chi^\lambda \psi_{22L_z}^\lambda)$
$1^4D_{\frac{1}{2}^+}$	$ 56, {}^4 10, 2, 2, \frac{1}{2}^+\rangle$	$\phi^s \chi^s \psi_{22L_z}^s$
$1^4D_{\frac{3}{2}^+}$	$ 56, {}^4 10, 2, 2, \frac{3}{2}^+\rangle$	$\phi^s \chi^s \psi_{22L_z}^s$
$1^4D_{\frac{5}{2}^+}$	$ 56, {}^4 10, 2, 2, \frac{5}{2}^+\rangle$	$\phi^s \chi^s \psi_{22L_z}^s$
$1^4D_{\frac{7}{2}^+}$	$ 56, {}^4 10, 2, 2, \frac{7}{2}^+\rangle$	$\phi^s \chi^s \psi_{22L_z}^s$

For the Ω spectrum, the flavor-spin wave functions are representations of $SU(6)$, which are denoted by $|N_6, {}^{2S+1}N_3\rangle$, where N_6 (N_3) represents the $SU(6)$ ($SU(3)$) representation and S stands for the total spin of the wave function, and the detailed wave function of $\Omega^{(*)}$ excited states are shown in Table I. In momentum space, we present the simple harmonic oscillator wave functions to describe the $\Omega^{(*)}$ baryon [11]. The explicit forms of the spatial wave function $\Psi_{NLM_L}^\sigma(\mathbf{p}_\rho, \mathbf{p}_\lambda)$ in the momentum space, up to the $N = 2$ shell, can be found in Table II. The superscript σ characterizes the same set of quantum numbers (N, L, M_L) arising from different combinations of the ρ and λ oscillation systems [106,108,109]. Specifically, the combinations are defined by $N = 2(n_\rho + n_\lambda) + l_\rho + l_\lambda$, $L = l_\rho + l_\lambda, \dots, |l_\rho - l_\lambda|$, and $M_L = m_\rho + m_\lambda$, where (n_i, l_i, m_i) with $i = \rho$ or λ are the principal, orbital, and magnetic quantum numbers, respectively. The internal momenta of the ρ - and λ -oscillators, $\mathbf{p}_\rho$ and $\mathbf{p}_\lambda$, are expressed as

$$\begin{aligned} \mathbf{p}_\rho &= \frac{\sqrt{2}}{2}(\mathbf{p}_1 - \mathbf{p}_2), \\ \mathbf{p}_\lambda &= \frac{\sqrt{6}}{2} \frac{m_5(\mathbf{p}_1 + \mathbf{p}_2) - (m_1 + m_2)\mathbf{p}_5}{m_1 + m_2 + m_5}, \end{aligned} \quad (17)$$

respectively.

In momentum space, $\psi_{nlm}(\mathbf{p})$ is expressed by

$$\begin{aligned} \psi_{nlm}(\mathbf{p}) &= (i)^l (-1)^n \left[\frac{2n!}{(n+l+1/2)!} \right]^{1/2} \frac{1}{\alpha^{l+3/2}} \\ &\times \exp\left(-\frac{\mathbf{p}^2}{2\alpha^2}\right) L_n^{l+1/2}(\mathbf{p}^2/\alpha^2) \mathcal{Y}_{lm}(\mathbf{p}), \end{aligned} \quad (18)$$

where the l th solid harmonic polynomial is defined as $\mathcal{Y}_{lm}(\mathbf{p}) = |\mathbf{p}|^l Y_{lm}(\hat{\mathbf{p}})$, and the oscillator parameter α can be either α_ρ or α_λ . For the $\Omega^{(*)}$ wave function, we define $\alpha \equiv \alpha_\rho = \alpha_\lambda$. In this work, we use the α value obtained from single Gaussian fitting in Ref. [10].

For Ω_b baryon, the total wave function can be represented as

$$\Psi_{\Omega_b}(\mathbf{p}_\rho, \mathbf{p}_\lambda) = \zeta_{\Omega_b} \varphi_{\Omega_b} \chi_{s_z}^\lambda \psi_{000}^S(\mathbf{p}_\rho, \mathbf{p}_\lambda), \quad (19)$$

where ζ_{Ω_b} , $\chi_{s_z}^\lambda$, and $\psi_{000}^S(\mathbf{p}_\rho, \mathbf{p}_\lambda)$ represent the color, spin, and spatial wave functions, respectively. It is worth noting that in momentum space, we employ the simple harmonic oscillator wave functions to describe $\psi_{000}^S(\mathbf{p}_\rho, \mathbf{p}_\lambda)$, which is represented as

$$\psi_{000}^S(\mathbf{p}_\rho, \mathbf{p}_\lambda) = \psi_{000}(\mathbf{p}_\rho) \psi_{000}(\mathbf{p}_\lambda) \quad (20)$$

where $\mathbf{p}_\rho$ and $\mathbf{p}_\lambda$ are the internal momenta of the ρ - and λ -oscillators, respectively. It is worth mentioning that the parameter α_λ of Ω_b wave function is given by $\alpha_\lambda = [(3m_b)/(2m_s + m_b)]^{1/4} \alpha_\rho$ [107,110].

For the wave function of J/ψ meson, which can be represented as

$$\Psi(\mathbf{p}'_3, \mathbf{p}_4) = \zeta_{J/\psi} \varphi_{J/\psi} \chi_{s_z}^1 \psi(\mathbf{p}'_3, \mathbf{p}_4), \quad (21)$$

where $\zeta_{J/\psi}$ is the color wave function, $\varphi_{J/\psi}$, $\chi_{s_z}^1$, and $\psi(\mathbf{p}'_3, \mathbf{p}_4)$ represent the flavor, spin, and spatial wave functions, respectively, which are given as

$$\begin{aligned} \chi_{1,0,-1}^1 &= (\uparrow\uparrow, (\uparrow\downarrow + \downarrow\uparrow)/\sqrt{2}, \downarrow\downarrow), \\ \varphi_{J/\psi} &= c\bar{c}, \\ \psi(\mathbf{p}'_3, \mathbf{p}_4) &= \frac{1}{\pi^{3/4} \beta^{3/2}} \exp\left[-\frac{(\mathbf{p}'_3 - \mathbf{p}_4)^2}{8\beta^2}\right]. \end{aligned} \quad (22)$$

In Eq. (22), s_z denotes the z component of the J/ψ spin and β in $\psi(\mathbf{p}'_3, \mathbf{p}_4)$ controls the width of Gaussian distribution in momentum space.

To convert $\mathcal{M}(\Omega_b \rightarrow J/\psi \Omega^{(*)})$ from Eqs. (13) and (14) into the decay width, we use the following equation:

$$\Gamma = 8\pi^2 \frac{|\mathbf{q}| E_{J/\psi} E_{\Omega^{(*)}}}{M_{\Omega_b}} \frac{1}{2J_{\Omega_b} + 1} \sum_{J_i^z, J_f^z} |\mathcal{M}_{J_i^z, J_f^z}|^2, \quad (23)$$

TABLE II. The spatial functions $\psi_{NLM_L}^{\sigma}(\mathbf{p}_\rho, \mathbf{p}_\lambda)$ as the linear combination of $\psi_{n_\rho l_\rho m_\rho}(\mathbf{p}_\rho) \psi_{n_\lambda l_\lambda m_\lambda}(\mathbf{p}_\lambda)$.

$$\begin{aligned}
\psi_{000}^S(\mathbf{p}_\rho, \mathbf{p}_\lambda) &= \psi_{000}(\mathbf{p}_\rho) \psi_{000}(\mathbf{p}_\lambda) \\
\psi_{11M_L}^{\rho}(\mathbf{p}_\rho, \mathbf{p}_\lambda) &= \psi_{01M_L}(\mathbf{p}_\rho) \psi_{000}(\mathbf{p}_\lambda) \\
\psi_{11M_L}^{\lambda}(\mathbf{p}_\rho, \mathbf{p}_\lambda) &= \psi_{000}(\mathbf{p}_\rho) \psi_{01M_L}(\mathbf{p}_\lambda) \\
\psi_{200}^S(\mathbf{p}_\rho, \mathbf{p}_\lambda) &= \frac{1}{\sqrt{2}} [\psi_{100}(\mathbf{p}_\rho) \psi_{000}(\mathbf{p}_\lambda) + \psi_{000}(\mathbf{p}_\rho) \psi_{100}(\mathbf{p}_\lambda)] \\
\psi_{200}^{\lambda}(\mathbf{p}_\rho, \mathbf{p}_\lambda) &= \frac{1}{\sqrt{2}} [-\psi_{100}(\mathbf{p}_\rho) \psi_{000}(\mathbf{p}_\lambda) + \psi_{000}(\mathbf{p}_\rho) \psi_{100}(\mathbf{p}_\lambda)] \\
&\quad \psi_{200}^{\rho}(\mathbf{p}_\rho, \mathbf{p}_\lambda) \\
&= \frac{1}{\sqrt{3}} [\psi_{011}(\mathbf{p}_\rho) \psi_{01-1}(\mathbf{p}_\lambda) - \psi_{010}(\mathbf{p}_\rho) \psi_{010}(\mathbf{p}_\lambda) + \psi_{01-1}(\mathbf{p}_\rho) \psi_{011}(\mathbf{p}_\lambda)] \\
\psi_{22M_L}^S(\mathbf{p}_\rho, \mathbf{p}_\lambda) &= \frac{1}{\sqrt{2}} [\psi_{02M_L}(\mathbf{p}_\rho) \psi_{000}(\mathbf{p}_\lambda) + \psi_{000}(\mathbf{p}_\rho) \psi_{02M_L}(\mathbf{p}_\lambda)] \\
\psi_{22M_L}^{\lambda}(\mathbf{p}_\rho, \mathbf{p}_\lambda) &= \frac{1}{\sqrt{2}} [\psi_{02M_L}(\mathbf{p}_\rho) \psi_{000}(\mathbf{p}_\lambda) - \psi_{000}(\mathbf{p}_\rho) \psi_{02M_L}(\mathbf{p}_\lambda)] \\
&\quad \psi_{222}^{\rho}(\mathbf{p}_\rho, \mathbf{p}_\lambda) = \psi_{011}(\mathbf{p}_\rho) \psi_{011}(\mathbf{p}_\lambda) \\
\psi_{221}^{\rho}(\mathbf{p}_\rho, \mathbf{p}_\lambda) &= \frac{1}{\sqrt{2}} [\psi_{010}(\mathbf{p}_\rho) \psi_{011}(\mathbf{p}_\lambda) + \psi_{011}(\mathbf{p}_\rho) \psi_{010}(\mathbf{p}_\lambda)] \\
&\quad \psi_{220}^{\rho}(\mathbf{p}_\rho, \mathbf{p}_\lambda) \\
&= \frac{1}{\sqrt{6}} [\psi_{01-1}(\mathbf{p}_\rho) \psi_{011}(\mathbf{p}_\lambda) + 2\psi_{010}(\mathbf{p}_\rho) \psi_{010}(\mathbf{p}_\lambda) + \psi_{011}(\mathbf{p}_\rho) \psi_{01-1}(\mathbf{p}_\lambda)] \\
\psi_{22-1}^{\rho}(\mathbf{p}_\rho, \mathbf{p}_\lambda) &= \frac{1}{\sqrt{2}} [\psi_{01-1}(\mathbf{p}_\rho) \psi_{010}(\mathbf{p}_\lambda) + \psi_{010}(\mathbf{p}_\rho) \psi_{01-1}(\mathbf{p}_\lambda)] \\
\psi_{22-2}^{\rho}(\mathbf{p}_\rho, \mathbf{p}_\lambda) &= \psi_{01-1}(\mathbf{p}_\rho) \psi_{01-1}(\mathbf{p}_\lambda)
\end{aligned}$$

which is applied in our numerical analysis. In Eq. (23), $\mathcal{M}_{J_i^{\tilde{c}}, J_f^{\tilde{c}}}$ is the transition amplitude.

III. NUMERICAL RESULTS

For our numerical analysis, we adopt the CKM matrix elements and the masses of Ω_b and J/ψ from PDG [4], while the quark masses are taken from [107], as follows:

$$\begin{aligned}
(V_{cb}, V_{cs}) &= (0.042, 0.987), \\
(m_{\Omega_b}, m_{\Omega}, m_{J/\psi}) &= (6.046, 1.672, 3.0969) \text{ GeV}, \\
(m_s, m_c, m_b) &= (0.45, 1.48, 5.0) \text{ GeV}.
\end{aligned} \tag{24}$$

TABLE III. Our results for $\Omega_b \rightarrow J/\psi \Omega^{(*)}$ using the constituent model, with the quantum numbers $n^{2S+1}J^P$ assigned to $\Omega^{(*)}$. The parameter α and the mass of $\Omega^{(*)}$ M_f are both given in units of MeV; Γ and $\mathcal{B}$ are in units of 10^{-17} GeV and 10^{-4} , respectively.

$\Omega^{(*)}$ hyperon	α [10]	M_f [10]	Γ	$\mathcal{B}$	$\frac{\mathcal{B}(\Omega_b^- \rightarrow J/\psi \Omega^{(*)})}{\mathcal{B}(\Omega_b^- \rightarrow J/\psi \Omega^-)}$
$\Omega(1^4S_{\frac{3}{2}^+})$	440	1672	36	8.8	1.00
$\Omega(1^2P_{\frac{1}{2}^-})$	428	1957	17	4.2	0.48
$\Omega(1^2P_{\frac{3}{2}^-})$	411	2012	42	11	1.25
$\Omega(2^2S_{\frac{1}{2}^+})$	387	2232	18	4.5	0.51
$\Omega(2^4S_{\frac{3}{2}^+})$	381	2159	79	20	2.3
$\Omega(1^2D_{\frac{3}{2}^+})$	394	2245	7.5	1.9	0.22
$\Omega(1^2D_{\frac{5}{2}^+})$	380	2303	17	4.2	0.48
$\Omega(1^4D_{\frac{1}{2}^+})$	413	2141	9.3	2.3	0.26
$\Omega(1^4D_{\frac{3}{2}^+})$	399	2188	21	5.4	0.61
$\Omega(1^4D_{\frac{5}{2}^+})$	383	2252	29	7.2	0.81
$\Omega(1^4D_{\frac{7}{2}^+})$	367	2321	14	3.3	0.38

The lifetime of the Ω_b state is also taken from PDG [4]: $\tau_{\Omega_b} = 1.64 \times 10^{-12}$ s. The Wilson coefficient $C_2 = -0.365$ is from [86,111,112]. For the harmonic oscillator parameters, we use $(\alpha_\lambda, \alpha_\rho) = (0.56, 0.44)$ GeV for Ω_b [113], and $\beta = 0.50$ GeV for J/ψ , as adopted in [114,115]. For the ground-state and excited Ω hyperon states, the masses and the oscillator parameters α , calculated in [10], are summarized in Table III.

Utilizing the inputs above, we calculate $\mathcal{B}(\Omega_b \rightarrow J/\psi \Omega)$ to compare it with the previous studies. Specifically, the decay widths and branching fractions of possible $\Omega_b \rightarrow J/\psi \Omega^{(*)}$ are presented in Table III. Additionally, we plot $\Gamma(\Omega_b \rightarrow J/\psi \Omega)$ as functions of the parameters α and β in Fig. 2 and $\Gamma(\Omega_b \rightarrow J/\psi \Omega^*)$ as a function of m_{Ω^*} in Fig. 3.

IV. DISCUSSIONS AND CONCLUSION

The branching fraction of $\Omega_b \rightarrow J/\psi \Omega$, on the order of 10^{-5} – 10^{-3} , indicates inconclusive calculations from [73,77,86,89,92]. On the other hand, the constituent quark model has been applied to Ω_c^0 decays [96], where $\mathcal{B}(\Omega_c^0 \rightarrow \pi^+ \Omega(2012)^-)/\mathcal{B}(\Omega_c^0 \rightarrow \pi^+ \Omega^-) = 0.21$ agrees with the experimental value of $0.220 \pm 0.059 \pm 0.035$ [7], suggesting its applicability to Ω_b decays.

In the constituent quark model, we obtain $\mathcal{B}(\Omega_b \rightarrow \Omega J/\psi) = 8.8 \times 10^{-4}$, which agrees with the calculations using the light-front quark model [86,87], covariant confined quark model [77] and the perturbative QCD approach [89] to the order of the magnitude, as displayed in Table IV. According to the partial observation [4], $f_{\Omega_b} \mathcal{B}(\Omega_b \rightarrow J/\psi \Omega) = (1.4_{-0.4}^{+0.5}) \times 10^{-6}$, and with the branching fraction substituted by our result, we estimate the fragmentation fraction $f_{\Omega_b} \simeq 0.16 \times 10^{-2}$ [86,90], which denotes the $b \rightarrow \Omega_b$ production rate.

TABLE IV. The branching fraction of $\Omega_b \rightarrow J/\psi\Omega$ is expressed in units of 10^{-4} , compared with results from different models.

Our work	[77]	[92]	[86]	[73]	[89]	[87]
8.8	8	0.45	5.3	16.7	$6.9^{+0.5+1.0}_{-0.0-0.3}$	$6.9^{+2.4}_{-1.7}$

It should be pointed out that our calculation of $\Omega_b \rightarrow \Omega J/\psi$ depends on α and β . To test the sensitivity, we depict $\Gamma(\Omega_b \rightarrow J/\psi\Omega)$ as a function of α and β , within the parameter space: $0.4 \text{ GeV} < \alpha < 0.5 \text{ GeV}$ and $0.48 \text{ GeV} < \beta < 0.55 \text{ GeV}$. In Fig. 2, it can be seen that $\Gamma(\Omega_b \rightarrow J/\psi\Omega)$ is more sensitive to β , varying from 3.0 to 4.2 times 10^{-16} GeV . However, it remains close to the central value: $3.6 \times 10^{-16} \text{ GeV}$ (see Table III).

There are two excited $1P$ -wave Ω states in the Ω hyperon spectroscopy: $\Omega(1^2P_{1/2-})$ and $\Omega(1^2P_{3/2-})$. It is interesting to note that the newly observed $\Omega(2012)$ state is more likely to be assigned to the latter: $\Omega(1^2P_{3/2-})$, based on the fact that the measured mass and decay width are consistent with the quark model predictions [9,10,12,13,15].

Based on this assignment, we obtain $\mathcal{B}(\Omega_b \rightarrow J/\psi\Omega(2012)) = 1.1 \times 10^{-3}$. Specifically, $\mathcal{B}(\Omega_b \rightarrow J/\psi\Omega(2012))/\mathcal{B}(\Omega_b \rightarrow J/\psi\Omega) \simeq 1.3$, which clearly avoids the uncertain fragmentation fraction f_{Ω_b} , making it beneficial for experimental examination. In fact, $\Omega(2012)$ has been observed [6] and reconfirmed in $\Omega_c^0 \rightarrow \pi^+\Omega(2012)^-, \Omega(2012)^- \rightarrow \Xi^0 K^-$ [7]. Likewise, this can be reconfirmed from the resonant Ω_b decays, such as $\Omega_b \rightarrow J/\psi\Omega(2012)$, followed by the subsequent decay $\Omega(2012) \rightarrow \Xi^0 K^-$. In particular, $\Omega(2012) \rightarrow \Xi K$ has a branching fraction of around 90% [116]. The $\Omega(1^2P_{1/2-})$ is a typically missing Ω hyperon, with a predicted mass around 1950 MeV [10,28,29,38]. In our evaluation, $\mathcal{B}(\Omega_b \rightarrow J/\psi\Omega(1^2P_{1/2-})) = 4.2 \times 10^{-4}$ indicates that $\Omega(1^2P_{1/2-})$ has a smaller but compatible production rate to its $1P$ -wave cousin state.

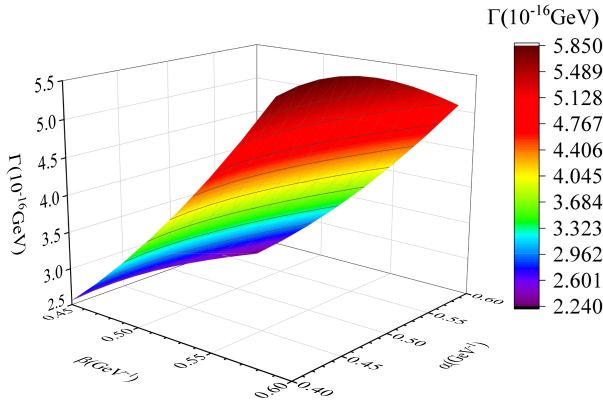


FIG. 2. The decay width of $\Omega_b \rightarrow J/\psi\Omega$ varies with the harmonic parameters α of Ω baryon and β of J/ψ meson.

In the quark model, the $1D$ -wave Ω^* hyperons are classified into the spin-doublet $\Omega(1^2D_{3/2+}, 1^2D_{5/2+})$ and the spin-quartet $\Omega(1^4D_{1/2+}, 1^4D_{3/2+}, 1^4D_{5/2+}, 1^4D_{7/2+})$, with masses evaluated to be around 2150 to 2250 MeV [9–11,96] in the mass spectrum. However, these states have yet to be firmly established experimentally.

In accordance with the masses from Ref. [10], we calculate $\Omega_b \rightarrow J/\psi\Omega(1D)$. Our results are listed in Table III, where $\Omega(1^2D_{5/2+}, 1^4D_{3/2+}, 1^4D_{5/2+}, 1^4D_{7/2+})$ states have production rates to be 0.5, 0.6, 0.8, 0.4, respectively, relative to their ground-state counterpart. We highlight that $\mathcal{B}(\Omega_b \rightarrow J/\psi\Omega(1^4D_{5/2+})) = 7.2 \times 10^{-4}$ is sufficiently large for a promising establishment of $\Omega(1^4D_{5/2+})$ through $\Omega_b \rightarrow \Omega(1^4D_{5/2+})J/\psi$, followed by $\Omega(1^4D_{5/2+}) \rightarrow \Xi(1530)^0 K^-$, where $\mathcal{B}(\Omega(1^4D_{5/2+}) \rightarrow \Xi(1530)K)$ is as large as 80% [9,10].

As for the existing $2S$ -wave Ω^* hyperon states, $\Omega(2^2S_{1/2+})$ and $\Omega(2^4S_{3/2+})$, we obtain $\mathcal{B}(\Omega_b \rightarrow J/\psi\Omega(2^2S_{1/2+}, 2^4S_{3/2+})) = (4.5, 20) \times 10^{-4}$. Remarkably, in the current Ω spectroscopy studies, $\Omega(2^4S_{3/2+})$ in $\Omega_b \rightarrow J/\psi\Omega^*$ is the only one that can have a production rate larger than that of its ground state counterpart.

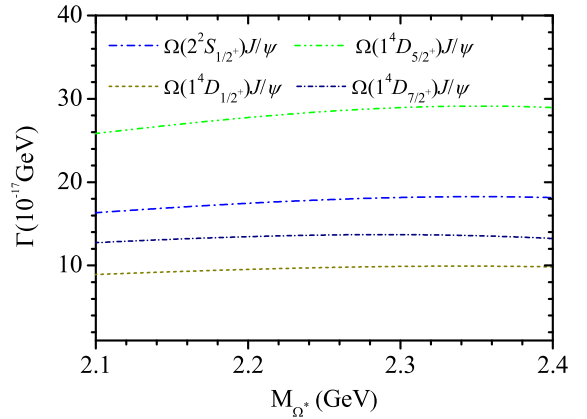
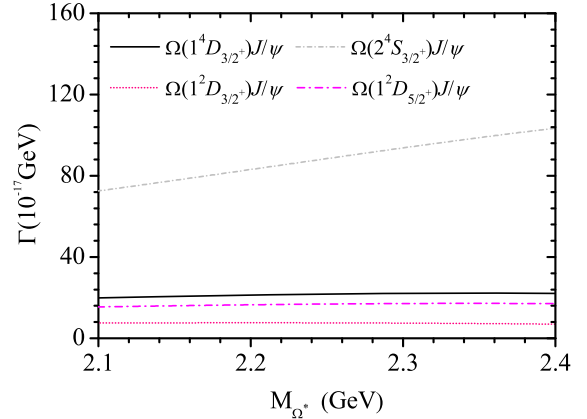


FIG. 3. The decay widths of $\Omega_b \rightarrow J/\psi\Omega^*$ as a function of M_{Ω^*} , with the range from the theoretical estimations.

The constituent quark model relies on theoretical inputs from the Ω^* masses. Due to the nature of these theoretical estimations, the masses carry some uncertainties. Therefore, in Figs. 3, we depict the decay widths involving various excited Ω hyperons as functions of M_{Ω^*} , with the possible ranges determined by the model calculations. It is found that the predicted decay widths for both $\Omega^*(1P, 1D)$ and $\Omega^*(2S)$ are not sensitive to the uncertainties arising from the mass estimations.

In conclusion, we have calculated the sextet b -baryon decays $\Omega_b \rightarrow J/\psi \Omega^{(*)}$ using the constituent quark model. We found that $\mathcal{B}(\Omega_b \rightarrow J/\psi \Omega) = 8.8 \times 10^{-4}$, which is consistent with the previous studies. With $\Omega(2012)$ identified as $\Omega(1^2P_{3/2-})$, we found that $\mathcal{B}(\Omega_b \rightarrow J/\psi \Omega(2012)) = 1.1 \times 10^{-3}$, which is compatible with $\mathcal{B}(\Omega_b \rightarrow J/\psi \Omega)$. Additionally, the production rates of the $\Omega(1^2D_{5/2+}, 1^4D_{3/2+}, 1^4D_{5/2+}, 1^4D_{7/2+})$ states have been calculated to be 0.5, 0.6, 0.8, 0.4, respectively, relative to their ground-state counterpart. We also found that $\mathcal{B}(\Omega_b \rightarrow J/\psi \Omega(2^2S_{1/2+}, 2^4S_{3/2+})) = (4.5, 20) \times 10^{-4}$, which is promising for measurement by LHCb. Since we

have demonstrated that our calculations are insensitive to the parameter inputs and uncertainties arising from the Ω^* masses, this provides a suitable test-bed to investigate the Ω hyperon spectroscopy.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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