

Probing spontaneous CP violation through precision Higgs observables

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We investigate the implications of spontaneous CP violation in the general two Higgs doublet model, which leads to a nondecoupling structure of the Higgs sector. All the masses of the Higgs bosons are purely determined by the vacuum expectation value of the Higgs fields, and are thus constrained to be smaller than $\mathcal{O}(500)$ GeV by the perturbative unitarity bound. Such a nondecoupling nature predicts sizable deviations from the Standard Model expectations in the observables of the discovered Higgs boson (h). We find that the magnitude of deviations in $\mathcal{B}_{h\rightarrow\gamma\gamma}$ ($\mathcal{B}_{h\rightarrow Z\gamma}$) are larger than $\sim 10\%$ (4%) in the Higgs alignment limit. Moreover, we show that a robust correlation emerges between the deviations in the one-loop corrected hhh coupling and $\mathcal{B}_{h\rightarrow\gamma\gamma}$ to be, e.g., 200% (50%) and -10.3% (-10.8%), respectively, under the constraints from theoretical bounds and current experimental data. Using a few benchmark points, we highlight that flavor-violating decays of the additional Higgs bosons can be sizable due to the constrained structure of the Yukawa interactions.

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I. INTRODUCTION

CP violation (CPV) observed in decays of K and B mesons can successfully be explained by the three-generation structure in the Standard Model (SM) of particle physics [1]. In the SM, the CP -violating phase or equivalently the Kobayashi-Maskawa (KM) phase is originated from complex phases of the Yukawa couplings of quarks [2]. Such CPV is called the explicit violation as the Lagrangian breaks the CP invariance.

There is an alternative possibility to explain the CPV, i.e., spontaneous CP violation (SCPV), where the Lagrangian respects the CP invariance, but the vacuum does not. SCPV cannot be realized in the SM, because the vacuum expectation value (VEV) is taken to be real without loss of generality. However, if the Higgs sector is extended from the minimal form with one isospin scalar doublet, the SCPV is possible via complex phases of VEVs. Since the true structure of the Higgs sector is still unknown, the origin of CPV, i.e., explicit or spontaneous, also remains an open question. Therefore, determination of the nature of CPV is quite important for uncovering the structure of the

Higgs sector, and to clarify the baryon asymmetry of the Universe because the CPV is one of the necessary ingredients [3].

The SCPV has originally been proposed by T. D. Lee [4] in two Higgs doublet models (2HDMs). The necessary and sufficient condition for the SCPV has been clarified in Ref. [5] in a basis independent way, i.e., at least one of three J -invariants [6,7] is nonzero and all the four I -invariants are zero. See also Ref. [8] for the physical conditions of SCPV. In 2HDMs, flavor alignments are often imposed to avoid flavor-changing neutral currents (FCNCs) via Higgs boson mediations at tree level, which can be realized by introducing a Z_2 symmetry [9] or Yukawa alignment [10]. However, such a scenario cannot generate the KM phase, because the CP phase from the Higgs VEV can be removed by rephasing quark fields [11]. We thus consider the 2HDM without imposing any additional symmetry, except for the CP invariance of the Lagrangian.

One of the striking features of the SCPV 2HDM is seen in masses of Higgs bosons which are purely given by the VEV, i.e., all the dimensionful parameters in the potential are eliminated after solving the vacuum conditions. Thus, there are upper limits on the masses provided by perturbative unitarity or perturbativity [11–15]. Such structure of the Higgs masses leads to a nondecoupling nature, in which we obtain sizable and correlated deviations in observables of the discovered Higgs boson (h) from the corresponding SM predictions [16,17]. In particular, the trilinear Higgs coupling, λ_{hhh} , receives large quantum corrections induced by extra Higgs bosons, while the decay rates of $h \rightarrow \gamma\gamma$ and

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$h \rightarrow Z\gamma$ are suppressed due to charged Higgs boson loops, yielding a robust negative correlation between these quantities. We show that there is a one-to-one correspondence between $\mathcal{B}_{h \rightarrow \gamma\gamma}$ and the charged Higgs mass ($m_{H^\pm}$) in the Higgs alignment limit, and it gives the lower limit on $m_{H^\pm}$ to be around 220 GeV from the current Higgs signal strength measured at the LHC. On the other hand, the upper limit on $m_{H^\pm}$ is given to be around 510 GeV coming from the combined constraints by theoretical bounds and current experimental data. We find that the measurement of λ_{hhhh} at the High-Luminosity LHC (HL-LHC) will put a stronger upper limit on $m_{H^\pm}$ compared to any theoretical bounds.

In addition to the physics of h , the constrained structure of the Yukawa couplings are predicted in this scenario, and gives rise to flavor-violating decays of the extra Higgs bosons, offering complementary discovery channels. Therefore, the SCPV could be explored at collider experiments such as the HL-LHC or future lepton colliders.

II. MODEL

We parametrize isospin scalar doublets $\Phi_{1,2}$ as

$$\Phi_a = e^{i\xi_a} \begin{pmatrix} \omega_a^+ \\ \frac{v_a + h_a + iz_a}{\sqrt{2}} \end{pmatrix}, \quad (a = 1, 2), \quad (1)$$

where $v_a \in \mathbb{R}$ are the VEVs and the phases ξ_a are taken to be $(\xi_1, \xi_2) = (0, \xi)$ using the global hypercharge symmetry without loss of generality. As we define our Lagrangian in a basis where all the parameters are taken to be real, the phase ξ ($\neq n\pi, n \in \mathbb{Z}$) breaks the CP symmetry spontaneously. In Refs. [18–20], it has been proven that when the spontaneously CP -breaking minimum exists, it is unique, and there is no tunneling to the other types of minima such as an electric charge-breaking one or a normal one, having two real VEVs from neutral components of two Higgs doublets.

It is convenient to define the so-called Higgs basis [21] as

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\xi} \end{pmatrix} \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \Phi \\ \Phi' \end{pmatrix}, \quad (2)$$

where $t_\beta = v_2/v_1$, $t_X = \tan X$, $c_X = \cos X$, $s_X = \sin X$, and

$$\Phi = \begin{pmatrix} G^+ \\ \frac{h'_1 + v + iG^0}{\sqrt{2}} \end{pmatrix}, \quad \Phi' = \begin{pmatrix} H^+ \\ \frac{h'_2 + ih'_3}{\sqrt{2}} \end{pmatrix}, \quad (3)$$

with $v = \sqrt{v_1^2 + v_2^2} \simeq 246$ GeV. In the above expression, $G^\pm$ and G^0 are the Nambu-Goldstone bosons, which are absorbed into the longitudinal component of $W^\pm$ and Z , respectively, while $H^\pm$ ($h'_{1,2,3}$) are the physical singly charged (neutral) Higgs bosons. The neutral Higgs bosons can be mixed with each other depending on the parameters in the Higgs potential.

The Higgs potential with explicit CP conservation can generally be written as

$$\begin{aligned} V = & m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - (m_3^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}) \\ & + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \\ & + \left[\frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2) + \lambda_6 |\Phi_1|^2 + \lambda_7 |\Phi_2|^2 \right] (\Phi_1^\dagger \Phi_2) + \text{H.c.}, \end{aligned} \quad (4)$$

where all the parameters are real. We can write down the potential in the Higgs basis by replacing $(\Phi_1, \Phi_2, m_i^2, \lambda_j) \rightarrow (\Phi, \Phi', M_i^2, \Lambda_j)$ with $i = 1-3$ and $j = 1-7$ in Eq. (4), where the new parameters M_i^2 and Λ_j are given as a function of m_{1-3}^2 and λ_{1-7} , respectively. Differently from the case with explicit CPV, M_i^2 and Λ_j are not completely independent due to the reality of the original parameters defined in Eq. (4). We also note that some of the potential parameters become complex in the Higgs basis, but it does not correspond to the explicit CPV because a basis still exists in which all Lagrangian parameters are real [5].

By solving the following vacuum conditions,

$$m_1^2 = -\frac{v^2}{2} (\lambda_1 c_\beta^2 + \lambda_3 s_\beta^2 - \lambda_4 s_\beta^2 - \lambda_5 s_\beta^2 + \lambda_6 s_{2\beta} c_\xi), \quad (5)$$

$$m_2^2 = -\frac{v^2}{2} (\lambda_2 s_\beta^2 + \lambda_3 c_\beta^2 + \lambda_4 c_\beta^2 - \lambda_5 c_\beta^2 + \lambda_7 s_{2\beta} c_\xi), \quad (6)$$

$$m_3^2 = \frac{v^2}{2} (\lambda_5 s_{2\beta} c_\xi + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2), \quad (7)$$

all the dimensionful parameters, m_i^2 , are constrained to be the order of $\mathcal{O}(v^2)$, so that all the Higgs boson masses are given by the Higgs VEV as seen below. The mass of charged Higgs boson is given by

$$m_{H^\pm}^2 = \frac{v^2}{2} (\lambda_5 - \lambda_4). \quad (8)$$

The mass matrix for the neutral Higgs bosons $\mathcal{M}$ can be written by the symmetric 3×3 form whose elements are given in the Higgs basis, i.e., (h'_1, h'_2, h'_3) in Appendix A. The mass eigenstates are defined as

$$\begin{pmatrix} h'_1 \\ h'_2 \\ h'_3 \end{pmatrix} = R \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}, \quad (9)$$

where R is the 3×3 orthogonal matrix determined as

$$R^T \mathcal{M} R = \text{diag}(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2). \quad (10)$$

We identify the $H_1(=h)$ state as the discovered Higgs boson with the mass of 125 GeV. The additional Higgs bosons H_2 and H_3 can be lighter than h in general, but we consider the case where the masses of $H_{2,3}$ as well as $H^\pm$ are larger than 125 GeV throughout the Letter.

Taking the determinant of $\mathcal{M}$, we obtain

$$\det[\mathcal{M}] \propto \frac{v^6 s_\xi^2}{(t_\beta + 1/t_\beta)^2}, \quad (11)$$

where the proportional factor is given by a function of λ_s . Since values of λ_i are constrained by perturbative unitarity bounds and/or perturbativity conditions, the above expression tells us the existence of upper limits on the masses of additional Higgs bosons, i.e., the decoupling limit does not exist. Also, the product of the three masses becomes zero if we take $s_\xi \rightarrow 0$, $t_\beta \rightarrow 0$ or $t_\beta \rightarrow \infty$, which provides the lower limit on $|s_\xi|$ and the lower and upper limit on $|t_\beta|$. See, e.g., Ref. [11] for bounds using the next-to-leading-order (NLO) perturbative unitarity.

Generally, h can be mixed with the other two neutral states $H_{2,3}$, by which the h couplings are modified from the SM predictions at tree level. Since the current LHC data show that the properties of the discovered Higgs boson are consistent with the SM ones [22], scenarios with an approximate Higgs alignment are favored. In the Higgs alignment limit, the R matrix is taken to be

$$R \xrightarrow{\text{Higgs alignment}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{pmatrix}. \quad (12)$$

This is realized by taking

$$\mathcal{M}_{12} = \mathcal{M}_{13} = 0, \quad (13)$$

and they determine $\lambda_{6,7}$ in terms of the other quartic couplings; see Eq. (A2).

Next, we discuss the Yukawa interactions. Generally, we have two independent 3×3 Yukawa matrices Y_1^f and Y_2^f ($Y_{1,2}^f \in \mathbb{R}$) for Φ_1 and Φ_2 , respectively, for each fermion type f with f being up-type quarks, down-type quarks, and charged leptons. Without loss of generality, the Yukawa interactions are written in the Higgs basis and the mass eigenbasis of fermions as

$$\begin{aligned} \mathcal{L}_Y = & - \left[\overline{Q}_L^d \left(\frac{\sqrt{2}M_d}{v} \Phi + \rho_d \Phi' \right) d_R \right. \\ & + \overline{Q}_L^u \left(\frac{\sqrt{2}M_u}{v} \Phi^c + \rho_u \Phi'^c \right) u_R \\ & \left. + \overline{L}_L \left(\frac{\sqrt{2}M_e}{v} \Phi + \rho_e \Phi' \right) e_R + \text{H.c.} \right], \quad (14) \end{aligned}$$

where M_f ($f = u, d, e$) are the diagonalized mass matrices with real-positive eigenvalues, and

$$Q_L^d = \begin{pmatrix} V^\dagger u_L \\ d_L \end{pmatrix}, \quad Q_L^u = \begin{pmatrix} u_L \\ V d_L \end{pmatrix}, \quad (15)$$

with V being the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In Eq. (14), ρ_f are the 3×3 complex matrices which are not completely general unlike the general 2HDM due to the reality of the original Yukawa matrices. The matrices ρ_f are constrained to be the following form:

$$\rho_f = \frac{\sqrt{2}}{s_{2\beta}v} \left[\left(c_{2\beta} + \frac{i}{\tan(2I_f\xi)} \right) M_f - \left(1 + \frac{i}{\tan(2I_f\xi)} \right) (V_f^\dagger V_f^* M_f U_f^T U_f) \right], \quad (16)$$

where $I_f = 1/2(-1/2)$ for $f = u(d, e)$, and V_f and U_f are the unitary matrices which appear via the biunitary transformation of fermions as $f_L \rightarrow V_f f_L$ and $f_R \rightarrow U_f f_R$ with the relation to the CKM matrix as $V = V_u^\dagger V_d$. The matrices ρ_f given in Eq. (16) are generally not diagonal, so that they induce FCNCs via the neutral Higgs boson mediation at tree level. In order to avoid such FCNCs, the Yukawa alignment [10] is often imposed, i.e., $Y_2^f = \tilde{\zeta}_f Y_1^f$ with $\tilde{\zeta}_f$ being an arbitrary real parameter. In this case, however, the mass matrices for fermions can be expressed by the real 3×3 matrices with an overall complex phase factor in the SCPV 2HDM, so that the KM phase cannot be reproduced as the physical phase. Therefore, the *Yukawa misalignment* is necessary to reproduce the KM phase [11]. This also means that a Z_2 symmetry cannot be imposed to the Higgs sector.

Although we cannot impose the Yukawa alignment on all the Yukawa interactions, we *can* impose it on the down-type quark and the charged lepton sectors. In this case, the mass matrices for down-type quarks and charged leptons are taken to be the real 3×3 matrices with overall phases, so that the unitary matrices $V_{d,e}$ and $U_{d,e}$ can be rewritten as

$$V_{d,e} = O_{d,e}^L, \quad U_{d,e} = O_{d,e}^R e^{-i\theta_{d,e}}, \quad (17)$$

where $O_{d,e}^{L,R}$ are the orthogonal matrices and

$$\theta_{d,e} = \arctan \left[\frac{t_\beta s_\xi \tilde{\zeta}_{d,e}}{1 + t_\beta c_\xi \tilde{\zeta}_{d,e}} \right]. \quad (18)$$

In this scenario, ρ_f given in Eq. (16) takes the following form:

$$\rho_u = \frac{\sqrt{2}}{s_{2\beta}v} \left[\left(c_{2\beta} + \frac{i}{t_\xi} \right) M_u - \left(1 + \frac{i}{t_\xi} \right) (V V^T M_u U_u^T U_u) \right], \quad (19)$$

$$\rho_{d,e} = \frac{\sqrt{2}M_{d,e}}{v}\zeta_{d,e}, \quad (20)$$

where we used $V_u^\dagger V_u^* = VV^T$, and

$$\begin{aligned} \zeta_{d,e} &= \frac{1}{s_{2\beta}} \left[\left(c_{2\beta} - \frac{i}{t_\xi} \right) - \left(1 - \frac{i}{t_\xi} \right) e^{-2i\theta_{d,e}} \right] \\ &= \frac{\tilde{\zeta}_{d,e} e^{i\xi} - t_\beta}{1 + t_\beta \tilde{\zeta}_{d,e} e^{i\xi}}. \end{aligned} \quad (21)$$

III. NONDECOUPLING EXTRA HIGGS BOSONS

As we have seen above, the masses of extra Higgs bosons purely come from the VEV. This gives significant loop effects of the extra Higgs bosons on the observed Higgs boson couplings. We here especially consider those for the Higgs self-coupling hhh and the loop-induced Higgs decays $h \rightarrow \gamma\gamma/Z\gamma$.

The hhh coupling is calculated by using the effective potential method at one-loop level; see Appendix B for details. In the Higgs alignment limit, the hhh coupling subtracted by its SM prediction is expressed as

$$\begin{aligned} \Delta\lambda_{hhh}^{1\text{-loop}} &\equiv (\lambda_{hhh}^{1\text{-loop}})_{\text{2HDM}} - (\lambda_{hhh}^{1\text{-loop}})_{\text{SM}} \\ &= \frac{1}{32\pi^2} \sum_{\phi=H^\pm, H_2, H_3} n_\phi \frac{\lambda_{\phi\phi h}^3}{m_\phi^2} \\ &= \frac{1}{32\pi^2} \sum_{\phi=H^\pm, H_2, H_3} n_\phi \frac{(2m_\phi^2 + m_h^2)^3}{v^3 m_\phi^2}, \end{aligned} \quad (22)$$

where $n_{H^\pm} = 2(1)$ for $\phi = H^\pm(H_{2,3})$ and $\lambda_{abc} \equiv \partial^3 V / (\partial a \partial b \partial c)|_0$ with $|_0$ representing all scalar fields being set to zero. We see that the difference is enhanced by the quartic-powerlike dependence of the masses of extra Higgs bosons [23,24].

The decay rates of $h \rightarrow \gamma\gamma$ and $h \rightarrow Z\gamma$ are given as

$$\begin{aligned} \Gamma_{h \rightarrow V\gamma} &= \frac{\sqrt{2}G_F \alpha_{\text{em}}^2 m_h^3}{8\pi^3 (1 + \delta_{V\gamma})} \left(1 - \frac{m_V^2}{m_h^2} \right)^3 \\ &\times \left| (t, W)_V + \frac{v\lambda_{H^\pm H^- h}}{m_h^2 - m_V^2} c_V F_V(m_{H^\pm}) \right|^2, \quad (V = Z, \gamma), \end{aligned} \quad (23)$$

where $c_V = 1(\cot 2\theta_W)$ with θ_W being the weak mixing angle for $V = \gamma(Z)$. The $(t, W)_V$ term inside the squared absolute value denotes the top and W boson loop contributions whose numerical values are given to be about 1.6(2.9) for $V = \gamma(Z)$ in the SM case; see, e.g., Ref. [25] for their analytic expressions. The loop function F_V is given by

$$\begin{aligned} F_V(m) &= \frac{m_V^2}{2(m_h^2 - m_V^2)} [B_0(m_V^2; m, m) - B_0(m_h^2; m, m)] \\ &\quad - \frac{1}{2} - m^2 C_0(m_V^2, 0, m_h^2; m, m, m), \end{aligned} \quad (24)$$

with B_0 and C_0 being the Passarino-Veltman's two-point and three-point scalar functions [26], respectively. In the Higgs alignment limit, we have

$$\lambda_{H^\pm H^- h} = \frac{1}{v} (2m_{H^\pm}^2 + m_h^2), \quad (25)$$

and the deviations of these decay rates purely come from the $H^\pm$ loop. For $m_h^2/m_{H^\pm}^2 \ll 1$, the $H^\pm$ loop contribution asymptotically becomes $-1/12$ for $h \rightarrow \gamma\gamma$ and $-\cot 2\theta_W/12$ for $h \rightarrow Z\gamma$, so that it gives a destructive interference with the $(t, W)_V$ part, and it makes the branching ratios of $h \rightarrow \gamma\gamma$ ($h \rightarrow Z\gamma$) to be about 10% (4%) smaller than the SM prediction.

IV. CONSTRAINTS

The scalar quartic couplings are constrained by the vacuum stability bound [27–29], the tree-level perturbative unitarity bound [30–32]¹ and the electroweak oblique S and T parameters [36,37]. In the general 2HDM, these S and T parameters have been calculated in Refs. [38–41]. Due to the Yukawa misalignment, ρ_u given in Eq. (19) is strongly constrained by flavor observables, including meson mixings [42] and $b \rightarrow s\gamma$ [43,44]. Direct searches at the LHC for charged [45,46] and neutral Higgs bosons [47–50] provide additional bounds on ρ_u . The CPV phase ξ contributes to the electric dipole moment (EDM). We impose the upper bound on the electron EDM, $|d_e| \leq 4.1 \times 10^{-30}$ e-cm (90% CL) [51]. For the constraints from the electron EDM, we compute the Barr-Zee diagrams [52] which give the dominant contribution to the EDM based on the formulas given in Ref. [53]. See also Ref. [54] for the recent calculation of the electron EDM including non-Barr-Zee types.

The ρ_u matrix depends on the unitary matrix U_u as shown in Eq. (19) which can be parametrized as follows:

$$U_u = R_{23}(\theta_{23}, \delta_{23}) R_{13}(\theta_{13}, \delta_{13}) R_{12}(\theta_{12}, \delta_{12}), \quad (26)$$

where $R_{ij}(\theta_{ij}, \delta_{ij})$ are the SU(3) rotation matrices with $i'-i'$ ($i' \neq i, j$), $i-i'/j-j$, and $i-j$ ($j-i$) components being 1, $\cos \theta_{ij}$, and $\sin \theta_{ij} e^{-i\delta_{ij}}$ ($-\sin \theta_{ij} e^{i\delta_{ij}}$), respectively, while all the other components are zero. In addition to these

¹The perturbative unitarity bound has been evaluated at NLO in 2HDMs [33–35], and it has been applied to the SCPV 2HDM in Ref. [11]. The upper limit on the masses of extra Higgs bosons has been given to be $m_{H^\pm} \lesssim 435$ (650) GeV, $m_{H_2} \lesssim 485$ (600) GeV and $m_{H_3} \lesssim 545$ (700) GeV by using the perturbative unitarity bound at NLO [11].

angles and phases of U_u , there are eight free parameters in our model, i.e., $\{m_{H^\pm}, m_{H_2}, m_{H_3}, \tan \beta, \tan \xi, \alpha, \tilde{\xi}_{d,e}\}$. All these parameters and their ranges used for our scan are detailed in Table I. We randomly scanned 1×10^6 parameter points, and found that approximately 1500 points satisfy all the bounds.

V. RESULTS

In Fig. 1, we show our main result, the correlation between $\Delta\mathcal{B}_{h \rightarrow \gamma\gamma}$ and $\Delta\kappa_\lambda$, i.e.,

$$\Delta\mathcal{B}_{h \rightarrow \gamma\gamma} = \frac{\mathcal{B}_{h \rightarrow \gamma\gamma}^{\text{2HDM}}}{\mathcal{B}_{h \rightarrow \gamma\gamma}^{\text{SM}}} - 1, \quad \Delta\kappa_\lambda = \frac{\Delta\lambda_{hhh}^{\text{1-loop}}}{(\lambda_{hhh})_{\text{SM}}}, \quad (27)$$

with $\mathcal{B}_{h \rightarrow \gamma\gamma}^{\text{2HDM(SM)}}$ being the branching ratio of $h \rightarrow \gamma\gamma$ and $\Delta\lambda_{hhh}^{\text{1-loop}}$ given in Eq. (22). The branching ratio of $h \rightarrow \gamma\gamma$ only depends on $m_{H^\pm}$ because of Eq. (25), so that we display the value of $m_{H^\pm}$ on the upper edge of the figure. From the limit given at the ATLAS experiment limit $-0.12 \leq \Delta\mathcal{B}_{h \rightarrow \gamma\gamma} \leq 0.18$ [55], we can extract $m_{H^\pm} \gtrsim 218$ GeV. The green points represent those satisfying the theoretical and experimental constraints discussed above. We find that the lightest neutral extra Higgs boson, H_2 , remains below the $t\bar{t}$ production threshold in all the

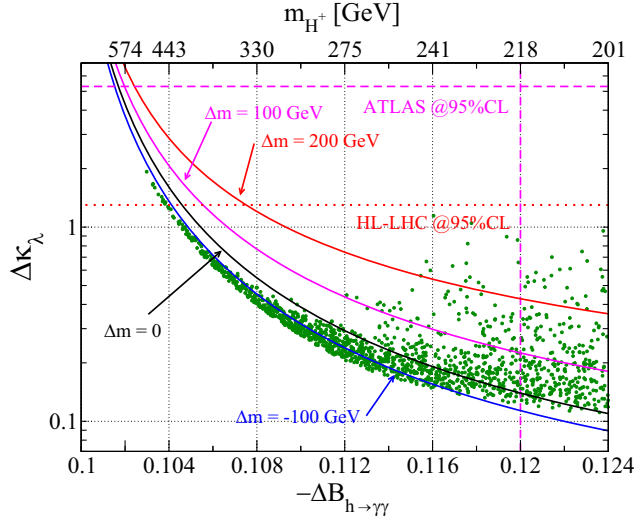


FIG. 1. Correlation between $\Delta\mathcal{B}_{h \rightarrow \gamma\gamma}$ and $\Delta\kappa_\lambda$ defined in Eq. (27) in the Higgs alignment limit. The blue, black, magenta, and red curves show the cases with $\Delta m (\equiv m_{H_2} - m_{H^\pm}) = -100, 0, 100$, and 200 GeV, respectively, and $m_{H^\pm} = m_{H_3}$ for all curves. The magenta dashed lines represent the current upper limit taken by the ATLAS experiments at 95% CL, while the red dotted line shows the expected upper limit on $\Delta\kappa_\lambda$ at 95% CL given at the HL-LHC. The green dots show the predictions allowed by the theoretical constraints (perturbative unitarity and vacuum stability) and the experimental constraints (S , T parameters, $b \rightarrow s\gamma$, meson mixings, electron EDM, and direct searches for charged and extra neutral Higgs bosons at the LHC).

benchmark points (BPs). For $m_{H^\pm} \gtrsim 300$ GeV, the masses of $H^\pm$ and H_3 tend to be nearly degenerate and larger than about 100 GeV than m_{H_2} by the constraints. This leads to a quite strong correlation, e.g., $(\Delta\mathcal{B}_{h \rightarrow \gamma\gamma}, \Delta\kappa_\lambda) = (-10.3\%, 200\%)$ for $m_{H^\pm} \simeq 500$ GeV and $(-10.8\%, 50\%)$ for $m_{H^\pm} \simeq 330$ GeV. The current constraint on $\Delta\kappa_\lambda$, i.e., $-1.4 \leq \Delta\kappa_\lambda \leq 5.3$ [22,56] is too weak to exclude the additional parameter space, but its future sensitivity at the HL-LHC is expected to be $\Delta\kappa_\lambda < 1.3$ at 95% CL with 3 ab^{-1} data [57]. Thus, some of our BPs can be explored by the precise measurement of the hhh coupling. On the other hand, $\Delta\mathcal{B}_{h \rightarrow \gamma\gamma}$ is expected to be measured with 4% accuracy at 1σ level [57], so that if the central value is given to be around zero, our scenario would be ruled out.

VI. PHENOMENOLOGY OF EXTRA HIGGS BOSONS

In Table II, we present a few selected BPs that illustrate the characteristic phenomenology in our model. As discussed above, the matrix ρ_u is constrained from flavor physics, and m_{H_2} remains below the $t\bar{t}$ production threshold. These constraints make the dominant decay mode of H_2 to be $H_2 \rightarrow tc$. Phenomenology of the heavier neutral Higgs boson (H_3) varies depending on the mass hierarchy of the extra Higgs bosons. In the degenerate case in masses, the decay of H_3 mimics that of H_2 . However, in the nondegenerate case, H_3 decays into bosonic channels, $H_3 \rightarrow H_2 Z$ or $H^\pm W^\mp$, leading to different collider signatures. The decay of $H^\pm$ also strongly depends on ρ_u and we find that along with the usual tb mode, $H^\pm$ decay substantially into unique albeit elusive cb final state [58,59]. In the case of the nondegenerate spectrum, $H^\pm \rightarrow H_2 W^\pm$ also opens up, which gives rise to distinctive phenomenology [60–63].

At the bottom of Table II, we also list the primary production channels for the extra Higgs bosons. The production cross section for $H^\pm$, $\sigma(pp \rightarrow H^\pm tb)$, is computed at the leading order using MadGraph5-aMC@NLO-3.5.3 [64,65]. The heavy neutral Higgs bosons, H_2 and H_3 , are produced predominantly via gluon fusion, and their cross sections are estimated using SusHi [66,67] at the next-to-next-to-leading-order level.

VII. CONCLUSION

Spontaneous CP violation provides a compelling framework for explaining the origin of CP violation beyond the Standard Model. The general two Higgs doublet model offers the simplest setting to realize this mechanism, leading to distinctive and testable predictions in Higgs-sector observables. We have explored the nondecoupling property of the SCPV 2HDM, where the hhh coupling and $h \rightarrow \gamma\gamma/Z\gamma$ decays significantly deviate from the SM predictions. After taking into account all relevant theoretical and experimental constraints, we have found that the

branching ratio of $h \rightarrow \gamma\gamma$ ($h \rightarrow Z\gamma$) decreases more than 10% (4%) for most of the parameter space, and the current signal strength for the $h \rightarrow \gamma\gamma$ mode gives a robust restriction on the mass of charged Higgs boson to be larger than 220 GeV. Such a large deviation in $h \rightarrow \gamma\gamma$ will be explored at the HL-LHC in the near future.

In this Letter, we have demonstrated for the first time the complementarity of the hhh coupling and the $h \rightarrow \gamma\gamma$ decay in testing the scenarios with SCPV. This interplay provides a novel, robust strategy to explore the heavy scalar spectrum and our analysis sets new exclusion bounds on heavy Higgs spectrum that up to now were in agreement with all relevant constraints. We also demonstrated the characteristic decay patterns of the heavy Higgs bosons such as $H_2 \rightarrow tc$ and $H^\pm \rightarrow cb$, which can serve as additional experimental targets. We therefore strongly advocate systematic investigations of these nonstandard channels in current and upcoming experiments such as the HL-LHC, as they offer crucial opportunities to test the SCPV paradigm.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: HIGGS BOSON MASSES

The mass matrix for the neutral Higgs bosons $\mathcal{M}$ (symmetric 3×3 form) can be written in the Higgs basis, i.e., (h'_1, h'_2, h'_3) as

$$\begin{aligned} \frac{\mathcal{M}_{11}}{v^2} &= \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2(\lambda_3 + \lambda_4 + \lambda_5 c_{2\xi}) c_\beta^2 s_\beta^2 \\ &\quad + 2(\lambda_6 c_\beta^2 + \lambda_7 s_\beta^2) s_{2\beta} c_\xi, \\ \frac{\mathcal{M}_{22}}{v^2} &= (\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4) c_\beta^2 s_\beta^2 + \frac{\lambda_5}{2} (1 + c_{2\beta}^2 c_{2\xi}) \\ &\quad - (\lambda_6 - \lambda_7) s_{2\beta} c_{2\beta} c_\xi, \\ \frac{\mathcal{M}_{33}}{v^2} &= \lambda_5 s_\xi^2, \\ \frac{\mathcal{M}_{12}}{v^2} &= (\lambda_2 s_\beta^2 - \lambda_1 c_\beta^2) c_\beta s_\beta + \frac{1}{4} (\lambda_3 + \lambda_4 + \lambda_5 c_{2\xi}) s_{4\beta} \\ &\quad + [\lambda_6 (2c_{2\beta} - 1) c_\beta^2 + \lambda_7 (2c_{2\beta} + 1) s_\beta^2] c_\xi, \\ \frac{\mathcal{M}_{13}}{v^2} &= -\frac{\lambda_5}{2} s_{2\beta} s_{2\xi} - (\lambda_6 c_\beta^2 + \lambda_7 s_\beta^2) s_\xi, \\ \frac{\mathcal{M}_{23}}{v^2} &= -\frac{1}{2} \lambda_5 c_{2\beta} s_{2\xi} + \frac{1}{2} (\lambda_6 - \lambda_7) s_{2\beta} s_\xi. \end{aligned} \quad (\text{A1})$$

The Higgs alignment condition, $\mathcal{M}_{12} = \mathcal{M}_{13} = 0$, can then be solved in terms of the $\lambda_{6,7}$ as follows:

$$\begin{aligned} \lambda_6 &= \left[-\lambda_1 c_\beta + \lambda_2 t_\beta s_\beta + (\lambda_3 + \lambda_4) \frac{c_{2\beta}}{c_\beta} \right. \\ &\quad \left. + \frac{\lambda_5}{c_\beta} (1 - 2c_\beta^2 (2 + c_{2\xi})) \right] \frac{s_\beta}{2c_\xi}, \\ \lambda_7 &= \left[\lambda_1 \frac{c_\beta}{t_\beta} - \lambda_2 s_\beta - (\lambda_3 + \lambda_4) \frac{c_{2\beta}}{s_\beta} \right. \\ &\quad \left. + \frac{\lambda_5}{s_\beta} (1 - 2s_\beta^2 (2 + c_{2\xi})) \right] \frac{c_\beta}{2c_\xi}. \end{aligned} \quad (\text{A2})$$

APPENDIX B: EFFECTIVE POTENTIAL

The effective potential is given by

$$V_{\text{eff}} = V_0 + V_1 + V_{\text{CT}}, \quad (\text{B1})$$

where V_0 , V_1 , and V_{CT} are, respectively, the contributions from the tree level diagrams, the one-loop diagrams, and the counterterms. Assuming no charge-breaking vacuum, we can parametrize the background fields in the Higgs basis as

$$\Phi = \begin{pmatrix} 0 \\ \frac{\varphi_1}{\sqrt{2}} \end{pmatrix}, \quad \Phi' = \begin{pmatrix} 0 \\ \frac{\varphi_2 + i\varphi_3}{\sqrt{2}} \end{pmatrix}. \quad (\text{B2})$$

We note that $\varphi_{2,3}$ are generally nonzero even in the Higgs basis due to the radiative corrections. We then obtain V_0 by using Eqs. (2), (4), and (B2). The counterterm V_{CT} can be obtained by replacing $m_i^2 \rightarrow \delta m_i^2$ and $\lambda_j \rightarrow \delta \lambda_j$ in V_0 .

The one-loop contribution is given by [68,69]

$$\begin{aligned} V_1 &= \sum_\phi V_1^\phi + \dots (\phi = H^\pm, H_{1,2,3}) \\ &= \frac{n_\phi}{64\pi^2} m_\phi^4(\varphi) \left[\ln \left(\frac{m_\phi^2(\varphi)}{\mu^2} \right) - c \right] + \dots, \end{aligned} \quad (\text{B3})$$

where $m_\phi^2(\varphi)$ denote the field dependent masses of ϕ , $n_\phi = 2(1)$ for $\phi = H^\pm (H_{1,2,3})$, and the ellipses show the other contributions from SM fermion and gauge boson loops. In the above expression, μ^2 is an arbitrary dimensionful parameter, and c is a constant including UV divergence given by

$$c = \frac{3}{2} + \frac{1}{\epsilon} - \gamma_E + \ln 4\pi, \quad (\text{B4})$$

with ϵ being defined via the D dimensional spacetime $D = 4 - 2\epsilon$ in the dimensional regularization and γ_E being the Euler-Mascheroni constant.

We impose the renormalization conditions [70]

$$(V_1 + V_{\text{CT}})_{\varphi_i} = 0, \quad (V_1 + V_{\text{CT}})_{\varphi_i \varphi_j} = 0, \quad (\text{B5})$$

where we introduced the following shorthand notation of the derivatives:

$$X_{\varphi_i} \equiv \frac{\partial X}{\partial \varphi_i} \Big|_v, \quad X_{\varphi_i \varphi_j} \equiv \frac{\partial^2 X}{\partial \varphi_i \partial \varphi_j} \Big|_v, \quad (\text{B6})$$

with $|_v$ denoting the replacement of $\{\varphi_1, \varphi_2, \varphi_3\} \rightarrow \{v, 0, 0\}$ after the derivative. These conditions determine nine counterterms, and ensure that the vacuum and masses are not changed from those given at tree level. In particular, if we take the third derivative with respect to φ_1 , we obtain

$$\begin{aligned} V_{\text{eff}, \varphi_1 \varphi_1 \varphi_1} &= \lambda_{h'_1 h'_1 h'_1} + \frac{1}{32\pi^2} \sum_{\phi} n_{\phi} \frac{[(m_{\phi}^2)_{\varphi_1}]^3}{m_{\phi}^2} \\ &+ \sum_{\phi} \left[\frac{3}{v^2} + \frac{(m_{\phi}^2)_{\varphi_1 \varphi_1 \varphi_1}}{(m_{\phi}^2)_{\varphi_1}} - \frac{3[(m_{\phi}^2)_{\varphi_1 \varphi_1}]^2}{[(m_{\phi}^2)_{\varphi_1}]^2} \right] V_{1, \varphi_1}^{\phi} \\ &+ \sum_{\phi} \left[-\frac{3}{v} + \frac{3(m_{\phi}^2)_{\varphi_1 \varphi_1}}{(m_{\phi}^2)_{\varphi_1}} \right] V_{1, \varphi_1 \varphi_1}^{\phi} + \dots \quad (\text{B7}) \end{aligned}$$

We note that the first derivative of the field dependent mass provides the corresponding scalar trilinear coupling, i.e., $(m_{\phi}^2)_{\varphi_i} = \lambda_{\phi \phi^* \varphi_i}$.

In the Higgs alignment case, we have the following relations:

$$(m_{\phi}^2)_{\varphi_1 \varphi_i} = \lambda_{\phi \phi^* h'_1 h'_i} = \frac{\lambda_{\phi \phi^* h'_i}}{v}, \quad (m_{\phi}^2)_{\varphi_1 \varphi_i \varphi_j} = 0. \quad (\text{B8})$$

Therefore, the second and third lines of Eq. (B7) vanish, and we obtain the simple form given in Eq. (22).

APPENDIX C: BENCHMARK POINTS

We show the list of the parameters scanned in the numerical analysis and their ranges in Table I. The BPs discussed in the main text are shown in Table II.

TABLE I. Parameters and their scan ranges used for this analysis.

Parameters	Range	Parameters	Range
$m_{H^{\pm}/H_2/H_3}$ (GeV)	200–800	α	$-\pi/2 - \pi/2$
$\tan \beta$	1–8	$\theta_{12/23/13}$	$0 - \pi$
$\tan \xi$	0.5–50	$\delta_{12/23/13}$	$0 - 2\pi$
$\tilde{\zeta}_d$	0–50	$\tilde{\zeta}_e$	1

TABLE II. BPs satisfying all theoretical and experimental constraints.

Parameters	BP1	BP2	BP3	BP4
$m_{H^{\pm}/H_2/H_3}$ (GeV)	455/228/454	264/212/223	236/248/446	246/243/245
$\tan \beta$	2.61	3.12	2.51	2.53
$\tan \xi$	33.13	7.95	13.45	42.31
α (rad)	0.0	−0.5	−0.05	0.35
$\tilde{\zeta}_d/\tilde{\zeta}_e$	32/1	38/1	41/1	31/1
$\theta_{12/23/13}$ (rad)	1.64/0.43/1.55	1.71/1.67/1.51	1.55/3.13/1.49	1.56/2.72/1.45
$\delta_{12/23/13}$ (rad)	1.93/6.02/1.60	5.41/0.88/1.69	2.02/4.47/1.64	1.11/3.04/1.60
$\mathcal{B}_{H_2 \rightarrow XY}$ (%)	$tc: 82.93$ $\tau\tau: 4.70$ $tu: 3.88$	$tc: 96.85$ $\tau\tau: 1.03$ $tu: 0.62$	$tc: 99.04$ $\tau\tau: 0.23$ $tu: 0.21$	$tc: 99.56$ $\dots$ $\dots$
$\mathcal{B}_{H_3 \rightarrow XY}$ (%)	$H_2 Z: 89.52$ $tt: 9.57$ $tc: 0.59$	$tc: 97.34$ $\tau\tau: 0.73$ $tu: 0.63$	$H^{\pm} W^{\mp}: 66.46$ $H_2 Z: 25.98$ $tt: 3.78$	$tc: 99.5$ $\dots$ $\dots$
$\mathcal{B}_{H^{\pm} \rightarrow XY}$ (%)	$H_2 W^{\pm}: 78.1$ $tb: 21.23$ $cb: 0.65$	$tb: 56.78$ $cb: 43.11$ $H_2 W^{\pm}: 0.06$	$cb: 61.28$ $tb: 38.67$ $\tau\nu: 0.03$	$cb: 77.95$ $tb: 22.03$ $\dots$
$\sigma(pp \rightarrow H^{\pm} tb/H^{\pm} cb)$ (pb)	0.087/0.088	0.253/6.367	0.471/19.762	0.371/38.671
$\sigma(pp \rightarrow H_{2/3})$ (pb)	4.277/4.247	4.854/7.022	3.702/4.657	4.163/8.348

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