

Chiral gravitational waves from multiphase magnetogenesis

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(Received 27 February 2026; accepted 27 April 2026; published 29 May 2026)

Cosmological vector fields are central to many early-Universe phenomena, including inflationary dynamics, primordial magnetogenesis, and dark-matter scenarios. However, constructing models able to generate cosmological magnetic fields while avoiding strong coupling, backreaction, and cosmic microwave background constraints remains challenging. We study a novel mechanism in which brief non-slow-roll phases during inflation amplify primordial magnetic fields at small scales, while maintaining theoretical consistency and observational viability. We incorporate parity-violating interactions in the vector sector and demonstrate, for the first time in a non-slow-roll framework, that chirality can significantly boost magnetic-field amplitudes and imprint distinctive polarization-dependent spectral features. We complement detailed numerical computations with an analytical treatment yielding compact expressions for chiral vector mode functions that reproduce the main spectral properties. We then develop a systematic formalism to evaluate the stochastic gravitational-wave background naturally induced at second order by these amplified fields, identifying both an intensity component and a circularly polarized contribution with characteristic frequency profiles. We discuss detection prospects with future multiband gravitational-wave observatories, showing that chiral signatures could provide a distinctive observational probe. Our results introduce new avenues for enhancing primordial magnetic fields and their associated gravitational-wave signals, opening promising possibilities for their future detection and interpretation, both with cosmological and gravitational-wave probes.

DOI: [10.1103/49qy-k7fy](https://doi.org/10.1103/49qy-k7fy)

I. INTRODUCTION

Cosmological vector fields play a central role in a wide range of early-Universe phenomena, from models of inflation (see, e.g., [1] for a review) to scenarios of primordial magnetogenesis (see [2–15] for seminal works) and dark-matter models (see, e.g., [16]). In particular, primordial magnetogenesis provides an appealing mechanism to account for the large-scale magnetic fields observed in galaxies and clusters. However, concrete realizations face significant theoretical and observational challenges, including strong-coupling regimes, backreaction effects, and bounds from the cosmic microwave background (CMB); see, e.g., [17,18] for reviews. Developing

scenarios that overcome these obstacles while still producing observable magnetic fields remains an important open problem.

In this work, we explore a novel framework designed to amplify small-scale primordial magnetic fields while simultaneously generating chirality and satisfying both observational constraints (notably those from the CMB) and theoretical consistency requirements such as backreaction control. Our mechanism exploits a brief phase of departure of the inflaton from slow-roll evolution, which can influence the effective couplings between scalar and vector sectors. Such non-slow-roll evolutions are known to enhance the spectrum of curvature perturbations—potentially leading to primordial black hole formation (see, e.g., [19] for a review)—and can likewise be used to boost inflationary magnetic fields. For the first time within a non-slow-roll setting, to the best of our knowledge, we consistently incorporate parity-violating effects in the vector sector and show that they can provide an additional and significant enhancement of the magnetic-field amplitude

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beyond that produced by the background dynamics alone, as well as adding new features to the magnetic-field profile.

After introducing the setup in Sec. II, we perform in Sec. III a detailed numerical analysis of the resulting magnetic-field spectrum. We demonstrate that the spectrum grows from large to small scales, extending and refining previous studies [20–22] of its growth properties. We also show that this behavior allows the model to evade both backreaction constraints and current CMB bounds. We then include parity-violating interactions and find that chirality significantly modifies the spectral shape, generating additional polarization-dependent peaks whose features distinguish the two helicity states. In Sec. IV, we complement the numerical study with an analytical approach based on [23,24], deriving compact formulas for the chiral vector mode functions. Despite their simplicity, these expressions successfully reproduce the main characteristics of the full numerical spectra obtained in Sec. III.

In Sec. V, we develop a systematic framework to compute the stochastic gravitational-wave (GW) background induced at second order by the enhanced magnetic fields on small scales. We show that the resulting energy-density spectrum can be decomposed into a standard contribution Ω_{GW} , governed by the total GW intensity, and a parity-violating component Ω_{GW}^V associated with circular polarization. Both contributions exhibit distinctive spectral shapes that can lead to observable signatures. We discuss prospects for detection and characterization with future multiband gravitational-wave observatories, emphasizing that chirality could be inferred from characteristic spectral features. Overall, our scenario provides new mechanisms for enhancing the GW signal and endowing it with properties that may facilitate its detection and interpretation.

II. OUR SETUP

Our work aims to develop both a numerical and analytical understanding of the properties of chiral magnetic fields generated during inflationary scenarios featuring a departure from slow-roll evolution. As we shall learn, a transient non-slow-roll epoch can lead to a significant rapid amplification of magnetic fields at small scales. This enhancement has important consequences for the generation of a background of chiral gravitational waves after inflation ends, which are sourced by the amplified magnetic-field spectrum.

We work in a Friedmann-Lemaître-Robertson-Walker background (with a mostly plus signature) and we denote conformal time by τ . The dynamics of the electromagnetic field during inflation is governed by the action [25,26]

$$S[A^\mu] = -\frac{1}{4} \int d\tau d^3x \sqrt{-g} J^2 \left[F_{\mu\nu} F^{\mu\nu} - \frac{\gamma}{2} F_{\mu\nu} \tilde{F}^{\mu\nu} \right], \quad (2.1)$$

where the electromagnetic-field strength tensor is defined in terms of the vector potential as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Its dual is given by

$$\tilde{F}^{\mu\nu} \equiv \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}, \quad (2.2)$$

with $\epsilon^{\mu\nu\alpha\beta}$ the totally antisymmetric Levi-Civita tensor. The function J , if dependent on coordinates, breaks the conformal invariance of the electromagnetic action allowing for vector-field amplification, while the parameter γ controls the strength of parity violation in the vector sector.

During inflation, both J and γ are typically modeled as functions of the inflaton or of a spectator field, leading to direct couplings between scalar degrees of freedom and the electromagnetic-field strength [20,21,26–35]. In scenarios that include brief non-slow-roll phases—often motivated by primordial black hole production (see, e.g., [36] for a review), or by punctuated inflation models [37,38]—the scalar-field velocities may vary rapidly over short time intervals. This behavior induces sharp time variations in both J and γ during the inflationary process. In addition to magnetogenesis, such scenarios are useful for building models of longitudinal vector dark matter [39,40]. In our setup, without committing to explicit model building, we treat these quantities as explicit functions of conformal time τ . We analyze two representative scenarios that capture their evolution in the presence of brief non-slow-roll epochs, during which the profiles of J and γ can change significantly. The class of models we wish to capture—such as those featuring violations of slow roll in scenarios of primordial black hole formation—share the characteristic that the decaying mode, which would otherwise be strongly suppressed on superhorizon scales, becomes significantly excited. A central result of our analysis is that, independent of the specific form or parametrization of the functions J and γ , the resulting spectra exhibit universal behavior. In particular, during a brief transition phase associated with sharp variations of these functions, we find that the slope of the spectrum follows a model-independent prediction. This universality manifests itself, for instance, in the existence of a bound on the maximal spectral slope, as well as in common qualitative features of the spectral profile. It is precisely this robustness and insensitivity to model details that render our results both general and compelling and ensure their applicability across a wide class of scenarios.

We work in the Coulomb gauge setting $A_0 = \partial_i A^i = 0$. We introduce the quantity $\mathcal{A}_k = J A_k$, where A_k is the mode function in Fourier space, so that its corresponding equation of motion is [20,21,25]

$$\mathcal{A}_k'' + \left(k^2 + 2\lambda\gamma k \frac{J'}{J} - \frac{J''}{J} \right) \mathcal{A}_k = 0, \quad (2.3)$$

where the index of polarization is $\lambda = [L, R] = \pm 1$, and a prime indicates derivative along conformal time. While the term $J''/J \propto \tau^{-2}$ introduces a demarcation of sub-Hubble and super-Hubble behavior to the mode function, the term $2\lambda\gamma kJ'/J$ induces chiral imbalance, enhancing the mode function of one polarization, while suppressing the other. The exact time, scale, and chiral dependence of the effects of these terms is associated with the behavior of $J(\tau)$. The magnetic power spectrum $\mathcal{P}_B^\lambda(k)$ for a given polarization index λ is then [8,26,29]

$$\mathcal{P}_B^\lambda(k) = \frac{J^2}{a^4} \frac{k^5}{2\pi^2} |A_k^\lambda|^2 = \frac{k^5}{2\pi^2 a^4} |\mathcal{A}_k^\lambda|^2, \quad (2.4)$$

which is evaluated at the end of inflation, when J is set to reach unity, restoring conformal invariance. For reference, the parity-preserving case ($\gamma = 0$) with monotonic evolution of $J = (a^2/a_e^2)$ throughout inflation (with a_e being the scale factor at the end of inflation) leads to equal and scale-invariant magnetic-field spectra for both helicities as [20,34]

$$\mathcal{P}_B^L(k) = \mathcal{P}_B^R(k) = \frac{9H^4}{8\pi^2}. \quad (2.5)$$

For $\gamma = 0$ we shall denote as $\mathcal{P}_B^0$ the corresponding total power spectrum $\mathcal{P}_B^{\gamma=0}(k) = \mathcal{P}_B^L(k) + \mathcal{P}_B^R(k) = 9H^4/(4\pi^2)$ and use it as as reference to contrast against the features induced due to deviating phase of J and/or parity violation ($\gamma \neq 0$), the topic we are mostly interested in.

In fact, we now pass to discuss two complementary scenarios of inflationary magnetogenesis, differing in the behavior of γ as a function of time. In Sec. III we investigate a first scenario, in which we extend the analysis of Ref. [22] by accounting for the finite duration of the non-slow-roll epoch and by including parity-violating effects through the parameter γ appearing in Eq. (2.1). We make the hypothesis that γ is a constant parameter turned on during the entire inflationary epoch. Our study of this case relies on a numerical analysis of the coupled mode equations, and it reveals new nontrivial features in the profile of the resulting magnetic-field spectrum. The second scenario, discussed in Sec. IV, further generalizes [22] by incorporating parity violation in a setup that remains analytically tractable. In this case, we assume that γ is switched on *only* during the brief non-slow-roll phase. We then derive a fully analytic expression for the gauge-field mode functions, as well as for the chiral magnetic-field power spectrum. The interesting implications of both scenarios for the generation of chiral induced gravitational waves are developed in Sec. V.

III. FIRST SCENARIO AND NUMERICAL ANALYSIS

In the first scenario, we model the behavior of the nonconformal coupling function J as often done in models of inflation with a brief phase of ultraslow roll (USR) sandwiched between slow-roll evolutions—see, e.g., the review [36] and references therein. We model the time evolution of J as involving three phases, such as $J(\tau) \sim \tau^{-2}, \tau^{-n_2}, \tau^{-2}$. The phases of $J \propto \tau^{-2}$ shall lead to scale-invariant $\mathcal{P}_B(k)$, while the intermediate phase shall lead to deviations from scale invariance. We may expect such a behavior of J when it is modeled in terms of the inflaton ϕ or the associated kinetic term $X = -\partial_\mu \phi \partial^\mu \phi / 2$ in USR models or models that have brief departure from slow-roll evolution (for related efforts, see [21,28,34]).

The exact form of $J(\tau)$ can be arrived at by matching conditions at transitions between phases, while ensuring continuity of the function,

$$J(\tau) = \begin{cases} \left(\frac{\tau_e}{\tau_2}\right)^2 \left(\frac{\tau_1}{\tau_2}\right)^{-n_2} \left(\frac{\tau}{\tau_1}\right)^{-2} & \text{for } \tau < \tau_1, \\ \left(\frac{\tau_e}{\tau_2}\right)^2 \left(\frac{\tau}{\tau_2}\right)^{-n_2} & \text{for } \tau_1 < \tau < \tau_2, \\ \left(\frac{\tau}{\tau_e}\right)^{-2} & \text{for } \tau > \tau_2, \end{cases} \quad (3.1)$$

where τ_1 and τ_2 are the times demarcating the phase junctions. We approximate the transition between the three phases as instantaneous. We ensure that $J(\tau)$ is continuous across phases and set $J(\tau_e) = 1$ at the end of inflation τ_e . The setup is very similar to the construction presented in [22]. The difference in the scenario of this section is that here the intermediate phase is not instantaneous but of finite duration, and the index describing the phase n_2 is of finite value. This fact has interesting consequences for phenomenology. For numerical purposes, we describe the transitions through a step function modeled using a hyperbolic tangent function in terms of e-folds $N \equiv \ln(a/a(\tau_e))$.

We solve the electromagnetic mode function numerically in terms of e-folds N for the three-phase model of J and compute the corresponding $\mathcal{P}_B(k)$ at the end of inflation. For carrying out numerics, the inflationary background dynamics can effectively be set as de Sitter expansion, parametrized by a constant Hubble parameter H . The scale factor is then $a(\tau) = -1/(H\tau)$, with $\tau < 0$ during inflation.

A. Nonhelical case: Steepest n_B

First, we analyze the three-phase model in the parity-preserving case ($\gamma = 0$), focusing on the steepest possible growth of $\mathcal{P}_B(k)$. This question is also of interest in comparison with the scalar case. For curvature perturbations, the growing mode during non-slow-roll evolution is well described by a power-law spectrum with spectral index

$n \simeq 4$, which represents the maximal slope attainable within this class of scenarios [41,42].

In contrast, the amplification of vector fluctuations exhibits a more intricate behavior. Although the magnetic power spectrum also grows steeply, with an effective spectral index initially close to $n \simeq 4$, its evolution is not accurately described by a pure power law. Instead, the slope can exceed that of the scalar case, reaching a maximal value of $n \simeq 4.75$, as derived analytically in [22] under the assumption of an infinitesimally brief departure from slow roll. Here we show that even larger values of n can be obtained when the non-slow-roll phase has a finite duration.

Specifically, we numerically analyze a setup featuring a short intermediate non-slow-roll phase of J with a large power-law index τ^{n_2} , following the construction of [22]. We characterize this intermediate stage by large negative values $n_2 \in [-20, -26]$ and a brief duration $\Delta N_2 \leq 0.5$. For these parameters, we compute $\mathcal{P}_B(k)$ and the corresponding spectral index $n_B \equiv \frac{d \ln \mathcal{P}_B(k)}{d \ln k}$ numerically, scanning the

parameter space to determine the largest attainable value of n_B .

Figure 1 shows that n_B can reach values as high as 5 for $n_2 \in [-20, -26]$ and $\Delta N_2 \gtrsim 0.4$. This slightly exceeds the analytic maximum $n_B = 4.75$ obtained in [22]. The discrepancy between the analytic and numerical maxima can be attributed to the finite duration ΔN_2 of the intermediate phase. Notably, a steeper spectral growth is phenomenologically advantageous when confronting the model with observations, as it more readily evades CMB constraints [17].

B. Magnetic power spectrum: The parity-violating case

We proceed to study the three-phase model of J with inclusion of the parity violation in the action. This is an important new ingredient of our work; see also [27,30,32,33]. In the first scenario of this section, we keep the parameter γ constant during inflation. Our fiducial values for the parameters are as follows. The parity-violating term in the action is set to $\gamma = 1$; the onset of

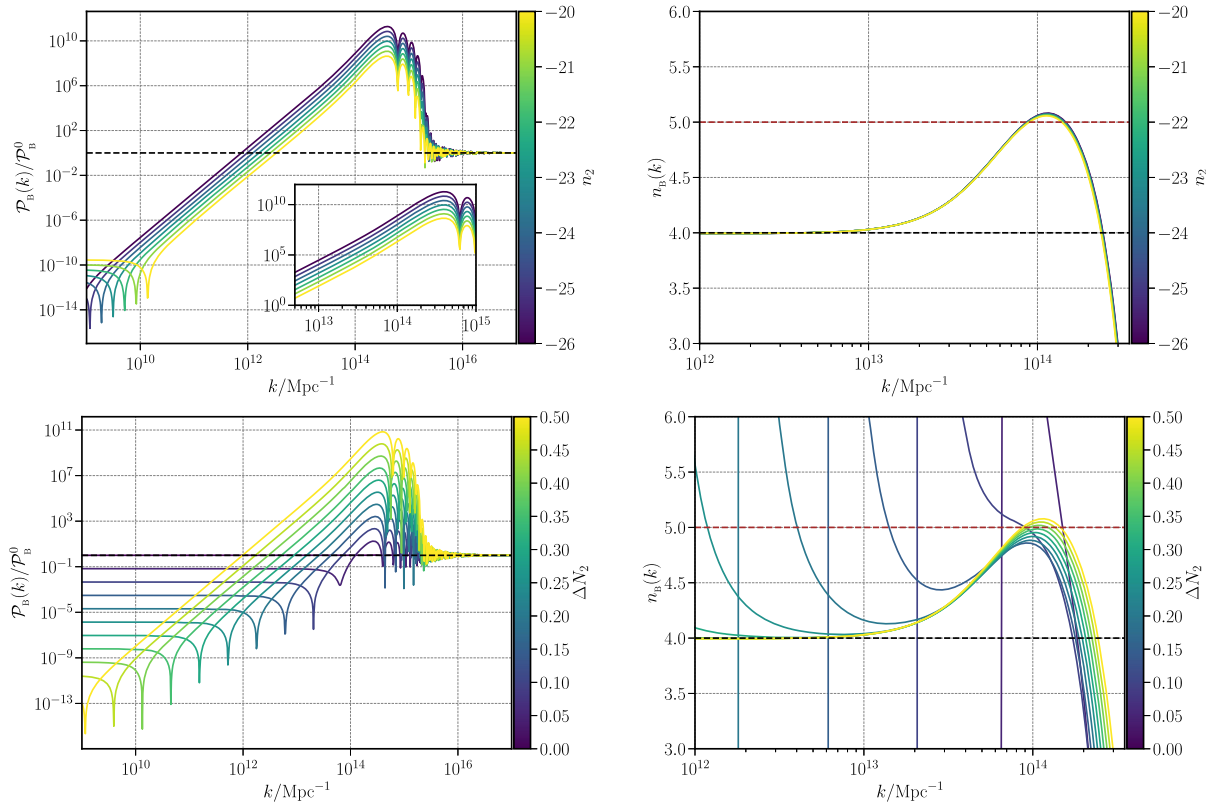


FIG. 1. The magnetic power spectrum $\mathcal{P}_B(k)$ (in terms of $\mathcal{P}_B^0$) and the associated spectral index n_B arising from the three-phase model of $J(N)$ are presented across variations of the parameters characterizing the intermediate phase, namely, n_2 and ΔN_2 . We compute them in the parity-preserving case of $\gamma = 0$ with a representative value of $N_1 = 15$ (counted from the end of inflation). For reference, we mark the value of $\mathcal{P}_B(k) = \mathcal{P}_B^0$ (in dashed black) in the plots of $\mathcal{P}_B(k)$. We examine the spectrum and n_B over the range of scales where the spectrum rises toward the peak, and we inspect the behavior across a range of large negative values of $n_2 = [-20, -26]$ (top). For this analysis we set the intermediate phase of J to be brief yet finite with $\Delta N_2 = 0.5$. We find that the spectral index $n_B \geq 5$ close to the peak. We also examine the spectrum and n_B across a range of $\Delta N_2 = [0, 0.5]$, setting $n_2 = -25$ (bottom). Once again, n_B easily reaches values ≥ 5 for values of $\Delta N_2 \geq 0.4$.

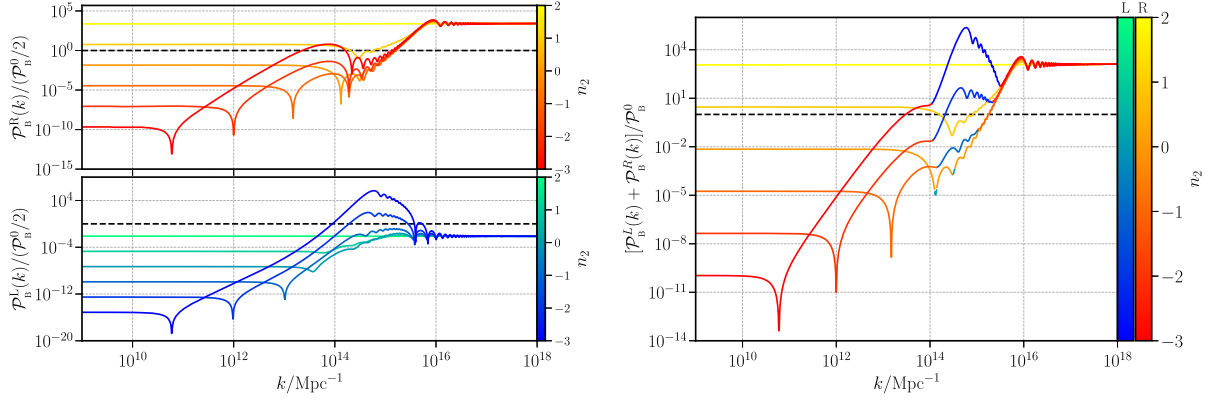


FIG. 2. We present $\mathcal{P}_B^L(k)/(\mathcal{P}_B^0/2)$ and $\mathcal{P}_B^R(k)/(\mathcal{P}_B^0/2)$ (on the left) and the total power spectrum $\sum_\lambda \mathcal{P}_B^\lambda(k)/\mathcal{P}_B^0$ (on the right) across a range of $n_2 = [-3, 2]$. For reference, we also plot the spectrum of the nonhelical case without the intermediate phase of J where $\mathcal{P}_B^L = \mathcal{P}_B^R = \mathcal{P}_B^0/2$ (in dashed black). We have chosen $\gamma = 1$, $N_1 = 15$, and $\Delta N_2 = 3$ in the functional form of $J(N)$. This choice of parameters leads to the wave number corresponding to the onset of the intermediate phase of J to be $k_1 = 9.92 \times 10^{13} \text{ Mpc}^{-1}$. For $n_2 = 2$, we obtain $\mathcal{P}_B^L(k)$ and $\mathcal{P}_B^R(k)$ to be scale invariant but with a significant difference in magnitude due to nonzero γ . With the decrease in n_2 , the features of dip, rise, and peak become prominent in both the spectra. The total power spectrum (on the right) is predominantly right circular but for a brief window of scales between the asymptotic values, where the power increases and turns left circular in polarization with decrease in n_2 . Hence, for $n_2 = -3$ with $\Delta N_2 = 3$, we obtain an interesting double-peaked spectrum with distinct polarization for each peak.

intermediate phase to $N_1 = 15$ (counted from the end of inflation); the duration of the phase to $\Delta N_2 = 3$; and the index to $n_2 = -3$. Note that the value of $n_2 = -3$ is the dual to the case of $n_2 = 2$ as they both lead to the same solution for $\mathcal{A}_k$ when $\gamma = 0$. The choices of parameters are made to produce the peak in $\mathcal{P}_B(k)$ over small scales of around $k \simeq 10^{14} \text{ Mpc}^{-1}$ and hence induce gravitational waves over corresponding frequencies of $f \simeq 0.1 \text{ Hz}$. If this setup is realized in an ultraslow-roll model suitable model of $J(\phi)$, N_1 denotes the onset of ultraslow-roll regime and our choice is consistent with the corresponding constraint on it from observational data over large scales [43]. To understand the effect of these model parameters on $\mathcal{P}_B^\lambda(k)$, we vary them around the aforementioned fiducial values and examine the associated implications.

We begin by inspecting the behavior of $\mathcal{P}_B^\lambda(k)$ for each helicity against the variation of n_2 . In Fig. 2, we present the spectra $\mathcal{P}_B^{L,R}(k)$ computed numerically, and the total power spectrum $\mathcal{P}_B(k) = \mathcal{P}_B^L(k) + \mathcal{P}_B^R(k)$ across a range of choices for n_2 . The spectrum is scale invariant and right circular for $n_2 = 2$. As we vary $n_2 = [-3, 2]$, the features of dip, rise, and peak become prominent in $\mathcal{P}_B^\lambda(k)$ of both helicities.

However, due to the parity-violating term in the action, the behavior around the peak is different for the two helicities. Although the overall power in the right-circular mode is enhanced and the left-circular mode suppressed—as expected with $\gamma \neq 0$ —the trend is *reversed* around the scales approaching the peak of the spectrum. This phenomenon leads to a double-peaked structure of total magnetic power, characterized by altering helicities. We interpret it as due to interplay of democratic enhancement

in power in both helicities due to the intermediate phase of $J(N)$ and a preferential enhancement arising from the parity-violating term. Since for values of $n_2 \neq 2$ the right-circular mode is suppressed with respect to the left-circular mode around the peak, the prominent peak of the power spectrum is left circular in an otherwise right-circular spectrum. We shall inspect this feature more closely in Sec. III C. See also [44] for related results in a similar context.

We then inspect the effect of the duration of the intermediate phase ΔN_2 on the features $\mathcal{P}_B(k)$. We set $n_2 = -3$ and $\gamma = 1$ and present the results in Fig. 3. Once again, as we vary $\Delta N_2 = [0, 3]$, we find the features of dip, rise, and peak becoming prominent in $\mathcal{P}_B^\lambda(k)$ for both helicities. Moreover, the altering helicity at the peak of the spectrum also becomes prominent with increase in ΔN_2 .

Finally, we repeat our analysis varying the constant parameter γ within the interval $\gamma = [0, 1]$. We set $n_2 = -3$ and $\Delta N_2 = 3$ and present the spectra in Fig. 4. We observe that for $\gamma = 0$, $\mathcal{P}_B^L(k) = \mathcal{P}_B^R(k)$ with features of dip, rise, and peak in the spectra, as described in [22]. For values of $\gamma > 0$, we notice an overall enhancement of $\mathcal{P}_B^R(k)$ and a suppression of $\mathcal{P}_B^L(k)$, except around the peak. Around the peak, in fact, we have a reversal of this trend, as noted earlier.

Additionally, in the nonhelical case of our setup ($\gamma = 0$), the spectrum of electric field at the end of inflation $\mathcal{P}_E^\lambda(k)$ behaves as $\mathcal{P}_E^\lambda(k) \simeq \mathcal{P}_B^\lambda(k)(k/k_e)^2$, with k_e corresponding to the wave number at the end of inflation. It turns out to be $\mathcal{P}_E^\lambda(k) \simeq \mathcal{P}_B^\lambda(k)$ when we turn on γ and becomes $\mathcal{P}_E^\lambda(k) = \mathcal{P}_B^\lambda(k)$ when $\gamma = 1$, consistently mimicking the features in the spectrum, including alteration of helicity. However, the

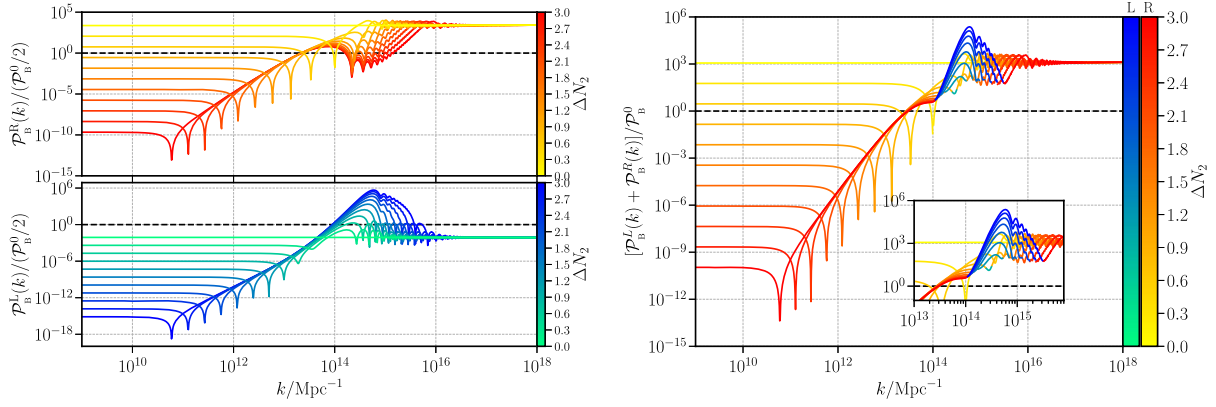


FIG. 3. We present $\mathcal{P}_B^L(k)$ and $\mathcal{P}_B^R(k)$ (on the left) and the total power spectrum $\sum_\lambda \mathcal{P}_B^\lambda(k)$ (on the right) across a range of $\Delta N_2 = [0, 3]$, in terms of $\mathcal{P}_B^0$ as in Fig. 2. We also plot the reference spectrum of $\mathcal{P}_B^L = \mathcal{P}_B^R = \mathcal{P}_B^0/2$ (in dashed black). We have chosen $\gamma = 1$, $N_1 = 15$, and $n_2 = -3$ in the functional form of $J(N)$. For $\Delta N_2 = 0$, we obtain $\mathcal{P}_B^L(k)$ and $\mathcal{P}_B^R(k)$ to be scale invariant but with $\mathcal{P}_B^R(k)$ dominating over $\mathcal{P}_B^L(k)$. With the increase in ΔN_2 , the features of dip, rise, and peak become prominent in both the spectra. Around the peak of the spectra, $\mathcal{P}_B^L(k)$ begins to dominate over $\mathcal{P}_B^R(k)$ with increase in ΔN_2 . The total power spectrum (on the right) is predominantly right circular but for the window of scales that is peaked and left circular in polarization (as focused in the inset).

electric field gets shorted during reheating with the production of charged particles and the associated high conductivity of the epoch [17,29].

Before proceeding further, we shall briefly remark about the observational bounds on the magnetic-field strength over large scales in the current Universe and the compatibility of our model with the bounds. The magnetic-field strength over scales of $\lambda \geq 1$ Mpc is bounded on the lower end as $B_{1 \text{ Mpc}} > 10^{-17}$ G, from the γ -ray observations of extragalactic sources [45,46]. On the other hand, it receives an upper bound from CMB anisotropies as $B_{1 \text{ Mpc}} < 10^{-9}$ G [47,48]. To examine these bounds in our model, we inspect the behavior of $\mathcal{P}_B(k)$ over large scales of CMB.

We can see that our model consistently leads to a scale-invariant $\mathcal{P}_B(k)$ over CMB scales. Further, the amplitude

$\mathcal{P}_B(k) \ll \mathcal{P}_B^0$ over such scales, compared to $\mathcal{P}_B^0$ over small scales. For the fiducial values of $n_2 = -3$ and $\Delta N_2 = 3$, with $\gamma = 0$, we obtain $\mathcal{P}_B(k) \simeq 10^{-13} \mathcal{P}_B^0$ over CMB scales. We may relate the present-day magnetic-field strength smoothed over a given scale λ , denoted as B_λ , to $\mathcal{P}_B(k)$ as [47–49]

$$B_\lambda^2 = \left(\frac{a_e}{a_0}\right)^4 \int \frac{d \ln k}{4\pi} e^{-k^2 \lambda^2} \mathcal{P}_B(k), \quad (3.2)$$

where a_e is the scale factor at the end of inflation and a_0 is the scale factor today. Since we are interested in estimating B_λ for $\lambda > 1$ Mpc, the field strength is assumed to follow the behavior of $1/a^2$ since the end of inflation, without any effects of turbulence; hence the ratio of scale factors in the

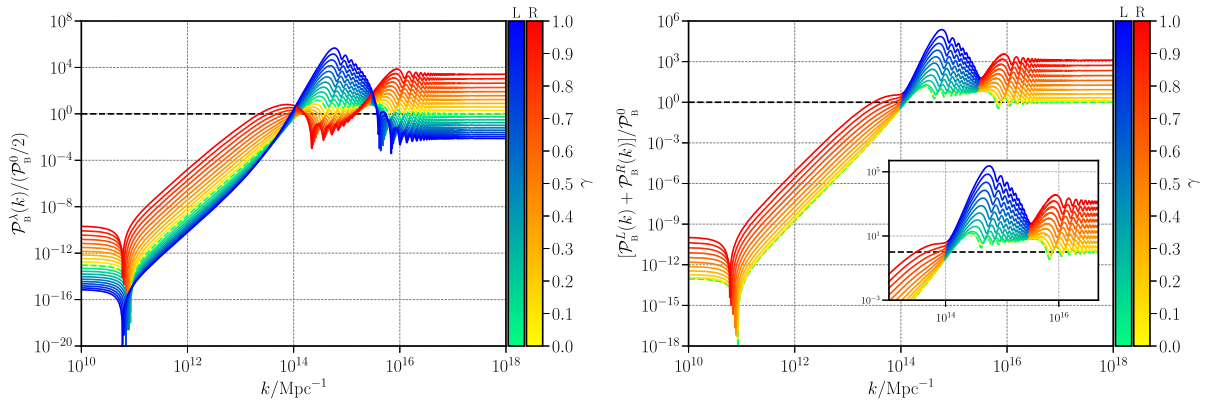


FIG. 4. We present the magnetic power spectrum $\mathcal{P}_B^\lambda(k)$ for a range of $\gamma = [0, 1]$ for individual helicities (on the left) and for the sum of both helicities (on the right) in terms of $\mathcal{P}_B^0$ as in Figs. 2 and 3. The parameters of the model are set to be $N_1 = 15$, $n_2 = -3$, $\Delta N_2 = 3$. Evidently, for $\gamma = 0$, we have both left and right helical modes having the same spectra. The inset on the right focuses on the range of scales around the peak that shall be relevant for the behavior of the spectra of induced GW. Note that the case of $\gamma = 1$ generates an amplification of about 10^5 compared to the nonhelical case of $\gamma = 0$ at the peak of the spectrum.

above expression. Using conservation of entropy, the ratio can be expressed in terms of the Hubble parameter during inflation as [20,21]

$$\frac{a_0}{a_e} = 2.8 \times 10^{28} \left(\frac{H}{10^{-5} M_{\text{Pl}}} \right)^{1/2}. \quad (3.3)$$

We have worked with the Hubble parameter to be $H \simeq 7 \times 10^{-6} M_{\text{Pl}}$, consistent with the bound on tensor-to-scalar ratio from CMB anisotropies. Using these values, we may estimate the magnetic-field strength arising from our model to be about $B_{1 \text{ Mpc}} \simeq 10^{-15} \text{ G}$. For $\gamma = 1$, we get $\mathcal{P}_B(k) \simeq 10^{-10} \mathcal{P}_B^0$ over CMB scales and hence $B_{1 \text{ Mpc}} \simeq 3 \times 10^{-14} \text{ G}$. These values are within the existing bounds on the parameter $10^{-17} < B_{1 \text{ Mpc}} < 10^{-9} \text{ G}$ [45–49].

C. Altering helicities at the peak: An analytical understanding

The curious feature of altering helicities around the peak value of $\mathcal{P}_B(k)$ warrants closer examination and we may understand this effect in the following way. Recall that the equation of motion governing $\mathcal{A}_k$ is Eq. (2.3),

$$\mathcal{A}_k^{\lambda'''} + \left(k^2 + 2\xi^\lambda \gamma k \frac{J'}{J} - \frac{J''}{J} \right) \mathcal{A}_k^\lambda = 0. \quad (3.4)$$

During the phase when $J \propto a^2 \propto \tau^{-2}$, we get $J'/J = |2/\tau|$ and $J''/J = 6/\tau^2$. Hence, the equation becomes

$$\mathcal{A}_k^{\lambda'''} + \left(k^2 + 4\xi^\lambda \gamma \left| \frac{k}{\tau} \right| - \frac{6}{\tau^2} \right) \mathcal{A}_k^\lambda = 0. \quad (3.5)$$

The parity-violating term enhances $\mathcal{A}_k^R$ (with $\xi^R = -1$) and suppresses $\mathcal{A}_k^L$ (with $\xi^L = 1$) for wave numbers exiting the Hubble radius, i.e., $|k\tau| \simeq \sqrt{6}$ during this regime. The behavior of the enhanced mode in the asymptotic regime of $|k\tau| \ll 1$ is [26]

$$\mathcal{A}_k^R \propto \frac{e^{2\pi\gamma}}{(2\gamma)^{3/2}}. \quad (3.6)$$

On the other hand, in the phase when $J \propto a^{-3} \propto \tau^3$, we get $J'/J = -|3/\tau|$, while $J''/J = 6/\tau^2$. This leads to the equation of motion being

$$\mathcal{A}_k^{\lambda'''} + \left(k^2 - 6\lambda\gamma \left| \frac{k}{\tau} \right| - \frac{6}{\tau^2} \right) \mathcal{A}_k^\lambda = 0. \quad (3.7)$$

The difference in sign of J'/J with respect to the earlier case leads to the opposite effect on helicities. The parity-violating term now enhances $\mathcal{A}_k^L$ and suppresses $\mathcal{A}_k^R$ for modes attaining $|k\tau| \simeq \sqrt{6}$ during this regime. The behavior of the enhanced mode in the asymptotic regime of $|k\tau| \ll 1$ is

$$\mathcal{A}_k^L \propto \frac{e^{3\pi\gamma}}{(3\gamma)^{3/2}}. \quad (3.8)$$

Thus, we have different helicities enhanced over wave numbers exiting Hubble radius in different regimes of $J(\tau)$.

Note that the enhancement of the mode function exiting the Hubble radius during the phase of $J \propto a^{-3}$ is larger by a factor of 3/2 in the exponent compared to the mode function exiting in the phase of $J \propto a^2$, i.e.,

$$\mathcal{A}_{k(J \sim a^{-3})}^L \propto [\mathcal{A}_{k(J \sim a^2)}^R]^{3/2}. \quad (3.9)$$

Hence, the corresponding power spectra for the two cases may be related as

$$\mathcal{P}_B^L(k)_{(J \sim a^{-3})} \propto [\mathcal{P}_B^R(k)_{(J \sim a^2)}]^{3/2}. \quad (3.10)$$

This relation explains the difference between enhancements of amplitudes of power over large scales and the power at the peak. For $\gamma = 1$, over large scales $\mathcal{P}_B^L(k \ll k_1)$ is enhanced by about 10^3 compared to the case of $\gamma = 0$. The value $\mathcal{P}_B^R(k \simeq k_1)$ around the peak is enhanced by about $10^{9/2} \simeq 3 \times 10^5$.

D. Backreaction

The issue of backreaction may become crucial in the presence of parity-violating interactions, which lead to chiral instabilities [20,30,31,50–52]. To trust our setup, we must ensure that the energy density of the electromagnetic field ρ_{EM} remains subdominant to the background energy density driving inflation ρ_I . In case of magnetogenesis with monotonic evolution of the coupling function J , value of $\gamma \gtrsim 2.5$ is shown to cause strong backreaction [20]. Interestingly, we do not encounter issues with backreaction in our scenario, making it a viable model for chiral magnetogenesis from inflation without issues related with backreaction.

In fact, let us examine the case of a three-phase evolution of $J(N)$ in the scenario we are considering here. Recall that the tensor power spectrum $\mathcal{P}_T(k) \simeq 2H^2/M_{\text{Pl}}^2$. Therefore, the background energy density during inflation is $\rho_I(\tau)$,

$$\rho_I(\tau) = 3H^2 M_{\text{Pl}}^2 \simeq M_{\text{Pl}}^4 \mathcal{P}_T(k). \quad (3.11)$$

On the other hand, the electromagnetic energy density is

$$\rho_{\text{EM}}(\tau) = \frac{1}{2} \int d \ln k [\mathcal{P}_B(k) + \mathcal{P}_E(k)] \simeq \int d \ln k \mathcal{P}_B(k), \quad (3.12)$$

where the last expression is justified in case of $\gamma = \mathcal{O}(1)$ leading to $\mathcal{P}_B(k) \simeq \mathcal{P}_E(k)$. Hence, $\mathcal{P}_B(k)/(\mathcal{P}_T(k)M_{\text{Pl}}^4)$ is an equivalent estimate of ρ_{EM}/ρ_I . Ensuring this ratio to

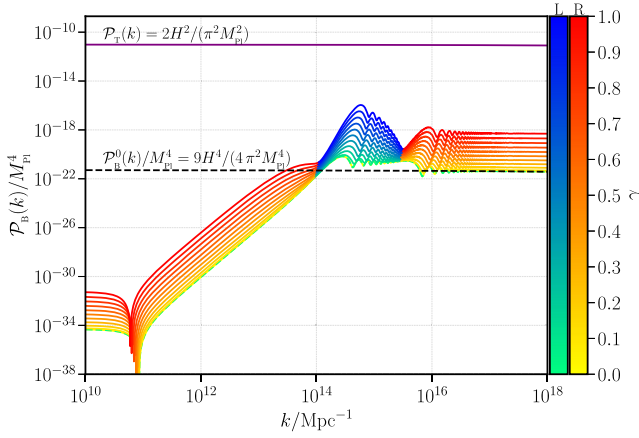


FIG. 5. The total power spectrum $\mathcal{P}_B(k) = \sum_\lambda \mathcal{P}_B^\lambda$ is presented in units of M_{Pl}^4 against the power spectrum of primary tensor perturbations $\mathcal{P}_T(k)$, for a range of $\gamma = [0, 1]$. We set the model parameters to be $n_2 = -3$, $N_1 = 15$, and $\Delta N_2 = 3$. The plot illustrates that the ratio of $\mathcal{P}_B(k)/(\mathcal{P}_T(k)M_{\text{Pl}}^4)$ is small even at the peak amplitude of $\mathcal{P}_B(k)$. At the asymptotic value over small scales, this ratio is approximately $\mathcal{P}_B(k)/(\mathcal{P}_T(k)M_{\text{Pl}}^4) \simeq f(\gamma)(H/M_{\text{Pl}})^2$, where $f(\gamma)$ is typically $\mathcal{O}(10^3)$ for $\gamma = 1$. So, as long as $H/M_{\text{Pl}} \leq 10^{-3/2}$, the backreaction due to electromagnetic energy density is small in the model of interest.

remain below unity across scales indicates control over backreaction.

We illustrate the comparison of $\mathcal{P}_B(k)$ against $M_{\text{Pl}}^4 \mathcal{P}_T(k)$ in Fig. 5 over the range of values of γ we work with. (We choose $H \simeq 7 \times 10^{-6} M_{\text{Pl}}$, to be consistent with bounds on the tensor to scalar ratio over CMB scales.) The spectrum $\mathcal{P}_B(k)$ is suppressed from $\mathcal{P}_B^0 = 9H^4/4\pi^2$ over large scales, and so it is very small compared to $M_{\text{Pl}}^4 \mathcal{P}_T(k)$. Crucially, the ratio of $\mathcal{P}_B(k)/(\mathcal{P}_T(k)M_{\text{Pl}}^4)$ is small at the peak amplitude of $\mathcal{P}_B(k)$, even for the maximum value of $\gamma = 1$. Over small scales, where $\mathcal{P}_B(k)$ settles to the asymptotic value of $\mathcal{P}_B^0$, this ratio becomes $\mathcal{P}_B(k)/(\mathcal{P}_T(k)M_{\text{Pl}}^4) \simeq f(\gamma)(H/M_{\text{Pl}})^2$, where $f(\gamma)$ is $\mathcal{O}(10^3)$ for $\gamma = 1$ [20,30,31]. So—as long as $H/M_{\text{Pl}} \leq 10^{-3/2}$ —thanks to the fact that the magnetic field is enhanced rapidly at small scales only, we conclude that the backreaction due to electromagnetic field is negligible, as stated above.

IV. SECOND SCENARIO: ANALYTICAL RESULTS

In this section, we present a second scenario for non-slow-roll magnetogenesis. We extend the analytical methods of [22] to a setup including parity violation, with the aim of obtaining analytic insight into the numerical results presented in Sec. III. The approach closely follows the techniques of [23,24], adapted to our context; differences and clarifications are discussed where relevant.

The starting point is the evolution equation for the helical modes during de Sitter inflation—Eq. (2.3) which we report here,

$$\mathcal{A}_k^{\lambda'''}(\tau) + \left(k^2 + 2\lambda k \gamma(\tau) \frac{J'(\tau)}{J(\tau)} - \frac{J''(\tau)}{J(\tau)} \right) \mathcal{A}_k^\lambda(\tau) = 0, \quad (4.1)$$

with $\lambda = \pm 1$, and where we allow $\gamma(\tau)$ to be time dependent.

As a reference, in the standard case in which $J(\tau) \propto a^2(\tau)$ and $\gamma(\tau) = 0$ during inflation, with $a(\tau) = -1/(H_I \tau)$, the solution connected to the Bunch-Davies vacuum is the well-known mode function

$$\mathcal{A}_k(\tau) = \frac{3a^2(\tau)H_I^2}{\sqrt{2}k^{5/2}} e^{-ik\tau} \left(1 + ik\tau - \frac{k^2\tau^2}{3} \right). \quad (4.2)$$

We are interested in a general class of models in which

$$J(\tau) = a^2(\tau) \sqrt{\omega(\tau)}, \quad (4.3)$$

with $\omega(\tau_R) = 1$ at the end of inflation. We assume the following time dependence for the functions J and γ :

$$\omega(\tau) = \begin{cases} \omega_0 & \text{for } \tau < \tau_1, \\ \text{continuous but rapidly varying} & \text{for } \tau_1 < \tau < \tau_2, \\ 1 & \text{for } \tau_2 < \tau < \tau_R, \end{cases} \quad (4.4)$$

$$\gamma(\tau) = \begin{cases} 0 & \text{for } \tau < \tau_1, \\ \text{continuous but rapidly varying} & \text{for } \tau_1 < \tau < \tau_2, \\ 0 & \text{for } \tau_2 < \tau < \tau_R, \end{cases} \quad (4.5)$$

The two times τ_1 and τ_2 are close to each other—in fact, we assume

$$\frac{\tau_2 - \tau_1}{\tau_1} \ll 1, \quad (4.6)$$

so that the interval $\tau_1 < \tau < \tau_2$ is short: within this brief period, $J(\tau)$ may significantly deviate from a power law and parity-violating effects encoded in $\gamma(\tau)$ can be active. [Later in this section, we consider the limit of Eq. (4.6) infinitesimally small.]

Physically, as described in the previous sections, this situation can be realized in inflationary models where the inflaton kinetic energy (or $\dot{\phi}$) changes rapidly for a short time. In such scenarios, one can envisage setup with $\gamma \sim \dot{\phi}^2$, so that γ is negligible during slow roll and becomes nonzero (and potentially large) only during the brief non-slow-roll phase. Notice that the choice of profile in Eq. (4.5) is different with respect to what was discussed in Sec. III B, where we instead assume $\gamma = 1$ throughout inflation.

To obtain full analytic solutions for this system, we adopt the following ansatz for the mode function,

$$\mathcal{A}_k(\tau) = \frac{3a^2(\tau)\sqrt{\omega(\tau)}H_1^2}{\sqrt{2}k^{5/2}}e^{-ik\tau}\left[1 + ik\tau - \frac{k^2\tau^2}{3} - k\tau_0\Gamma_1(\tau) + (ik\tau_0)^2(G_{(2)}(\tau) + i\Gamma_{(2)}(\tau)) + (ik\tau_0)^3(G_{(3)}(\tau) + i\Gamma_{(3)}(\tau)) + \dots\right], \quad (4.7)$$

where τ_0 is a pivot quantity whose explicit value will cancel out of final observables. The solution is constructed piecewise, extending the methods of [41]:

- (1) For $\tau < \tau_1$, we recover the standard solution ($J = a^2, \gamma = 0$) so all G_i, Γ_j vanish and we reduce to Eq. (4.2).
- (2) For $\tau_1 < \tau < \tau_2$, we determine the functions G_i, Γ_j by inserting the ansatz (4.7) into Eq. (4.1) and solving order by order in powers of k .
- (3) For $\tau_2 < \tau < \tau_R$, we again have the standard slow-roll form ($J = a^2, \gamma = 0$) and we impose Israel matching conditions at $\tau = \tau_2$ to fix the post-intermediate-stage coefficients.

Working within the short interval $\tau_1 < \tau < \tau_2$, we expand all quantities about τ_1 and define $\Delta\tau \equiv \tau - \tau_1$ with $0 \leq \Delta\tau \ll \tau_1$. From the k^1 order we obtain the differential relation

$$\frac{d}{d\tau} \left[\frac{\tau_0 \omega(\tau) \Gamma_1'(\tau)}{\tau^4} \right] = \lambda \gamma(\tau) \frac{d}{d\tau} \left[\frac{\omega(\tau)}{\tau^4} \right], \quad (4.8)$$

which is formally integrated to

$$\tau_0 \Gamma_1(\tau) = \lambda \int_{-\infty}^{\tau} d\tau_a \frac{\tau_a^4}{\omega(\tau_a)} \int_{-\infty}^{\tau_a} d\tau_b \gamma(\tau_b) \frac{d}{d\tau_b} \left[\frac{\omega(\tau_b)}{\tau_b^4} \right]. \quad (4.9)$$

We introduce the following parameters evaluated at $\tau = \tau_1$:

$$\alpha \equiv \left(\frac{d \ln \omega}{d \ln \tau} \right)_{\tau=\tau_1}, \quad (4.10)$$

$$\beta \equiv \left(\frac{d\gamma}{d \ln \tau} \right)_{\tau=\tau_1}. \quad (4.11)$$

Expanding the formal solution (4.9) in $\Delta\tau$ we find that the leading nonzero contribution to Γ_1 is cubic,

$$\tau_0 \Gamma_1(\tau) = \lambda \frac{\beta(\alpha-4)\Delta\tau^3}{6\tau_1^2} + \mathcal{O}(\Delta\tau^4). \quad (4.12)$$

From the term k^2 in the expansion we obtain two conditions, which once Taylor expanded in $\Delta\tau$ give

$$\tau_0^2 G_2(\tau) = \frac{\alpha}{6} \Delta\tau^2 + \mathcal{O}(\Delta\tau^3), \quad (4.13)$$

$$\tau_0^2 \Gamma_2(\tau) = \lambda \frac{\beta(\alpha-4)}{6\tau_1} \Delta\tau^3 + \mathcal{O}(\Delta\tau^4). \quad (4.14)$$

At k^3 order, the leading results are

$$\tau_0^3 G_3(\tau) = \frac{\alpha}{6} \tau_1 \Delta\tau^2 + \mathcal{O}(\Delta\tau^3), \quad (4.15)$$

$$\tau_0^3 \Gamma_3(\tau) = \lambda \frac{\beta(\alpha-4)}{18} \Delta\tau^3 + \mathcal{O}(\Delta\tau^4). \quad (4.16)$$

For higher orders k^n with $n \geq 4$ the coefficients obey simple recursion relations (for derivatives evaluated at τ_1),

$$\tau_0^n G_n^{(n-1)}(\tau_1) = 2\tau_0^{n-1} G_{n-1}^{(n-2)}(\tau_1), \quad (4.17)$$

$$\tau_0^n \Gamma_n^{(n)}(\tau_1) = 2\tau_0^n \Gamma_{n-1}^{(n-1)}(\tau_1), \quad (4.18)$$

which imply

$$\tau_0^n G_n^{(n-1)}(\tau_1) = 2^{n-3} \frac{\alpha \tau_1}{3}, \quad (4.19)$$

$$\tau_0^n \Gamma_n^{(n)}(\tau_1) = 2^{n-3} \lambda \beta \frac{(\alpha-4)}{3}. \quad (4.20)$$

Hence, keeping leading Taylor terms for each $n \geq 3$,

$$\tau_0^n G_n(\tau) = 2^{n-3} \frac{\alpha \tau_1}{3} \frac{\Delta\tau^{n-1}}{(n-1)!} + \dots, \quad (4.21)$$

$$\tau_0^n \Gamma_n(\tau) = 2^{n-3} \lambda \beta \frac{(\alpha-4)}{3} \frac{\Delta\tau^n}{n!} + \dots \quad (4.22)$$

Plugging all these results in ansatz (4.7), they lead to an exact resummation to exponentials. In fact, we use

$$\sum_{n=3}^{\infty} (ik\tau_0)^n G_n(\tau) = \frac{ik\tau_1 \alpha}{12} (e^{2ik\Delta\tau} - 1 - 2ik\Delta\tau), \quad (4.23)$$

$$\begin{aligned} & i \sum_{n=3}^{\infty} (ik\tau_0)^n \Gamma_n(\tau) \\ &= i\lambda\beta \frac{(\alpha-4)}{24} (e^{2ik\Delta\tau} - 1 - 2ik\Delta\tau + 2k^2\Delta\tau^2). \end{aligned} \quad (4.24)$$

To conclude, the resulting mode function in the short interval $\tau_1 < \tau < \tau_2$ can be written as

$$\mathcal{A}_k(\tau) = \frac{3a^2(\tau)\sqrt{\omega(\tau)}H_I^2}{\sqrt{2}k^{5/2}}e^{-ik\tau}\left[1 + ik\tau - \frac{k^2\tau^2}{3} - \lambda\frac{\beta(\alpha-4)k\Delta\tau^3}{6\tau_1^2} - k^2\left(\frac{\alpha}{6}\Delta\tau^2 + i\lambda\frac{\beta(\alpha-4)}{6\tau_1}\Delta\tau^3\right) + \frac{ik\tau_1\alpha}{12}(e^{2ik\Delta\tau} - 1 - 2ik\Delta\tau) + i\lambda\beta\frac{(\alpha-4)}{24}(e^{2ik\Delta\tau} - 1 - 2ik\Delta\tau + 2k^2\Delta\tau^2)\right]. \quad (4.25)$$

This expression displays the explicit chirality dependence through λ and captures the leading modifications arising from the short non-slow-roll episode.

Matching this intermediate solution to the final slow-roll phase using Israel junction conditions yields, for $\tau > \tau_2$, the standard form

$$\mathcal{A}_k(\tau) = \frac{3a^2(\tau)H_I^2}{\sqrt{2}k^{5/2}}\left[C_1\left(1 + ik\tau - \frac{k^2\tau^2}{3}\right)e^{-ik\tau} + C_2\left(1 - ik\tau - \frac{k^2\tau^2}{3}\right)e^{ik\tau}\right], \quad (4.26)$$

where the coefficients $C_{1,2}$ are fixed by the matching conditions of the function and its derivatives at τ_2 .

We proceed making additional steps that further simplify our expressions for the mode functions. We take the limits

$$\Delta\tau \rightarrow 0, \quad \alpha \rightarrow \infty, \quad \beta \rightarrow \infty, \quad \alpha\Delta\tau = 2\Pi_0, \quad \Pi_0\beta/\alpha = B_0, \quad (4.27)$$

keeping Π_0, B_0 finite. This is motivated by the 't Hooft limit [53] which one encounters in particle physics. We work in terms of the dimensionless quantity

$$\kappa = -k\tau_1, \quad (4.28)$$

with τ_1 the conformal time during inflation when the non-slow-roll phase occurs. Then, the quantities $C_{1,2}$ appearing in Eq. (4.26) reduce to

$$C_1 = 1 + \left(1 - \frac{3i}{\kappa^3} - \frac{2i}{\kappa} - \frac{iB_0(9 + 3\kappa^2 + \kappa^4)\lambda}{\kappa^4}\right)\Pi_0, \quad (4.29)$$

$$C_2 = -\frac{e^{2ik}(i + \kappa)(-3 + 3i\kappa + \kappa^2)\Pi_0}{\kappa^3} + \frac{iB_0e^{2ik}(-3 + 3i\kappa + \kappa^2)^2\lambda\Pi_0}{\kappa^4}. \quad (4.30)$$

These remarkably simple formulas, once plugged in Eq. (4.26), lead to a simple, exact expression for the parity-violating vector mode function in the final epoch $\tau \geq \tau_2$ after the non-slow-roll phase occurs.

The amplitude of the resulting power spectrum at the end of inflation, normalized by its value at large scales, results

$$\Pi(\kappa) = \frac{\mathcal{P}_B(\kappa)}{\mathcal{P}_B(\kappa \rightarrow 0)} \quad (4.31)$$

$$= |C_1 + C_2|^2. \quad (4.32)$$

We plot it in Fig. 6 for a representative choice of parameters. Notice that also in this analytical scenario 2 we find evidence for the phenomenon of “chirality flip” toward the peak, as found in Sec. III for scenario 1. More generally, for intermediate values of the parameter B_0

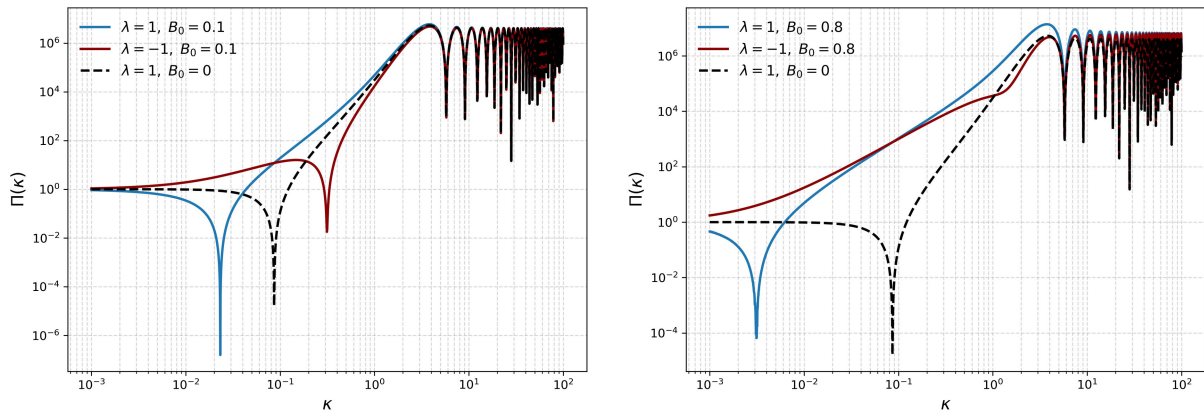


FIG. 6. Analytical spectra evaluated with the limiting formula (4.31).

connected to the parity-violating effects, we notice that the profiles of the resulting chiral spectra resemble the numerical results of Sec. III. We conclude that the analytical approach of this section is able to capture the most important features of the system in terms of very simple, analytical expressions for the parity-violating mode functions.

V. CHIRAL GRAVITATIONAL WAVES FROM MAGNETIC FIELDS

Our results so far indicate that a short phase of non-slow-roll dynamics can considerably enhance the spectrum of primordial magnetic field at small scales. In this section, we study the consequences of this phenomenon for the production of a chiral stochastic GW background induced at the second order by the chiral magnetic field. After their production at the time of reentry during the epoch of radiation domination, the gravitational waves propagate essentially freely due to the extreme weakness of their interactions, so that they traverse the perturbed cosmological background with negligible scattering or backreaction. As a consequence, their subsequent evolution is largely decoupled from, and therefore insensitive to, the later dynamics and possible nonlinear evolution of the magnetic fields, preserving to a very good approximation the information imprinted at the time of their generation.

The topic of vector-induced GWs has been developed in various works [54,55] and the role of helical magnetic fields investigated in [26]. Our discussion aims to develop more systematic arguments and general formulas that can be used in our context of non-slow-roll magnetogenesis. We generalize the approach of [22] to study the generation of GWs from the electromagnetic (EM) field during the radiation-dominated epoch following inflation. Recall that the conformal invariance of the EM field is restored post inflation, since $J = 1$ at the end of inflation. The Fourier decomposition of the electromagnetic vector potential is

$$A_i(\tau, \mathbf{x}) = \sum_{\lambda=L}^R \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} e_i^{(\lambda)}(\hat{k}) A_\lambda(\tau, \mathbf{k}), \quad (5.1)$$

where $e_i^{(\lambda)}(\hat{k})$ are two orthonormal polarization vectors satisfying $e_i^{(\lambda)} \hat{k}_i = 0$. Vectors with a hat denote unit vectors, $\hat{k} = \mathbf{k}/k$ with $k \equiv |\mathbf{k}|$. The helicity polarization vectors obey the eigenrelation

$$(i\varepsilon_{ijl}k_j)e_i^{(\lambda)}(\hat{k}) = \xi^\lambda k e_i^{(\lambda)}(\hat{k}), \quad \xi^L = -\xi^R = 1, \quad (5.2)$$

and the normalization $e_i^{(\lambda)} e_i^{(\sigma)*} = \delta^{\lambda\sigma}$. The comoving magnetic field is defined by

$$B_i(\tau, \mathbf{x}) = \varepsilon_{ijk} \partial_j A_k(\tau, \mathbf{x}), \quad B_i(\tau, \mathbf{k}) = i\varepsilon_{ijl} k_j A_l(\tau, \mathbf{k}). \quad (5.3)$$

Using (5.2) and the expansion (5.1) we obtain

$$B_i(\tau, \mathbf{k}) = \sum_{\lambda=L,R} \xi^\lambda k e_i^{(\lambda)}(\hat{k}) A_\lambda(\tau, \mathbf{k}). \quad (5.4)$$

We assume the two helicities of the vector potential have (in general) different spectra,

$$\langle A_\lambda(\tau, \mathbf{k}) A_\sigma^*(\tau, \mathbf{q}) \rangle' = \delta_{\lambda\sigma} \frac{2\pi^2}{k^3} \mathcal{P}_A^\lambda(k), \quad (5.5)$$

where the prime over the expectation value denotes the quantity barring the associated Dirac δ function, here $\delta^{(3)}(\mathbf{k} - \mathbf{q})$. We define symmetric and helical combinations of $\mathcal{P}_A(k)$ as

$$\begin{aligned} \mathcal{P}_A^S(k) &\equiv \frac{\mathcal{P}_A^L(k) + \mathcal{P}_A^R(k)}{2}, \\ \mathcal{P}_A^H(k) &\equiv \frac{\mathcal{P}_A^L(k) - \mathcal{P}_A^R(k)}{2}. \end{aligned} \quad (5.6)$$

The quantity $\mathcal{P}_A^H(k)$ controls the amount of helicity in the vector sector. It is controlled by the quantity γ in the specific scenarios discussed in Secs. III and IV. With these ingredients, we can compute the magnetic-field two-point functions,

$$\begin{aligned} \langle B_i(\mathbf{k}) B_j^*(\mathbf{q}) \rangle' &= \sum_{\lambda,\sigma} \xi^\lambda \xi^\sigma k^2 e_i^{(\lambda)}(\hat{k}) e_j^{(\sigma)*}(\hat{k}) \langle A_\lambda(\mathbf{k}) A_\sigma^*(\mathbf{q}) \rangle' \\ &= \sum_{\lambda} (\xi^\lambda)^2 k^2 e_i^{(\lambda)}(\hat{k}) e_j^{(\lambda)*}(\hat{k}) \frac{2\pi^2}{k^3} \mathcal{P}_A^\lambda(k) \\ &= \frac{2\pi^2}{k} \sum_{\lambda=L,R} \mathcal{P}_A^\lambda(k) e_i^{(\lambda)}(\hat{k}) e_j^{(\lambda)*}(\hat{k}). \end{aligned} \quad (5.7)$$

We now make use of the helicity-basis identity

$$e_i^{(\lambda)}(\hat{k}) e_j^{(\lambda)*}(\hat{k}) = \frac{1}{2} (\pi_{ij}(\hat{k}) + i\xi^\lambda \varepsilon_{ijl} \hat{k}_l), \quad (5.8)$$

to obtain

$$\begin{aligned} \langle B_i(\mathbf{k}) B_j^*(\mathbf{q}) \rangle' &= \frac{2\pi^2}{k} \frac{1}{2} \left[\pi_{ij}(\hat{k}) (\mathcal{P}_A^L(k) + \mathcal{P}_A^R(k)) \right. \\ &\quad \left. + i\varepsilon_{ijl} \hat{k}_l (\mathcal{P}_A^L(k) - \mathcal{P}_A^R(k)) \right] \\ &= \frac{2\pi^2}{k} \left[\pi_{ij}(\hat{k}) \mathcal{P}_A^S(k) + i\varepsilon_{ijl} \hat{k}_l \mathcal{P}_A^H(k) \right]. \end{aligned} \quad (5.9)$$

This is the usual decomposition into parity-even (symmetric) and parity-odd parts: note the explicit factor i in front

of the Levi-Civita tensor for the parity-odd contribution (this is required for the correlator to have the correct Hermiticity properties).

In order to map to the more common k^{-3} convention for the magnetic-field spectrum, we define the associated magnetic spectra by

$$\mathcal{P}_B^S(k) \equiv k^2 \mathcal{P}_A^S(k), \quad \mathcal{P}_B^H(k) \equiv k^2 \mathcal{P}_A^H(k). \quad (5.10)$$

Then

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{q}) \rangle' = \frac{2\pi^2}{k^3} [\pi_{ij}(\hat{k}) \mathcal{P}_B^S(k) + i \varepsilon_{ijl} \hat{k}_l \mathcal{P}_B^H(k)]. \quad (5.11)$$

We now proceed computing the GW spectrum induced at second order by the enhanced magnetic-field configuration. After the end of inflation, tensor perturbations h_{ij} obey

$$h_{ij}''(\tau, \mathbf{x}) + 2\mathcal{H}(\tau) h_{ij}'(\tau, \mathbf{x}) - \nabla^2 h_{ij}(\tau, \mathbf{x}) = \Pi_{ij}^{(T)}(\tau, \mathbf{x}), \quad (5.12)$$

where $\Pi_{ij}^{(T)}$ is the transverse-traceless (TT) part of the anisotropic stress associated with the amplified magnetic field. In Fourier space, we decompose

$$h_{ij}(\tau, \mathbf{x}) = \sum_{\lambda=\pm 2} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} e_{ij}^{(\lambda)}(\mathbf{k}) h_{\mathbf{k}}^{(\lambda)}(\tau), \quad (5.13)$$

where the spin-two polarization tensors $e_{ij}^{(\lambda)}(\mathbf{k})$ with $\lambda = [L, R] = \pm 2$, are given by the product of two spin-one tensors used to decompose the vector fields in Eq. (5.1): $e_{ij}^{(\lambda)} = e_i^{(\lambda)} e_j^{(\lambda)}$. The evolution equation for each Fourier mode for the spin-two GW is

$$h_{\mathbf{k}}^{(\lambda)''} + 2\mathcal{H} h_{\mathbf{k}}^{(\lambda)'} + k^2 h_{\mathbf{k}}^{(\lambda)} = S^{(\lambda)}(\tau, \mathbf{k}), \quad (5.14)$$

where the source term is the TT projection of the magnetic stress,

$$\begin{aligned} S^{(\lambda)}(\tau, \mathbf{k}) &\equiv e^{(\lambda)ij}(\hat{k}) \Pi_{ij}^{(T)}(\tau, \mathbf{k}) = \frac{e^{(\lambda)ij}(\hat{k}) \Lambda_{ij}^{mn} \tau_{mn}^B(\mathbf{k})}{a^2(\tau)}, \\ &= \frac{e^{(\lambda)ij}(\hat{k}) \tau_{ij}^B(\mathbf{k})}{a^2(\tau)}. \end{aligned} \quad (5.15)$$

To write these equalities, we used the definition of projector Λ ,

$$\Lambda_{ij}^{\ell m} = \frac{1}{2} \sum_{\lambda} e_{ij}^{(\lambda)} e^{(\lambda)\ell m} = \frac{1}{2} (\pi_i^{\ell} \pi_j^m + \pi_j^{\ell} \pi_i^m - \pi_{ij} \pi^{\ell m}), \quad \pi_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j. \quad (5.16)$$

The magnetic stress tensor is

$$\tau_{ij}^{(B)}(\mathbf{k}) = \frac{1}{4\pi} \int \frac{d^3 p}{(2\pi)^3} \left[B_i(\mathbf{p}) B_j(\mathbf{k} - \mathbf{p}) - \frac{\delta_{ij}}{2} B_m(\mathbf{p}) B_m(\mathbf{k} - \mathbf{p}) \right]. \quad (5.17)$$

We ignore the contribution from electric field in the source as it gets shorted during reheating, as mentioned earlier. The formal solution of Eq. (5.14) is

$$h_{\mathbf{k}}^{(\lambda)}(\tau) = \frac{1}{a(\tau)} \int d\tau' g_{\mathbf{k}}(\tau, \tau') a(\tau') S_{\mathbf{k}}^{(\lambda)}(\tau'), \quad (5.18)$$

where $g_{\mathbf{k}}(\tau, \tau')$ is the Green's function. During radiation domination, the Green's function is

$$g_{\mathbf{k}}(\tau, \tau') = \frac{1}{k} [\sin(k\tau) \cos(k\tau') - \sin(k\tau') \cos(k\tau)]. \quad (5.19)$$

The two-point function (rescaled toward the standard power-spectrum definition) reads

$$\frac{1}{2} \frac{k^3}{2\pi^2} \langle h_{\mathbf{k}}^{(\lambda)}(\tau) h_{\mathbf{q}}^{(\sigma)*}(\tau) \rangle'_{\mathbf{k}=\mathbf{q}} = \frac{k^3}{4\pi^2 a^2(\tau)} \int d\tau_1 d\tau_2 g_{\mathbf{k}}(\tau, \tau_1) g_{\mathbf{k}}(\tau, \tau_2) a(\tau_1) a(\tau_2) \langle S_{\mathbf{k}}^{(\lambda)}(\tau_1) S_{\mathbf{k}}^{(\sigma)*}(\tau_2) \rangle'. \quad (5.20)$$

We decompose GW correlators in terms of Stokes-like structures, focusing on intensity and circular polarization. Ignoring linear (spin-four) polarizations, the product of polarization tensors can be organized into parity-even and parity-odd parts as

$$e_{ij}^{(\lambda)} e_{mn}^{(\sigma)*} = \frac{\delta^{\lambda\sigma}}{2} \Lambda_{ijmn} + i \frac{\varepsilon^{\lambda\sigma}}{2} \Sigma_{ijmn}, \quad (5.21)$$

where Λ_{ijmn} is the symmetric TT combination [given in (5.16)] and $\varepsilon^{\lambda\sigma}$ is the Levi-Civita tensor in two dimensions with $\varepsilon^{LR} = +1$. We conveniently define

$$X_{ij}(\hat{v}) \equiv \varepsilon_{ijl} \hat{v}_l \quad (5.22)$$

and express Σ in the decomposition (5.21) as

$$\Sigma_{ijml}(\hat{k}) = \frac{1}{2} (\pi_{im}(\hat{k}) X_{jl}(\hat{k}) + \pi_{jl}(\hat{k}) X_{im}(\hat{k})). \quad (5.23)$$

We now collect these definitions and compute the source two-point function. We have

$$\begin{aligned} \langle S_{\mathbf{k}}^{(\lambda)}(\tau_1) S_{\mathbf{k}}^{(\sigma)*}(\tau_2) \rangle' &= \frac{e_{ij}^{(\lambda)} e_{mn}^{(\sigma)*}}{a^2(\tau_1) a^2(\tau_2)} \frac{(2\pi^2)^2}{(4\pi)^2} \int \frac{d^3 p}{(2\pi)^3 p^3 q^3} \\ &\times \left[(\pi_{im}(\hat{p}) \mathcal{P}_B^S(p) + i X_{im}(\hat{p}) \mathcal{P}_B^H(p)) (\pi_{jn}(\hat{q}) \mathcal{P}_B^S(q) + i X_{jn}(\hat{q}) \mathcal{P}_B^H(q)) + (m \leftrightarrow q) \right], \end{aligned} \quad (5.24)$$

with $\mathbf{q} = \mathbf{k} - \mathbf{p}$. We decompose $d^3 p = 2\pi p^2 dp d\mu$, with $\mu = \mathbf{p} \cdot \mathbf{k} / (pk)$. We can write

$$\langle S_{\mathbf{k}}^{(\lambda)}(\tau_1) S_{\mathbf{k}}^{(\sigma)*}(\tau_2) \rangle' = \frac{k^{-3}}{4a^2(\tau_1) a^2(\tau_2)} [\delta^{\lambda\sigma} \Gamma_{\text{even}}(k) - \varepsilon^{\lambda\sigma} \Gamma_{\text{odd}}(k)], \quad (5.25)$$

where the scalar functions Γ_{even} (parity even) and Γ_{odd} (parity odd) are real. Taking into account of the properties of the Levi-Civita tensor, they are obtained by expanding Eq. (5.24), keeping the nonvanishing contributions,

$$\begin{aligned} \Gamma_{\text{even}}(k) &= \int \frac{dp d\mu}{p |\mathbf{q}|^3 / k^3} \left\{ \Lambda_{ijmn}(\hat{k}) [\pi_{im}(p) \pi_{jn}(q) + \pi_{in}(p) \pi_{jm}(q)] \mathcal{P}_B^S(p) \mathcal{P}_B^S(q) \right. \\ &\quad \left. - \Lambda_{ijmn}(\hat{k}) [X_{im}(p) X_{jn}(q) + X_{in}(p) X_{jm}(q)] \mathcal{P}_B^H(p) \mathcal{P}_B^H(q) \right\} \end{aligned} \quad (5.26)$$

and

$$\begin{aligned} \Gamma_{\text{odd}}(k) &= \int \frac{dp d\mu}{p |\mathbf{q}|^3 / k^3} \left\{ \Sigma_{ijmn}(\hat{k}) [\pi_{im}(p) X_{jn}(q) + \pi_{in}(p) X_{jm}(q)] \mathcal{P}_B^S(p) \mathcal{P}_B^H(q) \right. \\ &\quad \left. + \Lambda_{ijmn}(\hat{k}) [X_{im}(p) \pi_{jn}(q) + X_{in}(p) \pi_{jm}(q)] \mathcal{P}_B^H(p) \mathcal{P}_B^S(q) \right\}. \end{aligned} \quad (5.27)$$

Using the dimensionless variables

$$u = \frac{|\mathbf{k} - \mathbf{p}|}{k}, \quad v = \frac{p}{k}, \quad \mu = \frac{\mathbf{k} \cdot \mathbf{p}}{kp} = \frac{1 - u^2 + v^2}{2v}, \quad (5.28)$$

we obtain the final convolution integrals

$$\Gamma_{\text{even}}(k; \tau_1, \tau_2) = \int_0^\infty dv \int_{-1}^1 d\mu \frac{1}{u^3 v} [\mathcal{C}_{SS}(u, v, \mu) \mathcal{P}_B^S(ku) \mathcal{P}_B^S(kv) + \mathcal{C}_{HH}(u, v, \mu) \mathcal{P}_B^H(ku) \mathcal{P}_B^H(kv)], \quad (5.29)$$

and

$$\Gamma_{\text{odd}}(k; \tau_1, \tau_2) = \int_0^\infty dv \int_{-1}^1 d\mu \frac{1}{u^3 v} [\mathcal{C}_{\text{mix}}^A(u, v, \mu) \mathcal{P}_B^S(ku) \mathcal{P}_B^H(kv) + \mathcal{C}_{\text{mix}}^B(u, v, \mu) \mathcal{P}_B^H(ku) \mathcal{P}_B^S(kv)]. \quad (5.30)$$

Here $\mathcal{C}_{SS}, \mathcal{C}_{HH}, \mathcal{C}_{\text{mix}}^{A,B}$ are scalar kernels obtained after contracting the geometric tensors with the TT projector and polarization tensors. After performing the necessary contractions, the scalar kernels result

$$\begin{aligned}\mathcal{C}_{SS}(u, v, \mu) &= \frac{(1 + \mu^2)(2 + v(v - 4\mu + v\mu^2))}{2(1 + v^2 - 2v\mu)}, \\ \mathcal{C}_{HH}(u, v, \mu) &= \frac{2\mu(-1 + v\mu)}{\sqrt{1 + v^2 - 2v\mu}},\end{aligned}\quad (5.31)$$

$$\begin{aligned}\mathcal{C}_{\text{mix}}^A(u, v, \mu) &= \frac{(1 - v\mu)(1 + \mu^2)}{\sqrt{1 + v^2 - 2v\mu}}, \\ \mathcal{C}_{\text{mix}}^B(u, v, \mu) &= \frac{\mu(2 + v(v - 4\mu + v\mu^2))}{1 + v^2 - 2v\mu}.\end{aligned}\quad (5.32)$$

Plugging these formulas into (5.20), the tensor two-point function can be compactly written as (a prime indicates correlators without the Dirac δ functions associated with momentum conservation)

$$\frac{1}{2} \frac{k^3}{2\pi^2} \langle h_{\mathbf{k}}^{(\lambda)}(\tau) h_{\mathbf{s}}^{(\sigma)*}(\tau) \rangle'_{\mathbf{k}=\mathbf{q}} = \frac{\pi}{2a^2(\tau)} \mathcal{I}_{\tau}^2 \mathcal{I}_{uv}^{(\lambda\sigma)}, \quad (5.33)$$

with

$$\mathcal{I}_{\tau}^2 = \left[\int_{\tau_R}^{\tau} \frac{d\tau_1}{a(\tau_1)} g_k(\tau, \tau_1) \right]^2, \quad (5.34)$$

$$\mathcal{I}_{uv}^{(\lambda\sigma)} = \delta^{\lambda\sigma} \Gamma_{\text{even}}(k; \tau_1, \tau_2) + \epsilon^{\lambda\sigma} \Gamma_{\text{odd}}(k; \tau_1, \tau_2). \quad (5.35)$$

Notice that, for vector-induced gravitational-wave spectra, the two-point correlator (5.33) factorizes into a product of a time integral and a momentum integral: the former depends only on Green's functions during radiation domination, while the latter encodes the properties of the magnetic-field source, see [22]. This separation allows us to estimate the two contributions independently.

(i) Time integral. During radiation domination, $a(\tau)/a(\tau_1) = \tau/\tau_1$ and $a(\tau)H(\tau) = 1/\tau$. Averaging over rapid oscillations yields

$$\overline{\mathcal{I}_{\tau}^2} = \frac{1}{2k^2 a^4 H^2} \left[\text{Ci}(-k\tau_R)^2 + \left(\frac{\pi}{2} - \text{Si}(-k\tau_R) \right)^2 \right], \quad (5.36)$$

where Ci and Si are the cosine and sine integral functions. τ_R is a fiducial conformal time when the radiation domination epoch starts.

(ii) Momentum integral. We introduce the standard variables t and s via

$$u = \frac{t + s + 1}{2}, \quad v = \frac{t - s + 1}{2}. \quad (5.37)$$

Including the Jacobian for this transformation, we can carry out the momentum integral for $0 \leq t \leq \infty$

and $-1 \leq s \leq 1$.¹ The resulting GW energy density is (for the moment, we keep the polarization indexes, later we define more “physical” quantities)

$$\begin{aligned}\Omega_{\text{GW}}^{(\lambda\sigma)} &= \frac{k^2}{12a^2 H^2} \times \frac{1}{2} \frac{k^3}{2\pi^2} \langle h_{\mathbf{k}}^{(\lambda)}(\tau) h_{\mathbf{q}}^{(\sigma)*}(\tau) \rangle' \\ &= \frac{\pi k^2}{24a^4 H^2} \overline{\mathcal{I}_{\tau}^2} \mathcal{I}_{uv}^{(\lambda\sigma)}.\end{aligned}\quad (5.38)$$

During radiation domination,

$$a(\tau) = H_0 \sqrt{\Omega_{\text{rd}}} \tau, \quad (5.39)$$

where H_0 and Ω_{rd} are the present-day Hubble parameter and radiation energy fraction. Defining $\hat{\Omega}_B = \rho_B/\rho_{\text{cr}} = \mathcal{P}_B^{\text{CMB}}/(3H_0^2)$ and including the redshift factor Ω_{rd} [56], we obtain

$$\Omega_{\text{GW}}^{(\lambda\sigma)}(\kappa) = \left[\frac{3\hat{\Omega}_B^2}{64\Omega_{\text{rd}}} \right] \left[\text{Ci}^2(\kappa x_{\star}) + \left(\frac{\pi}{2} - \text{Si}(\kappa x_{\star}) \right)^2 \right] \mathcal{I}_{uv}^{(\lambda\sigma)}(\kappa), \quad (5.40)$$

with $\kappa = -k\tau_1$, with $|\tau_1|$ the epoch during inflation associated with the non-slow-roll phase. The small parameter $x_{\star} = |\tau_R/\tau_1|$ controls the ratio between the relevant conformal times when radiation domination starts versus the non-slow-roll epoch and enters only logarithmically in the final result; we adopt $x_{\star} = 10^{-4}$ for definiteness. The prefactor $[3\hat{\Omega}_B^2/64\Omega_{\text{rd}}]$ is fixed to be 2×10^{-48} as per the arguments presented in [22].

We can now introduce more physically transparent combinations, by extracting the parity-even and parity-odd parts of the GW spectrum, defined as

$$\Omega_{\text{GW}} = \Omega_{\text{GW}}^{(LL)} + \Omega_{\text{GW}}^{(RR)}, \quad (5.41)$$

$$\Omega_{\text{GW}}^V = \Omega_{\text{GW}}^{(LL)} - \Omega_{\text{GW}}^{(RR)}. \quad (5.42)$$

The quantity Ω_{GW} is the traditional GW energy density computed through the GW intensity, while Ω_{GW}^V aims at characterizing the effects of helical magnetic fields that induce circular polarization in the GW background.

A. Results and implications for GW experiments

We compute the GW density parameter Ω_{GW} and associated chirality Ω_{GW}^V induced by the spectra $\mathcal{P}_B(k)$ using Eq. (5.40). We focus on our first scenario described in Sec. III—see the plot in Fig. 4. The behavior of the resulting GW spectra is illustrated in Fig. 7, where we plot Ω_{GW} and Ω_{GW}^V as functions of frequency. We set

¹For numerical implementation, we take the range of t to be $[10^{-2}, 200]$.

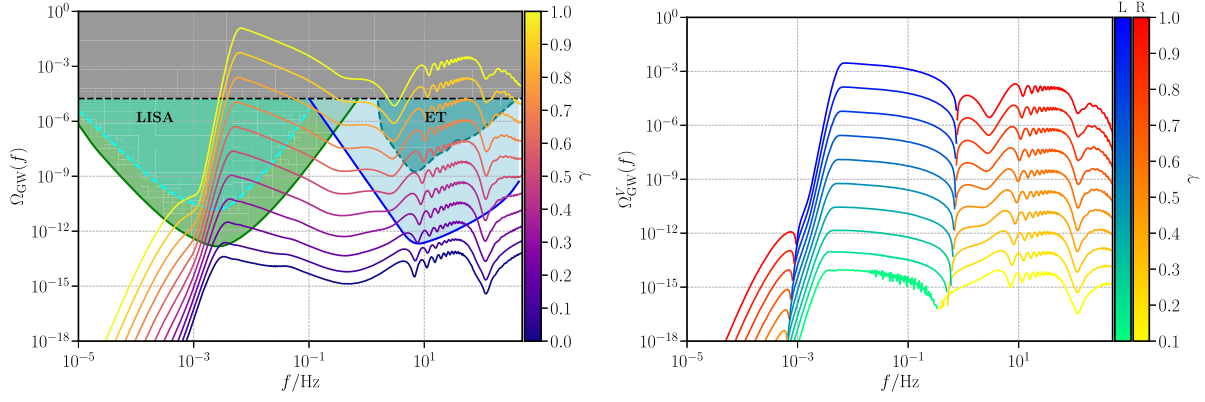


FIG. 7. The spectral density Ω_{GW} and the associated V mode Ω_{GW}^V are presented for a range of $\gamma = [0, 1]$, corresponding to the magnetic spectra presented in Fig. 4. For reference, we plot the sensitivity curves of LISA (the nominal form in cyan and for the broken-power-law case in green) and ET (the nominal form in teal and for the power-law case in blue) [57,58]. We also demarcate the bound of $\Omega_{\text{GW}} < 1.74 \times 10^{-5}$ from BBN (in black dashed line) [59]. Beyond the overall amplification of Ω_{GW} due to nonzero γ , we find the spectrum having significant helicity as evidenced in Ω_{GW}^V . The feature of altering helicity of $\mathcal{P}_B(k)$ induces the corresponding feature of altering chirality around the peak of GW spectra as seen in Ω_{GW}^V .

$n_2 = -3$, $\Delta N_2 = 3$ and the onset of the intermediate phase in J at $N_1 = 15$ such that the corresponding wave number $k_1 \simeq 10^{14} \text{ Mpc}^{-1}$. These choices are motivated by the requirement that the peak of Ω_{GW} enters in the range of interest for future GW experiments, once the units are converted into frequency expressed in hertz (more on this later).

A parity-violating term in the action, controlled by the parameter γ , significantly increases the induction of GW (see Fig. 7, left panel). For example, choosing $\gamma = 1$ leads to an amplification of about 10^{12} in the amplitude of Ω_{GW} , when compared to $\gamma = 0$. This behavior can be understood from the relation $\Omega_{\text{GW}} \propto \mathcal{P}_B^2$ and the amplification is about 10^6 at the peak of $\mathcal{P}_B$ (cf. Fig. 4). The bound from big bang nucleosynthesis (BBN) already rules out a range of values of $\gamma > 0.7$ for the case of $n_2 = -3$ and $\Delta N_2 = 3$. As to $\Omega_{\text{GW}}(f)$, we find that the values of $\gamma \in [0.2, 0.7]$ lead to amplitude significant enough to be detected by the sensitivity of the Laser Interferometer Space Antenna (LISA). Additionally, the features of the magnetic-field spectral growth, its prominent peak, a second subdominant peak, and its subsequent oscillations as seen in $\mathcal{P}_B(k)$ are well reflected and captured in the $\Omega_{\text{GW}}(f)$ profile.

Turning to the behavior of chirality in the GW spectrum $\Omega_{\text{GW}}^V(f)$, the signature of altering helicities around the peak of $\mathcal{P}_B(k)$ is reproduced in the chirality of induced GW (see Fig. 7, right panel). Note that it is nontrivial to expect this behavior *a priori*, due to the nonlinearities in the convolution integrals and the specific integral kernels. We may see from the structure of the kernels involved in integration of source functions [cf. Eqs. (5.29) and (5.30)] that both polarizations of $\mathcal{P}_B^\lambda(k)$ contribute to both chiralities of $\Omega_{\text{GW}}^\lambda(f)$. Nonetheless, we find that the peak amplitude of Ω_{GW}^V is positive (left chiral) and it switches to negative

values (right chiral) on either side. Moreover, we should note that, for any given value of γ , the spectrum is never maximally chiral, i.e., $\Omega_{\text{GW}}^V/\Omega_{\text{GW}} < 1$; the reason being, again, that both polarizations of $\mathcal{P}_B^{\text{L,R}}(k)$ contribute to a given chirality of induced GW.

Our findings open up interesting opportunities for GW experiments. It is well known that directly measuring GW circular polarization is challenging with planar detectors such as LISA [60,61] and the Einstein Telescope (ET) [62], see, e.g., [63] (as well as with pulsar timing arrays; see, e.g., [64]). Nevertheless, the two panels in Fig. 7 suggest at least two distinct strategies for detecting effects of parity violation in the GW spectrum:

- (1) Information about chirality can be extracted from measurements of Ω_{GW} with LISA and ET in synergy [58]. In particular, as shown in the left panel of Fig. 7, Ω_{GW} exhibits a characteristic double-peak structure that would be accessible through a multi-band detection with LISA and ET. Distinctive features of the second peak in the ET frequency band—such as the abrupt change in slope preceding the peak and the subsequent oscillatory behavior—depend sensitively on the value of the chirality parameter γ . Accurate measurements of the spectral shape of Ω_{GW} could therefore allow one to infer γ , potentially extending the reconstruction techniques developed in [65,66].
- (2) For suitable choices of parameters, the amplitude of Ω_{GW}^V can be quite large, and at the same time Ω_{GW} satisfies BBN bounds at frequencies between millihertz and tens of hertz. Although Ω_{GW}^V cannot be detected with measurements of the isotropic background, it can be measured from the properties of the

kinematic dipole anisotropy [64,67–70], which might be detectable in this regime.

A detailed quantitative investigation of both these possibilities is left for future work. Further, we must note that there are very few models of parity violation that are known to generate a strong signal of Ω_{GW} over a wide range of frequency with distinctly different chiralities of GW over different windows of frequencies (see [71,72] for such scenarios). The narrow set of candidate models predicting altering chirality shall help in breaking of degeneracy during comparison against observations.

ACKNOWLEDGMENTS

H. V. R. thanks Ye Shifeng for valuable queries and comments. H. V. R. acknowledges support by the MUR PRIN2022 Project “BROWSEPOL: Beyond standard model with cosmic microwave background polarization” 2022EJNZ53 financed by the European Union—Next

Generation EU. H. V. R. acknowledges the use of the computing cluster under University of Padova Strategic Research Infrastructure Grant 2017: “CAPRI: Calcolo ad Alte Prestazioni per la Ricerca e l’Innovazione.” G. T. is partially funded by the STFC Grants No. ST/T000813/1 and No. ST/X000648/1. L. S. wishes to thank the Indo-French Centre for the Promotion of Advanced Research (IFCPAR/CEFIPRA), New Delhi, India, for support of the proposal 6704-4 titled “Testing flavors of the early Universe beyond vanilla models with cosmological observations” under the Collaborative Scientific Research Programme.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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