

Tree-level NLO corrections to inclusive ψ' production in high energy factorization

A. A. Prokhorov¹, S. P. Baranov², A. V. Lipatov³, M. A. Malyshev^{3,4} and X. Chen^{5,6}

¹*Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia*

²*P. N. Lebedev Institute of Physics, 119991 Moscow, Russia*

³*Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, 119991 Moscow, Russia*

⁴*Moscow Aviation Institute, 125993 Moscow, Russia*

⁵*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*

⁶*School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China*



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We consider inclusive $\psi(2S)$ production in proton-proton collisions at collider energies in the framework of nonrelativistic QCD and high energy factorization beyond the low-order approximation. We utilize a matching scheme proposed earlier to merge the leading-order and tree-level next-to-leading-order (NLO) production amplitudes and now extend it to low transverse momenta and/or forward rapidities. With the improved scheme, we examine the possibility to simultaneously describe all unpolarized LHC data in the full kinematic range accessible to experimental measurements and try different transverse momentum-dependent gluon distributions in the proton. A global fit to the data is carried out to extract the color octet long-distance matrix elements for $\psi(2S)$ mesons. We show that taking the NLO corrections into account leads to a better description of the data.

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I. INTRODUCTION

Understanding the data on the inclusive and associated production of heavy quarks bound states—heavy quarkonia—is one of topical subjects of modern high energy physics [1–3]. Among the different approaches to the problem, the formalism of nonrelativistic QCD (NRQCD) [4–6] is commonly regarded as providing the most rigorous working framework. For the sake of completeness, one also should mention the color singlet model [7–9] (actually, a predecessor of NRQCD) and the color evaporation model [10–13]. The NRQCD formalism exploits a hypothesis that the perturbative production of heavy quarks in a hard parton-parton interaction factorizes from their subsequent nonperturbative evolution to a physical meson. The probabilities of these two sequential steps are given, accordingly, by the partonic cross section $\sigma(Q\bar{Q})$ and the long-distance matrix element (LDME).

In calculating the perturbative step, we employ the k_T -factorization [14] (or high energy factorization [15])

approach where the gluon distributions may obey Balitsky-Fadin-Kuraev-Lipatov [16] or Ciafaloni-Catani-Fiorani-Marchesini (CCFM) [17] evolution equations and are called transverse momentum dependent (TMD) or unintegrated. This approach can be considered as convenient alternative to explicit high-order perturbative QCD calculations. It effectively includes the leading-log part of higher-order corrections (namely, the terms proportional to $\alpha_s^n \ln^n s / \Lambda_{\text{QCD}}^2 \sim \alpha_s^n \ln^n 1/x$ up to infinitely large n) absorbing them into TMD gluon densities. These corrections correspond to the emission of real gluons in the initial state and play the dominant role at high energies s (or, equivalently, at small x). A detailed description of the k_T -factorization technique can be found, for example, in reviews [18,19].

The incorporation of the k_T -factorization approach with the NRQCD formalism enables one to achieve an improved overall description of the collider data on heavy quarkonia. In fact, the description obtained in the k_T -factorization already with leading-order (LO) off-shell amplitudes for inclusive charmonia and bottomonia production (see [20–25] and references therein) is of not poorer quality than that seen in the collinear factorization at next-to-leading order (NLO) [26–39] (see also [2]). Employing the special depolarization model [40] based on the classical multipole radiation theory made it possible to obtain consistent predictions for J/ψ and ψ' polarization observables, thus providing a solution to

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a long-standing “polarization puzzle.” Further progress has recently been made [41] in restoring the heavy quark spin symmetry (HQSS) relations [42,43] in the χ_c production. Violations of these relations were present in our previous studies [20,22,44], but have gone away after including the NLO contributions.

In the present study, we turn to the inclusive production of ψ' mesons at the LHC conditions. A recent analysis [45] performed in the conventional (collinear) factorization scheme at NLO shows that a reasonable agreement with the data (polarized and unpolarized) can only be achieved for a limited p_T region, $p_T(\psi') > 7$ TeV, while the theory strongly overestimates the data at smaller $p_T(\psi')$. The same kind of discrepancy is also seen in the k_T -factorization predictions at LO [20] for definite sets of TMD gluon distributions. Our present goal is to improve the theoretical calculations by implementing the NLO corrections.

A major difficulty in including higher-order corrections in the k_T -factorization is to avoid double counting, i.e., the cases when the same gluon emission is counted twice [41]: as part of the initial gluon evolution and as part of the hard partonic subprocess. Therefore, one needs to take care to separate the gluon emissions in the phase space and then to accurately merge the LO and NLO amplitudes. We basically follow the procedure [41], which has already been successfully used for inclusive χ_c production. Now, with some proper modifications, we extend it to relatively low transverse momenta and/or forward rapidities. We wish to perform a global fit to all available unpolarized ψ' data, leaving the polarization analysis for a following dedicated study.¹ The data included in the fit are taken at $\sqrt{s} = 5.02, 7, 8$ and 13 TeV and cover the entire rapidity range [49–58]. Such an extensive analysis of the ψ' production in the k_T -factorization approach with taking into account NLO terms is performed for the first time.

The paper is organized as follows. In Sec. II, we briefly describe our theoretical framework and the basic steps of our calculations. In Sec. III, we present the numerical results and discussions. Section IV sums up our conclusions.

II. THEORETICAL FRAMEWORK

This section provides a brief review of the k_T -factorization formalism applied to ψ' production, the essential calculation steps, and the fitting procedure. In general, our method of calculations resembles the usual NLO* (tree-level NLO) scheme, though with two notable exceptions.

¹The only known data on ψ' polarization are collected by the CMS [46,47] at $\sqrt{s} = 7, 13$ TeV and LHCb [48] Collaborations at $\sqrt{s} = 7$ TeV. These data contain a small amount of points in comparison with unpolarized (measurements of polarization observables in different frames may not be considered as independent) and show large experimental uncertainties. So, they cannot affect the global fit and, for the time being, can be ignored.

First, in the k_T -factorization approach, the true LO contribution to the CO production mechanism is represented by a $2 \rightarrow 1$ off-shell gluon-gluon fusion subprocess, while in the collinear case the perturbation series starts from $2 \rightarrow 2$ subprocesses (see below). Accordingly, our $2 \rightarrow 2$ subprocesses are regarded as NLO corrections. Second, our calculations are also made in the tree-level approximation, but for a different reasoning and in a different way as compared to the usual NLO*. The kinematic constraints that reject virtual and infrared singularities arise from a double-counting-exclusion (DCE) condition, which separates the LO and NLO contributions (see also [59,60]). For the sake of definiteness and clarity, we hereafter will refer to this scheme as to NLO[†] [41].

A. Basic formulas

The LO contributions to ψ' production are presented by several $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ off-shell gluon-gluon fusion subprocesses producing the $c\bar{c}$ pairs in the CO and CS states, respectively,

$$g^*(k_1) + g^*(k_2) \rightarrow c\bar{c} \left[{}^1S_0^{[8]}, {}^3P_J^{[8]}, {}^3S_1^{[8]} \right] (p), \quad (1)$$

$$g^*(k_1) + g^*(k_2) \rightarrow c\bar{c} \left[{}^3S_1^{[1]} \right] (p) + g(p_g), \quad (2)$$

where the four-momenta of all particles are indicated in the parentheses and $J = 0, 1, 2$. It is important that the interacting gluons have nonzero transverse momenta, non-zero virtualities $k_1^2 = -\mathbf{k}_{1T}^2 \neq 0$, $k_2^2 = -\mathbf{k}_{2T}^2 \neq 0$, and longitudinal components in their polarization vectors. The tree-level NLO corrections are represented by a $2 \rightarrow 2$ color octet $\mathcal{O}(\alpha_s^3)$ subprocess,

$$g^*(k_1) + g^*(k_2) \rightarrow c\bar{c} \left[{}^1S_0^{[8]}, {}^3P_J^{[8]}, {}^3S_1^{[8]} \right] (p) + g(p_g). \quad (3)$$

If the produced $c\bar{c}$ pair forms a color singlet state ${}^3S_1^{[1]}$, it can directly convert into ψ' meson with the probability given by the radial wave function at the origin of the coordinate space $|R_\psi(0)|^2$ (see [7–9]). The latter can be extracted from the measured ψ' leptonic decay width or calculated within the potential models [61,62]. The CO states evolve into real ψ' mesons through nonperturbative QCD transitions (via soft gluon emissions). According to the multipole radiation scenario [40], this step is described in terms of chromoelectric dipole (E1) transitions [63,64], which dominate the multipole expansion. In the case of ${}^3P_J^{[8]}$ states, a single E1 transition $c\bar{c} [{}^3P_J^{[8]}] \rightarrow c\bar{c} [{}^3S_1^{[1]}] + g$ is needed to transform them into ψ' mesons, whereas the transformation of ${}^3S_1^{[8]}$ state is treated as two sequential transitions: $c\bar{c} [{}^3S_1^{[8]}] \rightarrow c\bar{c} [{}^3P_J^{[8]}] + g$ and $c\bar{c} [{}^3P_J^{[8]}] \rightarrow c\bar{c} [{}^3S_1^{[1]}] + g$, proceeding via either of three intermediate CO states: $J = 0, 1, 2$. It is

essential in our model that the gluons are not infinitely soft, but carry some finite energy $E_g \sim \Lambda_{\text{QCD}}$, which ensures escaping the domain of confinement (see also [40]). The intermediate color octet state must then be heavier than the final state quarkonium. The energy required to overcome the confinement is a model parameter that brings additional uncertainty to numerical calculations (see discussion below).

The expressions for the off-shell production amplitudes (1)–(3) can be found in [20,41], where their gauge invariance has been specially tested. The latter is achieved by using effective vertices (see [65,66] and references therein) that ensure the exact gauge invariance even with the off-shell initial gluons. The probabilities to form definite bound states are given by the corresponding LDMEs $\langle \mathcal{O}^{\psi'}[n] \rangle$, where n is the proper Fock state. The details of calculations are described in [20,41]. Here we note that the proposed approach results in a good description of the available LHC data on charmonia and bottomonia polarizations (see [20–25]).

According to the k_T -factorization prescription [14,15], the cross section of a considered process is calculated as a convolution of an off-shell production amplitude $|\overline{\mathcal{A}}|^2$ and TMD gluon densities in the proton, $f_g(x, \mathbf{k}_T^2, \mu^2)$. Thus, the cross sections for the $2 \rightarrow 1$ and $2 \rightarrow 2$ subprocesses (1)–(3) can be written as

$$\sigma_{2 \rightarrow 1} = \int \frac{2\pi}{x_1 x_2 s F} f_g(x_1, \mathbf{k}_{1T}^2, \mu^2) f_g(x_2, \mathbf{k}_{2T}^2, \mu^2) |\overline{\mathcal{A}_{2 \rightarrow 1}}|^2 \times d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 dy \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi}, \quad (4)$$

$$\sigma_{2 \rightarrow 2} = \int \frac{1}{8\pi x_1 x_2 s F} f_g(x_1, \mathbf{k}_{1T}^2, \mu^2) f_g(x_2, \mathbf{k}_{2T}^2, \mu^2) |\overline{\mathcal{A}_{2 \rightarrow 2}}|^2 \times d\mathbf{p}_T^2 d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 dy dy_g \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi}, \quad (5)$$

where ϕ_1 and ϕ_2 are the azimuthal angles of the initial off-shell gluons, p_T and y are the transverse momentum and rapidity of the produced ψ' meson, y_g is the rapidity of the outgoing gluon, $\sqrt{s}$ is the pp center-of-mass energy, μ is the factorization scale, and $F = 2\lambda^{1/2}(\hat{s}, k_1^2, k_2^2)$ is the flux factor,² where $\hat{s} = (k_1 + k_2)^2$ [68]. The necessary merging procedure for $2 \rightarrow 1$ and $2 \rightarrow 2$ contributions is discussed below.

Our calculations refer to two sets of TMD gluon densities, namely, A0 [69] and JH'2013 set 2 [70]. All of these gluon distributions are obtained from a numerical solution of the CCFM equation [at the leading logarithmic approximation (LLA)³] and are widely used in phenomenological

applications (see, for example, [72–76] and references therein). The parameters of (rather empirical) input distributions used in the JH'2013 set 2 and A0 sets were derived from a fit of the proton structure functions $F_2(x, Q^2)$ and $F_2^c(x, Q^2)$ to HERA data at small x . All these TMD gluon densities are available from a commonly known package TMDlib [77], which is a C++ library providing a framework and an interface to many different parametrizations. They are implemented also into the Monte Carlo event generator PEGASUS [78].

B. Merging the LO and NLO contributions

In this section we discuss the procedure for merging the $2 \rightarrow 1$ (LO) and $2 \rightarrow 2$ (NLO) contributions, which is necessary to avoid double counting in the k_T -factorization calculations. We follow the scheme that was first proposed and successfully applied to χ_{cJ} production (see [41] for more details). The key idea of this method is to introduce a DCE cut for $2 \rightarrow 2$ subprocess: $|\mathbf{p}_{gT}| > \max(|\mathbf{k}_{1T}|, |\mathbf{k}_{2T}|)$. It excludes the gluons already generated by the CCFM evolution in $2 \rightarrow 1$ and ensures that the hardest gluon emission in $2 \rightarrow 2$ events comes from the hard matrix elements.

The method works well in the central region, but there exist specific kinematic conditions where such DCE cut may not be sufficient. Let us consider the forward production at low p_T (say, less than 5 or 10 GeV). Here, one of the incoming gluons k_2 (the one moving in the direction opposite to the produced quarkonium) carries significantly lower momentum fraction than the other gluon k_1 ($x_2 \ll x_1$, up to several orders of magnitude). The evolution of the k_1 gluon differs from that of the gluon k_2 . In particular, there is a smearing of the angular ordering,⁴ that affects the transverse momenta of the gluons belonging to the k_2 evolution branch. The primordial gluon transverse momentum⁵ $\mathbf{k}_T^{\text{init}}$ becomes more important and should be taken into account. To safely avoid double counting under these conditions, we should strengthen the DCE cut as $|\mathbf{p}_{gT}| > \max(|\mathbf{k}_{1T}|, |\mathbf{k}_{2T}| + |\overline{\mathbf{k}}_{2T}^{\text{init}}|)$. This shift is based on a simple kinematic relation for gluon emissions $\mathbf{k}_{2T}^{\text{init}} = \sum \mathbf{p}_{gT_i}^{\text{emission}} + \mathbf{k}_{2T}$ together with an estimation of the transverse momentum of the hardest gluon in the evolution $|\mathbf{p}_{gT}^{\text{emission}}| < |\mathbf{k}_{2T}| + |\mathbf{k}_{2T}^{\text{init}}|$. The parameter $|\overline{\mathbf{k}}_{2T}^{\text{init}}|$ represents the average transverse momentum of the primordial gluon and can be extracted from the TMD gluon distributions at the starting scale μ_0 : $|\overline{\mathbf{k}}_T^{\text{init}}| = 1.12$ and 1.77 GeV for A0 and JH'2013 set 2 gluons, respectively. Such DCE cut efficiently excludes $2 \rightarrow 2$ events where the gluon emitted at the last step of the evolution has smaller transverse momentum

²In the case of $2 \rightarrow 2$ processes, this formula approximates standard expression $F \sim x_1 x_2 s$, see discussion [67] for more details.

³Unfortunately, the next-to-leading logarithmic corrections to the CCFM equation are yet not known. However, it can be argued [71] that the CCFM evolution at the LLA leads to reasonable QCD predictions.

⁴In case of CCFM evolution, the gluon emissions obey the angular ordering condition arising from color coherence effect.

⁵We mean the momentum of the gluon at the starting scale μ_0 , before the first step of the evolution.

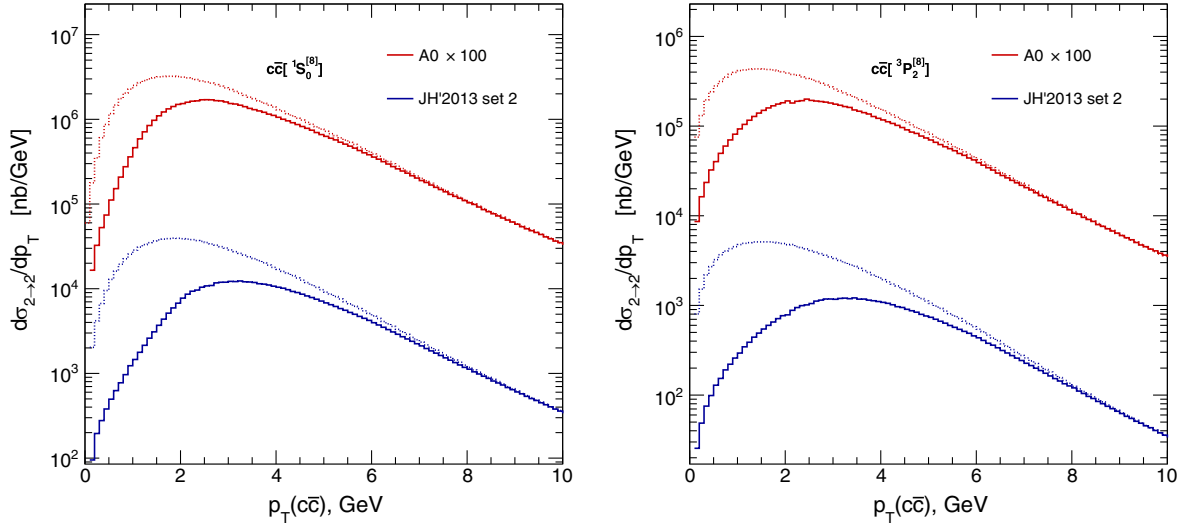


FIG. 1. Comparison of $d\sigma_{2\rightarrow 2}/dp_T$ contributions for color octet $c\bar{c}[^1S_0^{[8]}]$ (left) and $c\bar{c}[^3P_2^{[8]}]$ (right) with standard (dashed) and enhanced DCE cuts (solid) in the forward rapidity region $2 < y < 4.5$ at $\sqrt{s} = 13$ TeV.

than the primordial gluon. In other words, gluons acquiring in the evolution larger p_T than the gluons coming from the hard subprocess are rejected by the merging scheme. Such gluons are already taken into account in the LO part of the calculations. This way, we safely avoid double counting in the low p_T region.

The influence of the strengthened DCE cut on the NLO[†] contributions from $^1S_0^{[8]}$ and $^3P_2^{[8]}$ intermediate states is shown in Fig. 1. The calculations were done for relatively low p_T in the forward rapidity region, $2 < y^\psi < 4.5$. The modified (strengthened) scheme converges to the non-modified scheme at $p_T > 6$ GeV. We thus conclude that the modified DCE cut provides a somewhat more accurate treatment at relatively low p_T and/or forward rapidities and smoothly goes to the previous scenario [41] at larger transverse momenta. Note that using strengthened DCE cut is the only improvement of our present calculations.

C. Numerical parameters

Following [79], we set the meson mass to $m(\psi') = 3.686$ GeV and branching fraction $\text{Br}(\psi' \rightarrow \mu^+ \mu^-) = 0.8\%$. We use one-loop expression for the QCD coupling α_s with $n_f = 4$ quark flavors and $\Lambda_{\text{QCD}} = 250$ MeV for A0 gluon and two-loop expression with $n_f = 4$ and $\Lambda_{\text{QCD}} = 200$ MeV for JH'2013 set 2 gluon distribution [69,70]. Our default choice for the renormalization scale μ_R is the transverse mass m_T^ψ of the produced meson. The factorization scale μ_F was set to $\mu_F^2 = \hat{s} + \mathbf{Q}_T^2$, where $\mathbf{Q}_T$ is the net transverse momentum of the initial off-shell gluon pair. Such a choice of μ_F is specific for CCFM evolution (see [17] for more details). However, in accordance with the proposed merging LO + NLO scheme [41], we shift the

scale as $\mu_F \rightarrow \mu_F = m_T^\psi$ for $2 \rightarrow 2$ CO subprocesses. The CS LDME is determined from the meson decay width [80]: $\langle \mathcal{O}^{\psi'}[^3S_1^{[1]}] \rangle = 0.73 \text{ GeV}^3$.

D. LDME fitting procedure

The values of LDMEs for different CO intermediate states are free parameters not predictable by the theory and have to be found from the experimental data. As it was said, we perform a global fit that includes all of the available unpolarized LHC data on inclusive ψ' production. There are only three such free parameters, namely, $\langle \mathcal{O}^{\psi'}[^1S_0^{[8]}] \rangle$, $\langle \mathcal{O}^{\psi'}[^3P_0^{[8]}] \rangle$, and $\langle \mathcal{O}^{\psi'}[^3S_1^{[8]}] \rangle$, while the other LDMEs can be obtained from HQSS relations,

$$\langle \mathcal{O}^{\psi'}[^3P_J^{[8]}] \rangle = (2J+1) \langle \mathcal{O}^{\psi'}[^3P_0^{[8]}] \rangle, \quad (6)$$

where $J = 0, 1, 2$. According to the NRQCD formalism, the ψ' production cross section reads

$$\begin{aligned} \frac{d\sigma^{\psi'}}{dp_T} = & \frac{d\sigma(c\bar{c}[^3S_1^{[1]}])}{dp_T} \langle \mathcal{O}^{\psi'}[^3S_1^{[1]}] \rangle + \frac{d\sigma(c\bar{c}[^1S_0^{[8]}])}{dp_T} \langle \mathcal{O}^{\psi'}[^1S_0^{[8]}] \rangle \\ & + \sum_J (2J+1) \frac{d\sigma(c\bar{c}[^3P_J^{[8]}])}{dp_T} \langle \mathcal{O}^{\psi'}[^3P_0^{[8]}] \rangle \\ & + \frac{d\sigma(c\bar{c}[^3S_1^{[8]}])}{dp_T} \langle \mathcal{O}^{\psi'}[^3S_1^{[8]}] \rangle. \end{aligned} \quad (7)$$

The values of LDMEs are determined by minimizing the χ^2 function running over all data points. The uncertainties are calculated from the error matrix. The fitted values of

TABLE I. Values of r calculated in different rapidity regions for A0 and JH'2013 set 2 gluon densities in a proton.

TMD	Central	Forward	Combined
A0	0.51 ± 0.04	0.64 ± 0.07	0.54 ± 0.07
JH'2013 set 2	0.56 ± 0.04	0.65 ± 0.07	0.58 ± 0.06

LDMEs show strong negative correlation between $^1S_0^{[8]}$ and $^3P_J^{[8]}$ contributions for both considered TMD densities. This makes it impossible to extract these parameters with acceptable accuracy. Therefore, we replace the individual parameters with a linear combination,

$$\langle \mathcal{O}_{l.c.} \rangle = \langle \mathcal{O}[^3P_0^{[8]}] \rangle + r \times \langle \mathcal{O}[^1S_0^{[8]}] \rangle, \quad (8)$$

where r is defined as

$$r = \frac{d\sigma(c\bar{c}[^1S_0^{[8]}])/dp_T}{\sum (2J+1)d\sigma(c\bar{c}[^3P_J^{[8]}])/dp_T}. \quad (9)$$

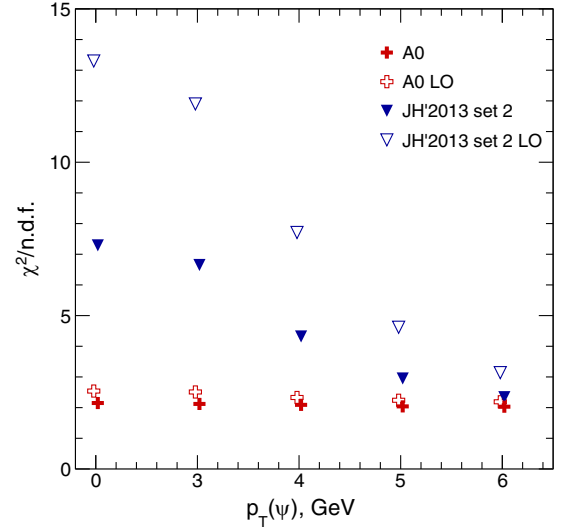
The coefficient r was calculated for every data point. Its values were averaged over the whole rapidity range and, separately, over the central and forward rapidity regions.⁶ We present them in Table I for both A0 and JH'2013 set 2 gluons.

The results presented in the next section are obtained with only two free parameters, $\langle \mathcal{O}_{l.c.} \rangle$ and $\langle \mathcal{O}[^3S_1^{[8]}] \rangle$, which can be extracted with a good accuracy.

III. NUMERICAL RESULTS

In this section, we present the results of our global fitting. In Fig. 2, we show the lowest values of $\chi^2/\text{n.d.f.}$ (number of degrees of freedom) obtained for different cutoffs $p_T^{\min}$ on the quarkonium transverse momentum. For A0 gluon density, a good simultaneous description of all available LHC data can be achieved without additional constraints on p_T . Moreover, the results depend very weakly on the chosen $p_T^{\min}$. An opposite situation is observed for JH'2013 set 2 gluon distribution, where the larger χ^2 values are obtained. An essential part of χ^2 comes from low $p_T < 6$ GeV at forward rapidities. The large overestimation seen there (up to 3 times) leaves no chance for a good simultaneous description of the data. At the same time, excluding the problematic area from the fitting yields results that are quite close to the A0 predictions. The observed difference in the behavior of the p_T spectra arises

⁶The differences in the values of r in the central and forward regions reflect only differences in the normalization in the different regions of the phase space, while the shapes of the transverse momentum distributions remain the same. This does not affect the obtained results but can be useful for upcoming studies.


 FIG. 2. Lowest values of $\chi^2/\text{n.d.f.}$ as function of minimal ψ' transverse momenta.

from the dynamics of the gluon evolution in the considered TMD gluon densities and could be used to distinguish between them (see below).

To highlight the importance of tree-level NLO corrections, we perform the same fitting procedure for the LO calculations. The inclusion of NLO corrections leads to changing of the shape and allows one to significantly soften the discrepancy, especially for JH'2013 set 2 gluons (see Fig. 2).

We wish to take a closer look at the comparison between the predictions based on A0 and JH'2013 set 2 gluons without any p_T cutoffs, and on JH'2013 set 2 gluons at $p_T > 6$ GeV, where we have reasonable agreement with the data. In Table II we show the fitted LDME values for both LO and NLO[†] analyses. The corresponding uncertainties are calculated as standard deviations from the error matrix. The uncertainties are noticeably small because of large number of data points and weak correlation between free parameters.

Table III exhibits the $\chi^2/\text{n.d.f.}$ values calculated separately for each available dataset (experiment + energy) with LDMEs from the Table II. The values in parentheses in the column “n.d.f.” show the number of data points remaining after imposing cuts on transverse momentum $p_T > 6$ GeV. In the case of JH'2013 set 2 gluon, removing the p_T constraints results in a significant increase of χ^2 for all LHCb and ALICE datasets and to an overall letdown in the fit quality. This tendency is expectedly worse for LO only (such problem was observed earlier [20]). It is worth noting the “ATLAS 13 TeV” dataset, which shows a significantly higher χ^2 value compared to other experiments in the central rapidity region. One of reasons can be attributed to a significantly increased accuracy of the data.

TABLE II. The values of LDMEs for ψ' meson fitted with A0 and JH'2013 gluon densities in a proton.

LDME	A0	JH'2013 set 2	JH'2013 set 2 ($p_T > 6$)
LO + NLO [†]			
$\langle \mathcal{O}_{l.c.} \rangle / \text{GeV}^5$	$(6.06 \pm 0.07) \times 10^{-3}$	$(6.17 \pm 0.06) \times 10^{-3}$	$(9.95 \pm 0.09) \times 10^{-3}$
$\langle \mathcal{O}^3 S_1^{[8]} \rangle / \text{GeV}^3$	$(10.2 \pm 0.2) \times 10^{-4}$	$(10.7 \pm 0.1) \times 10^{-4}$	$(7.46 \pm 0.13) \times 10^{-4}$
LO			
$\langle \mathcal{O}_{l.c.} \rangle / \text{GeV}^5$	$(10.4 \pm 0.1) \times 10^{-3}$	$(8.61 \pm 0.09) \times 10^{-3}$	$(18.9 \pm 0.2) \times 10^{-3}$
$\langle \mathcal{O}^3 S_1^{[8]} \rangle / \text{GeV}^3$	$(20.1 \pm 0.3) \times 10^{-4}$	$(29.3 \pm 0.2) \times 10^{-4}$	$(15.6 \pm 0.3) \times 10^{-4}$

TABLE III. $\chi^2/\text{n.d.f.}$ for available datasets at LO + NLO[†]|LO.

TeV (Ref.)	n.d.f.	A0	JH'2013 set 2	JH'2013 set 2 ($p_T > 6$)
ATLAS 5.02 [49]	15	0.87 0.96	1.69 3.13	1.02 1.52
ATLAS 7 [50]	168	1.05 1.12	2.77 5.88	1.28 1.65
ATLAS 8 [50]	172	1.53 2.25	2.15 4.28	1.51 1.70
ATLAS 13 [51]	84	5.54 5.53	9.07 16.4	4.89 4.46
CMS 5.02 [52]	10(8)	0.87 0.59	4.26 10.4	5.02 14.5
CMS 7 [53]	72	0.89 1.39	3.69 7.15	0.75 0.88
CMS 13 [54]	36	1.53 2.06	9.20 19.6	2.05 3.41
LHCb 7 [55]	55(40)	3.27 3.46	16.2 28.1	4.66 8.42
LHCb 13 [55]	79(59)	3.44 4.37	25.1 42.3	4.97 8.49
ALICE 7 [56]	9(3)	1.09 1.37	2.94 6.21	1.82 2.39
ALICE 8 [57]	9(3)	1.44 1.62	1.99 4.33	1.53 1.78
ALICE 13 [58]	12(6)	2.17 2.93	4.28 8.51	1.30 1.71
Summary	721(666)	2.15 2.54	7.29 13.3	2.34 3.14

We also note some discrepancy in the normalization between the ATLAS 13 TeV and “CMS 13 TeV” data. However, it is clearly seen that the overall improvement in fit quality is achieved for most datasets due to inclusion of tree-level NLO corrections.

The results of our calculations for the ψ' differential cross sections are displayed in Figs. 3–8 for all datasets in the form of “theory to data” ratios. The results are only presented for A0 gluons without cutoffs (red lines) and JH'2013 set 2 gluons with $p_T > 6$ GeV (blue lines). The theoretical uncertainty bands (shaded areas) are shown for A0 gluon density. The inner bands represent the reasonable variations in the soft gluon energy E_g emitted at the formation of bound states within the range $E_g = \Lambda_{\text{QCD}} \pm 50$ MeV. The outer bands include the renormalization scale uncertainties added in quadratures. As usual, variations in the renormalization scale were accompanied with using A0– and A0+ gluon densities for $\mu_R/2$ and $2\mu_R$ values, respectively (see [69]). We find no dramatic difference between the results obtained with A0 and JH'2013 set 2 gluon densities in the central and forward rapidity regions at moderate and large ψ' transverse momenta, $p_T > 6$ GeV. However, the region of relatively low p_T shows a pronounced sensitivity to the choice of TMD gluon

distribution.⁷ In addition, we show here the LO results for A0 gluons to emphasize the difference in shape of p_T spectra. The NLO tree-level corrections allow one to visibly reduce the slope of spectra. It leads to better agreement with data in the full available p_T region despite the comparable χ^2 for both LO and LO + NLO[†] cases.

One can conclude from Figs. 3–8 that the modified LO + NLO[†] merging scheme developed to avoid double counting has demonstrated its working power and is thus validated for future use.⁸ It will be implemented into the next version of the Monte Carlo event generator PEGASUS [78].

⁷The concept of TMD shape function has been proposed for an accurate theoretical description of charmonia production at low transverse momenta, where the soft gluon radiation plays a role (see, for example, [81–83] and references therein). The soft gluon radiation is not ignored in our work, but considered separately for the initial and the final states. The initial state effects are taken in through fixing the shape of the TMD gluon densities at the starting scale. The effects of final state radiation and the formation of quarkonium are taken into account through attributing some extra mass to the intermediate color octet states, as it was explained in Sec. II A.

⁸We leave the analysis of the ψ' polarization for a forthcoming dedicated study.

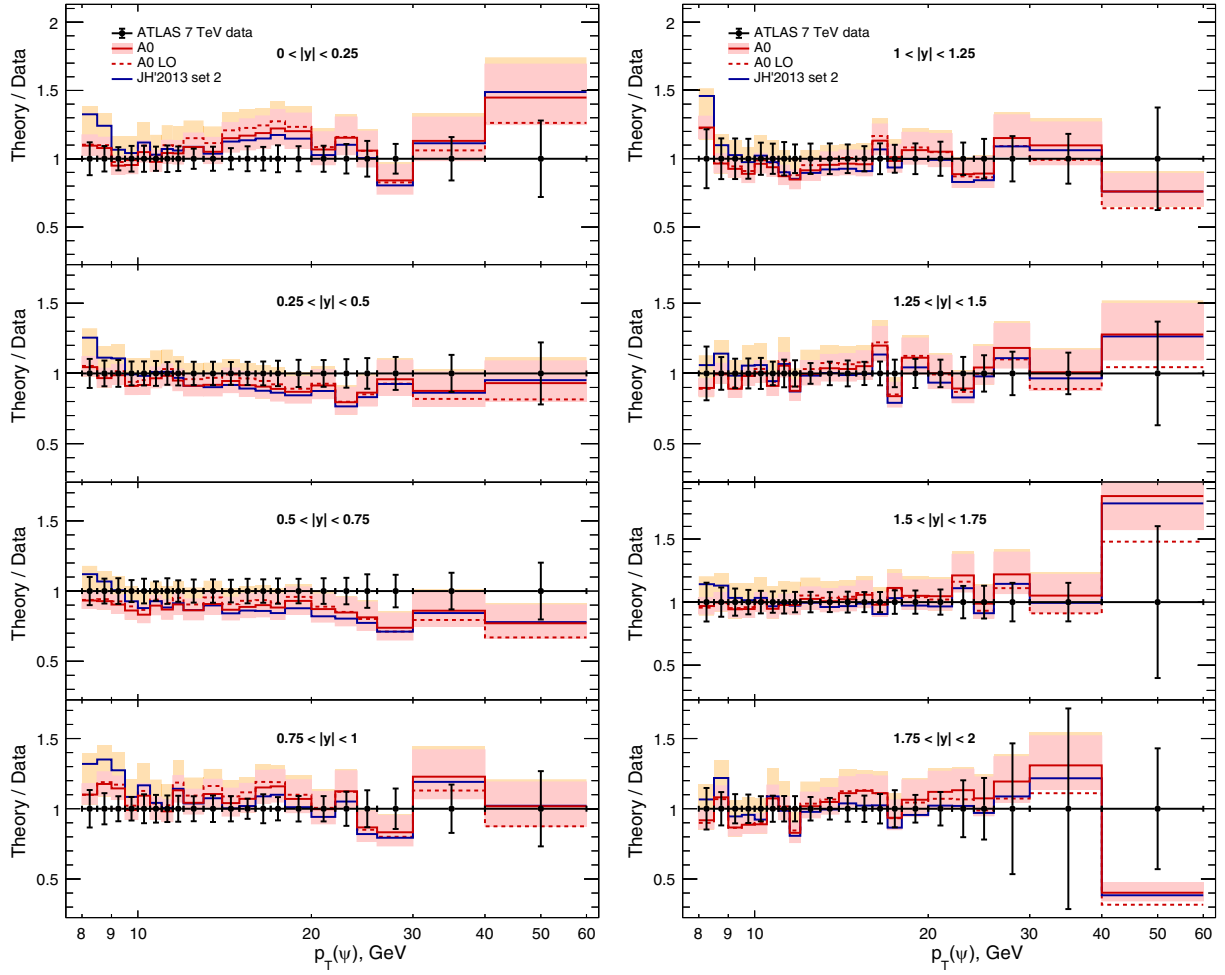


FIG. 3. Ratios of differential cross sections for inclusive ψ' production in pp collisions at $\sqrt{s} = 7$ TeV as functions of the ψ' transverse momentum at the central rapidity regions. The ATLAS data are taken from [50].

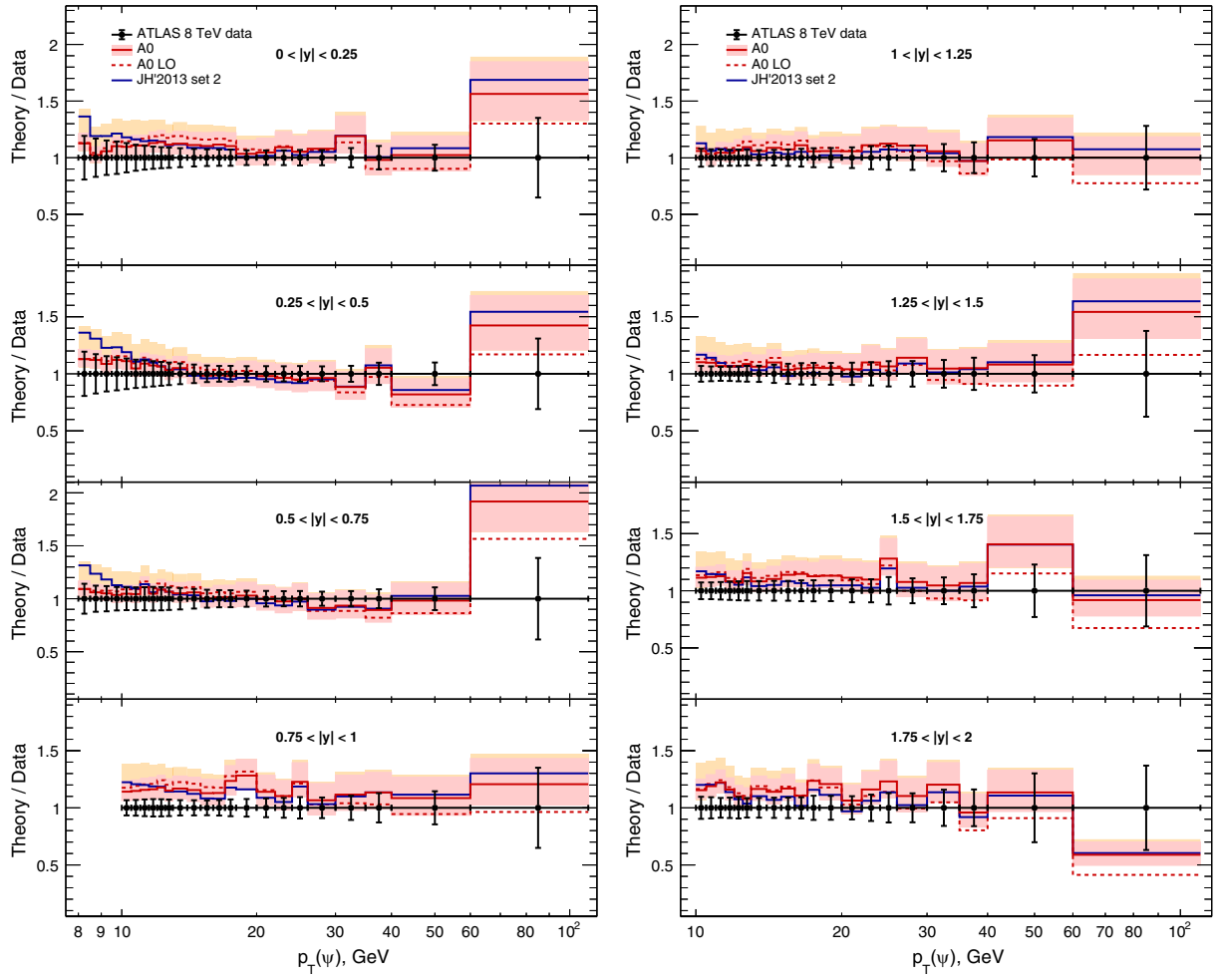


FIG. 4. Ratios of differential cross sections for inclusive ψ' production in pp collisions at $\sqrt{s} = 8$ TeV as functions of the ψ' transverse momentum at the central rapidity regions. The ATLAS data are taken from [50].

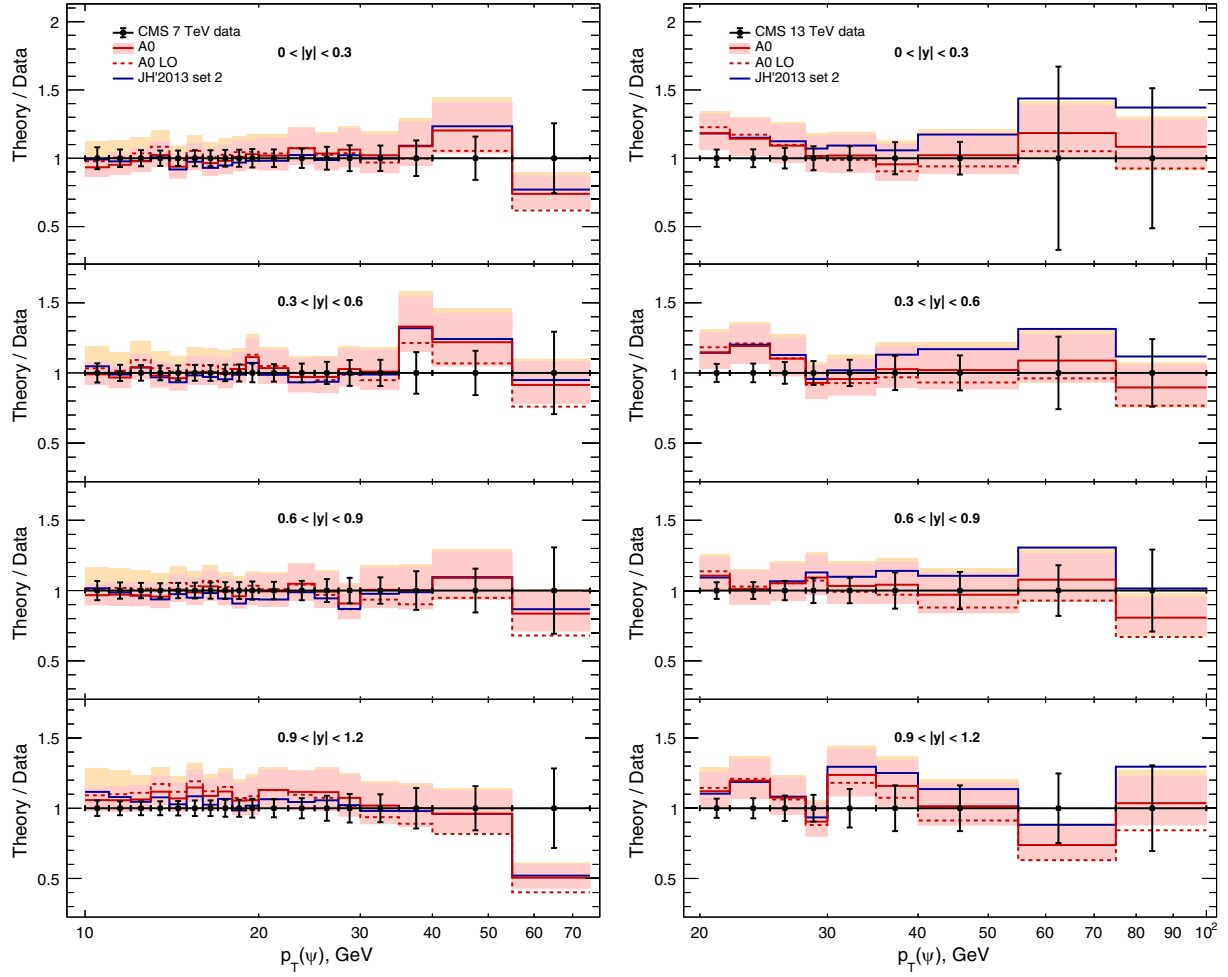


FIG. 5. Ratios of differential cross sections for inclusive ψ' production in pp collisions at $\sqrt{s} = 7$ TeV (left) and $\sqrt{s} = 13$ TeV (right) as functions of the ψ' transverse momentum at the central rapidity regions. The CMS data are taken from [53,54].

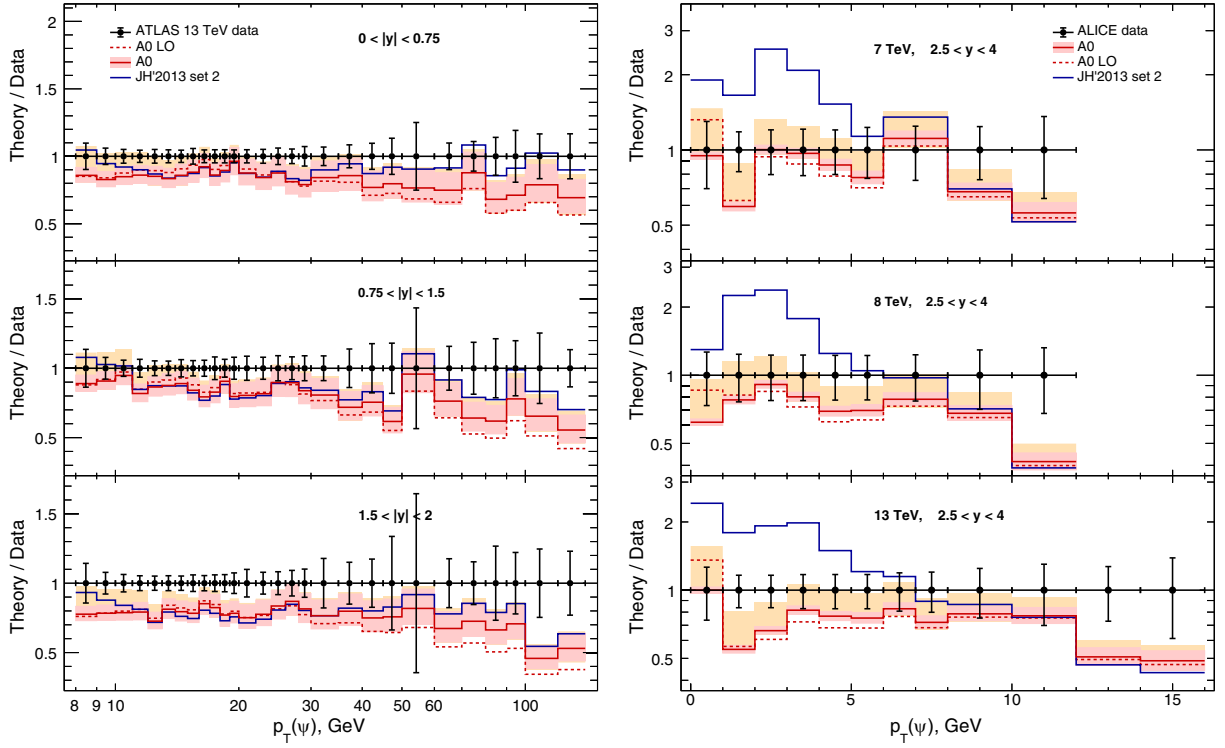


FIG. 6. Ratios of differential cross sections for inclusive ψ' production in pp collisions at $\sqrt{s} = 13$ TeV at the central rapidity regions (left) and $\sqrt{s} = 7, 8, 13$ TeV at the forward rapidity regions (right) as functions of the ψ' transverse momentum. The ATLAS data are taken from [51] and ALICE [56–58].

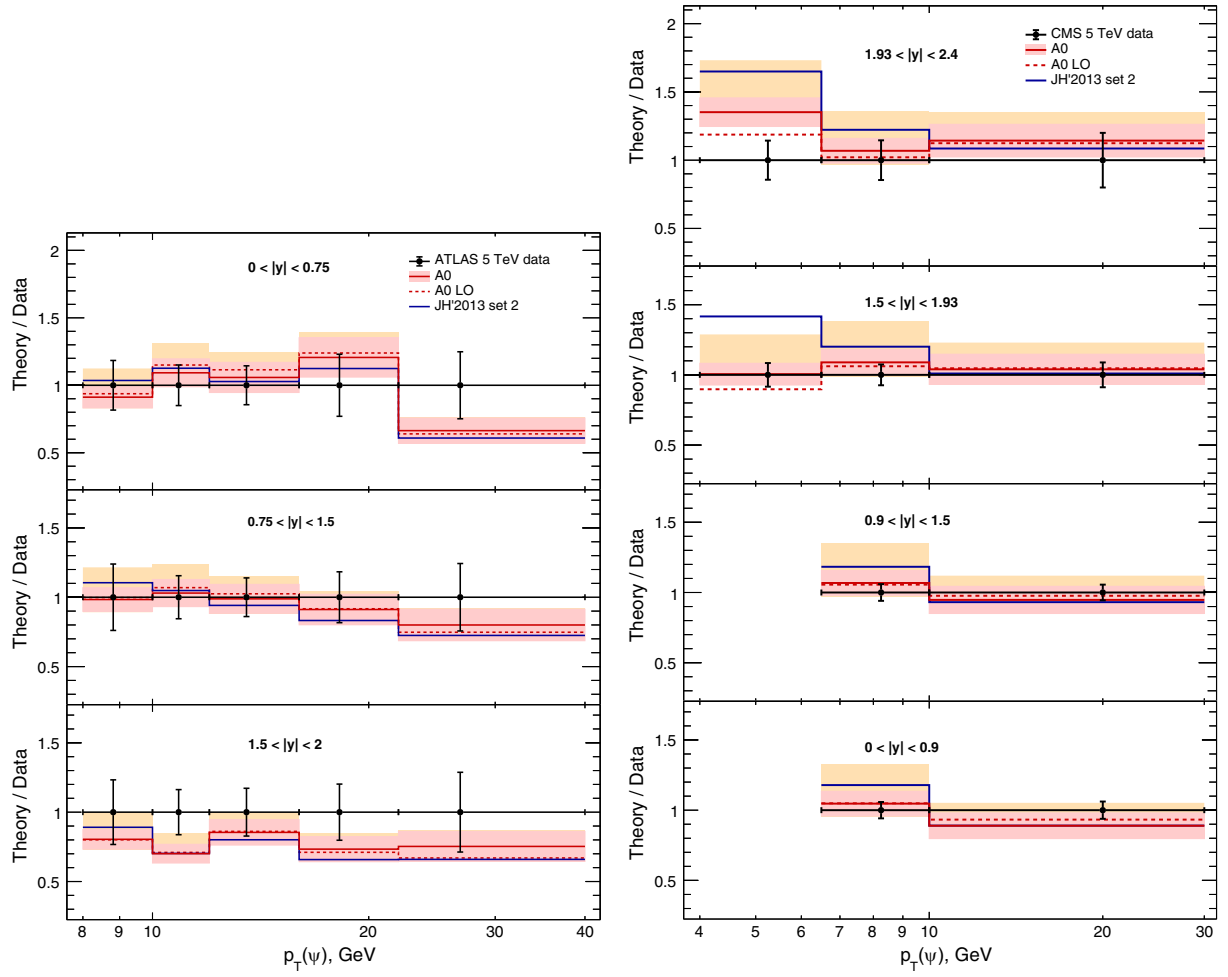


FIG. 7. Ratios of differential cross sections for inclusive ψ' production in pp collisions at $\sqrt{s} = 5.02$ TeV at the central rapidity regions as functions of the ψ' transverse momentum. The ATLAS data are taken from [51] and CMS [56–58].

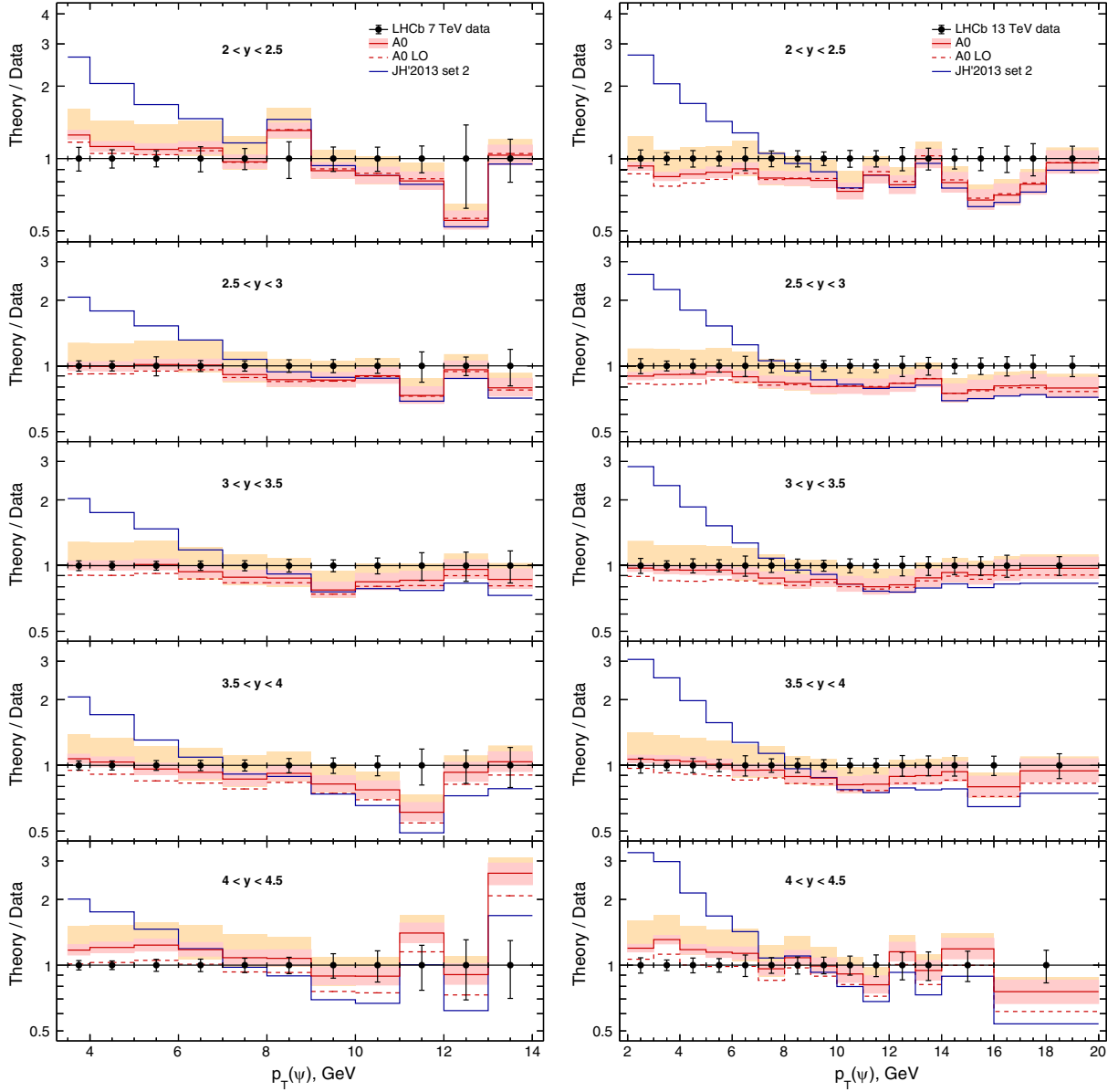


FIG. 8. Ratios of differential cross sections for inclusive ψ' production in pp collisions at $\sqrt{s} = 7$ TeV (left) and $\sqrt{s} = 13$ TeV (right) as functions of the ψ' transverse momentum in the forward rapidity region. The LHCb data are taken from [55].

IV. CONCLUSION

We have considered inclusive ψ' production in proton-proton collisions at the LHC energies in the k_T -factorization approach beyond the leading-order approximation. Our work represents the first attempt to carry out a simultaneous description of all available unpolarized data on ψ' meson production in the entire kinematic region within high energy factorization approach. The theoretical innovation consists of including the tree-level $\text{NLO}^\dagger$ contributions and in modifying the $\text{LO} + \text{NLO}$ merging scheme for specific kinematic conditions (forward rapidities at low p_T).

The results of our study can be summarized as follows.

- (i) Adding the tree-level NLO contributions to hard partonic subprocess helped to reach a reasonable agreement with the data in the whole kinematic range including the problematic region of low p_T , which has not yet been described satisfactorily in theoretical papers.
- (ii) The modified $\text{LO} + \text{NLO}^\dagger$ merging scheme developed to avoid double counting has demonstrated its working power and is thus validated for future use.

- (iii) A significant part of theoretical uncertainties (mainly regarding the overall normalization) comes from the meson formation mechanism, namely, from the energy required for a soft gluon to overcome the confinement.
- (iv) A comparison of the results obtained with different gluon densities shows no significant difference between A0 and JH'2013 set 2 gluons in the region of moderate and high p_T . The results obtained at small $p_T < 6$ GeV give clear preference to the A0 gluons.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [49–58].

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- [1] N. Brambilla *et al.*, *Eur. Phys. J. C* **71**, 1534 (2011).
 - [2] J.-P. Lansberg, *Phys. Rep.* **889**, 1 (2020).
 - [3] E. Chapon *et al.*, *Prog. Part. Nucl. Phys.* **122**, 103906 (2022).
 - [4] G. Bodwin, E. Braaten, and G. Lepage, *Phys. Rev. D* **51**, 1125 (1995).
 - [5] P. Cho and A. K. Leibovich, *Phys. Rev. D* **53**, 150 (1996).
 - [6] P. Cho and A. K. Leibovich, *Phys. Rev. D* **53**, 6203 (1996).
 - [7] C. Chang, *Nucl. Phys. B* **172**, 425 (1980).
 - [8] R. Baier and R. Ruckl, *Z. Phys. C* **19**, 251 (1983).
 - [9] E. L. Berger and D. L. Jones, *Phys. Rev. D* **23**, 1521 (1981).
 - [10] V. D. Barger, W.-Y. Keung, and R. J. N. Phillips, *Phys. Lett.* **91B**, 253 (1980).
 - [11] V. D. Barger, W.-Y. Keung, and R. J. N. Phillips, *Z. Phys. C* **6**, 169 (1980).
 - [12] R. Gavai, D. Kharzeev, H. Satz, G. A. Schuler, K. Sridhar, and R. Vogt, *Int. J. Mod. Phys. A* **10**, 3043 (1995).
 - [13] Y.-Q. Ma and R. Vogt, *Phys. Rev. D* **94**, 114029 (2016).
 - [14] L. V. Gribov, E. M. Levin, and M. G. Ryskin, *Phys. Rep.* **100**, 1 (1983); E. M. Levin, M. G. Ryskin, Yu. M. Shabelsky, and A. G. Shuvaev, *Sov. J. Nucl. Phys.* **53**, 657 (1991).
 - [15] S. Catani, M. Ciafaloni, and F. Hautmann, *Nucl. Phys. B* **366**, 135 (1991); J. C. Collins and R. K. Ellis, *Nucl. Phys. B* **360**, 3 (1991).
 - [16] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, *Sov. Phys. JETP* **44**, 443 (1976), <http://www.jetp.ras.ru/cgi-bin/e/index/e/44/3/p443?a=list>; **45**, 199 (1977), <http://www.jetp.ras.ru/cgi-bin/e/index/e/45/2/p199?a=list>; I. I. Balitsky and L. N. Lipatov, *Sov. J. Nucl. Phys.* **28**, 822 (1978).
 - [17] M. Ciafaloni, *Nucl. Phys. B* **296**, 49 (1988); S. Catani, F. Fiorani, and G. Marchesini, *Phys. Lett. B* **234**, 339 (1990); S. Catani, F. Fiorani, and G. Marchesini, *Nucl. Phys. B* **336**, 18 (1990); G. Marchesini, *Nucl. Phys. B* **445**, 49 (1995).
 - [18] R. Angeles-Martinez *et al.*, *Acta Phys. Pol. B* **46**, 2501 (2015).
 - [19] A. V. Lipatov, S. P. Baranov, and M. A. Malyshev, *Phys. Part. Nucl.* **55**, 256 (2024).
 - [20] S. P. Baranov and A. V. Lipatov, *Phys. Rev. D* **100**, 114021 (2019).
 - [21] S. P. Baranov and A. V. Lipatov, *Eur. Phys. J. C* **79**, 621 (2019).
 - [22] S. P. Baranov and A. V. Lipatov, *Eur. Phys. J. C* **80**, 1022 (2020).
 - [23] N. A. Abdulov and A. V. Lipatov, *Eur. Phys. J. C* **79**, 830 (2019).
 - [24] N. A. Abdulov and A. V. Lipatov, *Eur. Phys. J. C* **80**, 486 (2020).
 - [25] N. A. Abdulov and A. V. Lipatov, *Eur. Phys. J. C* **81**, 1085 (2021).
 - [26] B. Gong, X. Q. Li, and J.-X. Wang, *Phys. Lett. B* **673**, 197 (2009).
 - [27] M. Butenschön and B. A. Kniehl, *Phys. Rev. D* **84**, 051501 (2011).
 - [28] Y.-Q. Ma, K. Wang, K.-T. Chao, and H.-F. Zhang, *Phys. Rev. D* **83**, 111503 (2011).
 - [29] Y.-Q. Ma, K. Wang, and K.-T. Chao, *Phys. Rev. D* **84**, 114001 (2011).
 - [30] G. T. Bodwin, H. S. Chung, U.-R. Kim, and J. Lee, *Phys. Rev. Lett.* **113**, 022001 (2014).
 - [31] A. K. Likhoded, A. V. Luchinsky, and S. V. Poslavsky, *Phys. Rev. D* **90**, 074021 (2014).
 - [32] H.-F. Zhang, L. Yu, S.-X. Zhang, and L. Jia, *Phys. Rev. D* **93**, 054033 (2016).
 - [33] Y. Q. Ma, K. Wang, and K.-T. Chao, *Phys. Rev. Lett.* **106**, 042002 (2011).
 - [34] B. Gong, J.-X. Wang, and H.-F. Zhang, *Phys. Rev. D* **83**, 114021 (2011).
 - [35] K. Wang, Y.-Q. Ma, and K.-T. Chao, *Phys. Rev. D* **85**, 114003 (2012).
 - [36] B. Gong, L.-P. Wan, J.-X. Wang, and H.-F. Zhang, *Phys. Rev. Lett.* **112**, 032001 (2014).
 - [37] Y. Feng, B. Gong, L.-P. Wan, J.-X. Wang, and H.-F. Zhang, *Chin. Phys. C* **39**, 123102 (2015).
 - [38] H. Han, Y.-Q. Ma, C. Meng, H.-S. Shao, Y.-J. Zhang, and K.-T. Chao, *Phys. Rev. D* **94**, 014028 (2016).
 - [39] Y. Feng, B. Gong, C.-H. Chang, and J.-X. Wang, *Chin. Phys. C* **45**, 013117 (2021).
 - [40] S. P. Baranov, *Phys. Rev. D* **93**, 054037 (2016).
 - [41] S. P. Baranov, A. V. Lipatov, A. A. Prokhorov, and X. Chen, *Eur. Phys. J. C* **84**, 348 (2024).
 - [42] H.-F. Zhang, Z. Sun, W.-L. Sang, and R. Li, *Phys. Rev. Lett.* **114**, 092006 (2015).
 - [43] M. Butenschön, Z.-G. He, and B. A. Kniehl, *Phys. Rev. Lett.* **114**, 092004 (2015).

- [44] S. P. Baranov, *Phys. Rev. D* **83**, 034035 (2011).
- [45] M. Butenschön and B. A. Kniehl, *Phys. Rev. D* **107**, 034003 (2023).
- [46] CMS Collaboration, *Phys. Lett. B* **727**, 381 (2013).
- [47] CMS Collaboration, *Phys. Lett. B* **858**, 139044 (2024).
- [48] LHCb Collaboration, *Eur. Phys. J. C* **74**, 2872 (2014).
- [49] ATLAS Collaboration, *Eur. Phys. J. C* **78**, 171 (2018).
- [50] ATLAS Collaboration, *Eur. Phys. J. C* **76**, 283 (2016).
- [51] ATLAS Collaboration, *Eur. Phys. J. C* **84**, 169 (2024).
- [52] CMS Collaboration, *Phys. Lett. B* **790**, 509 (2019).
- [53] CMS Collaboration, *Phys. Lett. B* **114**, 191802 (2015).
- [54] CMS Collaboration, *Phys. Lett. B* **780**, 251 (2018).
- [55] LHCb Collaboration, *Eur. Phys. J. C* **80**, 185 (2020).
- [56] ALICE Collaboration, *Eur. Phys. J. C* **74**, 2974 (2014).
- [57] ALICE Collaboration, *Eur. Phys. J. C* **76**, 184 (2016).
- [58] ALICE Collaboration, *Eur. Phys. J. C* **77**, 392 (2017).
- [59] R. Maciula and A. Szczurek, *Phys. Rev. D* **100**, 054001 (2019).
- [60] A. V. Lipatov, M. A. Malyshev, and H. Jung, *Phys. Rev. D* **101**, 034022 (2020).
- [61] E. J. Eichten and C. Quigg, *Phys. Rev. D* **52**, 1726 (1995).
- [62] E. J. Eichten and C. Quigg, [arXiv:1904.11542](#).
- [63] H. Krasemann, *Z. Phys. C* **1**, 189 (1979).
- [64] G. Guberina, J. Kühn, R. Peccei, and R. Rückl, *Nucl. Phys.* **B174**, 317 (1980).
- [65] L. N. Lipatov and M. I. Vyazovsky, *Nucl. Phys.* **B597**, 399 (2001).
- [66] A. V. Bogdan and V. S. Fadin, *Nucl. Phys.* **B740**, 36 (2006).
- [67] S. P. Baranov and A. Szczurek, *Phys. Rev. D* **77**, 054016 (2008).
- [68] E. Bycling and K. Kajantie, *Particle Kinematics* (John Wiley and Sons, New York, 1973).
- [69] H. Jung, [arXiv:hep-ph/0411287](#).
- [70] F. Hautmann and H. Jung, *Nucl. Phys.* **B883**, 1 (2014).
- [71] H. Jung, S. P. Baranov, M. Deak, A. Grebenyuk, F. Hautmann, M. Hentschinski, A. Knutsson, M. Krämer, K. Kutak, A. V. Lipatov, and N. P. Zotov, *Eur. Phys. J. C* **70**, 1237 (2010).
- [72] K. Golec-Biernat, L. Motyka, and T. Stebel, *Phys. Rev. D* **103**, 034013 (2021).
- [73] A. V. Lipatov, M. A. Malyshev, and H. Jung, *Phys. Rev. D* **100**, 034028 (2019).
- [74] A. V. Lipatov and M. A. Malyshev, *Phys. Rev. D* **103**, 094021 (2001).
- [75] A. V. Lipatov, G. I. Lykasov, and M. A. Malyshev, *Phys. Lett. B* **839**, 137780 (2023).
- [76] A. V. Lipatov and M. A. Malyshev, *Phys. Rev. D* **108**, 014022 (2023).
- [77] N. A. Abdulov *et al.*, *Eur. Phys. J. C* **81**, 752 (2021).
- [78] A. V. Lipatov, M. A. Malyshev, and S. P. Baranov, *Eur. Phys. J. C* **80**, 330 (2020).
- [79] PDG Collaboration, *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
- [80] E. Braaten and J. Lee, *Phys. Rev. D* **67**, 054007 (2003).
- [81] M. G. Echevarria, *J. High Energy Phys.* **10** (2019) 144.
- [82] S. Fleming, Y. Makris, and T. Mehen, *J. High Energy Phys.* **04** (2020) 122.
- [83] L. Maxia, D. Boer, J. Bor, C. Pisano, and F. Yuan, *J. High Energy Phys.* **08** (2023) 105.