

Diffractive electroproduction of light vector particles: Leading Fock-state contribution in the presence of significant higher Fock-state effects

Chao Shi^{1,*}, Liming Lu,¹ Jian-feng Li,^{2,†} and Wenbao Jia¹

¹*Department of Nuclear Science and Technology, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*

²*School of Physics and Technology, Nantong University, Nantong 226019, China*



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We study exclusive diffractive production of vector mesons and photons using the color-dipole model with leading Fock-state light-front wave functions derived from Dyson-Schwinger and Bethe-Salpeter equations. New results for the ϕ meson and real photon are presented. Without data fitting, our calculation well matches HERA data in certain kinematical domains. The key finding of this paper is that in a color-dipole model study for ρ/γ and ϕ , where light quarks are involved, the leading $q\bar{q}$ approximation is valid only when Q^2 exceeds 20 and 10 GeV^2 respectively, unlike J/ψ which can be well described for $Q^2 \approx 0 \text{ GeV}^2$. This underscores the special role of ϕ electroproduction in the color-dipole picture: it strikes a balance between the large dipole size typical of light mesons and the smaller size associated with high- Q^2 photons, making it potentially well suited for probing gluon saturation effects.

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I. INTRODUCTION

Exclusive diffractive electroproduction of vector particles such as real photons and vector mesons constitutes an important probe of the transverse gluon density in hadrons and its saturation at small Bjorken- x [1–5]. In the color-dipole approach description, the incoming virtual photon fluctuates into a quark-antiquark dipole and then transforms back into an outgoing vector particle, both described by their $q\bar{q}$ light-front wave functions ($q\bar{q}$ -LFWFs). Therefore, the light vector particle production, such as ρ , $\phi(s\bar{s})$, and γ , has long faced great challenge due to the large uncertainty in their $q\bar{q}$ -LFWFs.

Although many existing studies achieved good agreement with data employing phenomenological light vector-meson wave function models [1,2,4,6–11], the connection between these models and realistic quantum chromodynamics (QCD) remains elusive. A primary problem, from our perspective, is that these wave functions assume that the $|q\bar{q}\rangle$ state saturates the meson state. The normalization condition $\langle q\bar{q}|q\bar{q}\rangle = 1$ is usually imposed, so there is no (or little) higher Fock-state content such as $|q\bar{q}g\rangle$ in light mesons. This is a rough approximation, given the complex

partonic structure of light hadrons. Meanwhile, recent next-to-leading-order calculations provided infrared-safe analytic expressions for exclusive ρ and ϕ production containing both $|q\bar{q}\rangle$ and $|q\bar{q}g\rangle$ contributions, yielding good agreement with HERA and LHC data [12,13]. Notably, therein the $|q\bar{q}g\rangle$ is obtained by perturbative emission of gluons from $|q\bar{q}\rangle$ at large Q^2 , and hence a suppression can be expected. On the other hand, there is a study that combines the color glass condensate (CGC) with the higher-twist formalism for exclusive processes, incorporating nonperturbative higher Fock-state effect [14,15].

The light vector particle electroproductions received particular interest as they were believed to be more sensitive to gluon saturation effects than heavy quarkonium such as J/ψ . This is due to the large dipole size of the light mesons that overlap with the flavor-independent $q\bar{q}$ -nucleon scattering amplitude. Searching the saturation effect with light vector-meson production in nuclei-nuclei and electron-nuclei collisions is thus appealing [16–18]. On the other hand, HERA data on light vector particle electroproduction serves as a complement to the heavy-meson case. It is important to utilize these data to further constrain or examine the theoretical understanding of nonperturbative QCD quantities as $q\bar{q}$ -LFWFs and/or color-dipole-nucleon scattering amplitudes.

In [19], we introduced a light front projection method that extracts the ρ and J/ψ meson $q\bar{q}$ -LFWFs from their covariant Bethe-Salpeter (BS) wave functions based on the Dyson-Schwinger equation (DSE) formalism. These $q\bar{q}$ -LFWFs were then put into the color-dipole study of the mesons' diffractive electroproduction. The key finding

*Contact author: cshi@nuaa.edu.cn

†Contact author: ljf169@ntu.edu.cn

therein was that in the ρ meson the $|q\bar{q}\rangle$ contributes less than 50%, i.e., $\langle q\bar{q}|q\bar{q}\rangle < 0.5$, implying significantly higher Fock-state contributions. Furthermore, the exclusive ρ electroproduction can be well described with our $q\bar{q}$ -LFWFs within the color-dipole approach, but only starting from $Q^2 \approx 10 \text{ GeV}^2$. This is reasonable in physics as twist suppression requires high Q^2 in exclusive processes. Nevertheless, we note that this scenario is demonstrated for the first time within the color-dipole approach as other studies all described data for Q^2 as low as 1 GeV^2 . For J/ψ , our study suggested $\langle q\bar{q}|q\bar{q}\rangle \approx 0.9$, and the $q\bar{q}$ -truncated color-dipole model well describes data for $Q^2 \approx 0 \text{ GeV}^2$. In this work, we supplement [19] with cases of the ϕ meson and real photon, which are all light vector particles. The novelty of this work is thus a first presentation of ϕ -meson $q\bar{q}$ -LFWFs based on the DSE formalism, as well as an exploration of the exclusive production of ϕ and γ using DS-BS equation-based $q\bar{q}$ -LFWFs. We also present a more detailed and careful analysis on differential cross sections, with focus on the light-quark sector. We note that the nonperturbative $q\bar{q}$ -LFWFs of real photons are adopted from [20], which was also based on the DSE formalism but with a simpler interaction model than the Maris-Tandy-like model [19] for vector mesons.

This paper is organized as follows. In Sec. II, we recapitulate the $q\bar{q}$ -LFWFs of vector mesons and photons and give the result for ϕ . In Sec. III, these $q\bar{q}$ -LFWFs are utilized in a color-dipole model study of vector mesons and photon electroproduction. We finally conclude in Sec. IV.

II. $q\bar{q}$ -LFWFs OF VECTOR MESONS AND PHOTONS

A. Formalism

A particle state takes a Fock-state expansion on the light front. For a vector meson V_M composed of a valence quark and antiquark of flavor f , the decomposition reads

$$|V_M\rangle = |q_f\bar{q}_f\rangle_{(V_M)} + |q_f\bar{q}_f g\rangle_{(V_M)} + \cdots \quad (1)$$

On the other hand, in the context of QCD plus quantum electrodynamics (QED), the photon Fock-state expansion reads

$$\begin{aligned} |\gamma_{\text{phys}}^*\rangle &= |\gamma_{\text{bare}}^*\rangle + |e^+e^-\rangle_{(\gamma^*)} + \sum_{f=u,d,s,\dots} |q_f\bar{q}_f\rangle_{(\gamma^*)} \\ &+ \sum_{f=u,d,s,\dots} |q_f\bar{q}_f g\rangle_{(\gamma^*)} + \cdots \end{aligned} \quad (2)$$

Every term on the right-hand sides of Eqs. (1) and (2) shares exactly the same quantum number as its parent particle state, and is mutually orthogonal to each other. The photon is an elementary particle and thus its Fock-state expansion contains an additional bare term. Meanwhile,

its $q\bar{q}$ -LFWFs run through all quark flavors. At first glance, the idea that the elementary particle photon has quark content may seem confusing, but this will soon be clarified.

Denoting $V = \gamma^*$ and V_M , a general decomposition of the leading Fock-state $|q_f\bar{q}_f\rangle_{(V)}$ reads

$$\begin{aligned} |q_f\bar{q}_f\rangle_{(V)}^\Lambda &= \sum_{\lambda,\lambda';i,j} \int \frac{d^2\mathbf{k}_T}{(2\pi)^3} \frac{dx}{2\sqrt{x\bar{x}}} \frac{\delta_{ij}}{\sqrt{3}} \Phi_{\lambda,\lambda',(V)}^{\Lambda,(f)}(x, \mathbf{k}_T) \\ &\times b_{f,\lambda,i}^\dagger(x, \mathbf{k}_T) d_{f,\lambda',j}^\dagger(\bar{x}, \bar{\mathbf{k}}_T) |0\rangle. \end{aligned} \quad (3)$$

The $\Phi_{\lambda,\lambda',(V)}^{\Lambda,(f)}$ is the $q\bar{q}$ -LFWF of particle V with helicity Λ and quark (antiquark) of flavor $f(\bar{f})$ and spin $\lambda(\lambda')$. $\Lambda = 0, \pm 1$ and $\lambda = \uparrow, \downarrow$, denoted as $\uparrow = +$ and $\downarrow = -$ in following. b^+ and d^+ are creation operators of the quark and antiquark, respectively. i and j are the color indices. $\mathbf{k}_T = (k^x, k^y)$ is the transverse momentum of the quark, and $\bar{\mathbf{k}}_T = -\mathbf{k}_T$ for the antiquark. The longitudinal momentum fraction carried by the quark is $x = k^+/P^+$, with $\bar{x} = 1 - x$ for the antiquark. We use the light-cone four-vector convention $A^\pm = \frac{1}{\sqrt{2}}(A^0 \pm A^3)$ and $\mathbf{A}_T = (A^1, A^2)$ throughout this paper.

By Eqs. (1)–(3), one can observe that the $q\bar{q}$ -LFWFs are essentially transition amplitudes of the parent particle V into the quark-antiquark state $b_{f,\lambda,i}^\dagger(x, \mathbf{k}_T) d_{f,\lambda',j}^\dagger(\bar{x}, \bar{\mathbf{k}}_T) |0\rangle$. So, the $q\bar{q}$ -LFWFs of the photon should not be viewed as the photon's bound state wave function, but rather the transition amplitude of the photon into a virtual $q\bar{q}$ pair by quantum fluctuation. Naturally, this interpretation also applies to vector mesons. It is also based on this idea that a connection can be built between the BS wave function, which is the transition amplitude of $V \rightarrow q\bar{q}$ in ordinary space-time coordinates, and the $q\bar{q}$ -LFWFs based on the light-front coordinate [21–23]. In [19], we introduced a light-front projection equation to obtain the $q\bar{q}$ -LFWFs of vector mesons from their BS wave functions, i.e.,

$$\begin{aligned} \Phi_{\lambda,\lambda',(V)}^{\Lambda,(f)}(x, \mathbf{k}_T) &= -\frac{1}{2\sqrt{3}} \int \frac{dk^- dk^+}{2\pi} \delta(xQ^+ - k_\eta^+) \\ &\times \text{Tr}\{\Gamma_{\lambda,\lambda'}\gamma^+ S_f(k_\eta) [\Gamma_{(V)}^{(f)}(k; Q) \cdot \epsilon_\Lambda(Q)] S_f(k_{\bar{\eta}})\}. \end{aligned} \quad (4)$$

$S_f(k)$ is the fully dressed quark propagator of flavor f , and $\Gamma_{(V),\mu}^{(f)}(k; Q)$ is the amputated $V \rightarrow q_f\bar{q}_f$ vertex, i.e., the BS amplitude, in momentum space. The BS wave function is defined as $\chi_{(V),\mu}^{(f)}(k; Q) \equiv S_f(k_\eta) \Gamma_{(V),\mu}^{(f)}(k; Q) S_f(k_{\bar{\eta}})$. Q is the four-momentum of vector meson. $k_\eta \equiv k + \eta Q$ is the momentum carried by the outgoing quark leg (corresponding to quark content) and $k_{\bar{\eta}} \equiv k - (1 - \eta)Q$ is that carried by the antiquark. The $\epsilon_\Lambda^\mu(Q)$ is the polarization vector for a vector particle. Choosing $\Gamma_{\pm,\mp} = I \pm \gamma_5$ or

$\Gamma_{\pm,\pm} = \mp(\gamma^1 \mp i\gamma^2)$,¹ we can project out an LFWF with a specific quark-antiquark helicity configuration. In color space, there is a unit matrix on the right-hand side of Eq. (4). The trace is taken in Dirac and color spaces.

Due to various symmetry constraints, the $\Phi_{\lambda,\lambda'}^{\Lambda}(x, \mathbf{k}_T)$'s can be further expressed with five independent scalar amplitudes $\psi(x, \mathbf{k}_T^2)$ [19,23,24], i.e.,

$$\Phi_{\pm,\mp}^0 = \psi_{(1)}^0, \quad \Phi_{\pm,\pm}^0 = \pm k_T^{(\mp)} \psi_{(2)}^0, \quad (5)$$

$$\Phi_{\pm,\pm}^{\pm 1} = \psi_{(1)}^{\pm 1}, \quad \Phi_{\pm,\mp}^{\pm 1} = \pm k_T^{(\pm)} \psi_{(2)}^{\pm 1},$$

$$\Phi_{\mp,\pm}^{\pm 1} = \pm k_T^{(\pm)} \psi_{(3)}^{\pm 1}, \quad \Phi_{\mp,\mp}^{\pm 1} = (k_T^{(\pm)})^2 \psi_{(4)}^{\pm 1}. \quad (6)$$

with $k_T^{(\pm)} = k^x \pm ik^y$ and $\psi_{(2)}^1(x, \mathbf{k}_T^2) = -\psi_{(3)}^1(1-x, \mathbf{k}_T^2)$. They form a complete set of possible $q\bar{q}$ -LFWFs of an unflavored vector meson and virtual photon. For a real photon, i.e., the limit $Q^2 \rightarrow 0$ of a virtual photon, only transverse components exist, and hence the $\psi_{(i)}^0$'s are all zero.

It is also customary to classify the $q\bar{q}$ -LFWFs by their quark-antiquark orbital angular momentum projected along the z axis, denoted by l_z . l_z can be determined by $\Lambda = \lambda + \lambda' + l_z$ due to angular momentum conservation. Given all possible spin configurations in $\Phi_{\lambda,\lambda'}^{\Lambda,(f)}$, l_z can be 0, ± 1 , and ± 2 , which are referred to as s -, p - and d -wave components in the literature. l_z can also be read directly from the power of $k_T^{(\pm)}$ in Eqs. (5) and (6) [24]. For example, from the last equation of Eq. (6) we read $\Phi_{+,+}^{-1} = (k_T^{(-)})^2 \psi_{(4)}^1$, and hence l_z is negative due to the minus sign in $k_T^{(-)}$, and there are two units of orbital angular momentum due to the power of 2, which preserves angular momentum conservation as both the quark and antiquark are spin-up.

B. Vector-meson $q\bar{q}$ -LFWFs

In [19], we obtained the $q\bar{q}$ -LFWFs of ρ and J/ψ with Eq. (4). Therein, the dressed quark propagator and meson BS amplitude are obtained by simultaneously solving the quark's DS and meson's BS equations; see Fig. 1 for a diagrammatic representation under the rainbow-ladder truncation. We employed a Maris-Tandy-like gluon propagator model [25,26] and physical current quark mass for quarks. In this paper, we employ the exact same model and truncation setup for the DSEs and BSEs. For the ϕ meson to be considered in this paper, the only change made is to change the current quark mass from $m_{u/d} = 5$ MeV for the ρ meson to $m_s = 95$ MeV for the ϕ meson. After solving

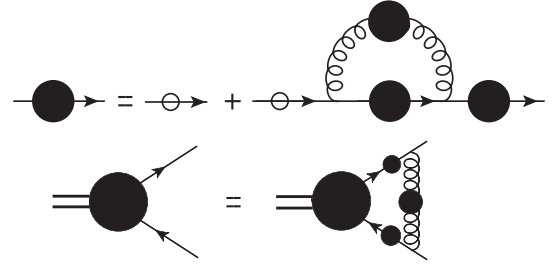


FIG. 1. The DSE of the quark's dressed propagator (upper plot) and vector meson's BSE of the BS amplitude (lower plot) under the rainbow-ladder truncation. The open circle is the bare (perturbative) quark propagator and the black blobs are the dressed quark and gluon propagators.

the DSEs and BSEs, the ϕ -meson mass is determined to be 1.01 GeV and the leptonic decay constant is 0.176 GeV, close to the Particle Data Group data 1.02 and 0.170 GeV, respectively [27]. The ϕ 's $q\bar{q}$ -LFWFs are then extracted based on Eq. (4), using a technique developed and explained in detail for ρ in [19].

In Fig. 2, we show the s -wave component $q\bar{q}$ -LFWFs of ϕ , as compared with the ρ meson's. There are significant differences between the two. The ϕ meson $q\bar{q}$ -LFWFs are generally narrower than ρ in longitudinal momentum fraction x . This is in line with the finding that $q\bar{q}$ -LFWFs with heavier quarks are generally more centered around $x = 1/2$, i.e., in heavier mesons the quark and antiquark tend to share the longitudinal momentum equally. In the transverse momentum $\mathbf{k}_T$, the ϕ -meson $q\bar{q}$ -LFWFs are more broadly distributed than the ρ meson, but not by much. This indicates that in coordinate space, the transverse size of the $q\bar{q}$ dipole of the ϕ meson is smaller than the ρ meson, but also not by much. This suggests that the ϕ meson can also be as sensitive a probe of saturation effects as ρ .

Generally speaking, the meson state is normalized, i.e., $\langle V_M | V_M \rangle = 1.0$. In all existing color-dipole model studies, this was used as a normalization condition for $q\bar{q}$ -LFWFs, i.e., enforcing $\langle q\bar{q} | q\bar{q} \rangle = 1.0$. In [19], we demonstrated with a DSE-BSE calculation that in ρ , $\langle q\bar{q} | q\bar{q} \rangle \lesssim 0.5$. Therein, we calculated

$$N_{\lambda,\lambda',(V_M)}^{\Lambda,(f)} \equiv \frac{\Lambda,\lambda,\lambda'}{(V_M)} \langle q_f \bar{q}_f | q_f \bar{q}_f \rangle_{(V_M)}^{\Lambda,\lambda,\lambda'} \\ = \int_0^1 dx \int \frac{d\mathbf{k}_T^2}{2(2\pi)^3} |\Phi_{\lambda,\lambda',(V)}^{\Lambda,(f)}(x, \mathbf{k}_T)|^2. \quad (7)$$

The results for the ϕ meson, along with the ρ and J/ψ mesons, are summarized in Table I. $N_{HF} \equiv 1 - \sum_{\lambda,\lambda'} N_{\lambda,\lambda'}^{\Lambda}$ indicates contribution from higher Fock states. We notice that in the ϕ meson, N_{HF} is around 50% and hence still significant. Meanwhile, by comparing ρ , ϕ , and J/ψ , one can notice that the s -wave $q\bar{q}$ -LFWFs' contribution increases with current quark mass, and the p - and d -wave

¹The convention is that the + and/or - signs in the same row should be simultaneously taken in one equation. This convention is adopted throughout this paper.

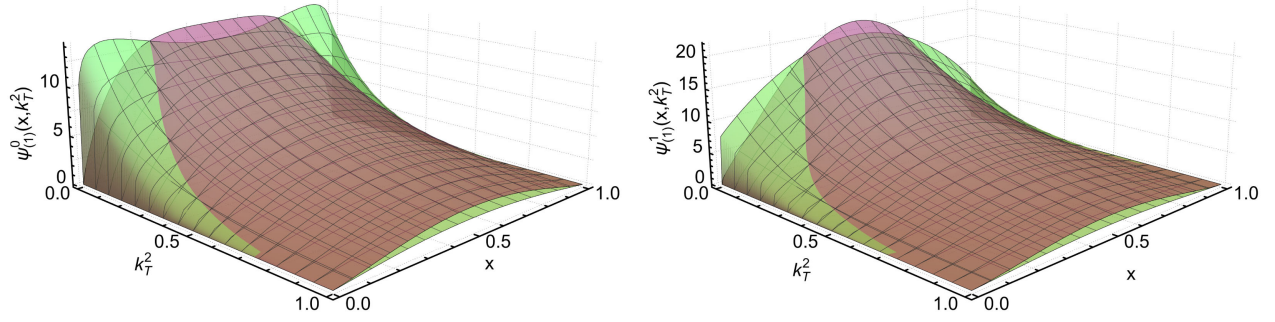


FIG. 2. Dominant s -wave component $q\bar{q}$ -LFWFs of ϕ (purple) and ρ (green). See Eqs. (3), (5), and (6) for the definitions of the ψ 's. Note the purple surfaces are larger than green ones in magnitude at $x = 0$ and $\mathbf{k}_T^2 = 0$ GeV².

components show the opposite behavior. The total effect is a reduction in N_{HF} . This is consistent with the finding that as the meson gets heavier, the higher Fock states and high orbital angular momentum states all get suppressed.

To conclude, we find that the ϕ meson $q\bar{q}$ -LFWFs are different from ρ in profile, but their $N_{\lambda,\lambda'}$'s are close. This reveals that the ϕ meson is essentially a light meson, e.g., it is closer to ρ instead of J/ψ . This is due to the fact that the dynamical chiral symmetry breaking of QCD reduces the difference between u/d and s quarks. For instance, although the current quark mass ratio is $m_s/m_u \approx 20$, after solving the quark's gap equation in Fig. 1, the ratio of their quark mass functions is approximately 1.2 by dynamical chiral symmetry breaking. The ϕ and ρ mesons therefore host comparable internal soft dynamics, reflected by overall quantities such as the $N_{\lambda,\lambda'}^\Lambda$'s. On the other hand, the $q\bar{q}$ -LFWFs characterize detailed structural information of hadrons, and hence are sensitive to the current quark mass. Analysis on leading twist parton distribution amplitudes² of the physical pion $\pi(u\bar{d})$ (130 MeV) and fictitious pion $\pi(s\bar{s})$ (690 MeV) had already shown significant discrepancy in their profiles [28].

C. Photon $q\bar{q}$ -LFWFs

In [20], we utilized Eq. (4) to obtain the photon's nonperturbative $q\bar{q}$ -LFWFs. The photon's BS wave function for $\gamma^* \rightarrow q\bar{q}$ is obtained by solving the contact interaction model within the DSE formalism. A diagrammatic representation of the photon's BSE under ladder truncation is shown in Fig. 3. As compared with Fig. 1, there is an additional bare term arising from leading-order QED. The gluon exchange ladder is important for low-virtuality Q^2 , and gets suppressed at large Q^2 , as a consequence of QCD's asymptotic freedom. It is therefore necessary to consider nonperturbative QCD effects in photons with low virtuality, including real photons where $Q^2 = 0$ GeV².

²The leading twist parton distribution amplitude is obtained by integrating over $\mathbf{k}_T$ in s -wave $q\bar{q}$ -LFWFs.

The extracted photon nonperturbative $q\bar{q}$ -LFWFs are summarized as follows:

$$\psi_{(1)}^{0,(f)}(x, \mathbf{k}_T^2) = e_f e P_T^{(f)}(Q^2) \frac{\sqrt{N_c}}{Q} \left(1 - \frac{2x(1-x)Q^2}{k_\perp^2 + \varepsilon_x^2} \right), \quad (8)$$

$$\psi_{(2)}^{0,(f)}(x, \mathbf{k}_T^2) = 0, \quad (9)$$

$$\psi_{(1)}^{1,(f)}(x, \mathbf{k}_T^2) = e_f e P_T^{(f)}(Q^2) \sqrt{2N_c} \frac{M_f}{k_\perp^2 + \varepsilon_x^2}, \quad (10)$$

$$\psi_{(2)}^{1,(f)}(x, \mathbf{k}_T^2) = e_f e P_T^{(f)}(Q^2) \sqrt{2N_c} \frac{x}{k_\perp^2 + \varepsilon_x^2} \quad (11)$$

$$= -\psi_{(3)}^{1,(f)}(1-x, \mathbf{k}_T^2), \quad (12)$$

$$\psi_{(4)}^{1,(f)}(x, \mathbf{k}_T^2) = 0, \quad (13)$$

with $\varepsilon_x \equiv \sqrt{Q^2 x(1-x) + M_f^2}$. $P_T^{(f)}(Q^2)$ is a dressing function of the quark-antiquark-photon vertex, i.e.,

$$P_T^{(f)}(Q^2) = \frac{1}{1 + K_\gamma^{(f)}(Q^2)}, \quad (14)$$

$$K_\gamma^{(f)}(Q^2) = \frac{4\alpha_{\text{IR}} M_f}{3\pi m_G^2} \int_0^1 d\alpha \alpha(1-\alpha) Q^2 \tilde{C}_1^{iu}(\varepsilon_\alpha), \quad (15)$$

TABLE I. The $q\bar{q}$ -LFWFs' contributions to Fock-state normalization. See Eq. (7) for the definition of N .

		$N_{\uparrow,\downarrow}$	$N_{\downarrow,\uparrow}$	$N_{\uparrow,\uparrow}$	$N_{\downarrow,\downarrow}$	N_{HF}
$\Lambda = 0$	ϕ	0.24	0.24	0.02	0.02	0.48
	ρ	0.19	0.19	0.04	0.04	0.54
	J/ψ	0.44	0.44	0.01	0.01	0.10
$\Lambda = 1$	ϕ	0.04	0.04	0.35	0.01	0.56
	ρ	0.04	0.04	0.24	0.02	0.66
	J/ψ	0.03	0.03	0.78	≈ 0.0	0.16

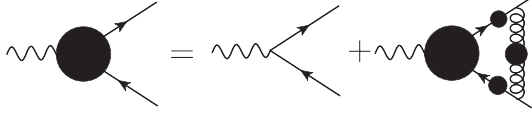


FIG. 3. The photon's inhomogeneous BSE of the photon's BS amplitude $\gamma^* \rightarrow q\bar{q}$ under the ladder truncation.

where $C^{iu}(M^2)/M^2 = \Gamma(-1, M^2\tau_{uv}^2) - \Gamma(-1, M^2\tau_{ir}^2)$, and $\Gamma(\alpha, y)$ is the incomplete gamma function. The notations $\tilde{C}_1^{iu}(z) = C_1^{iu}(z)/z$ and $\tilde{C}_1^{iu}(z) = -z(d/dz)C^{iu}(z)$ are used. The model parameters involved are the gluon mass $m_G = 0.5$ GeV, interaction strength $\alpha_{IR}/\pi = 0.36$, and regulators $\Lambda_{ir} = 0.24$ GeV and $\Lambda_{uv} = 0.91$ GeV. We

take current quark masses $m_{u/d} = 0.007$ GeV and $m_s = 0.095$ GeV, which produce $M_{u/d} = 0.37$ GeV and $M_s = 0.53$ GeV by solving the quark's gap equation in Fig. 1.

In the color-dipole model, the scattering amplitude is formulated in coordinate space, and hence the photon LFWFs are Fourier transformed to coordinate space, i.e.,

$$\tilde{\Phi}_{\lambda,\lambda', (V)}^{\Lambda, (f)}(x, \mathbf{r}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot\mathbf{r}} \Phi_{\lambda,\lambda', (V)}^{\Lambda, (f)}(x, \mathbf{k}). \quad (16)$$

Letting $\mathbf{r} = (r \cos \theta_r, r \sin \theta_r)$, Eqs. (8)–(13) and (16) yield

$$\tilde{\Phi}_{\lambda,\lambda', (\gamma^*)}^{0, (f)}(\mathbf{r}, x; Q) = e e_f P_T^{(f)}(Q^2) \frac{\sqrt{N_c}}{Q} \left(\delta^2(\mathbf{r}) - 2Q^2 x(1-x) \frac{K_0(\epsilon_x r)}{2\pi} \right) \delta_{\lambda, -\lambda'} \quad (17)$$

$$\tilde{\Phi}_{\lambda,\lambda', (\gamma^*)}^{\pm 1, (f)}(\mathbf{r}, x; Q) = e e_f P_T^{(f)}(Q^2) \sqrt{2N_c} \{ \mp i e^{\pm i\theta_r} [x \delta_{\lambda, \pm} \delta_{\lambda', \mp} - (1-x) \delta_{\lambda, \mp} \delta_{\lambda', \pm}] \partial_r + M_f \delta_{\lambda, \pm} \delta_{\lambda', \pm} \} \frac{K_0(\epsilon_x r)}{2\pi}. \quad (18)$$

The δ -function term is always omitted in color-dipole model studies due to $\mathcal{N}(x, r=0, b) = 0$; see Eq. (25) below. Equations (17) and (18) only work at low Q^2 , as the input photon BS wave function is solved using a low-energy effective interaction model. For high Q^2 , QCD effects get suppressed and the perturbative QED result can be used. The perturbative photon $q\bar{q}$ -LFWFs can be obtained by making the replacements $P_T^{(f)} \rightarrow 1$ and $M_f \rightarrow m_f$ in Eqs. (17) and (18) [29]. In [20] we introduced a transition function to interpolate the results in the low- and high- Q regions,

$$|\tilde{\Phi}_{\text{Full}}|^2 = F_{\text{part}}(Q^2) |\tilde{\Phi}_{\text{NP}}|^2 + [1 - F_{\text{part}}(Q^2)] |\tilde{\Phi}_{\text{P}}|^2. \quad (19)$$

$\tilde{\Phi}$ is the abbreviation for $\tilde{\Phi}_{\lambda,\lambda', (\gamma^*)}^{\Lambda, (f)}(r, x; Q)$, with the subscripts NP for nonperturbative and P for perturbative. The transition function takes the parametrization

$$F_{\text{part}}(Q^2) = \frac{Q_0^{2n}}{(Q^2 + Q_0^2)^n}. \quad (20)$$

The determination of the parameters Q_0 and n will be explained below Eq. (30).

For the purpose of the exclusive vector particle production study in this paper, we further rewrite Eq. (19) to get a direct expression for $\tilde{\Phi}_{\text{Full}}$. This is achievable, e.g., with

$$\tilde{\Phi}_{\text{Full}} = \tilde{\Phi}_{\text{NP}} / |\tilde{\Phi}_{\text{NP}}| \sqrt{F_{\text{part}}(Q^2) |\tilde{\Phi}_{\text{NP}}|^2 + [1 - F_{\text{part}}(Q^2)] |\tilde{\Phi}_{\text{P}}|^2}. \quad (21)$$

We employ Eq. (21) to calculate every spin component of the interpolated full $q\bar{q}$ -LFWFs.

III. EXCLUSIVE VECTOR-MESON AND PHOTON ELECTROPRODUCTION WITHIN THE COLOR-DIPOLE APPROACH UNDER $q\bar{q}$ TRUNCATION

A. Formalism

Diffractional vector-meson production is a class of exclusive processes in high-energy (virtual) photon-proton collisions $\gamma^* p \rightarrow V p$. Such events exhibit a large rapidity gap where no other particles are produced between the vector meson V and the intact target p . The vector particle can also be a real photon, namely, deeply virtual Compton scattering. In the color-dipole approach, the scattering amplitude for $\gamma^* p \rightarrow p V$ is written as an overlap of γ^* and the vector-meson (or photon) V 's $q\bar{q}$ -LFWFs convolved with the color dipole-proton scattering amplitude $\mathcal{N}(x, r, b)$ [30,31],

$$\mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p}(x, Q, \Delta) = 2i \int_0^1 \frac{dz}{4\pi} \int d^2\mathbf{r} \int d^2\mathbf{b} [\Psi_{(V)}^* \Psi_{(\gamma^*)}]_{T,L} \times e^{-i[\mathbf{b} - (\frac{1}{2}-z)\mathbf{r}] \cdot \Delta} \mathcal{N}(x, r, b). \quad (22)$$

Here $Q = \sqrt{-q^2}$ with virtual photon four-momentum q . $r = |\mathbf{r}|$ is the color-dipole size, $b = |\mathbf{b}|$ is the impact parameter, and $\Delta = |\Delta|$ is the momentum transfer between protons before and after scattering. $x = \frac{Q^2 + M_V^2}{Q^2 + W^2}$ is the longitudinal momentum fraction of the proton carried by the scattered gluon. The subscripts T and L represent transversely and longitudinally polarized vector particles,

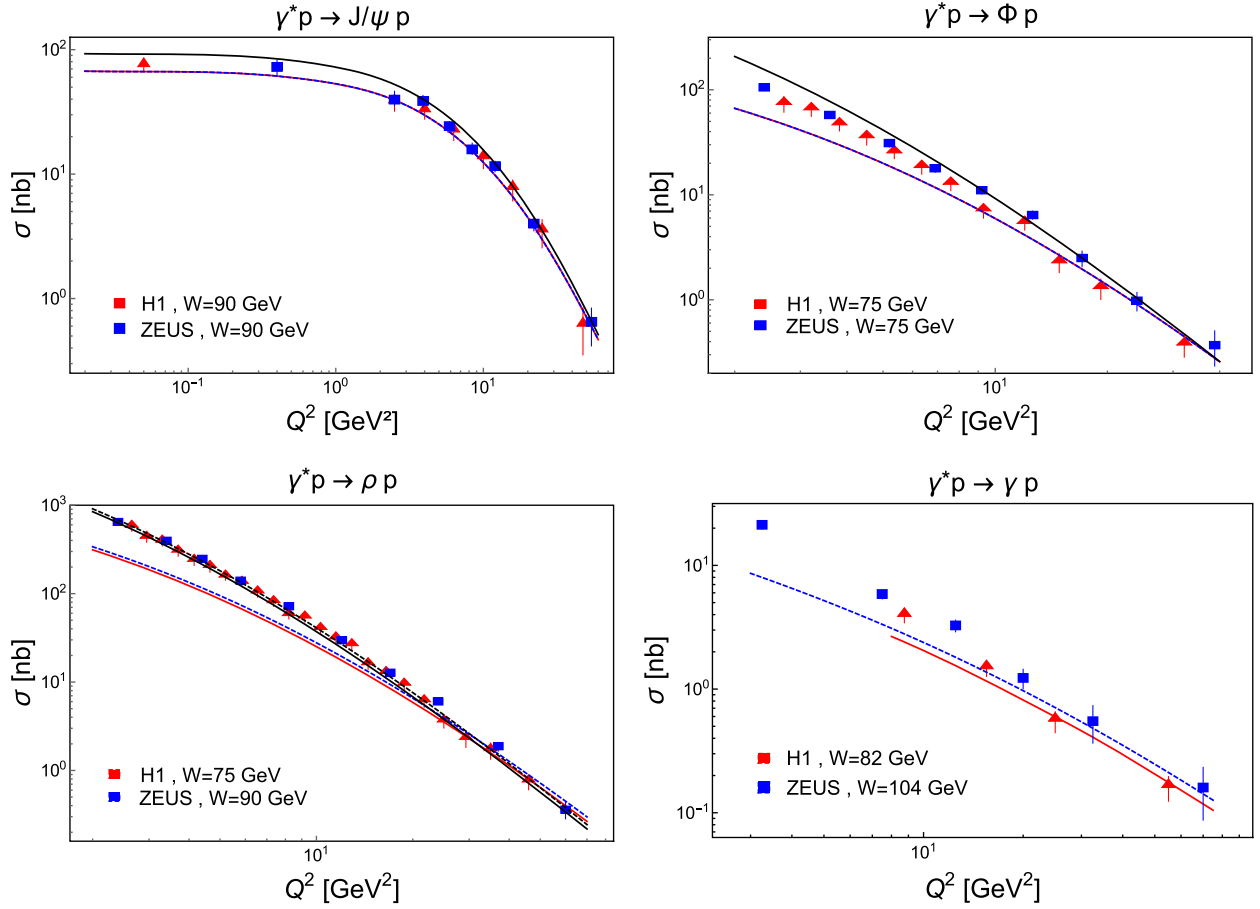


FIG. 4. Total cross sections for J/ψ , ϕ , ρ , and γ as functions of Q^2 . The red solid and blue dashed curves are calculations with our model to be compared with H1 and ZEUS data [40–46]. The black curves are obtained by replacing our $q\bar{q}$ -LFWFs with the boosted Gaussian model in [6]. Note the black curves are all larger than blue and red lines at $Q^2 \lesssim 10 \text{ GeV}^2$.

i.e., helicity $|\Lambda_V| = 1$ or $\Lambda_V = 0$, respectively. Note that $\mathcal{A}_L = 0$ for deeply virtual Compton scattering since the $q\bar{q}$ -LFWFs vanish for longitudinal real photons.

The overlap of $q\bar{q}$ -LFWFs takes the form

$$[\Psi_{(V)}^* \Psi_{(\gamma^*)}]_T = \frac{1}{2} \sum_{f=u,d,s,c} \sum_{\Lambda=\pm 1} \sum_{\lambda=\pm} \sum_{\lambda'=\pm} (\tilde{\Phi}_{\lambda,\lambda',(\gamma^*)}^{\Lambda,(f)}(z, \mathbf{r}))^* \times \tilde{\Phi}_{\lambda,\lambda',(V)}^{\Lambda,(f)}(z, \mathbf{r}, Q), \quad (23)$$

$$[\Psi_{(V)}^* \Psi_{(\gamma^*)}]_L = \sum_{f=u,d,s,c} \sum_{\Lambda=0} \sum_{\lambda=\pm} \sum_{\lambda'=\pm} (\tilde{\Phi}_{\lambda,\lambda',(V)}^{\Lambda,(f)}(z, \mathbf{r}))^* \times \tilde{\Phi}_{\lambda,\lambda',(\gamma^*)}^{\Lambda,(f)}(z, \mathbf{r}, Q). \quad (24)$$

Note that when V refers to a vector meson, the summation of flavor only covers the valence quarks, while when V refers to a real photon, the summation runs through all flavors. The real photon's $q\bar{q}$ -LFWFs can be obtained by setting $Q = 0$ in Eq. (21).

For the color-dipole-proton scattering amplitude $\mathcal{N}(x, r, b)$, we employ the impact parameter-dependent color glass condensate model [8,32], which reads

$$\mathcal{N}(x, r, b) = \begin{cases} N_0 \left(\frac{rQ_s}{2} \right)^{2\gamma_{\text{eff}}} & rQ_s \leq 2, \\ 1 - \exp[-\mathcal{A} \ln^2(\mathcal{B} r Q_s)] & rQ_s > 2, \end{cases} \quad (25)$$

with

$$Q_s(x, b) = \left(\frac{x_0}{x} \right)^{\frac{1}{\lambda}} \exp \left[-\frac{b^2}{4\gamma_s B_{\text{CGC}}} \right], \quad (26)$$

$$\gamma_{\text{eff}} = \gamma_s + \frac{1}{\kappa \lambda Y} \ln \left(\frac{2}{r Q_s} \right), \quad (27)$$

$$Y = \ln(1/x), \quad (28)$$

and

$$\mathcal{A} = -\frac{N_0^2 \gamma_s^2}{(1 - N_0)^2 \ln(1 - N_0)}, \quad (29)$$

$$\mathcal{B} = \frac{1}{2} (1 - N_0)^{\frac{1-N_0}{N_0 \gamma_s}}. \quad (30)$$

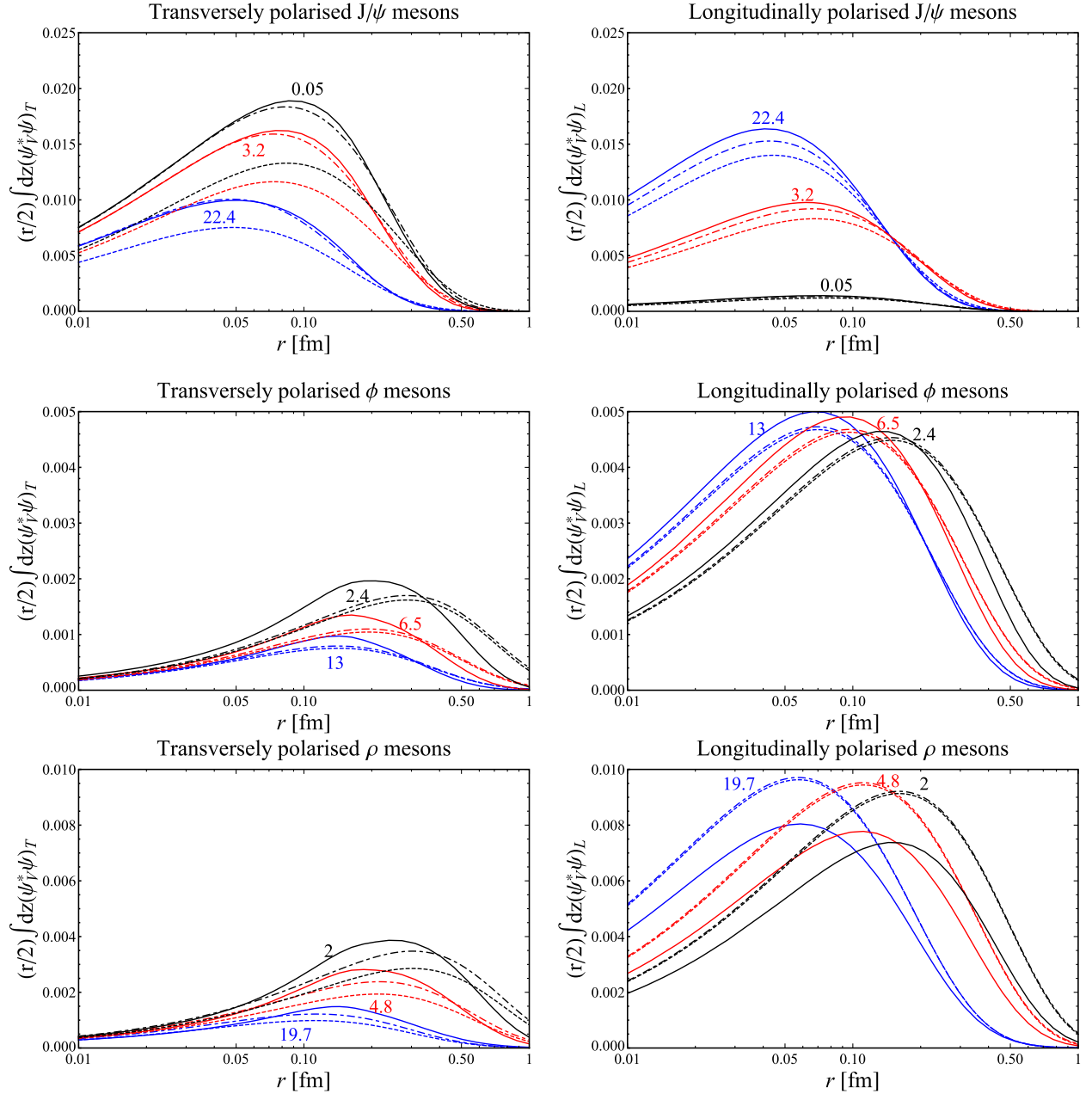


FIG. 5. Comparison between DSE-BSE-based vector-meson $q\bar{q}$ -LFWFs (solid) with the boosted Gaussian (dot-dashed) and Gaus-LC (dashed) models of [6] at different Q^2 (in different colors). At $r = |\mathbf{r}| = 0.5$ fm (all plots except longitudinally polarized J/ψ) the nine curves form three groups of three, colored black, red, and blue from top to bottom; for longitudinally polarized J/ψ the order is reversed at $r = 0.1$ fm (blue, red, black).

The model parameters of the impact parameter-dependent color glass condensate model are directly employed from [20]. $\kappa = 9.9$ and $B_{\text{CGC}} = 5.5 \text{ GeV}^{-2}$ are chosen following [8], and the other model parameters N_0 , γ_s , x_0 , and λ are combined with Q_0 and n in Eq. (20) to render a global fit to inclusive deep inelastic scattering reduced cross section data for $Q^2 \in [0.25, 50] \text{ GeV}^2$ [20]. The best fit yields $N_0 = 0.4596$, $\gamma_s = 0.6177$, $x_0 = 0.0001326$, and $\lambda = 0.1875$, along with $Q_0^2 = 1.052 \text{ GeV}^2$ and $n = 3.97$. Note that the physical current quark masses

$m_{u/d} \approx 0.005 \text{ GeV}$, $m_s = 0.095 \text{ GeV}$, and $m_c = 1.27 \text{ GeV}$ are used throughout the calculation, without introducing phenomenological values such as $m_{u/d/s} = 0.14 \text{ GeV}$ that were popular in color-dipole model studies [6,33–37].

The differential cross section of diffractive vector particle electroproduction is

$$\frac{d\sigma_{L,T}^{\gamma^* p \rightarrow V p}}{dt} = \frac{1}{16\pi} |\mathcal{A}_{L,T}^{\gamma^* p \rightarrow V p}(x, Q^2, \Delta)|^2 (1 + \beta^2) R_g^2. \quad (31)$$

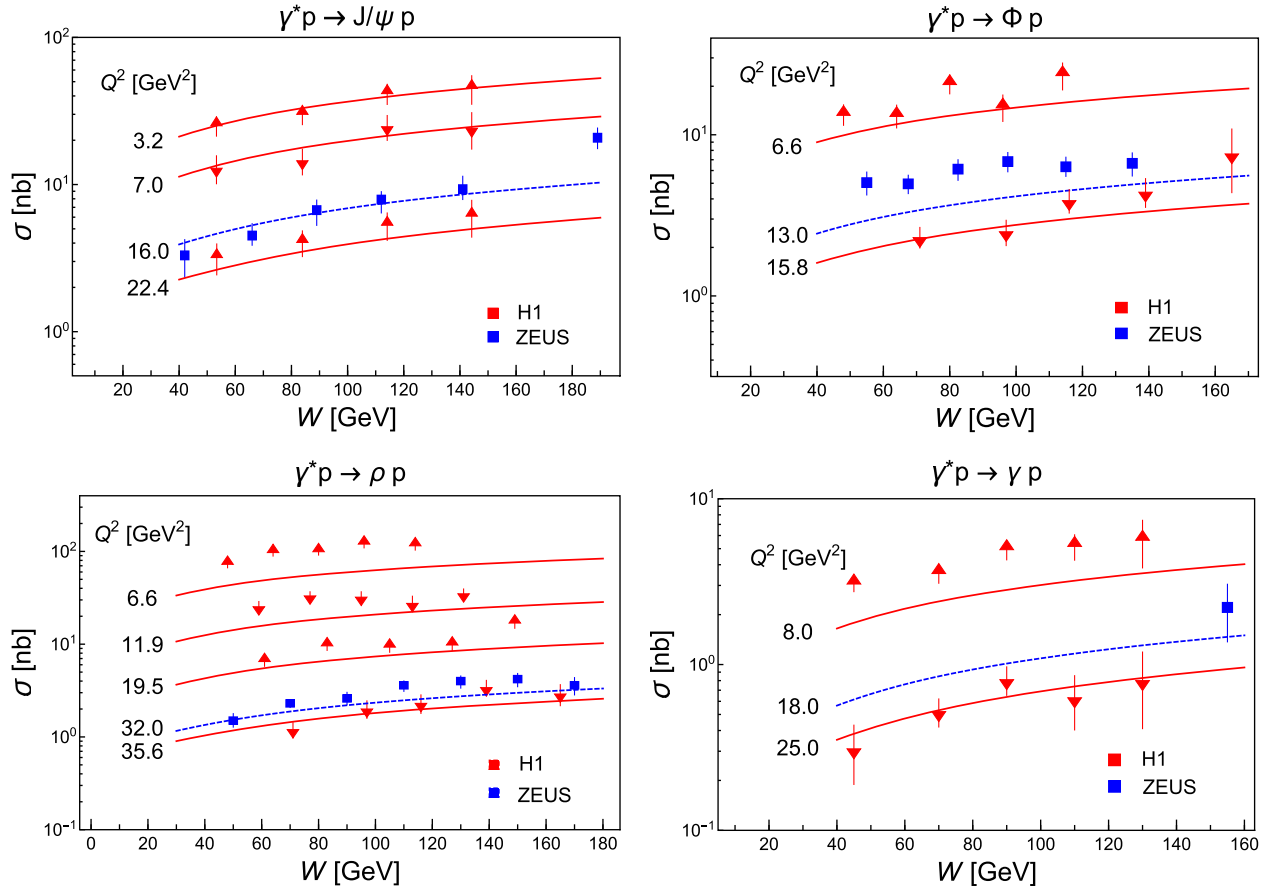


FIG. 6. Differential cross sections for J/ψ , ϕ , ρ , and γ as functions of W . The red solid and blue dashed curves are calculations to be compared with H1 and ZEUS data [40–46].

The modification factor $1 + \beta^2$ accounts for correction from the imaginary part of $\mathcal{N}$, with $\beta = \tan(\frac{\pi}{2}\lambda)$ and $\lambda = \frac{\partial \ln(\mathcal{A}_{T,L}^{*p \rightarrow Vp})}{\partial \ln(1/x)}$ [38]. $R_g(\lambda) = \frac{\gamma^{2\lambda+3} \Gamma(\lambda+\frac{5}{2})}{\sqrt{\pi} \Gamma(\lambda+4)}$ is a skewness factor that accounts for the skewness effect when gluons that interact with color dipoles carry different momentum fractions [39].

B. Results

In Fig. 4 we show the cross section of exclusive electroproduction of light and heavy vector mesons and photons as a function of Q^2 . For J/ψ , the result agrees well with data in almost the entire Q^2 range. As the meson gets lighter, the agreement shifts toward the larger Q^2 region. Denoting Q_c as the point where agreement starts, we find $Q_c^2 \approx 10, 20$, and 20 GeV^2 for ϕ ,³ ρ , and γ , respectively. This is reasonable, as in exclusive processes higher Fock-state contributions can only be suppressed by sufficiently high Q^2 . Based on Table I,

³For ϕ production, the ZEUS data is visibly larger than the H1 data. Since our calculation aligns more closely with the H1 data, the conclusions are drawn based on comparisons with those results.

the leading Fock-state approximation works fine for J/ψ since J/ψ is almost dominated by the $q\bar{q}$ component, but for light mesons, higher Fock states can not be ignored unless the high Q^2 suppresses their contribution. Meanwhile, from Fig. 4 we notice for the ϕ meson the agreement starts from $Q_c^2 \approx 10 \text{ GeV}^2$, which is lower than for the ρ meson. Hence, the ϕ meson is in a special position. On the one hand, it can be more sensitive to saturation effects than J/ψ for its large dipole size, which is comparable with ρ . On the other hand, the leading Fock-state truncation works better for ϕ than ρ and yields a Q_c that is not too high. We note that an incoming photon with a higher Q has a smaller dipole size, limiting the size of overlapping color dipoles in Eqs. (23) and (24), making the result less sensitive to saturation effects. In this sense, the ϕ meson exhibits a compromise between the light ρ meson and the heavy J/ψ meson, regarding the Q_c and color-dipole size. Finally, we notice that for exclusive photon production, the Q_c is close to the ρ meson's. This can be explained by noticing that the summation of photon $q\bar{q}$ -LFWFs runs through all four flavors in Eqs. (23) and (24). This includes the contribution from u/d quarks, which is potentially populated with higher Fock states at low scales.

We also compare our vector-meson $q\bar{q}$ -LFWFs with those commonly employed in color-dipole model studies,

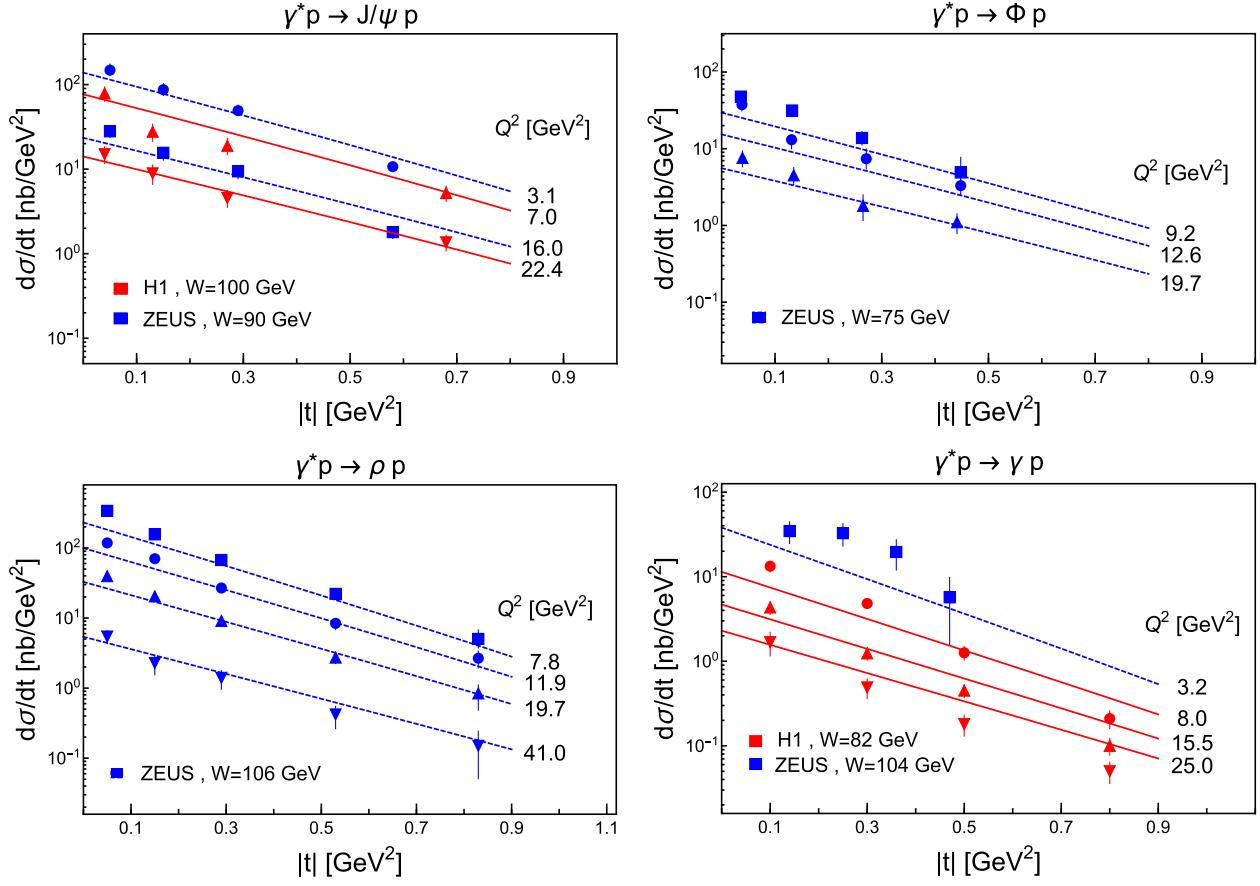


FIG. 7. Differential cross sections for J/ψ , ϕ , ρ , and γ as functions of t . The red solid and blue dashed curves are calculations to be compared with H1 and ZEUS data [40,41,43–46].

such as the Gaus-LC and boosted Gaussian models [6]. Figure 5 shows the quantity $r/2 \int dz [\Psi_{(V)}^*, \Psi_{(\gamma^*)}]_T$ for various vector mesons at different Q^2 . The solid curves correspond to the DSE-BSE $q\bar{q}$ LFWFs obtained in this work, while the dot-dashed and dashed curves represent the boosted Gaussian and Gaus-LC models, respectively. For J/ψ , the curves agree closely over a wide range of Q^2 , particularly between our wave functions and the boosted Gaussian ones. In contrast, for the ϕ and ρ mesons, the deviations from other models are more pronounced, though they diminish as Q^2 increases. This trend explains the convergence between the boosted Gaussian results (black curves) and ours at large Q^2 in Fig. 4. However, we emphasize that, as discussed in Sec. I, the boosted Gaussian model, despite its good agreement with data over a broad Q^2 range, relies on the oversimplified assumption that the $|q\bar{q}\rangle$ Fock state fully saturates the light meson.

In Fig. 6 the cross sections as functions of W are shown. The calculated J/ψ cross sections agree well with data from low to high Q^2 . For the ϕ meson, our calculation shows a preference for H1 data for sufficiently large $Q^2 = 15.8$ GeV². The $Q^2 = 13.0$ GeV² data is larger than our calculation, but we recall that the ZEUS data is

generally larger than H1 in this case. For the ρ meson, agreement shows up for $Q^2 = 19.5$ GeV. For deeply virtual Compton scattering, good agreement is found for $Q^2 = 18.0$ and 25.0 GeV², but deviation is prominent for $Q^2 = 8.0$ GeV. All of these results are in line with the conclusions drawn from Fig. 4.

In Fig. 7 we show the cross sections as functions of t . One can also notice that the agreement is better for larger Q^2 than small Q^2 . The t dependences of the curves are generally close to experiment data, except for the photon. This may be due to the oversimplified real photon $q\bar{q}$ -LFWFs obtained with the contact interaction model under the DSE approach. Employing realistic interaction models such as Maris-Tandy-like models can produce more realistic photon $q\bar{q}$ -LFWFs, and may bring the calculation closer to data.

IV. SUMMARY

Supplemented with real photon and ϕ meson DSE-BSE-based $q\bar{q}$ -LFWFs, we studied the diffractive electroproduction of various light and heavy vector particles, i.e., γ , ρ , ϕ , and J/ψ . We found that the light mesons host considerable higher Fock states, as summarized in Table I.

In particular, the ϕ meson, like the ρ meson, remains far from where the $\langle q\bar{q}|q\bar{q}\rangle = 1$ approximation can be seriously taken. On the other hand, the ϕ meson $q\bar{q}$ -LFWFs are quite different from the ρ meson's in profile, exhibiting ϕ 's distinctive and unique properties among the vector mesons.

Bearing in mind that light mesons contain significant higher Fock-state components, we studied exclusive vector meson production under the leading Fock-state truncation. Our key finding is shown in Fig. 4, i.e., while the $q\bar{q}$ truncated calculation agrees well with HERA J/ψ data for Q^2 as low as 0, the agreement for ρ , ϕ , and γ with HERA data only starts from a certain $Q_c^2 \gtrsim 10 \text{ GeV}^2$. This is in line with the twist expansion idea within the QCD factorization of exclusive processes. We emphasize that we did not introduce or tune any model parameter in this paper, and hence the calculation demonstrates a robust prediction as a joint effort of the color-dipole approach and nonperturbative DSE-BSE study. Our calculation therefore supports the validity of the leading Fock-state approximation for the photoproduction of J/ψ and Υ within the color-dipole framework in ultraperipheral collisions, while it remains inapplicable to the study of light vector-meson photoproduction at the present stage.

This study also highlights the importance of diffractive ϕ electroproduction studies. In color-dipole model studies, the lighter mesons have a larger dipole size and are more sensitive to saturation effects. Yet, as we have shown, they also push the Q_c toward larger values where the saturation effect gets weakened. The ϕ meson thus balances the advantages between light and heavy vector mesons. We note that existing simulations of exclusive ϕ production in electron-nuclei collisions at electron-ion colliders shows

that saturation can have a visible effect in the domain of $Q^2 \approx 10 \text{ GeV}^2$ [47]. This result could be revisited with DSE-BSE-based $q\bar{q}$ -LFWFs. If it stays true, then a kinematic window is open for studying saturation effects in nuclei, based on a novel determination of the leading Fock-state contribution to diffractive light vector particle electroproduction in the presence of complex higher Fock states.

Finally, we note that the critical scale Q_c for light vector-meson electroproduction may be reduced by incorporating higher Fock components, such as $|q\bar{q}g\rangle$, into the color-dipole framework. Formalisms accounting for such higher Fock-state effects within the color glass condensate effective theory have been developed, addressing both perturbative emission contributions [12,13] and nonperturbative genuine higher-twist effects [14,15]. This extension could enhance sensitivity to gluon saturation phenomena by effectively increasing the dipole size and thereby lowering Q_c .

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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