

## Gluonic contributions to the pion parton distribution functions

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We investigate gluonic contributions to the pion beyond the valence sector using basis light-front quantization (BLFQ) and the Minkowski Bethe-Salpeter equation (BSE). Starting from the valence light-front wave function, we construct the quark-antiquark-gluon component and derive the corresponding quark and gluon parton distribution functions (PDFs). A proposed operator generating this higher Fock sector is validated against BLFQ and BSE results. We identify dynamical chiral symmetry breaking through enhanced spin-flip transitions, which amplify the low- $x$  quark PDF and underlie the  $\pi$ - $\rho$  mass splitting. The framework also reveals the impact of a vanishing gluon mass on the gluon PDF, highlighting sensitivity to nonperturbative gluon dynamics.

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**Introduction.** The light-front (LF) representation of hadrons encodes the full complexity of quantum chromodynamics (QCD), with strongly interacting dressed quarks and gluons forming eigenstates of the mass-squared operator [1,2]. This requires the dynamical coupling of an infinite tower of Fock sectors, which is truncated in practical basis LF quantization (BLFQ) calculations [3]. In recent BLFQ studies, the valence and valence-plus-gluon sectors have been coupled to describe light mesons [4], where confinement acts in the valence sector and the  $|q\bar{q}g\rangle$  state arises via off-diagonal QCD LF Hamiltonian terms [1]. For the pion, the  $q\bar{q}$  and  $q\bar{q}g$  components each contribute about 50% to the total normalization [4], underscoring the dynamical role of the gluon even with confinement. Dynamical chiral symmetry breaking, responsible for the large  $\pi$ - $\rho$  mass splitting, requires coupling to higher Fock sectors. This coupling can be recast as an effective interaction, as in the

iterative resolvent method [1], which enhances the spin-flip matrix element via a large effective quark vertex mass [5], a mechanism successfully implemented in BLFQ [4].

The relevance of higher Fock components in the pion has been recently explored phenomenologically in Ref. [6], where a parametrization including  $q\bar{q}$ ,  $q\bar{q}q\bar{q}$ ,  $q\bar{q}g$ , and  $q\bar{q}gg$  components was fitted to reproduce experimental data on the pion parton distribution functions (PDFs) [7,8] and electromagnetic form factor. In contrast, four-dimensional nonperturbative approaches such as Dyson-Schwinger (DS) and Bethe-Salpeter (BS) equations describe the pion as a Goldstone boson from spontaneous chiral symmetry breaking, providing successful descriptions of pseudoscalar and vector mesons in the SU(3) sector [9,10]. However, these formulations are in Euclidean space and require additional steps to connect with the LF Fock expansion and extract parton distributions [11–13]. A Minkowski-space solution of the Bethe-Salpeter equation (BSE), as developed in Ref. [14], allows access to both the valence component and inclusive effects of higher Fock sectors [15], showing that the valence state accounts for about 70% of the normalization. The BSE in ladder approximation couples the valence state to an infinite Fock tower [16–20]. While only the valence wave function can be explicitly extracted—by integrating out the LF energy—the method retains the longitudinal and transverse momentum dependence of the LF variables [14,21].

Ideally, BLFQ and DS/BS approaches to QCD should yield consistent hadron observables, but differences in truncation

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schemes and confinement implementations lead to discrepancies. In BLFQ, the enhanced spin-flip matrix element leaves a distinct imprint on the pion structure at low  $x$  through the  $|q\bar{q}g\rangle$  component. Notably, BLFQ eigenvalue equations without confinement and truncated to  $q\bar{q}g$  resemble the ladder BSE in the Leibbrandt-Mandelstam gauge without self-energy corrections [22], corresponding to the lowest-order kernel in LF projected BSE expansions [16,19,20].

This work has two main goals: (i) to analyze the quark and gluon PDFs from the  $q\bar{q}g$  component using continuous parametrizations, bypassing the discretization in BLFQ, and (ii) to compare BLFQ and BSE results for the higher Fock contributions to the quark PDF, isolating the nonvalence part. We focus on characterizing these contributions, particularly the asymmetry around  $x = 1/2$ . The BSE model reproduces the pion electromagnetic form factor [23]. Additionally, we provide a practical method to generate the  $q\bar{q}g$  component from the leading spin-antialigned valence wave function, enabling broader applications.

*Theoretical framework.* The LF bound state is obtained by solving the Hamiltonian eigenvalue problem in a frame with total transverse momentum,  $\vec{P}_\perp = 0$ ,

$$P^- P^+ |\Psi\rangle = M^2 |\Psi\rangle, \quad (1)$$

where  $P^\pm = P^0 \pm P^3$  denote the LF Hamiltonian ( $P^-$ ) and longitudinal momentum ( $P^+$ ); the eigenvalue  $M^2$  gives the bound state mass squared.

The LF Hamiltonian we use contains the LF QCD Hamiltonian and confinement,  $P^- = P_{\text{QCD}}^- + P_{\text{C}}^-$  [4]. With one dynamical gluon, the LF QCD Hamiltonian in the LF gauge  $A^+ = 0$  [1,4] reads

$$\begin{aligned} P_{\text{QCD}}^- = & \int dx^- d^2x^\perp \left[ \frac{1}{2} \bar{\psi} \gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+} \psi \right. \\ & + \frac{1}{2} A_a^i [m_g^2 + (i\partial^\perp)^2] A_a^i + g_s \bar{\psi} \gamma_\mu T^a A_a^\mu \psi \\ & \left. + \frac{1}{2} g_s^2 \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi \right], \quad (2) \end{aligned}$$

where  $\psi$  and  $A^\mu$  are the quark and gluon fields, respectively.  $T^a$  is the half Gell-Mann matrix,  $T^a = \lambda^a/2$ , and  $\gamma^+ = \gamma^0 + \gamma^3$ , where  $\gamma^\mu$  represents the Dirac matrix. The first two terms in Eq. (2) are the kinetic energies of the quark and gluon; the last two describe their interactions via

the coupling  $g_s$ . Here,  $m_0$  and  $m_g$  denote the bare quark mass and model gluon mass, respectively. The explicit gluon mass term  $m_g^2$  breaks gauge invariance—already affected by Fock space truncation—but is introduced as an effective parameter to regularize infrared divergences and mimic confinement. While the gluon remains massless at the Lagrangian level, the Schwinger mechanism generates a gauge-invariant, dynamically dressed gluon propagator with effective mass behavior [24–26]. Our use of  $m_g^2$  reflects this dressed gluon picture, consistent with infrared regularization in Hamiltonian frameworks and effective models.

Using the Fock sector dependent renormalization scheme [27,28], we introduce a mass counterterm  $\delta m_q = m_0 - m_q$  in the leading Fock sector to regularize the quark self-energy, where  $m_q$  is the renormalized quark mass. Additionally, a separate quark mass  $m_f$  is introduced to parametrize nonperturbative effects in the vertex interactions [5,29]. The confinement in the leading Fock sector includes transverse and longitudinal confining potentials [4],

$$P_{\text{C}}^- P^+ = \kappa^4 \left\{ x(1-x) \vec{r}_\perp^2 - \frac{\partial_x [x(1-x) \partial_x]}{(m_q + m_{\bar{q}})^2} \right\}, \quad (3)$$

where  $\kappa$  is the strength of the confinement, and  $\vec{r}_\perp = \sqrt{x(1-x)}(\vec{r}_{\perp q} - \vec{r}_{\perp \bar{q}})$  represents the holographic variable [30].

In BLFQ, longitudinal dynamics use a discretized plane-wave basis with  $p^+ = \frac{2\pi k}{L}$ , where  $k$  is the half-integer (fermions) or integer (bosons), and transverse dynamics use Two-dimensional harmonic oscillator (2D-HO) wave functions  $\phi_{n,m}(k_\perp; b)$ . The longitudinal box of length  $2L$  enforces antiperiodic (fermions) or periodic (bosons) boundary conditions, with total momentum  $P^+ = \frac{2\pi K}{L}$  ( $K = \sum_i k_i$ ). Many-body states  $\{x, n, m, \lambda\}$  have total angular momentum projection  $m_J = \sum_i (m_i + \lambda_i)$ .

Basis truncations are set by  $K$  (longitudinal) and  $N_{\text{max}}$  (transverse), where  $N_{\text{max}} \geq 2n_i + |m_i| + 1$  imposes UV/IR cutoffs  $\Lambda_{\text{UV}} = b\sqrt{N_{\text{max}}}$ ,  $\Lambda_{\text{IR}} = \frac{b}{\sqrt{N_{\text{max}}}}$ . Numerical parameters:  $N_{\text{max}} = 14$ ,  $K = 15$ ,  $b = 0.29 \text{ GeV}$ . Fitted model parameters  $\{m_q, m_g, \kappa, m_f, g_s\} = \{0.39, 0.60, 0.65, 5.69, 1.92\} \text{ GeV}$  (except  $g_s$ ) reproduce unflavored light meson spectra [4].

The pion state vector obeys the eigenvalue equation (1) with  $M^2 \equiv M_\pi^2$  and admits a Fock expansion on the LF ( $x^+ = 0$ ) [1,31]

$$\begin{aligned} |\Psi\rangle = & \sum_{i_q s_q} \sum_{i_{\bar{q}} s_{\bar{q}}} \int \left[ \prod_{j=q,\bar{q}} \frac{d^3 p_j}{(2\pi)^3 2p_j^+} \right] 2P^+ (2\pi)^3 \delta^{(3)}(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{P}) \psi_{q\bar{q};(i_q, i_{\bar{q}})}^{(s_q, s_{\bar{q}})}(x_q, \vec{p}_{q\perp}, x_{\bar{q}}, \vec{p}_{\bar{q}\perp}) b_{i_q s_q}^\dagger(\vec{p}_q) d_{i_{\bar{q}} s_{\bar{q}}}^\dagger(\vec{p}_{\bar{q}}) |0\rangle \\ & + \sum_{i_q s_q} \sum_{i_{\bar{q}} s_{\bar{q}}} \sum_{\lambda a} \int \left[ \prod_{j=q,\bar{q},g} \frac{d^3 p_j}{(2\pi)^3 2p_j^+} \right] 2P^+ (2\pi)^3 \delta^{(3)}(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{P}) \psi_{q\bar{q}g;(i_q, i_{\bar{q}}, a)}^{(s_q, s_{\bar{q}}, \lambda)}(\{x, \vec{p}_\perp\}) \\ & \times b_{i_q s_q}^\dagger(\vec{p}_q) d_{i_{\bar{q}} s_{\bar{q}}}^\dagger(\vec{p}_{\bar{q}}) a_{\lambda a}^\dagger(\vec{p}_g) |0\rangle + \dots, \quad (4) \end{aligned}$$

where  $\{x, \vec{p}_\perp\} \equiv \{x_q, \vec{p}_{q\perp}, x_{\bar{q}}, \vec{p}_{\bar{q}\perp}, x_g, \vec{p}_{g\perp}\}$  satisfy the momentum conservation,

$$\vec{p}_{q\perp} + \vec{p}_{\bar{q}\perp} + \vec{p}_{g\perp} = 0 \quad \text{and} \quad x_q + x_{\bar{q}} + x_g = 1. \quad (5)$$

Here  $b_{i_q s_q}^\dagger (d_{i_{\bar{q}} s_{\bar{q}}}^\dagger)$  creates a (anti)quark with color  $i_q (i_{\bar{q}})$  and helicity  $s_q (s_{\bar{q}})$ , while  $a_{\lambda a}^\dagger$  creates a gluon with color  $a$  and helicity  $\lambda = \pm 1$ .

The BLFQ LF Hamiltonian features confinement only in the leading Fock sector, while higher sectors use Eq. (1) with constituent-mass dressed quarks and gluons. Truncating the Fock expansion (4) at second order yields the  $q\bar{q}g$  sector LFWF equation, valid within BLFQ,

$$\psi_{q\bar{q}g;(i_q, i_{\bar{q}}, a)}^{(s_q, s_{\bar{q}}, \lambda)} = 1/(M_\pi^2 - M_{0,q\bar{q}g}^2) [V \psi_{q\bar{q}g;(i_q, i_{\bar{q}}, a)}^{(s_q, s_{\bar{q}}, \lambda)}], \quad (6)$$

where the  $M_{0,q\bar{q}g}^2$  operator of the free  $q\bar{q}g$  system reads

$$M_{0,q\bar{q}g}^2 = \sum_{j=q,\bar{q},g} (\vec{p}_{j\perp}^2 + m_j^2)/x_j, \quad (7)$$

and  $V$  couples the  $q\bar{q}$  and  $q\bar{q}g$  sectors. We truncate the Fock space at  $q\bar{q}g$ ; in light-cone gauge, the sector-coupling interaction is,

$$\begin{aligned} [V \psi_{q\bar{q}g;(i_q, i_{\bar{q}}, a)}^{(s_q, s_{\bar{q}}, \lambda)}] &= \frac{g_s \sqrt{2}}{x_q + x_g} \sum_{i_1 s_1} T_{i_1 i_{\bar{q}}}^a W_\lambda^{(s_q, s_1)}(p_q, p_g) \\ &\times \psi_{q\bar{q}g;(i_1, i_{\bar{q}}, a)}^{(s_1, s_{\bar{q}}, \lambda)}(x_{\bar{q}}, \vec{p}_{\bar{q}\perp}) - \frac{g_s \sqrt{2}}{x_{\bar{q}} + x_g} \\ &\times \sum_{i_1 s_1} T_{i_1 i_{\bar{q}}}^a \bar{W}_\lambda^{(s_1, s_{\bar{q}})}(p_{\bar{q}}, p_g) \psi_{q\bar{q}g;(i_q, i_1, a)}^{(s_q, s_1)}(x_q, \vec{p}_{q\perp}), \end{aligned} \quad (8)$$

where  $p_{q(\bar{q})} \equiv \{p_{q(\bar{q})}^+, \vec{p}_{q(\bar{q})\perp}\}$  and  $p_g \equiv \{p_g^+, \vec{p}_{g\perp}\}$ . It should be noted that the arguments of the valence wave function correspond to those of the spectator quark/antiquark in the gluon radiation process.

The basic quark-gluon-quark matrix element is

$$W_\lambda^{(s_q, s_1)}(p_q, p_g) = \frac{1}{\sqrt{2}} \bar{u}_{s_q}(p_q) \epsilon_\lambda^*(p_g) u_{s_1}(p_q + p_g), \quad (9)$$

and the corresponding interaction for antiquarks reads

$$\bar{W}_\lambda^{(s_1, s_{\bar{q}})}(p_{\bar{q}}, p_g) = \frac{1}{\sqrt{2}} \bar{v}_{s_1}(p_{\bar{q}} + p_g) \epsilon_\lambda^*(p_g) v_{s_{\bar{q}}}(p_{\bar{q}}), \quad (10)$$

where  $u_{s_q}$  and  $v_{s_{\bar{q}}}$  are the light-cone helicity spinors.

The matrix elements (9) and (10) have been tabulated in Ref. [1]. For clarity, we write the spin-flip matrix elements,

$$\begin{aligned} W_\lambda^{(+,-)}(p_q, p_g) &= m_f(x_1 - x_q)/\sqrt{x_1 x_q} \delta_{\lambda,-}, \\ W_\lambda^{(-,+)}(p_q, p_g) &= -m_f(x_1 - x_q)/\sqrt{x_1 x_q} \delta_{\lambda,+}, \\ \bar{W}_\lambda^{(+,-)}(p_{\bar{q}}, p_g) &= \lambda m_f(x_1 - x_{\bar{q}})/\sqrt{x_1 x_{\bar{q}}}, \\ \bar{W}_\lambda^{(-,+)}(p_{\bar{q}}, p_g) &= 0, \end{aligned} \quad (11)$$

where  $x_1 = x_q + x_g (x_{\bar{q}} + x_g)$  for (anti)quark matrix elements. The parameter  $m_f$  controls their magnitude which, in BLFQ, rules the split between the pion and rho meson masses, lowering the pion to its small mass in the hadronic scale.

The contribution from  $q\bar{q}g$  sector to the quark PDF is

$$\begin{aligned} \Delta u_q(x_q) &= \frac{1}{(2\pi)^6} \sum_{i_q s_q} \sum_{i_{\bar{q}} s_{\bar{q}}} \sum_{\lambda a} \int \frac{d^2 p_{q\perp} d^2 p_{g\perp} dx_g}{4x_q x_g (1 - x_q - x_g)} \\ &\times |\psi_{q\bar{q}g;(i_q, i_{\bar{q}}, a)}^{(s_q, s_{\bar{q}}, \lambda)}(\{x, \vec{p}_\perp\})|^2. \end{aligned} \quad (12)$$

The corresponding expression for the antiquark PDF is obtained through the exchange  $q \rightarrow \bar{q}$ . The gluon PDF is similarly given by

$$\begin{aligned} u_g(x_g) &= \frac{1}{(2\pi)^6} \sum_{i_q s_q} \sum_{i_{\bar{q}} s_{\bar{q}}} \sum_{\lambda a} \int \frac{d^2 p_{q\perp} dx_q d^2 p_{g\perp}}{4x_q x_g (1 - x_q - x_g)} \\ &\times |\psi_{q\bar{q}g;(i_q, i_{\bar{q}}, a)}^{(s_q, s_{\bar{q}}, \lambda)}(\{x, \vec{p}_\perp\})|^2. \end{aligned} \quad (13)$$

The summed squared  $q\bar{q}g$  LF wave function entering Eqs. (12) and (13) takes the form

$$\begin{aligned} &\sum_{i_q s_q} \sum_{i_{\bar{q}} s_{\bar{q}}} \sum_{\lambda a} |\psi_{q\bar{q}g;(i_q, i_{\bar{q}}, a)}^{(s_q, s_{\bar{q}}, \lambda)}(\{x, \vec{p}_\perp\})|^2 \\ &= \frac{N(N-1)g_s^2}{(M_\pi^2 - M_{0,q\bar{q}g}^2)^2} \sum_{s_q s_{\bar{q}}} \sum_{\lambda} \sum_{s_1 s_2} \left\{ \frac{[W_\lambda^{(s_q, s_1)}(p_q, p_g)]^* [W_\lambda^{(s_q, s_2)}(p_q, p_g)]}{(x_q + x_g)^2} [\psi_{q\bar{q}}^{(s_1, s_{\bar{q}})}(x_{\bar{q}}, \vec{p}_{\bar{q}\perp})]^* \psi_{q\bar{q}}^{(s_2, s_{\bar{q}})}(x_{\bar{q}}, \vec{p}_{\bar{q}\perp}) \right. \\ &\quad + \frac{[\bar{W}_\lambda^{(s_1, s_{\bar{q}})}(p_{\bar{q}}, p_g)]^* [\bar{W}_\lambda^{(s_2, s_{\bar{q}})}(p_{\bar{q}}, p_g)]}{(x_{\bar{q}} + x_g)^2} [\psi_{q\bar{q}}^{(s_q, s_1)}(x_q, \vec{p}_{q\perp})]^* \psi_{q\bar{q}}^{(s_q, s_2)}(x_q, \vec{p}_{q\perp}) \\ &\quad \left. - 2\text{Re} \left[ \frac{[W_\lambda^{(s_q, s_1)}(p_q, p_g)]^* [\bar{W}_\lambda^{(s_2, s_{\bar{q}})}(p_{\bar{q}}, p_g)]}{(x_q + x_g)(x_{\bar{q}} + x_g)} [\psi_{q\bar{q}}^{(s_1, s_{\bar{q}})}(x_{\bar{q}}, \vec{p}_{\bar{q}\perp})]^* \psi_{q\bar{q}}^{(s_q, s_2)}(x_q, \vec{p}_{q\perp}) \right] \right\}, \end{aligned} \quad (14)$$

where in the valence wave function, we denote spectator (anti)quark momenta during gluon radiation. We assume dominance of the spin-antialigned component  $\psi_{q\bar{q}}^{(+,-)} = -\psi_{q\bar{q}}^{(-,+)}$  [14], neglecting aligned contributions.

*Results and discussion.* We study gluon contributions to pion PDFs by calculating the  $q\bar{q}g$  sector's impact on quark and gluon PDFs using the formalism described above, comparing results with BLFQ [4] and BSE approaches. Our analysis employs three parameter sets (Table I): Models I and II adopt BLFQ's kinetic quark mass  $m_q$  and gluon mass  $m_g$  [4], differing in their interaction mass  $m_f$  (Model I uses  $m_f = m_q$  while Model II's  $m_f = 5.69$  GeV reproduces the  $\pi$ - $\rho$  mass splitting). This effective  $m_f$  incorporates non-perturbative dressing effects despite its large numerical value. Model III instead uses BSE mass parameters [14]. All sets share the BLFQ coupling  $g_s = 1.92$  and reference parameters from [4].

For simplicity, we adopt a power-law form [32]:

$$\psi_{\text{pl}}(x, \vec{p}_\perp) = N[1 + (A_{0,\text{eff}}(x, \vec{p}_\perp)/4 - m_q^2)/\beta^2]^{-s}, \quad (15)$$

in the place of the valence amplitude.  $N$  is a normalization constant and the parameters  $s$  and  $\beta$  will be determined through a fit to either BLFQ or BSE results. Moreover, the effective function  $A_{0,\text{eff}}(x, \vec{p}_\perp)$  is chosen as

$$A_{0,\text{eff}}(x_q, \vec{p}_{q\perp}) = (p_{q\perp}^2 + m_q^2)/x_q + (p_{\bar{q}\perp}^2 + m_q^2)/x_{\bar{q}}. \quad (16)$$

For the  $q\bar{q}g$  momentum distributions, we use,

$$\vec{p}_{\bar{q}} = -\vec{p}_{q\perp} - \vec{p}_{g\perp}, \quad x_{\bar{q}} = 1 - x_q - x_g,$$

representing the antiquark's final momentum after gluon emission with  $q$  as spectator; the roles reverse if  $\bar{q}$  is the spectator. The chosen power-law form in Eq. (15) balances simplicity, physical relevance, and consistency with pQCD, ensuring smooth behavior at low transverse momentum and power-law falloff at high momentum, with exponent  $s$  determined phenomenologically or by quark counting rules [32–35]. Although Eq. (16) differs from standard valence wave function models using the free mass operator, its substitution into Eq. (14) satisfactorily reproduces the  $q\bar{q}g$  contribution to the quark PDF from BLFQ. This approach accounts for damping near  $x_g \rightarrow 1$  in loop integrals and the gluon distribution endpoint behavior.

TABLE I. Quark and gluon masses (GeV) for three parameter sets, with  $q\bar{q}g$  probabilities.

| Model | $m_q$ (GeV) | $m_f$ (GeV) | $m_g$ (GeV) | $1-P_{\text{val}}$ |
|-------|-------------|-------------|-------------|--------------------|
| I     | 0.390       | 0.390       | 0.600       | 0.508              |
| II    | 0.390       | 5.69        | 0.600       | 0.508              |
| III   | 0.255       | 0.255       | 0.638       | 0.300              |

Future work will employ more general valence wave functions reflecting the BLFQ Hamiltonian and BSE dynamics.

Note that we fit  $s$  and  $\beta$  parameters with the contribution of the valence state to the quark PDF. In this case, the function  $A_{0,\text{eff}}(x_q, \vec{p}_{q\perp})$  reduces to the standard mass squared function, namely,

$$M_{0,q\bar{q}}^2(x, \vec{p}_\perp) = (\vec{p}_\perp^2 + m_q^2)/[x(1-x)], \quad (17)$$

which was used in the fitting of the  $q\bar{q}$  leading Fock-sector momentum distributions from the valence state obtained with BLFQ and BSE calculations.

We find that the parameters  $s = 1.4$  and  $\beta/m_q = 1.16$  reproduce well the valence PDF of both the BLFQ and the BSE as shown in the top-left panel of Fig. 1.

The upper-right panel of Fig. 1 compares the  $q\bar{q}g$  contributions to the quark PDF from Models I–III with the BSE beyond-valence result and BLFQ. Model II qualitatively agrees with BLFQ, reproducing the large low- $x$  bump linked to its large  $m_f = 5.69$  GeV, necessary for the small pion mass [4]. The BSE result differs significantly from Model III, reflecting its inclusion of infinite  $q\bar{q}ng$  contributions and use of Feynman gauge. Differences between Model II and BLFQ likely arise from the simple analytic form and BLFQ's discretization of longitudinal fractions. The BLFQ framework highlights the second Fock sector's importance at low  $x$ , especially with large  $m_f$ . Discrepancies with BSE emphasize sensitivity to gluonic treatment, gauge choice, and possibly large- $N_c$  approximations, motivating future BSE studies in the LC gauge [22]. The lower panel of Fig. 1 compares the valence

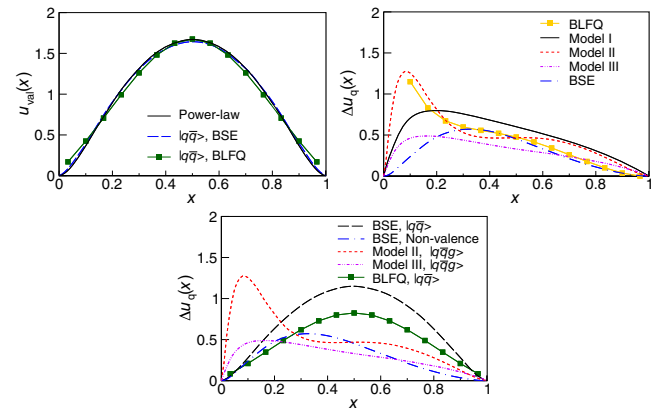


FIG. 1. Upper left: Valence quark PDF from the power-law wave function, Eq. (15), compared with BSE (long dashed) and BLFQ (squares with solid line), all normalized to unit area for comparison. Upper right:  $q\bar{q}g$  contribution to the quark PDF from the perturbative approach using parameter sets I (solid), II (dashed), and III (dash-double dotted), compared with BSE and BLFQ. Lower: Valence and  $q\bar{q}$  contributions from BLFQ and BSE compared with  $q\bar{q}g$  from Models II and III; BSE's nonvalence contribution shown with long dashed-single dotted line.

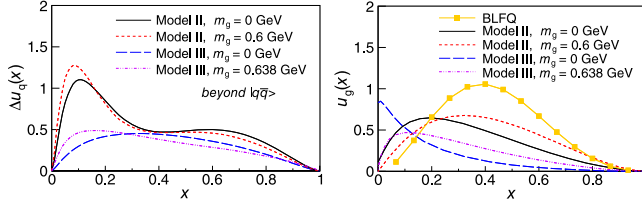


FIG. 2. Left: Quark PDF from the  $q\bar{q}g$  sector for Model II with  $m_g = 0$  (solid) and  $m_g$  from Table I (dashed), and for Model III with  $m_g = 0$  (long dashed) and  $m_g$  from Table I (long dash-double dotted). Right: Corresponding gluon PDFs for Models II and III with same  $m_g$  values. BLFQ result shown for comparison (squares with solid line).

$q\bar{q}$  PDFs from BLFQ and BSE with the  $q\bar{q}g$  contributions from Models II and III. The results show the second Fock sector is significant at small  $x$ , while the valence component dominates at larger  $x$ .

We analyze the first moments of the quark, antiquark, and gluon PDFs from Model III, finding  $\langle x_q^{q\bar{q}g} \rangle = \langle x_{\bar{q}}^{q\bar{q}g} \rangle \approx 0.1$  and  $\langle x_g^{q\bar{q}g} \rangle \approx 0.1$ , in reasonable agreement with BSE results (0.12 and 0.06, respectively) [36]. This indicates that gauge choice has limited impact on the qualitative features, justifying comparisons between BSE and BLFQ in the LC gauge.

We examine the effect of gluon mass  $m_g$  on the pion's quark and gluon PDFs. The left panel of Fig. 2 shows the  $q\bar{q}g$  contribution to the quark PDF for Models II and III, each with different  $m_g$  values (from Table I) and with  $m_g = 0$ . Increasing  $m_g$  shifts the quark PDF toward lower  $x$ , more noticeably in Model III ( $m_f = m_q$ ) than in Model II (large  $m_f$ ). The right panel shows the gluon PDF shifts oppositely—toward higher  $x$  with increasing  $m_g$ . The results qualitatively agree with BLFQ, though BLFQ PDFs are slightly shifted to higher  $x$ . Model II produces a gluon PDF more peaked at higher  $x$  than Model III, indicating that a larger  $m_f$  increases  $\langle x \rangle_g$ .

The upper panels of Fig. 3 compare our evolved pion valence quark PDF with global QCD analyses (JAM21 [37], MAP [6], xFitter [38]). We determine the initial scales  $\mu_0^2$  by matching the first moment  $\langle x \rangle_{\text{valence}} = 0.48 \pm 0.01$  at  $\mu^2 = 5 \text{ GeV}^2$  [7], obtaining  $\mu_{\text{BLFQ}}^2 = 0.34 \pm 0.03 \text{ GeV}^2$  for BLFQ [4],  $\mu_{\text{II}}^2 = 0.30 \pm 0.03 \text{ GeV}^2$  for Model II, and  $\mu_{\text{III}}^2 = 0.26 \pm 0.03 \text{ GeV}^2$  for Model III. Our results, including 10% initial-scale uncertainty bands, show good agreement with these global analyses.

As shown in the upper-right panel of Fig. 3, our predictions agree well with the JAM21 analysis using the expansion method for threshold resummation [37], but deviate at large  $x$  when the double Mellin method is used [37]. Our valence pion PDF falls as  $(1-x)^{1.77}$ , slightly slower than the pQCD prediction  $(1-x)^2$  [39] and Dyson–Schwinger Equations (DSE) results [40], but

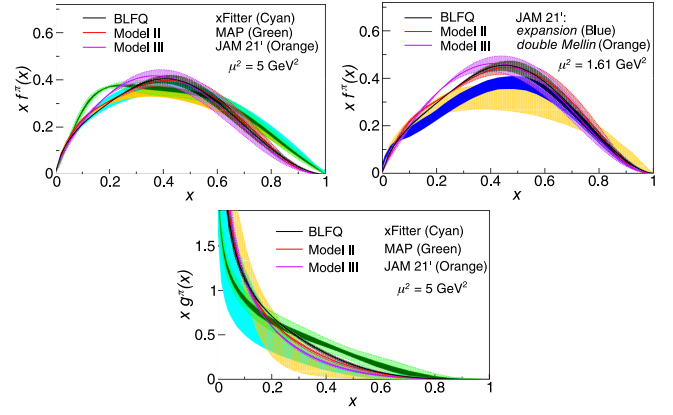


FIG. 3. Upper left: Valence quark PDFs evolved to  $5 \text{ GeV}^2$  from BLFQ (black), Model II (red), and Model III (violet), compared with global QCD fits: JAM21 [37] (orange band), MAP [6] (green), and xFitter [38] (cyan). Upper right: Evolved valence PDFs at  $1.61 \text{ GeV}^2$  compared with JAM21 results using expansion (blue) and double Mellin (orange) resummation methods. Lower: Gluon PDFs at  $5 \text{ GeV}^2$  compared with the same global analyses.

consistent with several other studies [41–43] and the JAM21 expansion method. In contrast, the double Mellin result shows a linear falloff,  $(1-x)^1$ . Our gluon PDF also shows good agreement with global QCD analyses [6], as seen in the lower panel of Fig. 3.

As a final remark, we confirmed that the BLFQ results for the higher Fock component contributions to the quark and gluon PDFs, computed using Model I and III parameters from Table I, agree with those in Figs. 1 and 2 when renormalized to 0.508 and 0.3, respectively. These values correspond to the nonvalence component probabilities in the BLFQ [4,44] and BSE [45] frameworks, obtained from their respective LFWFs. The present results were derived by applying Eq. (6) to compute the  $q\bar{q}g$  component from the valence wave function.

**Conclusion and outlook.** We studied the pion's  $|q\bar{q}g\rangle$  contributions to the quark PDF beyond the leading  $|q\bar{q}\rangle$  Fock sector using BLFQ and developed a perturbative method to generate these from the valence component. Comparisons were made with direct BLFQ and Minkowski-space BSE calculations, exploring the impact of gluon and quark mass variations. In BLFQ, we identified that dynamical chiral symmetry breaking enhances the spin-flip matrix element, which amplifies the  $|q\bar{q}g\rangle$  component and the gluon PDF. We showed that the low- $x$  peak in the quark PDF arises from this large spin-flip term linked to the  $\pi$ – $\rho$  mass splitting. The framework also revealed that the gluon PDF changes significantly when the gluon mass is set to zero.

The proposed framework in the light-cone gauge can be applied to other pion valence models to construct the  $|q\bar{q}g\rangle$  component and explore higher Fock-state effects. It offers a

practical method for computing quark and gluon PDFs, incorporating higher Fock contributions. We validated this approach by comparing the resulting PDFs, derived from applying the operator to the valence wave function, with those from BLFQ and Minkowski-space BSE calculations.

Future steps include extracting the  $q\bar{q}g$  component from phenomenological parametrizations, such as Ref. [6], to further support the spin-flip enhancement. The method could also be applied to valence pion wave functions from DS/BSE approaches in covariant gauges. These directions aim to deepen our understanding of the pion light-front wave function with dressed constituents in Fock space.

Another clear signature of the higher Fock component with massive gluons is the separation between the peaks of the gluon and quark PDFs when the spin-flip matrix element is enhanced to generate the  $\pi$ - $\rho$  mass splitting. In contrast, using parameters from the pion BS model in Minkowski space, where the spin-flip enhancement is mild, both PDFs from the  $q\bar{q}g$  component peak near  $x \sim 0.15$ , and the nonvalence contribution to the quark PDF peaks around  $x \sim 0.3$ . This shift may be attributed to the structure of the quark-gluon vertex, which was tested qualitatively here. Extending this approach to compute the  $qqqg$  component in the nucleon, using recent BLFQ results and valence models from Minkowski space dynamics [46–48], presents a promising future direction.

Based on these intercomparisons, we speculate that increasing the number of gluons in the retained Fock sectors within the BLFQ approach will: (i) drive the

$\pi$ - $\rho$  splitting, (ii) reduce the effective vertex mass of fermions, and (iii) increase the nonvalence sector contributions to the quark PDF in the low- $x$  region.

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*Data availability.* The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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