

Burgers equation for the bulk viscous pressure of quark matter

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The dissipative properties of relativistic strongly interacting nuclear matter significantly influence the damping of stellar oscillations and density fluctuations during compact star mergers. In this work, we derive the evolution equation for the bulk viscous pressure in unpaired quark matter under small deviations from equilibrium. Our analysis reveals that it behaves like a two-component Burgers fluid. We identify four key transport coefficients—two relaxation times and two bulk viscosity coefficients—expressed in terms of equilibrium parameters and electroweak nonleptonic and semileptonic decay rates. The transport coefficients are evaluated for two distinct equations of state: one based on perturbative quantum chromodynamics and the other on a modified MIT bag model, valid in different density regimes. We also determine the temperature and density region where nonleptonic electroweak processes dominate the dissipation. Our formulation establishes a new way of describing bulk viscous effects in quark matter, applicable for numerical simulations of compact star mergers.

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I. INTRODUCTION

Quantum chromodynamics (QCD), the fundamental theory describing strong nuclear interactions, predicts the existence of exotic phases of matter under extreme conditions [1–3]. In particular, at the high densities anticipated in the cores of compact stars quarks may become deconfined. There are many methods of detecting these novel forms of matter [4]. While the equation of state (EOS) of dense matter constrains the mass-to-radius ratio of compact objects, out-of-equilibrium phenomena may give the key to understanding their dynamical behavior. With recent and future advances in gravitational-wave astronomy [5–8], both the EOS and the transport properties of dense matter will be constrained more stringently beyond what is possible through electromagnetic observations alone. It is then important to deepen our understanding of the distinctive signatures of the quark matter phases in these compact star scenarios, and also provide suitable methods for their study.

In this article we focus on the bulk viscous effects of unpaired quark matter, that is, with the dissipation arising from compression and rarefaction. These effects play a crucial role in damping the rapid oscillations that follow the merger of compact stars [9–11], as well as various stellar oscillation modes in isolated stars [12,13], both of which could influence gravitational wave signals [14]. Previous studies have computed bulk viscosity coefficients for different hadronic [15–20] and quark matter phases [21–25] (see [26] for a review and a more complete set of references), typically as functions of the oscillation frequency, allowing quick estimates of energy dissipation. However, such formulations are not readily implementable in numerical simulations of the relativistic hydrodynamics associated with the relevant astrophysical settings. Only recently have efforts begun to incorporate bulk viscous dissipation into simulations of neutron star mergers, typically assuming that the bulk viscous stress tensor satisfies an Israel-Stewart (IS) equation [27–34]. This second-order hydrodynamic framework introduces a relaxation time, an additional transport coefficient, in addition to the bulk viscosity itself, ensuring the causal propagation of hydrodynamical perturbations, in contrast to standard first-order formulations.

In this work we show that the bulk viscous pressure in unpaired quark matter evolves according to a Burgers-type equation, also used in viscoelastic media [35]. In [36], Gavassino has shown that the bulk pressure of systems with three independent chemical potentials and two distinct reaction rates, such as hadronic matter composed of

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neutrons, protons, electrons and muons, also follows Burgers-type evolution. While quark matter does not fulfill the above mentioned conditions, in this article we show that its bulk viscous pressure is also governed by a Burgers equation. Only in certain temperature and density regimes does it reduce to the more familiar IS form. We compute the full set of second-order transport coefficients entering the Burgers equation, conveniently expressed in terms of two partial bulk viscosity coefficients and two relaxation times. Although our formulation is general, we explicitly evaluate these transport coefficients for two distinct quark matter EOSs at finite densities, temperatures, and for a finite strange quark mass: one derived from perturbative QCD (pQCD), applicable at very high densities [37–42], and another based on a modified MIT bag model [43,44], more suitable for lower densities. Our approach provides a versatile framework that can accommodate more accurate EOS models or updated reaction rates as they become available, and can provide the proper description of the limiting IS case as well.

We use metric conventions $(+, -, -, -)$ and natural units $k_B = \hbar = c = 1$ throughout, unless otherwise stated.

II. SETUP

For definiteness let us consider a situation relevant to the astrophysical settings. We take into account the presence of five species of particles: massless u , d quarks, massive s quarks, massless electrons e , each with a chemical potential μ_i , as well as neutrinos ν . After a disturbance, the chemical (beta) equilibrium is achieved by W -boson-mediated electroweak processes, namely,

$$\begin{aligned} u + d &\leftrightarrow u + s, \\ u + e &\longrightarrow d + \nu, & d &\longrightarrow u + e + \bar{\nu}, \\ u + e &\longrightarrow s + \nu, & s &\longrightarrow u + e + \bar{\nu}. \end{aligned} \quad (1)$$

Of these, the first one is the only *nonleptonic* process, whereas the remaining four are *semileptonic*. We assume that neutrinos remain untrapped [45], restricting to temperatures below 10 MeV, ignoring neutrino chemical potentials. The flavor-changing processes above lead to two linearly independent chemical potentials which characterize deviation from equilibrium. We define

$$\mu_1 \equiv \mu_s - \mu_d, \quad \mu_2 \equiv \mu_u - \mu_d + \mu_e, \quad (2)$$

noting that the chemical potential that characterizes the reaction $u + e \leftrightarrow s + \nu$ is $\mu_u - \mu_s + \mu_e = \mu_2 - \mu_1$. In terms of these two chemical potentials, we can write the linearized reaction rates,

$$\begin{aligned} \Gamma_{s+u \rightarrow d+u} - \Gamma_{d+u \rightarrow s+u} &\equiv \mu_1 \lambda_1, \\ \Gamma_{d \rightarrow u+e+\bar{\nu}_e} - \Gamma_{u+e \rightarrow d+\nu_e} &\equiv -\mu_2 \lambda_2, \\ \Gamma_{s \rightarrow u+e+\bar{\nu}_e} - \Gamma_{u+e \rightarrow s+\nu_e} &\equiv (\mu_2 - \mu_1) \lambda_3. \end{aligned} \quad (3)$$

The transport properties will also depend on (the partial derivatives of) the equilibrium pressure p , specifically on the particle number densities n_a and susceptibilities χ_{ab} defined as

$$n_a \equiv \frac{\partial p}{\partial \mu_a}, \quad \chi_{ab} \equiv \frac{\partial n_a}{\partial \mu_b} = \frac{\partial^2 p}{\partial \mu_a \partial \mu_b}, \quad a, b \in \{u, d, s, e\}. \quad (4)$$

As we will linearize our equations in μ_1, μ_2 , susceptibilities and densities are always taken to be evaluated in equilibrium, with the chemical potentials subject to two additional beta-equilibrium constraints, $\mu_s = \mu_d = \mu_u + \mu_e$. In addition to constraints arising from the reactions, the chemical potentials have to fulfill the requirement of charge neutrality, which is most conveniently written down as $n_Q \equiv (2n_u - n_d - n_s)/3 - n_e = 0$.

III. BURGERS EQUATION

In order to study the out-of-equilibrium evolution of quark matter, we write down equations for the bulk scalar $\Pi = p_{\text{non-eq}} - p$, measuring the difference of the nonequilibrium and equilibrium pressures, as well as the fluid velocity u^μ and $\vartheta \equiv \partial_\mu u^\mu$. In the system we consider,

$$\begin{aligned} \Pi &= n_u(\delta\mu_u + \delta\mu_e - \delta\mu_s) \\ &\quad + n_d(\delta\mu_d - \delta\mu_s) + n_q \left(\delta\mu_s - \frac{1}{3} \delta\mu_e \right), \end{aligned} \quad (5)$$

where $\delta\mu_a$ measures the deviation of the chemical potential of the particle a with respect to its equilibrium value, and $n_q \equiv n_u + n_d + n_s$ is the quark density. Constraints of charge neutrality and conservation of baryon number have to be imposed [see Eqs. (B2), (B3) of Appendix B] to express Π in terms of the algebraically independent chemical potentials μ_1, μ_2 . This amounts to writing the chemical potentials appearing in Eq. (5) in terms of μ_1, μ_2 , densities n_a , and (inverse) susceptibilities $\chi_{ab}^{(-1)}$. As a consequence of the constraints, the equilibrium system is characterized by a single free chemical potential which we choose to be μ_d .

We follow the logic of [36], adapting the notation and derivation for our purposes. To obtain a differential equation for the chemical potentials, we generalize the equations found in the supplemental material of [24] to nonoscillatory deformations of the fluid. That is, we start from the evolution equations of the different particle fractions, and after linearizing them around equilibrium, we find that the chemical potentials fulfill the system of differential equations

$$D_u \mu_\Gamma = [M \lambda \hat{\mu}]_\Gamma - [M \mathbf{X}]_\Gamma n_q \vartheta, \quad \Gamma \in \{1, 2, s, e\}, \quad (6)$$

where $D_u = u_\mu \partial^\mu$ is the convective derivative, and the indices $\{u, d, s, e\}$ refer to the particle species, whereas $\{1, 2\}$ refer to combinations corresponding to μ_1, μ_2 defined above. The matrix M contains elements of the inverse susceptibility matrix, and λ is a matrix of the rates of the weak reactions, explicitly

$$(M)_\Gamma^a \equiv (\chi^{-1})_\Gamma^a, \quad (7)$$

$$(\chi^{-1})_1^a \equiv (\chi^{-1})_s^a - (\chi^{-1})_d^a, \quad (8)$$

$$(\chi^{-1})_2^a \equiv (\chi^{-1})_u^a + (\chi^{-1})_e^a - (\chi^{-1})_d^a; \quad (9)$$

$$\lambda \equiv \begin{pmatrix} \lambda_3 & -\lambda_2 - \lambda_3 \\ \lambda_1 & \lambda_2 \\ -\lambda_1 - \lambda_3 & \lambda_3 \\ \lambda_3 & -\lambda_2 - \lambda_3 \end{pmatrix}. \quad (10)$$

Lastly, $\mathbf{X} \equiv (n_u, n_d, n_q - n_u - n_d, n_u - n_q/3)/n_q$ is a vector of the (equilibrium) particle fractions and $\hat{\mu} \equiv (\mu_1, \mu_2)$. Using the above quantities as well as $\hat{M}$, the reduction of M into its first two rows, we now define a two-by-two matrix $\mathcal{T}$,¹ and a two-component vector $\mathbf{b}$:

$$\mathcal{T}^{-1} \equiv -\hat{M} \lambda, \quad \mathbf{b} \equiv n_q \mathcal{T} \hat{M} \mathbf{X}. \quad (11)$$

We now readily find that

$$(\text{id}_2 + \mathcal{T} D_u) \hat{\mu} = -\vartheta \mathbf{b}. \quad (12)$$

Here, id_2 is the two-by-two identity matrix. By taking the second convective derivative of this equation, noting that the convective derivatives of equilibrium quantities vanish, multiplying the equation by $\det(\mathcal{T}) \mathcal{T}^{-1}$, and using the identity $\text{Tr}(\mathcal{T}) \text{id}_2 - \det(\mathcal{T}) \mathcal{T}^{-1} = \mathcal{T}$, valid for any non-singular two-by-two matrix, we obtain

$$\det(\mathcal{T}) D_u^2 \hat{\mu} + \text{Tr}(\mathcal{T}) D_u \hat{\mu} + \hat{\mu} = -\vartheta \mathbf{b} - (D_u \vartheta) \det(\mathcal{T}) \mathcal{T}^{-1} \mathbf{b}. \quad (13)$$

If we now write $\tau_\pm$ for the eigenvalues of $\mathcal{T}$, define $(\Pi_1, \Pi_2) \equiv \Pi$ via $\Pi = \Pi_1 \mu_1 + \Pi_2 \mu_2$, and contract (13) with it, we get the Burgers equation valid for our system:

$$\tau_+ \tau_- D_u^2 \Pi + (\tau_+ + \tau_-) D_u \Pi + \Pi = -\zeta \vartheta - \xi D_u \vartheta, \quad (14)$$

with transport coefficients

¹The matrix is well defined for reasonable EOSs as long as the reaction rates are all nonzero; this is discussed in more detail in [36].

$$\zeta = \langle \Pi, \mathbf{b} \rangle, \quad \xi = \tau_+ \tau_- \langle \Pi, \mathcal{T}^{-1} \mathbf{b} \rangle, \quad (15)$$

where $\langle -, - \rangle$ denotes the (here, two-dimensional) scalar product. In Eq. (15) the coefficients Π^a can be explicitly solved after taking into account the constraints of electrical neutrality and baryon number conservation. The explicit but rather cumbersome forms of the bulk scalar and the transport coefficients can be found in Appendix C.

The four quantities $\{\tau_+, \tau_-, \zeta, \xi\}$ characterize the system. However, their interplay is somewhat subtle. A more quantitative account can be obtained by examining the corresponding Green's function. As [36] points out, the Green's function of the Burgers equation in the fluid rest frame reads, as a function of time t ,

$$G(t) = G_+(t) + G_-(t), \quad G_\pm(t) \equiv \frac{\zeta_\pm}{\tau_\pm} \theta_H(t) e^{-t/\tau_\pm}, \quad (16)$$

where θ_H is the step function, and we have defined the partial viscosities,

$$\zeta_+ = \frac{\zeta \tau_+ - \xi}{\tau_+ - \tau_-}, \quad \zeta_- = \frac{\zeta \tau_- - \xi}{\tau_- - \tau_+}. \quad (17)$$

We will find it more useful to use $\{\tau_+, \zeta_+, \tau_-, \zeta_-\}$. Further, while the above derivation is not formally valid when $\mathcal{T}$ is singular, in the limit where one of the relaxation times is very large, $\tau_\pm \rightarrow \infty$, one finds that the hydrodynamical equation reduces to $\tau_\mp D_u \Pi + \Pi = -\zeta_\mp \vartheta$ and the IS limit is recovered. In particular, this occurs if one neglects the semileptonic processes, as when $\lambda_2, \lambda_3 \rightarrow 0$, then $\tau_- \rightarrow \infty$. Otherwise, all three rates enter into the expressions of the two relaxation times and viscosities.

IV. RESULTS

We apply the formulas derived above to two specific EOSs, with different applicabilities. We consider a modified bag model at intermediate densities ($\mu_d \approx 400\text{--}600$ MeV) and perturbative QCD (pQCD) at high densities ($\mu_d \gtrsim 850$ MeV), both at finite but small temperatures. For reference, for the range of temperatures we cover, below $T \lesssim 10$ MeV, the lower bound of the applicability range corresponds approximately to $n_B \sim 3.6 n_{\text{sat}}$ and $n_B \sim 37 n_{\text{sat}}\text{--}38 n_{\text{sat}}$, where $n_{\text{sat}} \approx 0.16 \text{ fm}^{-3}$ is the value of the nuclear saturation density, for the bag model and pQCD, respectively.

In addition to fixing the strong dynamics, we must fix the electroweak rates λ_i . These are presently known only in a leading-order expansion in the limit of small temperature and strange quark mass (i.e., with the quark chemical potentials as the dominant scales). Making such an assumption, we have [45–52]

$$\lambda_1 \approx \frac{64}{5\pi^3} G_F^2 \sin^2 \Theta_C \cos^2 \Theta_C \mu_d^5 T^2, \quad (18)$$

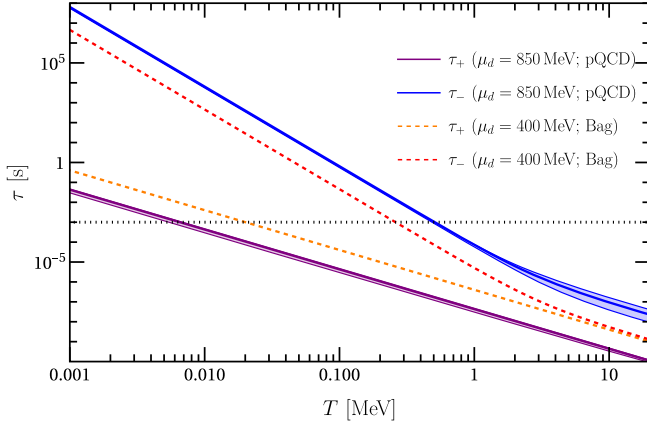


FIG. 1. The relaxation times $\tau_{\pm}$ in seconds, for the bag model and pQCD at $\mu_d = 400$ and 850 MeV, respectively. We have marked the millisecond scale, relevant for mergers or stellar oscillation modes, and shown the uncertainty bands obtained by varying the renormalization scale in pQCD.

$$\lambda_2 \approx \frac{17}{15\pi^2} G_F^2 \cos^2 \Theta_C \alpha_s \mu_d \mu_u \mu_e T^4, \quad (19)$$

$$\lambda_3 \approx \frac{17}{40\pi} G_F^2 \sin^2 \Theta_C m_s^2 T^4, \quad (20)$$

where $G_F \approx 1.166 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi coupling constant, $\Theta_C \approx 13.02^\circ$ is the Cabibbo angle, α_s is the strong fine-structure constant, and m_s is the strange quark mass, $m_s(\Lambda = 2 \text{ GeV}) \approx 94 \text{ MeV}$ [53], evaluated at a fixed renormalization scale Λ and evolved according to the renormalization group equations. Notably, the only rate scaling with a fifth power of the quark chemical potentials is λ_1 , with the semileptonic processes remaining formally subdominant—nevertheless, we do not neglect them as they turn out to be crucial for understanding the bulk viscosity ζ_- . We emphasize that in the expression for the rates the values for m_s and α_s must be replaced by their model counterparts in the bag model. In the figures shown in the main text, we have always fixed m_s to its bare value and set the bag model coupling $a_4 \approx 1 - 2\alpha_s/\pi$ [54] to $a_4 = 0.7$. We discuss the parameter dependence in more detail in Appendix B.

The predictions we obtain appear qualitatively unified, with a caveat for the coefficient τ_- obtained in the bag model at high densities. In Fig. 1, we see the relaxation times $\tau_{\pm}$. For both EOSs τ_+ is linear in T on a log–log scale, while τ_- is piecewise linear with an inflection point tempering its slope at an $O(\text{MeV})$ -scale temperature. There is a qualitative difference here present between the bag model and pQCD, with the pQCD values for τ_- saturating to a nearly density-independent form above τ_+ . Meanwhile, the τ_- remains strongly density dependent and attains much smaller values in the bag model, becoming nearly equal to τ_+ past the inflection point.

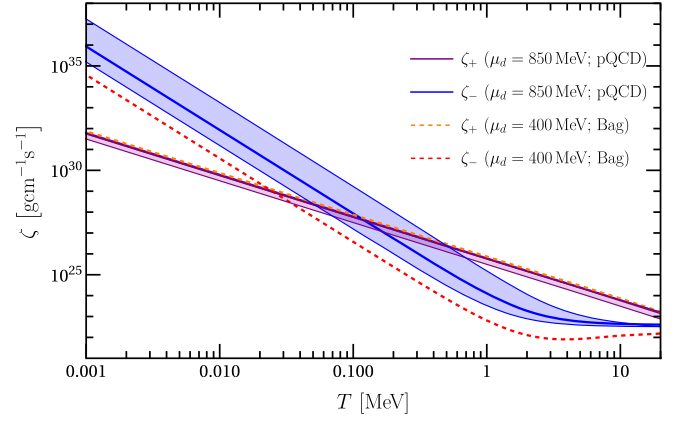


FIG. 2. The bulk viscosity components $\zeta_{\pm}$ for the bag model and pQCD at $\mu_d = 400$ and 850 MeV, respectively. Values of the bulk viscosity are given in CGS units. The uncertainty bands describe the dependence on the renormalization scale in pQCD.

We believe that this is in large part due to the form of the rates, where the (constant) effective coupling is used for the bag model, while the standard running coupling is used for pQCD. Thus, at larger temperatures, particularly in combination with relatively large densities, the coupling-dependent λ_2 is overestimated in the bag model.

The bulk viscosities $\zeta_{\pm}$ behave similarly (see Fig. 2), but ζ_- saturates to a nearly constant value and, starting at approximately $T = 0.1 \text{ MeV}$, ζ_+ overtakes ζ_- . The pQCD error bands, obtained by varying the renormalization scale (see Appendix B for details), are sizable for ζ_- before the inflection point, and considerably more modest for all other quantities.

The Green’s functions encode the effects of the transport coefficients in nontrivial ways. In practice, at a fixed density and time t' , the system will start with $G_+(t')$ dominating over $G_-(t')$, and as the fluid heats up, the picture rapidly flips, with $G_-(t)$ crossing over $G_+(t')$ and becoming the dominant contribution. We denote the temperature where this happens, i.e., where $G_+(t') = G_-(t')$, as T_{cross} . At extremely high temperatures, beyond those considered here, neutrinos are trapped, and the behavior of the system is further complicated.

Furthermore, we find that G_+ is well approximated by the nonleptonic limit of $\lambda_2, \lambda_3 \ll \lambda_1$, but G_- admits no simple approximation. Thus, we find that bulk viscous effects in quark matter require taking into account both channels above T_{cross} , and are well approximated by a single-component fluid, ignoring the semileptonic processes, below T_{cross} , with a narrow intermediate region.

We also note that these temperatures are of relevance for physical neutron star mergers. In merger simulations, the relevant timescales are of millisecond order [10,11]. In Fig. 3, we show the crossing temperature as a function of

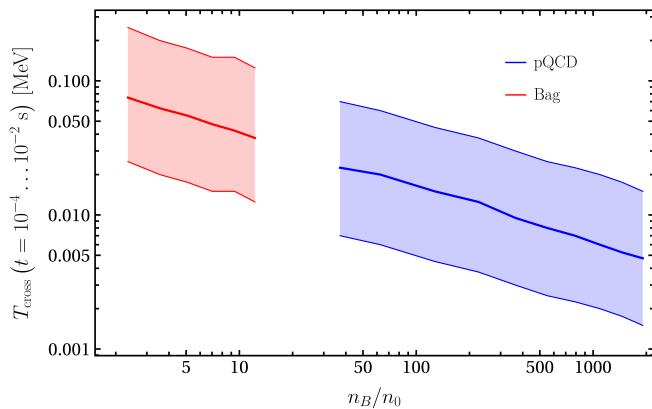


FIG. 3. The approximate crossing temperature T_{cross} at which the two Green's functions G_α coincide as a function of baryon density n_B . Only the central value of the renormalization scale is shown for pQCD. The band represents the timescales $t = 10^{-4} \dots 10^{-2}$ s, with the thick line corresponding to the millisecond scale. For $T \ll T_{\text{cross}}$ dissipation is dominated by the nonleptonic processes.

the baryon density,² with a band whose width is determined by the relevant timescales, here taken to be from 10^{-4} s to 10^{-2} s for illustrative purposes. The region below the band corresponds to the nonleptonic regime dominated by G_+ , the region above it corresponds to the regime dominated by G_- , and the band to the region where either or both components G_α can be relevant.

V. DISCUSSION

Simple relativistic hydrodynamic formulations of viscous effects suffer from significant shortcomings, including acausal signal propagation and inherent instabilities. These issues were addressed long ago through the development of a more advanced framework, incorporating second-order gradients of hydrodynamic variables. In second-order hydrodynamics, the dissipative component of the stress-energy tensor is elevated to the status of an independent dynamical variable, alleviating the problems of first-order hydrodynamics. In this work, we have derived an evolution equation for the bulk viscous pressure in unpaired quark matter, to be incorporated in second-order hydrodynamical formulations. While bulk viscous dissipation in the simulation of mergers of neutron stars has been studied with an IS evolution equation [10,11], we have shown that unpaired quark matter is more accurately described with a Burgers equation. It would be very interesting to extend our formulation to other quark matter phases [55–58].

After using two different EOSs, valid for either very large or moderate densities, we delineate the temperature

²To be specific, as we have finite resolution due to the numerics required to solve μ_e , we find for each density the temperature minimizing $1 - G_+(t)/G_-(t)$.

and density regimes where the nonleptonic electroweak processes dominate the dissipation, in timescales that may be relevant for mergers, and could influence the damping of oscillation modes in the star. Let us recall here that in these events the effect associated to the shear viscosity [59], dominated by the quark-quark scattering mediated by one-gluon exchange, is negligible [14] in the temperature regime we considered.

Finally, let us comment that while the Burgers equation (14) describes the close-to-chemical-equilibrium evolution of quark matter, the same effects could be described by taking into account the individual evolution of the densities of the quarks and electrons, after imposing the constraints of charge neutrality and baryon number conservation in their evolution. However, our proposed formulation is considerably simpler and will be of great use in analytical models and could facilitate numerical studies of both stellar oscillation modes and mergers of quark or hybrid stars. In the future, the treatment found here could also be further extended by considering the variety of superconducting phases found in dense matter.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [60].

APPENDIX A: FREQUENCY-DEPENDENT BULK VISCOSITY

From the Burgers equation, one can recover the same value of the frequency-dependent bulk viscosities as found using first-order hydrodynamics. One simply has to assume that the time dependence of the bulk viscous pressure is harmonic $\Pi \propto e^{i\omega t}$. After solving the Burgers equation, and computing the bulk viscosity as $\zeta_{\text{eff}}(\omega) = -\text{Re}\Pi/\vartheta$, one finds

$$\zeta_{\text{eff}}(\omega) = \frac{\kappa_1 + \kappa_2 \omega^2}{\kappa_3 + \kappa_4 \omega^2 + \omega^4}, \quad (\text{A1})$$

with the following identifications:

$$\begin{aligned}\kappa_3 &= \frac{1}{(\tau_+ \tau_-)^2}, & \frac{\kappa_1}{\kappa_3} &= \zeta_+ + \zeta_-, \\ \frac{\kappa_4}{\kappa_3} &= \tau_+^2 + \tau_-^2, & \frac{\kappa_2}{\kappa_3} &= \zeta_+ \tau_-^2 + \zeta_- \tau_+^2.\end{aligned}\quad (\text{A2})$$

We have checked that with the values of the second-order coefficients found in this manuscript we reproduce the frequency-dependent bulk viscosity computed in [25].³ One can even use the above relations to obtain the associated Burgers equation fulfilled by any system that has a frequency-dependent bulk viscosity with the same dependence as in Eq. (A1).

When $\tau_- \rightarrow \infty$, the frequency-dependent viscosity reduces to

$$\zeta_{\text{eff}}(\omega) \rightarrow \frac{\zeta_+}{\omega^2 + \tau_+^2}. \quad (\text{A3})$$

This frequency-dependent bulk viscosity can be recovered from the IS equation $\tau_+ D_u \Pi + \Pi = -\zeta_+ \vartheta$.

APPENDIX B: DETAILS OF CONSTRAINTS AND THE EQUATIONS OF STATE

As we explain in the main text, linearizing quantities in deviations of chemical potentials $\delta\mu_a$ from their equilibrium values leads to formulas dependent only on *equilibrium* susceptibilities and densities. To evaluate them, we enforce beta-equilibrium constraints of $\mu_s = \mu_d = \mu_u + \mu_e$ as well as a charge neutrality constraint $n_Q = (2n_u - n_d - n_s)/3 - n_e = 0$. These three constraints leave us with only a single free equilibrium chemical potential used to characterize quark matter close to equilibrium. In practice, we choose this to be the d -quark chemical potential μ_d , and solve the (small) electron chemical potential μ_e from the charge-equilibrium constraint numerically. We do so by assuming that the pressure of the electrons decouples from that of the quarks and that the electrons can be treated as free particles,

$$p_e = \frac{1}{12} \left(\frac{\mu_e^4}{\pi^2} + 2\mu_e^2 T^2 + \frac{7}{15} \pi^2 T^4 \right). \quad (\text{B1})$$

While a simple form of the electron pressure simplifies matters, solving the electron chemical potential remains the most numerically involved part of the calculation. Once it is known, the equilibrium densities and susceptibilities are readily evaluated in both pQCD and the bag model, and their values can be substituted to the formulas for the transport coefficients presented in the main text. In addition, there are constraints for the *deviations* $\delta\mu_a$ of chemical

potentials from their equilibrium values. We impose that these constraints also keep the system electrically neutral, and that the total baryon number is conserved. The constraints can be written in terms of the (equilibrium) susceptibilities as

$$0 = \delta n_Q = \sum_a (\chi_e^a - \chi_u^a) \delta\mu_a, \quad (\text{B2})$$

$$0 = \delta n_q = \sum_a (\chi_d^a + \chi_s^a - \chi_e^a) \delta\mu_a. \quad (\text{B3})$$

Combined with the definitions of μ_a from (2), we can obtain the resulting $\delta\mu_a$ in terms of the μ_a . They, on the other hand, can be substituted into the definition of bulk scalar (5), which then uniquely gives us the coefficients Π_1 , Π_2 used for defining the transport coefficients ζ , ξ .

As evidenced by the main text, the two equations of state provide us with a unified qualitative picture of the behavior of the transport coefficients.

A standard way of gauging the uncertainty of the pQCD description is by varying the renormalization scale by a factor of 2 around a central value, which results in the band shown in the main text (see, e.g., [37] for details). We note that this also affects the rates, as the coupling and the masses run with the scale (see [37] for details). The ζ_- in Fig. 2, strongly dependent on the semileptonic rates, shows a considerable uncertainty band, but the other quantities are kept well under check at these densities.⁴ This is to be expected from other pQCD computations.

On the other hand, the bag model depends on certain parameters: The bag model pressure has the general form [43,44]

$$p = p_e + a_4 \sum_{f \in \{u,d,s\}} p_f + B_{\text{eff}}, \quad (\text{B4})$$

where p_e is the electron pressure defined above, p_f are the individual quark contributions (each taken to be a free fermion), and a_4 and B_{eff} are model parameters. Of the quark pressures, the u , d quarks have the same form as the electron pressure, while the massive s -quark pressure cannot be expressed in closed form for general T, μ_s , and reads

$$\begin{aligned}p_s &= \frac{3T}{\pi^2} \int_0^\infty dk k^2 \left\{ \ln \left[1 + \exp \left(-\frac{\sqrt{k^2 + m_s^2} - \mu_s}{T} \right) \right] \right. \\ &\quad \left. + \ln \left[1 + \exp \left(-\frac{\sqrt{k^2 + m_s^2} + \mu_s}{T} \right) \right] \right\}.\end{aligned}\quad (\text{B5})$$

³Note that in [25], the chemical potential μ_2 is denoted $-\mu_3$, and the chemical potential μ_2 found there corresponds to our linear combination $\mu_2 - \mu_1$. The rates $\lambda_{2,3}$ are likewise swapped.

⁴We have also checked that the expected behavior persists with increasing densities, with the band shrinking and the transport quantities gradually shifting.

Here, the m_s parameter should be thought to represent the constituent quark mass, and acts as a third model parameter. Indeed, as the parameter responsible for breaking conformal invariance, it is the one most relevant for bulk viscosities. Densities and therefore also susceptibilities are independent of the bag constant, whereas the pressure depends approximately linearly on a_4 . For small variations of a_4 , its impact is essentially imperceptible on the doubly logarithmic scale used here for the bulk viscosities. To be more quantitative, we set $a_4 = 0.7$ [14] having checked explicitly that varying it between 0.6–0.8 changes τ_+ by $\lesssim 15\%$, ζ_+ by $\lesssim 30\%$, and both τ_- and ζ_- orders of magnitude less.

In contrast to a_4 , varying m_s significantly changes the $\zeta_{\pm}$ in particular. We show this in Fig. 4, displaying the two viscosities for different values of the mass parameter with the a_4 parameter fixed to 0.7. The behavior seen here can be approximated quite well by simple scaling laws, with ζ_+ scaling as $\sim(m_s^2)^2$, and ζ_- scaling as $\sim(m_s^2)^5$ for small temperatures (below the inflection temperature) and as $\sim(m_s^2)^{3/2}$ above the inflection temperature. Regardless of the sensitivity of the ζ to m_s , we have checked that T_{cross} is not significantly affected by the value of m_s .

APPENDIX C: EXPLICIT FORMULAS

For the sake of completeness and to possibly give the reader some insight on the explicit form of the bulk scalar and the transport coefficients, we list them below. We note that, as the forms are rather complicated, it is often simpler to obtain them using the methods outlined in

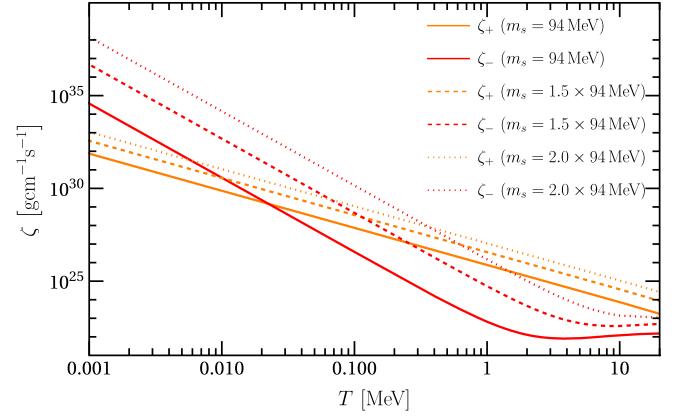


FIG. 4. $\zeta_{\pm}$ at $m_s \in \{94, 1.5 \times 94, 2.0 \times 94\}$ MeV in the bag model. Everything evaluated at $\mu_d = 400$ MeV, $a_4 = 0.7$.

the main text, as they require merely simple matrix operations. To simplify formulas, we define the shorthands

$$\begin{aligned}\chi_A^{-1} &\equiv (\chi^{-1})_d^d - 2(\chi^{-1})_d^s + (\chi^{-1})_s^s, \\ \chi_B^{-1} &\equiv (\chi^{-1})_u^u - 2(\chi^{-1})_u^d + (\chi^{-1})_d^d + (\chi^{-1})_e^e, \\ \chi_C^{-1} &\equiv (\chi^{-1})_u^u - 2(\chi^{-1})_u^s + (\chi^{-1})_s^s + (\chi^{-1})_e^e,\end{aligned}\quad (\text{C1})$$

and will make use of the Källén function $K(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$ ⁵ and the pairwise sum $Q(x, y, z) = xy + yz + zx$. In terms of these, we find the components of the bulk scalar under the constraints outlined above to read

$$\begin{aligned}\Pi_1 K(\chi_A^{-1}, \chi_B^{-1}, \chi_C^{-1}) &= n_s K(\chi_A^{-1}, \chi_B^{-1}, \chi_C^{-1}) + n_q \left\{ \left[(\chi^{-1})_u^u + \frac{1}{3}(\chi^{-1})_e^e \right] (\chi_A^{-1} + \chi_B^{-1} - \chi_C^{-1}) \right. \\ &\quad \left. - (\chi^{-1})_d^d (\chi_A^{-1} - \chi_B^{-1} - \chi_C^{-1}) - 2(\chi^{-1})_s^s \chi_B^{-1} + (\chi_A^{-1} - \chi_B^{-1} + \chi_C^{-1}) \chi_B^{-1} \right\}\end{aligned}\quad (\text{C2})$$

$$\begin{aligned}\Pi_2 K(\chi_A^{-1}, \chi_B^{-1}, \chi_C^{-1}) &= n_u K(\chi_A^{-1}, \chi_B^{-1}, \chi_C^{-1}) + n_q \left\{ -2 \left[(\chi^{-1})_u^u + \frac{1}{3}(\chi^{-1})_e^e \right] \chi_A^{-1} \right. \\ &\quad \left. + (\chi^{-1})_d^d (\chi_A^{-1} - \chi_B^{-1} + \chi_C^{-1}) + (\chi^{-1})_s^s (\chi_A^{-1} + \chi_B^{-1} - \chi_C^{-1}) - (\chi_A^{-1} - \chi_B^{-1} - \chi_C^{-1}) \chi_A^{-1} \right\}\end{aligned}\quad (\text{C3})$$

whereas the (inverse) relaxation times are

$$2\tau_{\pm}^{-1} = (\chi_A^{-1}\lambda_1 + \chi_B^{-1}\lambda_2 + \chi_C^{-1}\lambda_3) \pm \sqrt{(\chi_A^{-1}\lambda_1 + \chi_B^{-1}\lambda_2 + \chi_C^{-1}\lambda_3)^2 + Q(\lambda_1, \lambda_2, \lambda_3)K(\chi_A^{-1}, \chi_B^{-1}, \chi_C^{-1})}.\quad (\text{C4})$$

Lastly, the coefficients ζ , ξ are given by

⁵We opt for the notation K instead of the more common λ to avoid confusion with the rates.

$$\begin{aligned}
\zeta Q(\lambda_1, \lambda_2, \lambda_3) K(\chi_A^{-1}, \chi_B^{-1}, \chi_C^{-1}) &= [(n_u + n_s)\lambda_3(\Pi_1 + \Pi_2) + n_s\lambda_2\Pi_1 + n_u\lambda_1\Pi_2] K(\chi_A^{-1}, \chi_B^{-1}, \chi_C^{-1}) \\
&+ \left\{ -(\chi_A^{-1} - \chi_B^{-1})^2 + (\chi_A^{-1} + \chi_B^{-1})\chi_C^{-1} - (\chi_A^{-1} - \chi_B^{-1} + \chi_C^{-1}) \left[(\chi_u^u)^{-1} + \frac{1}{3}(\chi_e^e)^{-1} \right] \right. \\
&+ 2\chi_C^{-1}(\chi_d^d)^{-1} + (\chi_A^{-1} - \chi_B^{-1} - \chi_C^{-1})(\chi_s^s)^{-1} \left. \right\} n_q \lambda_3 (\Pi_1 + \Pi_2) \\
&+ \left\{ (\chi_A^{-1} - \chi_B^{-1} + \chi_C^{-1})\chi_B^{-1} + (\chi_A^{-1} + \chi_B^{-1} - \chi_C^{-1}) \left[(\chi_u^u)^{-1} + \frac{1}{3}(\chi_e^e)^{-1} \right] \right. \\
&- (\chi_A^{-1} - \chi_B^{-1} - \chi_C^{-1})(\chi_d^d)^{-1} - 2\chi_B^{-1}(\chi_s^s)^{-1} \left. \right\} n_q \lambda_2 \Pi_1 \\
&+ \left\{ -(\chi_A^{-1} - \chi_B^{-1} - \chi_C^{-1})\chi_A^{-1} - 2\chi_A^{-1} \left[(\chi_u^u)^{-1} + \frac{1}{3}(\chi_e^e)^{-1} \right] \right. \\
&+ (\chi_A^{-1} - \chi_B^{-1} + \chi_C^{-1})(\chi_d^d)^{-1} + (\chi_A^{-1} + \chi_B^{-1} - \chi_C^{-1})(\chi_s^s)^{-1} \left. \right\} n_q \lambda_1 \Pi_2, \tag{C5}
\end{aligned}$$

$$\begin{aligned}
\frac{\xi}{2} Q(\lambda_1, \lambda_2, \lambda_3) K(\chi_A^{-1}, \chi_B^{-1}, \chi_C^{-1}) &= \{n_q[(\chi^{-1})_d^d - (\chi^{-1})_s^s] + (n_d - n_s)\chi_A^{-1} - n_u(\chi_B^{-1} - \chi_C^{-1})\} \Pi_1 \\
&+ \left\{ n_q \left[(\chi^{-1})_d^d - (\chi^{-1})_u^u - \frac{1}{3}(\chi^{-1})_e^e \right] + (n_d - n_u)\chi_B^{-1} - n_s(\chi_A^{-1} - \chi_C^{-1}) \right\} \Pi_2, \tag{C6}
\end{aligned}$$

from which one may evaluate $\zeta_{\pm}$ using Eq. (17).

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