

Glueball mass spectrum at finite temperature revisited: Constant contribution in glueball correlators in the deconfinement phase

Toshizo Arikawa,^{*} Keita Sakai[†], and Shoichi Sasaki[‡]

Department of Physics, Tohoku University, Sendai 980-8578, Japan



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We study the glueball properties at finite temperature from the temporal correlation in $SU(3)$ Yang-Mills theory using anisotropic lattice QCD. Although the existence of a constant contribution to the meson correlation function appearing in the deconfinement phase is known, its effect on the glueball correlation function at finite temperature has not been considered in previous studies. The present study reveals that the constant contribution to the glueball correlation function actually occurs in the three lowest-lying glueball states, corresponding to the 0^{++} , 2^{++} , and 0^{-+} glueballs, from near the critical temperature of the deconfinement phase transition T_C to the high-temperature side. If the existence of constant terms is taken into account in the standard pole-mass analysis, it is observed that the pole mass of the glueball ground state remains unchanged below T_C and then increases with temperature above T_C . The result indicates that the true temperature dependence of the glueball mass above T_C is opposite to the results of the previous studies.

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I. INTRODUCTION

One of the interesting topics in finite temperature QCD is the possible mass shift and the width broadening of hadrons. In particular, the mass modification due to deconfinement and chiral phase transitions has been the subject of many related studies, because it is related to the mass origin of hadrons. In particular, since in pure $SU(3)$ Yang-Mills theory the deconfinement phase transition becomes a first-order phase transition, we expect to clarify the direct relationship between the hadron mass change and the deconfinement phase transition by studying the thermal mass shifts of glueballs, which are well established as the only hadronic excitations at zero temperature in the theory [1]. In previous studies [2–4], lattice QCD calculations reported that the mass of glueballs “decreases” with increasing temperature T from near the phase transition temperature T_C .

On the other hand, in Ref. [5], it was pointed out that the direct effect of the deconfinement phase transition appears as a constant term at $T \gtrsim T_C$ in the calculation of the two-point correlation function of mesons, which are ordinary hadrons. This is intuitively understood to be caused by the contribution of quarks wrap rounding in the temporal

direction, as shown in Fig. 1(b), after the phase transition, since quarks are liberated from the confinement. Unless the Dirac matrix Γ in the bilinear current operator $\bar{q}\Gamma q$ for mesons satisfies $\{\Gamma, \gamma_4\} = 0$ [6], a constant contribution of such a quark winding appears in the time direction for mesons with zero total momentum. Thus, there is no constant term for pseudoscalar mesons ($\Gamma = \gamma_5$) or vector mesons ($\Gamma = \gamma_i$), but it appears for scalar mesons ($\Gamma = 1$) or axial-vector mesons ($\Gamma = \gamma_5\gamma_i$) [5,6].

Since the glueball with the charge-conjugation parity $C = +1$ contains two gluons as the leading Fock state [7], simply replacing the quark lines by gluons as in the lower panels of Fig. 1 will lead to a contribution from the gluon winding in the time direction due to the deconfinement, as shown in Fig. 1(d). Therefore, the constant term is expected to appear universally in the $C = +1$ glueball two-point correlation functions, independent of the other quanta, such as the total spin J and the parity P , without any restrictions due to the nature of the Dirac spinor, as was the case for mesons [5,6]. Furthermore, the gluon winding contribution as shown in Fig. 1(d) has a Z_3 center symmetry as well as Figs. 1(a) and 1(c), while the quark winding contribution as shown in Fig. 1(b) is not invariant under the Z_3 transformation.

As the previous studies [2,3] were earlier than Ref. [5], the analysis did not take into account the presence of the constant terms that are likely to appear in the low-lying glueball two-point functions above T_C as well. Instead, in Refs. [3,4], the glueball two-point functions are analyzed under the assumption that the spectral function may be affected by the thermal width.

In this study, we first attempt to confirm that constant terms do indeed appear in the glueball two-point functions

^{*}Contact author: toshizo.arikawa.s3@alumni.tohoku.ac.jp

[†]Contact author: keita.sakai.s4@alumni.tohoku.ac.jp

[‡]Contact author: ssasaki@nucl.phys.tohoku.ac.jp

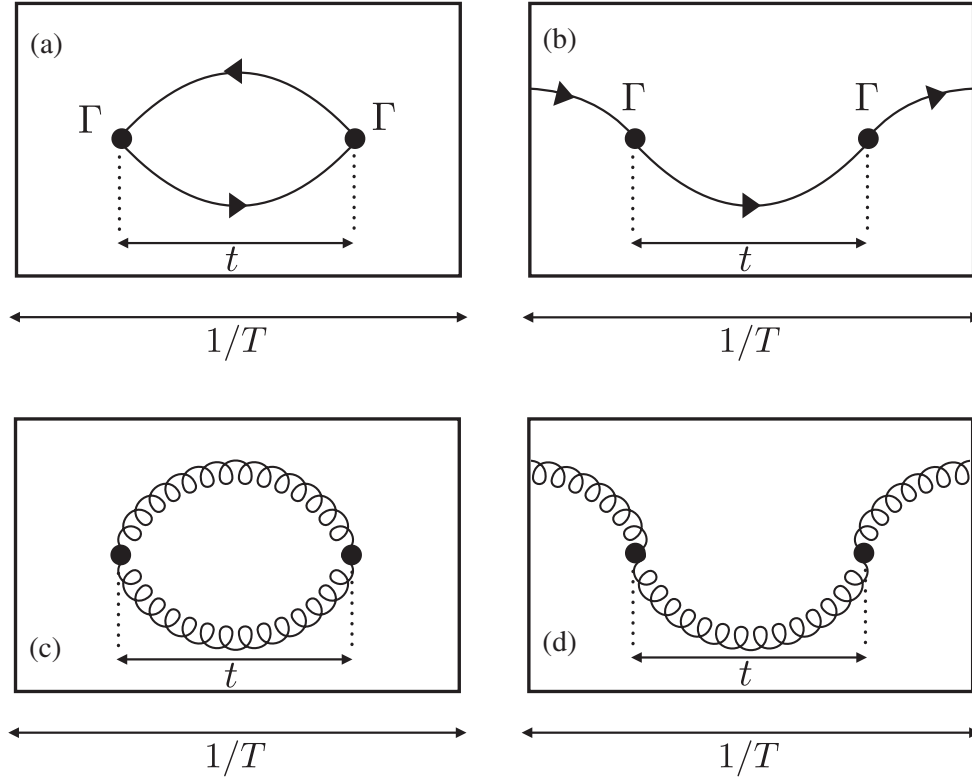


FIG. 1. Schematic diagrams for a mesonlike correlation (a), (b) and a glueball-like correlation (c), (d) in a system with period $N_t = 1/(a_t T)$. The quark winding (b) and the gluon winding (d) in the time direction due to the deconfinement may lead to the constant contributions in hadron two-point correlations $C(t)$ above T_C .

with respect to the deconfinement phase transition, and then we will perform the standard pole-mass analysis to determine the glueball ground-state masses at finite temperature, taking into account the presence of such contributions. Recall that three lowest-lying glueball states ($J^{PC} = 0^{++}$, 2^{++} , and 0^{-+}) have the C parity $C = +1$ [1].

This paper is organized as follows. In Sec. II, we first briefly summarize our numerical simulations for the glueball spectroscopy at zero temperature using anisotropic lattice QCD. Section III presents the numerical results of the glueball two-point functions at the finite temperature including the deconfinement phase above the critical temperature T_C . We verify the presence of constant terms in the glueball two-point functions above T_C and then examine the true temperature dependence of the glueball ground-state masses for the three lowest-lying channels ($J^{PC} = 0^{++}$, 2^{++} , and 0^{-+}) above T_C with the constant term subtracted. Finally, we close with a summary in Sec. IV.

II. LATTICE SETUP

A. Gauge ensembles

We perform the pure Yang-Mills lattice simulations on anisotropic lattices using the anisotropic Wilson gauge action

$$S_W[U] = \frac{1}{\xi_0} S_{\text{sp}}[U] + \xi_0 S_{\text{tp}}[U], \quad (1)$$

where the parameter ξ_0 is the bare anisotropy [8]. The explicit expressions of the spatial plaquette (sp) action $S_{\text{sp}}[U]$ and the temporal plaquette (tp) action $S_{\text{tp}}[U]$ are given by

$$S_{\text{sp}}[U] = \frac{\beta}{3} \sum_{i < j \leq 3} \text{ReTr}[1 - P_{ij}(x)], \quad (2)$$

$$S_{\text{tp}}[U] = \frac{\beta}{3} \sum_{i \leq 3} \text{ReTr}[1 - P_{i4}(x)], \quad (3)$$

where β is associated with the bare QCD coupling constant g_0 as $\beta = 6/g_0^2$ and $P_{\mu\nu}(x)$ denotes the plaquette operator defined as

$$P_{\mu\nu}(x) = U_\mu(x) U_\nu(x + \mu) U_\mu^\dagger(x + \nu) U_\nu^\dagger(x) \quad (4)$$

in the $\mu - \nu$ plane. If $\xi_0 > 1$ is chosen, the temporal lattice spacing a_t is finer than the spatial lattice spacing a_s . Since the true anisotropy $\xi \equiv a_s/a_t$ is equal to ξ_0 only at the

TABLE I. Details of the simulation parameters for the Wilson action adopted on our anisotropic lattice, following the simulations performed in Refs. [2,3].

β	$N_s^3 \times N_t$	ξ_0	ξ	a_s^{-1} (GeV)	$[a_s \text{ (fm)}]$	a_t^{-1} (GeV)	$[a_t \text{ (fm)}]$
6.25	$20^3 \times 80$	3.2552	4	2.341(16)	(~ 0.084)	9.365(66)	(~ 0.021)

classical level, one needs to determine the renormalized anisotropy ξ as a function of the bare anisotropy ξ_0 at a given β [8]. In this study, we adopt the parameter set of $\beta = 6.25$ and $\xi_0 = 3.2552$, which yields the renormalized anisotropy $\xi = 4$ [8], following the simulations performed in Refs. [2,3].

We generate anisotropic pure gauge ensemble with lattice size of $N_s^3 \times N_t$ using the anisotropic Wilson gauge action at $\beta = 6.25$ and $\xi_0 = 3.2552$. The gauge configurations are separated by 200 sweeps after 5000 sweeps for thermalization in all simulations. Each sweep consists of one pseudo-heat-bath [9] combined with four over-relaxation [10] steps. For the “zero-temperature” simulation ($N_s^3 \times N_t = 20^3 \times 80$), 4000 gauge configurations are totally accumulated. Details of the Wilson gauge action adopted on our anisotropic lattice at zero temperature are shown in Table I.

B. Glueball spectroscopy at zero temperature

For the glueball spectroscopy, we will calculate four lowest-lying glueball states which carry quantum numbers $R^{PC} = A_1^{++}, A_1^{-+}, E^{++}$, and T_2^{++} , where the irreducible representations (irreps) R of the octahedral point group O_h are the counterparts of the spin J for the continuum rotational group, and P and C represent the parity and the charge-conjugation parity, respectively. The quantum number R^{PC} is expected to have the following correspondence: $0^{++} \leftrightarrow A_1^{++}$, $0^{-+} \leftrightarrow A_1^{-+}$, and $2^{++} \leftrightarrow E_1^{++} \oplus T_2^{++}$ in the continuum limit [11,12].

We choose a “fish-shaped” operator (denoted as $\mathcal{O}_{\text{GB}}$), i.e., spacelike Wilson loops of eight links as shown in Fig. 2, to construct the glueball operators for $R^{PC} = A_1^{++}$, A_1^{-+} , E^{++} , and T_2^{++} .¹ We then get the irreducible contents of the representation R^P with fixed C parity from the operator $\mathcal{O}_{\text{GB}}$ according to Ref. [12].

The operator $\mathcal{O}_{\text{GB}}$ is classified with the shape characterized by the ordered closed path $\mathcal{C}$ and its associated orientation $[X, Y, X, Z, -X, -Z, -X, -Y]$. A linear combination of 48 copies with different orientations of Wilson loops $\mathcal{O}_{\text{GB}}[\hat{S}_i \mathcal{C}]$ where 48 symmetry operators $\hat{S}_i$ are applied to the original orientation is performed as

¹In Ref. [13], after applying high diffusion by the stout-link smearing method, it was found that the shape dependence of the Wilson loop almost disappears in the resulting two-point function.

$$\mathcal{O}_{\text{GB}}^\Gamma = \hat{P}_\Gamma \mathcal{O}_{\text{GB}}[\mathcal{C}] = \frac{1}{48} \sum_{i=1}^{48} \chi_\Gamma^*(S_i) \mathcal{O}_{\text{GB}}[\hat{S}_i \mathcal{C}], \quad (5)$$

where $\chi_\Gamma(S_i)$ denotes the characters of the irreps $\Gamma = A_1^+, A_1^-, E^+$, and T_2^+ . We then construct two-point correlation functions of the glueball states for given irreps Γ as

$$C(t) = \sum_{t'} \langle 0 | \tilde{\mathcal{O}}_{\text{GB}}^\Gamma(t + t') \tilde{\mathcal{O}}_{\text{GB}}^\Gamma(t')^\dagger | 0 \rangle, \quad (6)$$

where the tilde over $\mathcal{O}_{\text{GB}}$ implies the vacuum-subtracted operator defined as $\tilde{\mathcal{O}}_{\text{GB}} = \mathcal{O}_{\text{GB}} - \langle 0 | \mathcal{O}_{\text{GB}} | 0 \rangle$. See Ref. [13] for further details.

As is well known, ultraviolet noise from the gauge fields makes it difficult to perform the glueball mass calculation without noise reduction technique. In this study, we adopt the stout-link smearing method for the construction of the extended glueball operators which have a better overlap with the glueball ground states [13].

The stout-link smearing is a well-established smearing scheme and is defined as the following recursive procedure [14]. In this work, for the stout-smearing parameters, the spatially isotropic three-dimensional parameter set is chosen to be $\rho_{ij} = \rho$ and $\rho_{4\mu} = \rho_{\mu 4} = 0$. Therefore, only the spatial link variables $U_i^{(k)}(x)$ at step k are mapped into the link variables $U_i^{(k+1)}(x)$ as below,

$$U_i^{(k+1)}(x) = \exp\left(i\rho Q_i^{(k)}(x)\right) U_i^{(k)}(x), \quad (7)$$

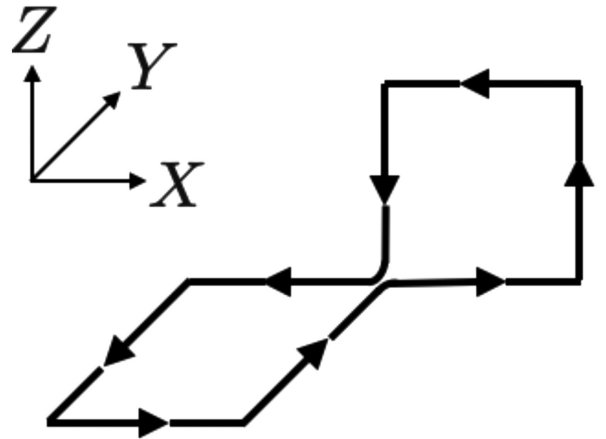


FIG. 2. Our choice of a fish-shaped Wilson loop with eight links for the construction of the glueball operator in this study.

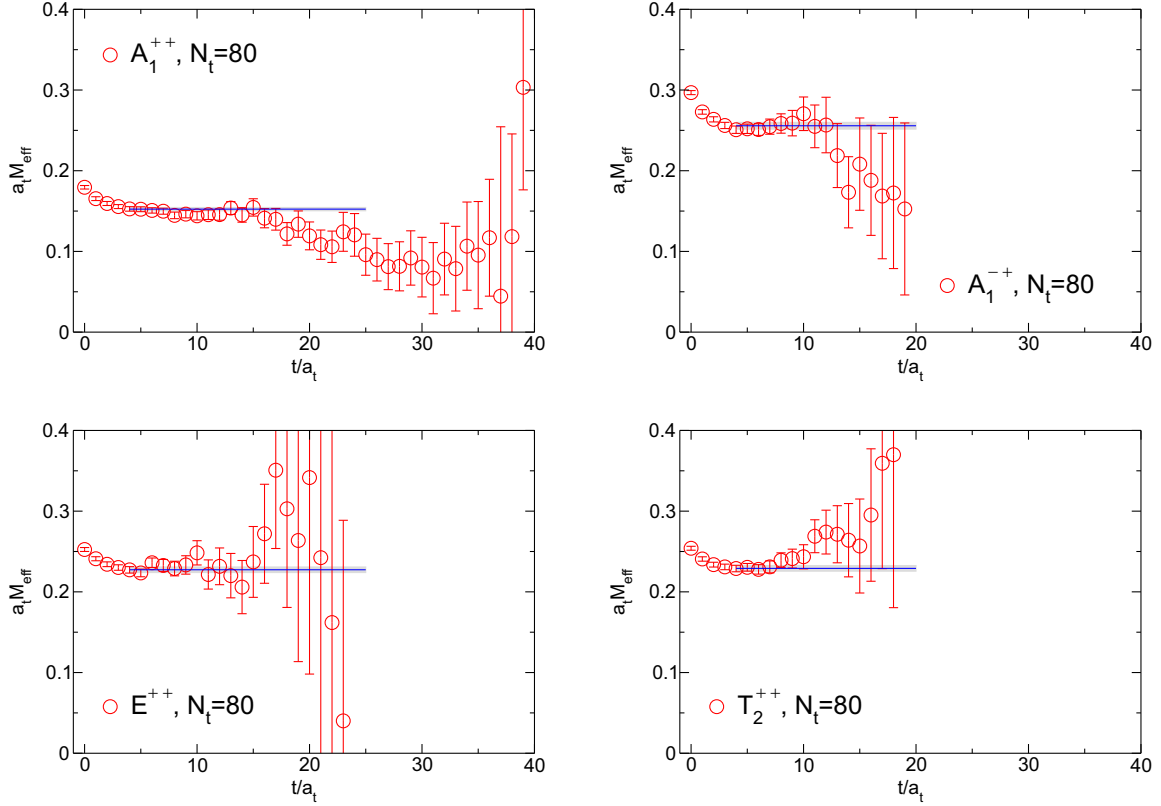


FIG. 3. Effective mass plots for A_1^{++} (upper left panel), A_1^{-+} (upper right panel), E^{++} (lower left panel), and T_2^{++} (lower right panel) irreps with $N_{\text{smr}} = 70$. Solid lines indicate fit results and shaded bands display the fitting range and one standard deviation estimated by the jackknife method.

where $Q_i^{(k)}(x)$ corresponds to a Lie-algebra-valued quantity given by

$$Q_i^{(k)}(x) = ig_0^2 \partial_{x,i} S_{\text{sp}}[U^{(k)}] \quad (8)$$

with the spatial plaquette action S_{sp} in terms of the stout-link variables.² The operator $\partial_{x,\mu}$ stands for the Lie-algebra-valued differential operator with respect to the link variable [15].

In this study, we use $\rho = 0.1$ for the smearing parameter. It is worth mentioning that when the number of smearing steps is N_{smr} , the spatial diffusion radius (denoted as R_d) resulting from the smearing process is approximately given by $R_d = \sqrt{6\rho N_{\text{smr}}}$ [13,16].

After applying the stout-link smearing, the glueball two-point function can be dominated by the ground state (its mass and spectral weight, denoted as M and W , respectively) and then expressed as $C(t) \sim W e^{-Mt}$ for

the relatively large Euclidean time t . However, in the time direction, the periodic boundary condition (PBC) is imposed with the finite extent N_t so that the two-point function is supposed to behave as below,

$$C(t) \xrightarrow[t/a_t \gg 1]{\text{PBC}} \sum_{n=-\infty}^{\infty} W e^{-M(t+nN_t)} = \frac{W}{1 - e^{-MN_t}} \left(e^{-Mt} + e^{-M(N_t-t)} \right) \quad (9)$$

$$= W \frac{\cosh[M(t - N_t/2)]}{\sinh[MN_t/2]}, \quad (10)$$

where the “wraparound effect” across the time boundary can be represented as $\sum_{n=-\infty}^{\infty} C(t + nN_t)$ (see Ref. [17] for more details). Therefore, to properly account for the wraparound effect, we use the effective mass $M_{\text{eff}}(t)$ that is defined as a solution of

$$\frac{C(t)}{C(t+1)} = \frac{\cosh[M_{\text{eff}}(t)(t - N_t/2)]}{\cosh[M_{\text{eff}}(t)(t+1 - N_t/2)]}. \quad (11)$$

Figure 3 shows the effective masses for A_1^{++} (upper left panel), A_1^{-+} (upper right panel), E^{++} (lower left panel), and

²For an isotropic four-dimensional parameter set as $\rho_{\mu\nu} = \rho$ with the case of $\xi_0 = 1$, $Q_\mu^{(k)}$ can be expressed by $Q_\mu^{(k)} = ig_0^2 \partial_{x,\mu} S_W[U^{(k)}]$, which was first derived in Ref. [15], and its particular form has recently led to a direct analytical proof of the equivalence between stout-link smearing and Wilson flow [16].

TABLE II. Masses of the ground state of the A_1^{++} , E^{++} , T_2^{++} , and A_1^{-+} irreps with $N_{\text{smr}} = 70$ at zero temperature on $20^3 \times 80$ lattice. Degrees of freedom (d.o.f.).

Irreps	$a_t M_G$	M_G (GeV)	Fit range	$\chi^2/\text{d.o.f.}$
A_1^{++}	0.1523(21)	1.426(20)	[4, 25]	1.41
E^{++}	0.2274(35)	2.130(33)	[4, 25]	1.55
T_2^{++}	0.2291(35)	2.146(33)	[4, 20]	0.73
A_1^{-+}	0.2509(42)	2.350(39)	[4, 20]	0.44

T_2^{++} (lower right panel) irreps with $N_{\text{smr}} = 70$. In each panel of this figure, the horizontal solid lines represent each fit result obtained by a correlated fit using a single-cosh function form, $A \cosh[M_G(t - N_t/2)]$ with two parameters of A and M_G on $\tilde{C}(t) \equiv C(t)/C(0)$, and shaded bands display the fit range and one standard deviation. In Table II, we summarize the results of the ground-state mass in all four channels.

The amplitude A is not normalized due to the wraparound effect even if the two-point function is solely described by the ground-state contribution. Therefore, we define the normalized amplitude $A_{\text{norm}} = A \cosh[M_G N_t/2]$, which is defined such that $0 \leq A_{\text{norm}} \leq 1$ regardless of the size of N_t . In Fig. 4, we show the normalized amplitude A_{norm} for A_1^{++} (upper left panel), A_1^{-+} (upper right panel), E^{++} (lower left

TABLE III. Number of gauge configurations (N_{cfg}) are used in our lattice simulations at target temperature T with temporal lattice size of N_t . The critical temperature was estimated as $T_c \simeq 280$ MeV in Refs. [2,3].

$N_s^3 \times N_t$	T (MeV)	N_{cfg}
$20^3 \times 72$ ($24^3 \times 72$, $16^3 \times 72$)	130 ($T \approx 0.5T_c$)	2000
$20^3 \times 48$	195 ($T \approx 0.75T_c$)	4000
$20^3 \times 40$	234	4000
$20^3 \times 36$	260	10000
$20^3 \times 34$	275 ($T \approx T_c$)	7500
$20^3 \times 32$	293	7500
$20^3 \times 28$	334	7500
$20^3 \times 24$ ($24^3 \times 24$, $16^3 \times 24$)	390 ($T \approx 1.5T_c$)	7500
$20^3 \times 20$	468	7500

panel), and T_2^{++} (lower right panel) irreps as a function of N_{smr} . In all four channels, the magnitude of the normalized amplitude is well saturated near unity ($A_{\text{norm}} \sim 0.9$ – 0.95) around $N_{\text{smr}} = 70$, where the fit mass does not change within the statistical uncertainty as N_{smr} is varied. The stout-link smearing method has been demonstrated to produce highly stable results with respect to the smearing number N_{smr} . The resulting two-point function composed of smeared operators is efficiently dominated by the contribution of the ground state.

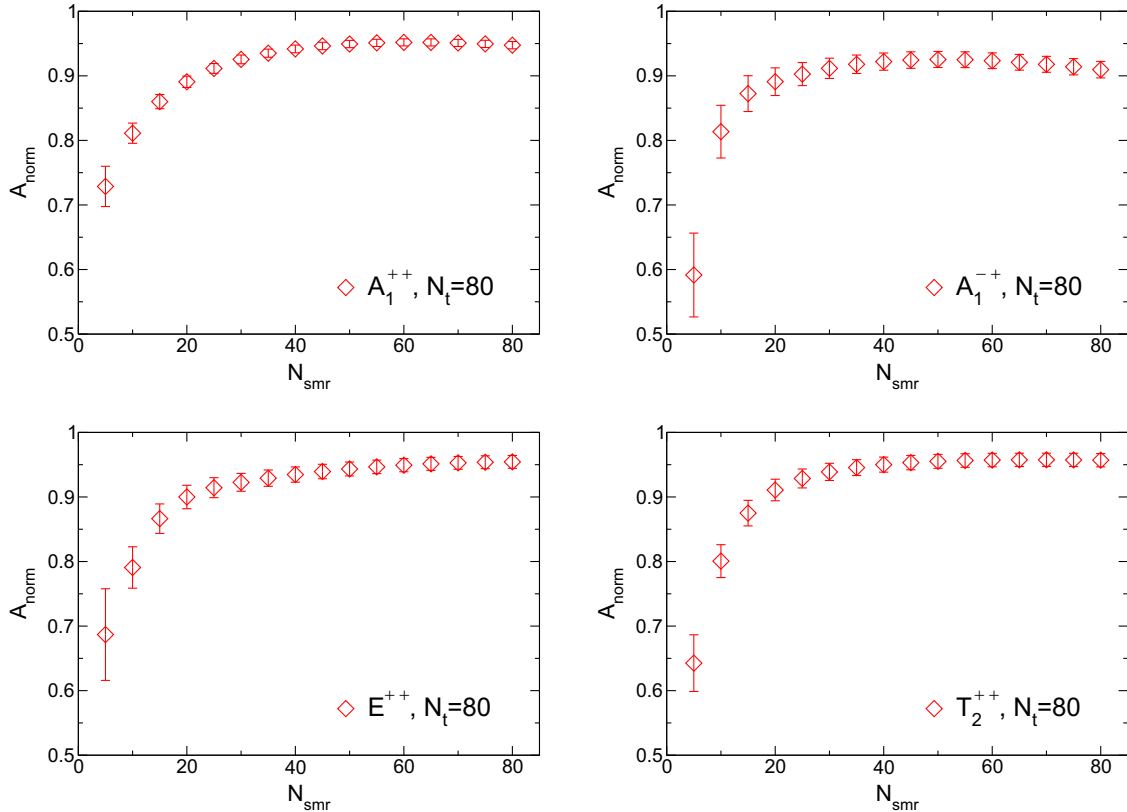


FIG. 4. Normalized amplitude, $A_{\text{norm}} = A \cosh[M_G N_t/2]$, for A_1^{++} (upper left panel), A_1^{-+} (upper right panel), E^{++} (lower left panel), and T_2^{++} (lower right panel) irreps as a function of N_{smr} .

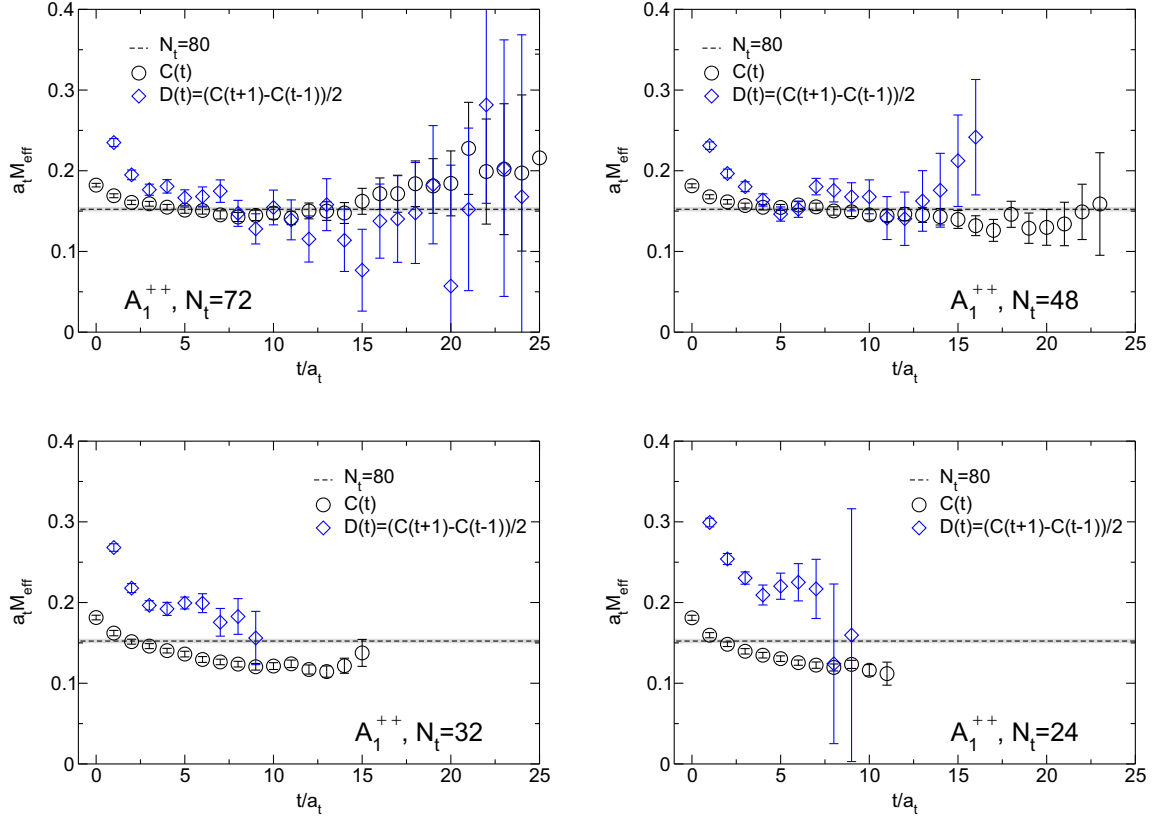


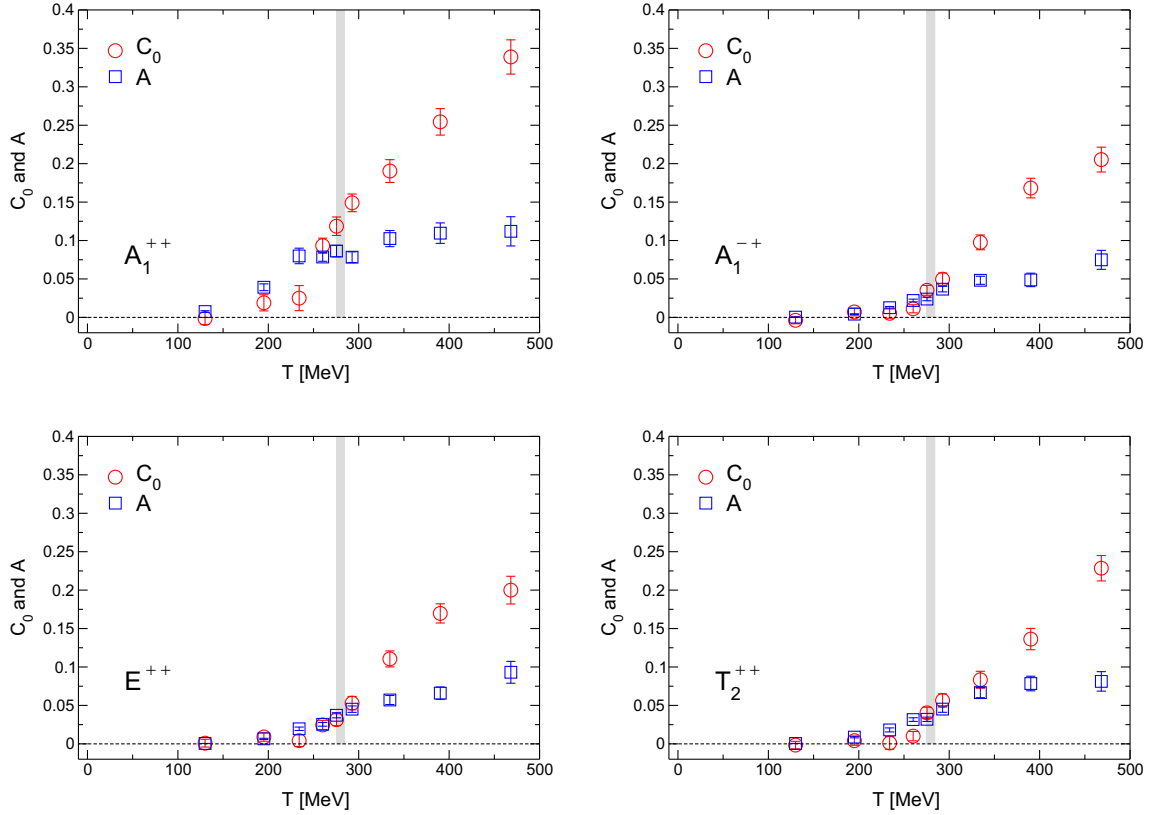
FIG. 5. Effective mass plots of the A_1^{++} irreps for $N_t = 72$ (upper left panel), $N_t = 48$ (upper right panel), $N_t = 32$ (lower left panel), and $N_t = 24$ (lower right panel) with $N_{\text{smr}} = 70$. Black circle and blue diamond symbols are given by the two-point correlator $C(t)$ and its time-derivative one $D(t) = (C(t+1) - C(t-1))/2$. Dotted lines with shaded bands indicate the fit result for $N_t = 80$ as $T = 0$.

TABLE IV. Masses of the ground state of the A_1^{++} and A_1^{-+} irreps at finite temperature on $20^3 \times N_t$ lattices.

Irreps	N_t	C_0	A	$a_t M_G$	Fit range	$\chi^2/\text{d.o.f.}$
A_1^{++}	72	-0.0015(78)	0.0076(11)	0.1534(41)	[4,25]	1.06
	48	0.0189(105)	0.0392(44)	0.1613(46)	[4,18]	0.70
	40	0.0251(163)	0.0799(101)	0.1573(60)	[4,18]	1.54
	36	0.0933(97)	0.0785(56)	0.1725(36)	[3,16]	1.67
	34	0.1186(120)	0.0863(73)	0.1753(46)	[3,16]	0.76
	32	0.1490(114)	0.0783(69)	0.1901(52)	[3,15]	1.29
	28	0.1904(148)	0.1025(105)	0.1934(66)	[3,13]	1.21
	24	0.2544(172)	0.1096(133)	0.2127(91)	[3,11]	0.74
	20	0.3388(222)	0.1120(190)	0.2392(153)	[3, 9]	1.06
A_1^{-+}	72	-0.0038(45)	0.0003(1)	0.2425(71)	[4,25]	0.77
	48	0.0071(50)	0.0042(5)	0.2539(55)	[3,20]	1.04
	40	0.0053(65)	0.0126(15)	0.2498(62)	[3,20]	1.53
	36	0.0112(53)	0.0220(18)	0.2457(46)	[3,18]	1.50
	34	0.0352(65)	0.0239(22)	0.2550(56)	[3,17]	0.98
	32	0.0497(80)	0.0368(36)	0.2415(62)	[3,16]	1.11
	28	0.0976(96)	0.0483(52)	0.2520(79)	[3,14]	0.71
	24	0.1683(127)	0.0488(88)	0.2859(167)	[4,12]	1.45
	20	0.2053(162)	0.0749(123)	0.2939(167)	[3, 9]	0.24

TABLE V. Masses of the ground state of the E^{++} and T_2^{++} irreps at finite temperature on $20^3 \times N_t$ lattices.

Irreps	N_t	C_0	A	$a_t M_G$	Fit range	$\chi^2/\text{d.o.f.}$
E^{++}	72	0.0009(50)	0.0005(1)	0.2317(52)	[4,25]	0.96
	48	0.0089(55)	0.0065(7)	0.2372(50)	[3,20]	0.65
	40	0.0041(77)	0.0196(22)	0.2290(58)	[3,18]	0.72
	36	0.0246(60)	0.0254(24)	0.2359(56)	[4,16]	0.32
	34	0.0314(77)	0.0372(33)	0.2298(53)	[3,16]	1.54
	32	0.0530(88)	0.0451(34)	0.2306(57)	[3,14]	1.66
	28	0.1105(104)	0.0571(59)	0.2415(73)	[3,14]	0.73
	24	0.1697(125)	0.0659(80)	0.2644(101)	[3,12]	1.50
	20	0.2000(180)	0.0931(80)	0.2771(150)	[3,10]	0.65
T_2^{++}	72	-0.0017(48)	0.0005(1)	0.2311(52)	[3,28]	1.65
	48	0.0041(59)	0.0087(10)	0.2239(48)	[3,20]	0.69
	40	-0.0017(82)	0.0182(26)	0.2335(76)	[4,20]	1.42
	36	0.0100(61)	0.0316(24)	0.2276(42)	[3,18]	1.06
	34	0.0403(73)	0.0319(28)	0.2392(53)	[3,17]	0.57
	32	0.0564(86)	0.0452(33)	0.2299(58)	[3,15]	1.63
	28	0.0834(110)	0.0667(66)	0.2330(69)	[3,14]	0.28
	24	0.1363(139)	0.0785(95)	0.2525(99)	[3,12]	1.34
	20	0.2285(166)	0.0812(127)	0.2877(156)	[3,10]	1.62

FIG. 6. Fit results of C_0 (red circles) and A (blue squares) as a function of temperature T for the A_1^{++} (upper left panel), A_1^{-+} (upper right panel), E^{++} (lower left panel), and T_2^{++} (lower right panel) irreps. The vertical gray-shaded band denotes the critical temperature, $T_C \approx 280$ MeV.

Therefore, in the following analysis, we simply set $N_{\text{smr}} = 70$, which corresponds to the spatial diffusion radius, $R_d \approx 0.54$ fm.

III. NUMERICAL RESULTS AT FINITE TEMPERATURE

We next calculate the glueball two-point functions on the lattice sizes of $20^3 \times N_t$ with $N_t = 72, 48, 40, 36, 34, 32, 28, 24$, and 20, covering a wide range of $130 \leq T \leq 468$ (MeV), where the temperature T is defined by $T = \frac{1}{a_t N_t}$. The critical temperature T_C for the deconfinement phase transition on our anisotropic lattice was estimated as $T_C \approx 280$ MeV in the previous work [2,3]. In addition, we also use the lattice sizes of $16^3 \times N_t$ and $24^3 \times N_t$ with $N_t = 24$ ($T \approx 0.5T_C < T_C$) and 72 ($T \approx 1.5T_C > T_C$), which are reserved for the finite volume study. The number of configurations analyzed is $\mathcal{O}(2000 - 10000)$ as shown in Table III.

A. Verification of the constant term

To determine the presence of a constant term in the two-point correlation function above T_C , we first define the time-derivative two-point function as

$$D(t) \equiv \frac{1}{2}(C(t+1) - C(t-1)), \quad (12)$$

which eliminates the constant term in $C(t)$ and then has the following functional form: $D(t) \propto \sinh[M_G(t - \frac{N_t}{2})]$. For this time-derivative correlator $D(t)$, the effective mass is given by a solution of

$$\frac{D(t)}{D(t+1)} = \frac{\sinh[\tilde{M}_{\text{eff}}(t)(t - N_t/2)]}{\sinh[\tilde{M}_{\text{eff}}(t)(t + 1 - N_t/2)]}. \quad (13)$$

If it is the case of $\tilde{M}_{\text{eff}}(t) = M_{\text{eff}}(t)$, one can confirm that there is no constant term in the original two-point function $C(t)$.

Figure 5 shows effective mass plots of the A_1^{++} irrep for $N_t = 72$ (upper left panel), $N_t = 48$ (upper right panel), $N_t = 32$ (lower left panel), and $N_t = 24$ (lower right panel) with $N_{\text{smr}} = 70$ as a typical case. The dotted lines with shaded bands indicate the $T = 0$ result obtained from the simulation with $N_t = 80$. For the case of $T < T_C$ such as $N_t = 72$ (upper left panel) and $N_t = 48$ (upper right panel), the standard effective masses (denoted by circle symbols) corresponding to the values of M_{eff} are consistent with the diamond symbols corresponding to $\tilde{M}_{\text{eff}}$. Both effective

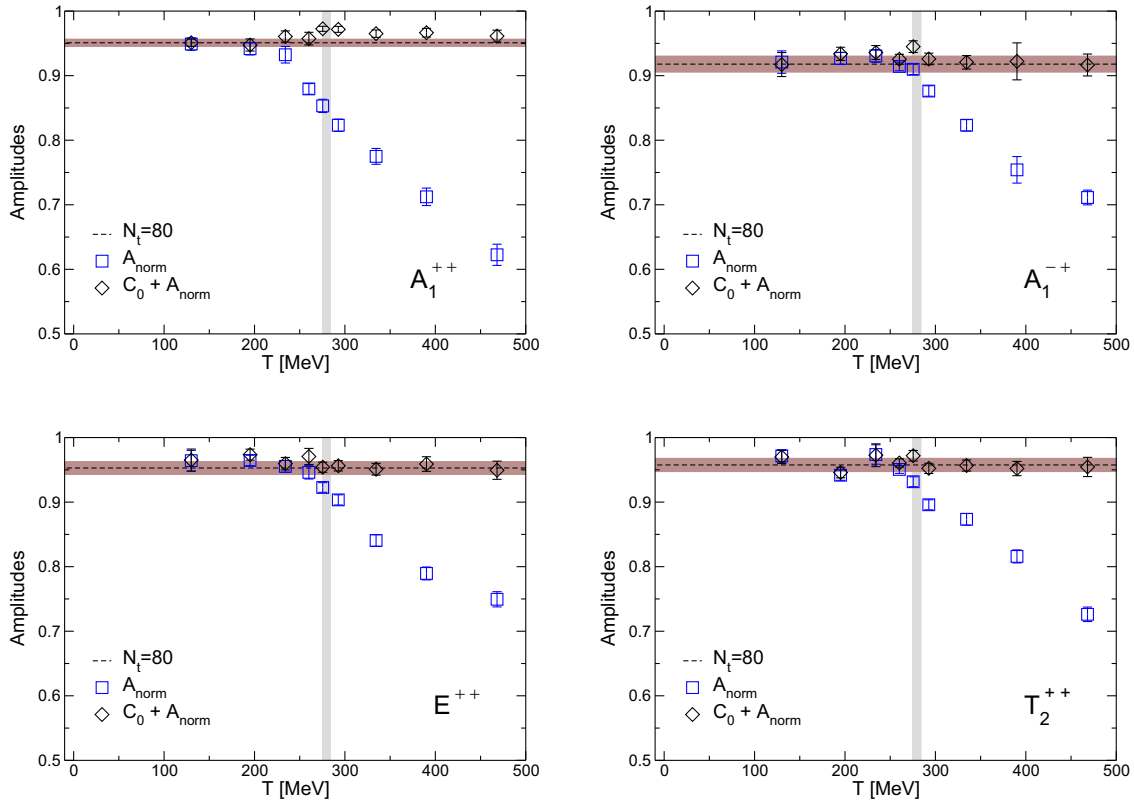


FIG. 7. Normalized amplitude A_{norm} and total sum of $C_0 + A_{\text{norm}}$ for the A_1^{++} (upper left panel), A_1^{-+} (upper right panel), E^{++} (lower left panel), and T_2^{++} (lower right panel) irreps as a function of temperature T . The dotted horizontal lines with brown-shaded bands indicate the corresponding normalized amplitudes obtained for $N_t = 80$ as $T = 0$. The vertical gray-shaded bands denote the critical temperature, $T_C \approx 280$ MeV.

masses are close to the dotted lines. On the other hand, for the case of $T > T_C$ such as $N_t = 32$ (lower left panel) and $N_t = 24$ (lower right panel), the standard effective masses are observed to shift below the dotted lines for large t . This trend is consistent with previous studies [2,3], but differs from the effective mass given by the time-differential correlator, which appears well above the dotted line. This implies that the original two-point function $C(t)$ contains the constant term.

To confirm the presence of the constant term in $C(t)$, we next employ a correlated fit on $\tilde{C}(t) = C(t)/C(0)$ using the following functional form:

$$\tilde{C}(t) = C_0 + A \cosh[M_G(t - N_t/2)] \quad (14)$$

with three parameters, C_0 , A , and M_G . The fit results are tabulated in Tables IV and V.

In Fig. 6, we plot the fit results of C_0 compared with the value of A as a function of T . A vertical gray-shaded band denotes the critical temperature, $T_C \approx 280$ MeV. In the range of $T < T_C$, the value of C_0 is observed to be

consistent with zero within statistical uncertainty, while in the range above T_C , a nonzero value of C_0 is identified in all four channels. Indeed, the fit results strongly suggest that there is a constant term in $C(t)$ at temperatures above T_C .

Figure 7 illustrates how the glueball ground-state contribution varies with temperature in the two-point function due to the manifestation of the constant term. In all four channels, the normalized amplitude A_{norm} shows no change from zero temperature to near the phase transition temperature, but suddenly begins to monotonically decrease with increasing temperature from near the phase transition point. In particular, the lowest-lying scalar (A_1^{++}) glueball shows the strongest tendency compared to the other glueball states. Furthermore, it is observed that the sum of C_0 and A_{norm} shows no change with increasing temperature. This implies that C_0 can be regarded as the “zero-energy” contribution that is normalized regardless of the size of N_t . Indeed, since it is the zero-energy contribution, the variational analysis [18,19] using the correlation matrix constructed by the $\tilde{O}_{\text{GB}}$ operator with different smearing steps

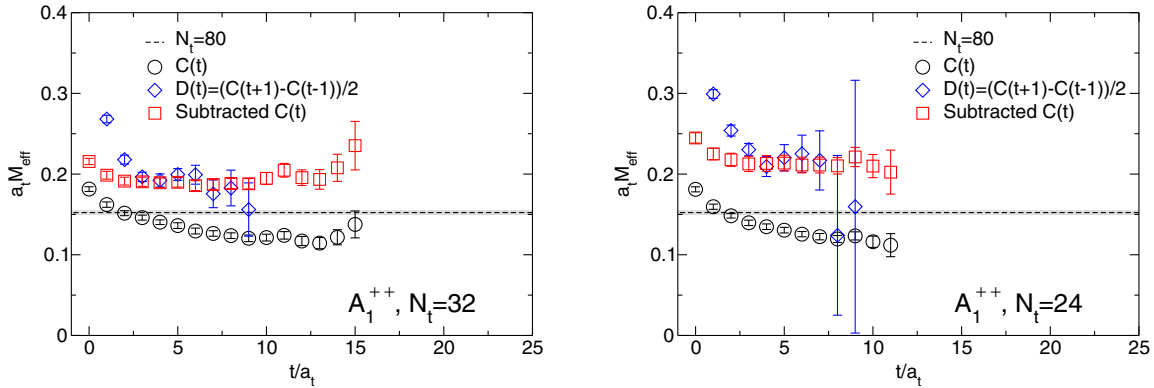


FIG. 8. Effective mass plots for the A_1^{++} irrep for $T > T_C$. Left: the case for $N_t = 32$ ($T \approx 1.1T_C$). Right: the case for $N_t = 24$ ($T \approx 1.5T_C$). Red square symbols are given by the two-point function with the constant term subtracted, while black circle and blue diamond symbols are given by the original two-point correlator and its time-derivative one. Dotted lines with shaded bands indicate the fit result for $N_t = 80$ as $T = 0$.

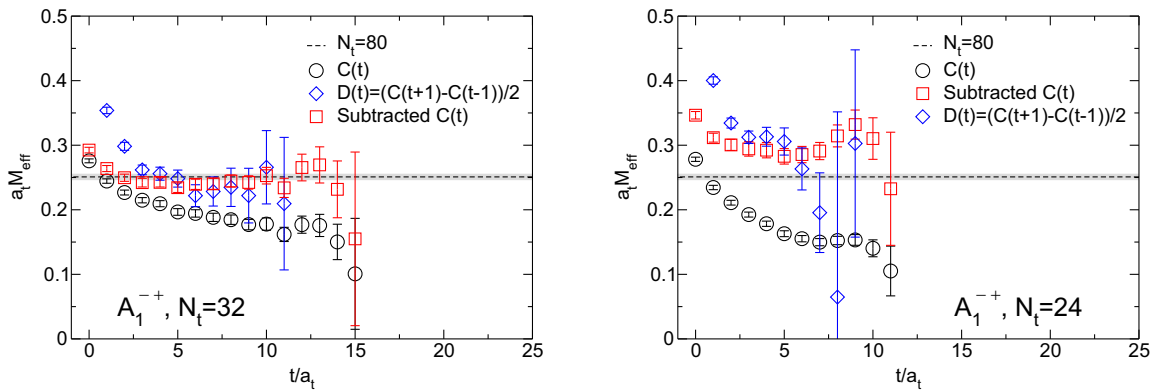
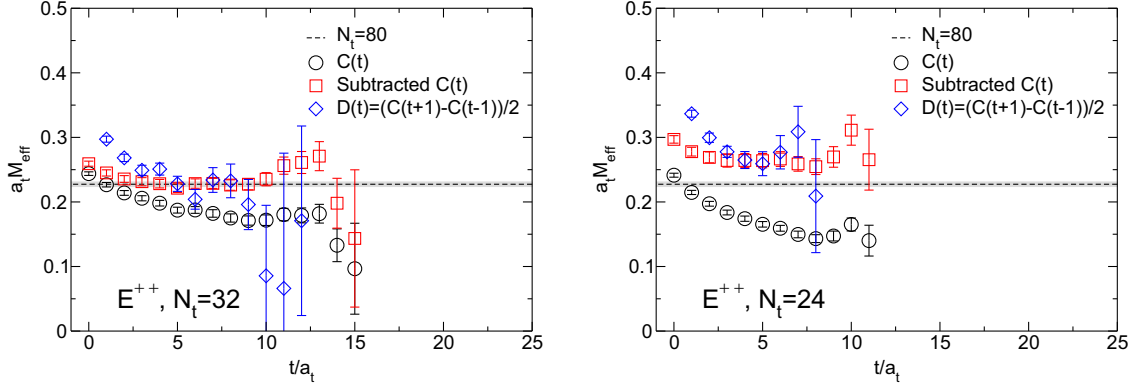
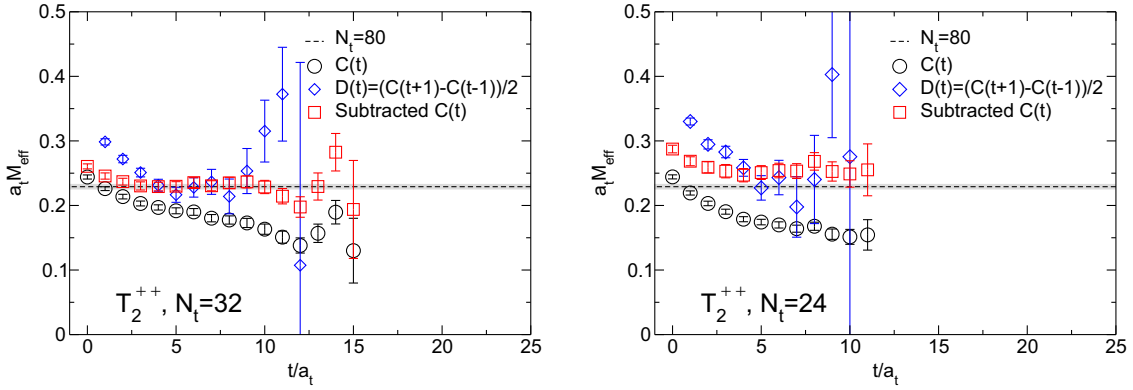


FIG. 9. Same as Fig. 8 for the A_1^{-+} irrep.

FIG. 10. Same as Fig. 8 for the E^{++} irrep.FIG. 11. Same as Fig. 8 for the T_2^{++} irrep.

could not separate the constant contribution from the finite energy modes.

B. Glueball mass at finite temperature

Next, let us subtract the constant term from each two-point function using the fit value of C_0 and then evaluate the effective mass defined in Eq. (11) for all four channels. The resulting effective mass is plotted over the two results obtained from $C(t)$ and $D(t)$ in each panel of Figs. 8–11. It is evident that the effective mass obtained from the two-point function with the constant term subtracted is in good agreement with the effective mass calculated from $D(t)$ in all four channels. Furthermore, the plateau behavior of the effective mass becomes more pronounced after subtracting the constant term.

Finally, the temperature dependence of the glueball ground-state masses for the A_1^{++} (red circles), E^{++} (blue diamonds), T_2^{++} (green squares), and A_1^{-+} (magenta triangles) irreps is shown in Fig. 12. It is observed that all glueball masses remain almost constant with increasing temperature until just before the phase transition. Subsequent to the phase transition, the mass is also found to increase with increasing temperature T . In comparison to tensor glueball (E^{++} and T_2^{++}) and pseudoscalar glueball

(A_1^{-+}), the aforementioned trend is particularly pronounced for scalar glueball, whose mass appears to vary linearly with temperature. Therefore, the mass difference between scalar glueball and the others seems to be smaller in the higher temperature region.

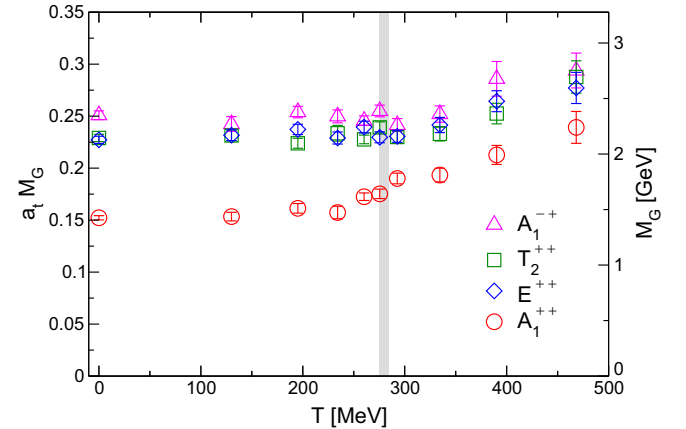


FIG. 12. Masses of the glueball ground states for the A_1^{++} (red circles), E^{++} (blue diamonds), T_2^{++} (green squares), and A_1^{-+} (magenta triangles) irreps as a function of temperature T . The vertical gray-shaded band denotes the critical temperature, $T_c \approx 280$ MeV.

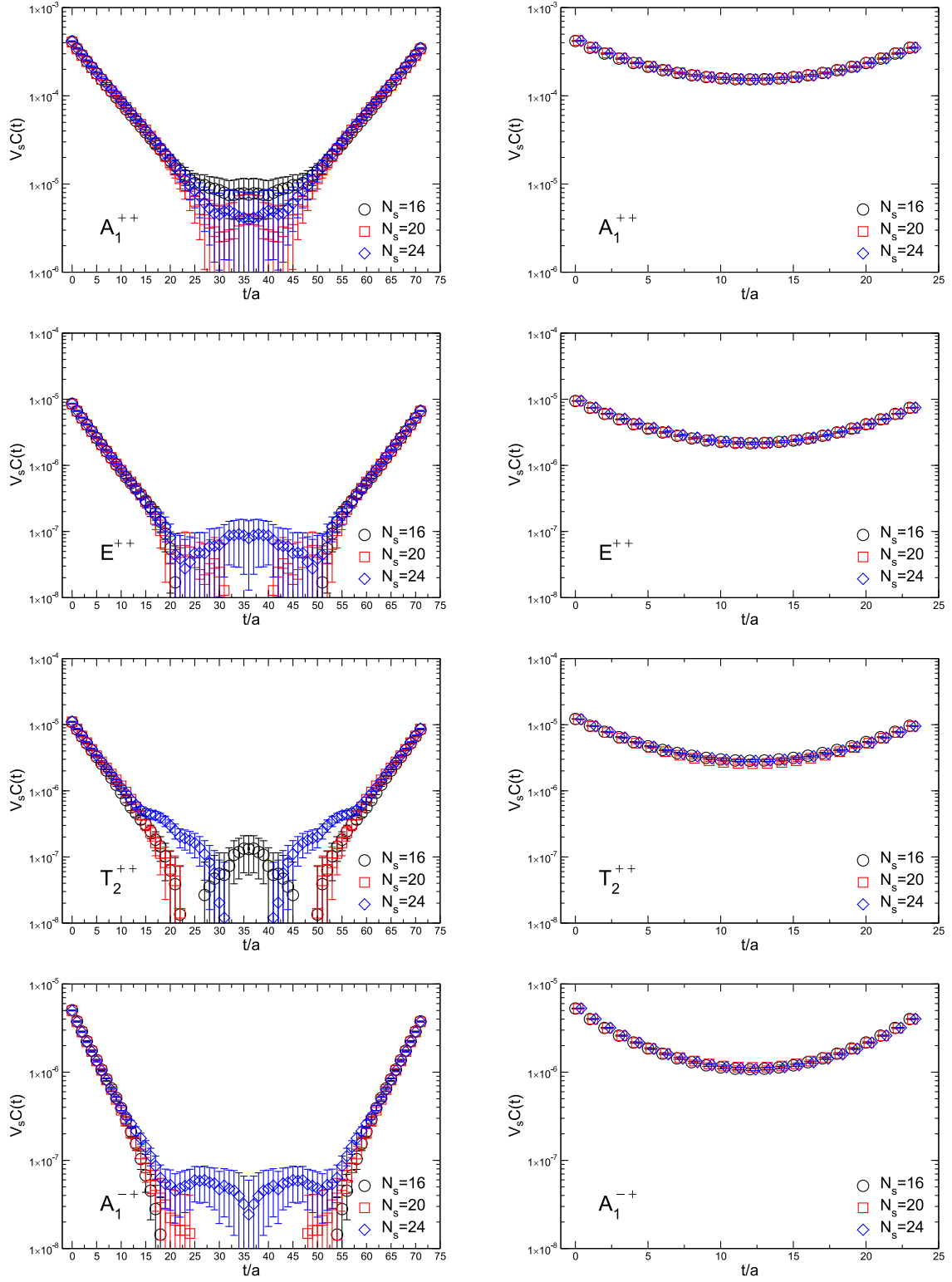


FIG. 13. Finite volume dependence of the two-point function on the low- and high-temperature sides. Left: the case for $N_t = 72$ ($T \approx 0.5T_C$). Right: the case for $N_t = 24$ ($T \approx 1.5T_C$).

C. Volume dependence of the spectral weight

The analysis performed in the previous two subsections shows that the constant term appears near the phase transition point, and the mass modification of the glueball states is definitely triggered by the phase transition. Therefore, the inquiry arises as to whether the glueball state persists as a bound state of gluons in the deconfined phase subsequent to the phase transition.

If the “glueball” becomes a multiparticle state of liberated gluons instead of a single-hadron state, it is anticipated that the volume dependence of the two-point function may exhibit discrepancies between the low- and high-temperature sides across the phase transition. These discrepancies can be attributed to the different normalization conditions applied to single- and multiparticle states in a finite box. For the single-particle case, the two-point function may be scaled by $1/V_s$ with the spatial volume factor $V_s = N_s^3$, while the two-particle states may receive an extra factor of $1/V_s$ in their spectral weight. See Appendix B in Ref. [20] for more details.

In Fig. 13, the two-point functions $C(t)$ multiplied by the spatial volume factor V_s are shown on both the low- and

high-temperature sides. The left panel shows the case for $N_t = 72$ ($T \approx 0.5T_C$) and the right panel shows the case for $N_t = 24$ ($T \approx 1.5T_C$). For ease of viewing, the three data points in the right panel are slightly offset on the x axis so that they do not overlap. There is no particular finite volume dependence on either side in all four channels. Although calculations based solely on different volumes do not seem to lead to a definitive conclusion, our results may suggest that the two-point functions $C(t)$ at both high and low temperatures are dominated by a single-hadron state whose finite volume spectral weight should be scaled as $1/V_s$. In addition, it is also found that the constant contribution is also scaled by $1/V_s$ in all four channels.

Next let us take a closer look at the scaling of the finite volume spectral weight of the glueball ground state with respect to the spatial volume V_s . Taking into account the wraparound effect for the glueball ground state, the two-point function $C(t)$ is expressed by

$$C(t) = C'_0 + \frac{W_G}{\sinh[M_G N_t/2]} \cosh[M_G(t - N_t/2)], \quad (15)$$

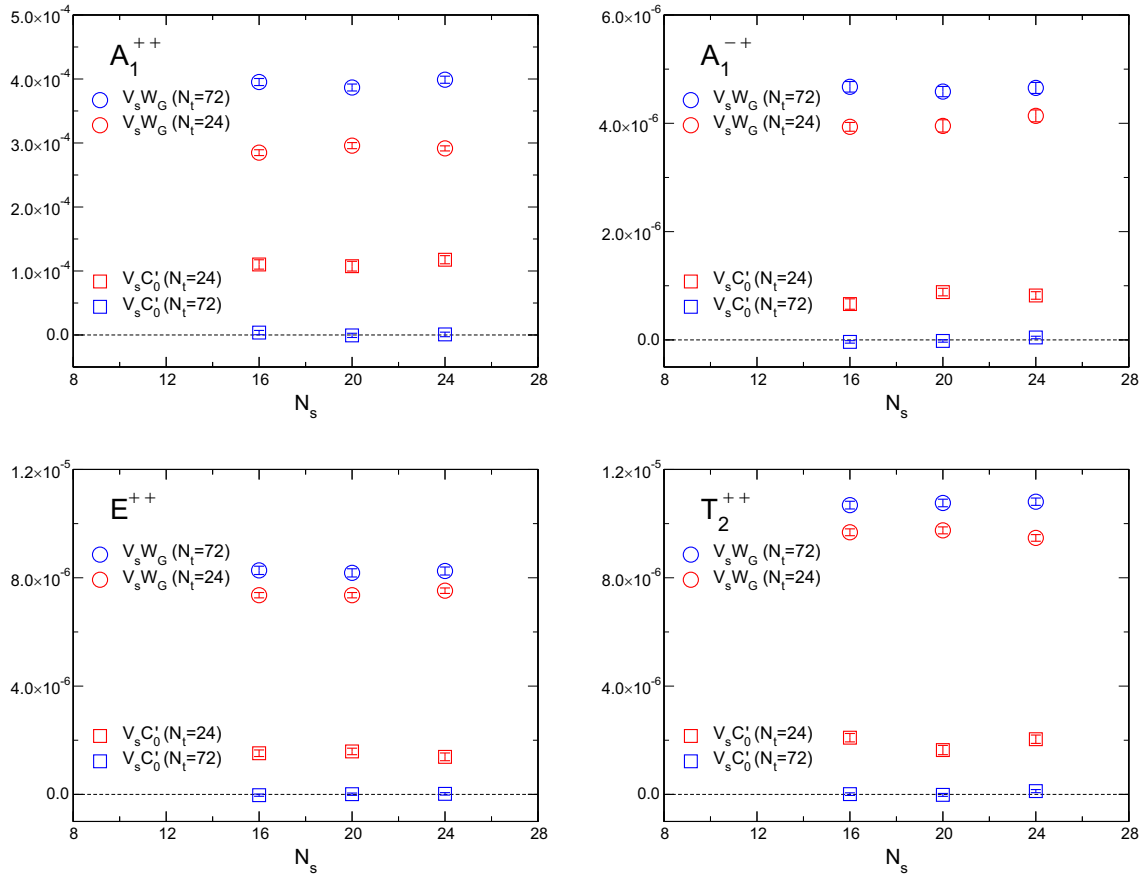


FIG. 14. Finite volume dependence of both the spectral weight of the glueball bound state (W_G) and the constant term (C'_0) on the low- and high-temperature sides. In each panel, W_G (circle symbols) and C'_0 (square symbols) scaled by the spatial volume $V_s = N_s^3$ are plotted as a function of N_s . The blue symbols denote the case for $N_t = 72$ ($T \approx 0.5T_C$), while the red symbols denote the case for $N_t = 24$ ($T \approx 1.5T_C$).

where W_G denotes the finite volume spectral weight of the glueball ground state, while C'_0 corresponds to the constant contribution. Recall that the spectral weight W_G and the constant C'_0 can be determined by setting $W_G = A \sinh[M_G N_t/2] C(0)$ and $C'_0 = C_0 C(0)$, where C_0 , A , and M_G are three fit parameters defined in Eq. (14) and the value of $C(0)$ is also known.

In Fig. 14, the finite volume spectral weight W_G and the constant contribution C'_0 scaled by the spatial volume factor V_s are plotted as a function of N_s . First of all, there is no significant dependence on N_s for the glueball ground state (denoted by circle symbols) at either low or high temperatures in all four channels. This implies that the volume dependence assuming a single-hadron state persists even after the phase transition. Furthermore, the squared symbols demonstrate analogous volume scaling in all four channels for the constant contribution, which manifests exclusively after the phase transition.

IV. SUMMARY

We have studied the glueball properties at finite temperature from the temporal correlation in pure $SU(3)$ Yang-Mills theory using anisotropic lattice QCD. The deconfinement phase transition becomes first order for the pure $SU(3)$ Yang-Mills theory without quarks. Accordingly, the glueball state, which is the sole hadronic excitation in this theory, permits direct study of how the mass modifications of hadrons stemming from the deconfinement phase transition.

The previous studies [2–4] have indeed shown that the glueball ground states survive as a hadronic state after the deconfinement phase transition, and their masses decrease with temperature above the critical temperature T_C . However, the presence of the constant contribution appearing in the glueball two-point functions with respect to the deconfinement phase transition was not taken into account in their analysis. It has been observed that constant contributions have been evident only for mesonic correlators in the scalar and axial-vector channels [5,6]. However, given the underlying mechanism that governs these contributions, it is reasonable to deduce that they are inevitable in the $C = +1$ glueball two-point functions.

To confirm the existence of the constant contribution, we have compared the effective mass plots given by two methods. One is the effective mass obtained from the original two-point function $C(t)$, and the other is the effective mass obtained by the time-differential one as

$D(t) = \frac{1}{2}(C(t+1) - C(t-1))$, where the constant contribution can be eliminated. It was observed that the effective masses of the two methods coincide just before the phase transition point, but a difference appears after the phase transition and then increases with increasing temperature T .

By performing the standard pole-mass analysis, which includes the constant contribution not taken into account in the previous studies, it is concluded that the glueball ground-state masses in the three lowest-lying channels increase with temperature T after the deconfinement phase transition. Furthermore, their normalized amplitudes, which correspond to the strength of the spectral weights, are reduced with increasing T after the deconfinement phase transition. Our result indicates that the true temperature dependence of the glueball mass above T_C is opposite to the results of the previous studies [2–4].

We have also investigated whether the transition from a single-hadron state to the multigluon state occurred before and after the deconfinement phase transition, using simulations in three different volumes, and found that the volume dependence assuming a single-hadron state persists after the phase transition. This seems to be consistent with the conclusion of the previous studies that the “glueball-like modes” survive up to around $T \approx 1.5T_C$. A more definitive conclusion might be to check for spatial localization by, for example, examining wave functions of glueball-like modes [21,22]. We plan to conduct such a study in the future.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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