

(2 + 1)D quantum electrodynamics at finite density on a quantum computer

Emil Otis Rosanowski^{1,*}, Arianna Crippa², Lena Funcke¹,
 Paulo Vitor Itaboraf^{3,2}, Karl Jansen^{3,2} and Simran Singh¹

¹*Transdisciplinary Research Area “Building Blocks of Matter and Fundamental Interactions”
 (TRA Matter) and Helmholtz Institute for Radiation and Nuclear Physics (HISKP),
 University of Bonn, Nussallee 14-16, 53115 Bonn, Germany*

²*Deutsches Elektronen-Synchrotron DESY, Platanenallee 6, 15738 Zeuthen, Germany*

³*Computation-Based Science and Technology Research Center,
 The Cyprus Institute, 20 Kavafi Street, 2121 Nicosia, Cyprus*



(Received 2 October 2025; accepted 27 May 2026; published 1 July 2026)

In this paper, we explore (2 + 1)D quantum electrodynamics at finite density on a quantum computer, including two fermion flavors. Our method employs an established gauge-invariant *Ansatz* together with a quantum circuit structure that enforces Gauss’s law. As a proof of principle, we benchmark our simulation protocol on a small lattice system of 2×2 , demonstrating the identification of density-induced level crossings in terms of the particle number of the fermion flavors. Classical simulations are used to obtain optimized variational parameters, which are then deployed in inference runs on IBM quantum hardware. We conclude by discussing hardware limitations and prospects for scaling this method to larger systems.

DOI: [10.1103/3ntp-13s7](https://doi.org/10.1103/3ntp-13s7)

I. INTRODUCTION

Gauge theories play a fundamental role in the description of nature. In the Standard Model, non-Abelian gauge theories describe the strong interactions (QCD) and the electroweak interactions, while Abelian gauge theories account for QED. Among these, simulating QCD poses particular challenges due to its strongly interacting nature. Lattice gauge theory has provided us with various insights into QCD, but regimes such as finite density, the θ -term, and transport properties of the quark-gluon plasma remain difficult to access. Furthermore, real-time dynamics, essential for understanding phenomena such as the out-of-equilibrium evolution after heavy-ion or proton collisions, are inaccessible with conventional lattice QCD techniques.

To address this challenge, there has been growing interest in studying the Hamiltonian formulation of lattice theories as a complementary approach (see, e.g., Refs. [1–3] for recent reviews). However, current methods such as exact diagonalization, tensor networks, and variational quantum algorithms remain limited to small systems due to both hardware constraints and the challenge of discretizing the Hilbert space. As a result, a full Hamiltonian treatment of

QCD in (3 + 1)D is still out of reach. A natural strategy is therefore to incrementally increase the dimensionality of the system, studying lower-dimensional models that preserve key features of QCD. The (1 + 1)D Schwinger model has long served this role, exhibiting confinement, chiral symmetry breaking, and a mass gap [4]. Recently, it has been simulated with multiple fermion flavors at finite density on superconducting quantum hardware [5] and on a trapped-ion quantum processor [6].

A natural next step is QED in 2 + 1D (QED₃), which retains key features such as confinement [7]. Beyond being a testbed for QCD, QED₃ is physically rich in its own right. In condensed matter systems, QED₃ with multiple fermion flavors and chemical potentials can serve as an effective description of the experimentally elusive quantum spin liquids [8–12] and cuprate superconductors [13–17]. This interplay between QED₃ and fermionic matter has recently been explored with quantum Monte Carlo methods [18,19]. In addition, quantum spin liquids have also been studied using exact diagonalization [20]. The first quantum simulations of QED₃ were recently reported in Refs. [21,22], focusing on a single fermion flavor at zero density. QED₃ with two fermion flavors at finite density has been simulated using variational Monte Carlo methods [23], while QED₃ with a single fermion flavor and nonzero total charge has been investigated with neural flow wave functions [24] and tensor networks [25]. Complementary to these matter-focused studies, further recent works on 2 + 1D QED focusing on the gauge sector, specifically truncation schemes, include [26,27], while cold-atom

*Contact author: rosanowski@hiskp.uni-bonn.de

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI. Funded by SCOAP³.

simulator proposals exploiting higher-spin representations have also been put forward [28].

In this work, we present the first proof-of-principle demonstration of QED₃ with multiple fermion flavors at finite density on quantum hardware. To this end, we employ the variational quantum eigensolver (VQE) [29], a resource-efficient alternative to quantum phase estimation [30] that is well-suited for noisy intermediate-scale quantum (NISQ) devices [31]. For the circuit *Ansatz*, we generalize the QED₃ construction of Ref. [32] to incorporate two fermion flavors at finite density. We classically simulate the VQE and perform inference runs on quantum devices, thereby benchmarking our finite-density state-preparation protocol on small lattice sizes of 2×2 . Our $(2+1)$ D finite-density protocol marks a first step toward real-time evolution of finite-density systems in $(2+1)$ D, relevant for quench dynamics (see, e.g., Ref. [33,34]). This approach is scalable to larger lattices through advanced adaptive *Ansätze* such as (SC)-ADAPT-VQE [35] and (SC)²-ADAPT-VQE [36], which we leave for future study. The paper is structured as follows. In Sec. II, we first introduce the Hamiltonian in Sec. II A and discuss the phase structure in Sec. II B. The VQE protocol is detailed in Sec. III, where Sec. III A presents the quantum circuit and Sec. III B describes the classical optimizer. The results are given in Sec. IV, with classical simulations in Sec. IV A and inference runs in Sec. IV C. We conclude with a discussion of the results in Sec. V. The source code is available at Ref. [37].

II. MODEL AND METHODS

A. Lattice Hamiltonian

The model studied in this work is an extension of the one presented in Ref. [32] for multiple flavors of fermions with different masses and chemical potentials.

Thus, we consider a $(2+1)$ -dimensional lattice formulation of QED with staggered fermions of multiple flavors and compact gauge links. The total Hamiltonian is given by

$$\hat{H} = \hat{H}_E + \hat{H}_B + \hat{H}_m + \hat{H}_{\text{kin}} + \hat{H}_\mu. \quad (1)$$

The electric field contribution reads

$$\hat{H}_E = \frac{g^2}{2} \sum_{\vec{r}} (\hat{E}_{\vec{r},x}^2 + \hat{E}_{\vec{r},y}^2). \quad (2)$$

Here, $\hat{E}_{\vec{r},\nu}$ is the electric field on the gauge link starting at site $\vec{r}$ and going in ν direction. The coupling strength is denoted with g . As in Ref. [32], the electric fields are truncated to

$$\hat{E}|e\rangle = e|e\rangle, \quad e \in [-l, \dots, l], \quad (3)$$

where the integer l is the truncation parameter. The effect of this truncation is discussed in Appendix A. The magnetic plaquette term is

$$\hat{H}_B = -\frac{1}{2a^2 g^2} \sum_{\vec{r}} (\hat{P}_{\vec{r}} + \hat{P}_{\vec{r}}^\dagger), \quad (4)$$

with the plaquette operator defined as

$$\hat{P}_{\vec{r}} = \hat{U}_{\vec{r},x} \hat{U}_{\vec{r}+x,y} \hat{U}_{\vec{r}+y,x}^\dagger \hat{U}_{\vec{r},y}^\dagger, \quad (5)$$

where $\hat{U}_{\vec{r},\nu}$ acts as a ladder operator for $\hat{E}_{\vec{r},\nu}$ in its eigenbasis, and a denotes the lattice spacing. For all numerical results, we set $a = 1$. The fermionic sector includes the fermionic mass term,

$$\hat{H}_m = \sum_{\vec{r},f} (-1)^{r_x+r_y} m_f \hat{\phi}_{\vec{r},f}^\dagger \hat{\phi}_{\vec{r},f}, \quad (6)$$

with $\hat{\phi}_{\vec{r},f}$ being a one-component spinor at site $\vec{r}$ of flavor f and mass m_f . Furthermore, there is the kinetic hopping term,

$$\begin{aligned} \hat{H}_{\text{kin}} = & \frac{i}{2a} \sum_{\vec{r},f} (\hat{\phi}_{\vec{r},f}^\dagger \hat{U}_{\vec{r},x} \hat{\phi}_{\vec{r}+x,f} - \text{H.c.}) \\ & - \frac{1}{2a} \sum_{\vec{r},f} (-1)^{r_x+r_y} \left(\hat{\phi}_{\vec{r},f}^\dagger \hat{U}_{\vec{r},y} \hat{\phi}_{\vec{r}+y,f} + \text{H.c.} \right). \end{aligned} \quad (7)$$

Finally, the chemical potential contribution reads

$$\hat{H}_\mu = \sum_{\vec{r},f} \mu_f \hat{\phi}_{\vec{r},f}^\dagger \hat{\phi}_{\vec{r},f}, \quad (8)$$

with μ_f being the chemical potential of flavor f . Note that our sign convention for the chemical potential differs from the standard grand-canonical form $(H - \mu N)$, following instead the convention of Ref. [23]. This does not affect the physical content of our analysis; we work at fixed total particle number $N_1 + N_2$, and our interest is solely in the redistribution of particles between the two flavors as the relative chemical potential $\mu_1 - \mu_2$ is varied. A global sign redefinition of both chemical potentials leaves this quantity unchanged, and therefore does not affect any physically relevant observable or intuition in our study. Turning to the gauge sector, physical states $|\Phi\rangle$ must satisfy Gauss' law at each lattice site,

$$\left[\sum_{\mu=x,y} (\hat{E}_{\vec{r},\mu} - \hat{E}_{\vec{r}-\mu,\mu}) - \sum_f \hat{q}_{\vec{r},f} \right] |\Phi\rangle = 0, \quad (9)$$

where the local dynamical charge is

$$\hat{q}_{\vec{r},f} = \hat{\phi}_{\vec{r},f}^\dagger \hat{\phi}_{\vec{r},f} - \frac{1}{2}[1 + (-1)^{r_x+r_y+1}]. \quad (10)$$

Following Ref. [32], we restrict our simulation to the gauge-invariant subspace by explicitly encoding Gauss's law into the *Ansatz*. This means that the equations are explicitly solved, such that some gauge links can be expressed in terms of others. In the case of a 2×2 lattice, Gauss' law corresponds to four individual equations. With open boundary conditions (OBC), there are four links and four charges. Solving the equations then eliminates three dynamical links and sets the total charge to zero. Consequently, for this system, only one instead of four gauge links is unconstrained. This link is simulated, while the other gauge links are expressed in terms of this link. In the following, we elaborate on this minimal setup, as it provides a substantial reduction of the gauge-field Hilbert space.

Despite this simplification, which results in simulating just a single link, the matter sector retains a genuine two-dimensional structure. In particular, the fermionic Hamiltonian includes hopping terms along both lattice directions, such that the fermions still probe the full 2D connectivity of the lattice. Furthermore, the gauge sector remains nontrivial at the level of plaquette interactions. After imposing Gauss's law on the 2×2 lattice with open boundary conditions, the plaquette operator $\hat{P}_{\vec{r}} + \hat{P}_{\vec{r}}^\dagger$ reduces to an off-diagonal operator acting on the remaining dynamical link. In the electric-flux basis, this corresponds to a raising and lowering operator, thereby coupling different flux sectors.

To assess the extent to which the observed density-induced level crossings are influenced by the minimal 2×2 OBC geometry, we additionally perform exact-diagonalization (ED) calculations for systems with a less constrained gauge structure, including a 2×2 lattice with periodic boundary conditions (PBC) as well as 2×3 and 2×4 lattices with OBC (see Appendix C). For the same gauge coupling $g = 5$, we find that the qualitative level-crossing structure persists across all geometries considered. This indicates that the observed behavior is not an artifact of the specific Gauss-law reduction in the 2×2 OBC system.

B. Phase structure

The zero-temperature phase diagram of two massless fermions at finite density can be analyzed through the particle number of the fermion flavors, following the approach of the (1 + 1)D study in Refs. [5,38], but now applied to a (2 + 1)D theory. For this analysis, we rewrite the flavor chemical potentials in terms of the chemical potential coupled to the total charge and in terms of the charge difference (isospin) of the two species [see Ref. [39] and Eq. (13) below].

In (1 + 1)D, the partition function and therefore the phase diagram is independent of the sum of the two

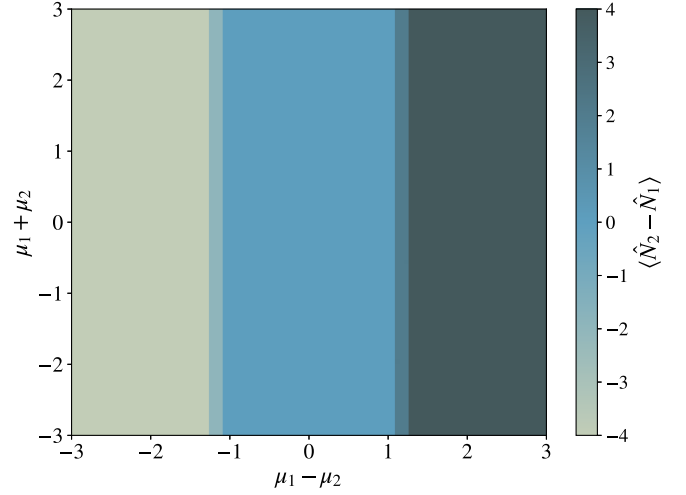


FIG. 1. Difference in particle number $\langle \hat{N}_2 - \hat{N}_1 \rangle$ depending on the sum and difference of μ_1 and μ_2 for a 2×2 system with open boundary conditions at $g = 2$ with two flavors of massless fermions.

chemical potentials, therefore, the net charge can be set to zero [39]. Indeed, Ref. [39] set the net charge to zero and obtained the density-induced level-crossing points in the sign-problem free region, in terms of the isospin chemical potential. References [38,5] went beyond and studied the same phase diagram in the sign-problem-afflicted regime of nonzero net chemical potential, using tensor networks and quantum computing, respectively.

In (2 + 1)D, no theoretical predictions exist but many considerations parallel those in lower dimensions. Indeed, also in (2 + 1)D, the phase diagram is independent of the sum of the two chemical potentials, as we show in Fig. 1 using exact diagonalization.

For simulating the sign-problem-afflicted regime of nonzero net chemical potential in our work, we now derive a formula for the locations of the density-induced level crossings in (2 + 1)D, following the analogous derivation in (1 + 1)D from Ref. [5].

For this, we first define the total number operator for each of the F flavors,

$$\hat{N}_f = \sum_{\vec{r}} \hat{\phi}_{\vec{r},f}^\dagger \hat{\phi}_{\vec{r},f}, \quad (11)$$

and note that $[\hat{H}, \hat{N}_f] = 0$. This enables us to express the ground-state energy in each phase sector as

$$E_{(N_f)}(\mu_f) = \sum_{f=1}^F \mu_f N_f + E_{\text{ind}}, \quad (12)$$

where E_{ind} is a constant for each sector and N_f is the eigenvalue of the operator $\hat{N}_f$. From now on, we only consider two fermion flavors [40]. Thus, we define

$$\mu_1 = 0, \quad \mu_2 = \mu, \quad N = N_1 + N_2, \quad \Delta N = N_2 - N_1, \quad (13)$$

yielding the ground-state energy

$$E_{(N_1, N_2)}(\mu) = \frac{1}{2}\mu N + \frac{1}{2}\mu \Delta N + E_{\min}. \quad (14)$$

The density-induced level-crossing occurs at the critical chemical potential μ_{LC} , where energies in different sectors become degenerate, i.e.,

$$E_{(N_1, N_2)}(\mu_{\text{LC}}) = E_{(\tilde{N}_1, \tilde{N}_2)}(\mu_{\text{LC}}). \quad (15)$$

Here, the second sector is denoted by a tilde. This equation can then be solved for

$$\mu_{\text{LC}} = 2 \frac{E_{\min}(N_1, N_2) - E_{\min}(\tilde{N}_1, \tilde{N}_2)}{\Delta N(\tilde{N}_1, \tilde{N}_2) - \Delta N(N_1, N_2)}. \quad (16)$$

To determine the constant values of each phase, arbitrary points μ^* and $\tilde{\mu}^*$ in both phases can be used. This way, the final expression for the critical point reads

$$\mu_{\text{LC}} = 2 \cdot \frac{E_{(N_1, N_2)}(\mu^*) - E_{(\tilde{N}_1, \tilde{N}_2)}(\tilde{\mu}^*) - N_2 \mu^* + \tilde{N}_2 \tilde{\mu}^*}{\Delta N(\tilde{N}_1, \tilde{N}_2) - \Delta N(N_1, N_2)}. \quad (17)$$

This expression will be used later on to determine the location of the density-induced level crossing from the data obtained on quantum hardware.

III. VQE PROTOCOL

The VQE needs a parametrized *Ansatz* that is able to capture the ground state wave function as a quantum circuit. Moreover, an optimizer is required to find the parameters minimizing the energy. Both components will be presented in the following.

A. Quantum circuit *Ansatz*

The variational *Ansatz* comprises three components; (i) fermion state encoding, (ii) gauge link representation, and (iii) entangling gates ensuring the matter-gauge coupling. The schematic structure is shown in Fig. 2. Due to limitations on the available hardware, this work focuses on a 2×2 system with OBC. As discussed above, this means that three out of four links can be fixed by Gauss's law and a total of eight fermions are present. However, the *Ansatz* can be extended to larger systems or PBC as well.

The mapping of the lattice to a string of qubits can be performed as indicated by Fig. 3. By applying Jordan-Wigner mapping [41,42] the fermionic degrees of freedom can be mapped to qubits using the ordering indicated by the

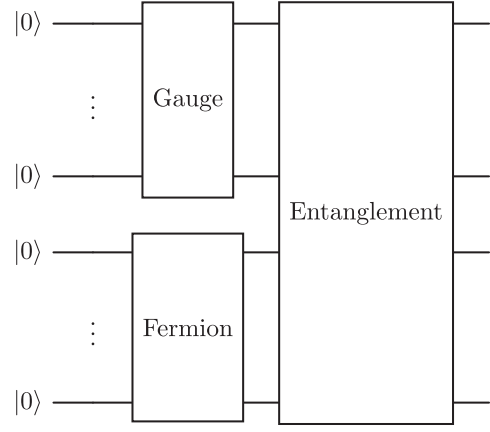


FIG. 2. Schematic structure of the variational *Ansatz* combining gauge, fermion, and entanglement subcircuits.

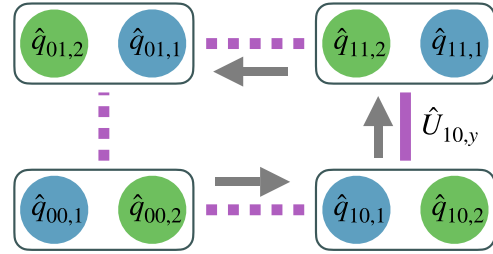


FIG. 3. The 2×2 system with open boundary conditions. Dotted purple lines represent fixed gauge links, while the solid purple line depicts a free gauge link. At each physical lattice site, two flavors of fermions are present and thus it is mapped to two qubits. The gray arrows indicate the ordering of the fermionic degrees of freedom in the Jordan-Wigner transformation.

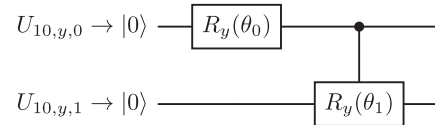


FIG. 4. Gauge field encoding circuit using Gray code at truncation $l = 1$, following Ref. [32].

arrows. Each lattice site includes two flavors of fermions and thus is mapped to two qubits. Therefore, the simulated lattice is effectively of double the width of the physical lattice.

For the gauge sector, we adopt the Gray encoding at truncation $l = 1$ from Ref. [32] (see Fig. 4). Since $l = 1$ requires three states ($|-1\rangle$, $|0\rangle$ and $|1\rangle$) [43], but any number of qubits results in a number of basis states being a power of two, one basis state needs to be excluded. This encoding on two qubits removes the state $|01\rangle$, as the state of the second qubit can only be changed when the first is in state $|1\rangle$. An important point to note here is that hardware

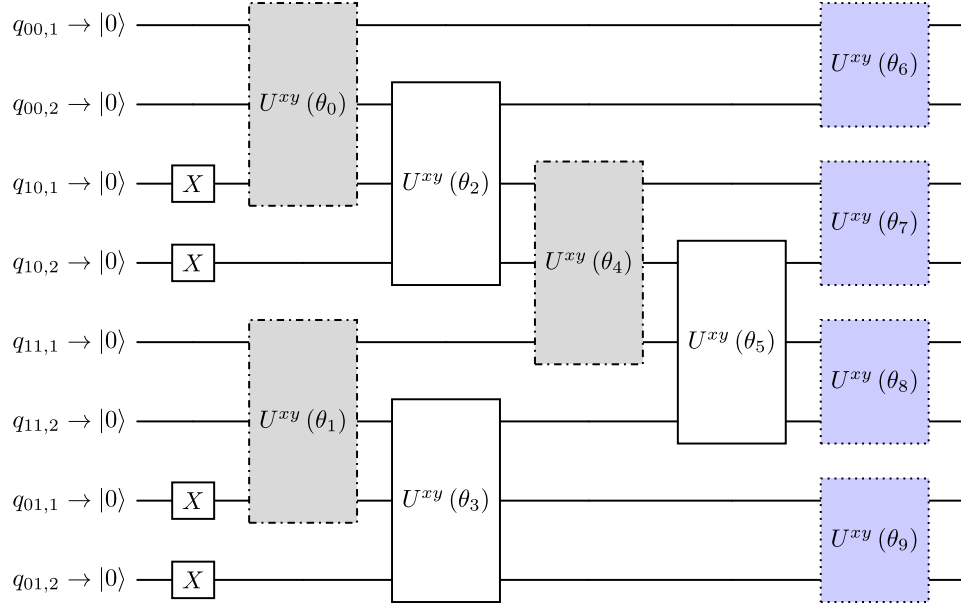


FIG. 5. Fermion encoding circuit for a 2×2 system with two fermion flavors. The different parts are highlighted by their color. The cascade structure is shown in light gray and white (dash dotted and solid), while the interflavor gates are shown in light blue (dotted). The different line style and color is only for highlighting and the gates are defined in the same way.

noise can cause violations of Gauss law causing the excluded state $|01\rangle$ to be reached with a small but nonzero probability. We have quantified this effect in Appendix D.

The fermionic subcircuit generalizes the structure in Ref. [5] to two flavors, with additional interflavor mixing layers (see Fig. 5). Here, the $U^{xy}(\theta)$ gates are defined as

$$U_{ij}^{xy}(\theta) = e^{-i\frac{\theta}{2}(X_i X_j + Y_i Y_j)}. \quad (18)$$

These gates ensure that only states with the same number of qubits in state $|0\rangle$ and $|1\rangle$ can be reached. The structure of the circuit in Ref. [5] can be seen shaded as gray and white for each flavor in Fig. 5. The interflavor mixing layer can be seen in light blue. The last layer is made up of $R_Z(\theta)$ gates for each qubit to correct the phases. This layer is not shown for better visibility.

Controlled U^{xy} -gates entangle the gauge and matter sectors while maintaining gauge invariance as shown in [21].

B. Classical optimization

The *Ansatz* contains 24 variational parameters optimized using the COBYLA algorithm [44]. The fermionic circuit shown in Fig. 5 contains 10 parameters, while the gauge circuit in Fig. 4 has two variational parameters. The additional R_Z -layer adds another eight parameters. The remaining four parameters are hidden in the entanglement circuit shown in Fig. 2. While VQE-specific optimizers such as NFT [45], can offer faster convergence, their additional requirements significantly increase the circuit

depth in our case. Specifically, for NFT, all gates of the form

$$R_j(\theta_j) = \exp\left(-\frac{i\theta_j}{2}A_j\right), \quad (19)$$

need to fulfill the requirement $A_j^2 = \mathbb{I}$. However, this is not the case for the gates considered in this work. The generator A_j for $U_{ij}^{xy}(\theta)$ is $X_i X_j + Y_i Y_j$. Squaring this operator yields

$$(X_i X_j + Y_i Y_j)^2 = 2\mathbb{I} - 2Z_i Z_j. \quad (20)$$

Consequently, the $U_{ij}^{xy}(\theta)$ gates in Eq. (18) satisfy $A_j^2 \neq \mathbb{I}$. To make NFT applicable, one would need $A_j^2 = \mathbb{I}$, which in turn would require adding an additional $Z_i Z_j$ term to the exponent in Eq. (18), thereby increasing the circuit depth. Consequently, COBYLA is employed with a maximum of 1,000 iterations, which ensures convergence. To obtain the optimal parameters, the simulation of the quantum computer is performed in the limit of infinite measurements.

C. Scaling of the *Ansatz* circuit

In this work a 2×2 system with open boundary conditions is considered. However, the *Ansatz* circuit can also be extended to different system sizes and boundary conditions. In this section, a few different systems will be discussed in terms of qubits and depth of the circuit. For this purpose Table I shows the properties of the *Ansatz* circuits for $n_{\text{flavors}} = 2$ and $n_{\text{flavors}} = 1$, respectively. It can

TABLE I. *Ansatz* circuit properties for different system sizes at $n_{\text{flavors}} = 2$. Here, *Depth* refers to the layers of multiqubit gates. The compilation of the circuits is performed for the device `ibm_marrakesh` that is introduced in Sec. IV C.

System	N_f	Depth	No. qubits	No. parameters
2×2 OBC	2	69	10	24
2×2 PBC	2	644	18	88
2×2 OBC	1	26	6	11
2×2 PBC	1	601	14	67
2×4 OBC	1	211	14	42

be seen that going from periodic to open boundary conditions reduces the circuit depth by a factor of $\mathcal{O}(10)$, while the circuit depth only moderately increases when going from $N_f = 1$ to $N_f = 2$ flavors. This is because increasing the number of flavors only affects the fermionic sector, while switching from open to periodic boundary conditions also affects the gauge sector.

An analytic description of the scaling of some parts can be given. For simplicity we restrict this to $N_f = 2$. Then, the number of qubits for the fermionic part is given as $n_{q,f} = 2 \cdot L_x \cdot L_y$. The number of gauge links is $n_g = L_x \cdot L_y + 1 + (-L_x - L_y)$ for PBC (OBC). Thus, the number of qubits for the gauge links is $n_{q,p} = \lceil \log_2(2l + 1) \rceil \cdot n_g$. Moreover, the number of U^{xy} -gates in the fermionic part scales as

$$n(U^{xy}) = \frac{n_{q,f}}{4} \left(\frac{n_{q,f}}{2} + 1 \right) + L_x \cdot L_y \quad (21)$$

$$= \frac{1}{2} (L_x \cdot L_y)^2 + \frac{3}{2} L_x \cdot L_y. \quad (22)$$

The depth is $d(U^{xy}) = L_x \cdot L_y + 1$. The depth of the other circuits cannot be expressed in closed form, as the circuit encoding a gauge link changes depending on the truncation.

We emphasize that the *Ansatz* can, in principle, be efficiently scaled to larger lattice sizes by employing advanced adaptive methods such as (SC)-ADAPT-VQE [35] and (SC)²-ADAPT-VQE [36]. A detailed investigation of this scaling is left to future work.

IV. RESULTS

A. Classical simulation

To benchmark the protocol, we perform noiseless classical simulations for a 2×2 lattice with open boundary conditions, gauge coupling $g = 5$, and truncation $l = 1$. In this large coupling regime, the support of the electric field remains small and this truncation suffices. The effect of this truncation in our setup is discussed in Appendix A. In principle, various combinations of these parameters can be

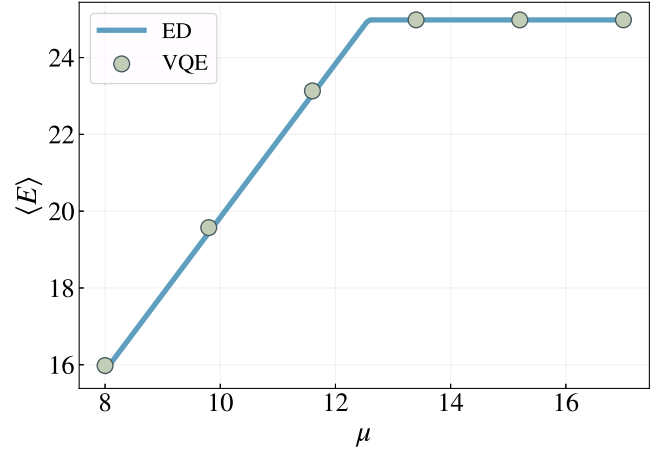


FIG. 6. Ground-state energies $\langle E \rangle$ against the chemical potential μ from the classical VQE simulation for a 2×2 system. As these are calculations with infinite shots, there are no error bars.

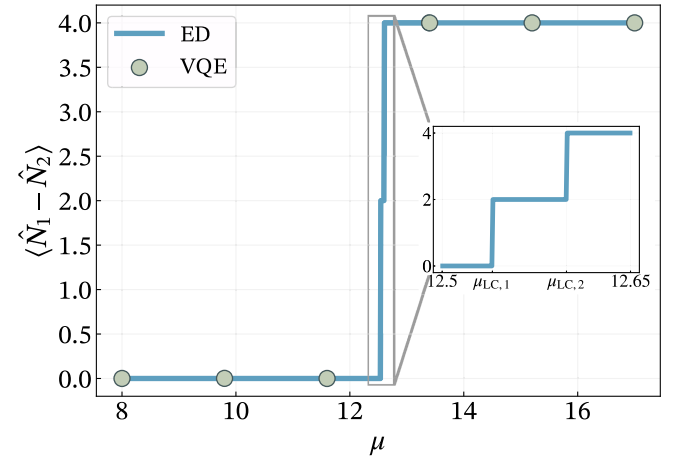


FIG. 7. Difference in fermion number ΔN against the chemical potential μ from the classical VQE simulation. As these are calculations with infinite shots, there are no error bars.

studied. However, smaller couplings lead to a larger entanglement which then requires a deeper circuit.

Therefore, we focus on the large-coupling region of $g = 5$. The resulting ground-state energies and particle number differences are shown in Figs. 6 and 7. The emergence of level crossings is clearly visible, similar to [5]. A small intermediate phase appears near the transition region, which cannot be well resolved with the VQE. The width of this region increases with decreasing coupling strength. An analysis of this region can be found in Sec. IV B.

The difference in particle numbers N_1 and N_2 shows the expected steps of height $\Delta N_{\text{first region}} - \Delta N_{\text{second region}} = 2$ as we consider two flavors of fermions and thus each increase in N_1 corresponds to a decrease of N_2 and vice versa.

Due to computational limitations, we restrict our study to a 2×2 lattice. While the positions of these level crossings are expected to shift with increasing system size, their qualitative nature and magnitude, $\Delta N_{\text{first region}} - \Delta N_{\text{second region}} = 2$, remain unaffected.

B. Analysis of the intermediate region

In Fig. 7, the intermediate region with $\langle \hat{N}_1 - \hat{N}_2 \rangle = 2$ can be seen, although it appears to be of a small width. If the coupling is modified, the size of the intermediate region changes. This can be seen in Fig. 8 where the positions of both level crossings are shown. The difference of these decreases with g , making the region difficult to resolve. Additionally, as g increases, the center of the intermediate region shifts noticeably towards larger values. This would naturally make it an obvious choice to simulate g as small as possible. However, in order to keep the circuit depth as small as possible, the entanglement in the system should be as small as possible, as this allows for fewer layers. Thus, in Fig. 9, the von Neumann entropy between the gauge link and the fermion system,

$$S = -\text{Tr}\{\rho_G \log_2 \rho_G\} \quad \text{with} \quad \rho_G = \text{Tr}_F\{\rho\}, \quad (23)$$

is calculated for the ground state. Here, ρ is the density matrix describing the full subsystem and the subscripts G and F denote the gauge and fermion subsystem, respectively. Consequently, $S = 0$ mean completely decoupled fermion and gauge systems.

In Fig. 9, it can be seen that the intermediate region exhibits a comparatively large entanglement regardless of

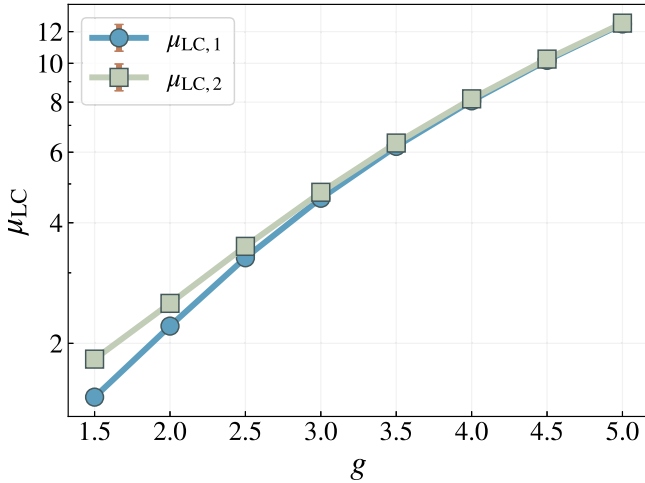


FIG. 8. Critical values of the chemical potential μ_{pt} , as computed in Sec. IV A using ED, as a function of the gauge coupling g . Here, $\mu_{\text{LC},1}$ and $\mu_{\text{LC},2}$ correspond to the edges of the intermediate region characterized by $\langle \hat{N}_1 - \hat{N}_2 \rangle = 2$, as shown for $g = 5$ in Fig. 7. Note that the intermediate region has a decreasing width and an increasing center with increasing g . The uncertainties are smaller than the markers.

the coupling, while the entanglement entropy in the other regions approaches $S = 0$ for increasing g . Therefore, investigating a coupling at which the region with high entanglement cannot be resolved is favorable for simulation on real hardware.

C. Inference run on quantum hardware

The optimized parameters obtained from classical simulation are deployed for inference runs on IBM quantum hardware. Due to noise and limited circuit depth, practical results highlight the challenges in preserving gauge symmetry and capturing the full phase structure with current devices.

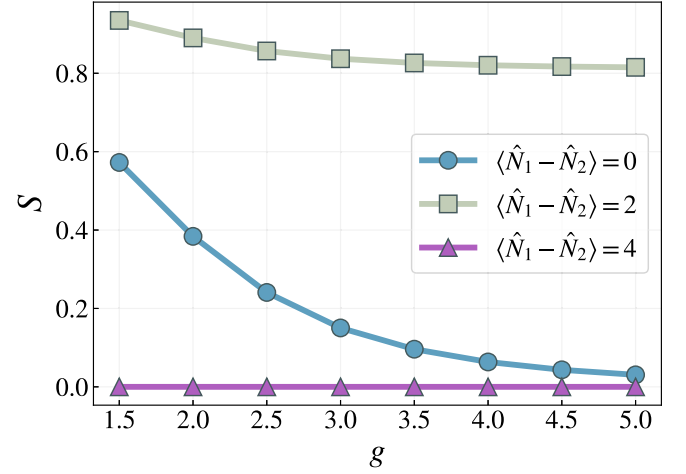


FIG. 9. Von Neumann entropy S between the gauge link and the fermion system for different coupling strengths g in all three regions. The ground state was calculated with ED in each of the three particle number sectors and the von Neumann entropy was obtained using Eq. (23).

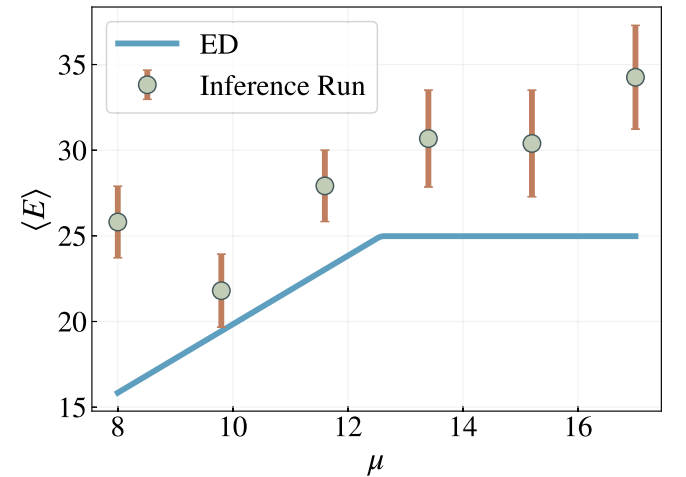


FIG. 10. The energy $\langle E \rangle$ as a function of the chemical potential μ measured on `ibm_marrakesh` with $n_{\text{shots}} = 8,192$ after applying ZNE for the system described above.

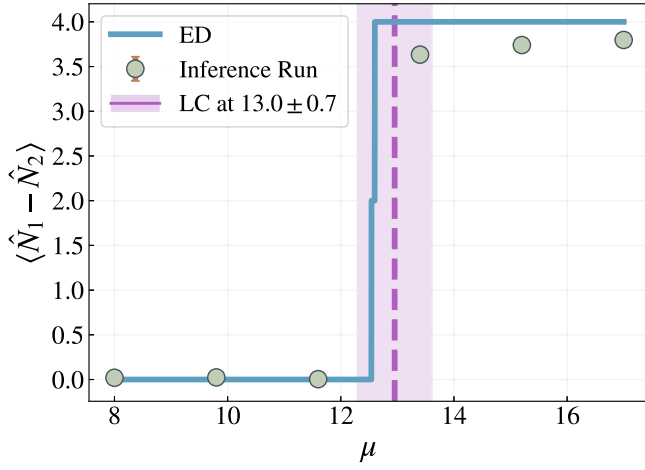


FIG. 11. $\hat{N}_1$ and $\hat{N}_2$ as a function of the chemical potential μ measured on `ibm_marrakesh` with $n_{\text{shots}} = 8,192$ after applying ZNE for the system described above. The LC is determined with Eq. (17) from the data shown in Fig. 10 and here.

For the inference run, the device `ibm_marrakesh` is used with $n_{\text{shots}} = 8,192$ shots for each data point. The three observables E , N_1 , and N_2 are measured and the results are shown in Figs. 10 and 11, respectively. All data shown include a zero-noise extrapolation (ZNE) [46]. This has been done by applying a linear fit to different noise levels ($\lambda = [1, 1.2, 1.5]$), which have been achieved using gate folding. Further details are shown in Appendix E.

One notable observation is that hardware noise has a smaller effect on the number operator than on the Hamiltonian. This can be understood in terms of the condition number, which is lower for the number operator. Further details are provided in Sec. IV D.

Subsequently, the data from this inference run are then used to calculate the critical point of the level crossing (LC) using Eq. (17). The result can be seen in Fig. 11. Both critical points can only be determined using ED, which means no exact values are present. The first is at $\mu = 12.540 \pm 0.001$ and the second at $\mu = 12.599 \pm 0.001$. Thus, both values agree with the critical point determined by the inference run data of 13 ± 0.7 .

To resolve both level crossings, data points in the intermediate region have to be taken, which is currently not possible due to limitations on the circuit depth. Consequently, even with advanced error mitigation techniques or a larger number of measurements, the uncertainty would be reduced, but the result would still correspond to an average of the two transition points.

We note that no analytical or numerical results are available for the level-crossing points in $(2+1)$ -dimensional QED, neither at finite nor at infinite volume, for the parameter regime studied in this work. The location of the critical points is expected to shift to larger values with increasing system size due to finite-size effects, as observed in the $(1+1)$ D case [5]. For the $(2+1)$ D case,

classical simulations of QED at finite density based on variational Monte Carlo methods [23] show that 8×8 lattices are sufficient to avoid finite-size effects. However, a direct comparison of our results to their findings is not possible as they studied a different parameter regime unavailable to us. Therefore, the shift of the location of the transition points due to finite-size effects remain an open question for the parameters chosen in our study.

D. Noise sensitivity of operators

In Sec. IV C it can be seen that the expectation values of the energy are noticeably affected by the hardware noise, while the number operator gives results close to the expected values. To give a quantifiable reason for this, one can define a noise sensitivity analogous to a condition number, which measures how much an arbitrary perturbation can change the measured expectation value.

For an observable O with operator norm $\|O\|_\infty$, the *state-sensitivity* quantifies the worst case amplification of state-preparation errors, where the error is measured in trace distance $T(\rho, \tilde{\rho}) = \frac{1}{2} \|\rho - \tilde{\rho}\|_1$ [47],

$$\kappa_{\text{state}}(O) = 2\|O\|_\infty = \sup_{T(\rho, \tilde{\rho})=1} |\text{Tr}[O(\rho - \tilde{\rho})]|. \quad (24)$$

This gives the bound

$$|\langle O \rangle_{\tilde{\rho}} - \langle O \rangle_{\rho}| \leq \kappa_{\text{state}}(O) T(\rho, \tilde{\rho}). \quad (25)$$

Similarly, the *channel-sensitivity* measures the worst case amplification of hardware (channel) errors, quantified by the diamond norm $\|\Lambda - \Lambda_0\|_\diamond$ [48] between the implemented channel Λ and the ideal channel Λ_0 [49],

$$\kappa_{\text{chan}}(O) = \|O\|_\infty = \sup_{\|\Lambda - \Lambda_0\|_\diamond=1} |\langle O \rangle_\Lambda - \langle O \rangle_{\Lambda_0}|, \quad (26)$$

which gives the bound

$$|\langle O \rangle_\Lambda - \langle O \rangle_{\Lambda_0}| \leq \kappa_{\text{chan}}(O) \|\Lambda - \Lambda_0\|_\diamond. \quad (27)$$

For the Hamiltonian $\hat{H}$ in our problem, the operator norm varies with the chemical potential μ . Over the interval $\mu \in [8, 17]$ we find

$$2174 \leq \|\hat{H}\|_\infty \leq 2201, \quad (28)$$

yielding a state condition number

$$4348 \leq \kappa_{\text{state}}(\hat{H}) \leq 4402, \quad (29)$$

and

$$2174 \leq \kappa_{\text{chan}}(\hat{H}) \leq 2201. \quad (30)$$

For the number operator $\hat{N}_i$, the operator norm is independent of μ and takes the value

$$\|\hat{N}_i\|_\infty = 4, \quad (31)$$

leading to

$$\kappa_{\text{state}}(\hat{N}_i) = 8 \quad \text{and} \quad \kappa_{\text{chan}}(\hat{N}_i) = 4. \quad (32)$$

These values show that small deviations have a much greater impact on the result for the Hamiltonian than for the number operator compliant with our experimental data.

V. CONCLUSIONS

In this paper, we presented the first proof-of-principle implementation of QED₃ with multiple fermion flavors at finite density on quantum hardware. To this end, we developed a VQE protocol for lattice QED₃ with two fermion flavors, enforcing gauge invariance directly at the circuit level. Furthermore, we showed the emergence of density-induced level-crossings in the particle number operator as a function of the chemical potential. To this end, we identified three distinct regions in the chemical potential, distinguished by differences in the number operator for each flavor. Additionally, using ED, we showed that the phases are independent of the sum of the chemical potentials.

In (2 + 1)D, no analytical predictions were available for the critical points; thus, we restricted ourselves to small systems involving 10 qubits in our proof-of-concept demonstration. These systems remain tractable via ED for lattice sizes of 2×2 , which we used to benchmark and verify our quantum simulations. While current limitations in circuit depth and hardware noise lead to small discrepancies in number-operator differences and larger deviations in the measured energies, our results demonstrate the feasibility of capturing essential features of level crossings, including critical points, on small lattice sizes.

In the future, we aim to investigate the scalability of our method to larger lattices using advanced adaptive ansätze such as (SC)-ADAPT-VQE [35] and (SC)²-ADAPT-VQE [36], since the scaling of the proposed VQE *Ansatz* remains prohibitive on current quantum devices. This direction is particularly relevant since the location of critical points is expected to shift with increasing system size due to finite-size effects, as observed in the (1 + 1)D case [5]. Variational Monte Carlo studies of (2 + 1)D QED at finite density [23] indicate that 8×8 lattices suffice to avoid finite-size effects at smaller couplings, whereas the required system size for the couplings considered here remains an open question.

ACKNOWLEDGMENTS

We wish to thank Tobias Hartung, Stefan Kühn, Paolo Stornati, and Ivano Tavernelli for valuable discussions and feedback on the manuscript. The authors gratefully acknowledge the granted access to the Marvin cluster hosted by the University of Bonn. We acknowledge the use of IBM Quantum services for this work. The views expressed are those of the authors, and do not reflect the official policy or position of IBM or the IBM Quantum team. This project was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) as part of the CRC 1639 NuMerIQS—Project No. 511713970 and under Germany's Excellence Strategy—Cluster of Excellence Matter and Light for Quantum Computing (ML4Q) EXC 2004/1–390534769. This work is supported with funds from the Ministry of Science, Research and Culture of the State of Brandenburg within the Center for Quantum Technology and Applications (CQTA). This work is part of the Quantum Computing for High-Energy Physics (QC4HEP) working group. This work is supported by funds from the European Union's Horizon Europe Framework Programme (HORIZON) under the ERA Chair scheme with Grant Agreement No. 101087126.

DATA AVAILABILITY

The data that support the findings of this article are openly available in Ref. [50].

APPENDIX A: CONVERGENCE OF TRUNCATION

In order to reduce the depth of the circuit, only the smallest truncation of $l = 1$ is applied. To study the effect of this truncation on the physical results, the relative error of the ground-state energy with respect to the next higher

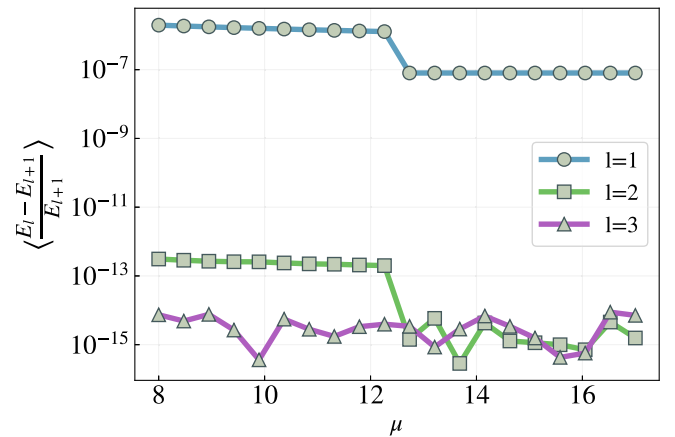


FIG. 12. Relative difference $\langle (E_l - E_{l+1}) / E_{l+1} \rangle$ of the ground-state energy for different levels of the truncation as a function of the chemical potential μ .

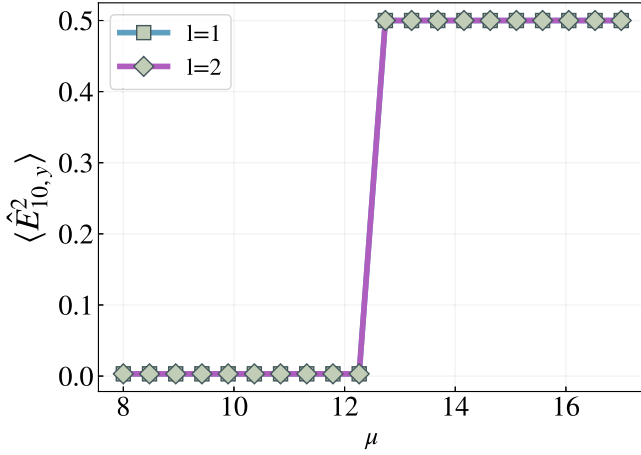


FIG. 13. Expectation value of the squared electric field on the dynamical link in the system, $\langle \hat{E}_{10,y}^2 \rangle$, against the chemical potential μ . The data is obtained with two different truncation levels of $l = 1, 2$.

truncation is calculated using exact diagonalization. This is shown in Fig. 12. It can be observed that for the given system, the relative error reaches machine precision already for $l = 3$. Even at $l = 1$, only small deviations can be seen.

Furthermore, the expectation value of the squared electric field on the remaining dynamical link, $\langle \hat{E}_{10,y}^2 \rangle$, as a function of the chemical potential μ (see Fig. 13), shows that even for truncations $l = 1, 2$ the maximum value attained is $\langle \hat{E}_{10,y}^2 \rangle = 0.5$ (up to deviations of order 10^{-5}). A more detailed analysis for $l = 1, 2, 3, 4$ reveals that, for $\mu > \mu_{LC}$, the ground state is always an equal superposition of two states with $\langle E_{10,y}^2 \rangle = 1$ and $\langle E_{10,y}^2 \rangle = 0$, respectively (up to contributions with amplitudes smaller than 10^{-7}). This implies that, in the parameter regime considered in this work, a truncation as small as $l = 1$ is sufficient to accurately describe the ground states. Thus, this truncation provides results close to the full $U(1)$ theory for the parameter regimes used in this work.

APPENDIX B: PHASES WITH $N_f = 3$ FLAVORS OF FERMIONS

Although the extension to $N_f = 3$ flavors of fermions is not useful for simulations on currently available quantum hardware, results using exact diagonalization can be obtained. The chemical potentials are chosen as $\mu_1 = -\mu$, $\mu_2 = 2$, and $\mu_3 = \mu$, while the coupling is set to $g = 2$. The difference in particle numbers between fermions of flavors one and three now changes in steps of one instead of two as can be seen in Fig. 14. Consequently, there are five different phases visible now, which are also labeled by their color. These phases can also be identified as linear regions in the energy as shown in Fig. 15. Thus, the same qualitative behavior as in Ref. [5] is found. Although currently available quantum hardware is not suitable for this system yet, future

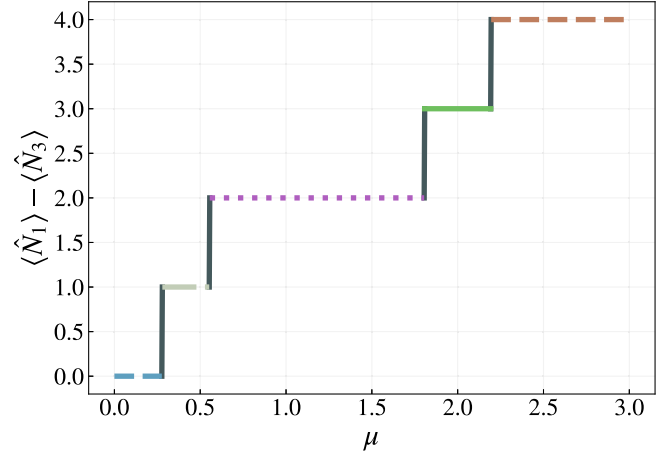


FIG. 14. Difference in particle number $\langle \hat{N}_1 - \hat{N}_3 \rangle$ in a system with three flavors of massless fermions against the chemical potential μ , where $\mu_1 = -\mu$, $\mu_2 = 2$ and $\mu_3 = \mu$ at $g = 2$.

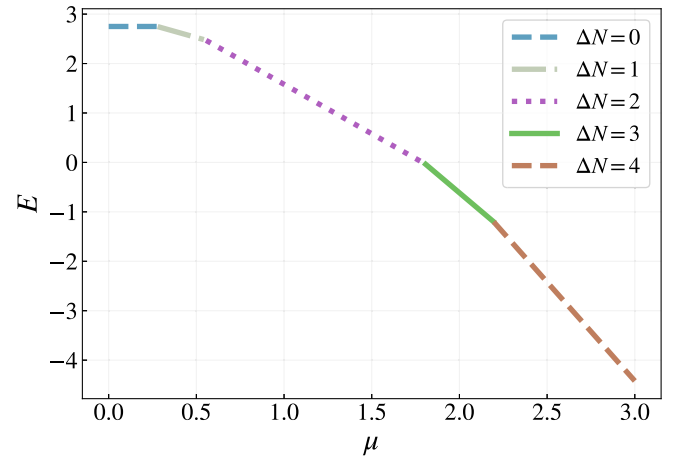


FIG. 15. Energy E in a system with three flavors of massless fermions against the chemical potential μ , where $\mu_1 = -\mu$, $\mu_2 = 2$ and $\mu_3 = \mu$ at $g = 2$. The different regions according to the difference in particle number $\Delta N = \langle \hat{N}_1 - \hat{N}_3 \rangle$ are colored as in Fig. 14.

work might consider performing inference runs for this as well.

APPENDIX C: FINITE-SIZE EFFECTS

Throughout this work, we focus on 2×2 lattices with open boundary conditions, since larger systems are not yet accessible on current quantum hardware. In this setup, only a single gauge link remains dynamical after imposing Gauss's law. Nevertheless, larger lattices can still be investigated using ED. In the following, we consider a 2×2 lattice with open and periodic boundary conditions as well as 2×3 and 2×4 lattices with open boundary conditions. These systems contain one, five, two, and three dynamical links, respectively.

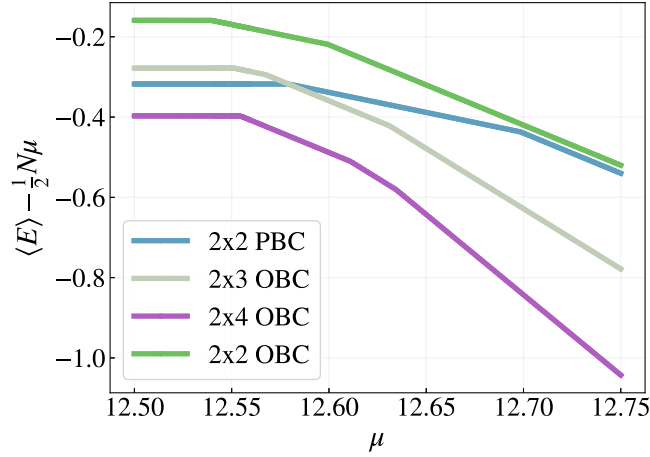


FIG. 16. Expectation value of the Hamiltonian $\langle E \rangle$ as a function of the chemical potential μ for systems with $N_f = 2$ fermion flavors. For better visibility, all curves are shifted by subtracting $\frac{1}{2}N\mu$, where $N = N_1 + N_2 = \text{const}$ within each phase. All results are obtained for truncation $l = 1$, with lattice sizes and boundary conditions as indicated in the legend.

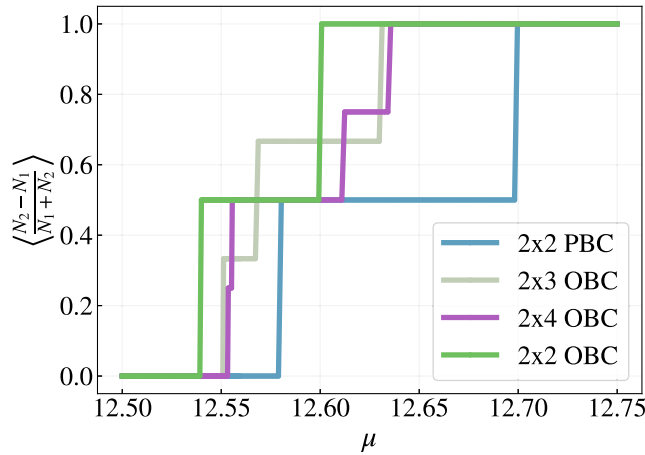


FIG. 17. Ratio of the particle-number difference to the total particle number, $\langle \frac{N_1 - N_2}{N_1 + N_2} \rangle$, as a function of the chemical potential μ for various system sizes at truncation $l = 1$. The lattice sizes and boundary conditions are indicated in the legend.

The corresponding ground-state energy $\langle E \rangle$ as a function of the chemical potential μ is shown in Fig. 16. The same qualitative behavior as in Fig. 6 is observed. At the same time, the larger systems exhibit additional kinks, reflecting the fact that more fermions can be accommodated and therefore more level crossings appear.

Similarly, Fig. 17 shows the ratio of the particle-number difference to the total particle number, $\langle \frac{N_1 - N_2}{N_1 + N_2} \rangle$, as a function of μ for the same systems. As in Fig. 7, a step-like structure is observed. For the larger lattices, however, additional plateaus appear due to the larger number of accessible fermionic configurations.

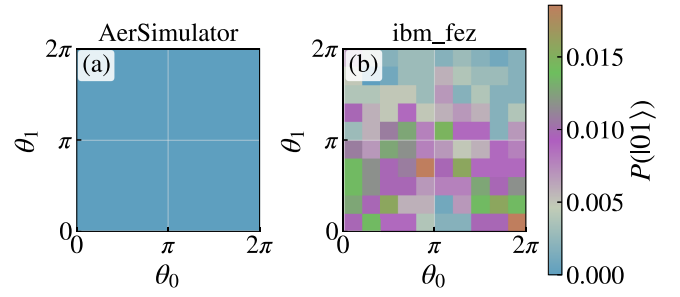


FIG. 18. Probability of measuring the excluded gauge-link state $|01\rangle$ in the Gray encoding as a function of the variational parameters θ_0 and θ_1 . The left panel shows the result obtained in a noiseless simulation with AerSimulator, while the right panel shows the corresponding data measured on ibm_fez. In both cases, $n_{\text{shots}} = 1,024$ shots are used. (a) shows the result obtained in a noiseless simulation with AerSimulator, (b) shows the corresponding data measured on ibm_fez.

Overall, the larger lattices show the same qualitative behavior as the 2×2 system with open boundary conditions. While the positions and number of level crossings depend on the system size and boundary conditions, the characteristic step-like behavior in the particle-number observables persists. This indicates that the small-volume system studied in the main text already captures the qualitative structure of the finite-density level crossings also present on larger lattices.

APPENDIX D: LEAKAGE OUT OF THE GAUGE-INVARIANT SECTOR

The Gray-encoding circuit shown in Fig. 4 restricts the accessible states to the physical gauge-invariant subspace on an ideal quantum device. On real hardware, however, noise can induce transitions into states that are excluded by construction in the noiseless case. Although hardware errors affect the results in several ways, this specific type of leakage can be quantified directly.

To this end, Fig. 18 shows the probability of measuring the excluded state $P(|01\rangle)$ as a function of the two variational parameters θ_0 and θ_1 . We compare noiseless simulations performed with AerSimulator to data obtained on the ibm_fez device. In both cases, $n_{\text{shots}} = 1,024$ shots are used. As expected, the excluded state is not populated in the noiseless simulation. On quantum hardware, by contrast, a nonzero leakage probability is observed, reaching values of up to approximately 2%. This behavior is consistent with the noise levels of current NISQ devices.

APPENDIX E: ZERO-NOISE EXTRAPOLATION

The ZNE has been used in Sec. IV C to reduce the influence of hardware errors. We applied partial gate folding to achieve noise factors of $\lambda \in [1, 1.2, 1.5]$. The extrapolation

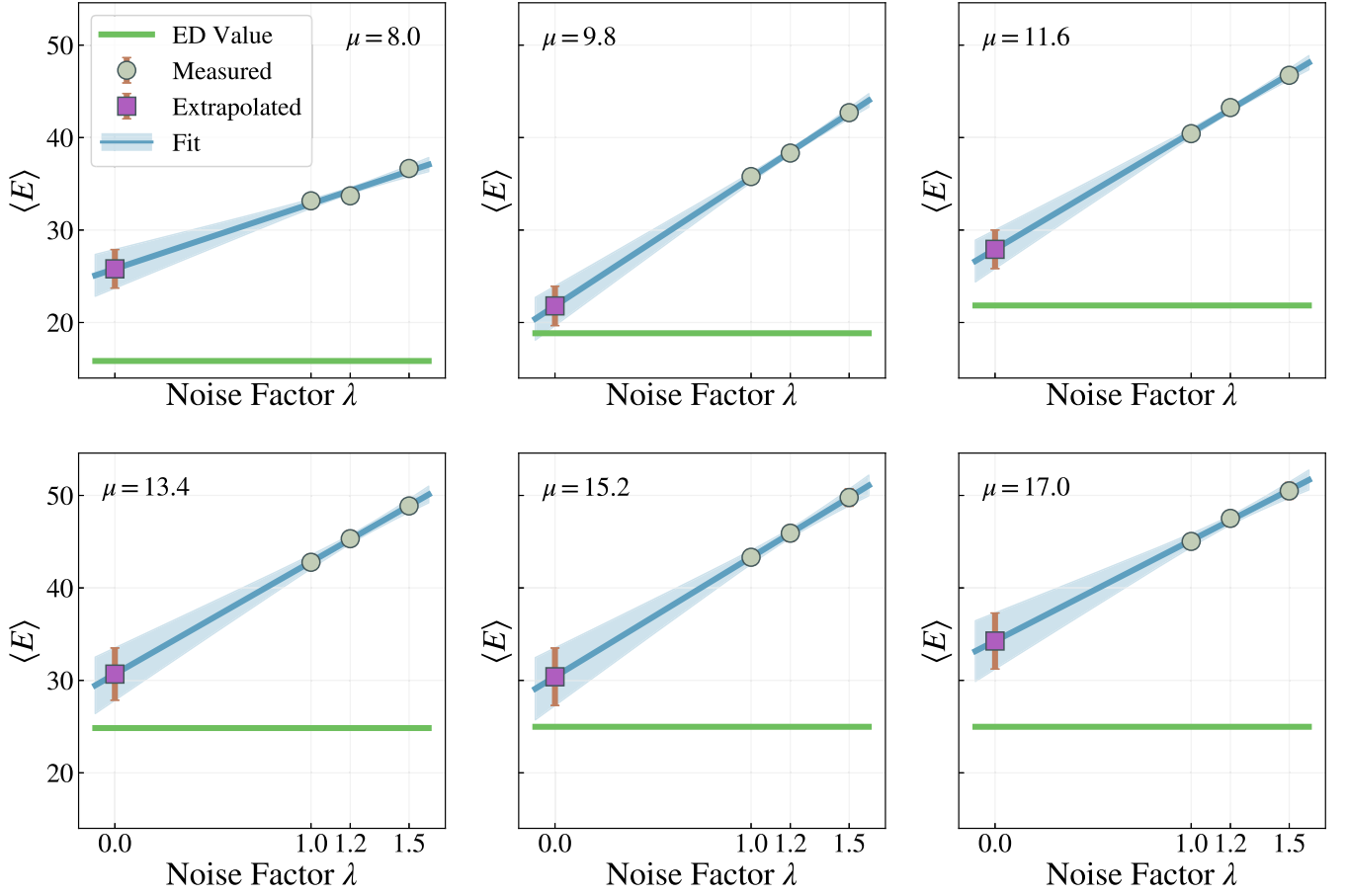


FIG. 19. ZNE for the observable $\langle E \rangle$ with the three noise factors $\lambda \in [1, 1.2, 1.5]$. Extrapolation is performed with a linear fit to the data points measured on `ibm_marrakesh` with 8,192 shots. The different panels correspond to the different values of μ being used.

is done using a linear fit as this described the data points best. Here, in Fig. 19 the extrapolation for the energy is explicitly shown. The different panels correspond to the different values of $\mu \in [8, 9.8, 11.6, 13.4, 15.2, 17]$ being used in Fig. 10. Since $n_{\text{shots}} = 8,192$ shots have been used, the uncertainties on the data points are smaller than the markers.

-
- [1] A. Di Meglio *et al.*, Quantum computing for high-energy physics: State of the art and challenges, *PRX Quantum* **5**, 037001 (2024).
 - [2] L. Funcke, T. Hartung, K. Jansen, and S. Kühn, Review on quantum computing for lattice field theory, *Proc. Sci. LATTICE2022* (2023) 228 [arXiv:2302.00467].
 - [3] G. Magnifico, G. Cataldi, M. Rigobello, P. Majcen, D. Jaschke, P. Silvi, and S. Montangero, Tensor networks for lattice gauge theories beyond one dimension, *Commun. Phys.* **8**, 322 (2025).
 - [4] S. R. Coleman, More about the massive Schwinger model, *Ann. Phys. (N.Y.)* **101**, 239 (1976).
 - [5] S. Schuster, S. Kühn, L. Funcke, T. Hartung, M.-O. Pleinert, J. von Zanthier, and K. Jansen, Studying the phase diagram of the three-flavor Schwinger model in the presence of a chemical potential with measurement- and gate-based quantum computing, *Phys. Rev. D* **109**, 114508 (2024).
 - [6] C. Melzer, S. Schuster, D. A. O. Millán, J. Hilder, U. Poschinger, K. Jansen, and F. Schmidt-Kaler, Variational quantum simulation of the interacting Schwinger model on a trapped-ion quantum processor, arXiv:2504.20824.
 - [7] T. Appelquist and R. D. Pisarski, High-temperature Yang-Mills theories and three-dimensional quantum chromodynamics, *Phys. Rev. D* **23**, 2305 (1981).
 - [8] M. Hermele, T. Senthil, M. P. A. Fisher, P. A. Lee, N. Nagaosa, and X.-G. Wen, Stability of $U(1)$ spin liquids in two dimensions, *Phys. Rev. B* **70**, 214437 (2004).
 - [9] Z. Y. Meng, T. C. Lang, S. Wessel, F. F. Assaad, and A. Muramatsu, Quantum spin liquid emerging in two-dimensional correlated Dirac fermions, *Nature (London)* **464**, 847 (2010).

- [10] M. Hermele, P. A. Lee, and X.-G. Wen, Algebraic spin liquid as the mother of many competing orders, *Phys. Rev. B* **72**, 104404 (2005).
- [11] M. Hermele, T. Senthil, and M. P. A. Fisher, Erratum: Algebraic spin liquid as the mother of many competing orders, *Phys. Rev. B* **72**, 104404 (2005); **76**, 149906(E) (2007).
- [12] Y.-C. He, M. P. Zaletel, M. Oshikawa, and F. Pollmann, Signatures of Dirac cones in a DMRG study of the Kagome Heisenberg model, *Phys. Rev. X* **7**, 031020 (2017).
- [13] I. Affleck and J. B. Marston, Large- n limit of the Heisenberg-Hubbard model: Implications for high- T_c superconductors, *Phys. Rev. B* **37**, 3774 (1988).
- [14] J. B. Marston and I. Affleck, Large- n limit of the Hubbard-Heisenberg model, *Phys. Rev. B* **39**, 11538 (1989).
- [15] P. A. Lee, N. Nagaosa, T.-K. Ng, and X.-G. Wen, SU(2) formulation of the $t - J$ model: Application to underdoped cuprates, *Phys. Rev. B* **57**, 6003 (1998).
- [16] W. Rantner and X.-G. Wen, Electron spectral function and algebraic spin liquid for the normal state of underdoped high T_c superconductors, *Phys. Rev. Lett.* **86**, 3871 (2001).
- [17] W. Rantner and X.-G. Wen, Spin correlations in the algebraic spin liquid: Implications for high- T_c superconductors, *Phys. Rev. B* **66**, 144501 (2002).
- [18] X. Y. Xu, Y. Qi, L. Zhang, F. F. Assaad, C. Xu, and Z. Y. Meng, Monte Carlo study of lattice compact quantum electrodynamics with fermionic matter: The parent state of quantum phases, *Phys. Rev. X* **9**, 021022 (2019).
- [19] C. Chen, U. F. P. Seifert, K. Feng, O. A. Starykh, L. Balents, and Z. Y. Meng, Emergent gauge flux in QED₃ with flavor chemical potential: Application to magnetized U(1) Dirac spin liquids, [arXiv:2508.08528](https://arxiv.org/abs/2508.08528).
- [20] A. Wietek, S. Capponi, and A. M. Läuchli, Quantum electrodynamics in 2 + 1 dimensions as the organizing principle of a triangular lattice antiferromagnet, *Phys. Rev. X* **14**, 021010 (2024).
- [21] A. Crippa, K. Jansen, and E. Rinaldi, Analysis of the confinement string in (2 + 1)-dimensional quantum electrodynamics with a trapped-ion quantum computer, *Commun. Phys.* **9**, 46 (2026).
- [22] M. Meth, J. Zhang, J. F. Haase, C. Edmunds, L. Postler, A. J. Jena, A. Steiner, L. Dellantonio, R. Blatt, P. Zoller, T. Monz, P. Schindler, C. Muschik, and M. Ringbauer, Simulating two-dimensional lattice gauge theories on a qudit quantum computer, *Nat. Phys.* **21**, 570 (2025).
- [23] J. Bender, P. Emonts, and J. I. Cirac, Variational Monte Carlo algorithm for lattice gauge theories with continuous gauge groups: A study of (2 + 1)-dimensional compact QED with dynamical fermions at finite density, *Phys. Rev. Res.* **5**, 043128 (2023).
- [24] Z. Chen, D. Luo, K. Hu, and B. K. Clark, Simulating 2 + 1D lattice quantum electrodynamics at finite density with neural flow wavefunctions, [arXiv:2212.06835](https://arxiv.org/abs/2212.06835).
- [25] T. Felser, P. Silvi, M. Collura, and S. Montangero, Two-dimensional quantum-link lattice quantum electrodynamics at finite density, *Phys. Rev. X* **10**, 041040 (2020).
- [26] M. Miranda-Riaza, P. Fontana, and A. Celi, Renormalized dual basis for scalable simulations of 2 + 1d compact quantum electrodynamics, *Phys. Rev. D* **113**, 074518 (2026).
- [27] Z.-G. Mou and B. Chakraborty, Scalable quantum computation of quantum electrodynamics beyond one spatial dimension, [arXiv:2510.27668](https://arxiv.org/abs/2510.27668).
- [28] P. Majcen, J. J. Osborne, P. Hauke, B. Yang, S. Montangero, and J. C. Halimeh, Towards 2 + 1d quantum electrodynamics on a cold-atom quantum simulator, [arXiv:2602.04948](https://arxiv.org/abs/2602.04948).
- [29] A. Peruzzo, J. McClean, P. Shadbolt, M.-H. Yung, X.-Q. Zhou, P. J. Love, A. Aspuru-Guzik, and J. L. O'Brien, A variational eigenvalue solver on a photonic quantum processor, *Nat. Commun.* **5**, 4213 (2014).
- [30] A. Y. Kitaev, Quantum measurements and the Abelian stabilizer problem, [arXiv:quant-ph/9511026](https://arxiv.org/abs/quant-ph/9511026).
- [31] J. Preskill, Quantum Computing in the NISQ era and beyond, *Quantum* **2**, 79 (2018).
- [32] A. Crippa, S. Romiti, L. Funcke, K. Jansen, S. Kühn, P. Stornati, and C. Urbach, Towards determining the (2 + 1)-dimensional quantum electrodynamics running coupling with Monte Carlo and quantum computing methods, *Commun. Phys.* **8**, 367 (2025).
- [33] J. Cobos, J. Fraxanet, C. Benito, F. di Marcantonio, P. Rivero, K. Kapás, M. A. Werner, Ö. Legeza, A. Bermudez, and E. Rico, Real-time dynamics in a (2 + 1)-D gauge theory: The stringy nature on a superconducting quantum simulator, [arXiv:2507.08088](https://arxiv.org/abs/2507.08088).
- [34] J. Cao, R. Joshi, Y. Tian, N. S. Srivatsa, and J. C. Halimeh, String breaking and glueball dynamics in 2 + 1d quantum link electrodynamics, [arXiv:2601.16166](https://arxiv.org/abs/2601.16166).
- [35] R. C. Farrell, M. Illa, A. N. Ciavarella, and M. J. Savage, Scalable circuits for preparing ground states on digital quantum computers: The Schwinger model vacuum on 100 qubits, *PRX Quantum* **5**, 020315 (2024).
- [36] E. Gustafson, K. Sherbert, A. Florio, K. Shirali, Y. Chen, H. Lamm, S. Valgushev, A. Weichselbaum, S. E. Economou, R. D. Pisarski, and N. M. Tubman, Surrogate constructed scalable circuits ADAPT-VQE in the Schwinger model, *Phys. Rev. Appl.* **23**, 064002 (2025).
- [37] A. Crippa, P. V. Itaborai, and E. Rosanowski, Lattice QED Hamiltonian for Quantum Computing, (v0.1.0) (Zenodo, 2025), [10.5281/zenodo.15536986](https://doi.org/10.5281/zenodo.15536986).
- [38] M. C. Bañuls, K. Cichy, J. I. Cirac, K. Jansen, and S. Kühn, Density induced phase transitions in the Schwinger model: A study with matrix product states, *Phys. Rev. Lett.* **118**, 071601 (2017).
- [39] R. Narayanan, Two flavor massless Schwinger model on a torus at a finite chemical potential, *Phys. Rev. D* **86**, 125008 (2012).
- [40] However, in Appendix B a short discussion of the case including three fermion flavors can be found.
- [41] P. Jordan and E. Wigner, Über das paulische äquivalenzverbot, *Z. Phys.* **47**, 631 (1928).
- [42] F. Verstraete and J. I. Cirac, Mapping local Hamiltonians of fermions to local Hamiltonians of spins, *J. Stat. Mech.* (2005) P09012.
- [43] These states refer to the physical states to mark a distinction from the encoded ones.
- [44] M. J. D. Powell, A direct search optimization method that models the objective and constraint functions by linear interpolation, in *Advances in Optimization and Numerical*

- Analysis*, edited by S. Gomez and J.-P. Hennart (Springer Netherlands, Dordrecht, 1994), pp. 51–67.
- [45] K. M. Nakanishi, K. Fujii, and S. Todo, Sequential minimal optimization for quantum-classical hybrid algorithms, *Phys. Rev. Res.* **2**, 043158 (2020).
- [46] K. Temme, S. Bravyi, and J. M. Gambetta, Error mitigation for short-depth quantum circuits, *Phys. Rev. Lett.* **119**, 180509 (2017).
- [47] P. J. Coles, M. Cerezo, and L. Cincio, Strong bound between trace distance and Hilbert-Schmidt distance for low-rank states, *Phys. Rev. A* **100**, 022103 (2019).
- [48] G. Benenti and G. Strini, Computing the distance between quantum channels: Usefulness of the Fano representation, *J. Phys. B* **43**, 215508 (2010).
- [49] A. Hashim, S. Seritan, T. Proctor, K. Rudinger, N. Goss, R. K. Naik, J. M. Kreikebaum, D. I. Santiago, and I. Siddiqi, Benchmarking quantum logic operations relative to thresholds for fault tolerance, *npj Quantum Inf.* **9**, 109 (2023).
- [50] E. O. Rosanowski, Data for (2+1)D Quantum Electrodynamics at Finite Density on a Quantum Computer (Zenodo, 2026), [10.5281/zenodo.20444068](https://zenodo.org/record/20444068).