

Meson-nucleus bound states with neural-network quantum states

Wei-Lin Wu^{1,*}, Liang-Zhen Wen^{1,†} and Shi-Lin Zhu^{2,‡}

¹*School of Physics, Peking University, Beijing 100871, China*

²*School of Physics and Center of High Energy Physics, Peking University, Beijing 100871, China*



(Received 11 June 2026; accepted 18 September 2026; published 30 September 2026)

We present the first systematic calculations of ϕ -, η_c -, and J/ψ -nucleus ground states up to mass number $A = 12$ based on the HAL QCD meson-nucleon potentials at near-physical point. The $(A + 1)$ -body Schrödinger equation is solved with a neural-network variational Monte Carlo framework, generalized to incorporate mesonic degrees of freedom. Benchmarking on light nuclei from ${}^2\text{H}$ to ${}^{12}\text{C}$ yields ground-state energies consistent with the experiment. Meson-nucleus bound states emerge at $A \geq 2$ for ϕ , $A \geq 4$ for J/ψ , and $A \geq 6$ for η_c . The ϕ -nucleus systems exhibit the strongest binding, with binding energies reaching tens of MeV. The J/ψ -nucleus and η_c -nucleus systems are weakly bound at the few-MeV and sub-MeV scale, respectively. The binding energy per nucleon deepens nearly linearly with A for charmonium systems, whereas the ϕ -nucleus system exhibits a nonmonotonic behavior peaking at ${}^4\text{He}$ —a distinctive hallmark of the short-range and strongly attractive ϕN interaction. The meson compresses the nucleon distribution relative to the parent nucleus, and evolves from a halo configuration to one embedded inside the nucleus with increasing A . Our results provide predictions for future experimental searches, and establish a quantitative bridge between lattice QCD meson-nucleon interactions and the emergent many-body phenomena in meson-nucleus bound states.

DOI: 10.1103/3nrk-c5c7

I. INTRODUCTION

Understanding the interactions between hadrons is a central problem in particle physics. A particularly interesting class is the interaction between quarkonium M ($c\bar{c}$, $s\bar{s}$) and nucleon N , which share no valence quarks. Single-meson exchange is suppressed by the Okubo-Zweig-Iizuka rule, so the leading contribution may arise from short-range multigluon exchange [1]. The MN interaction therefore provides a unique window into gluonic dynamics, revealing different aspects of the strong force compared with the NN interaction. Moreover, the MN system is closely tied to exotic hadron spectroscopy: $J/\psi p$ is the discovery channel of the hidden-charm pentaquark states P_c [2,3], and the ϕp channel has been proposed to host their strange analogues [4,5].

The meson-nucleus bound states serve as a testing ground to study the MN interaction. Brodsky *et al.* [1] first predicted η_c -nucleus bound states for $A \geq 3$ using the

QCD van der Waals force picture [6]. Subsequent works adopted various frameworks [7–19], yielding widely divergent predictions for the binding energies. Although no meson-nucleus bound state has been observed experimentally, photoproduction cross sections of $\gamma p \rightarrow \phi p$ [20,21], $\gamma p \rightarrow J/\psi p$ [22–24], and J/ψ off light nuclei [25], together with the p - ϕ correlation function [26], have been measured. Analyses of these data support attractive ϕN and $J/\psi N$ interactions, yet the extracted two-body scattering lengths span several orders of magnitude depending on the analysis framework [27–33]. The discrepancies in the predicted spectrum and two-body scattering lengths call for a systematic, first-principles study of meson-nucleus bound states, which is crucial for understanding the underlying dynamics and guiding future experimental searches.

Recently, HAL QCD Collaboration extracted the $\phi N({}^4S_{3/2})$ [34] and charmonium- N [35] potentials from lattice QCD at near-physical pion mass, providing first-principles input for calculations of meson-nucleus bound states. Some subsequent studies have employed these potentials [36–39], but they are restricted to three-body systems or rely on α -cluster approximations, introducing uncontrolled model uncertainty. A full dynamical treatment of meson-nucleus systems across light nuclei is still missing. Such calculations are challenging, as they require solving the $(A + 1)$ -body Schrödinger equation with short-range interactions while accommodating possible halo meson configurations.

* Contact author: wlwu@pku.edu.cn

† Contact author: wenlzh_hep-th@stu.pku.edu.cn

‡ Contact author: zhushl@pku.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

The neural-network-based variational Monte Carlo (NN-VMC) approach is well suited to such a many-body problem. Owing to the capacity of deep neural networks to approximate high-dimensional functions, neural-network quantum states have emerged as a powerful many-body wave-function ansatz [40]. The VMC approach optimizes the network parameters by minimizing the energy expectation value, so that the trial wave function converges to the ground state. NN-VMC has been successfully applied to solve electronic structures [41,42], nuclear ground states [43–46], hypernuclei [47,48], and multi-quark states [49], reaching accuracies competitive with or surpassing established many-body methods, with computational costs that scale polynomially with the particle number. These features enable accurate solutions of meson-nucleus bound states without dynamical approximations commonly used to simplify many-body calculations.

In this work, we carry out the first calculations of ϕ -, η_c -, and J/ψ -nucleus ground states up to $A = 12$ based on the HAL QCD interactions using NN-VMC framework. We first benchmark the framework on light nuclei from ${}^2\text{H}$ to ${}^{12}\text{C}$, then perform a systematic study of the meson-nucleus bound states, examining their binding energies, spatial structures, and spin splittings, and discussing how the results reflect the underlying meson-nucleon dynamics.

II. HAMILTONIAN

The nuclear interactions in this work are described by a leading-order pionless effective field theory (LO $\not\equiv$ EFT) [50], which assumes that nucleon momenta are well below the pion mass, so that pion exchange is absorbed into short-range contact interactions.

$$V_N = \sum_{i<j} (v_{ij}^{\text{CI}} + v_{ij}^{\text{EM}}) + \sum_{i<j<k} V_{ijk}. \quad (1)$$

The charge-independent (CI) two-nucleon potential takes the form

$$v^{\text{CI}} = C_{01}C_1(r)P_0^\sigma P_1^\tau + C_{10}C_0(r)P_1^\sigma P_0^\tau, \quad (2)$$

where P_S^σ (P_T^τ) is the spin (isospin) projector onto total spin (isospin) S (T), and C_{01} , C_{10} are the low-energy constants (LECs) for the 1S_0 and 3S_1 channels, respectively. $C_i(r) = \frac{1}{\pi^{3/2}R_i^3} e^{-(r/R_i)^2}$ is the Gaussian regularized function. The electromagnetic term includes only the finite-size proton-proton Coulomb repulsion [51],

$$v^{\text{EM}}(r_{ij}) = \alpha \frac{F_c(r_{ij}) (1 + \tau_{iz})(1 + \tau_{jz})}{r_{ij}}, \quad (3)$$

where α is the fine-structure constant, and the isospin projector restricts the interaction to proton-proton pairs.

The finite-size form factor is

$$F_c(r) = 1 - \left(1 + \frac{11}{16}br + \frac{3}{16}(br)^2 + \frac{1}{48}(br)^3\right)e^{-br}, \quad (4)$$

with $b = 4.27 \text{ fm}^{-1}$. The three-nucleon force is a Gaussian contact term,

$$V_{ijk} = \frac{c_E}{f_\pi^4 \Lambda_\chi} \frac{(\hbar c)^6}{\pi^3 R_3^6} \sum_{\text{cyc}} e^{-(r_{ij}^2 + r_{jk}^2)/R_3^2}. \quad (5)$$

All parameters are taken from the model-“o” fit of Ref. [52], where the LECs and cutoff parameters were fitted to reproduce the np effective-range expansion and the binding energies of ${}^2\text{H}$ and ${}^3\text{H}$.

The meson-nucleon interactions are described by the HAL QCD potential at $m_\pi = 146 \text{ MeV}$ [34,35]. The J/ψ - N and η_c - N potentials are parametrized as sums of three Gaussian functions,

$$\begin{aligned} V_{J/\psi N}^{4S_{3/2}}(r) &= \sum_{i=1}^3 a_{i1} e^{-(r/b_{i1})^2}, \\ V_{J/\psi N}^{2S_{1/2}}(r) &= \sum_{i=1}^3 a_{i2} e^{-(r/b_{i2})^2}, \\ V_{\eta_c N}^{2S_{1/2}}(r) &= \sum_{i=1}^3 a_{i3} e^{-(r/b_{i3})^2}. \end{aligned} \quad (6)$$

The ϕ - N potential in the ${}^4S_{3/2}$ channel includes two Gaussian functions and a two-pion exchange tail,

$$V_{\phi N}^{4S_{3/2}}(r) = \sum_{i=1}^2 \alpha_i e^{-(r/\beta_i)^2} + \alpha_3 m_\pi^4 f(r, \beta_3) \left(\frac{e^{-m_\pi r}}{r} \right)^2. \quad (7)$$

In the ${}^2S_{1/2}$ ϕ - N channel, no direct lattice QCD potential is available due to the complex coupled-channel effects. We construct the potential by assuming the meson-nucleon

TABLE I. The hadron masses in lattice configurations with $m_\pi = 146 \text{ MeV}$ [34,35], together with the corresponding experimental values [53]. For charmonia, Ref. [35] uses two sets of parameters to interpolate the physical charm quark mass, and we therefore adopt the experimental masses for J/ψ and η_c in this work.

Hadron	This work (MeV)	Expt. (MeV)
π	146	138.0
N	954	938.9
ϕ	1048	1019.5
J/ψ	3096	3096.9
η_c	2984	2984.1

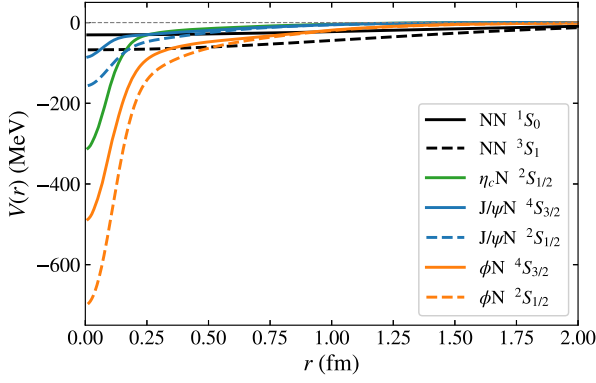


FIG. 1. The NN potentials in 1S_0 and 3S_1 channels of the LO χ EFT (black), and the HAL QCD meson-nucleon potentials for η_c - N (green), J/ψ - N (blue), and ϕ - N (orange) in the $^4S_{3/2}$ (solid) and $^2S_{1/2}$ (dashed) channels.

potential can be decomposed as

$$V_{MN}(r) = V^c(r) + V^s(r)\vec{S}_M \cdot \vec{S}_N, \quad (8)$$

where V^c and V^s are the central and spin-spin components. We further assume that the spin-dependent interactions in the J/ψ - N and ϕ - N potentials are inversely proportional to the meson mass. The $^2S_{1/2}$ ϕ - N potential is then obtained as [37]

$$V_{\phi N}^s(r) = \frac{m_{J/\psi}}{m_\phi} V_{J/\psi N}^s(r) = \frac{m_{J/\psi}}{m_\phi} \frac{2(V_{J/\psi N}^{4S_{3/2}} - V_{J/\psi N}^{2S_{1/2}})}{3},$$

$$V_{\phi N}^{2S_{1/2}}(r) = V_{\phi N}^{4S_{3/2}}(r) - \frac{3}{2} V_{\phi N}^s(r). \quad (9)$$

The full Hamiltonian reads,

$$H = \sum_i \frac{-\nabla_i^2}{2m_N} + \frac{-\nabla_M^2}{2m_M} + V_N + V_{MN}, \quad (10)$$

where m_N and m_M are the nucleon and meson masses in lattice configurations with $m_\pi = 146$ MeV, as listed in Table I. It is worth noting that they lie close to the physical values. The two-body NN and MN potentials are shown in Fig. 1.

III. NEURAL-NETWORK WAVE FUNCTION

The neural-network trial wave function adopts the Slater-Jastrow form. A Deep Sets module [43,54,55] constructs a permutation-invariant Jastrow correlation factor. A message-passing neural network (MPNN) [56,57] generates backflow-transformed nucleonic and mesonic orbitals that encode many-body correlations. Slater determinants of the nucleonic orbitals enforce antisymmetry, while the mesonic orbitals enter as additional factors. The architecture is illustrated in Fig. 2.

The inputs of the neural networks are constructed as one-particle and two-particle features $\mathbf{x}_i = (\vec{r}_i, s_i, t_i)$, $\mathbf{x}_{ij} = (\mathbf{r}_{ij}, r_{ij}, s_i, s_j, t_i, t_j)$, where $\vec{r}_i$ is the nucleon coordinate in the center-of-mass (c.m.) frame, $\mathbf{r}_{ij} = \vec{r}_i - \vec{r}_j$, $r_{ij} = |\mathbf{r}_{ij}|$, and s_i, t_i are the nucleon spin and isospin projections along the z axis. Mesonic features are defined analogously. Working in the c.m. frame ensures translational invariance of the wave function and removes the c.m. motion.

The permutation-invariant Jastrow correlation factor is written in the Deep Sets form,

$$J = \rho\left(\sum_{i \neq j} u_{NN}(\mathbf{x}_{ij}), \sum_i u_{MN}(\mathbf{x}_{iM})\right), \quad (11)$$

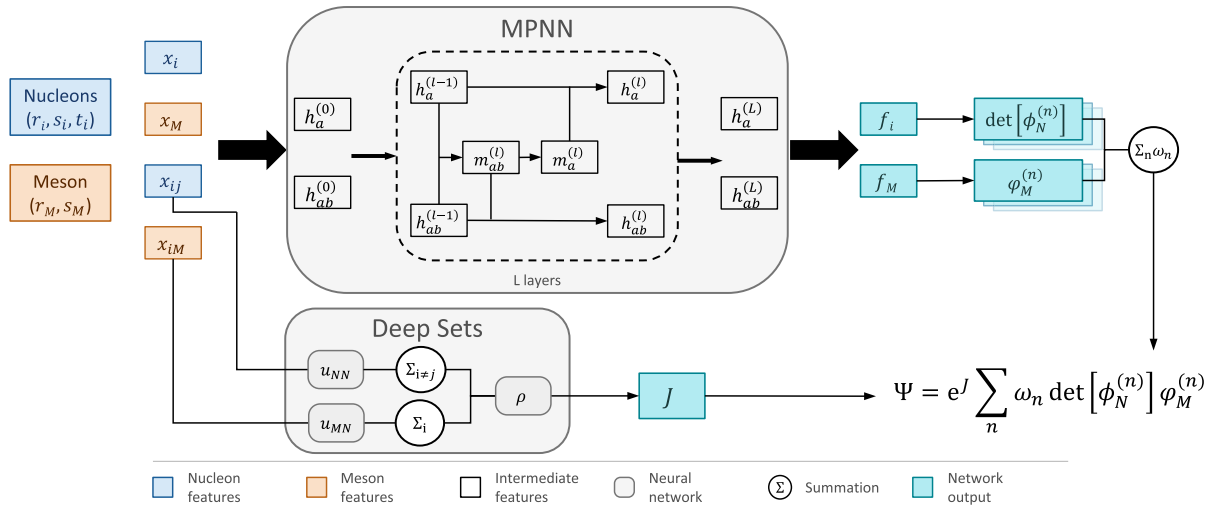


FIG. 2. Architecture of the meson-nucleus neural-network wave function. An MPNN produces many-body dressed features f_i and f_M , which are further transformed into nucleonic and mesonic orbitals ϕ_N and ϕ_M . The Deep Sets module gives a permutation-invariant Jastrow factor J . The wave function is constructed in the Slater-Jastrow form.

where u_{NN} and u_{MN} are residual networks [58] embedding two-particle features into a latent space $\mathbb{R}^F$, and $\rho: \mathbb{R}^{2F} \rightarrow \mathbb{R}$ decodes the pooled features into a scalar Jastrow.

The backflow transformation [41,42,45,59] generalizes the single-particle orbitals to functions depending on the features of all particles $\phi_{i\mu}(\mathbf{x}_1, \dots, \mathbf{x}_A, \mathbf{x}_M)$, encoding inter-particle correlations within the determinant. Antisymmetry requires that nucleon exchange $i \leftrightarrow j$ swaps the corresponding orbital rows $\phi_{i\mu}$ and $\phi_{j\mu}$, while leaving the remaining rows unchanged, thereby flipping the sign of $\det \phi$. The construction proceeds in two stages.

First, an MPNN maps the input features $\mathbf{x}_i, \mathbf{x}_M, \mathbf{x}_{ij}, \mathbf{x}_{iM}$ into many-body dressed single-particle features $\mathbf{f}_i$ and $\mathbf{f}_M$. Linear transformations $\mathbf{h}^{(0)} = W\mathbf{x} + \mathbf{b}$ project the input features into hidden features of dimension F , with W and $\mathbf{b}$ being learnable parameters. The hidden features are then updated as

$$\begin{aligned} \mathbf{m}_{ab}^{(l)} &= M^{(l)}(\mathbf{h}_a^{(l-1)}, \mathbf{h}_b^{(l-1)}, \mathbf{h}_{ab}^{(l-1)}), \\ \mathbf{m}_a^{(l)} &= \sum_{b \neq a} \mathbf{h}_b^{(l-1)} \odot \mathbf{m}_{ab}^{(l)}, \\ \mathbf{h}_a^{(l)} &= \mathbf{h}_a^{(l-1)} + U^{(l)}(\mathbf{h}_a^{(l-1)}, \mathbf{m}_a^{(l)}), \\ \mathbf{h}_{ab}^{(l)} &= \mathbf{h}_{ab}^{(l-1)} + V^{(l)}(\mathbf{h}_{ab}^{(l-1)}, \mathbf{m}_{ab}^{(l)}). \end{aligned} \quad (12)$$

Here, $\mathbf{m}_{ab}^{(l)}, \mathbf{m}_a^{(l)} \in \mathbb{R}^F$ are the message features, $M^{(l)}, U^{(l)}, V^{(l)}$ are fully connected neural networks, $\odot$ denotes the Hadamard product, and the indices a, b run over the $A + 1$ particles. After L iterations, the many-body dressed features $\mathbf{f}_i$ and $\mathbf{f}_M$ are constructed as

$$\mathbf{f}_a = \mathbf{h}_a^{(L)} + \sum_{b=1}^{A+1} \lambda(\mathbf{h}_b^{(L)}), \quad (13)$$

with λ being a residual network. This permutation-equivariant MPNN structure ensures that the nucleon exchange $i \leftrightarrow j$ amounts to permuting of output features $\mathbf{f}_i \leftrightarrow \mathbf{f}_j$, while encoding many-body correlations through the message passing.

Second, the many-body dressed features are mapped into N_{det} sets of nucleonic and mesonic orbitals,

$$\begin{aligned} \phi_{i\mu}^{(n)} &= O_N^{(n)}(\mathbf{f}_i)_\mu \exp(-\xi^{(n)2} \tilde{r}_i^2), \\ \phi_M^{(n)} &= O_M^{(n)}(\mathbf{f}_M) \exp\left(-\sum_{\alpha=1}^3 \eta_\alpha^{(n)2} \tilde{r}_{M,\alpha}^2\right), \end{aligned} \quad (14)$$

where $O_N^{(n)}$ and $O_M^{(n)}$ are residual networks, and the Gaussian envelopes ensure normalizability of the wave function. The learnable decay rates ξ and η adapt to the typical size of each system during training, with the mesonic envelope kept anisotropic to accommodate nonspherical tails of the meson distribution [48].

Combining all components, the trial wave function reads

$$\Psi = e^J \sum_{n=1}^{N_{\text{det}}} w_n \det[\phi^{(n)}] \phi_M^{(n)}, \quad (15)$$

where w_n are learnable mixing coefficients. Multiple determinants are found to be essential for accurately describing larger nuclei with nontrivial shell structure [44]. Moreover, the z components of the total spin S and isospin T are fixed as $S_z = S$ and $T_z = T$. All neural networks in the architecture use tanh as the activation function. The pure-nucleon ansatz used for the bare nuclei is recovered by removing the meson inputs $\mathbf{x}_M, \mathbf{x}_{iM}$ and the meson orbital $\phi_M^{(n)}$.

The network parameters are optimized variationally by minimizing the loss function $\mathcal{L} = \langle H \rangle$, evaluated with Monte Carlo samples drawn from $|\Psi|^2$ via the Metropolis-Hastings algorithm. The parameters are then updated using the stochastic reconfiguration method [60]. Further details on the training procedure can be found in Ref. [49]. The codes are developed on top of the NetKet package [61,62].

IV. RESULTS AND DISCUSSION

We first benchmark the NN-VMC framework on the nuclei ${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$, ${}^6\text{Li}$, ${}^8\text{Be}$, and ${}^{12}\text{C}$. The ground-state energies, computed with both the lattice nucleon mass $m_N = 954$ MeV and the physical value $m_N = 938.9$ MeV, together with total-spin expectation values are collected in Table II. The statistical errors of the energies are below 0.1%, demonstrating the high numerical precision of the framework. The wave functions reproduce the exact spin quantum numbers $\langle S^2 \rangle = S(S+1)$ for ${}^2\text{H}$, ${}^6\text{Li}$ ($S=1$), ${}^3\text{H}$, ${}^3\text{He}$ ($S=1/2$), and ${}^4\text{He}$, ${}^8\text{Be}$, ${}^{12}\text{C}$ ($S=0$). With the physical nucleon mass, the energies for $A \leq 4$ are in close agreement with the experiment, while ${}^6\text{Li}$ and ${}^{12}\text{C}$ show a slight under-binding. This under-binding is a known feature of the LO $\not\equiv$ EFT, which produces excessive repulsion in p -shell systems [46]. Our calculation gives $E({}^8\text{Be}) > 2E({}^4\text{He})$, which is in agreement with the unbound nature of ${}^8\text{Be}$.

TABLE II. Nuclear ground-state energies and S^2 expectation values. E uses the lattice nucleon mass $m_N = 954$ MeV, while E_{phys} uses the physical $m_N = 938.9$ MeV. Statistical errors are shown in parentheses. Experimental values from [63] are listed for comparison.

Nucleus	E (MeV)	E_{phys} (MeV)	$\langle S^2 \rangle$	Exp. (MeV)
${}^2\text{H}$	-2.416(0)	-2.240(1)	2.00	-2.224
${}^3\text{H}$	-8.921(1)	-8.470(3)	0.75	-8.482
${}^3\text{He}$	-8.250(1)	-7.808(2)	0.75	-7.718
${}^4\text{He}$	-29.072(1)	-28.161(3)	0.00	-28.30
${}^6\text{Li}$	-32.478(4)	-31.134(12)	2.01	-31.99
${}^8\text{Be}$	-57.873(7)	-55.689(26)	0.02	...
${}^{12}\text{C}$	-91.621(16)	-87.724(28)	0.03	-92.16

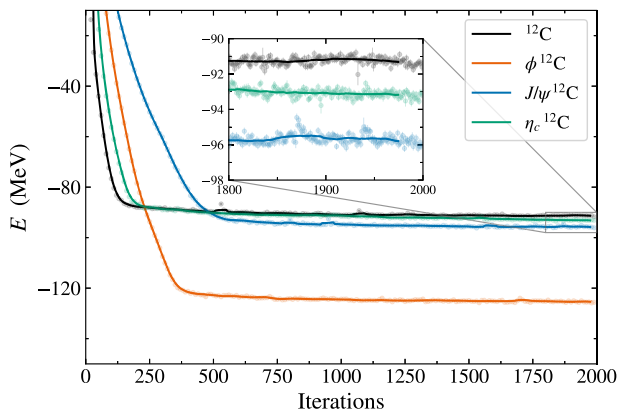


FIG. 3. Energy convergence for meson- ^{12}C system. Scatter points with error bars represent the raw VMC energy estimates and their Monte Carlo statistical uncertainties; solid curves are 50-step moving averages, which suppress stochastic fluctuations and highlight the convergence trend.

Since the two-cluster scattering state cannot be well described by our trial wave function designed for bound states, the ground-state energy stalls slightly above the threshold. The lattice results are systematically lower than the physical ones, because the heavier nucleon mass reduces the kinetic energy and deepens the binding.

We now turn to the meson-nucleus bound states. Energy convergence of the training process is shown in Fig. 3, taking $M^{12}\text{C}$ ($M = \phi, J/\psi, \eta_c$) as an example. The ground-state binding energies $E_B = E_{M-A} - E_A$, total-spin expectation values, and root-mean-square (rms) radii are collected in Table III. For each bound state two uncertainties are quoted: the parenthetical σ_{stat} is the VMC statistical error, while $\pm\sigma_V$ denotes the uncertainty propagated from the HAL QCD potential parameters (see Appendix A for details).

None of the three mesons forms a bound state with a single nucleon. As A increases, the meson-nucleus attraction grows stronger and bound states emerge. The ϕ - A systems are already bound at $A = 2$, with E_B deepening from 1.4 MeV at $A = 2$ to 34 MeV at $A = 12$. The J/ψ - A systems are shallowly bound for $A \geq 4$. The existence of the η_c - ${}^4\text{He}$ bound state remains inconclusive given the potential uncertainty; η_c - A binding is firmly established only for $A \geq 6$, and even η_c - ${}^{12}\text{C}$ is bound by merely ~ 2 MeV. The hierarchy $|E_B^\phi| \gg |E_B^{J/\psi}| > |E_B^{\eta_c}|$ reflects the relative strengths of the meson-nucleon interactions, consistent with the ordering of the scattering lengths $|a_0^{\phi N}| \gg |a_0^{J/\psi N}| > |a_0^{\eta_c N}|$ [34,35]. It is noteworthy that, although bare ${}^8\text{Be}$ is an unbound resonance above the $\alpha + \alpha$ threshold, the meson-nucleon attraction is strong enough to stabilize all three M - ${}^8\text{Be}$ systems below $2E({}^4\text{He})$.

The spatial configurations of the meson-nucleus systems are characterized by the rms radii in Table III and illustrated by the representative ϕ - and η_c -nucleus density distributions

TABLE III. Meson-nucleus ground-state binding energies E_B (in MeV), S^2 expectation values and rms radii (in fm). r_{NN}^{bare} is the rms internucleon distance of the bare parent nucleus. For each bound state, σ_{stat} (compact parenthetical on last digit) is the VMC statistical error and $\pm\sigma_V$ is the potential-parameter uncertainty. E_B for ${}^8\text{Be}$ systems are referenced to $2E({}^4\text{He})$ since bare ${}^8\text{Be}$ lies above the $\alpha + \alpha$ threshold.

Nucleus	ϕ -A				J/ψ -A				η_c -A			
	E_B	$\langle S^2 \rangle$	r_{MN}	r_{NN}	E_B	$\langle S^2 \rangle$	r_{MN}	r_{NN}	E_B	$\langle S^2 \rangle$	r_{MN}	r_{NN}
^2H	$-1.400(5) \pm 0.640$	0.01	4.09	2.83	>0				>0			3.71
^3H	$-7.342(8) \pm 1.604$	0.75	2.42	2.28	>0				>0			2.85
^4He	$-7.236(6) \pm 1.609$	0.75	2.45	2.30	>0				>0			2.90
^6He	$-17.010(8) \pm 2.737$	2.00	1.97	1.99	$-1.059(3) \pm 0.294$	2.00	3.22	2.24	$-0.041(2) \pm 0.077$	0.00	7.04	2.28
^7Li	$-19.952(13) \pm 2.991$	0.01	2.58	2.97	$-1.814(5) \pm 0.377$	0.02	3.45	3.50	$-0.344(5) \pm 0.133$	2.01	4.89	3.69
^8Be	$-23.576(11) \pm 3.402$	2.02	2.77	3.11	$-2.632(9) \pm 0.455$	2.01	3.49	3.65	$-0.726(7) \pm 0.193$	0.02	4.15	3.81
^{12}C	$-34.00(5) \pm 4.30$	2.03	2.73	3.18	$-4.49(3) \pm 0.93$	2.07	3.15	3.47	$-2.06(3) \pm 0.46$	0.03	3.38	3.51

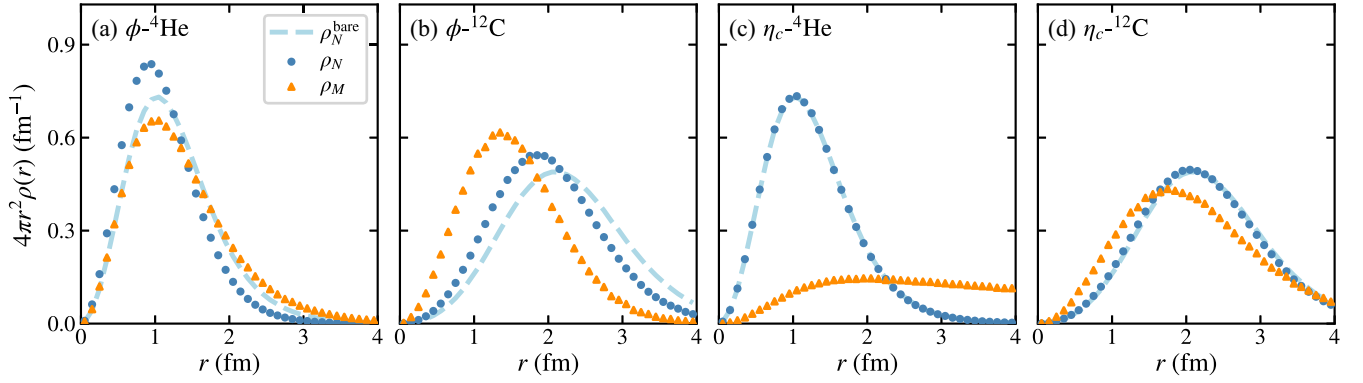


FIG. 4. Radial densities for (a) ϕ - ^4He , (b) ϕ - ^{12}C , (c) η_c - ^4He , and (d) η_c - ^{12}C . Each panel shows the nucleon radial density (blue circles), meson radial density (orange triangles) of the meson-nucleus system, and the nucleon density of the corresponding parent nucleus (blue dashed line).

in Fig. 4. The nucleon and meson radial densities are defined as

$$\rho_N(r) = \frac{1}{4\pi r^2} \frac{\langle \Psi | \sum_i \delta(|\mathbf{r}_i - \mathbf{R}_n| - r) | \Psi \rangle}{A \langle \Psi | \Psi \rangle},$$

$$\rho_M(r) = \frac{1}{4\pi r^2} \frac{\langle \Psi | \delta(|\mathbf{r}_M - \mathbf{R}_n| - r) | \Psi \rangle}{\langle \Psi | \Psi \rangle}, \quad (16)$$

where $\mathbf{R}_n$ is the nuclear c.m. coordinate. The meson-nucleon rms radii follow the ordering $r_{\phi N} < r_{J/\psi N} < r_{\eta_c N}$ in all systems. In the M - ^4He systems, the ϕ is embedded inside the nucleus with $r_{\phi N} \lesssim r_{NN}$, and its density peak coincides with the nucleon density peak; the η_c instead forms a halo configuration in which the meson is broadly distributed with a long tail extending well beyond the nucleon distribution. As A increases, the mesons move closer to the nuclear interior, as can be seen by comparing Figs. 4(a) and (c) with Figs. 4(b) and (d). The effect is most striking for the η_c , which evolves from a halo configuration at $A = 4$ to a configuration contracted inside the nucleus at $A = 12$. The meson also acts as a “glue” analogous to the Λ hyperon in light hypernuclei, pulling the nucleons closer to each other [39,47]: in every meson-nucleus bound state r_{NN} is smaller than r_{NN}^{bare} of the parent nucleus. This contraction is most pronounced in the deeply bound ϕ - A systems: even for the tightly bound ^4He , the ϕ meson visibly compresses the nucleon distribution, as shown in Fig. 4(a), demonstrating that treating α as an inert cluster would miss sizable polarization effects. In the weakly bound states, by contrast, the nucleon distribution remains close to that of the parent nucleus, as seen in Figs. 4(c) and 4(d). The density distributions of other meson-nucleus systems are provided in Appendix B.

Figure 5 shows the meson-nucleus binding energy per nucleon E_B/A , which measures the average meson-nucleon interaction strength. For the charmonium-nucleus systems, E_B/A deepens nearly linearly with A from $A = 4$ to $A = 12$. By contrast, the ϕ - A systems exhibit an interesting

behavior: E_B/A deepens linearly for $A \leq 4$, and then decreases in magnitude toward a saturated value of ~ -2.8 MeV. These trends can be understood from the spatial structure discussed above. Since all three meson-nucleon potentials are dominated by the short-range interaction at $r \lesssim 0.5$ fm, as shown in Fig. 1, the interaction strength is largely determined by the overlap between the meson and nucleon distributions. In the J/ψ - A and η_c - A systems, the meson evolves from outside the nucleus toward its interior as A increases, so that the overlap and the interaction strength grow accordingly. The same holds for the ϕ - A systems at $A \leq 4$. For $A \geq 6$, however, the ϕ meson already resides inside the nucleus, while the nuclear volume keeps growing with A ; this dilutes the nucleon

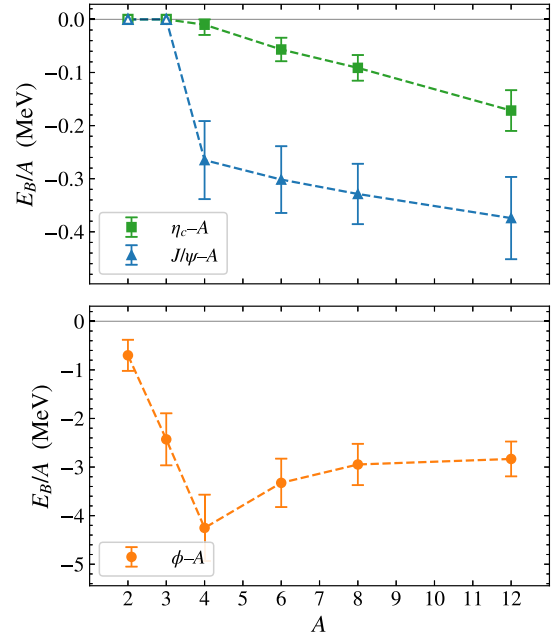


FIG. 5. Meson-nucleus binding energy per nucleon E_B/A as a function of nuclear mass number A for the η_c - A , J/ψ - A (top) and ϕ - A (bottom) systems.

TABLE IV. Binding energies and rms radii for J/ψ - ${}^6\text{Li}$, ϕ - ${}^6\text{Li}$, and ϕ - ${}^2\text{H}$ in different spin channels, evaluated with the central values of the meson-nucleon potential parameters.

System	$\langle S^2 \rangle$	E_B (MeV)	r_{NN} (fm)	r_{MN} (fm)
J/ψ - ${}^6\text{Li}$	0.02	-1.815(6)	3.50	3.45
	2.01	-1.816(6)	3.53	3.48
	6.00	-1.739(6)	3.57	3.54
ϕ - ${}^6\text{Li}$	0.01	-19.952(13)	2.97	2.58
	2.01	-19.711(12)	3.01	2.59
	6.01	-19.169(12)	3.08	2.65
ϕ - ${}^2\text{H}$	0.01	-1.400(5)	2.83	4.09
	2.00	-0.807(4)	3.00	4.76
	6.00	-0.062(2)	3.31	6.69

density and weakens the interaction strength. The maximum of E_B/A therefore lies at ${}^4\text{He}$, the most compact light nucleus, which offers the largest meson-nucleon overlap. This nonmonotonic behavior is a distinctive signature of the short-range, strongly attractive character of the ϕN interaction.

Finally, we examine the spin splittings of the meson-nucleus bound states, specifically the ϕ - ${}^2\text{H}$, ϕ - ${}^6\text{Li}$, and J/ψ - ${}^6\text{Li}$ systems. In systems with near-degenerate spin states, the optimized neural-network wave function may not be an eigenstate of the total-spin operator S^2 , a phenomenon known as spin contamination [64]. To overcome this, we augment the loss function with a spin penalty term,

$$\mathcal{L} = \langle H \rangle \rightarrow \langle H \rangle + \omega \langle S^2 \rangle, \quad \omega = 1 \text{ MeV}. \quad (17)$$

The penalty term effectively enlarges the spin splittings, thereby suppressing the contributions from higher-spin states $S > S_z$ and ensuring that the wave function converges to the $S = S_z$ sector. The resulting binding energies and radii in the different spin channels are collected in Table IV.

In the J/ψ - ${}^6\text{Li}$ system, the $S = 0$ and $S = 1$ states are degenerate within statistical precision, and the $S = 2$ state lies only ~ 77 keV above. The near-degeneracy follows from the geometry: the typical range of spin-dependent $V_{J/\psi N}^s$ is less than 0.5 fm, whereas $r_{MN} \approx 3.5$ fm places the meson well outside its range, strongly suppressing $\langle V_{J/\psi N}^s \rangle$. On the other hand, $V_{\phi N}^s$ in Eq. (9) is enhanced by the mass ratio $m_{J/\psi}/m_\phi \approx 2.95$, and the smaller $r_{MN} \approx 2.6$ fm weakens this geometric suppression. As a result, the ϕ - ${}^6\text{Li}$ system shows a larger splitting of several hundred keV. The spin splitting of ϕ - ${}^2\text{H}$ states is the largest among the three systems, and the wave-function configurations in different channels are substantially altered, as reflected by the large variation of r_{MN} in Table IV.

V. CONCLUSIONS AND OUTLOOKS

We carry out the first systematic calculations of ϕ -, η_c -, and J/ψ -nucleus ground states up to mass number $A = 12$, using HAL QCD meson-nucleon potentials at near-physical pion mass [34,35] and an LO $\not\equiv$ EFT description of the nuclear interaction [52]. The $\phi N(^2S_{1/2})$ interaction unavailable in lattice results is obtained by a model of spin-dependent interactions. The many-body Schrödinger equation is solved using the NN-VMC framework, with a Slater-Jastrow neural-network quantum state generalized to incorporate mesonic degrees of freedom. The framework is benchmarked on light nuclei from ${}^2\text{H}$ to ${}^{12}\text{C}$, yielding ground-state energies consistent with experiment. In meson-nucleus systems, we observe the hierarchy of mesonic binding energies $|E_B^\phi| \gg |E_B^{J/\psi}| > |E_B^{\eta_c}|$, and obtain the critical mass number for bound state formation: ϕ -nucleus are bound for $A \geq 2$, J/ψ for $A \geq 4$, and η_c for $A \geq 6$, with η_c - ${}^4\text{He}$ inconclusive under potential uncertainties. Interestingly, E_B/A deepens nearly linearly with A for the charmonium systems, whereas the ϕ -nucleus system exhibits a nonmonotonic behavior peaking at ${}^4\text{He}$ —a hallmark of the short-range and strongly attractive ϕN interaction. We also compute the rms radii and density distributions to investigate the spatial structures. With increasing A , the meson evolves from a halo to a configuration embedded inside the nucleus. The meson also acts as a “glue,” compressing the nucleon distribution. Moreover, the spin splittings of J/ψ - ${}^6\text{Li}$, ϕ - ${}^6\text{Li}$, and ϕ - ${}^2\text{H}$ systems are found to be relatively small, which is another characteristic of the short-range meson-nucleon interaction. Since the spin-dependent ϕN interaction has not yet been determined in lattice QCD and is obtained from model assumptions, experimental measurements of ϕ -nucleus spin splittings would directly constrain this interaction.

Our calculations are performed at near-physical point, and the uncertainty from extrapolating to physical values is expected to be small. Future experiments at J-PARC [65], Jefferson Lab [66], and FAIR [67] may provide the first probes of meson-nucleus bound states [68,69]. A quantitative comparison between theoretical predictions and experimental data will shed light on the underlying meson-nucleon dynamics.

ACKNOWLEDGMENTS

We are grateful to Yan Lyu for providing HAL QCD potential data and the helpful discussions on the uncertainty analysis. We also thank Lu Meng for the helpful discussions. This project was supported by the National Natural Science Foundation of China (Grants No. 12475137 and No. 125B2105). The computational resources are supported by High-Performance Computing Platform of Peking University.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: UNCERTAINTY OF BINDING ENERGIES

In this work, two types of uncertainty for the meson-nucleus binding energies are reported: the VMC statistical error σ_{stat} , and the uncertainty propagated from the HAL QCD meson-nucleon potential parameters σ_V .

In the VMC method, the energy expectation value $\langle E \rangle$ is estimated as the sample mean $\bar{E} = \frac{1}{N} \sum_{t=1}^N E_{L,t}$ over N Monte Carlo configurations, where $E_{L,t} = \Psi^{-1} H \Psi$ is the local energy evaluated on configuration t . The statistical error measures the numerical error due to finite and random Monte Carlo sampling. It is estimated using the blocking method. The N samples are divided into m batches of length l , and the batch-average energies are

$$\bar{E}^i = \frac{1}{l} \sum_{t=(i-1)l+1}^{il} E_{L,t}, \quad i = 1, \dots, m. \quad (\text{A1})$$

The statistical error of $\bar{E}$ is defined as

$$\sigma_{\text{stat}} = \frac{\sqrt{\text{var}(\bar{E})}}{\sqrt{m}} = \sqrt{\frac{1}{m^2} \sum_{i=1}^m (\bar{E}^i - \bar{E})^2}, \quad (\text{A2})$$

where $\text{var}(\bar{E})$ is the variance of batch means. The batch size l is chosen sufficiently large that the batch means are effectively uncorrelated. In this work, all statistical errors are below 0.1% of the corresponding energy values, confirming that the Monte Carlo sample sizes are adequate for precise estimates.

The HAL QCD meson-nucleon potentials [34,35] are obtained by fitting the parametrized forms [Eqs. (6) and (7)] to lattice QCD data. The fitting uncertainties are estimated via the jackknife method: N_{jk} sets of reduced ensemble are generated by removing subsets of gauge configurations, and the fit is repeated to yield N_{jk} sets of potential parameters $\{p^{(k)}\}_{k=1}^{N_{\text{jk}}}$. The central parameters and the uncertainties are given by

$$\bar{p} = \frac{1}{N_{\text{jk}}} \sum_{k=1}^{N_{\text{jk}}} p^{(k)},$$

$$\sigma_p = \sqrt{\frac{N_{\text{jk}} - 1}{N_{\text{jk}}} \sum_{k=1}^{N_{\text{jk}}} (p^{(k)} - \bar{p})^2}. \quad (\text{A3})$$

The neural-network quantum state is optimized to the ground state of $H(\bar{p})$. To estimate the uncertainties propagated to the meson-nucleus binding energies, we evaluated the energy on

each jackknife parameter set $p^{(k)}$ using the optimized neural-network wave function Ψ :

$$E^{(k)} = \frac{\langle \Psi | H(p^{(k)}) | \Psi \rangle}{\langle \Psi | \Psi \rangle}, \quad k = 1, \dots, N_{\text{jk}}. \quad (\text{A4})$$

The uncertainty is then given by the jackknife error estimate,

$$\sigma_V = \sqrt{\frac{N_{\text{jk}} - 1}{N_{\text{jk}}} \sum_{k=1}^{N_{\text{jk}}} (E^{(k)} - \bar{E}_{\text{jk}})^2}, \quad \bar{E}_{\text{jk}} = \frac{1}{N_{\text{jk}}} \sum_{k=1}^{N_{\text{jk}}} E^{(k)}. \quad (\text{A5})$$

This approach assumes that the wave function does not change appreciably under the potential variations, which is justified by the small uncertainty of the potentials. We have also verified this assumption by performing full reoptimizations for selected jackknife parameter sets, finding consistent results with the estimates. The jackknife method automatically preserves the correlation among the fit parameters, whereas a naïve analysis assuming independent parameter distribution would overestimate the uncertainty by a large factor. The $\phi N(^2S_{1/2})$ potential in Eq. (9) depends on parameters of both $\phi N(^4S_{3/2})$ and $J/\psi N$ potential. For simplicity, the jackknife uncertainty is evaluated by varying only the $\phi N(^4S_{3/2})$ parameters, while keeping the $J/\psi N$ parameters at their central values. The N_{jk} sets of parameters are provided by the authors of Refs. [34,35].

In addition to the uncertainty, we also briefly discuss the variance of the local energy,

$$\sigma^2 = \frac{\langle \Psi | (H - \langle E \rangle)^2 | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \frac{1}{N} \sum_{t=1}^N (E_{L,t} - \bar{E})^2. \quad (\text{A6})$$

The variance measures the fluctuation of E_L over the Monte Carlo samples. For an exact eigenstate, E_L is constant in the configuration space and $\sigma^2 = 0$; a small σ^2 therefore indicates that the wave function closely approximates a true eigenstate.

In the present calculations, we find that the short-range, strongly attractive meson-nucleon potentials—particularly the ϕN interaction—give rise to a large variance. This is a consequence of the sampling method: the Monte Carlo samples are distributed by $|\Psi|^2$, and the short-range sampling is suppressed by the r^2 factor in the radial volume element $d^3r = 4\pi r^2 dr$. As a result, the wave function is less thoroughly optimized in the short-range regions [70], where the meson-nucleon potential is strongest, and E_L may deviate substantially from $\langle E \rangle$. These few deviant short-range samples result in a large variance. Nevertheless, the short-range contributions to the expectation values are also suppressed by the r^2 geometric factor in the integral. Therefore, the binding energies reported in this work are not appreciably biased by the inflated variance and remain reliable.

APPENDIX B: MESON-NUCLEUS DENSITY DISTRIBUTION

Figure 6 shows the radial density distributions of the M - ^4He , M - ^6Li , M - ^8Be , and M - ^{12}C systems. With increasing A , all three mesons move closer to the nuclear interior.

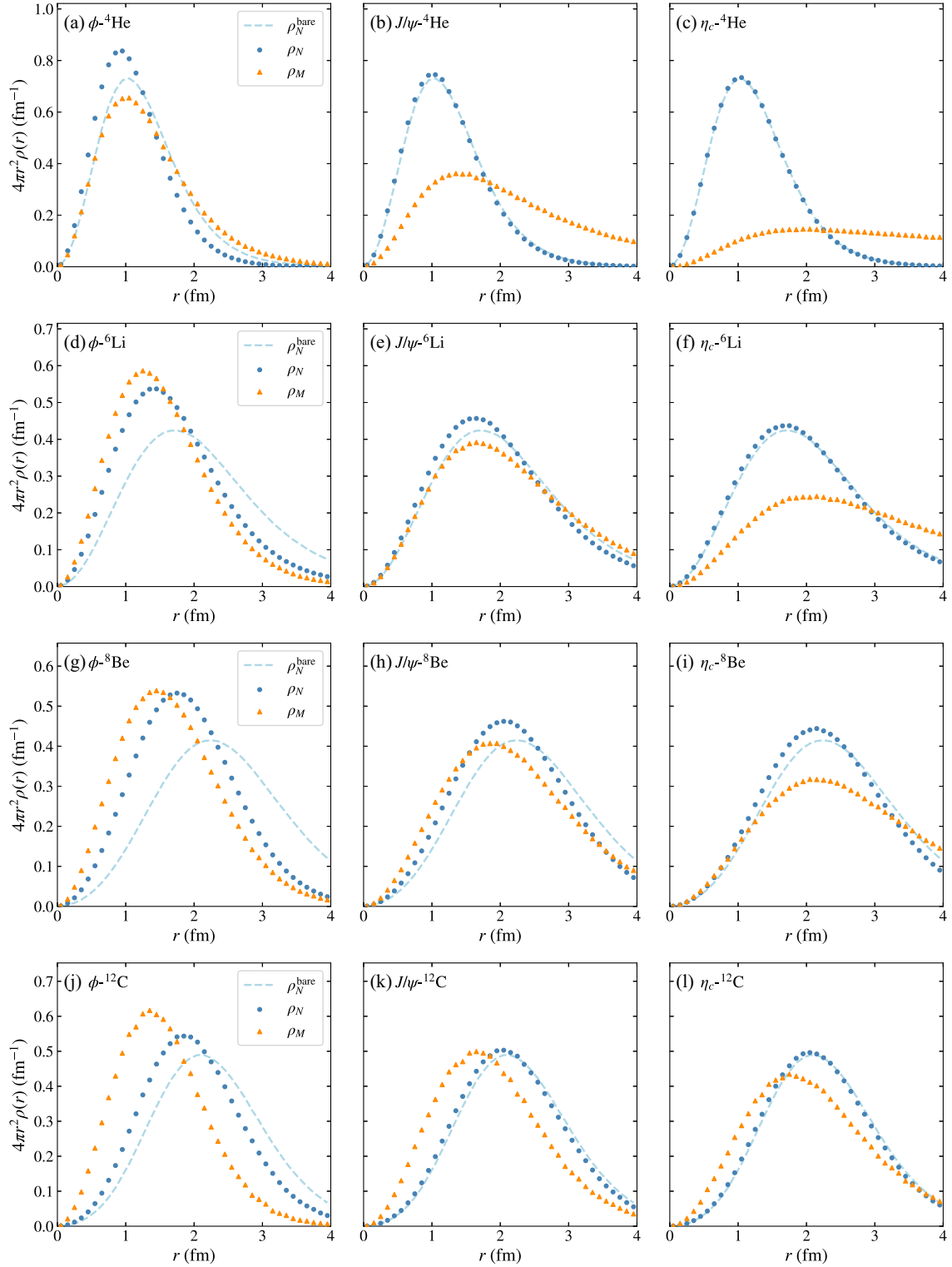


FIG. 6. Radial density of the M - ^4He , M - ^6Li , M - ^8Be , and M - ^{12}C . Each panel shows the nucleon radial density (blue circles), meson radial density (orange triangles) of the meson-nucleus system, and the nucleon density in the parent nucleus (blue dashed line).

- [1] S. J. Brodsky, I. A. Schmidt, and G. F. de Téramond, Nuclear bound quarkonium, *Phys. Rev. Lett.* **64**, 1011 (1990).
- [2] R. Aaij *et al.* (LHCb Collaboration), Observation of J/ψ resonances consistent with pentaquark states in $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays, *Phys. Rev. Lett.* **115**, 072001 (2015).
- [3] R. Aaij *et al.* (LHCb Collaboration), Observation of a narrow pentaquark state, $P_c(4312)^+$, and of two-peak structure of the $P_c(4450)^+$, *Phys. Rev. Lett.* **122**, 222001 (2019).
- [4] S.-M. Wu, F. Wang, and B.-S. Zou, Strange molecular partners of P_c states in the $\gamma p \rightarrow \phi p$ reaction, *Phys. Rev. C* **108**, 045201 (2023).
- [5] W.-Y. Tian, N.-C. Wei, Y.-F. Wang, F. Huang, and B.-S. Zou, Effects of the hadronic molecule $N(2080)3/2^-$ on $K^{*+}\Lambda$ photoproduction, *Phys. Rev. C* **112**, 055203 (2025).
- [6] T. Appelquist and W. Fischler, Some remarks on van der Waals forces in QCD, *Phys. Lett.* **77B**, 405 (1978).
- [7] D. A. Wasson, Comment on ‘Nuclear bound quarkonium.’, *Phys. Rev. Lett.* **67**, 2237 (1991).
- [8] M. E. Luke, A. V. Manohar, and M. J. Savage, A QCD Calculation of the interaction of quarkonium with nuclei, *Phys. Lett. B* **288**, 355 (1992).
- [9] A. B. Kaidalov and P. E. Volkovitsky, Heavy quarkonia interactions with nucleons and nuclei, *Phys. Rev. Lett.* **69**, 3155 (1992).
- [10] G. F. de Téramond, R. Espinoza, and M. Ortega-Rodriguez, Proton proton spin correlations at charm threshold and quarkonium bound to nuclei, *Phys. Rev. D* **58**, 034012 (1998).
- [11] H. Gao, T. S. H. Lee, and V. Marinov, Φ 0—N bound state, *Phys. Rev. C* **63**, 022201(R) (2001).
- [12] A. Sibirtsev and M. B. Voloshin, The interaction of slow J/ψ and ψ' with nucleons, *Phys. Rev. D* **71**, 076005 (2005).
- [13] F. Huang, Z. Y. Zhang, and Y. W. Yu, N ϕ state in chiral quark model, *Phys. Rev. C* **73**, 025207 (2006).
- [14] V. B. Belyaev, N. V. Shevchenko, A. I. Fix, and W. Sandhas, Binding of charmonium with two- and three-body nuclei, *Nucl. Phys. A* **780**, 100 (2006).
- [15] K. Tsushima, D. H. Lu, G. Krein, and A. W. Thomas, J/Ψ -nuclear bound states, *Phys. Rev. C* **83**, 065208 (2011).
- [16] J.-J. Wu and T. S. H. Lee, Photo-production of bound states with hidden charms, *Phys. Rev. C* **86**, 065203 (2012).
- [17] A. Yokota, E. Hiyama, and M. Oka, Possible existence of charmonium–nucleus bound states, *Prog. Theor. Exp. Phys.* **2013**, 113D01 (2013).
- [18] S. R. Beane, E. Chang, S. D. Cohen, W. Detmold, H. W. Lin, K. Orginos, A. Parreño, and M. J. Savage, Quarkonium-nucleus bound states from lattice QCD, *Phys. Rev. D* **91**, 114503 (2015).
- [19] J. J. Cobos-Martínez, K. Tsushima, G. Krein, and A. W. Thomas, Φ -meson-nucleus bound states, *Phys. Rev. C* **96**, 035201 (2017).
- [20] T. Mibe *et al.* (LEPS Collaboration), Diffractive ϕ -meson photoproduction on proton near threshold, *Phys. Rev. Lett.* **95**, 182001 (2005).
- [21] B. Dey, C. A. Meyer, M. Bellis, and M. Williams (CLAS Collaboration), Data analysis techniques, differential cross sections, and spin density matrix elements for the reaction $\gamma p \rightarrow \phi p$, *Phys. Rev. C* **89**, 055208 (2014); **90**, 019901(A) (2014).
- [22] A. Ali *et al.* (GlueX Collaboration), First measurement of near-threshold J/ψ exclusive photoproduction off the proton, *Phys. Rev. Lett.* **123**, 072001 (2019).
- [23] S. Adhikari *et al.* (GlueX Collaboration), Measurement of the J/ψ photoproduction cross section over the full near-threshold kinematic region, *Phys. Rev. C* **108**, 025201 (2023).
- [24] B. Duran *et al.*, Determining the gluonic gravitational form factors of the proton, *Nature (London)* **615**, 813 (2023).
- [25] J. R. Pybus *et al.*, First measurement of near-threshold and subthreshold J/ψ photoproduction off nuclei, *Phys. Rev. Lett.* **134**, 201903 (2025).
- [26] S. Acharya *et al.* (ALICE Collaboration), Experimental evidence for an attractive p - ϕ interaction, *Phys. Rev. Lett.* **127**, 172301 (2021).
- [27] I. I. Strakovsky, L. Pentchev, and A. I. Titov, Comparative analysis of ωp , ϕp , and $J/\psi p$ scattering lengths from A2, CLAS, and GlueX threshold measurements, *Phys. Rev. C* **101**, 045201 (2020).
- [28] O. Gryniuk and M. Vanderhaeghen, Accessing the real part of the forward J/ψ - p scattering amplitude from J/ψ photoproduction on protons around threshold, *Phys. Rev. D* **94**, 074001 (2016).
- [29] I. I. Strakovsky, D. Epifanov, and L. Pentchev, $J/\psi p$ scattering length from GlueX threshold measurements, *Phys. Rev. C* **101**, 042201(R) (2020).
- [30] D. Winney *et al.* (Joint Physics Analysis Center Collaboration), Dynamics in near-threshold J/ψ photoproduction, *Phys. Rev. D* **108**, 054018 (2023).
- [31] E. Chizzali, Y. Kamiya, R. Del Grande, T. Doi, L. Fabbietti, T. Hatsuda, and Y. Lyu, Indication of a p - ϕ bound state from a correlation function analysis, *Phys. Lett. B* **848**, 138358 (2024).
- [32] B. Wu, X.-K. Dong, M.-L. Du, F.-K. Guo, and B.-S. Zou, Deciphering the mechanism of J/ψ -nucleon scattering, *Fund. Res.* **5**, 2530 (2025).
- [33] I. I. Strakovsky, W. J. Briscoe, P. Gubler, J. R. Pybus, A. Schmidt, and A. Somov, J/ψ -meson nucleon scattering length from threshold photoproduction on light nuclei, *Phys. Rev. C* **113**, 065201 (2026).
- [34] Y. Lyu, T. Doi, T. Hatsuda, Y. Ikeda, J. Meng, K. Sasaki, and T. Sugiura, Attractive N - ϕ interaction and two-pion tail from lattice QCD near physical point, *Phys. Rev. D* **106**, 074507 (2022).
- [35] Y. Lyu, T. Doi, T. Hatsuda, and T. Sugiura, Nucleon-charmonium interactions from lattice QCD, *Phys. Lett. B* **860**, 139178 (2025).
- [36] I. Filikhin, R. Y. Kezerashvili, and B. Vlahovic, Possible $H\phi 3$ hypernucleus with the HAL QCD interaction, *Phys. Rev. D* **110**, L031502 (2024).
- [37] L.-Z. Wen, Y. Ma, L. Meng, and S.-L. Zhu, ϕ NN, J/ψ NN, η cNN systems based on HAL QCD interactions, *Phys. Rev. D* **111**, 114004 (2025); **112**, 039901(E) (2025).
- [38] F. Etmann, Bound states of Becc^{-9} within $cc^{-} + \alpha + \alpha$ cluster models based on state-of-the-art QCD charmonium-nucleon interactions, *Phys. Rev. C* **112**, 064001 (2025).

- [39] H. Zhou and X. Liu, Three-body resonances of $\alpha\alpha M$ clusters ($M = \phi, J/\psi, \eta c$) in BeM8 nuclei, *Phys. Rev. D* **113**, 014031 (2026).
- [40] G. Carleo and M. Troyer, Solving the quantum many-body problem with artificial neural networks, *Science* **355**, 602 (2017).
- [41] D. Pfau, J. S. Spencer, A. G. d. G. Matthews, and W. M. C. Foulkes, Ab-initio solution of the many-electron Schrödinger equation with deep neural networks, *Phys. Rev. Res.* **2**, 033429 (2020).
- [42] J. Hermann, Z. Schätzle, and F. Noé, Deep-neural-network solution of the electronic Schrödinger equation, *Nat. Chem.* **12**, 891 (2020).
- [43] C. Adams, G. Carleo, A. Lovato, and N. Rocco, Variational Monte Carlo calculations of $A \leq 4$ nuclei with an artificial neural-network correlator ansatz, *Phys. Rev. Lett.* **127**, 022502 (2021).
- [44] A. Gnech, C. Adams, N. Brawand, G. Carleo, A. Lovato, and N. Rocco, Nuclei with up to $A = 6$ nucleons with artificial neural network wave functions, *Few Body Syst.* **63**, 7 (2022).
- [45] Y. L. Yang and P. W. Zhao, Deep-neural-network approach to solving the *ab initio* nuclear structure problem, *Phys. Rev. C* **107**, 034320 (2023).
- [46] A. Gnech, B. Fore, A. J. Tropiano, and A. Lovato, Distilling the essential elements of nuclear binding via neural-network quantum states, *Phys. Rev. Lett.* **133**, 142501 (2024).
- [47] A. Di Donna, L. Contessi, A. Lovato, and F. Pederiva, Hypernuclei with neural network quantum states, *Phys. Rev. Res.* **8**, 013160 (2026).
- [48] Z.-X. Zhang, Y.-L. Yang, W.-B. He, P.-W. Zhao, B.-N. Lu, and Y.-G. Ma, Machine learning the single- Λ hypernuclei with neural-network quantum states, *Phys. Lett. B* **874**, 140285 (2026).
- [49] W.-L. Wu, L. Meng, and S.-L. Zhu, DeepQuark: A deep-neural-network approach to multiquark bound states, *Phys. Rev. Lett.* **136**, 071901 (2026).
- [50] H. W. Hammer, S. König, and U. van Kolck, Nuclear effective field theory: Status and perspectives, *Rev. Mod. Phys.* **92**, 025004 (2020).
- [51] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, An Accurate nucleon-nucleon potential with charge independence breaking, *Phys. Rev. C* **51**, 38 (1995).
- [52] R. Schiavilla, L. Girlanda, A. Gnech, A. Kievsky, A. Lovato, L. E. Marcucci, M. Piarulli, and M. Viviani, Two- and three-nucleon contact interactions and ground-state energies of light- and medium-mass nuclei, *Phys. Rev. C* **103**, 054003 (2021).
- [53] S. Navas *et al.* (Particle Data Group Collaboration), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [54] M. Zaheer, S. Kottur, S. Ravanbakhsh, B. Póczos, R. Salakhutdinov, and A. Smola, Deep sets, in *Advances in Neural Information Processing Systems* (2017), Vol. 30, pp. 3391–3401, [arXiv:1703.06114](https://arxiv.org/abs/1703.06114).
- [55] E. Wagstaff, F. B. Fuchs, M. Engelcke, I. Posner, and M. Osborne, On the limitations of representing functions on sets, in *Proceedings of the 36th International Conference on Machine Learning (ICML)* (2019), pp. 6487–6494, [arXiv:1901.09006](https://arxiv.org/abs/1901.09006).
- [56] J. Gilmer, S. S. Schoenholz, P. F. Riley, O. Vinyals, and G. E. Dahl, Neural message passing for quantum chemistry, in *Proceedings of the 34th International Conference on Machine Learning (ICML)* (2017), pp. 1263–1272, [arXiv:1704.01212](https://arxiv.org/abs/1704.01212).
- [57] G. Pescia, J. Nys, J. Kim, A. Lovato, and G. Carleo, Message-passing neural quantum states for the homogeneous electron gas, *Phys. Rev. B* **110**, 035108 (2024).
- [58] K. He, X. Zhang, S. Ren, and J. Sun, Deep residual learning for image recognition, in *2016 IEEE Conference on Computer Vision and Pattern Recognition (CVPR)* (2016), pp. 770–778, [arXiv:1512.03385](https://arxiv.org/abs/1512.03385).
- [59] D. Luo and B. K. Clark, Backflow transformations via neural networks for quantum many-body wave functions, *Phys. Rev. Lett.* **122**, 226401 (2019).
- [60] S. Sorella, Wave function optimization in the variational Monte Carlo method, *Phys. Rev. B* **71**, 241103(R) (2005).
- [61] G. Carleo, K. Choo, D. Hofmann, J. E. Smith, T. Westerhout, F. Alet, E. J. Davis, S. Efthymiou, I. Glasser, S.-H. Lin, M. Mauri, G. Mazzola, C. B. Pereira, and F. Vicentini, Netket: A machine learning toolkit for many-body quantum systems, *SoftwareX* **10**, 100311 (2019).
- [62] F. Vicentini, D. Hofmann, A. Szabó, D. Wu, C. Roth, C. Giuliani, G. Pescia, J. Nys, V. Vargas-Calderón, N. Astrakhantsev, and G. Carleo, Netket 3: Machine learning toolbox for many-body quantum systems, *SciPost Phys. Codebases* **7** (2022).
- [63] M. Wang, W. J. Huang, F. G. Kondev, G. Audi, and S. Naimi, The AME 2020 atomic mass evaluation (II). Tables, graphs and references, *Chin. Phys. C* **45**, 030003 (2021).
- [64] J. S. Andrews, D. Jayatilaka, R. G. Bone, N. C. Handy, and R. D. Amos, Spin contamination in single-determinant wavefunctions, *Chem. Phys. Lett.* **183**, 423 (1991).
- [65] H. Ohnishi, F. Sakuma, and T. Takahashi, Hadron physics at J-PARC, *Prog. Part. Nucl. Phys.* **113**, 103773 (2020).
- [66] JLab PAC42, PAC42 final report, Technical Report, Jefferson Lab, 2014.
- [67] U. Wiedner, Future prospects for hadron physics at PANDA, *Prog. Part. Nucl. Phys.* **66**, 477 (2011).
- [68] G. Krein, A. W. Thomas, and K. Tsushima, Nuclear-bound quarkonia and heavy-flavor hadrons, *Prog. Part. Nucl. Phys.* **100**, 161 (2018).
- [69] V. Metag, M. Nanova, and E. Y. Paryev, Meson-nucleus potentials and the search for meson–nucleus bound states, *Prog. Part. Nucl. Phys.* **97**, 199 (2017).
- [70] J. W. T. Keeble and A. Rios, Machine learning the deuteron, *Phys. Lett. B* **809**, 135743 (2020).