

Graviton stimulated emission in squeezed vacuum states

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We study the dynamics of gravitons in a squeezed vacuum state under a thermal radiation background. Unlike traditional treatments that rely on the Boltzmann equation, we employ the Heisenberg equation and average it over general quantum states. In contrast to the usual Boltzmann-based descriptions, our approach captures the subtleties arising from quantum coherence in different number eigenstates, which is essential for soft graviton modes in the squeezed vacuum state. Our new method successfully reproduces the previous one-loop results within the in-in formalism when the expansion parameter is small and deviates significantly as the parameter increases, indicating that our results extend beyond the one-loop in-in formalism. We examine the implications of graviton emission effects stimulated by quantum coherence in both flat and expanding backgrounds. In the flat background, it is found that backreaction of radiation on the spacetime dynamics is crucial for significant stimulated emission. In the expanding background, to avoid the subtleties associated with superhorizon modes, we investigate the effect of emission within the horizon immediately after reheating and find a significant effect. We also examined the IR graviton evolution from a symmetry perspective and propose a regularization prescription to eliminate the secular growth problem.

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I. INTRODUCTION

Gravity is omnipresent, intricately coupled across all scales of known physics, yet it is the weakest in coupling strength among all fundamental forces, making it exceedingly difficult to capture its presence. While the elegant framework of general relativity was established already a century ago, the direct detection of gravitational waves was only recently confirmed with extremely sensitive experiments [1,2]. In inflationary cosmology, although the curvature fluctuations predicted by inflation are already observed in the anisotropies of the cosmic microwave background, primordial gravitational waves remain elusive. Despite the fact that inflation might be the highest energy scale accessible to us, due to Planck mass suppression, the strength of such signals is extremely small, and now the tensor-to-scalar ratio is constrained to $r \lesssim \mathcal{O}(0.01)$ [3]. To capture more signals of gravitational waves, we either keep

improving the sensitivity of experiments or look for more dramatic sources, such as the merger of more massive compact objects, large scalar perturbations acting as secondary sources [4–8], and so on.

Under the assumption of general relativity, the coupling between matter and gravitons is ubiquitous. The extreme weakness of gravitational interactions stems from the smallness of the gravitational coupling constant. As a consequence, scattering processes in vacuum are exceedingly rare, rendering gravitational interactions difficult to detect. A natural way to compensate for the small interaction strength is to increase the number of target particles that interact with gravitons. For example, the relatively large gravitational waves can be generated if the source particles are produced in large numbers through certain instabilities [9–15]. Another natural possibility is to consider the high temperature thermal bath, since particle occupation numbers increase with temperature. In an extreme scenario where the temperature of the thermal bath is high enough to drive spacetime evolution, the Universe enters a radiation-dominated phase. In this regime, the Friedmann equation then relates the temperature to the Hubble parameter H as $T \sim \sqrt{M_{\text{pl}} H}$. Nevertheless, as demonstrated in the previous work [16], these thermal induced gravitational waves may still be too weak, since their amplitude remains suppressed by H/M_{pl} .

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Gravitational waves can also be generated through other cosmological processes, including first-order phase transitions and topological defects. For further details and additional possibilities, we refer the reader to the comprehensive review and the references therein [17]. In this work, we aim to address the following question: Is there an alternative mechanism to further enhance the amplitude of gravitational waves? Motivated by light amplification via stimulated emission in atomic systems, we will consider the possibility of graviton stimulated emission from a thermal bath [18].

To fully understand the mechanism underlying the possibility of graviton stimulated emission, it is essential to carefully specify the quantum states of the gravitons. If we assume that gravitons are in a mixed state expanded in terms of Hamiltonian eigenstates as adopted in many previous studies, then the Heisenberg equation of quantum operators can be reduced to a Boltzmann equation for the phase space distribution. This method, sometimes referred to as the quantum Boltzmann equation, was developed in the study of neutrino mixing [19,20] and has also been applied to the polarization of the cosmic microwave background [21] (for recent developments, see also [22–30]). Such an ansatz for a quantum state permits counting graviton numbers classically and applying a kinematic description. In such a framework, the phase space distributions of each polarization mode fully characterize statistical graviton states. However, this is not necessarily the case for general quantum states of gravitons. For instance, the distribution of primordial gravitons generated during inflation reside in a two-mode squeezed vacuum state [31,32], which is not solely determined by graviton number, as the coherence of different number eigenstates is nonvanishing. As in Ref. [18], it was suggested that squeezed vacuum states of gravitons could stimulate graviton emission, potentially enhancing the initial graviton population. In fact, the role of initial excitations in triggering stimulated creation of quanta, and thereby leaving potentially observable imprints, has already been emphasized in seminal works [33,34]. Another interesting direction concerns graviton production in a thermal background at hydrodynamical scales [35–37], whereas in this paper we focus on the quantum effects in preoccupied graviton states. For a broader overview of cosmological particle production mechanisms, including stimulated processes, see the review by Ford [38].

As highlighted above, the current setup is incompatible with the usual assumptions in kinetic theory and requires the inclusion of additional information. To be more specific, rather than focusing solely on the evolution of the number operator as in the conventional Boltzmann approach, we will introduce a new operator $L_{\mathbf{k}}$ that encodes the coherence between different number eigenstates. We will show that the dynamics of the number operator and this new operator are coupled and governed by a set of

first-order differential equations. We can thus evaluate the stimulated emission of gravitons in squeezed states by solving this set of time-evolution differential equations. This new approach is also different from the earlier work [16,18], which is based on perturbation theory and used the in-in formalism. Instead of dealing with differential equations, that method involves multiple nested time integrals, making it difficult to go beyond one-loop calculations.

The above effect, termed *cosmological stimulated emission*, presents two key challenges. First, the emission rate is proportional not to the number density but to the radiation pressure. As a result, the usual Planck suppression of the emission rate is canceled, potentially causing a breakdown of perturbative analysis. Second, the additional mass dimension introduced by radiation pressure is compensated by graviton momentum, implying that infrared (IR) gravitons are more sensitive to the process. Such an IR sensitivity is common for the stimulated emission for photons [39]. In lasers, the finite gap of atomic electron eigenstates limits the maximum possible IR photon wavelength. However, in a cosmological situation, there is no such constraint since the radiation momenta can form arbitrary squeezed triangle configurations in momentum space, which results in a secular growth of the IR graviton power spectrum [16], which has also been confirmed recently by [40]. It should be emphasized that assuming a completely pure quantum state for gravitons is an idealization. In realistic settings, interactions with sub-Hubble modes or minimal gravitational nonlinearities induce decoherence, rapidly suppressing quantum coherence and driving the system toward classical stochastic behavior [41–45]. These studies consistently show that decoherence of primordial gravitons takes place on time scales of order one to a few Hubble times after horizon crossing. As a result, modes that exited the horizon long before reheating are already effectively classical by the onset of radiation domination. In contrast, modes reentering the horizon within the first few Hubble times after the onset of radiation domination may still retain quantum coherence at that stage. It is precisely in this regime that stimulated graviton emission can operate, before decoherence renders the fluctuations effectively classical.

In this paper, to investigate those subtle issues of the cosmological stimulated emission, we first outline the details of setup in Sec. II, and then derive the evolution equations governing both the number density and internal quantum phase of squeezed-state gravitons using the Heisenberg equation in Sec. III. Similar methods have also been applied to the study of particle production [46,47] and neutrino kinematics [48–52]. The derivation follows an approach similar to the quantum-mechanical formulation of the Boltzmann equation, with the primary difference being the ansatz for quantum states. The sensitivity to IR gravitons is subtle in an expanding universe, as the physical interpretation of gravitons on superhorizon scales remains

unclear. To address this, we begin with a careful analysis in a flat background where the issue is absent in Sec. IV. Then, we will show that the backreaction of radiation to the background spacetime is essential when discussing the sizable stimulated emission. As an alternative approach, we analyze cosmological stimulated emission in an expanding universe under a reasonable parameter space in Sec. V, where the initial graviton wavelength is not significantly larger than the Hubble horizon size, ensuring that uncertainties from large gauge transformation do not arise. We employ the averaged Heisenberg equation (distinct from the usual Boltzmann equation based on the kinematical description) and compute the evolution of graviton number, comparing the results obtained in the in-in formalism [18]. When the interaction coupling parameter is small, this new method accurately reproduces the results obtained from perturbation theory based on the in-in formalism. However, as the parameter increases, we observe significant deviations from the previous one-loop results, this indicates that our approach extends beyond conventional perturbative methods. In Sec. VI, to address the secular IR issue, we propose a regularization method to eliminate this secular growth with a special focus on the large gauge symmetry. Finally, we conclude and outline possible directions for future work in Sec. VII.

II. BASIC SETUP

Let us consider an Friedmann–Lemaître–Robertson–Walker (FLRW) background spacetime and its traceless transverse perturbations,

$$ds^2 = a(\tau)^2(-d\tau^2 + \gamma_{ij}dx^i dx^j), \quad (2.1)$$

$$\gamma_{ij} \equiv \delta_{ij} + \frac{2}{M_{\text{pl}}} h_{ij} + \frac{2}{M_{\text{pl}}^2} h_{ik} h^k_j + \dots, \quad (2.2)$$

$$h^i_i = \partial_i h^i_j = 0. \quad (2.3)$$

Here, Latin indices refer to spatial coordinates and are raised and lowered using the background spatial metric δ^{ij} and δ_{ij} . M_{pl} denotes the reduced Planck mass, and $a(\tau)$ is the isotropic scale factor. Expanding the Einstein-Hilbert action to the second order in h^i_j yields the action for the free graviton,

$$S = \int d\tau L[h^i_j, h^i_j, \tau], \quad (2.4)$$

$$L[h^i_j, h^i_j, \tau] \equiv \frac{1}{2} \int d^3x a(\tau)^2 [(h^i_j)^2 - (\partial_k h^i_j)^2]. \quad (2.5)$$

We consider a thermal free scalar field χ that is minimally coupled to the graviton. Interactions between χ and h_{ij} arise from the kinetic term,

$$-\frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \supset M_{\text{pl}}^{-1} \int d^4x a^2 h^{ij} \partial_i \chi \partial_j \chi. \quad (2.6)$$

It has been found that the four-point interaction is eliminated by perturbing a tadpole diagram; hence, we truncate the interaction Hamiltonian at this order [16].

Define the conjugate momentum as

$$\pi^j_i \equiv \frac{\delta L[h^i_j, h^i_j, \tau]}{\delta h^i_j} = a^2 h^j_i. \quad (2.7)$$

The Fourier expansion of these tensor perturbations is given by

$$h^i_j(\tau, \mathbf{x}) = \sum_{s=\pm} \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} e^i_j(\mathbf{k}, s) h_{\mathbf{k}}^{(s)}(\tau), \quad (2.8)$$

$$\pi^i_j(\tau, \mathbf{x}) = \sum_{s=\pm} \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} e^i_j(\mathbf{k}, s) \pi_{\mathbf{k}}^{(s)}(\tau), \quad (2.9)$$

where the polarization tensors satisfy

$$e^i_j(\mathbf{k}, s) (e^i_j(\mathbf{k}, s'))^* = \delta_{ss'}, \quad (2.10)$$

$$(e^i_j(\mathbf{k}, s))^* = e^i_j(-\mathbf{k}, s) = e^i_j(\mathbf{k}, -s). \quad (2.11)$$

Next, we impose the canonical commutation relation for the Fourier modes,

$$[h_{\mathbf{k}}^{(s)}, \pi_{\mathbf{k}'}^{(s')}] = i\hbar \delta^{ss'} (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}'). \quad (2.12)$$

Hereafter, we set $\hbar = 1$.

Equation (2.5) yields the free Hamiltonian

$$H_0[h^i_j, \pi^j_i, \tau] = \frac{1}{2} \sum_s \int \frac{d^3k}{(2\pi)^3} \left(\frac{\pi_{\mathbf{k}}^{(s)} \pi_{-\mathbf{k}}^{(s)}}{a^2} + a^2 k^2 h_{\mathbf{k}}^{(s)} h_{-\mathbf{k}}^{(s)} \right). \quad (2.13)$$

Using the *instantaneous* annihilation operator [53]

$$d_{\mathbf{k}}^{(s)} \equiv a(\tau) \sqrt{\frac{k}{2}} h_{\mathbf{k}}^{(s)} + \frac{i}{a(\tau) \sqrt{2k}} \pi_{\mathbf{k}}^{(s)}, \quad (2.14)$$

one can diagonalize the free Hamiltonian operator as

$$H_0[h^i_j, \pi^j_i, \tau] = \sum_s \int \frac{d^3k}{(2\pi)^3} \frac{k}{2} (d_{\mathbf{k}}^{(s)} d_{\mathbf{k}}^{(s)\dagger} + d_{\mathbf{k}}^{(s)\dagger} d_{\mathbf{k}}^{(s)}). \quad (2.15)$$

From this, one can define the instantaneous number operator,

$$N_{\mathbf{k}}^{(s)}(\tau) \equiv d_{\mathbf{k}}^{(s)\dagger}(\tau)d_{\mathbf{k}}^{(s)}(\tau), \quad (2.16)$$

which allows us to express the Hamiltonian as

$$H_0[h^i_j, \pi^j_i, \tau] = \sum_{s=\pm} \int \frac{d^3k}{(2\pi)^3} k N_{\mathbf{k}}^{(s)}(\tau) + \mathcal{C}, \quad (2.17)$$

where the last term $\mathcal{C}$ represents a constant energy shift. In addition to the number operator, we introduce

$$L_{\mathbf{k}}^{(s)}(\tau) \equiv d_{\mathbf{k}}^{(s)}(\tau)d_{-\mathbf{k}}^{(s)}(\tau). \quad (2.18)$$

The expectation value of L characterizes the coherence between different number eigenstates, which does not appear in standard kinetic theory formulated in terms of separable number eigenstates.

III. HEISENBERG EQUATION

In this section, we derive the Heisenberg equation averaged over given quantum states. We do not call this equation the Boltzmann equation to emphasize that we retain the quantum field theory perspective without fully reducing the system to kinetic theory. The following equations describe the equations of motion for field operators in QFT where the “size” of each graviton is too large to regard them as point particles.

A. Free theory

Let us start with the free theory. The free Heisenberg equation is given by

$$\frac{d\mathcal{O}}{d\tau} = i[H_0, \mathcal{O}] + \frac{\partial\mathcal{O}}{\partial\tau}. \quad (3.1)$$

In an expanding universe, operators depend explicitly on time via the scale factor, which gives rise to the last term.

Using Eq. (2.14), we obtain

$$[H_0, N_{\mathbf{k}}^{(s)}] = 0, \quad (3.2)$$

$$[H_0, L_{\mathbf{k}}^{(s)}] = -2kL_{\mathbf{k}}^{(s)}. \quad (3.3)$$

Next, to evaluate the partial derivatives of the operators, we compute

$$\begin{aligned} \frac{\partial}{\partial\tau} d_{\mathbf{k}}^{(s)}(\tau) &= \frac{1}{a(\tau)} \frac{\partial a(\tau)}{\partial\tau} \left[a(\tau) \sqrt{\frac{k}{2}} h_{\mathbf{k}}^{(s)} - \frac{i}{a(\tau)\sqrt{2k}} \pi_{\mathbf{k}}^{(s)} \right] \\ &= \frac{1}{a(\tau)} \frac{\partial a(\tau)}{\partial\tau} \left[a(\tau) \sqrt{\frac{k}{2}} h_{-\mathbf{k}}^{(s)} + \frac{i}{a(\tau)\sqrt{2k}} \pi_{-\mathbf{k}}^{(s)} \right]^\dagger. \end{aligned} \quad (3.4)$$

Note that the canonical variables do not explicitly depend on time. Summarizing, with $\mathcal{H} \equiv \partial_\tau a/a$, we obtain

$$\frac{\partial}{\partial\tau} d_{\mathbf{k}}^{(s)}(\tau) = \mathcal{H} d_{-\mathbf{k}}^{(s)\dagger}(\tau), \quad (3.5)$$

$$\frac{\partial}{\partial\tau} d_{\mathbf{k}}^{(s)\dagger}(\tau) = \mathcal{H} d_{-\mathbf{k}}^{(s)}(\tau). \quad (3.6)$$

Using these, we find

$$\begin{aligned} \frac{dN_{\mathbf{k}}^{(s)}}{d\tau} &= \mathcal{H}(L_{\mathbf{k}}^{(s)} + L_{\mathbf{k}}^{(s)\dagger}), \\ \frac{dL_{\mathbf{k}}^{(s)}}{d\tau} &= -2ikL_{\mathbf{k}}^{(s)} + \mathcal{H}(N_{\mathbf{k}}^{(s)} + N_{-\mathbf{k}}^{(s)} + V). \end{aligned} \quad (3.7)$$

Now, consider the initial vacuum state $\varrho_0 = |0\rangle\langle 0|$ and expand the instantaneous ladder operator with respect to this initial vacuum state,

$$d_{\mathbf{k}}^{(s)}(\tau) = \mu_k(\tau, \tau_0) d_{\mathbf{k}}^{(s)}(\tau_0) + \nu_k(\tau, \tau_0) d_{-\mathbf{k}}^{(s)\dagger}(\tau_0). \quad (3.8)$$

Taking the expectation value of Eq. (3.7) with respect to ϱ_0 , we obtain

$$\frac{d|\nu_k|^2}{d\tau} = \mathcal{H}(\mu_k \nu_k + \mu_k^* \nu_k^*), \quad (3.9)$$

$$\frac{d(\mu_k \nu_k)}{d\tau} = -2ik\mu_k \nu_k + \mathcal{H}(2|\nu_k|^2 + 1). \quad (3.10)$$

These equations are consistent with the equation of motion for the mode functions, as the Bogoliubov coefficients depend on time through the canonical variables.

B. Interaction theory

In the interaction picture, the field operators remain free, while the density operator follows the von Neumann equation with respect to the interaction Hamiltonian H_I ,

$$\frac{d\varrho}{d\tau} = -i[H_I, \varrho]. \quad (3.11)$$

The evolution equation for the expectation value of $\mathcal{O}$ is

$$\begin{aligned} \frac{d}{d\tau} \text{Tr}[\varrho \mathcal{O}] &= \text{Tr} \left[\frac{d\varrho}{d\tau} \mathcal{O} + \varrho \frac{d\mathcal{O}}{d\tau} \right] \\ &= \text{Tr} \left[-i[H_I, \varrho] \mathcal{O} + i\varrho [H_0, \mathcal{O}] + \varrho \frac{\partial\mathcal{O}}{\partial\tau} \right] \\ &= \text{Tr} \left[i\varrho [H_I, \mathcal{O}] + i\varrho [H_0, \mathcal{O}] + \varrho \frac{\partial\mathcal{O}}{\partial\tau} \right]. \end{aligned} \quad (3.12)$$

The first equality follows from the linearity of the trace and the Leibniz rule for the total derivative. Next, we define

$$n \equiv \frac{1}{V} \text{Tr} \left[\varrho N_{\mathbf{k}}^{(s)} \right], \quad (3.13)$$

$$\lambda \equiv \frac{1}{V} \text{Tr} \left[\varrho L_{\mathbf{k}}^{(s)} \right], \quad (3.14)$$

where $V = (2\pi)^3 \delta^{(3)}(0)$. In the absence of interactions, these reduce to $n_0 = |\nu_k|^2$ and $\lambda_0 = \mu_k \nu_k$, with the subscript implying the order in the interaction Hamiltonians. Since ϱ includes the effect of interactions to all orders, so do n and λ . The total time derivatives are given by

$$\frac{1}{V} \frac{d}{d\tau} \text{Tr} \left[\varrho N_{\mathbf{k}}^{(s)} \right] = \frac{dn}{d\tau}, \quad (3.15)$$

$$\frac{1}{V} \frac{d}{d\tau} \text{Tr} \left[\varrho L_{\mathbf{k}}^{(s)} \right] = \frac{d\lambda}{d\tau}. \quad (3.16)$$

Moreover, we have

$$\frac{1}{V} \text{Tr} \left[\varrho i \left[H_0, N_{\mathbf{k}}^{(s)} \right] \right] = 0, \quad (3.17)$$

$$\frac{1}{V} \text{Tr} \left[\varrho i \left[H_0, L_{\mathbf{k}}^{(s)} \right] \right] = -2ik\lambda, \quad (3.18)$$

and also

$$\frac{1}{V} \text{Tr} \left[\varrho \frac{\partial N_{\mathbf{k}}^{(s)}}{\partial \tau} \right] = 2\mathcal{H} \text{Re}[\lambda], \quad (3.19)$$

$$\frac{1}{V} \text{Tr} \left[\varrho \frac{\partial L_{\mathbf{k}}^{(s)}}{\partial \tau} \right] = \mathcal{H}(2n + 1). \quad (3.20)$$

Thus, the last two terms in Eq. (3.12) are evaluated nonperturbatively. In Ref. [47], particle production from a scalar condensate was considered for $H_I = \tilde{\phi}(t)\chi\chi$. In that case, the interaction term was also evaluated nonperturbatively, which differs from the setup here.

In general, a perturbative expansion is needed for the interaction term. The interaction-state evolution is given by $\varrho \rightarrow F\varrho_0 F^{-1}$ with the initial state ϱ_0 and the evolution operator

$$F(\tau) \equiv T \exp \left(-i \int_0^\tau d\tau' H_I(\tau') \right). \quad (3.21)$$

Then, we obtain

$$\text{Tr}[i\varrho[H_I(\tau), \mathcal{O}(\tau)]] = \text{Tr}[i\varrho_0 F(\tau)^{-1} [H_I(\tau), \mathcal{O}(\tau)] F(\tau)], \quad (3.22)$$

which can be expanded as [54]

$$\text{Tr}[i\varrho[H_I(\tau), \mathcal{O}(\tau)]] = \sum_{n=0}^{\infty} i^n \int^\tau d\tau_1 \cdots \int^{\tau_{n-1}} d\tau_n \text{Tr}[\varrho_0 [H_I(\tau_n), \cdots [H_I(\tau_1), i[H_I(\tau), \mathcal{O}(\tau)]]]]. \quad (3.23)$$

Under the Born-Markov approximation [29], the interaction term simplifies to

$$\text{Tr}[i\varrho[H_I(\tau), \mathcal{O}(\tau)]] \simeq \text{Tr}[i\varrho_0[H_I(\tau), \mathcal{O}(\tau)]] - \int_0^\infty d\tilde{\tau} \text{Tr}[\varrho_0 [H_I(\tau - \tilde{\tau}), [H_I(\tau), \mathcal{O}(\tau)]]]. \quad (3.24)$$

For clarity, we briefly review the Born-Markov approximation. The Born approximation assumes that the coupling between the system and the environment is weak. A direct consequence of this assumption is that, in a perturbative expansion in the interaction Hamiltonian, we can safely truncate the series at the second order. The Markovian approximation is based on a separation of timescales. The expectation values change on the macroscopic timescale τ , and the microscopic timescale of the individual interaction is denoted by $\tilde{\tau}$. Changing the time variable from τ_1 to $\tilde{\tau}$, the upper integral domain becomes τ , which is sufficiently larger than the microscopic timescale. We therefore set it to infinity, following the convention in kinetic theory. In fact, the following calculation shows that the relevant time window is a very short interval $\beta \ll \tau$; thus, this prescription is well justified.

C. Backward scattering

The forward scattering vanishes for the three-point interaction, so we only need to evaluate the second term in Eq. (3.24). From Eq. (2.6), the interaction Hamiltonian is given by

$$H_I = -\frac{a^2}{M_{\text{pl}}} \int d^3x h^{ij} \partial_i \chi \partial_j \chi. \quad (3.25)$$

Using Eq. (2.8) and

$$\chi(\tau, x) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{x}} \chi_{\mathbf{k}}(\tau), \quad (3.26)$$

we can rewrite the interaction Hamiltonian as

$$H_I = \frac{a^2}{M_{\text{pl}}} \int \frac{d^3 k d^3 p_1 d^3 p_2}{(2\pi)^9} (2\pi)^3 \delta(\mathbf{k} + \mathbf{p}_1 + \mathbf{p}_2) \times \sum_s e_{ij}^{(s)}(\mathbf{k}) h_{\mathbf{k}}^{(s)} p_{1i} p_{2j} \chi_{\mathbf{p}_1} \chi_{\mathbf{p}_2}. \quad (3.27)$$

Let us define the energy-momentum tensor projected onto the graviton polarization plane as

$$T_{\mathbf{k}}^{(s)} \equiv -a^2 \int \frac{d^3 l d^3 p}{(2\pi)^3} \delta(\mathbf{k} + \mathbf{l} + \mathbf{p}) e_{ij}^{(s)}(\mathbf{k}) l_i p_j \chi_{\mathbf{l}} \chi_{\mathbf{p}}. \quad (3.28)$$

Then, the interaction Hamiltonian can be further simplified to

$$H_I = -M_{\text{pl}}^{-1} \sum_{s=\pm} \int \frac{d^3 k}{(2\pi)^3} h_{\mathbf{k}}^{(s)} T_{\mathbf{k}}^{(s)}, \quad (3.29)$$

or more concisely,

$$H_I = -M_{\text{pl}}^{-1} h^S T_S. \quad (3.30)$$

Here, S encapsulates both the momentum and polarization indices and repeated S implies summation over polarization and integration over momentum. We will use the notations in Eqs. (3.29) and (3.30) interchangeably in the following discussion.

For $\mathcal{O} = N^S, L^S$, Eq. (3.24) expands as

$$-\int_0^\infty d\tilde{\tau} [H_I(\tau - \tilde{\tau}), [H_I(\tau), \mathcal{O}(\tau)]] = A[\mathcal{O}] + B[\mathcal{O}], \quad (3.31)$$

where

$$A[\mathcal{O}] = -\frac{1}{2M_{\text{pl}}^2} \int_0^\infty d\tilde{\tau} [T_{S_2}(\tau - \tilde{\tau}), T_{S_1}(\tau)] \times \{h^{S_2}(\tau - \tilde{\tau})[h^{S_1}(\tau), \mathcal{O}(\tau)] + [h^{S_1}(\tau), \mathcal{O}(\tau)]h^{S_2}(\tau - \tilde{\tau})\}, \quad (3.32)$$

$$B[\mathcal{O}] = -\frac{1}{2M_{\text{pl}}^2} \int_0^\infty d\tilde{\tau} [h^{S_2}(\tau - \tilde{\tau}), [h^{S_1}(\tau), \mathcal{O}(\tau)]] \times \{T_{S_2}(\tau - \tilde{\tau})T_{S_1}(\tau) + T_{S_1}(\tau)T_{S_2}(\tau - \tilde{\tau})\}. \quad (3.33)$$

The term $B[\mathcal{O}]$ is independent of the graviton quantum state, corresponding to the spontaneous emission from the scalar field. This type of contribution is often considered in the graviton production in thermal plasma [35] and induced gravitational waves from density fluctuations [6]. In contrast, $A[\mathcal{O}]$ depends on the initial graviton state and thus represents stimulated emission. One can show that $A[\mathcal{O}]$ dominates for infrared (IR) graviton modes. Therefore, we will neglect spontaneous emission in the subsequent analysis.

D. Stimulated emission

The evaluation of Eq. (3.32) is similar to that in Ref. [18], and it is simplified due to the Markov approximation. Using the retarded and Keldysh propagators,

$$G_{p_1}^R(\tau_1, \tau_2)(2\pi)^3 \delta(\mathbf{p}_1 + \mathbf{p}_2) = ia^2(\tau_2) \Theta(\tau_1 - \tau_2) [\chi_{\mathbf{p}_1}(\tau_1), \chi_{\mathbf{p}_2}(\tau_2)], \quad (3.34)$$

$$G_{l_1}^K(\tau_1, \tau_2)(2\pi)^3 \delta(\mathbf{l}_1 + \mathbf{l}_2) = a^2(\tau_2) \text{Tr}[\varrho(\chi_{l_2}(\tau_2) \chi_{l_1}(\tau_1) + \chi_{l_1}(\tau_1) \chi_{l_2}(\tau_2))]_{\chi}, \quad (3.35)$$

we find

$$\text{Tr}[\varrho_{\chi}[T_{S_2}(\tau - \tilde{\tau}), T_{S_1}(\tau)]] = 2i\delta_{s_1 s_2} (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) X(\tau, \tau - \tilde{\tau}), \quad (3.36)$$

where we define

$$X(\tau_1, \tau_2) \equiv \frac{2}{15} \frac{a^2(\tau_1)}{a^2(\tau_2)} \int \frac{p^2 dp}{2\pi^2} p^4 G_p^K(\tau_1, \tau_2) G_p^R(\tau_1, \tau_2). \quad (3.37)$$

Since the thermal scalar field is well inside the horizon, the propagators take the Minkowski form rescaled by the scale factor,

$$G_p^R(\tau_1, \tau_2) = \frac{a(\tau_2) \sin p(\tau_1 - \tau_2)}{a(\tau_1) p} \Theta(\tau_1 - \tau_2), \quad (3.38)$$

$$G_p^K(\tau_1, \tau_2) \simeq \frac{a(\tau_2) \cos p(\tau_1 - \tau_2)}{a(\tau_1) p} (1 + 2f_{\beta p}), \quad (3.39)$$

where $f_{\beta p} = \frac{1}{e^{\beta p} - 1}$ stands for the thermal distribution of bosonic particles, with β denoting the inverse of the comoving temperature. Thus, the scale factor cancels out, and X becomes a function of $\tilde{\tau}$,

$$X(\tilde{\tau}) = \frac{1}{15\pi^2} \left[-\frac{3}{8\tilde{\tau}^5} + \text{csch}^5\left(\frac{2\pi\tilde{\tau}}{\beta}\right) \frac{\pi^5}{\beta^5} \left(11 \cosh\left(\frac{2\pi\tilde{\tau}}{\beta}\right) + \cosh\left(\frac{6\pi\tilde{\tau}}{\beta}\right) \right) \right]. \quad (3.40)$$

$X(\tilde{\tau})$ acts as a window function open for $\tilde{\tau} \sim \beta/(2\pi)$ and vanishes for $\tilde{\tau}/\beta \gg 1$, justifying the Markov approximation. By integrating $X(\tilde{\tau})$, we obtain

$$\lim_{\beta/\tau \rightarrow 0} \int_0^\infty d\tilde{\tau} X(\tilde{\tau}) = \frac{\pi^2}{450\beta^4}. \quad (3.41)$$

Thanks to the Markov approximation, the graviton operators reduce to a local operator product,

$$A[\mathcal{O}] \simeq -\frac{i\pi^2}{450M_{\text{pl}}^2\beta^4} \delta_{S_1 S_2} \{h^{S_2}(\tau)[h^{S_1}(\tau), \mathcal{O}(\tau)] + [h^{S_1}(\tau), \mathcal{O}(\tau)]h^{S_2}(\tau)\}. \quad (3.42)$$

We have also verified that the non-Markovian correction is $\mathcal{O}(\beta^{-3})$, which we may ignore in the present case. Using the instantaneous operator (2.14), the field operator takes the form

$$h_{\mathbf{k}}^{(s)} = \frac{1}{a} \frac{1}{\sqrt{2k}} (d_{\mathbf{k}}^{(s)} + d_{-\mathbf{k}}^{(s)\dagger}), \quad (3.43)$$

which yields

$$\langle 0 | h^{S_2} [h^{S_1}, N^S] + [h^{S_1}, N^S] h^{S_2} | 0 \rangle = \frac{V}{ka^2} \delta^{SS_1} \delta^{SS_2} (\lambda_0^{S_2} - \lambda_0^{S_2*}), \quad (3.44)$$

$$\begin{aligned} & \langle 0 | h^{S_2} [h^{S_1}, L^S] + [h^{S_1}, L^S] h^{S_2} | 0 \rangle \\ &= -\frac{V}{ka^2} \delta^{SS_1} \delta^{SS_2} (2\lambda_0^{S_2} + 2n_0^{S_2} + 1), \end{aligned} \quad (3.45)$$

where the repeated S indices on the lhs are not summed. Contracting S_1 and S_2 , we obtain

$$V^{-1} A[N^S] = \frac{\pi^2}{225M_{\text{pl}}^2 ka^2 \beta^4} \text{Im}[\lambda_0^S], \quad (3.46)$$

$$V^{-1} A[L^S] = i \frac{\pi^2}{225M_{\text{pl}}^2 ka^2 \beta^4} \left(\lambda_0^S + n_0^S + \frac{1}{2} \right). \quad (3.47)$$

Due to the Markov approximation, terms of $\mathcal{O}(1/(M_{\text{pl}}\beta))$ are neglected, which is justified unless the system is near the Planck scale.

E. Summary so far

Combining Eqs. (3.18)–(3.20) and (3.46)–(3.47), as well as elevating free variables to their nonperturbative counterparts by matching their initial values, we obtain the Heisenberg equations averaged over a general quantum state,

$$n' = 2 \frac{a'}{a} \text{Re}[\lambda] + \alpha \text{Im}[\lambda], \quad (3.48)$$

$$\lambda' = -2i\lambda + 2 \frac{a'}{a} \left(n + \frac{1}{2} \right) + i\alpha \left(\lambda + n + \frac{1}{2} \right), \quad (3.49)$$

where we define the dimensionless time variables normalized by k : $x = k\tau$, $x_\beta = k\beta$, $x_{\text{pl}} = k/M_{\text{pl}}$, and $\iota \equiv d/dx$, along with

$$\alpha \equiv \frac{\pi^2 x_{\text{pl}}^2}{225 a^2 x_\beta^4}. \quad (3.50)$$

Note that in the interaction terms in Eqs. (3.46) and (3.47), n and λ correspond to their free values indexed as 0, while they can be identified with their nonperturbative counterparts at the initial time. At each time step, the perturbed values of n and λ are computed and used as initial conditions for the next step. This iterative procedure continues until the final values are determined. A similar prescription is employed in deriving the standard Boltzmann equations based on the Born-Markov approximation.

To simplify the equations further, we separate the real and imaginary parts of variable λ as

$$\lambda = \xi + i\eta, \quad (3.51)$$

where ξ and η are real numbers. This leads to the following set of equations:

$$n' = 2 \frac{a'}{a} \xi + \alpha \eta, \quad (3.52)$$

$$\xi' = 2\eta + 2 \frac{a'}{a} \left(n + \frac{1}{2} \right) - \alpha \eta, \quad (3.53)$$

$$\eta' = -2\xi + \alpha \left(\xi + n + \frac{1}{2} \right). \quad (3.54)$$

For example, during the radiation dominant era, we have $a'/a = 1/x$.

IV. FLAT BACKGROUND

While the above formulation applies to a general FLRW background, we first consider a simple Minkowski background to examine the physical properties of graviton emission. In the Minkowski limit, we set $a = 1$. Here, β^{-1} represents the physical temperature, and τ becomes the physical time. The emission rate then simplifies to

$$\alpha = \frac{\pi^2 x_{\text{pl}}^2}{225 x_\beta^4}. \quad (4.1)$$

Since the Hubble parameter vanishes in this limit, the Heisenberg equations (3.52)–(3.54) reduce to

$$\frac{dn}{dx} = \alpha \eta, \quad (4.2)$$

$$\frac{d\xi}{dx} = 2\eta - \alpha \eta, \quad (4.3)$$

$$\frac{d\eta}{dx} = -2\xi + \alpha \left(n + \xi + \frac{1}{2} \right). \quad (4.4)$$

A general squeezed vacuum state initial condition is given by

$$\xi(0) = \sqrt{n(0)(n(0) + 1)} \cos \theta(0), \quad (4.5)$$

$$\eta(0) = \sqrt{n(0)(n(0) + 1)} \sin \theta(0). \quad (4.6)$$

We recall that ξ and η represent the real and imaginary parts of $\lambda \sim \text{Tr}[\rho L_{\mathbf{k}}]$, satisfying the relation: $|\lambda|^2 = n(n + 1)$, where n is the average particle number. Although we impose the above initial condition as an illustrative example, the effects discussed here are not restricted to the squeezed vacuum state. In fact, any initial conditions

corresponding to more general initial states can be considered. In a Minkowski background, the vacuum state is uniquely defined as $n(0) = 0$. For $n(0) \geq 1$ and arbitrary θ , the states correspond to excited states, which can be expressed as squeezed vacuum states. Without interaction with the scalar field ($\alpha = 0$), n_0 remains constant, and λ_0 evolves as $\lambda_0 = e^{-2ix} \lambda_0(0)$. Thus, the graviton number remains independent of the internal phase (i.e., the intrinsic phase of λ). In the left panel of Fig. 1, we plot the time evolution of n_0 , ξ_0 , and η_0 for $n_0(0) = \xi_0(0) = \eta_0(0) = 1$.

When turning on interactions ($\alpha \neq 0$), the internal phase and number are coupled. For a typical parameter choice relevant to LIGO/Virgo-like gravitational wave interferometers [1], we take,

- (i) $\ell \sim k^{-1} = \mathcal{O}(10^6)m$: the LIGO-Virgo detectable scale is $\mathcal{O}(100)$ Hz.
- (ii) $x_{\text{pl}} = k/M_{\text{pl}} = 10^{-41}$.
- (iii) $x_\beta = \frac{k}{M_{\text{pl}}} \beta M_{\text{pl}} \sim 10^{-41} \times 10^{20} = 10^{-21}$: $\beta^{-1} = \mathcal{O}(10^{-1})$ GeV.

This flat-space calculation, using LIGO/Virgo parameters, serves as a simplified illustration of the stimulated emission mechanism and helps build intuition for the relevant physical scales. Given that the primordial gravitational wave background is expected to span a vast range of frequencies, exploring a potential amplification mechanism at these terrestrial scales is of independent physical interest.

The numerical result, shown in the right panel of Fig. 1, demonstrates a small fluctuation in n . Oscillations in the phase transfer into variations in the graviton number via Eq. (4.2). In the right panel of Fig. 1, the periodic average value of n decreases, corresponding to stimulated absorption. Depending on the initial phase θ , the effect can manifest as either emission or absorption.

In the flat background case, the system of first-order differential equations (4.2)–(4.4) can be solved exactly. Given the initial conditions $n(0) = 1$ and $\theta = \pi/4$, the exact solution takes the form,

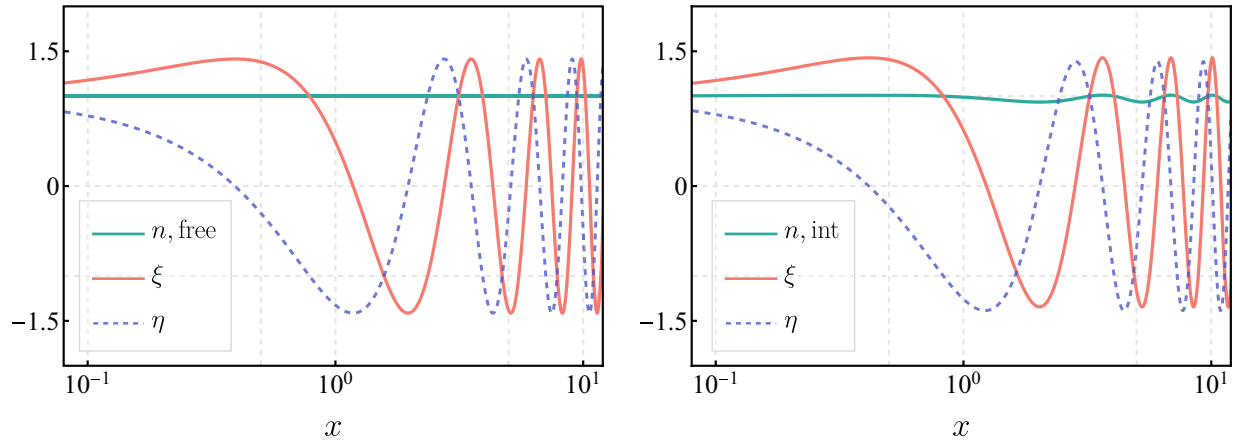


FIG. 1. *Left*: Free evolution of n (green), ξ (red), η (blue dashed). *Right*: Interacting evolution of n (green), ξ (red), η (blue dashed), with the parameters $x_{\text{pl}} = 10^{-41}$, $x_\beta = 3 \times 10^{-21}$ and the initial conditions are set to $n = 1$, $\theta = \pi/4$.

$$\begin{aligned}
n(x) = & \frac{\alpha}{2\sqrt{1-\alpha}} \sin(2x\sqrt{1-\alpha}) \\
& + \frac{\alpha(5\alpha-4)}{8(\alpha-1)} \cos(2x\sqrt{1-\alpha}) \\
& + \frac{1}{8} \left(7 - 5\alpha + \frac{1}{1-\alpha} \right). \quad (4.7)
\end{aligned}$$

When $\alpha < 1$, the particle number oscillates over time and the behavior remains perturbative. However, for $\alpha > 1$, the trigonometric functions reduce to their hyperbolic counterparts and then instability arises, causing the number density to grow exponentially. Neglecting the exponentially suppressed terms, one can approximately write $n(x)$ as

$$n(x) \simeq \frac{\alpha}{16} \left(5 + \frac{4}{(\alpha-1)^{1/2}} + \frac{1}{\alpha-1} \right) e^{2\sqrt{\alpha-1}x}. \quad (4.8)$$

For small α , our prediction aligns with the perturbation theory based on the in-in formalism. Indeed, by expanding Eq. (4.7) to the leading order in α , it reproduces the analytical expression derived in previous work [18] using

the in-in formalism,

$$n^{\text{PT},1\text{-loop}} = -\alpha\sqrt{n_0(n_0+1)} \sin(x) \sin(x-\theta_0), \quad (4.9)$$

which indicates that our new method provides a resummation of the in-in formalism, see also Appendix for a summary of previous results.

In Fig. 1, we present the solutions for $\alpha \sim 0.054$, corresponding to $x_\beta = 3 \times 10^{-21}$. This parameter α is small, and the result closely resembles those obtained in perturbation theory using the in-in formalism. The limitation of the in-in formalism arises because the unperturbed operators in the integral evolve with α . If their nonlinear evolution deviates significantly from their background values, the traditional perturbative estimation fails, and our new approach based on the effective equation of motion becomes advantageous.

A comparison between one-loop perturbation theory and the Heisenberg equation is shown in Fig. 2 for different choices of the parameter α . When α is smaller than $\mathcal{O}(10^{-2})$, our method matches the previous one-loop

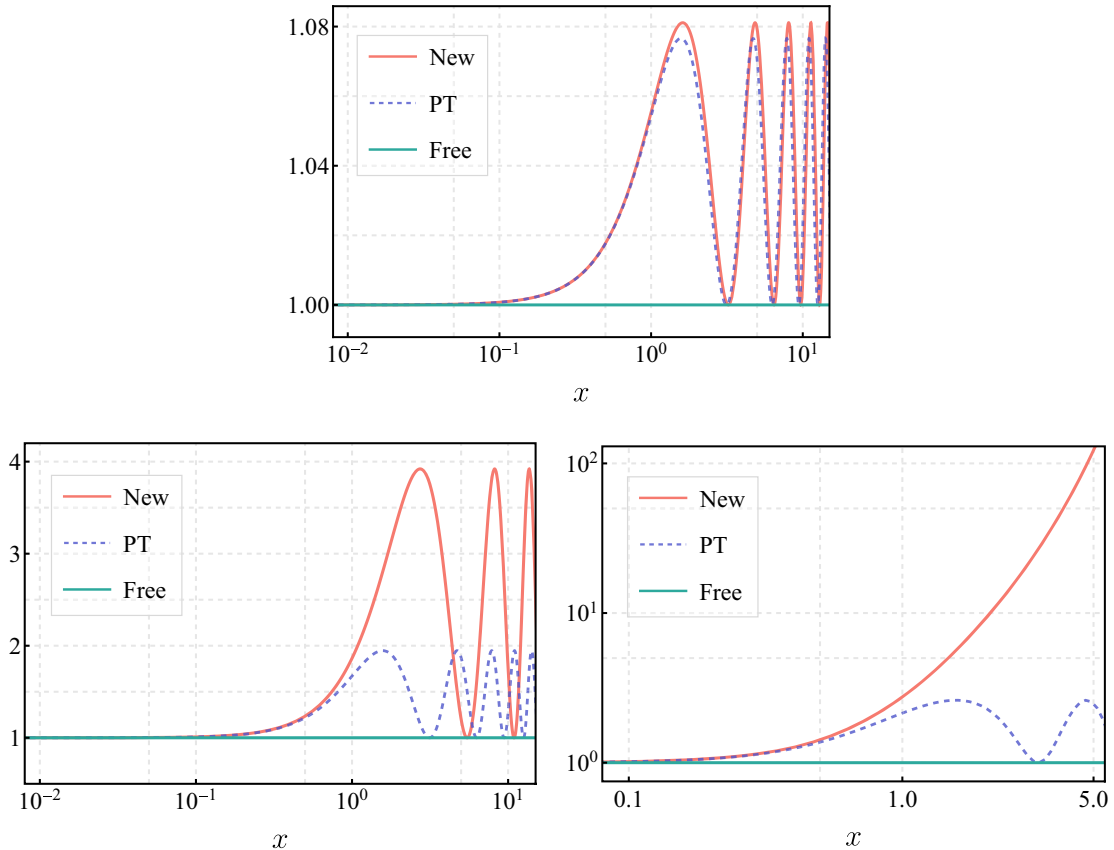


FIG. 2. Graviton number is computed using the Heisenberg equation (red solid line) and 1-loop perturbation theory (blue dashed line) in the Minkowski background. The initial conditions are set to $n(0) = 1$, $\theta(0) = \pi$, and the parameter x_{pl} is set to 10^{-41} . *Top*: $x_\beta = 3 \times 10^{-21}$, i.e., $\alpha \sim 0.054$. The emission effect is small and remains perturbative. *Bottom left*: $x_\beta = 1.6 \times 10^{-21}$, i.e., $\alpha \sim 0.67$. The deviation between the Heisenberg equation and the 1-loop perturbation theory becomes apparent. *Bottom right*: $x_\beta = 1.4 \times 10^{-21}$, i.e., $\alpha \sim 1.14$, where a resonance effect is observed.

perturbative calculation within the in-in formalism very well, with only a few percent difference. As α increases to $\mathcal{O}(10^{-1})$, the deviation becomes more significant, differing by an order of magnitude, suggesting that our approach extends beyond conventional perturbative methods. However, when α exceeds unity, the differential equations become unstable, indicating the breakdown of the Born approximation. Although we numerically compared our results with the in-in calculation up to one-loop order, the deeper connection between our method and the in-in formalism deserves further investigation, which we leave as a direction for future work.

Higher temperatures or longer graviton wavelengths enhance stimulated effects. As the temperature increases, the deviation from perturbation theory becomes more pronounced. As we discussed before (4.7), the stable regime corresponds to $\alpha < 1$. For $\alpha > 1$, the graviton number undergoes resonant exponential amplification. The resonance time scale is given by $(k\alpha)^{-1}$.

So far, we have considered a Minkowski background, treating the thermal scalar field as a spectator field. In this setup, the thermal bath temperature is a free parameter. However, the backreaction to spacetime dynamics cannot be neglected if the energy density $\rho_\chi = \pi^2/(30\beta^4)$ is large. Consider a thermal bath of size $\ell = 2\pi\gamma/k$ with some numerical factor γ , and k is the wavenumber of the emitted graviton. For $\gamma > 1$, a graviton can be enclosed within the bath. The time scale ℓ is related to the curvature radius, and the physical Hubble parameter is given by $H = 1/(2\ell)$. To ensure that the backreaction remains small, we require

$$3M_{\text{pl}}^2 H^2 \gg \rho_\chi. \quad (4.10)$$

This condition can be rewritten as

$$\alpha \ll \frac{1}{40\pi^2\gamma^2}. \quad (4.11)$$

Thus, α must remain at most at the subpercent level to keep the backreaction to spacetime negligible. Since α is determined by the graviton wavenumber and the thermal bath temperature, the stimulated emission effect remains well within the perturbative regime as long as the backreaction is insignificant.

Before closing this section, we offer several comments on the equations of motion we obtained. Firstly, it turns out that $\lambda = 0$ is a valid solution to the differential equations in the flat background, that corresponding to the expectation value of $L_{\mathbf{k}}$ in any particle number eigenstates. Thus, one can safely assume that graviton mixed states can be expanded in terms of the Hamiltonian eigenstates. With this initial condition, stimulated emission does not occur. In other words, assuming the graviton is initially in a mixed state that can be expanded in terms of the particle number eigenstates, stimulated emission will never occur, and n will remain constant while λ stays at zero. Secondly, at

next-to-leading order in the Markov approximation, a higher-order correction in x_{pl}/x_β appears as a time-ordered modification to Eq. (3.42). The correction to Eq. (4.2) at this order corresponds to the $\Gamma_{\chi\chi h}$ term discussed in the introduction, with an overall minus factor. This contribution generally leads to absorption, which persists even in mixed states. However, since x_{pl}/x_β is typically very small, this effect remains negligible in most cases.

V. FLRW BACKGROUND

In the flat background, a higher thermal bath temperature or a longer graviton wavelength enhances stimulated graviton emission. Such behavior is analogous to stimulated emission in quantum electrodynamics (QED). In QED, the population of excited atomic electrons also decreases more significantly for lower atomic energy levels. Thus, α has a cutoff, and the softest photon is constrained by the minimum energy gap in the atomic state that stimulates emission.

However, in the present case, k and β remain unconstrained. In the previous section, we found that the temperature or wavelength must be sufficiently small to neglect the backreaction on spacetime dynamics. At this stage, an FLRW background is essential for discussing significant stimulated emission. As α increases, the backreaction of the thermal bath on spacetime dynamics becomes inevitable. While such a situation is already interesting as a thought experiment, it is also of practical importance for gravitons predicted by inflation, as the scale-invariant spectrum is generated at the onset of the radiation-dominated universe.

In a general FLRW background, the quantities β , τ , and k are comoving. If the thermal bath of χ drives the background evolution, the energy density of χ is given by

$$\rho_\chi = \frac{\pi^2}{30(a\beta)^4}. \quad (5.1)$$

During radiation domination, the physical Hubble parameter H and the scale factor a satisfy the relation $aH = \tau^{-1}$. Substituting this into the Friedmann equation

$$3M_{\text{pl}}^2 H^2 = \rho_\chi, \quad (5.2)$$

we obtain the simple expression for the coefficient

$$\alpha \equiv \frac{2}{5x^2}, \quad (5.3)$$

where $x = k\tau$. The Heisenberg equations are then given by

$$n' = \frac{2\xi}{x} + \frac{2\epsilon\eta}{5x^2}, \quad (5.4)$$

$$\xi' = 2\eta + \frac{2}{x} \left(n + \frac{1}{2} \right) - \frac{2\epsilon\eta}{5x^2}, \quad (5.5)$$

$$\eta' = -2\xi + \frac{2\epsilon}{5x^2} \left(\xi + n + \frac{1}{2} \right), \quad (5.6)$$

where we have introduced an order-counting parameter ϵ , which denotes the strength of the interactions. With $\epsilon = 1$, stimulated emission dominates over the Hubble friction term on superhorizon scales ($x \ll 1$). This behavior has been reported in perturbative analyses based on in-in formalism [18], where it results in secular, scale-invariant growth in the graviton power spectrum. This secular growth

behavior has also been confirmed by recent work [40]. The breakdown of perturbative analysis was suggested on superhorizon scales. Here, the Heisenberg equations incorporate nonlinear corrections in α as an accumulated effect, potentially leading to a partial resummation of the series in α as explicitly shown in the flat spacetime example in Sec. IV.

Assuming instantaneous reheating, if the interaction is turned off $\epsilon = 0$, the expressions denote the free evolution for n and λ , which are then given by [18]

$$n_0(x) = \frac{1}{8x^2x_R^4} [1 + 2x^2 + 2x_R^4 + 2(x_R - x + 2x_R^2x) \sin(2x - 2x_R) + (2x_R(x_R - 2x) - 1) \cos(2x - 2x_R)], \quad (5.7)$$

$$\begin{aligned} \lambda_0(x) = \frac{1}{8x^2x_R^4} [& +(4x^2x_R^2 + 4ix_Rx(x - x_R) - 2(x - x_R)^2 + 2ix + 1) \cos(2x - 2x_R) \\ & + 2(x(x - i)(2x_R - 2ix_R^2 + i) - x_R) \sin(2x - 2x_R) - (2ix + 1)(2x_R^4 + 1)], \end{aligned} \quad (5.8)$$

here τ_R denotes the reheating time, while $x_R = k\tau_R$ is the corresponding dimensionless variable. These equations are derived by matching the inflationary mode functions with those during the radiation-dominated era. Consequently, the initial conditions for the differential equations above can be chosen by evaluating the given expressions at x_R , yielding

$$n(x_R) = \frac{1}{4x_R^2}, \quad \lambda(x_R) = \frac{1}{4x_R^2} - \frac{i}{2x_R}. \quad (5.9)$$

We illustrate the results for various values of ϵ in Fig. 3. For smaller ϵ , the numerical solutions of the Heisenberg equations remain consistent with perturbation theory based on the in-in formalism. On subhorizon scales, the dynamics also agree with perturbation theory except for their

amplitudes, as the interaction terms vanish for large x . On the superhorizon scale, the interaction term becomes dominant, and exponential growth occurs, just like the case in the Minkowski background.

As pointed out in Ref. [16], the secular growth in the superhorizon limit is subtle. Unlike the Minkowski background, superhorizon gravitons exhibit subtle behaviors. In the linear theory, superhorizon tensor modes are constant, and they can be eliminated by a large gauge transformation, which is an asymptotic symmetry in the FLRW background [55]. Recently, it has been suggested that soft gravitons reside in a symmetric vacuum state, expressed as a superposition of infinitely degenerate vacua [56]. Eliminating soft gravitons implies that we take the symmetric vacuum with $n = 0$ at the initial time; however, this is not a simple solution. After horizon reentry, the graviton spectrum is

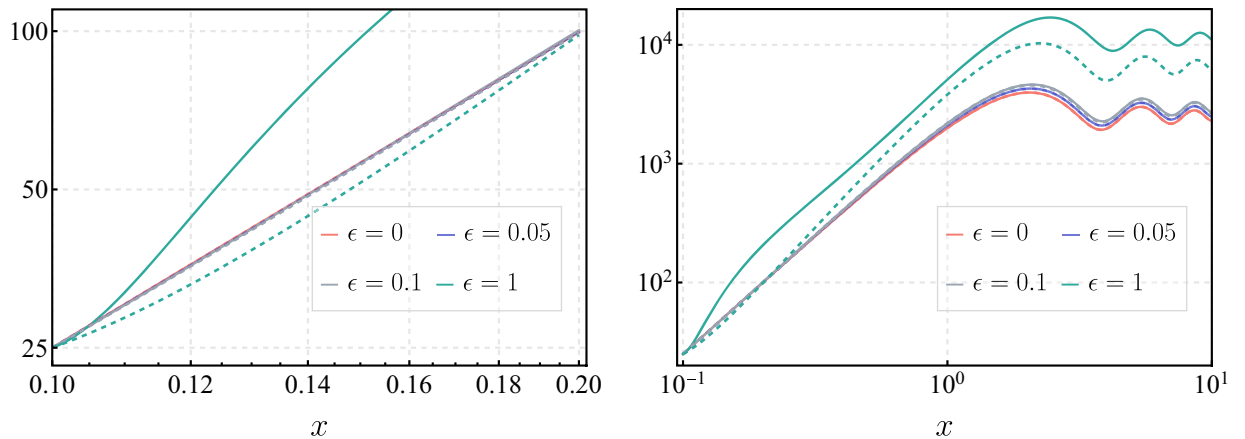


FIG. 3. *Left and Right:* n as a function of $x = k\tau$ with $x_R = 0.1$ for new (solid) and perturbation theory (dashed) solutions for $\epsilon = 0, 0.05, 0.1, 1$. The left figure focuses on $x < 0.2$.

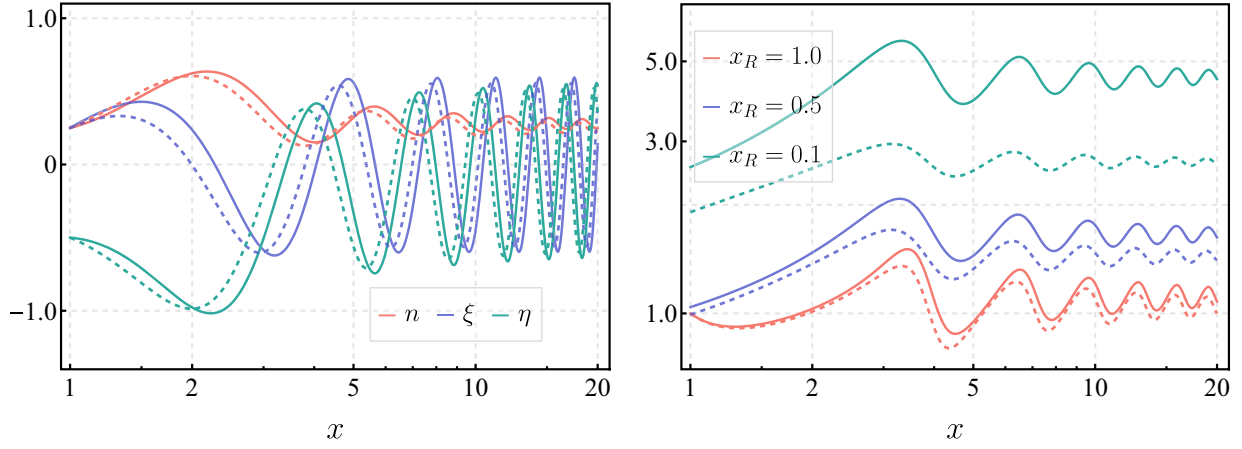


FIG. 4. *Left:* The evolution of n (red), ξ (blue), η (green) for free theory (dashed) and interaction theory (solid) with $\epsilon = 1$ and $x_R = k\tau_R = 1$. *Right:* n/n_0 as a function of $x = k\tau$ with $\epsilon = 1$, and $x_R = 1, 0.5, 0.1$, respectively. Solid lines represent the Heisenberg equation, and dashed lines represent perturbation theory results.

expected to persist as a remnant of the primordial gravitons, requiring a return to the original symmetry-broken phase. Thus, the final spectrum depends on the choice of vacuum, and one must project the symmetric state onto the symmetry-broken vacuum. The challenge lies in the implementation of this projection, as stimulated emission becomes significant prior to horizon reentry. Even if we choose $n_0 = 0$, corresponding to the symmetric vacuum, vacuum-stimulated emission still arises due to the vacuum fluctuations. This effect is then resonantly amplified, ultimately leading to secular growth in the IR limit.

The second issue concerns the robustness of perturbation theory. One might think that the Heisenberg equation is useful because it can partially resum the loop spectrum at each time step, making it valid in the nonlinear regime, similar to the Boltzmann equation. However, we argue that the Born approximation, assumed at the outset, may break down. The interaction term is proportional to x^{-2} , suggesting a breakdown of perturbative expansion on super-horizon scales. Rigorously speaking, we cannot justify the Born approximation in Eq. (3.24) without considering the other dimensional constants, while we can only do this after obtaining the Heisenberg equation. This ambiguity arises from the dimensional coupling in general relativity. Naively, the correction terms can be expanded in a series of $1/(\beta M_{\text{pl}}) \sim \sqrt{H/M_{\text{pl}}}$. However, any negative power of x may arise, which completely breaks perturbativity for $x < 1$ at the level of the Heisenberg equation before the Born approximation.

As discussed above, the subtlety of gravitons in general relativity and the issue of perturbativity suggests that the reliable regime is only a few Hubble times after reheating, i.e., $x \gtrsim 1$. In Fig. 4, we illustrate the stimulated emission for $x_R = 1$, which corresponds to the gravitons inside the horizon when inflation ends with instantaneous reheating. In this regime, the issues discussed above are absent, and

our equation of motion is reliable. Even for $x_R = k\tau_R = 1$, stimulated emission is observed. In the right panel of Fig. 4, n/n_0 is shown as a function of $x = k\tau$ and $x_R = 1, 0.5, 0.1$. Solid lines represent the Heisenberg equation, and dashed lines represent perturbation theory. As x_R decreases, the initial gravitons become softer, leading to a larger stimulated emission.

Finally, we close this section with a comment. Even without interaction ($\epsilon = 0$), n and $\lambda = \xi + i\eta$ couple due to cosmic expansion, meaning that $\lambda = 0$ cannot be taken as a consistent ansatz in the expanding background where $x \sim 1$ is allowed. This indicates the failure of the assumption behind the standard kinetic theory, where $x \rightarrow \infty$ is implicit. Quantum mechanical states are inevitably squeezed in a time-dependent background, even if they are initially separable.

VI. IR REGULARIZATION AND LARGE GAUGE TRANSFORMATION

The secular growth in the graviton number indicates the superhorizon evolution of the IR mode [16]. In this section, we discuss the IR dynamics and present a prescription for eliminating the secular term from a symmetry perspective.

Free IR graviton modes obey the linearized Einstein equation,

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} = 0. \quad (6.1)$$

This equation of motion is invariant under the large gauge transformation

$$\xi^i = \frac{1}{2}\epsilon^{ij}x_j, \quad (6.2)$$

which turns into a constant shift in h_{ij} :

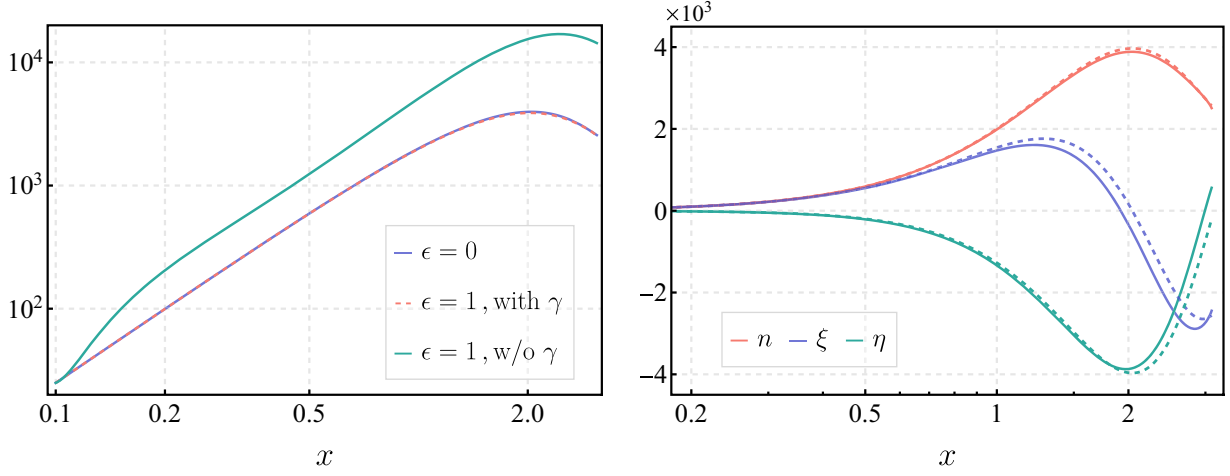


FIG. 5. *Left:* n as a function of $x = \kappa\tau$ with $x_R = 0.1$ for the free theory (blue), the interaction theory without regulator (green), and the interaction theory with the regulator γ (red dashed). The parameter ϵ is set to 1. *Right:* The evolution of n (red), ξ (blue), η (green) for the free theory (dashed) and the interaction theory with regulator γ (solid), here parameters $\epsilon = 1$ and $x_R = 0.1$.

$$h_{ij} \rightarrow h_{ij} + \epsilon_{ij}. \quad (6.3)$$

More precisely, ϵ_{ij} can correspond to either a constant or a decaying adiabatic mode [57]. The latter results in a constant shift for the conjugate momentum π_{ij} . Since these adiabatic modes can be eliminated by a large gauge transformation, free IR graviton modes are considered gauge dependent. As the secular term is on the IR scale, we need to understand if the effect is physical or not in our setup.

The residual gauge symmetry is regarded as a global anisotropic rescaling symmetry, which can be spontaneously broken by the quantum state chosen by inflation. Primordial gravitons are quantum-mechanically generated during inflation. When generated, they were within the horizon and the adiabatic vacuum is chosen as a vacuum state in the remote past. After the horizon exit, the quantum state remains adiabatic, which is regarded as a symmetry-broken phase in the IR regime [56].

In the present case, the stimulated emission is regarded as a tachyonic instability due to the negative mass squared. In fact, one may consider an effective Hamiltonian

$$H_{\text{eff}} = -\frac{1}{5}a^4H^2 \int d^3x h^{ij}h_{ij}, \quad (6.4)$$

which reproduces the Heisenberg equation we found in Sec. III E. From Eq. (6.4), one can read an effective graviton mass due to the coupling to radiation¹:

$$m_{\text{eff}}^2 = -\frac{2}{5}H^2, \quad (6.5)$$

where H is the physical Hubble parameter. For massive gravitons, the explicit symmetry breaking takes place even

for the IR equation of motion due to the mass, rather than the gradient. Equation (6.1) with this negative effective mass squared is given by

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \frac{2}{5}\left(\frac{a'}{a}\right)^2 h_{ij} = 0. \quad (6.6)$$

Equation (6.6) implies that large gauge symmetry is explicitly broken in the dynamics of the IR graviton modes.

By regarding the tensor mode as the Nambu-Goldstone mode of the spontaneously broken anisotropic rescaling symmetry, one can naturally implement its nonlinear realization around the background field $h(x_0)$,

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \frac{2}{5}\left(\frac{a'}{a}\right)^2 (h_{ij} - h_{ij}(x_0)) = 0. \quad (6.7)$$

In this way, the shift symmetry of the effective mass term can be restored. With this choice, the superhorizon h_{ij} remains constant, eliminating the tachyonic instability on superhorizon scales.

We now derive the effective interaction Hamiltonian corresponding to Eq. (6.7). Solving the linear dynamics during the radiation-dominated era gives massless evolution

$$h_{\mathbf{k}}^{(s)}(x) = u(x - x_0)h_{\mathbf{k}}^{(s)}(x_0), \quad (6.8)$$

where

$$u(x) \equiv \frac{\sin x}{x}, \quad (6.9)$$

and $h_{\mathbf{k}}(x_0)$ is the amplitude at the initial time. To be more precise, we need to take the linear solution for Eq. (6.7), Eq. (6.9) is enough to discuss the role as a regulator. In our discussion, we take $x_0 = x_R$. While $u(x)$ represents the graviton mode function, its imaginary part is suppressed for

¹The same tachyonic mass has also been discussed in [40].

$x_R = k\tau_R \rightarrow 0$. Thus, when considering superhorizon modes at reheating, we may neglect the imaginary part. Then, the effective mass after gauge fixing (6.7) is recast into γm_{eff}^2 with

$$\gamma \equiv 1 - u^{-1}. \quad (6.10)$$

Consequently, we simply replace α with $\gamma\alpha$ in Sec. III E. Since $\gamma \rightarrow 0$ in the superhorizon limit, γ acts as an IR regulator, effectively eliminating the secular term. After removing the secular term, the stimulated emission rate is at the percent level, consistent with calculations in the Minkowski background, as illustrated in Fig. 5.

We acknowledge that the present IR prescription is *ad hoc*, in the sense that it only aims at eliminating the IR secular growth by enforcing large gauge symmetry. The effective mass in the IR region is scale independent, and therefore the subtraction of the mass term is unique. However, its extrapolation to subhorizon scales is not straightforward, since the regulator implies a degeneracy of constant solutions in the IR regime. As the introduction of boundary data in Eq. (6.7) resolved this issue, the secular growth we encountered may in fact be a consequence of the nonlocality of the process under consideration. Moreover, when deriving α in Sec. III C, our explicit calculations suggest that there is no room to introduce such additional information. In practice, our prescription of multiplying γ amounts to replacing h^{ij} in Eq. (3.25) with $h^{ij} - h^{ij}(x_0)$. In this way, one can restore the large gauge symmetry in the interaction Hamiltonian and eliminate the secular growth. A more consistent prescription will be discussed elsewhere.

VII. CONCLUSIONS

In this paper, we considered the evolution of the graviton number operator in a thermal radiation background modeled by a scalar field. The crucial assumption was that the graviton quantum state is in a squeezed vacuum state at the initial time. This quantum state cannot be described by a classical distribution function in phase space, as the concept of particle number does not uniquely specify the state. Then, we derived the Heisenberg equation averaged over a general quantum state—the quantum state-averaged field equation of motion—rather than relying on the standard Boltzmann equation.

The advantage of this approach is that it allows us to derive the equation of motion applicable to a nonlinear regime, similar to the Boltzmann equation. This could potentially enable partial resummation of the loop calculations in the quantum field theory approach based on the in-in formalism. Compared to previous analyses based on perturbation theory, our calculations indicate that the behavior of the system can be more accurately captured in this framework.

Depending on the initial squeezed state, the graviton number can either increase or decrease. This phenomenon,

known as cosmological stimulated emission, was pointed out in Ref. [18] based on perturbation theory. Like stimulated emission in QED, stronger emission is expected for more IR modes and higher temperatures. Unlike in QED, however, the soft graviton mode is unconstrained in this context. In particular, the inflationary paradigm predicts a scale-invariant primordial graviton spectrum as the initial condition.

Although we used a scalar field as a toy model of thermal radiation for simplicity, this situation is ubiquitous in the early Universe, which was dominated by radiation. Therefore, our setup is quite general. We found that superhorizon gravitons are resonantly amplified in the present analysis, which contradicts the current observational status of primordial gravitational waves [3]. A potential issue with this approach is the robustness of perturbation theory in the superhorizon regime, as well as the subtlety of superhorizon gravitons.

To explore the physical picture, we first considered the same setup in the Minkowski background, where these issues do not arise. We confirmed similar stimulated emission effects in this case. We also confirmed that our new calculations reproduce the perturbative results for small interactions, and we observed deviation from the one-loop perturbation theory as we increase the thermal bath temperature. However, in the parameter space where stimulated emission is significant, the backreaction to spacetime dynamics cannot be neglected. Thus, the FLRW background is essential for discussing these effects.

As a conservative scenario, we considered a radiation-dominated universe just after instantaneous reheating, focusing on the graviton modes inside the horizon at that time. These modes cannot be gauged away, and the system remains in the perturbative regime. We confirmed that stimulated emission significantly alters the initial graviton spectrum in this context.

Regarding the secular growth of superhorizon modes,² there may be mechanisms to suppress the effect or schemes to find a reasonable resummed value consistent with the current nondetection of primordial B modes. A promising direction could involve considering the treatment of soft gravitons in an FLRW background. Free soft gravitons are considered an artifact of large gauge symmetry for the exact $k = 0$ mode. For finite-momentum modes in a decelerating universe, these gravitons eventually reenter the horizon and generate an observable signature in the sky. Hence, the role of large gauge transformations is more subtle for finite-momentum modes. Based on this perspective, we propose a regularization prescription from a symmetry-based viewpoint. In our interpretation, the residual large gauge symmetry in the IR effectively behaves as a global symmetry, which is *explicitly* broken by an effective mass

²Similar secular growth is reported in Refs. [58–60] in other contexts of cosmological loop analysis.

term induced by the thermal background. This breaking introduces a natural IR regulator, determined by a boundary condition set at reheating. With this regularization, the IR graviton modes remain constant on superhorizon scales, and the secular growth is eliminated.

In this paper, we did not consider the decoherence of inflationary gravitons and assumed that $\lambda \neq 0$, which may be a crucial simplification, and a more detailed analysis is left for future work. From an engineering perspective, preparing localized radiation with high temperatures is more feasible, similar to the quark-gluon plasma generated in colliders. While this scenario differs from the homogeneous radiation background setup assumed in our work, it remains of interest, as it may offer a practical way to realize specific thermal bath configurations that can induce strong stimulated emission without significant backreaction effects.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: THE FORMULAS OBTAINED IN THE IN-IN-FORMALISM

The graviton number and internal phase at second order in the interaction Hamiltonian are given by [18],

$$n_2 = \text{Re}[\zeta(2n_0 + 1) + 2\sigma\lambda_0^*], \quad (\text{A1})$$

$$\lambda_2 = 2\zeta\lambda_0 + \sigma(2n_0 + 1), \quad (\text{A2})$$

where we define

$$\zeta \equiv 2i \int^\tau d\tau_2 \int_{\tau_2}^\tau d\tau_1 u_k(\tau_2, \tau) u_k^*(\tau_1, \tau) \frac{X(\tau_1, \tau_2)}{M_{\text{pl}}^2}, \quad (\text{A3})$$

$$\sigma \equiv 2i \int^\tau d\tau_2 \int_{\tau_2}^\tau d\tau_1 u_k^*(\tau_2, \tau) u_k^*(\tau_1, \tau) \frac{X(\tau_1, \tau_2)}{M_{\text{pl}}^2}. \quad (\text{A4})$$

The positive frequency mode function for the field operator at τ , with respect to the vacuum state at τ_0 , is denoted by $u_k(\tau, \tau_0)$. Unlike the calculation in the main text, here we integrate overtime twice, and the operators inside the integrand are time ordered. While λ_2 is not explicitly derived in the reference, its derivation closely follows that of n_2 .

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