

Regular geometries from singular matter in quasitopological gravity

Pablo Bueno^{1,*}, Robie A. Hennigar^{2,†}, Ángel J. Murcia^{1,‡} and Aitor Vicente-Cano^{1,§}

¹*Departament de Física Quàntica i Astrofísica, Institut de Ciències del Cosmos Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain*

²*Centre for Particle Theory, Department of Mathematical Sciences, Durham University, Durham DH1 3LE, United Kingdom*



(Received 1 April 2026; accepted 20 May 2026; published 27 July 2026)

Vacuum quasitopological gravity with infinitely many terms in the action satisfies Markov's limiting curvature hypothesis: the spherically symmetric solutions are regular and all curvature invariants are bounded by solution-independent scales. We study how this picture changes when the theory is coupled to matter. We find that minimally coupled matter spoils the scaling properties of the vacuum equations that lead to the validity of Markov's hypothesis but find that the corresponding geometries often remain regular. We make this precise by developing a set of sufficient conditions on general static, spherically symmetric stress tensors such that the corresponding solutions have bounded curvature. These conditions cover regular matter sectors but also singular matter profiles that are sufficiently singular in a sense we quantify. Our conclusions hold independently of the matter field equations and include configurations in which matter exhibits divergent energy density and pressure at finite radius or at Killing horizons, results that may have implications for mass inflation in these models. We then explore nonminimal couplings, focusing on theories with infinite towers of higher-curvature and electromagnetic terms in the action. In this class, Markov's hypothesis can be restored: we present theories admitting a universal upper bound on curvature, independent of the mass and charge. Overall, our results highlight subtleties in coupling quasitopological gravity to matter and suggest Markov's hypothesis as a potential selection criterion for resummed gravity-matter effective theories.

DOI: [10.1103/3mk3-j8vs](https://doi.org/10.1103/3mk3-j8vs)

I. INTRODUCTION

According to general relativity (GR), black holes possess spacetime singularities in their interiors [1,2]. As experimental evidence for the existence of black holes [3–6]—or objects that closely resemble them [7]—continues to accumulate, the theoretical question concerning the fate of singularities becomes increasingly pressing, despite their experimental inaccessibility.

Among the possible alternatives, a conceptually conservative yet physically radical approach is provided by regular black holes (RBHs): spacetimes which preserve an exterior event horizon while replacing the central singularity with a regular core concealed behind an inner

horizon [8–12]. Remarkable progress regarding the physical properties of RBHs has been achieved relying on purely kinematical considerations [13–17]. However, the absence of a general dynamical framework has remained the Achilles' heel of the RBHs program since its inception more than half a century ago.

To date, the most widely investigated approach to realizing RBHs as solutions of concrete theories has been to couple GR to suitably chosen—more or less exotic—matter [18–35]. Nevertheless, generic black hole solutions in these models remain singular, with RBHs occurring only in highly restricted corners of parameter space. In particular, since all vacuum solutions of GR persist as solutions, the underlying singularity problem remains essentially unaltered. Alternative routes and mechanisms have been proposed, e.g., in [36–61].

Recently, static RBHs corresponding to multiparametric generalizations of the Schwarzschild solution in general dimensions have been shown to arise as the unique spherically symmetric solutions of broad classes of theories involving GR coupled to infinite towers of higher-curvature corrections [62]. All such theories belong to the so-called quasitopological (QT) class [63–71], characterized by possessing second-order equations on spherically

*Contact author: pablobueno@ub.edu

†Contact author: robie.a.hennigar@durham.ac.uk

‡Contact author: angelmurcia@icc.ub.edu

§Contact author: avicentecano@icc.ub.edu

symmetric backgrounds, as well as for satisfying versions of the Birkhoff theorem [72]. With the exception of the $D = 4$ case—which requires nonpolynomial densities [73]¹—the theories are perturbatively equivalent, via field redefinitions, to the most general gravitational effective action [62,75]. Remarkably, within this framework, such RBHs arise as the by-product of the gravitational collapse of ordinary matter [76–78]. Thus, QT theories offer a general dynamical framework within which a wide range of questions concerning the physical properties of RBHs can be systematically probed for the first time.² In this paper we address one such question: namely, how does singular matter affect the regularity of spacetimes that are nonsingular in the vacuum case?

As shown in [107], (vacuum) RBHs in any QT theory satisfy a version of Markov’s limiting curvature hypothesis (LCH) [36,108]; i.e., each of their curvature invariants remains bounded above by a certain universal mass-independent quantity.³ The situation changes in the presence of matter. Indeed, as shown in [109], perfect-fluid stars in QT theories may reach arbitrarily high curvatures as generalized versions of the Buchdahl limit are approached, unless the dominant energy condition is imposed. Nevertheless, the interplay between singular matter and regular geometry turns out to be remarkably more subtle and counterintuitive, in general. Indeed, in this paper we perform a general analysis of this interplay for minimally coupled, static, spherically symmetric matter, remaining largely agnostic about the details of the matter model. We observe that regular matter sectors produce regular spacetimes, and so do matter configurations with central power-law density singularities. On the other hand, spacetime singularities appear in the presence of matter with mildly singular energy densities at finite radii. Interestingly, the trend is reversed for sufficiently singular matter. In that case, which includes configurations with divergent energy density and pressure at finite radii or at Killing horizons, the regularity of spacetime can be preserved.

Our results align with a recent striking finding concerning the fate of mass inflation at the inner horizon of RBHs in this setting [110]. Specifically, by analyzing a model of two null shells colliding near the inner horizon, Frolov and Zelnikov demonstrated that the mass inflation instability does not develop for RBHs in QT theories, at least within the regime where a classical metric description remains valid.

In another vein, understanding the actual interplay of matter and geometry in general effective theories requires the consideration of higher-order terms in curvature and

matter, including nonminimal couplings. As a first step in this program, we also study special theories of gravity and a gauge $(D - 3)$ -form which admit magnetically charged black hole solutions characterized by a single metric function fulfilling an algebraic equation. Such electromagnetic quasitopological gravities [54,55,57,61,111–113] have been identified in general spacetime dimensions $D \geq 3$ and may incorporate nonminimally coupled terms of any order in the curvature and gauge field strength. We will discuss in which cases these theories admit magnetically charged regular black holes, as well as specify sufficient conditions for Markov’s limiting curvature hypothesis to hold in electrovacuum solutions.

The structure of the remainder of the paper is the following. In Sec. II we explain the general setup which we exploit throughout most of the paper. This entails considering general static and spherically symmetric solutions of QT gravities coupled to a minimally coupled—but otherwise general—matter stress tensor. We obtain the general form of all possible combinations of the metric functions susceptible to appearing in general algebraic curvature invariants. In Sec. III we review the argument which establishes the fulfillment of Markov’s limiting curvature hypothesis for general vacuum spherically symmetric regular black holes in QT theories, as well as the conditions needed for such solutions to exist. In Sec. IV we analyze the interplay between singular matter and regularity in the case of matter satisfying $T^t_t = T^r_r$. We show that singular matter leads to singular geometries whenever the singularity is mild enough, whereas regular geometries are produced by sufficiently singular matter. In Sec. V we consider general stress tensors. We find a similar pattern to the one observed in the restricted case of the previous section. In particular, we show that inner and cosmological horizons remain regular in the presence of matter densities and radial pressures which blow up at such horizons. Some explicit examples corresponding to minimally coupled electromagnetic and scalar fields are worked out in Sec. VI. In Sec. VII we move beyond minimally coupled matter and analyze various models of QT theories coupled to electromagnetic QT gravities. We show that for some of them there exist universal bounds for curvature invariants—both independent of the mass and the charge of the solutions. We argue that such bounds are lost when the nonminimally coupled matter is turned off and exclusively minimally coupled matter is kept. We conclude with several comments and future directions in Sec. VIII. Various intermediate calculations regarding the effects of different types of singular matter for several families of qualitatively different QT models are relegated to Appendixes A and B. Finally, in Appendix C we remove the staticity condition on the matter models and verify the boundedness of curvature invariants for cosmological spacetimes filled with perfect-fluid matter.

¹See also [49,52,74] for previous results involving RBHs as solutions to nonpolynomial gravities.

²For additional progress on the properties of RBHs in QT models, see [79–106].

³This is a highly nontrivial property which many proposed RBH models, such as the Bardeen black hole [9], fail to satisfy.

II. STATIC SPHERICALLY SYMMETRIC SOLUTIONS WITH MATTER

A general higher-curvature theory built from contractions of the metric and Riemann tensor can be written as

$$I = \int d^D x \sqrt{|g|} \left[\frac{\mathcal{L}(R_{abcd}, g^{ef})}{16\pi G_N} + \mathcal{L}_{\text{matter}} \right]. \quad (1)$$

The corresponding equations of motion read [114]

$$P_a{}^{cde} R_{bcde} - \frac{g_{ab} \mathcal{L}}{2} - 2\nabla^c \nabla^d P_{acdb} = 8\pi G_N T_{ab}, \quad (2)$$

where

$$P^{abcd} \equiv \left[\frac{\partial \mathcal{L}}{\partial R_{abcd}} \right], \quad (3)$$

and, in general, the field equations are fourth order in derivatives.

In what follows we will focus on QT gravities [63–72], a broad class of higher-curvature theories with nontrivial representatives at essentially every curvature order in general dimensions $D \geq 5$ (the only exception occurs at $n = D/2$ in even dimensions, for which the corresponding densities are trivial). A key feature of QT gravities is that, despite their higher-derivative nature, their spherically symmetric sector is typically governed by markedly simpler equations and—except for a measure-zero subset—these theories satisfy a Birkhoff theorem, so their most general spherically symmetric vacuum solutions are static and take a Schwarzschild-Tangherlini-like form.

We consider Lagrangians of the form

$$\mathcal{L}(R_{abcd}, g^{ef}) = R + \sum_{n=2}^{n_{\max}} \tilde{\alpha}_n \mathcal{Z}_n, \quad (4)$$

where $\tilde{\alpha}_n$ are arbitrary couplings with dimensions of $\text{length}^{2(n-1)}$, introducing new length scales in the theory. In this work, we shall work in the limit $n_{\max} \rightarrow \infty$, for which the vacuum solutions of the corresponding theories are regular black holes under very mild assumptions [62].

The QT densities $\mathcal{Z}_n$ can be obtained at arbitrary order starting from the first five, $\mathcal{Z}_i$, $\{i = 1, \dots, 5\}$, using the recursive formula [68]

$$\begin{aligned} \mathcal{Z}_{n+5} = & \frac{3(n+3)\mathcal{Z}_1\mathcal{Z}_{n+4}}{D(D-1)(n+1)} - \frac{3(n+4)\mathcal{Z}_2\mathcal{Z}_{n+3}}{D(D-1)n} \\ & + \frac{(n+3)(n+4)\mathcal{Z}_3\mathcal{Z}_{n+2}}{D(D-1)n(n+1)}. \end{aligned} \quad (5)$$

The explicit form of the first five densities is not particularly illuminating and can be found, for instance, in Eq. (7) of [76].

We study general static and spherically symmetric metrics, which can be written as

$$ds^2 = -N(r)^2 f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{D-2}^2. \quad (6)$$

In this setting, we consider quasitopological gravity minimally coupled to matter. The general form of the stress tensor compatible with spherical and time-reflection symmetry reads

$$T_a^b = \text{diag}(-\rho, p_r, p_t, \dots, p_t). \quad (7)$$

In a spacetime region where t is the time coordinate, then $\rho(r)$ is the energy density, $p_r(r)$ is the radial pressure, and $p_t(r)$ is the tangential pressure. The same form of the stress tensor also applies in regions where $f(r) < 0$, such as black hole interiors, though $\rho(r)$ is no longer an energy density in that case.⁴ Throughout, we shall refer to $\rho(r)$ as the energy density with the understanding that this name derives from the physical interpretation of $\rho(r)$ in the static coordinate chart. If $p_r = p_t$, then the stress tensor describes a perfect fluid, but we shall not, in general, impose this requirement. Local conservation of the stress tensor implies that

$$\begin{aligned} \frac{dp_r}{dr} = & -\frac{1}{2}(\rho + p_r) \frac{d \log(N^2 f)}{dr} \\ & - \frac{(D-2)(p_r - p_t)}{r}. \end{aligned} \quad (8)$$

For this system, there are two independent gravitational field equations, which read

$$\frac{d}{dr} [r^{D-1} h(\psi)] = -\frac{16\pi G_N}{(D-2)} r^{D-2} T_t^t, \quad (9)$$

$$\frac{N'}{rN} = \frac{8\pi G_N}{(D-2)f h'(\psi)} (T_r^r - T_t^t). \quad (10)$$

The characteristic function $h(\psi)$ encodes all the information about the quasitopological theory under consideration. It takes the form

$$h(\psi) \equiv \psi + \sum_{n=2}^{n_{\max}} \alpha_n \psi^n, \quad (11)$$

where we defined

$$\psi \equiv \frac{1-f(r)}{r^2}, \quad \alpha_n \equiv \tilde{\alpha}_n \frac{(D-2n)}{(D-2)}, \quad (12)$$

with $\tilde{\alpha}_n$ the couplings appearing in the action.

⁴Similarly, we will refer to r as the radial coordinate throughout, even inside horizons where it is timelike.

Combined with the generalized Bianchi identity and local conservation of the stress tensor, one finds the angular field equations are satisfied. We therefore have a system of three differential equations for five unknown functions $\{N(r), f(r), \rho(r), p_r(r), p_t(r)\}$. Hence, two more conditions must be provided to obtain a closed system. These can be taken to be the equation of state of the system and a prescription for the pressure anisotropy $p_r - p_t$. If the matter arises from an action principle, then these conditions are automatically provided as a consequence of the matter field equations. An important observation for our work here is that we will be able to characterize the behavior of the spacetime curvature *without* imposing matter field equations. Hence, we shall obtain results that hold independent of the matter model, requiring only certain behaviors for ρ and p_r .

Let us now proceed to obtain the form of $f(r)$ and $N(r)$. It will be convenient to introduce rescaled versions of the energy density and pressure according to

$$\{\varrho, \mathbf{p}_r, \mathbf{p}_t\} \equiv \frac{8\pi G_N}{(D-2)} \{\rho, p_r, p_t\}. \quad (13)$$

The solution of the first field equation is expressed most simply after the introduction of a new function $S(r)$ defined as [86]

$$S(r) = \frac{2M}{r^{D-1}} + \frac{2}{r^{D-1}} \int_{\infty}^r x^{D-2} \varrho(x) dx, \quad (14)$$

where

$$M \equiv \frac{8\pi G_N}{(D-2)\Omega_{D-2}} M \quad (15)$$

with M the ADM mass of the solution. The first field equation then reads

$$h(\psi) = S(r), \quad (16)$$

which we can formally solve by introducing the inverse function of $h(x)$ which we denote $H(x)$, i.e., $H(h(x)) = x$,

$$\psi = H(S) \Rightarrow f(r) = 1 - r^2 H(S). \quad (17)$$

While the characteristic function $h(x)$ is what is prescribed by the theory, we shall see that it is the inverse function $H(x)$ that is most useful for assessing the regularity of solutions.

For $N(r)$, we have the equation

$$\frac{N'}{rN} = \frac{(\mathbf{p}_r + \varrho)}{fh'(\psi)}. \quad (18)$$

Taking the normalization such that $N(\infty) = 1$ and writing the equation in terms of $H(x)$,⁵ we can solve the above as

$$N(r) = \exp \left[\int_{\infty}^r \frac{x H'(S(x)) (\mathbf{p}_r(x) + \varrho(x))}{1 - x^2 H(S(x))} dx \right]. \quad (19)$$

We note that we have not used the conservation equation in arriving at the above results. Hence, they hold generally, independent of the matter equations.

Our main objective is to determine the conditions under which solutions to this system have bounded curvature invariants. To this end, we now introduce the Riemann tensor for the general static and spherically symmetric metric (6), which is most conveniently written as

$$R_{ab}{}^{cd} = \mathcal{R}^{(1)} \tau_{[a}^{[c} \rho_{b]}^{d]} + \mathcal{R}^{(2)} \tau_{[a}^{[c} \sigma_{b]}^{d]} + \mathcal{R}^{(3)} \rho_{[a}^{[c} \sigma_{b]}^{d]} + \mathcal{R}^{(4)} \sigma_{[a}^{[c} \sigma_{b]}^{d]}, \quad (20)$$

where we have defined

$$\begin{aligned} \mathcal{R}^{(1)} &= -\frac{6f'N' + 2Nf'' + 4fN''}{N}, & \mathcal{R}^{(2)} &= -\frac{2Nf' + 4fN'}{rN}, \\ \mathcal{R}^{(3)} &= -\frac{2f'}{r}, & \mathcal{R}^{(4)} &= 2\frac{(1-f)}{r^2}. \end{aligned} \quad (21)$$

Here, $\tau_a^b = \delta_a^t \delta_t^b$, $\rho_a^b = \delta_a^r \delta_r^b$, and $\sigma_a^b = \delta_a^b - \rho_a^b - \tau_a^b$ are projectors on the t , r , and sphere directions, respectively. As they are projectors, we have $\tau_a^c \tau_c^b = \tau_a^b$, $\rho_a^c \rho_c^b = \rho_a^b$, and $\sigma_a^c \sigma_c^b = \sigma_a^b$. Since they project onto orthogonal subspaces, we have that the contraction between any two different projectors vanishes, $\tau_a^b \rho_b^c = \tau_a^b \sigma_b^c = \rho_a^b \sigma_b^c = 0$. Finally, the traces satisfy $\tau_a^a = 1 = \rho_a^a$ and $\sigma_a^a = D - 2$.

Any algebraic curvature invariant of order n —built from contractions of the metric and curvature tensors and products thereof—can be expressed as a homogeneous polynomial of degree n in the $\mathcal{R}^{(i)}$. For instance, the Kretschmann scalar is an $n = 2$ invariant and reads

$$\begin{aligned} R_{abcd} R^{abcd} &= \frac{1}{4} \left[(\mathcal{R}^{(1)})^2 + (D-2)((\mathcal{R}^{(2)})^2 + (\mathcal{R}^{(3)})^2) \right] \\ &\quad + \frac{(D-2)(D-3)}{2} (\mathcal{R}^{(4)})^2. \end{aligned} \quad (22)$$

Therefore, a sufficient condition for all polynomial curvature invariants to remain bounded is that each $\mathcal{R}^{(i)}$ be bounded. This is in fact also necessary: the $\mathcal{R}^{(i)}$ are real, and there exist invariants (e.g., the Kretschmann scalar above) that involve only even powers of the $\mathcal{R}^{(i)}$ with strictly positive numerical coefficients, precluding any generic cancellations among terms.

⁵Here we use the relationship between the derivative of a function and its inverse, $h'(\psi) = 1/H'(S)$.

We must evaluate the above curvature components on the metrics under consideration. Noting the identities

$$S' = -\frac{(D-1)S}{r} + \frac{2Q}{r}, \quad S'' = \frac{D(D-1)S}{r^2} - \frac{2DQ}{r^2} + \frac{2Q'}{r}, \quad (23)$$

and using the solution for $N(r)$ and $f(r)$ presented above, we can obtain the following expressions for the various Riemann tensor components:

$$\begin{aligned} \mathcal{R}^{(1)} = & 4H(S) + 2(D-1)(D-4)SH'(S) - 4(D-4)QH'(S) \\ & + 4rQ'H'(S) + 2((D-1)S - 2Q)^2H''(S) \\ & - 6r[((D-1)S - 2Q)H'(S) - 2H(S)]W \\ & - 4(1 - r^2H(S))(W' + W^2), \end{aligned} \quad (24)$$

$$\mathcal{R}^{(2)} = 4H(S) - 2(D-1)SH'(S) - 4p_rH'(S), \quad (25)$$

$$\mathcal{R}^{(3)} = 4H(S) - 2(D-1)SH'(S) + 4QH'(S), \quad (26)$$

$$\mathcal{R}^{(4)} = 2H(S), \quad (27)$$

where we introduced the shorthand

$$W(r) = \frac{N'(r)}{N(r)} = \frac{rH'(S)(p_r + Q)}{1 - r^2H(S)}. \quad (28)$$

The $\mathcal{R}^{(i)}$ are complicated. To obtain a better understanding of the boundedness conditions, we shall study different scenarios, beginning with the vacuum case.

To help orient the discussion, let us introduce two representative classes of characteristic functions. The Hayward-like family of resummations is given by

$$h(x) = \frac{x}{(1 - \alpha^N x^N)^{1/N}} \Rightarrow H(x) = \frac{x}{(1 + \alpha^N x^N)^{1/N}} \quad (29)$$

where N is a positive integer and α is the length-squared scale of the theory. In the case $N = 1$, the vacuum solution of the theory is the D -dimensional Hayward regular black hole. A second example is the arctanh resummation for which we have

$$h(x) = \frac{\text{arctanh}(\alpha x)}{\alpha} \Rightarrow H(x) = \frac{1}{\alpha} \tanh(\alpha x). \quad (30)$$

As it will be important later, let us note that as $|x| \rightarrow \infty$ in the Hayward-like models, we have that the inverse function decays polynomially as $H'(x) \sim |x|^{-\beta}$ and

$H''(x) \sim |x|^{-(\beta+1)}$ with $\beta = N + 1$,⁶ while in the arctanh model the decay is superpolynomial with $H'(x), H''(x) \sim \exp(-2|x|)$ as $|x| \rightarrow \infty$. There are, of course, many other possible models—see, e.g., [62,86] for additional examples. However, here the main point is to illustrate that models with regular black holes exist for which the inverse function decays either polynomially or superpolynomially for large argument.

III. LIMITING CURVATURE HYPOTHESIS IN VACUUM

In vacuum we have $Q = p_r = p_t = 0$. The field equations simplify so that we have $N(r) = 1$ and

$$f(r) = 1 - r^2H(S) \quad \text{with} \quad S(r) = \frac{2M}{r^{D-1}}. \quad (31)$$

The $\mathcal{R}^{(i)}$ also simplify to

$$\begin{aligned} \mathcal{R}^{(1)} = & 4H(S) + 2(D-1)(D-4)SH'(S) \\ & + 2(D-1)^2S^2H''(S), \end{aligned} \quad (32)$$

$$\mathcal{R}^{(2)} = \mathcal{R}^{(3)} = 4H(S) - 2(D-1)SH'(S), \quad (33)$$

$$\mathcal{R}^{(4)} = 2H(S). \quad (34)$$

The crucial observation here (which was first made in [107] in a different notation) is that in vacuum the $\mathcal{R}^{(i)}$ involve only $H(S)$, its derivatives, and S —other parameters of the solution (such as the mass) make no explicit appearance. Since any algebraic curvature invariant will be a homogeneous function of the $\mathcal{R}^{(i)}$, any such invariant will inherit this property. As a result, if any polynomial curvature invariant is bounded, then that bound cannot depend on the mass in vacuum. This is the essence of Markov's limiting curvature hypothesis: a universal upper bound on the curvature.

So, then, under which circumstances are the algebraic curvature invariants bounded? As we have described, an algebraic curvature invariant of order n will necessarily be a homogeneous polynomial of order n in $\mathcal{R}^{(i)}$. In vacuum, the $\mathcal{R}^{(i)}$ themselves are bounded provided that each of

$$\{H(S), SH'(S), S^2H''(S)\} \quad (35)$$

are all bounded on their corresponding domain. In vacuum, $S \in (0, \infty)$, and this gives the domain of H . The inverse function H must be such that it is bounded on that domain. Combined with the boundedness of $H(x)$, the second and

⁶Unless otherwise stated, $f(x) \sim g(x)$ denotes asymptotic equivalence up to multiplicative constants (not necessarily with unit ratio). Whenever the multiplicative constant is relevant, it will be written explicitly.

third conditions tell us that $H'(x)$ and $H''(x)$ are not permitted to have poles of order 1 and 2, respectively, at $x = 0$. More importantly, those conditions also tell us that as $x \rightarrow \infty$ we must have $|H'(x)|$ decay faster than $1/|x|$ and $|H''(x)|$ decay faster than $1/x^2$. If these conditions are met, then all polynomial curvature invariants will be bounded (and therefore regular).⁷ The example resummations we gave in Eqs. (29) and (30) satisfy these conditions.

The regularity conditions are most easily expressed in terms of the inverse function $H(x)$, but what the theories actually give us is the characteristic function $h(x)$. So, what do these conditions imply for $h(x)$? The existence and uniqueness of the inverse function $H(x)$ can be ensured provided that $h(x)$ is strictly monotonic on its domain. The existence of an Einstein gravity limit with a positive Newton constant further requires $h'(0) = 1$, and hence $h(x)$ should be monotone increasing. The requirement that $H(x)$ has a range which is bounded implies that the domain of $h(x)$ is finite. Moreover, the fact that the (vacuum) domain of $H(x)$ is $[0, \infty)$ indicates that $h(x)$ must have this interval as its range, on a finite domain. Hence, the function $h(x)$ should be a strictly monotone increasing function defined on an interval $x \in [0, b)$ with a divergence at the endpoint of the interval. The conditions on the decay of the derivatives of $H(x)$ can be translated into the following endpoint requirements on $h(x)$ and its derivatives:

$$\lim_{x \rightarrow b^-} \left| \frac{h(x)}{h'(x)} \right| = 0, \quad \lim_{x \rightarrow b^-} \left| \frac{h(x)^2 h''(x)}{(h'(x))^3} \right| = 0. \quad (36)$$

It is difficult to translate these requirements into useful necessary/sufficient conditions on the coupling constants themselves, and we shall not do that here.⁸

IV. MATTER SATISFYING $T'_t = T'_r$

Before addressing the most general case, let us first consider minimally coupled matter satisfying $T'_t = T'_r$. This condition implies $\mathbf{p}_r = -\mathbf{q}$; then, (19) yields $N'(r) = 0$, so $N(r)$ is a constant that we may set to unity. Hence, for $T'_t = T'_r$ the most general static solution is characterized by a single metric function, $f(r)$. This constraint may seem severe but, for example, Maxwell theory, as well as any theory of nonlinear electrodynamics, satisfies this property [86].

Under these assumptions, the $\mathcal{R}^{(i)}$ simplify considerably and read

⁷The marginal decay $H'(x) \sim 1/|x|$ would keep $xH'(x)$ bounded but forces $H(x) \sim \log|x|$, contradicting bounded $H(x)$ (and yielding divergent curvature).

⁸A set of such conditions was proposed in [62], but those conditions were incorrect. A correct set of *sufficient* conditions on the coupling constants appears in [88]. We thank Pedro G. S. Fernandes for bringing this to our attention. None of the results of [62] or follow-ups are affected by this correction.

$$\begin{aligned} \mathcal{R}^{(1)} &= 4H(S) + 2(D-1)(D-4)SH'(S) \\ &\quad - 4(D-4)\varrho H'(S) + 4r\varrho' H'(S) \\ &\quad + 2((D-1)S - 2\varrho)^2 H''(S), \end{aligned} \quad (37)$$

$$\mathcal{R}^{(2)} = \mathcal{R}^{(3)} = 4H(S) - 2(D-1)SH'(S) + 4\varrho H'(S), \quad (38)$$

$$\mathcal{R}^{(4)} = 2H(S). \quad (39)$$

Once minimally coupled matter is introduced, it is already clear that any parameter-independent curvature bounds will generally be lost. Indeed, the curvature coefficients now acquire explicit dependence on r and on the matter profile ϱ , in addition to H and S . This raises a more basic question: do the curvature invariants remain bounded at all? If so, what (if any) conditions must the matter sector satisfy in order to ensure bounded curvature?

A new feature arises in the presence of matter. Just as in vacuum, the range of $h(x)$ (and therefore the domain of H) is determined by $S(r)$. In vacuum $S(r) \in (0, \infty)$, but with matter this domain can differ as $S(r)$ can take on negative values. In fact, not only can this happen, but if the matter energy density is positive this will happen generally. Therefore, let us make a few assumptions to allow progress. First, let us suppose that the domain of $H(x)$ is the real numbers. This means we are allowing for $S(r) \in \mathbb{R}$. In certain cases this assumption will be stronger than necessary, but this will allow us to avoid worrying here about the range of $S(r)$ for matter models in a case-by-case analysis. Our assumption that the domain of $H(x)$ contains the real numbers therefore ensures that the original function $h(x)$ was invertible for all $S(r) \in \mathbb{R}$. This is not true for the Hayward(-like) theories in Eq. (29) with *odd* N , but it does hold, for example, for

$$h(x) = \frac{x}{\sqrt{1 - \alpha^2 x^2}} \Rightarrow H(x) = \frac{x}{\sqrt{1 + \alpha^2 x^2}} \quad (40)$$

as well as for many others [62,86], including the Hayward (-like) theories in Eq. (29) with *even* N as well as the arctanh model (30). Let us further suppose that we have chosen the theory such that the polynomial curvature invariants are all bounded in vacuum, satisfying the limiting curvature hypothesis in that case. Finally, we assume that the matter is well behaved in the sense that ϱ decays sufficiently fast near infinity that the integral required for $S(r)$ converges there.

We then find that boundedness of $\mathcal{R}^{(i)}$ requires boundedness of

$$\begin{aligned} \varrho(r)H'(S), \quad r\varrho'(r)H'(S), \\ \varrho(r)^2 H''(S), \quad \varrho(r)SH''(S). \end{aligned} \quad (41)$$

The question is whether the vacuum boundedness conditions imply these relationships or whether the boundedness

of these quantities requires additional restrictions on the matter content of the theory. What makes this somewhat nontrivial is that $S(r)$ itself depends on q in a complicated manner, and this can lead to surprising consequences. We collect our calculations in Appendixes A and B and comment here on the results.

A little thought reveals that the vacuum boundedness conditions on their own do not, in general, imply boundedness of the quantities in (41). The underlying reason is simple: the resummation suppresses matter contributions only when the argument $S(r)$ is driven to large magnitude, $|S| \rightarrow \infty$, so that $H'(S)$ and $H''(S)$ are small. If instead the density diverges while $S(r)$ remains finite, then $H'(S)$ and $H''(S)$ evaluate to ordinary finite numbers and cannot compensate the blowup of q or q' .

A simple counterexample is to take $q(r)$ to have an *integrable* singularity at some $r = r_p > 0$. In such a case, $S(r_p)$ remains perfectly finite yet the density is divergent; hence, the curvature terms grow unbounded, reflecting the divergent matter energy density. To make this more concrete, let us take an explicit example. Suppose we take the $N = 2$ Hayward model, determined by (40), which meets all the conditions we have specified. For the density profile, let us take

$$q(r) = \frac{M_p}{\pi r^{D-1}} \sqrt{\frac{r_p}{|r - r_p|}}, \quad (42)$$

where M_p is some constant with units of mass. Then, it is easy to check that $S(r_p) = (2M - 2M_p)/r_p^{D-1}$, which is clearly finite. Moreover,

$$H'(x) = \frac{1}{(1 + \alpha^2 x^2)^{3/2}} \quad (43)$$

which is also finite when evaluated for $x = S(r_p)$. Hence, $q(r)H'(S)$ diverges at $r = r_p$ —and it is a simple exercise to check that $q^2 H''(S)$, $qSH''(S)$ and $rq'(r)H'(S)$ are also divergent in this example—and therefore curvature scalars will generically be divergent at $r = r_p$ in such a model.

We have concluded that even in the simplest setting of minimally coupled matter satisfying $T_t^t = T_r^r$, the boundedness of the vacuum curvature invariants does not imply the boundedness of those same invariants in the presence of matter. However, this is not particularly surprising: at this point we have demanded essentially nothing from the matter sector. One would like to understand what additional constraints must be placed on $q(r)$ so that curvature invariants remain bounded.

Clearly, if the matter sector itself satisfies certain boundedness properties, e.g., $|q| < q_{\max}$ along with the boundedness of $q'(r)$, then the curvature invariants will also be bounded. However, the full situation is actually (and perhaps surprisingly) far less constraining than this. The key point is whether the matter blowup forces $|S| \rightarrow \infty$,

since only then does the large- $|S|$ decay of $H'(S)$ and $H''(S)$ become effective.

First, if $q(r)$ is finite for all $r > 0$ and exhibits a power-law divergence at the center,

$$q(r) \sim \frac{1}{r^{\gamma_1}} \quad (r \rightarrow 0^+; \gamma_1 > 0), \quad (44)$$

then $|S(r)| \rightarrow \infty$ as $r \rightarrow 0$ (either because the vacuum term $2M/r^{D-1}$ dominates or because the matter contribution does). In this regime the resummation suppresses the potentially dangerous combinations in (41). In particular, assuming the same large- $|S|$ behavior that is sufficient to ensure bounded curvature in vacuum [namely, $H'(S) \sim |S|^{-\beta}$ and $H''(S) \sim |S|^{-(\beta+1)}$ with $\beta > 1$ or faster], one finds that all density-driven terms in (41) remain bounded as $r \rightarrow 0^+$ when $T_t^t = T_r^r$. We provide the detailed estimates in Appendix A.

Second, one can even consider more severe circumstances in which $q(r)$ has a nonintegrable power-law singularity at some finite radius $r = r_p > 0$,

$$q(r) \sim \frac{1}{(r - r_p)^{\gamma_1}} \quad (r \rightarrow r_p^+; \gamma_1 > 1). \quad (45)$$

In this case the integral defining $S(r)$ diverges; thus, $|S| \rightarrow \infty$ as $r \rightarrow r_p$, and the resummation can again suppress the matter blowup. For resummations with polynomial large- $|S|$ decay (as in the Hayward-like models), Appendix A shows that bounded curvature is ensured provided that $H'(S) \sim |S|^{-\beta}$ with $\beta > 1$ and that the blowup is sufficiently strong:

$$\gamma_1 \geq \frac{\beta + 1}{\beta - 1}. \quad (46)$$

The direction of this inequality is not a typo: for fixed β there is a *lower* bound on the strength of the matter singularity required for it to be regularized. In cases where derivatives of the inverse function decay faster than polynomial for a large argument, the above becomes a sufficient but not necessary condition: polynomial matter blowups are regularized for any $\gamma_1 > 1$ in such a case, as explained in Appendix B. Schematically, these cases work because the divergences of q occur simultaneously with divergences of $S(r)$, so, for example, $qH'(S)$ inherits the same asymptotic behavior as $SH'(S)$, and similarly for the other terms in (41).

Finally, the marginal finite-radius case $\gamma_1 = 1$ is more subtle: $|S|$ diverges only logarithmically, and the outcome depends on the *rate* of decay of $H'(S)$ and $H''(S)$ (see Appendix B for a summary). For polynomial decays, the suppression is not strong enough, and the curvature invariants diverge, as shown in Appendix A.

Putting these cases together, we find a simple organizing principle in the $T'_t = T'_r$ sector: singular matter leads to singular curvature precisely when the singularity is mild enough that it does not drive $|S| \rightarrow \infty$ (as for integrable finite-radius blowups). Conversely, when the matter or vacuum blowup forces $|S|$ large (as at the center, or for nonintegrable finite-radius divergences), the resummation can suppress the would-be divergences and yield bounded curvature. Thus, we reach the somewhat surprising conclusion that quasitopological gravity gives rise to regular geometries sourced by singular matter provided that the matter energy density is sufficiently singular.

V. GENERAL CASE

Let us now move on to the general case, for which we will not have $\mathbf{p}_r(r) = -q(r)$. We then need to deal with the general components of the Riemann tensor given in (24). In addition to the boundedness of the vacuum curvature invariants, we must also have the boundedness of the terms in (41) along with the new terms

$$\begin{aligned} \mathbf{p}_r(r)H'(S), \quad rW(r)H(S), \quad rW(r)qH'(S), \\ rW(r)SH'(S), \quad f(r)W'(r), \quad f(r)W(r)^2. \end{aligned} \quad (47)$$

Before turning to the regimes where the resummation can suppress matter divergences, it is useful to isolate a few situations in which curvature singularities are essentially unavoidable. First, if q has an *integrable* singularity at some $r_0 > 0$, then $S(r_0)$ remains finite, and $H'(S(r_0))$ is just a finite number; thus, terms such as $qH'(S)$ will diverge, and curvature invariants will be singular. Similarly, if $\mathbf{p}_r$ has a singularity at some $r_0 \geq 0$ while $q(r_0)$ remains either finite or has an integrable singularity, then terms such as $\mathbf{p}_r H'(S)$ diverge and the geometry is singular. This latter case includes, for example, constant-density stars with divergent internal pressure [109].

So far, these are singularities driven directly by corresponding (mild) divergences in the matter sector. Let us now assume that the energy density and radial pressure are singular at $r = 0$ with behavior

$$q \sim \frac{1}{r^{\gamma_1}}, \quad \mathbf{p}_r \sim \frac{1}{r^{\gamma_2}}, \quad (48)$$

with $\gamma_1 > 0$ and $\gamma_2 \in \mathbb{R}$. As we discovered in the previous section (see Appendix A for details), all the terms involving the energy density in (41) are automatically finite. The only subtleties arise from the pressure-dependent terms, and as we show in Appendix A, the solutions will be regular in the case of polynomial decay provided that

$$\begin{aligned} \gamma_2 \leq \beta(D-1) \quad \text{if } \gamma_1 \leq D-1, \\ \gamma_2 \leq \beta\gamma_1 \quad \text{if } \gamma_1 > D-1. \end{aligned} \quad (49)$$

Because $\beta > 1$ by the vacuum boundedness conditions, this means that (within these sufficient conditions) the pressure may be either less or more singular than the energy density at $r = 0$ while maintaining a regular geometry. On the other hand, if the inverse function decays faster than polynomial, the curvature invariants will be bounded with no further constraints on γ_2 (see Appendix B).

Let us now suppose that both q and $\mathbf{p}_r$ have power-law singularities at some $r = r_p > 0$, characterized by different strengths,

$$q \sim \frac{1}{(r-r_p)^{\gamma_1}}, \quad \mathbf{p}_r \sim \frac{1}{(r-r_p)^{\gamma_2}} \quad (r \rightarrow r_p^+), \quad (50)$$

with $\gamma_1 > 1$, $\gamma_2 \in \mathbb{R}$, and $f(r_p) \neq 0$. We know from the above that all terms in the $\mathcal{R}^{(i)}$ involving factors of q will be bounded provided that

$$\gamma_1 \geq \frac{\beta+1}{\beta-1}. \quad (51)$$

Examining the additional pressure- and W -dependent terms, one finds (Appendix A) that if $\gamma_2 \leq \gamma_1$ then all of the terms are bounded without any further constraint on γ_2 . However, if $\gamma_2 > \gamma_1$ then one must also have

$$\gamma_2 \leq \beta(\gamma_1 - 1) - 1 \quad (52)$$

to ensure boundedness of all terms. These conditions are derived in Appendix A. In the case of faster than polynomial decay, then there are no constraints imposed on γ_2 , and the configurations will be regular in general.

The only remaining case to examine is the marginal one, corresponding to $\gamma_1 = 1$ in (50). In that case, the behavior of $S(r)$ as we approach r_p reads

$$S(r) \sim \lambda_{\log} \log(r-r_p)(r \rightarrow r_p^+), \quad (53)$$

where $\lambda_{\log} = \frac{2}{r_p} \lim_{r \rightarrow r_p^+} (r-r_p)q(r)$. This behavior causes the matter fields to diverge exponentially in $|S|$, resulting in a curvature singularity, since no finite β can suppress this growth. See Appendix A for more details. However, as we discuss in Appendix B, if the decay of the inverse function is superpolynomial, in many circumstances this marginal blowup can also be regularized.

A qualitatively different divergence can arise in the general case due to the W -dependent terms in $\mathcal{R}^{(1)}$. In particular, the contribution

$$\frac{\mathbf{p}_r + q}{f(r)} \quad (54)$$

is dangerous. In the presence of a horizon $r = r_h$ where $f(r_h) = 0$, one generically finds divergent curvature

invariants in any theory of gravity unless the horizon regularity condition

$$p_r(r_h) = -\varrho(r_h) \quad (55)$$

holds (assuming ρ and p_r are themselves finite there). Physically, this reflects a “blueshift” divergence. To see this, consider the local energy density recorded by a freely falling radial observer. The D -velocity satisfies $u_a u^a = -1$ with nontrivial components

$$u^t = \frac{E}{N^2 f}, \quad u^r = -\sqrt{\frac{E^2}{N^2} - f}, \quad (56)$$

and the local energy density perceived by this observer is

$$\rho_{\text{loc}} = T_{ab} u^a u^b = \frac{E^2 (p_r + \rho)}{N^2 f} - p_r, \quad (57)$$

which diverges at $f = 0$ unless $p_r = -\rho$ at the horizon. In most constructions of static black holes sourced by matter, $p_r(r_h) = -\rho(r_h)$ is imposed as a regularity condition [115–117].

Let us now consider an alternative possibility, motivated by the previous results: sufficiently singular matter can sometimes yield bounded curvature invariants in quasitopological gravity. Specifically, we can ask whether ϱ and p_r may blow up at a horizon while the geometry nevertheless remains regular. Mathematically, one finds that this is only possible provided that (i) the horizon is an inner or cosmological horizon and (ii) ϱ is negative in its vicinity. The reason is that for matter to diverge at a horizon in a manner that can be suppressed in the geometry, one must have $S(r) \rightarrow +\infty$ as $r \rightarrow r_h$, which requires a locally negative ϱ , and the resulting near-horizon form of $f(r)$ forces $f'(r_h) < 0$.

To see this explicitly, suppose that near $r = r_h$ the matter behaves as

$$\varrho(r) \sim -\frac{A}{(r - r_h)^{\gamma_1}}, \quad p_r(r) \sim \frac{B}{(r - r_h)^{\gamma_2}}, \quad (58)$$

with constants A, B and exponents γ_1, γ_2 such that $\gamma_1 > 1$. [Note that the marginal $\gamma_1 = 1$ results in a curvature singularity for polynomially decaying $H(x)$, as in the finite-radius case.] Extracting the most divergent part of $S(r)$ near $r = r_h$, we find

$$S(r) \sim \frac{2A}{(\gamma_1 - 1)r_h} \frac{1}{(r - r_h)^{\gamma_1 - 1}}. \quad (59)$$

Clearly, if $A > 0$ then $S(r) \rightarrow +\infty$ as $r \rightarrow r_h$, while $S(r) \rightarrow -\infty$ if $A < 0$. Let us denote

$$\psi_{\pm} = \lim_{x \rightarrow \pm\infty} H(x). \quad (60)$$

Then,

$$f(r_h) = 1 - r_h^2 \psi_{\pm}. \quad (61)$$

Requiring $f(r_h) = 0$ implies $\psi_{\pm} = 1/r_h^2$, which is only consistent when $\psi_{\pm} > 0$. For the relevant examples under consideration [i.e., monotonically increasing and bounded functions $H(S)$ for every $S \in \mathbb{R}$, like those in (29) and (30)] this selects ψ_+ , and therefore we must have $S(r) \rightarrow +\infty$ as $r \rightarrow r_h$, which is where the requirement $A > 0$ comes from.⁹

What does this imply for the boundedness of curvature invariants? Without modification, we can leverage the results of the previous section to conclude that all terms appearing in (41) are bounded provided that

$$\gamma_1 \geq \frac{\beta + 1}{\beta - 1}, \quad (62)$$

for polynomially decaying inverse functions, while $\gamma_1 > 1$ suffices if the resummation decays superpolynomially. In any event, we note that (62) forces

$$f(r) \approx -\frac{2(r - r_h)}{r_h} + \dots, \quad (63)$$

so the horizon must be an inner or cosmological one.

On the other hand, one finds

$$f(r)W(r)^2 \sim \frac{C_1}{(r - r_h)^{2\theta_1 + 1}} + \frac{C_2}{(r - r_h)^{2\theta_2 + 1}}, \quad (64)$$

$$f(r)W'(r) \sim \frac{C_3}{(r - r_h)^{\theta_1 + 1}} + \frac{C_4}{(r - r_h)^{\theta_2 + 1}}, \quad (65)$$

where $\theta_1 = \gamma_1 - \beta(\gamma_1 - 1)$, $\theta_2 = \gamma_2 - \beta(\gamma_1 - 1)$, and C_1, C_2, C_3 , and C_4 are constants depending on A, B , and other factors. If $\gamma_2 \leq \gamma_1$, (62) is enough to ensure the boundedness of fW^2 and fW' . If $\gamma_2 > \gamma_1$, W is bounded provided (62) holds and

$$\gamma_2 \leq \beta(\gamma_1 - 1) - 1. \quad (66)$$

We therefore conclude that, despite a slightly more involved analysis for the case of a blowup at a horizon, the boundedness conditions are exactly the same as in the more general finite-radius blowup case [see (51) and (52)].

⁹Note that for an inner horizon residing in a black hole interior or “outside” a cosmological horizon, t is not a time coordinate and hence $\rho(r)$ is not the matter energy density. The requirement of $A > 0$ remains perfectly consistent with positive local energy density.

It is then relatively easy to verify that all terms entering the $\mathcal{R}^{(i)}$ are individually bounded, and hence the solution is regular at $r = r_h$. We therefore see another example of how quasitopological gravity can yield regular geometries in cases where the matter fields are sufficiently singular: the blueshift singularity described above generically results in a singular horizon *except* in the case where the matter fields are themselves singular at the horizon. This curious result may have implications for understanding the mass inflation instability of regular black holes in quasitopological gravity—see [110].

Above we have assumed that $N(r_h)$ takes on a finite constant value at a zero of $f(r_h)$. One may wonder what happens if this is no longer the case. For example, if $N(r)$ exhibits a $N(r) \sim 1/\sqrt{r-r_h}$ blowup at r_h , then the interpretation is different and represents a breakdown of the chart (6). In such a case, it is easy to verify that the metric is not singular and the surface $r = r_h$ represents a kind of “throat.” On the other hand, if $N(r) \sim (r-r_h)^\sigma$ for any $\sigma \neq 0, -\frac{1}{2}$, one sees that the geometry will be singular at $r = r_h$.

The discussion pertaining to horizons has so far focused on the case where the inverse function exhibits polynomial decay at large argument. When the inverse function decays in superpolynomial fashion, the constraints for horizons are identical to those for matter singularities at finite radius—see Appendix B. In particular, sufficiently strong superpolynomial decay can regularize even the case of a marginal ($\gamma_1 = 1$) matter divergence at an inner horizon, depending on the details of the matter model.

VI. EXAMPLES

We now illustrate the general results obtained so far with explicit examples. We consider electrodynamics and a minimally coupled scalar field.

A. Electrodynamics

Let us restrict ourselves to purely electric spherically symmetric Ansätze. In this case, a general second-order theory of electrodynamics $\mathcal{L}^{\text{EM}}(s)$ can be written as a general function of the electromagnetic invariant

$$s = -\frac{1}{4}F_{ab}F^{ab}. \quad (67)$$

The corresponding stress tensor of such a theory takes the form of (7) with

$$\rho = 2s\mathcal{L}_s^{\text{EM}} - \mathcal{L}^{\text{EM}}, \quad (68)$$

$$p_r = \mathcal{L}^{\text{EM}} - 2s\mathcal{L}_s^{\text{EM}}, \quad (69)$$

$$p_t = \mathcal{L}^{\text{EM}}, \quad (70)$$

where $\mathcal{L}_s^{\text{EM}} = \partial_s \mathcal{L}^{\text{EM}}$. Hence, any theory of electrodynamics satisfies the $T'_t = T'_r$ condition in the entire spacetime.

The simplest and most interesting example is Maxwell electrodynamics, for which $\mathcal{L}^{\text{EM}}(s) = s$. The Maxwell equations are solved by $F = dA$ with

$$A = \sqrt{\frac{(D-2)}{16\pi G_N(D-3)}} \frac{Q}{r^{D-3}} dt, \quad (71)$$

where Q is a constant of integration. We have

$$\varrho = \frac{D-3}{2} \frac{Q^2}{r^{2(D-2)}} = -p_r \quad (72)$$

and hence

$$S(r) = \frac{2M}{r^{D-1}} - \frac{Q^2}{r^{2(D-2)}}. \quad (73)$$

For fixed (M, Q) , the function $S(r)$ takes values in $(-\infty, S_{\max}]$ with

$$S_{\max} = \frac{D-3}{D-2} M \left(\frac{(D-1)M}{(D-2)Q^2} \right)^{\frac{D-1}{D-3}}. \quad (74)$$

Since M and Q^2 are freely specifiable, positive real numbers, the union of these ranges over all allowed parameter choices is $\mathbb{R}$. Hence, when quasitopological gravity is coupled to Maxwell electrodynamics, regularity requires resummations for which $H(x)$ has $\mathbb{R}$ as the domain. This holds true for the Hayward-like models with N even and the arctanh model we have considered here.

The matter fields have power-law divergent energy density and pressure at $r=0$ with the same exponents $\gamma_1 = \gamma_2 = 2(D-2)$. Hence, by our general results the regularity of the solution is ensured by the vacuum regularity conditions, provided $H(S)$ has the domain described above. We refer to [86] for additional details and examples of regular black holes in quasitopological gravity coupled to nonlinear electrodynamics.

In another vein, we can actually argue that regular black hole solutions in these theories will not satisfy the limiting curvature hypothesis. To this end, let us write $S_0 = \frac{2M}{r^{D-1}}$ and $q^2 S_0^\sigma = \frac{Q^2}{r^{2(D-2)}}$ with $\sigma = 2\frac{(D-2)}{D-1}$ and $q^2(2M)^\sigma = Q^2$ (we consider $M > 0$). Writing $H(S) = H_q(S_0)$, it is clear from (32)–(33) that all curvature invariants will be a function of $\{H(S_0), S_0 H'_q(S_0), S_0^2 H''_q(S_0)\}$, where the prime denotes a derivative with respect to S_0 . Let $S_0^{(0)} = q^{-2/(\sigma-1)}$ be the value of S_0 such that $S = 0$. Direct computation shows that

$$H'_q(S_0^{(0)}) = H'(0)(1-\sigma). \quad (75)$$

Now, as $2 \geq \sigma \geq 4/3$ if $D \geq 4$, we have that $S_0^{(0)} \rightarrow +\infty$ as $q^2 \rightarrow 0$, while $H'(0)(1-\sigma)$ need not be small. As a result, $S_0^{(0)}H'_q(S_0^{(0)})$ can become arbitrarily large as $q^2 \rightarrow 0$, implying that Markov's proposal will not be satisfied. Although the failure of the limiting curvature hypothesis arises as one considers a zero-charge configuration, note that neutral regular black holes will, of course, satisfy Markov's proposal. These ideas can be reconciled by noting that $S \neq 0$ for vanishing charge, so no $S_0^{(0)}$ exists.

B. Scalar field

The action of a scalar field ϕ minimally coupled to gravity reads

$$\mathcal{L}^{\text{field}} = -\frac{1}{2}\partial_a\phi\partial^a\phi - V(\phi) \quad (76)$$

where $V(\phi)$ is the scalar potential. In spherical symmetry, the stress tensor that follows from this action takes the form of (7) with

$$\begin{aligned} \rho &= \frac{1}{2}f(r)(\phi')^2 + V, & p_r &= \frac{1}{2}f(r)(\phi')^2 - V, \\ p_t &= -\left(\frac{1}{2}f(r)(\phi')^2 + V\right). \end{aligned} \quad (77)$$

In this case we have $T'_t \neq T'_r$, and, in general, the solutions will have $N(r) \neq 1$. Because of the form of the stress tensor, provided that $f(r)(\phi')^2 \rightarrow 0$ at any horizon, we will have $p_r(r_h) = -\rho(r_h)$, and the horizon regularity condition will be satisfied. On the other hand, it could happen that $\phi'(r_h)$ blows up, in which case horizon regularity would depend on the strength of the blowup.

To understand this better, let us examine the behavior of the scalar in the vicinity of a horizon (which may be an event or inner horizon). The scalar equation of motion reads

$$\frac{1}{Nr^{D-2}}(Nr^{D-2}f\phi')' = V_{,\phi}(\phi), \quad (78)$$

where a “ $'$ ” denotes the radial derivative and $V_{,\phi}$ is the ϕ derivative of the potential. Near a horizon we can expand

$$f(r) = f_1(r - r_h) + \mathcal{O}(r - r_h)^2, \quad (79)$$

$$N(r) = N_0 + \mathcal{O}(r - r_h). \quad (80)$$

Then, sufficiently close to any such horizon we can approximate the scalar equation by

$$(f_1(r - r_h)\phi')' = V_{,\phi}(\phi). \quad (81)$$

Since $V_{,\phi}$ depends on ϕ , assessing the behavior of $\phi(r)$ near the horizon requires knowledge of the potential. Here,

for simplicity, we shall assume a massive scalar field with $V(\phi) = (\mu/2)\phi^2$. In that case, we can solve analytically for the behavior of the scalar near the horizon:¹⁰

$$\phi(r) = \phi_h + b_h \log \frac{|\mu|(r - r_h)}{|f_1|} + \dots, \quad (82)$$

where ϕ_h and b_h are constants of integration. We attach the subscript h to these quantities to emphasize that the analysis is local to a given horizon. If the spacetime contains multiple horizons, the same local form applies near each one but with horizon-specific values of ϕ_h and b_h .

It is clear that when $b_h = 0$ we will have a (locally) regular solution. For this case it is easy to verify that the energy density and pressure are finite and the horizon regularity condition $p_r(r_h) = -\rho(r_h)$ is satisfied with

$$\rho(r_h) = V(\phi_h) = \frac{\mu\phi_h^2}{2}. \quad (83)$$

On the other hand, when $b_h \neq 0$ the scalar will be singular at the horizon. The energy density and pressure will diverge marginally,

$$\rho \sim p_r \sim \frac{f_1 b_h^2}{2(r - r_h)}. \quad (84)$$

To contextualize these findings, it is helpful to recall how one typically obtains black hole solutions sourced by a real scalar field $\phi(r)$. Such solutions can rarely be obtained analytically, and numerical methods must be used. The approach employed is to assume the scalar is regular at the event horizon and obtain a power series solution for the scalar and metric functions there. The series expansion (for given values of the free parameters) is then used to prescribe initial data to a numerical integration scheme to integrate the solution either toward the asymptotic region or into the black hole interior.

What this means for our local solution above is that, at the event horizon, one would set $b_h = 0$ to obtain a regular event horizon. However, it will generically not be possible to control the value of b_h at more than one horizon in the spacetime. Hence, for the case of a real massive scalar, solutions with a regular event horizon will generally have $b_h \neq 0$ at any inner horizons. At such inner horizons, the scalar and its stress tensor components will diverge in the marginal manner studied above.

Applying the results of our analysis we conclude that when considering quasitopological gravity coupled to a real massive scalar, if the inverse function $H(x)$ decays

¹⁰We allow for the possibility that $\mu < 0$. This is permissible in anti-de Sitter spacetimes, and since our analysis is local, it is ultimately insensitive to the asymptotic structure.

polynomially, then the corresponding inner horizons will always be singular. However, if the inverse function decays superpolynomially, then our results indicate the inner horizon can remain regular. For example, for an inverse function that decays exponentially with $H'(S) \sim \exp(-a|S|)$, the geometry will remain regular provided

$$\frac{4\pi G_N}{(D-2)} \frac{a|f_1|b_h^2}{r_h} > 1. \quad (85)$$

Alternatively, if $H'(S) \sim \exp(-a|S|^\sigma)$ with $\sigma > 1$, then the geometry will generically be regular, requiring no further constraints on parameters.

These conclusions are of course specific to the case of a real massive scalar. For different choices of potential, the degree of divergence will differ and may therefore lead to different conclusions for which types of resummation will yield regular or singular geometries. It would be an interesting task for future work to construct such solutions numerically (e.g., with AdS asymptotics) and to classify the behavior of more general scalar potentials.

VII. ELECTROMAGNETIC QUASITOPOLOGICAL GRAVITIES

We have so far focused on minimally coupled matter in QT gravities. However, as the gravitational sector contains an infinite tower of higher-curvature terms, the minimal coupling prescription is, at best, unnatural. In a more physically plausible setup, one would simultaneously consider higher-order corrections in the matter sector along with nonminimal couplings between curvature and matter. We now turn our attention to this problem.

As an incipient endeavor in this direction, we will devote this section to the study of regular (both in matter and geometry) configurations in electromagnetic quasitopological (EMQT) gravities in $D \geq 4$ [54,55,111,113]. These theories generalize purely gravitational QTs with the inclusion of a nonminimally coupled $(D-3)$ -form $\mathcal{A}$ which appears in the action via its field strength $\mathcal{F} = d\mathcal{A}$, thus ensuring gauge invariance. More precisely, EMQTs are characterized by admitting static and spherically symmetric black hole solutions (6) with a magnetic charge such that $N(r) = 1$ and $f(r)$ is determined by a first-order differential equation which may be directly integrated into an algebraic equation, just like the usual QTs. There is an important difference, though: despite the fact that it is possible to define EMQTs to arbitrary order in the curvature and the gauge field strength, it is not clear whether these theories provide a basis for effective theories of gravity and a gauge $(D-3)$ -form. Hence, we should interpret the results in this section as a first foray into the interplay of matter and geometry in general effective theories of gravity and matter.

With these provisos in mind, let us consider the following theories of gravity and a $(D-3)$ -form comprised of

purely gravitational higher-curvature terms, higher-order matter terms, and nonminimally coupled terms:¹¹

$$\mathcal{L} = R - \frac{\mathcal{F}^2}{(D-4)!} + \sum_{m=2}^{m_{\max}} \frac{\tilde{\lambda}_m}{((D-2)!)^m} (\mathcal{F}^2)^m + \sum_{n=2}^{n_{\max}} \tilde{\alpha}_n \mathcal{Z}_n + \sum_{n=1}^{n'_{\max}} \sum_{m=1}^{m'_{\max}} \frac{\tilde{\kappa}_{n,m}}{((D-2)!)^m} \mathcal{L}_{\text{EMQT}}^{(n,m)}, \quad (86)$$

$$\begin{aligned} \mathcal{L}_{\text{EMQT}}^{(n,m)} = & n(\mathcal{F}^2)^{m-1} (\mathcal{F}^2)^{cd} [g_{n,m} (R^n)^{ab}_{cd} \\ & + (R^{n-1})^{eb}_{cd} (R \delta_e^a + 2s_{n,m} R_e^a) \\ & + 2(n-1) (R^{n-2})^{fb}_{ed} R_e^c R_f^a], \end{aligned} \quad (87)$$

where we have made the following definitions:

$$\begin{aligned} (R^n)^{cd}_{ab} &= R^{e_1 f_1}_{ab} R^{e_2 f_2}_{e_1 f_1} \dots R^{cd}_{e_{n-1} f_{n-1}}, \\ (\mathcal{F}^2)^{cd}_{ab} &= \mathcal{F}_{abe_1 \dots e_{D-4}} \mathcal{F}^{cde_1 \dots e_{D-4}}, \quad \mathcal{F}^2 = (\mathcal{F}^2)^{ab}_{ab}, \end{aligned} \quad (88)$$

$$s_{n,m} = 4m + n - 1 - (2m + n - 1)D, \quad g_{n,m} = \frac{(s_{n,m} + 1)s_{n,m}}{2n}. \quad (89)$$

The couplings $\tilde{\lambda}_m$ and $\tilde{\kappa}_{n,m}$ are arbitrary, with dimensions of $\text{length}^{2(m-1)}$ and $\text{length}^{2(m+n-1)}$, respectively. These theories comprise a special subclass of the two families of EMQTs identified in [111,113] which proves to be useful for our purposes. Throughout this section, we will consider the class of purely gravitational QTs to be contained in the EMQT family, arising in the limit of a vanishing magnetic charge.

A. Limiting curvature hypothesis in electrovacuum

Consider the equations of motion of the theory (87), without further matter content (in electrovacuum). We will be interested in static and spherically symmetric solutions (6) endowed with a magnetic field strength:

$$\mathcal{F} = P \text{vol}_{S^{D-2}}, \quad (90)$$

where P is the magnetic charge and $\text{vol}_{S^{D-2}}$ is the volume form of the $(D-2)$ -dimensional round sphere. As it turns out, EMQTs of the form (87) admit spherically symmetric solutions determined by [111,113]

$$N = 1, \quad \frac{d}{dr} (r^{D-1} h_y(\psi)) = 0, \quad (91)$$

¹¹Note that here we are allowing for the possibility of finite or infinite towers of corrections in each sector, depending on whether one takes the terms $m_{\max}$, $n_{\max}$, $m'_{\max}$, $n'_{\max}$ to infinity or keeps some set of them finite.

where we have defined

$$h_y(\psi) \equiv h_0(\psi) + h_y(0) + \sum_{n=1}^{n'_{\max}} \sum_{m=1}^{m'_{\max}} \kappa_{n,m} \psi^n y^m, \quad (92)$$

$$h_0(\psi) \equiv \psi + \sum_{n=2}^{n_{\max}} \alpha_n \psi^n, \quad (93)$$

$$h_y(0) \equiv y + \sum_{m=2}^{m_{\max}} \lambda_m y^m, \quad (94)$$

$$\kappa_{n,m} \equiv \frac{(D-2n-2m(D-2))\tilde{\kappa}_{n,m}}{2^{-n+1}(D-2)}, \quad (95)$$

$$\lambda_m \equiv \frac{\tilde{\lambda}_m}{(D-2)(D-1-2m(D-2))}, \quad (96)$$

$$y \equiv \frac{P^2}{r^{2(D-2)}}, \quad \psi \equiv \frac{1-f(r)}{r^2}. \quad (97)$$

Consequently, electrovacuum solutions are given by

$$h_y(\psi) = S_0(r), \quad S_0(r) = \frac{2M}{r^{D-1}}, \quad (98)$$

where M is related to the mass M of the solution as in (15). Restricting to positive mass $M > 0$ solutions, it is clear that $y = p^2 S_0^\sigma$ with $\sigma = 2\frac{D-2}{D-1}$ and $(2M)^\sigma p^2 = P^2$. Assuming $\partial_\psi h_y(\psi) \geq 0$ —which is achieved, e.g., by imposing $\kappa_{n,m} \geq 0$ —, it is possible to express ψ in terms of S_0 in (98) (at least implicitly), obtaining

$$\psi = H_p(S_0), \quad (99)$$

where we include the subscript p to indicate that we are working with magnetically charged solutions. This expression is formally equivalent to the equation determining black hole solutions of purely gravitational QTs [62], except for the fact that $H_p(S_0)$ depends parametrically on the magnetic charge. As a result, expressions (32)–(33) apply after exchanging $H(S)$ and derivatives thereof by $H_p(S_0)$ and its derivatives with respect to S_0 (denoted also with primes). Therefore, if

$$\{H_p(S_0), S_0 H'_p(S_0), S_0^2 H''_p(S_0)\} \quad (100)$$

remain bounded for $S_0 \in (0, +\infty)$ and $p^2 \in [0, +\infty)$, we conclude that all curvature invariants will be finite and (black hole) solutions will be regular.

A relevant question arises: will the limiting curvature hypothesis be satisfied by EMQTs with (magnetically charged) regular black hole solutions? By the assumptions (100), it turns out that curvature invariants will be bounded

for every $p^2 \in [0, +\infty)$. However, this does not guarantee that the limiting curvature hypothesis will be satisfied, as one needs to show the existence of a universal bound (independent of the charge and mass) for *any* curvature invariant. Specifically, the limiting curvature hypothesis will be satisfied if there exists $K \in \mathbb{R}$ independent of the mass and charge such that

$$K \geq \max \{H_p(S_0), S_0 H'_p(S_0), S_0^2 H''_p(S_0)\} \quad (101)$$

for every $S_0 \in [0, \infty)$ and $p^2 \in [0, \infty)$. In the following, examples of theories with (magnetically charged) regular black holes that fulfill and do not fulfill the limiting curvature hypothesis are presented:

- (1) *RBHs without LCH.* Choose¹² $\lambda_m = 0$ and $\kappa_{n,m} = 0$ if $m^2 + n^2 > 2$, $\kappa_{1,1} = \kappa > 0$, and $\alpha_n = \alpha^{n-1}$ for $\alpha > 0$. In such a case, the characteristic function reads

$$h_y^I(\psi) = \frac{\psi}{1 - \alpha\psi} + y + \kappa y \psi. \quad (102)$$

Substituting in (98), solving for ψ , and selecting the solution smoothly connected to the Hayward black hole as $p^2 \rightarrow 0$, one finds

$$\psi = H_p^I(S_0) = \frac{\tilde{S}_0 - \sqrt{4p^2 S_0^\sigma (p^2 S_0^\sigma - S_0) \alpha \kappa + \tilde{S}_0^2}}{2p^2 \alpha \kappa S_0^\sigma}, \quad (103)$$

where $\tilde{S}_0 = 1 + \alpha S_0 - p^2 S_0^\sigma (\alpha - \kappa)$. Observe that the radicand appearing in (103) will always be non-negative. For any finite p , it is easy to see that the terms (100) for H_p^I will be bounded for every $S_0 \in (0, +\infty)$ and $p^2 \in [0, +\infty)$. Hence, the solutions always represent regular black holes. Nevertheless, this is not enough to guarantee that the limiting curvature hypothesis is satisfied. As a matter of fact, let us first focus on the limit of sufficiently large p , satisfying $p^2 S_0^\sigma \gg S_0$. In such a regime, (103) simplifies to

$$H_p^I = -\frac{1}{\kappa} + \frac{1 + S_0(\alpha + \kappa)}{\kappa(\alpha + \kappa)p^2 S_0^\sigma} + \mathcal{O}(p^{-4}). \quad (104)$$

On the other hand, let us now consider the vanishing charge limit. One directly finds

¹²In these examples, for the sake of simplicity, we will be considering theories whose uncharged solutions correspond to the D -dimensional Hayward black hole for odd $D \geq 5$ [62]. Nevertheless, other instances corresponding to different resummations of the purely gravitational sector may be found.

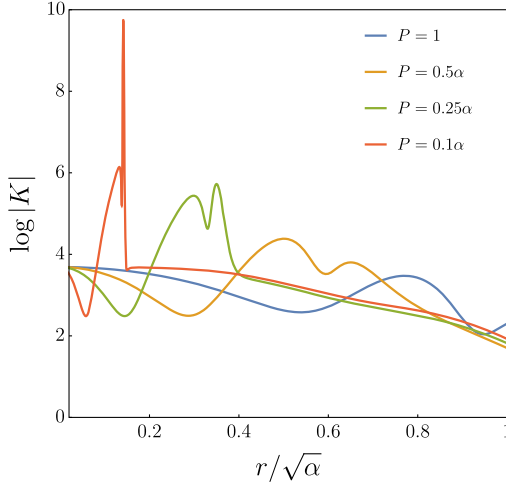


FIG. 1. Logarithm of the Kretschmann scalar associated with various solutions of a five-dimensional EMQT with characteristic function $h_y^I(\psi)$, as defined in (102). We explicitly observe that the Kretschmann scalar attains larger and larger values as $P \rightarrow 0$. In addition, we see that the maximum appears for smaller values of r as the charge diminishes. For the sake of concreteness, we have used $\kappa = \alpha = 2M$.

$$H_p^I = \frac{S_0}{1 + \alpha S_0} - \frac{(1 + (\alpha + \kappa)S_0)p^2 S_0^\sigma}{(1 + \alpha S_0)^3} + \mathcal{O}(p^4). \quad (105)$$

Let us then examine the behavior of H_p^I as $S_0 \rightarrow +\infty$ for a sufficiently small magnetic charge. Indeed, if S_0 becomes large but $p^2 S_0^\sigma$ is comparatively small—which indeed happens for a small magnetic charge—the approximation (105) applies and will only transition into the regime governed by (104) as S_0 is even larger. The crossover between these two regimes, taking place around the (nonzero) minimum value of the radicand in (103), has drastic consequences on $S_0(H_p^I)'$, which develops a steep jump near the transition as the magnetic charge goes to zero—we have checked this numerically. As a result, $S_0^2(H_p^I)''$ becomes arbitrarily large, and the theory does not satisfy the limiting curvature hypothesis. This is illustrated in Fig. 1.

- (2) *RBHs and LCH.* Take $\lambda_{2m} = 0$, $\lambda_{2m+1} = (-1)^m \kappa^m$, $\alpha_n = \alpha^{n-1}$, $\kappa_{n,2m} = 0$, and $\kappa_{n,2m+1} = \alpha^{n-1}(-1)^m \kappa^m$ for $\alpha, \kappa > 0$. The characteristic function is

$$h_y^{\text{II}}(\psi) = \frac{1}{1 - \alpha\psi} \left(\psi + \frac{y}{1 + \kappa y^2} \right). \quad (106)$$

We can invert analytically to obtain

$$\psi = H_p^{\text{II}}(S_0) = \frac{S_0 - p^2 S_0^\sigma (1 - \kappa p^2 S_0^{1+\sigma})}{(1 + \alpha S_0)(1 + \kappa p^4 S_0^{2\sigma})}. \quad (107)$$

It is straightforward to see that $H_p^{\text{II}}(S_0)$ will give rise to finite curvature invariants for every $p \in [0, \infty)$. On the one hand, as the magnetic charge $p^2 \rightarrow +\infty$, one has

$$H_p^{\text{II}}(S_0) = \frac{S_0}{1 + \alpha S_0} - \frac{1}{p^2 S_0^\sigma \kappa (1 + \alpha S_0)} + \mathcal{O}(p^{-6}), \quad (108)$$

so the infinite-charge limit will give rise to bounded curvature invariants—amusingly, the solution tends to the *neutral* black hole solution with the charge contributions effectively decoupling. Furthermore, the limit of a vanishing magnetic charge is well defined:

$$H_p^{\text{II}}(S_0) = \frac{S_0}{1 + \alpha S_0} - \frac{p^2 S_0^\sigma}{1 + \alpha S_0} + \mathcal{O}(p^6). \quad (109)$$

As the leading terms in (108) and (109) coincide, H_p^{II} does not feature the issues of the previous example as the magnetic charge goes to zero. As a result, there exists a universal bound for curvature invariants, independent of the charge and mass of the solution. This is explicitly shown in Fig. 2.

We conclude this section with two remarks. First, we note that magnetically charged regular black holes can arise without the need for an infinite tower of higher-curvature terms, as long as the charge of the solution is *nonzero* [54,55]. Indeed, take $\alpha_n = 0$ for $n > 2$, $\lambda_m = 0$ for $m > 2$, and $\kappa_{n,m} = 0$ for every $n^2 + m^2 > 2$, while $\kappa_{1,1} = 1$. In this case, we trivially have that

$$h_y^{\text{III}}(\psi) = \psi + y + \psi y, \quad (110)$$

while $\psi = H_p^{\text{III}}(S_0)$ is obtained as

$$H_p^{\text{III}}(S_0) = \frac{S_0 - p^2 S_0^\sigma}{1 + p^2 S_0^\sigma}. \quad (111)$$

As long as $p \neq 0$, the solution will have finite curvature invariants.

Second, one may ask about the case of Einstein gravity coupled to a nonlinear theory of electrodynamics. In this latter case, the characteristic function adopts the form

$$h_y^{\text{IV}}(\psi) = \psi + Q(y), \quad (112)$$

where $Q(y)$ is a certain function of y . One then has

$$\psi = H_p^{\text{IV}}(S_0) = S_0 - Q(p^2 S_0^\sigma). \quad (113)$$

Since $2 \geq \sigma > 1$ for every $D \geq 4$, no analytical choice for $Q(y)$ may render a bounded H_p^{IV} . Furthermore, even if

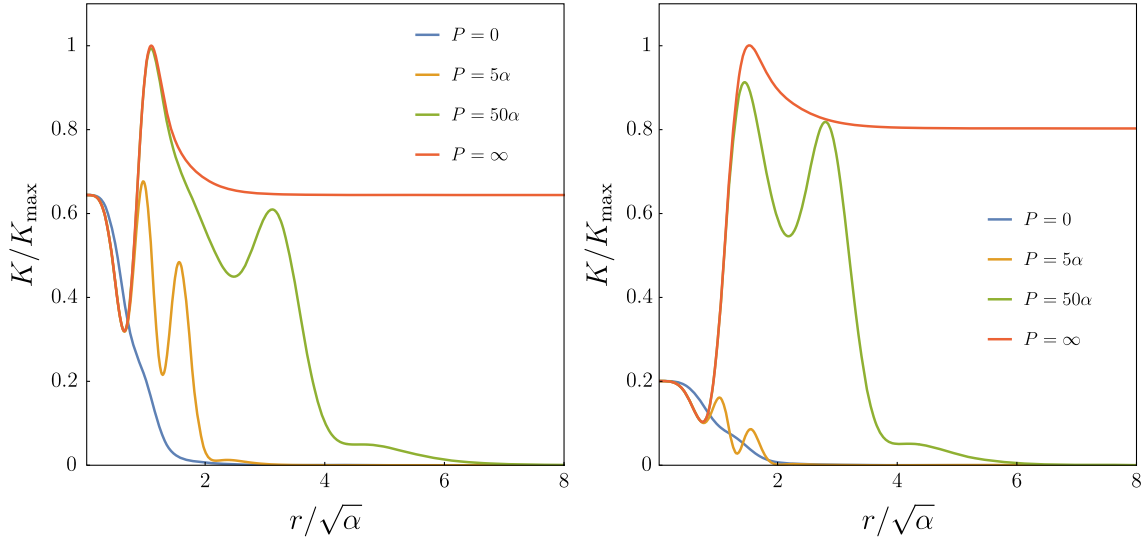


FIG. 2. Kretschmann scalar associated with various solutions of a five-dimensional EMQT with characteristic function $h_y^{\text{II}}(\psi)$, as defined in (106). In the left plot, the maximum value of the Kretschmann scalar, K_{max} , corresponds to $\sim 62/\alpha^2$, and we have used $\kappa = M = \alpha$. In the right plot, $K_{\text{max}} \sim 200/\alpha^2$, and we have used $2\kappa = M/3 = \alpha$. We explicitly observe that the limiting curvature hypothesis is satisfied: curvature invariants are bounded by a universal quantity, independent of the mass and the charge.

some nonanalytical choice for $Q(y)$ is made so that a term proportional to S_0 in $Q(y)$ is generated, the cancellation of the divergence of ψ as $S_0 \rightarrow \infty$ may only happen if the magnetic charge P and the mass M are related. This is the prototypical case of regular black hole models of GR coupled to a theory of nonlinear electrodynamics [18,19,23,118].

VIII. DISCUSSION

In this paper we have studied how matter distorts the regularity of static and spherically symmetric spacetimes in QT gravities. The bulk of our results concerned minimally coupled matter, and a summary of our main findings can be found in Table I. We also performed a preliminary investigation of nonminimal coupling to matter.

We established a largely model-independent understanding of when minimally coupled matter preserves or destroys the regularity of static, spherically symmetric solutions in QT gravities. The key object is

$$S(r) = \frac{2M}{r^{D-1}} + \frac{2}{r^{D-1}} \int_{\infty}^r x^{D-2} Q(x) dx, \quad (114)$$

which sets the argument of the inverse characteristic function $H(S)$ through $\psi = H(S)$. Curvature singularities arise precisely when the matter sector diverges while $S(r)$ remains finite, so $H'(S)$ and $H''(S)$ evaluate to ordinary finite numbers and cannot suppress the blowup. Conversely, whenever a divergence in the matter sector forces $|S(r)| \rightarrow \infty$, the large- $|S|$ decay of $H'(S)$ and $H''(S)$ generically eliminates the would-be divergence in the curvature.

A central and somewhat counterintuitive consequence is that “mild” matter singularities (integrable finite-radius singularities in the energy density) are the most problematic cases insofar as regularity is concerned. By contrast, central $r=0$ power-law divergences and nonintegrable finite-radius divergences are generally compatible with bounded curvature, with the geometry remaining regular even when the stress tensor itself is singular (Table I). These conclusions require no equation of state and do not use matter field equations. They rely only on local asymptotic behavior of ρ, p_r, p_t , and hence hold for general matter models.

In the presence of minimally coupled matter, Markov’s limiting curvature hypothesis no longer holds. Importantly, this failure is not synonymous with the appearance of a spacetime singularity. Rather, it means that the curvature scales of the geometry become solution dependent (e.g., controlled by the mass, charge, or other parameters).

Our results also highlight the subtlety of coupling QT gravity to matter. Studying minimally coupled matter is convenient for pragmatic purposes: this setting provides a tractable arena in which to study deformations of the corresponding vacuum models while remaining largely agnostic about the particular details of the matter content. What is much less clear is how such results directly inform the fundamental interplay between matter and geometry in these theories. Recall that in five and higher dimensions the quasitopological theories furnish a basis for the gravitational effective action in vacuum. From this perspective, incorporation of matter would generally proceed not by minimal coupling but by including higher-derivative operators in the matter sector together with nonminimal couplings between

TABLE I. Summary of results for minimally coupled matter. We describe the different scenarios studied in this work, expressing for each matter configuration the resulting curvature behavior along with sufficient conditions to ensure that outcome. The theories are defined by an invertible characteristic function $h(\psi)$ with $\psi = H(S) = h^{-1}(S)$. The parameter $\beta > 1$ is defined by the large- $|S|$ decays $H'(S) \sim |S|^{-\beta}$ and $H''(S) \sim |S|^{-(\beta+1)}$ that are required to ensure boundedness of vacuum curvature invariants. For faster-than-polynomial decays of $H'(S)$, the geometry remains regular for a broader class of singular matter distributions than in the polynomial decay case.

Where	Local matter behavior	Curvature outcome	Comments
Polynomial decay of the inverse function: $H'(S) \sim S ^{-\beta}$, $H''(S) \sim S ^{-(\beta+1)}$, with $\beta > 1$			
Anywhere	Regular matter: $\varrho, \mathbf{p}_r, \mathbf{p}_t$ bounded (and derivatives entering $\mathcal{R}^{(i)}$ not singular)	Regular geometry	Assuming the underlying vacuum theory already satisfies the bounded- H conditions (so vacuum polynomial invariants are bounded)
$r \rightarrow 0^+$ $f(0) = 1$	Central power-law density blowup with $T_t^t = T_r^r \Leftrightarrow \varrho = -\mathbf{p}_r$: $\varrho \sim r^{-\gamma_1}$, $\gamma_1 > 0$	Regular geometry	$ S \rightarrow \infty$ and the resummation suppresses the blowup
	Central power-law density blowup: $\varrho \sim r^{-\gamma_1}$, $\gamma_1 > 0$ $\mathbf{p}_r \sim r^{-\gamma_2}$, $\gamma_2 \in \mathbb{R}$	Regular geometry iff pressure is not too singular	Boundedness requires: $\gamma_2 \leq \beta(D-1)$ if $\gamma_1 \leq D-1$ $\gamma_2 \leq \beta\gamma_1$ if $\gamma_1 > D-1$
	Central pressure blowup: $\mathbf{p}_r \sim r^{-\gamma_2}$, $\gamma_2 > 0$ with $\varrho(0^+)$ finite, e.g., $\varrho \sim r^{-\gamma_1}$, $\gamma_1 \leq 0$	Curvature singularity	$\mathbf{p}_r H'(S)$ diverges while $H'(S)$ is finite
$r \rightarrow r_0$ s.t. either: $r_0 = r_p^+ > 0$ or $r_0 = r_h^+ > 0$ $f(r_h^+) = 0^-$ (Inner or cosmological horizon)	Nonintegrable density blowup with $T_t^t = T_r^r \Leftrightarrow \varrho = -\mathbf{p}_r$: $\varrho \sim (r-r_0)^{-\gamma_1}$, $\gamma_1 > 1$	Regular geometry iff density and pressure are singular enough	Boundedness requires: $\gamma_1 \geq \frac{\beta+1}{\beta-1}$
	Nonintegrable density blowup: $\varrho \sim (r-r_0)^{-\gamma_1}$, $\gamma_1 > 1$	Regular geometry iff density is singular enough and pressure is not too singular	Boundedness requires: $\gamma_1 \geq \frac{\beta+1}{\beta-1}$ if $\gamma_2 \leq \gamma_1$ $\gamma_1 \geq \frac{\beta+1}{\beta-1}$ and $\gamma_2 \leq \beta(\gamma_1-1)-1$ if $\gamma_2 > \gamma_1$
	$\mathbf{p}_r \sim (r-r_0)^{-\gamma_2}$, $\gamma_2 \in \mathbb{R}$		Matter fields grow exponentially in $ S $
	Marginal density blowup: $\varrho \sim (r-r_0)^{-\gamma_1}$, $\gamma_1 = 1$	Curvature singularity	
	Integrable density blowup, e.g., $\varrho \sim (r-r_0)^{-\gamma_1}$, $0 < \gamma_1 < 1$		Terms such as $\varrho H'(S)$, $r \varrho' H'(S)$ diverge while $H'(S)$ is finite
	Pressure blowup: $\mathbf{p}_r \sim (r-r_0)^{-\gamma_2}$, $\gamma_2 > 0$ with $\varrho(r_0)$ finite or integrable		$\mathbf{p}_r H'(S)$ diverges while $H'(S)$ is finite
$r \rightarrow r_h^+$ $f(r_h^+) = 0^\pm$	Static matter at a Killing horizon with finite ϱ and $\mathbf{p}_r$	Curvature singularity due to “blueshift” divergence	Avoided if the horizon regularity condition holds: $T_t^t(r_h) = T_r^r(r_h) \Leftrightarrow \mathbf{p}_r(r_h) = -\varrho(r_h)$
Faster than polynomial decay of the inverse function: $H'(S) \sim e^{-a S }$, with $a > 0$			
Main differences between polynomial (lhs of the arrows) and faster than polynomial decays (rhs of the arrows):			
Regular geometry	$\Rightarrow$	Regular geometry	
Regular geometry under some conditions	$\Rightarrow$	Regular geometry	
Curvature singularity due to exponential growth of matter fields	$\Rightarrow$	Regular geometry under some conditions (see below)	
Other types of curvature singularities	$\Rightarrow$	Curvature singularities	
Where	Local matter behavior	Curvature outcome	Comments
$r \rightarrow r_0$ s.t. either $r_0 = r_p^+ > 0$ or $r_0 = r_h^+$ $f(r_h^+) = 0^-$	Marginal density blowup: $\varrho \sim (r-r_0)^{-\gamma_1}$, $\gamma_1 = 1$ $\mathbf{p}_r \sim (r-r_0)^{-\gamma_2}$, $\gamma_2 \in \mathbb{R}$ $S(r) \sim \lambda_{\log} \log(r-r_0)$ where $\lambda_{\log} = \lim_{r \rightarrow r_0} \frac{2}{r-r_0} (r-r_0) \varrho(r)$	Regular geometry iff the geometry decays fast enough	Exponential behavior of the matter fields is suppressed by the geometry invariants if $a \lambda_{\log} \geq \max\{2, 1+\gamma_2\}$

matter and curvature. This is a substantially more difficult problem, and at present we do not know of any Einstein–matter effective field theories that can be mapped at all orders to a “quasitopological-like” frame in which the equations of motion remain comparably accessible.

Nevertheless, we performed a preliminary exploration of the effects of nonminimally coupled matter using EMQT gravities. These include nonminimal couplings to a $(D - 3)$ -form, which may be defined at any order in the curvature and the gauge field strength and generically allow for magnetically charged regular black hole solutions that are continuous deformations of the vacuum QT ones. We have presented explicit examples of EMQTs in which the electrovacuum solutions are regular and all curvature invariants are bounded by mass- and charge-independent scales; i.e., Markov’s limiting curvature hypothesis holds. This underscores the limitation of conclusions based on minimally coupled matter: a singular minimal matter sector—or failure of Markov’s hypothesis there—need not reflect the true behavior once the matter sector is resummed. Our results also show that the limiting curvature hypothesis can serve as a criterion by which to select resumptions of gravity plus matter actions.

Our results may have implications for the study of mass inflation in these models. Minimal matter couplings can be valuable as a proof of principle: the possibility of matter singularities yet regular geometry at Killing horizons suggests that QT gravity might evade mass-inflation-type instabilities, as recently argued in [110]. At the same time, our broader conclusion—that coupling QT gravity to matter is subtle—tempers how definitive such analyses can be when they rely on minimal matter sectors. In this setting, a positive result (e.g., the absence of mass inflation in [110]) is encouraging insofar as it demonstrates that avoiding the instability is possible, in principle. A negative result, however, is much less informative: as we have explicitly shown, higher-order matter corrections can play an essential role and could qualitatively change the conclusion. Overall, a conceptually complete treatment of mass inflation in QT gravity likely requires a framework that systematically incorporates higher-order matter terms.

There are a number of directions for future work. A natural extension would be to exploit the connection between QT gravities in spherical symmetry and two-dimensional Horndeski theories to consider the implications of (non)minimal matter for broader classes of theories. While QT gravities can be dimensionally reduced in spherical symmetry to a particular class of two-dimensional Horndeski theories [76], there is a much broader set of two-dimensional theories with similar properties [106,119]. It would be interesting to assess the generality of our results in that broader context and, moreover, to determine the necessary constraints (if any) for Markov’s hypothesis to be realized in vacuum or otherwise in those models. In that setting, making contact with a broader class of theories than

those defined by quasitopological gravities requires considering terms nonpolynomial in the curvature [73,102]. Such theories are on a more speculative footing than the polynomial quasitopological actions, which can arise as perturbative corrections to general relativity within an effective field theory approach. Hence, limiting curvature may serve as a selection criterion to discriminate between different choices of nonpolynomial gravitational actions, singling out representatives that may be better behaved.

Concerning nonminimal matter models, it would be particularly interesting to determine whether EMQTs are sufficiently general to capture electrovacuum effective field theory. While this is at present unclear, these models could serve, at the very least, as useful proxies towards the systematic study of the interplay between matter and geometry in general effective theories. Interestingly enough, for the class of EMQTs we examined, we observed that these lead to (magnetically charged) regular black hole solutions under analogous conditions to those of purely gravitational QTs. However, differently from the latter, ensuring the existence of regular black holes does not guarantee the fulfillment of Markov’s limiting curvature hypothesis in electrovacuum. Rather than a drawback, this provides us with a powerful physical criterion for further refining the class of EMQTs.

Another important question concerns the role of curvature invariants involving covariant derivatives of curvature—see [106]. Even in vacuum these will only be bounded provided that the metric function under consideration is C^∞ near $r = 0$. More concretely, this translates to the requirement that, should the metric function admit a series expansion in the vicinity of $r = 0$, then that series expansion should have no odd powers of r [120]. For example, this holds for the Hayward black holes provided that D is odd. Here we note that curvature invariants featuring covariant derivatives are still bounded for C^∞ regular black holes, but the bound is not independent of the mass. This is because the covariant derivative introduces new objects which cannot be written in terms of just S and (derivatives of) $H(S)$, and hence the scaling property of the polynomial invariants is lost. In the case of the quasitopological theories we have studied here, the issue is not so important as the actions of the theories we consider do not feature covariant derivatives, and hence both the solutions and the actions are bounded in a mass-independent manner. Nonetheless, it would be interesting—though more involved—to study the boundedness of invariants containing covariant derivatives of curvature. It would be especially interesting to identify a regular black hole metric for which *all* curvature invariants have a mass-independent bound—or to prove that such a construction is impossible.

ACKNOWLEDGMENTS

We thank Pablo A. Cano and Pedro G. S. Fernandes for useful discussions. P. B. was supported by a Proyecto de

Consolidación Investigadora (CNS 2023-143822) from Spain's Ministry of Science, Innovation and Universities, and by Grant No. PID2022-136224NB-C22, funded by MCIN/AEI/10.13039/501100011033/FEDER, UE. Á.J.M. was supported by Juan de la Cierva Contract No. JDC2023-050770-I from Spain's Ministry of Science, Innovation and Universities. A.V.C. was supported by Spanish Government Fellowship No. FPU24/00899. The authors also acknowledge financial support from Grant No. CEX2024-001451-M funded by MICIU/AEI/10.13039/501100011033.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: REGULARITY FOR POWER-LAW MATTER BLOWUPS

This appendix collects the regularity estimates for stress tensors that develop power-law divergences either at the center $r = 0$ or at a finite radius $r = r_p$. The two cases discussed in the main text are recovered as special cases:

- (i) $T_t^t = T_r^r$: the anisotropy function $W(r)$ vanishes, and the analysis reduces to bounds involving only ϱ .
- (ii) $T_t^t \neq T_r^r$: $W(r)$ contributes additional terms, leading to extra inequalities involving the pressure falloff exponent.

Throughout we assume $D \geq 4$ and use the same definitions as in the main text. We assume that $f(r)$ is nonzero at the location of any matter blowup—the case where the matter blows up at a Killing horizon was treated in the main text. The potentially dangerous matter-sourced contributions to curvature invariants arise as combinations of the terms

$$\begin{aligned} &\rho H'(S), \quad r\rho H'(S), \quad \rho^2 H''(S), \quad \rho S H''(S), \\ &p_r H'(S), \quad rW(r)H(S), \quad rW(r)\rho H'(S), \\ &rW(r)SH'(S), \quad W'(r), \quad W^2(r). \end{aligned} \quad (\text{A1})$$

We assume the same large- $|S|$ decay that is sufficient to bound the *vacuum* curvature invariants:

$$H'(S) \sim \frac{1}{|S|^\beta}, \quad H''(S) \sim \frac{1}{|S|^{\beta+1}}, \quad |S| \rightarrow \infty, \quad (\text{A2})$$

with $\beta > 1$. Of course, some resummations have $H(S)$ decay faster than any polynomial. In these cases, sometimes less restrictive conditions than what we will find below will hold. However, it is more difficult to assess all such possibilities in a coherent manner. We therefore stress that for resummations with faster than polynomial decays,

the following results will correspond to sufficient, but not necessary, conditions.

1. Central power-law blowup at $r = 0$

Assume that near the origin the energy density diverges as

$$\varrho(r) \sim \frac{1}{r^{\gamma_1}}, \quad r \rightarrow 0^+, \quad (\text{A3})$$

with $\gamma_1 > 0$ and ϱ otherwise smooth and such that the integral defining $S(r)$ is well behaved away from $r = 0$. In the anisotropic case we also allow

$$p_r(r) \sim \frac{1}{r^{\gamma_2}}, \quad r \rightarrow 0^+, \quad (\text{A4})$$

for some exponent γ_2 , while $W(r)$ is the same anisotropy quantity used in the main text (so that $W = 0$ when $T_t^t = T_r^r$).

We can then isolate the divergent part of $S(r)$ at small r :

$$S(r) \sim \frac{2M}{r^{D-1}} - \frac{\lambda}{r^{\gamma_1}} + \text{finite}, \quad r \rightarrow 0^+, \quad (\text{A5})$$

for some constant λ . We then consider two possibilities in turn.

a. Case 1: Matter-dominated center, $\gamma_1 > D - 1$

When $\gamma_1 > D - 1$, the matter term dominates S at small r , and one may invert

$$r \sim \left(\frac{|\lambda|}{|S|} \right)^{1/\gamma_1}. \quad (\text{A6})$$

Then

$$\varrho \sim |S|, \quad r\varrho' \sim |S|, \quad p_r \sim |S|^{\gamma_2/\gamma_1}. \quad (\text{A7})$$

Using (A2), the density-driven terms in (A1) scale as

$$\varrho H'(S) \sim |S|^{1-\beta}, \quad (\text{A8})$$

$$r\varrho' H'(S) \sim |S|^{1-\beta}, \quad (\text{A9})$$

$$\varrho^2 H''(S) \sim |S|^{1-\beta}, \quad (\text{A10})$$

$$\varrho S H''(S) \sim |S|^{1-\beta}, \quad (\text{A11})$$

which are bounded for $\beta > 1$. If $T_t^t = T_r^r$, then the argument concludes at this stage.

If $T_t^t \neq T_r^r$, a potentially new requirement comes from the pressure sector:

$$p_r H'(S) \sim |S|^{\gamma_2/\gamma_1 - \beta}, \quad (\text{A12})$$

so $p_r H'(S)$ is bounded provided

$$\gamma_2 \leq \beta \gamma_1. \quad (\text{A13})$$

If $T'_l \neq T'_r$, terms involving $W(r)$ appear [cf. (A1)]. Given that the terms involving the energy density are bounded, several of the terms involving $W(r)$ simplify, and we need to require only the boundedness of

$$W(r) \quad \text{and} \quad W'(r). \quad (\text{A14})$$

We can work out these terms to find that

$$W(r) \sim C_1 |S|^{\frac{-1+(1-\beta)\gamma_1}{\gamma_1}} + C_2 |S|^{\frac{-1+\gamma_2-\beta\gamma_1}{\gamma_1}}, \quad (\text{A15})$$

$$W'(r) \sim C_3 |S|^{1-\beta} + C_4 |S|^{\frac{\gamma_2-\beta\gamma_1}{\gamma_1}}. \quad (\text{A16})$$

If Eq. (A13) holds, then the decay of these terms introduces no new requirements.

b. Case 2: Vacuum-dominated center, $\gamma_1 < D-1$

Now the mass term dominates S , giving

$$r \sim \left(\frac{2M}{S} \right)^{1/(D-1)}. \quad (\text{A17})$$

Hence

$$q \sim S^{\frac{\gamma_1}{(D-1)}}, \quad r q' \sim S^{\frac{\gamma_1}{(D-1)}}, \quad p_r \sim S^{\frac{\gamma_2}{(D-1)}}. \quad (\text{A18})$$

Then the density-driven terms behave as

$$q H'(S) \sim S^{\frac{\gamma_1}{(D-1)}-\beta}, \quad (\text{A19})$$

$$r q' H'(S) \sim S^{\frac{\gamma_1}{(D-1)}-\beta}, \quad (\text{A20})$$

$$q^2 H''(S) \sim S^{\frac{2\gamma_1}{(D-1)}-\beta-1}, \quad (\text{A21})$$

$$q S H''(S) \sim S^{\frac{\gamma_1}{(D-1)}-\beta}, \quad (\text{A22})$$

which are bounded under the same vacuum requirement $\beta > 1$ because $\gamma_1 < D-1$ implies $\gamma_1/(D-1) < 1$. If $T'_l = T'_r$, then the argument concludes here: the vacuum boundedness conditions ensure the boundedness in the presence of matter with a central power-law singularity.

For the pressure term one has

$$p_r H'(S) \sim S^{\frac{\gamma_2}{(D-1)}-\beta}, \quad (\text{A23})$$

so boundedness is ensured if

$$\gamma_2 \leq \beta(D-1). \quad (\text{A24})$$

As in Case 1, many of the W -dependent terms simplify upon using the boundedness of the above density-driven terms. The conditions reduce to requiring boundedness of

$$W(r) \quad \text{and} \quad W'(r), \quad (\text{A25})$$

which in this case work out to be

$$W(r) \sim C_1 S^{\frac{\gamma_1-1}{D-1}-\beta} + C_2 S^{\frac{\gamma_2-1}{D-1}-\beta}, \quad (\text{A26})$$

$$W'(r) \sim C_3 S^{\frac{\gamma_1}{D-1}-\beta} + C_4 S^{\frac{\gamma_2}{D-1}-\beta}. \quad (\text{A27})$$

There are no further conditions beyond Eq. (A24).

c. Case 3: Marginal central blowup, $\gamma_1 = D-1$

In this section we treat the marginal case for a central divergence, $\gamma_1 = D-1$, which is not covered by Cases 1 and 2. Assume

$$q(r) \sim \frac{A}{r^{D-1}}, \quad r \rightarrow 0^+, \quad (\text{A28})$$

and in the anisotropic case, allow for

$$p_r(r) \sim \frac{B}{r^{\gamma_2}} \quad (\text{A29})$$

as in (A4). Then the matter integrand in $S(r)$ behaves as $x^{D-2}q(x) \sim A/x$, so the matter integral develops a logarithmic divergence. More precisely, for any fixed $r_0 > 0$ one may write

$$\begin{aligned} \int_{\infty}^r x^{D-2} q(x) dx &= \int_{\infty}^{r_0} x^{D-2} q(x) dx \\ &\quad + \int_{r_0}^r \left(x^{D-2} q(x) - \frac{A}{x} \right) dx + A \int_{r_0}^r \frac{dx}{x} \\ &\equiv C + A \log\left(\frac{r}{r_0}\right), \quad r \rightarrow 0^+, \end{aligned} \quad (\text{A30})$$

where C is finite by construction. Substituting into the definition of S gives

$$\begin{aligned} S(r) &\sim \frac{2M + 2C + 2A \log(\frac{r}{r_0})}{r^{D-1}} \\ &\equiv \frac{2M_{\text{eff}} + 2A \log(\frac{r}{r_0})}{r^{D-1}}, \quad r \rightarrow 0^+, \end{aligned} \quad (\text{A31})$$

so that $|S| \rightarrow \infty$ and, for sufficiently small r ,

$$|S(r)| \sim \frac{2|A| |\log r|}{r^{D-1}}. \quad (\text{A32})$$

The large- $|S|$ decay (A2) then implies the useful small- r estimates

$$H'(S) \sim \frac{r^{\beta(D-1)}}{|\log r|^\beta}, \quad H''(S) \sim \frac{r^{(\beta+1)(D-1)}}{|\log r|^{\beta+1}}, \quad r \rightarrow 0^+. \quad (\text{A33})$$

Using $q \sim r^{-(D-1)}$ and $rq' \sim -(D-1)q$, the density-driven terms in (A1) scale as

$$qH'(S) \sim \frac{r^{(\beta-1)(D-1)}}{|\log r|^\beta}, \quad (\text{A34})$$

$$rq'H'(S) \sim \frac{r^{(\beta-1)(D-1)}}{|\log r|^\beta}, \quad (\text{A35})$$

$$q^2H''(S) \sim \frac{r^{(\beta-1)(D-1)}}{|\log r|^{\beta+1}}, \quad (\text{A36})$$

$$qSH''(S) \sim \frac{r^{(\beta-1)(D-1)}}{|\log r|^\beta}, \quad (\text{A37})$$

which are bounded (in fact, they vanish) as $r \rightarrow 0^+$ provided $\beta > 1$. Hence, when $T'_t = T'_r$ (so $W = 0$), the marginal blowup $\gamma_1 = D - 1$ does not cause curvature singularities provided there are none already in the vacuum case.

If $T'_t \neq T'_r$, the pressure sector contributes

$$p_r H'(S) \sim \frac{r^{\beta(D-1)-\gamma_2}}{|\log r|^\beta}, \quad (\text{A38})$$

so $p_r H'(S)$ is bounded provided

$$\gamma_2 \leq \beta(D-1). \quad (\text{A39})$$

Assuming the boundedness of the density-driven terms (A37), the remaining W -dependent contributions in (A1) reduce, as in Cases 1 and 2, to requiring boundedness of $W(r)$ and $W'(r)$. Using the same scaling that led to (A37), their dominant contributions behave as

$$W(r) \sim C_1 \frac{r^{(\beta-1)(D-1)+1}}{|\log r|^\beta} + C_2 \frac{r^{\beta(D-1)+1-\gamma_2}}{|\log r|^\beta}, \quad (\text{A40})$$

$$W'(r) \sim C_3 \frac{r^{(\beta-1)(D-1)}}{|\log r|^\beta} + C_4 \frac{r^{\beta(D-1)-\gamma_2}}{|\log r|^\beta}. \quad (\text{A41})$$

The first terms in (A40) and (A41) are bounded for $\beta > 1$. The second term in $W'(r)$ is bounded precisely under (A39), and no further restrictions arise. In particular, the marginal case $\gamma_1 = D - 1$ yields the same pressure bound as the vacuum-dominated branch (A24).

d. Summary of central power-law blowups

For $T'_t = T'_r$, a central power-law blowup of the form $\rho \sim r^{-\gamma_1}$ does not force curvature divergences provided vacuum curvature is bounded ($\beta > 1$). If $T'_t \neq T'_r$, one

additionally requires the pressure exponent to satisfy

$$\begin{aligned} \gamma_2 &\leq \beta(D-1) \quad \text{if } \gamma_1 \leq D-1, \\ \gamma_2 &\leq \beta\gamma_1 \quad \text{if } \gamma_1 > D-1. \end{aligned} \quad (\text{A42})$$

2. Finite-radius blowup at $r=r_p$

a. Power-law blowup with $\gamma_1 > 1$

Assume the energy density diverges at some finite radius $r_p > 0$ as

$$q(r) \sim \frac{A}{(r-r_p)^{\gamma_1}}, \quad r \rightarrow r_p^+, \quad (\text{A43})$$

with $\gamma_1 > 0$, while ρ is otherwise smooth for $r > r_p$ and the integral defining $S(r)$ converges. In the anisotropic case we also allow

$$p_r(r) \sim \frac{B}{(r-r_p)^{\gamma_2}}, \quad r \rightarrow r_p^+, \quad (\text{A44})$$

for some exponent γ_2 and constant B , while $W(r)$ is the same anisotropy quantity used in the main text (so that $W = 0$ when $T'_t = T'_r$).

The most singular piece of the matter integral

$$I(r) = \int_{\infty}^r x^{D-2} q(x) dx \quad (\text{A45})$$

behaves, for $\gamma_1 \neq 1$, as

$$I(r) \sim -\frac{r_p^{D-2} A}{(\gamma_1 - 1)(r - r_p)^{\gamma_1 - 1}}, \quad r \rightarrow r_p^+, \quad (\text{A46})$$

and therefore

$$S(r) \sim \frac{\lambda}{(r - r_p)^{\gamma_1 - 1}} \quad (\text{A47})$$

for some constant λ .

If $0 < \gamma_1 < 1$, the integral $I(r)$ stays finite as $r \rightarrow r_p$, so $S(r)$ does not run to $\pm\infty$ and the resummation cannot suppress the matter blowup. The curvature invariants diverge in this case, as discussed in the main text. Henceforth let us assume $\gamma_1 > 1$.

If $\gamma_1 > 1$, then $|S| \rightarrow \infty$ as $r \rightarrow r_p$, and one can trade $(r - r_p)$ for $|S|$:

$$(r - r_p) \sim |S|^{-1/(\gamma_1 - 1)}. \quad (\text{A48})$$

Then

$$q \sim |S|^{\frac{\gamma_1}{\gamma_1 - 1}}, \quad rq' \sim |S|^{\frac{\gamma_1 + 1}{\gamma_1 - 1}}, \quad p_r \sim |S|^{\frac{\gamma_2}{\gamma_1 - 1}}. \quad (\text{A49})$$

The potentially dangerous terms involving the density scale as

$$\varrho H'(S) \sim |S|^{\frac{\gamma_1}{(\gamma_1-1)}-\beta}, \quad (\text{A50})$$

$$r\varrho' H'(S) \sim |S|^{\frac{\gamma_1+1}{(\gamma_1-1)}-\beta}, \quad (\text{A51})$$

$$\varrho^2 H''(S) \sim |S|^{\frac{2\gamma_1}{(\gamma_1-1)}-(\beta+1)}, \quad (\text{A52})$$

$$\varrho S H''(S) \sim |S|^{\frac{\gamma_1}{(\gamma_1-1)}-\beta}. \quad (\text{A53})$$

All of these are bounded provided

$$\gamma_1 \geq \frac{\beta+1}{\beta-1}. \quad (\text{A54})$$

For the pressure term one has

$$\mathfrak{p}_r H'(S) \sim |S|^{\frac{\gamma_2}{(\gamma_1-1)}-\beta} \quad (\text{A55})$$

which is bounded provided that $\gamma_2 \leq \beta(\gamma_1 - 1)$. Assuming the boundedness of the terms involving the density, various terms involving $W(r)$ further simplify, and the problem reduces to the boundedness of $W(r)$ and $W'(r)$. For the present case, the dominant contributions can be worked out to be

$$W(r) \sim C_1 |S|^{\frac{\gamma_1}{\gamma_1-1}-\beta} + C_2 |S|^{\frac{\gamma_2}{\gamma_1-1}-\beta}, \quad (\text{A56})$$

$$W'(r) \sim C_3 |S|^{\frac{\gamma_1+1}{\gamma_1-1}-\beta} + C_4 |S|^{\frac{\gamma_2+1}{\gamma_1-1}-\beta}. \quad (\text{A57})$$

These terms give stronger constraints than the pressure term above. In general, we have that if $\gamma_2 \leq \gamma_1$, then all of the terms are bounded without additional constraints on γ_2 . However, if $\gamma_2 > \gamma_1$ then we must also have

$$\gamma_2 \leq \beta(\gamma_1 - 1) - 1, \quad (\text{A58})$$

to ensure boundedness of all terms.

b. Marginal case $\gamma_1 = 1$

The estimate (A46) excludes $\gamma_1 = 1$. In this case the leading singularity of the matter integral is logarithmic. To simplify the following, let us write $\epsilon \equiv r - r_p \rightarrow 0^+$. Using $x^{D-2} \simeq r_p^{D-2}$ in the singular region, one finds

$$\begin{aligned} I(r) &= \int_{\infty}^r x^{D-2} \varrho(x) dx \\ &\sim A r_p^{D-2} \log \epsilon + C, \quad r \rightarrow r_p^+, \end{aligned} \quad (\text{A59})$$

for some finite constant C . Substituting into the definition of $S(r)$ then gives

$$\begin{aligned} S(r) &\sim S_0 + \frac{2A}{r_p} \log \epsilon \equiv S_0 + \lambda_{\log} \log \epsilon, \\ \lambda_{\log} &\equiv \frac{2A}{r_p}, \quad (r \rightarrow r_p^+), \end{aligned} \quad (\text{A60})$$

so that $|S| \rightarrow \infty$ as $r \rightarrow r_p^+$, but only logarithmically. In particular,

$$|S| \sim |\lambda_{\log}| |\log \epsilon|, \quad \Rightarrow \epsilon \sim e^{-|S|/|\lambda_{\log}|}. \quad (\text{A61})$$

Consequently, the matter fields grow exponentially in $|S|$:

$$\begin{aligned} \varrho &\sim \frac{A}{\epsilon} \sim e^{|S|/|\lambda_{\log}|}, \quad r\varrho' \sim \frac{1}{\epsilon^2} \sim e^{2|S|/|\lambda_{\log}|}, \\ \mathfrak{p}_r &\sim \epsilon^{-\gamma_2} \sim e^{\gamma_2 |S|/|\lambda_{\log}|}. \end{aligned} \quad (\text{A62})$$

Under the polynomial decay assumption (A2), these scalings are too strong to be suppressed by $H'(S)$ and $H''(S)$. For example,

$$\begin{aligned} \varrho H'(S) &\sim e^{|S|/|\lambda_{\log}|} |S|^{-\beta} \rightarrow \infty, \\ r\varrho' H'(S) &\sim e^{2|S|/|\lambda_{\log}|} |S|^{-\beta} \rightarrow \infty, \end{aligned} \quad (\text{A63})$$

and similarly $\varrho^2 H''(S)$ and $\varrho S H''(S)$ diverge. Thus, already in the case with $T'_t = T'_r$ (so that $W = 0$), the combinations listed in (A1) are unbounded for $\gamma_1 = 1$, and the curvature invariants diverge. The anisotropic case only worsens this conclusion through the additional contributions involving $\mathfrak{p}_r$ and W .

APPENDIX B: FASTER-THAN-POLYNOMIAL DECAYS

Appendix A assumes the power-law decay (A2), which is sufficient to bound the *vacuum* curvature invariants and leads to the various exponent inequalities on γ_1 and γ_2 . Here we summarize what changes if instead the resummation decays faster than any polynomial at large $|S|$, in the sense that

$$\begin{aligned} \lim_{|S| \rightarrow \infty} |S|^n H'(S) &= 0, \quad \forall n > 0, \\ \lim_{|S| \rightarrow \infty} |S|^n H''(S) &= 0, \quad \forall n > 0. \end{aligned} \quad (\text{B1})$$

A representative example is $H'(S) \sim e^{-a|S|}$, or more generally $H'(S) \sim e^{-a|S|^\sigma}$ with $\sigma > 0$. The results of this section are rather simple to obtain from the previous section—one must just track the various powers of β that appear and replace them with the new superpolynomial decays. We therefore present a summary of the key facts.

1. Power-law blowups that drive $|S| \rightarrow \infty$

In all power-law blowups treated in Appendix A for which $|S| \rightarrow \infty$ and the matter fields grow at most polynomially in $|S|$ (possibly with logarithmic corrections), the faster-than-polynomial decay (B1) implies that any factor of the form $|S|^k H'(S)$ or $|S|^k H''(S)$ is bounded for every fixed $k \geq 0$. Consequently, once $|S| \rightarrow \infty$, the density- and pressure-driven combinations in (A1) are automatically bounded, and the remaining W -dependent terms are likewise suppressed after using the fact that W carries an explicit factor of $H'(S)$ through the field equations.

In particular, the exponent inequalities derived in Appendix A become sufficient but not necessary in the following regimes:

- (i) Central power-law blowups at $r = 0$ (including the marginal case $\gamma_1 = D - 1$): once vacuum curvature is bounded, no additional restrictions on γ_1 are needed, and the pressure bounds on γ_2 found in Case 1 correspond to sufficient conditions only.
- (ii) Nonintegrable finite-radius blowups with $\gamma_1 > 1$: the conditions such as (A54) and the associated pressure and W' bounds are again sufficient but not necessary.

In these cases, vacuum boundedness implies boundedness in the presence of matter.

2. Cases where faster decay does not automatically help

There remain two situations in which (B1) does not, by itself, guarantee regularity:

- (1) *Integrable finite-radius blowup* $0 < \gamma_1 < 1$. As discussed in Appendix A, the integral $I(r)$ stays finite, and $S(r)$ does not run to $\pm\infty$. In this case the large- $|S|$ behavior of H is irrelevant and the curvature invariants still diverge.
- (2) *Marginal finite-radius blowup* $\gamma_1 = 1$. From (A60) and (A61) one has $S(r) \sim S_0 + \lambda_{\log} \log \epsilon$ with $\epsilon = r - r_p \rightarrow 0^+$; hence, $\epsilon \sim e^{-|S|/|\lambda_{\log}|}$, and the matter fields grow exponentially in $|S|$, see (A62). In this case the outcome depends on the rate of decay of $H'(S)$ and $H''(S)$, not just that it is faster than polynomial. For example, if $H'(S) \sim e^{-a|S|}$ (and similarly for H''), then

$$H'(S) \sim \epsilon^{a|\lambda_{\log}|}, \quad (\text{B2})$$

and the most singular density-driven terms behave as

$$r q' H'(S) \sim \epsilon^{-2} \epsilon^{a|\lambda_{\log}|} = \epsilon^{a|\lambda_{\log}|-2}, \quad (\text{B3})$$

$$q^2 H''(S) \sim \epsilon^{-2} \epsilon^{a|\lambda_{\log}|} = \epsilon^{a|\lambda_{\log}|-2}. \quad (\text{B4})$$

Thus, a sufficient condition in the $T_t^t = T_r^r$ case is $a|\lambda_{\log}| \geq 2$. In the anisotropic case (at finite radius r_p or at the inner horizon r_h), one also needs to control

terms involving $\mathbf{p}_r$ and $W'(r)$; a simple sufficient condition is

$$a|\lambda_{\log}| \geq \max\{2, \gamma_2 + 1\}. \quad (\text{B5})$$

For stronger decays such as $H'(S) \sim e^{-a|S|^\sigma}$ with $\sigma > 1$, the suppression is stronger still, and the marginal case is correspondingly easier to regularize.

APPENDIX C: COSMOLOGIES

In this appendix we remove the staticity condition and consider the fate of curvature invariants for cosmological spacetimes filled with a combination of perfect fluids.

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric is given by

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_{D-2}^2 \right) \quad (\text{C1})$$

where $a(t)$ is the scale factor and $k = 0, \pm 1$ the spatial curvature. As argued in [78], spherical FLRW spacetimes can be easily studied within the two-dimensional Horndeski framework which captures the dynamical information of the spherically symmetric sector of QT theories. In particular, the scale factor satisfies second-order equations of motion for such theories.

Because of the isotropy of the spacetime metric, matter can be described by a perfect fluid, so the stress tensor takes the form (7) with $p_r = p_t \equiv p$. We then assume the presence of n different matter sources, each characterized by an energy density ρ_i and pressure p_i . The conservation law for each stress-energy tensor provides the following system of equations:

$$\dot{q}_i + (D-1)(q_i + \mathbf{p}_i) \frac{\dot{a}(t)}{a(t)} = 0, \quad (\text{C2})$$

where the dot refers to the derivative with respect to t . The modified Friedmann equation then reads

$$h(\Phi) = S(t), \quad (\text{C3})$$

where

$$\Phi = \frac{k + \dot{a}(t)^2}{a(t)^2}, \quad S(t) = \frac{2}{D-1} \sum_{i=1}^n q_i(t), \quad (\text{C4})$$

where $h(x)$ is the characteristic function of QT gravities defined in (11).

As argued in Sec. II, to determine the conditions on the matter for bounded curvature invariants, it is necessary to analyze each term of the Riemann tensor, which in this case can be written as

$$R_{ab}^{cd} = \mathcal{R}^{(1)} \tau_{[a}^{[c} \pi_{b]}^{d]} + \mathcal{R}^{(2)} \pi_{[a}^{[c} \pi_{b]}^{d]}, \quad (\text{C5})$$

where we have defined

$$\mathcal{R}^{(1)} = 4 \frac{\ddot{a}(t)}{a(t)}, \quad \mathcal{R}^{(2)} = 2 \frac{k + \dot{a}(t)^2}{a(t)^2}. \quad (\text{C6})$$

Here τ_a^b and π_a^b are projectors onto the t and spatial directions, respectively, defined in an analogous way to those that appear in (20). The Kretschmann scalar reads

$$R_{abcd}R^{abcd} = \frac{D-1}{4} (\mathcal{R}^{(1)})^2 + \frac{(D-1)(D-2)}{2} (\mathcal{R}^{(2)})^2. \quad (\text{C7})$$

Using the Friedmann equation together with the conservation law of the stress-energy tensor, the $\mathcal{R}^{(i)}$ terms (C6) can be expressed in terms of the inverse of the characteristic function $H(x) = h^{-1}(x)$ as

$$\mathcal{R}^{(1)} = 4H(S) - 4 \sum_{i=1}^n (q_i + p_i) H'(S), \quad (\text{C8})$$

$$\mathcal{R}^{(2)} = 2H(S). \quad (\text{C9})$$

Hence, in order to determine the validity of the limiting curvature hypothesis for this type of spacetime, we have to study the boundedness of the following quantities:

$$\{H(S), SH'(S), p_i H'(S)\}. \quad (\text{C10})$$

The first two are bounded by the analogous discussion in the static case. In order to study the third one, we assume that each pressure can be described by a polytropic equation of state with polytropic exponent $\Gamma_i \geq 1$ at least for larger values of $|S|$, that is,

$$|p_i| \sim |q_i|^{\Gamma_i}. \quad (\text{C11})$$

The sum of the pressures in $\mathcal{R}^{(1)}$ will then be dominated by the j th fluid with exponent $\Gamma_j = \max\{\Gamma_i : i = 1, \dots, n\}$ in the limit $|S| \rightarrow \infty$,

$$\left| \sum_{i=1}^n p_i \right| \sim |p_j| \sim |q_j|^{\Gamma_j} \sim |S|^{\Gamma_j}. \quad (\text{C12})$$

Therefore, since $H'(S) \sim |S|^{-\beta}$ with $\beta > 1$, curvature remains bounded if and only if $\Gamma_i \leq \beta$ for all species of matter.

It is worth mentioning that for the usual cosmological equation of state $p = wq$, corresponding to $\Gamma = 1$, the curvature of the cosmological solution is bounded provided

that the curvature of the vacuum solution of the theory is bounded.

1. Oppenheimer-Snyder collapse

In a previous paper we studied the problem of gravitational collapse for a spherically symmetric perfect-fluid pressureless star—analogous to the original setup of Oppenheimer and Snyder [121]—in QT theories whose vacuum solutions are regular black holes [78]. We showed that the stars undergo a never-ending process of contractions and expansions throughout infinite universes and produce regular black holes as by-products.

Since this process consists of matching the vacuum metric to the dust cosmological one at the star's boundary, it is straightforward that the limiting curvature hypothesis is satisfied during the collapse because both spacetimes satisfy it independently.

Nonetheless, another question remains: is the global upper curvature bound given by the vacuum solution or by the matter solution? In order to address this question we should compare both Riemann tensors, corresponding to the interior and the exterior, during the collapse process.

Making use of the scaling property in the vacuum and the homogeneity of the cosmological solutions, and expressing the couplings $\alpha_n = \alpha^{n-1} \beta_n$ for a given squared-length scale $\alpha > 0$, both dimensionless Riemann tensors αR_{abcd} —namely, the one corresponding to the vacuum metric as well as the one corresponding to the FLRW metric—can be

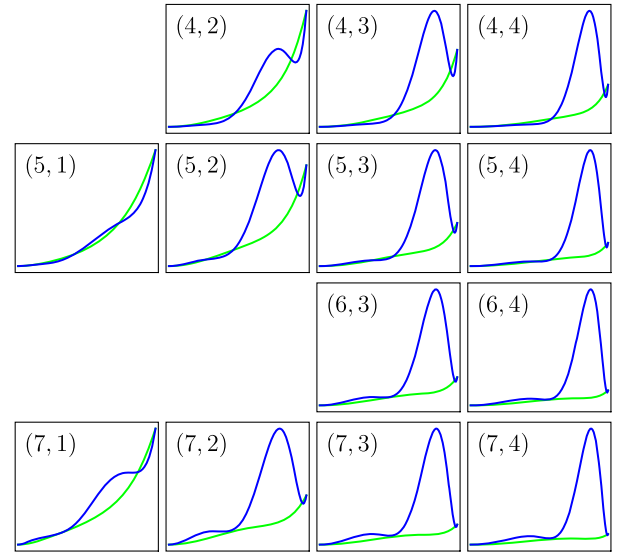


FIG. 3. Plots of the Kretschmann scalar $\alpha^2 R_{abcd} R^{abcd}$ for various (D, N) -Hayward-like QT theories. In each graph, the curvature invariant is depicted in blue for the vacuum spacetime and in green for the FLRW spacetime, as a function of $\alpha\psi$ or $\alpha\Phi$, respectively, over their domain $(0, 1)$. In all the cases presented, the maximum curvature is reached by the vacuum solution, and it becomes more pronounced as D and/or N increase.

expressed as functions of $\alpha\psi$ and $\alpha\Phi$ alone. Given that both quantities depend in a complicated way on the characteristic polynomial, let us consider Hayward solutions in order to analyze whether the maximum value of the Kretschmann scalar during a collapse process corresponds to a fixed value given by the vacuum theory. Since the collapse depends on the initial conditions, we propose an alternative approach: comparing the two curvature invariants separately within their corresponding domains for Hayward solutions. This problem cannot be tackled analytically, so

we address it graphically. In Fig. 3 we plot both curvature invariants for different dimensions D and orders N for Hayward-like solutions. We observe that the Kretschmann scalar in the FLRW spacetime always lies below its maximum value in vacuum. Furthermore, for higher D and/or N , the maximum curvature in vacuum becomes significantly larger than in FLRW. We thus conclude that Markov's hypothesis holds in Hayward-like cosmologies with dust and therefore also in this collapsing scenario.

-
- [1] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 2023).
 - [2] J. M. M. Senovilla, Singularity theorems and their consequences, *Gen. Relativ. Gravit.* **30**, 701 (1998).
 - [3] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Observation of gravitational waves from a binary black hole merger, *Phys. Rev. Lett.* **116**, 061102 (2016).
 - [4] A. G. Abac *et al.* (LIGO Scientific, VIRGO, and KAGRA Collaborations), GWTC-4.0: Updating the gravitational-wave transient catalog with observations from the first part of the fourth LIGO-Virgo-KAGRA observing run, [arXiv:2508.18082](https://arxiv.org/abs/2508.18082).
 - [5] A. G. Abac *et al.* (LIGO Scientific, Virgo, and KAGRA Collaborations), GW250114: Testing Hawking's area law and the Kerr nature of black holes, *Phys. Rev. Lett.* **135**, 111403 (2025).
 - [6] K. Akiyama *et al.* (Event Horizon Telescope Collaboration), First M87 Event Horizon Telescope results. I. The shadow of the supermassive black hole, *Astrophys. J. Lett.* **875**, L1 (2019).
 - [7] V. Cardoso and P. Pani, Testing the nature of dark compact objects: A status report, *Living Rev. Relativity* **22**, 4 (2019).
 - [8] A. D. Sakharov, Nachal'naia stadija rasshirenija Vselennoj i vozniknovenije neodnorodnosti raspredelenija veshchestva, *Sov. Phys. JETP* **22**, 241 (1966), <https://ui.adsabs.harvard.edu/abs/1966JETP...22..241S/abstract>.
 - [9] J. Bardeen, Non-singular general relativistic gravitational collapse, in *Proceedings of the 5th International Conference on Gravitation and the Theory of Relativity* (House of Tbilisi University, 1968), p. 87.
 - [10] E. Poisson and W. Israel, Structure of the black hole nucleus, *Classical Quantum Gravity* **5**, L201 (1988).
 - [11] I. Dymnikova, Vacuum nonsingular black hole, *Gen. Relativ. Gravit.* **24**, 235 (1992).
 - [12] S. A. Hayward, Formation and evaporation of regular black holes, *Phys. Rev. Lett.* **96**, 031103 (2006).
 - [13] R. Carballo-Rubio, F. Di Filippo, S. Liberati, C. Pacilio, and M. Visser, On the viability of regular black holes, *J. High Energy Phys.* **07** (2018) 023.
 - [14] R. Carballo-Rubio, F. Di Filippo, S. Liberati, C. Pacilio, and M. Visser, Inner horizon instability and the unstable cores of regular black holes, *J. High Energy Phys.* **05** (2021) 132.
 - [15] R. Carballo-Rubio, F. Di Filippo, S. Liberati, C. Pacilio, and M. Visser, Regular black holes without mass inflation instability, *J. High Energy Phys.* **09** (2022) 118.
 - [16] R. Carballo-Rubio *et al.*, Towards a non-singular paradigm of black hole physics, *J. Cosmol. Astropart. Phys.* **05** (2025) 003.
 - [17] H. C. D. Lima Junior, R. B. Magalhães, G. J. Olmo, and D. Rubiera-Garcia, Resolving space-time singularities in spherically symmetric black holes: Geodesic completeness, curvature scalars, and tidal forces, [arXiv:2506.17918](https://arxiv.org/abs/2506.17918).
 - [18] E. Ayon-Beato and A. Garcia, Regular black hole in general relativity coupled to nonlinear electrodynamics, *Phys. Rev. Lett.* **80**, 5056 (1998).
 - [19] K. A. Bronnikov, Regular magnetic black holes and monopoles from nonlinear electrodynamics, *Phys. Rev. D* **63**, 044005 (2001).
 - [20] E. Ayon-Beato and A. Garcia, The Bardeen model as a nonlinear magnetic monopole, *Phys. Lett. B* **493**, 149 (2000).
 - [21] K. A. Bronnikov, Comment on "Regular black hole in general relativity coupled to nonlinear electrodynamics", *Phys. Rev. Lett.* **85**, 4641 (2000).
 - [22] E. Ayon-Beato and A. Garcia, Four parametric regular black hole solution, *Gen. Relativ. Gravit.* **37**, 635 (2005).
 - [23] I. Dymnikova, Regular electrically charged structures in nonlinear electrodynamics coupled to general relativity, *Classical Quantum Gravity* **21**, 4417 (2004).
 - [24] W. Berej, J. Matyjasek, D. Tryniecki, and M. Woronowicz, Regular black holes in quadratic gravity, *Gen. Relativ. Gravit.* **38**, 885 (2006).
 - [25] L. Balart and E. C. Vagenas, Regular black hole metrics and the weak energy condition, *Phys. Lett. B* **730**, 14 (2014).
 - [26] Z.-Y. Fan, Critical phenomena of regular black holes in anti-de Sitter space-time, *Eur. Phys. J. C* **77**, 266 (2017).
 - [27] K. A. Bronnikov, Nonlinear electrodynamics, regular black holes and wormholes, *Int. J. Mod. Phys. D* **27**, 1841005 (2018).

- [28] J. T. S. S. Junior, F. S. N. Lobo, and M. E. Rodrigues, (Regular) Black holes in conformal Killing gravity coupled to nonlinear electrodynamics and scalar fields, *Classical Quantum Gravity* **41**, 055012 (2024).
- [29] G. Alencar, K. A. Bronnikov, M. E. Rodrigues, D. Sáez-Chillón Gómez, and M. V. de S. Silva, On black bounce space-times in non-linear electrodynamics, *Eur. Phys. J. C* **84**, 745 (2024).
- [30] K. A. Bronnikov, Regular black holes as an alternative to black bounce, *Phys. Rev. D* **110**, 024021 (2024).
- [31] S. V. Bolokhov, K. A. Bronnikov, and M. V. Skvortsova, A regular center instead of a black bounce, *Gravitation Cosmol.* **30**, 265 (2024).
- [32] M. Skvortsova, Ringing of extreme regular black holes, *Gravitation Cosmol.* **30**, 279 (2024).
- [33] S. Murk and I. Soranidis, Light rings and causality for nonsingular ultracompact objects sourced by nonlinear electrodynamics, *Phys. Rev. D* **110**, 044064 (2024).
- [34] Z.-C. Li and H. Lu, Regular electric black holes from EMS gravity, [arXiv:2407.07952](https://arxiv.org/abs/2407.07952).
- [35] X.-Y. Zhang, L. Zhao, and Y.-Q. Wang, Bardeen-Dirac stars in AdS spacetime, [arXiv:2409.14402](https://arxiv.org/abs/2409.14402).
- [36] V. P. Frolov, M. A. Markov, and V. F. Mukhanov, Through a black hole into a new universe?, *Phys. Lett. B* **216**, 272 (1989).
- [37] C. Barrabes and V. P. Frolov, How many new worlds are inside a black hole?, *Phys. Rev. D* **53**, 3215 (1996).
- [38] A. Bonanno and M. Reuter, Renormalization group improved black hole space-times, *Phys. Rev. D* **62**, 043008 (2000).
- [39] P. Nicolini, A. Smailagic, and E. Spallucci, Noncommutative geometry inspired Schwarzschild black hole, *Phys. Lett. B* **632**, 547 (2006).
- [40] E. Greenwood and D. Stojkovic, Quantum gravitational collapse: Non-singularity and non-locality, *J. High Energy Phys.* **06** (2008) 042.
- [41] G. J. Olmo and D. Rubiera-Garcia, Reissner-Nordström black holes in extended Palatini theories, *Phys. Rev. D* **86**, 044014 (2012).
- [42] A. Saini and D. Stojkovic, Nonlocal (but also nonsingular) physics at the last stages of gravitational collapse, *Phys. Rev. D* **89**, 044003 (2014).
- [43] V. P. Frolov, Information loss problem and a “black hole” model with a closed apparent horizon, *J. High Energy Phys.* **05** (2014) 049.
- [44] A. B. Balakin, J. P. S. Lemos, and A. E. Zayats, Regular nonminimal magnetic black holes in spacetimes with a cosmological constant, *Phys. Rev. D* **93**, 024008 (2016).
- [45] D. Bazeia, L. Losano, G. J. Olmo, D. Rubiera-Garcia, and A. Sanchez-Puente, Classical resolution of black hole singularities in arbitrary dimension, *Phys. Rev. D* **92**, 044018 (2015).
- [46] A. H. Chamseddine and V. Mukhanov, Nonsingular black hole, *Eur. Phys. J. C* **77**, 183 (2017).
- [47] C. Bambi, D. Rubiera-Garcia, and Y. Wang, Black hole solutions in functional extensions of Born-Infeld gravity, *Phys. Rev. D* **94**, 064002 (2016).
- [48] C. Bejarano, G. J. Olmo, and D. Rubiera-Garcia, What is a singular black hole beyond General Relativity?, *Phys. Rev. D* **95**, 064043 (2017).
- [49] A. Colléaux, S. Chinaglia, and S. Zerbini, Nonpolynomial Lagrangian approach to regular black holes, *Int. J. Mod. Phys. D* **27**, 1830002 (2018).
- [50] H. Kawai and Y. Yokokura, A model of black hole evaporation and 4D Weyl anomaly, *Universe* **3**, 51 (2017).
- [51] P. A. Cano, S. Chimento, T. Ortín, and A. Ruipérez, Regular stringy black holes?, *Phys. Rev. D* **99**, 046014 (2019).
- [52] A. Colleaux, Regular black hole and cosmological space-times in non-polynomial gravity theories, Ph.D. thesis, Trento University, 2019.
- [53] M. Guerrero and D. Rubiera-Garcia, Nonsingular black holes in nonlinear gravity coupled to Euler-Heisenberg electrodynamics, *Phys. Rev. D* **102**, 024005 (2020).
- [54] P. A. Cano and Á. Murcia, Electromagnetic quasitopological gravities, *J. High Energy Phys.* **10** (2020) 125.
- [55] P. A. Cano and Á. Murcia, Resolution of Reissner-Nordström singularities by higher-derivative corrections, *Classical Quantum Gravity* **38**, 075014 (2021).
- [56] R. Brandenberger, L. Heisenberg, and J. Robnik, Through a black hole into a new universe, *Int. J. Mod. Phys. D* **30**, 2142001 (2021).
- [57] P. Bueno, P. A. Cano, J. Moreno, and G. van der Velde, Regular black holes in three dimensions, *Phys. Rev. D* **104**, L021501 (2021).
- [58] G. J. Olmo and D. Rubiera-Garcia, Regular black holes in Palatini gravity, [arXiv:2209.05061](https://arxiv.org/abs/2209.05061).
- [59] J. T. S. S. Junior, F. S. N. Lobo, and M. E. Rodrigues, Black holes and regular black holes in coincident $f(Q, \mathbb{B}_Q)$ gravity coupled to nonlinear electrodynamics, [arXiv:2402.02534](https://arxiv.org/abs/2402.02534).
- [60] T. Harada, C.-M. Chen, and R. Mandal, Singularity resolution and regular black hole formation in gravitational collapse in asymptotically safe gravity, *Phys. Rev. D* **111**, 126017 (2025).
- [61] P. Bueno, O. Lasso Andino, J. Moreno, and G. van der Velde, On regular charged black holes in three dimensions, *J. High Energy Phys.* **08** (2025) 132.
- [62] P. Bueno, P. A. Cano, and R. A. Hennigar, Regular black holes from pure gravity, *Phys. Lett. B* **861**, 139260 (2025).
- [63] J. Oliva and S. Ray, A new cubic theory of gravity in five dimensions: Black hole, Birkhoff’s theorem and C-function, *Classical Quantum Gravity* **27**, 225002 (2010).
- [64] R. C. Myers and B. Robinson, Black holes in quasi-topological gravity, *J. High Energy Phys.* **08** (2010) 067.
- [65] M. H. Dehghani, A. Bazrafshan, R. B. Mann, M. R. Mehdizadeh, M. Ghanaatian, and M. H. Vahidinia, Black holes in quartic quasitopological gravity, *Phys. Rev. D* **85**, 104009 (2012).
- [66] J. Ahmed, R. A. Hennigar, R. B. Mann, and M. Mir, Quintessential quartic quasi-topological quartet, *J. High Energy Phys.* **05** (2017) 134.
- [67] A. Cisterna, L. Guajardo, M. Hassaine, and J. Oliva, Quintic quasi-topological gravity, *J. High Energy Phys.* **04** (2017) 066.

- [68] P. Bueno, P. A. Cano, and R. A. Hennigar, (Generalized) quasi-topological gravities at all orders, *Classical Quantum Gravity* **37**, 015002 (2020).
- [69] P. Bueno, P. A. Cano, R. A. Hennigar, M. Lu, and J. Moreno, Generalized quasi-topological gravities: The whole shebang, *Classical Quantum Gravity* **40**, 015004 (2023).
- [70] J. Moreno and Á. J. Murcia, Classification of generalized quasitopological gravities, *Phys. Rev. D* **108**, 044016 (2023).
- [71] J. Moreno and Á. J. Murcia, Cosmological higher-curvature gravities, *Classical Quantum Gravity* **41**, 135017 (2024).
- [72] P. Bueno, R. A. Hennigar, and Á. J. Murcia, Birkhoff implies quasi-topological, [arXiv:2510.25823](https://arxiv.org/abs/2510.25823).
- [73] P. Bueno, P. A. Cano, R. A. Hennigar, and Á. J. Murcia, Regular black hole formation in four-dimensional non-polynomial gravities, *Phys. Rev. D* **113**, 024019 (2026).
- [74] S. Chinaglia, A. Colléaux, and S. Zerbini, A non-polynomial gravity formulation for loop quantum cosmology bounce, *Galaxies* **5**, 51 (2017).
- [75] P. Bueno, P. A. Cano, J. Moreno, and Á. Murcia, All higher-curvature gravities as generalized quasi-topological gravities, *J. High Energy Phys.* **11** (2019) 062.
- [76] P. Bueno, P. A. Cano, R. A. Hennigar, and Á. J. Murcia, Regular black holes from thin-shell collapse, *Phys. Rev. D* **111**, 104009 (2025).
- [77] P. Bueno, P. A. Cano, R. A. Hennigar, and Á. J. Murcia, Dynamical formation of regular black holes, *Phys. Rev. Lett.* **134**, 181401 (2025).
- [78] P. Bueno, P. A. Cano, R. A. Hennigar, Á. J. Murcia, and A. Vicente-Cano, Regular black holes from Oppenheimer-Snyder collapse, *Phys. Rev. D* **112**, 064039 (2025).
- [79] R. A. Konoplya and A. Zhidenko, Dymnikova black hole from an infinite tower of higher-curvature corrections, *Phys. Lett. B* **856**, 138945 (2024).
- [80] R. A. Konoplya and A. Zhidenko, Infinite tower of higher-curvature corrections: Quasinormal modes and late-time behavior of D-dimensional regular black holes, *Phys. Rev. D* **109**, 104005 (2024).
- [81] F. Di Filippo, I. Kolář, and D. Kubiznak, Inner-extremal regular black holes from pure gravity, *Phys. Rev. D* **111**, L041505 (2025).
- [82] T.-X. Ma, T.-F. Fang, and Y.-Q. Wang, Boson stars and their frozen states in an infinite tower of higher-derivative gravity, *Eur. Phys. J. C* **85**, 542 (2025).
- [83] A. Cisterna, N. Grandi, and J. Oliva, de Sitter geometric inflation from dynamical singularities, *Phys. Rev. D* **110**, 084043 (2024).
- [84] A. Ditta, A. Bouzenada, G. Mustafa, Y. M. Alanazi, and F. Mushtaq, Particle motion, shadows and thermodynamics of regular black hole in pure gravity, *Phys. Dark Universe* **46**, 101573 (2024).
- [85] S.-W. Wang, S.-P. Wu, and S.-W. Wei, Are regular black holes from pure gravity classified within the same thermodynamical topology?, *Phys. Lett. B* **864**, 139402 (2025).
- [86] R. A. Hennigar, D. Kubizňák, S. Murk, and I. Soranidis, Thermodynamics of regular black holes in anti-de Sitter space, *J. High Energy Phys.* **11** (2020) 121.
- [87] P. G. S. Fernandes, Regular BTZ black holes from an infinite tower of corrections, *Phys. Lett. B* **868**, 139772 (2025).
- [88] P. G. S. Fernandes, Singularity resolution and inflation from an infinite tower of regularized curvature corrections, *Phys. Rev. D* **112**, 084028 (2025).
- [89] A. Cisterna, M. Hassaine, and U. Hernandez-Vera, Thermodynamics of four-dimensional regular black holes with an infinite tower of regularized curvature corrections, *Phys. Rev. D* **112**, 064036 (2025).
- [90] Y. Ling and Z. Yu, Big bounce and black bounce in quasi-topological gravity, [arXiv:2509.00137](https://arxiv.org/abs/2509.00137).
- [91] A. Eichhorn and P. G. S. Fernandes, Regular black holes without mass-inflation instability and gravastars from modified gravity, [arXiv:2508.00686](https://arxiv.org/abs/2508.00686).
- [92] P. G. S. Fernandes, J. Gou, L. Heisenberg, and N. Nussbaumer, Inflation, black holes with primary hair, and regular planar black holes from an infinite tower of regularized Lovelock-Proca corrections, [arXiv:2511.22798](https://arxiv.org/abs/2511.22798).
- [93] M. Aguayo, L. Gajardo, N. Grandi, J. Moreno, J. Oliva, and M. Reyes, Holographic explorations of regular black holes in pure gravity, *J. High Energy Phys.* **09** (2025) 030.
- [94] V. Boyanov and R. Carballo-Rubio, Regular Vaidya solutions of effective gravitational theories, [arXiv:2506.14875](https://arxiv.org/abs/2506.14875).
- [95] R. A. Konoplya and A. Zhidenko, Convergence of higher-curvature expansions near the horizon: Hawking radiation from regular black holes, *Int. J. Grav. Theor. Phys.* **1**, 5 (2025).
- [96] C.-H. Hao, J. Jing, and J. Wang, Charged regular black holes from quasi-topological gravities in $D \geq 5$, [arXiv:2512.04604](https://arxiv.org/abs/2512.04604).
- [97] C. Tan and Y.-Q. Wang, Frozen neutron stars in four-dimensional non-polynomial gravities, [arXiv:2512.23525](https://arxiv.org/abs/2512.23525).
- [98] V. P. Frolov, Quasitopological gravity and double-copy formalism, [arXiv:2512.14674](https://arxiv.org/abs/2512.14674).
- [99] J.-R. Chen and Y.-Q. Wang, Proca stars and their frozen states in an infinite tower of higher-derivative gravity, [arXiv:2512.24584](https://arxiv.org/abs/2512.24584).
- [100] J. P. Arbelaez, Grey-body factors of higher dimensional regular black holes in quasi-topological theories, [arXiv:2601.22340](https://arxiv.org/abs/2601.22340).
- [101] G. Li and Y.-Q. Wang, Ellis–Bronnikov wormhole in quasi-topological gravity, [arXiv:2602.01029](https://arxiv.org/abs/2602.01029).
- [102] J. Borissova and R. Carballo-Rubio, Regular black holes from pure gravity in four dimensions, [arXiv:2602.16773](https://arxiv.org/abs/2602.16773).
- [103] R. Tsuda, R. Suzuki, and S. Tomizawa, Fan-Wang type regular black holes in quasi-topological gravity, [arXiv:2602.16754](https://arxiv.org/abs/2602.16754).
- [104] R. A. Konoplya, Quasinormal modes of four-dimensional regular black holes in quasi-topological gravity: Overtones’ outburst via WKB method, [arXiv:2603.03189](https://arxiv.org/abs/2603.03189).
- [105] J. M. M. Senovilla, Junction conditions for general gravitational theories, [arXiv:2603.04645](https://arxiv.org/abs/2603.04645).
- [106] J. Borissova, All 2D generalised dilaton theories from $d \geq 4$ gravities, [arXiv:2603.06786](https://arxiv.org/abs/2603.06786).
- [107] V. P. Frolov, A. Koek, J. P. Soto, and A. Zelnikov, Regular black holes inspired by quasitopological gravity, *Phys. Rev. D* **111**, 044034 (2025).
- [108] M. A. Markov, Limiting density of matter as a universal law of nature, *Lett. ZhETF* **36**, 214 (1982), <https://ui.adsabs.harvard.edu/abs/1982ZhPmR..36..214M/abstract>.

- [109] P. Bueno, R. A. Hennigar, Á. J. Murcia, and A. Vicente-Cano, Buchdahl limits in theories with regular black holes, [arXiv:2512.19796](#).
- [110] V. P. Frolov and A. Zelnikov, Regular black holes in quasitopological gravity: Null shells and mass inflation, [arXiv:2601.01861](#).
- [111] P. A. Cano, Á. J. Murcia, A. Rivadulla Sánchez, and X. Zhang, Higher-derivative holography with a chemical potential, *J. High Energy Phys.* **07** (2022) 010.
- [112] P. Bueno, P. A. Cano, J. Moreno, and G. van der Velde, Electromagnetic generalized quasitopological gravities in $(2 + 1)$ dimensions, *Phys. Rev. D* **107**, 064050 (2023).
- [113] P. Bueno, P. A. Cano, Á. Murcia, and A. Rivadulla Sánchez, Universal feature of charged entanglement entropy, *Phys. Rev. Lett.* **129**, 021601 (2022).
- [114] T. Padmanabhan, Some aspects of field equations in generalised theories of gravity, *Phys. Rev. D* **84**, 124041 (2011).
- [115] M. Visser, Dirty black holes: Thermodynamics and horizon structure, *Phys. Rev. D* **46**, 2445 (1992).
- [116] T. Padmanabhan, Classical and quantum thermodynamics of horizons in spherically symmetric space-times, *Classical Quantum Gravity* **19**, 5387 (2002).
- [117] A. J. M. Medved, D. Martin, and M. Visser, Dirty black holes: Space-time geometry and near horizon symmetries, *Classical Quantum Gravity* **21**, 3111 (2004).
- [118] K. A. Bronnikov, Regular black holes sourced by nonlinear electrodynamics, [arXiv:2211.00743](#).
- [119] R. Carballo-Rubio, Master field equations for spherically symmetric gravitational fields beyond general relativity, *Nat. Commun.* **17**, 1399 (2026).
- [120] B. L. Giacchini, T. d. P. Netto, and L. Modesto, Action principle selection of regular black holes, *Phys. Rev. D* **104**, 084072 (2021).
- [121] J. R. Oppenheimer and H. Snyder, On continued gravitational contraction, *Phys. Rev.* **56**, 455 (1939).