

Dynamical generation of charmoniumlike tetraquarks in an off-shell coupled-channel formalism

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We investigate the dynamical generation of isoscalar charmoniumlike states with quantum numbers $J^{PC} = 0^{++}, 1^{++}, 2^{++}$, and 3^{--} in the mass range of 3.6 to 4.3 GeV. We employ the off-shell coupled-channel formalism, constructing kernel amplitudes from effective Lagrangians that respect heavy-quark spin-flavor and chiral symmetries. To focus solely on dynamically generated states, we explicitly exclude s -channel pole diagrams and include only t - and u -channel meson exchanges. Solving the integral equations, we identify six poles in the complex energy plane. In the scalar (0^{++}) sector, we find a bound state below the $D\bar{D}$ threshold and a resonance at $\sqrt{s_R} = (3861 - i23)$ MeV. For the axial-vector (1^{++}) sector, the experimentally observed $\chi_{c1}(3872)$ is reproduced as a bound state near the $D\bar{D}^*$ threshold, alongside a broader resonance at $(3961 - i32)$ MeV, which is a plausible candidate for the $X(3940)$. Furthermore, we find a narrow tensor (2^{++}) pole at $(4005 - i6)$ MeV and a ψ_3 -like (3^{--}) pole at $(4030 - i33)$ MeV. The present results demonstrate that coupled-channel dynamics, particularly involving the $D^*\bar{D}^*$ channel, play a crucial role in the formation of these charmoniumlike exotic states.

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I. INTRODUCTION

The discovery of the $\chi_{c1}(3872)$ [also known as $X(3872)$] by the Belle Collaboration [1] opened a new era in the study of the heavy hadrons. Its extremely narrow width, isospin-violating decays, and proximity to the $D^0\bar{D}^{*0}$ threshold cannot be accommodated by a simple interpretation of conventional quark models. Since then, a large number of charmoniumlike structures, often referred to as the XYZ states, have been reported by the Belle, BABAR, BESIII, and LHCb Collaborations in various hadronic processes such as B decays, initial-state radiation, two-photon fusion, and e^+e^- annihilation (see the reviews [2–7]). While a few of the observed structures can still be understood as ordinary charmonia [8–10], e.g., $\psi_2(3823)$ and $\psi_3(3842)$ as the $1D$ spin-triplet [11] or $\chi_{c2}(3930)$ as the 2^3P_2 $c\bar{c}$ state [12,13], many others lie close to open-charm thresholds and show behaviors that are difficult to describe with a naive $c\bar{c}$ interpretation. Several of these states have unexpectedly narrow widths, decay modes or branching ratios that do not follow quark-model predictions.

Their properties often depend sensitively on nearby thresholds, for instance $D^{(*)}\bar{D}^{(*)}$ channels, indicating that coupled-channel dynamics play a crucial role. As a result, the spectrum in the charmonium sector (especially in the $\sqrt{s} \approx 3.7$ to 4.1 GeV region) cannot be described solely in terms of radial and orbital excitations of $c\bar{c}$ states. Instead, it has become a mixture of various structures affected by threshold effects and channel mixing [3,4]. In such a regime, the investigation of coupled-channel dynamics provides essential information, since the properties of these states are directly influenced by how each state couples to nearby open-flavor channels, such as through threshold effects or triangle singularities, offering a practical guide to the underlying configurations of these hidden-charm candidates. The theoretical interpretation of the charmoniumlike spectrum remains highly non-trivial, and many models have been developed to describe the observed structures. Beyond the description of the conventional quark models, compact tetraquarks [14–20] and hadronic molecules generated by near-threshold dynamics [3–7] constitute two of the most widely discussed pictures. Additional mechanisms such as hybrid charmonia, hadroquarkonium configurations, and kinematic effects (cusplike structures or triangle singularities) have also been considered in describing specific candidates.

A particularly important aspect of this sector is that various channels are open near $D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}$, and $D^*\bar{D}^*$

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thresholds, a feature that requires the consideration of coupled-channel dynamics. While the Bethe-Salpeter (BS) equation provides a proper theoretical framework for treating multichannel hadronic scattering, it is practically too complicated to deal with the four-dimensional BS equation. To overcome the complexities of solving the full BS equation, various approximations have been developed such as the on-shell approximation and three-dimensional reductions. In the current work, we will take a three-dimensional reduction known as the Blankenbecler-Sugar (BbS) scheme, which preserves both the unitarity and the off-shell contributions of the coupled-channel integral equations. The kernel amplitudes for the BbS equation are constructed by computing the Feynman invariant amplitudes based on effective Lagrangians. Since we are mainly interested in how the tetraquark states are dynamically generated, we only include the t - and u -channel amplitudes. To preserve the unitarity and to consider the structure of hadrons, it is essential to introduce form factors at each vertex, which causes uncertainties in numerical results. To minimize them, we introduce the reduced cutoff mass defined by the subtraction of the corresponding cutoff mass and exchange hadron mass, which will be discussed in more detail later. Having solved the three-dimensional coupled BbS equations, we can derive the transition amplitudes containing the coupled-channel effects. Surveying pole positions of the transition amplitudes in the complex energy plane, we can determine the masses and widths of the tetraquark states corresponding to the poles. This coupled-channel formalism has been successfully applied to describe various hadronic reactions. In Refs. [21–23], it was shown how the axial-vector mesons such as $a_1(1260)$, $b_1(1235)$, and $h_1(1415)$ are dynamically generated from pseudoscalar-vector meson scattering. The production mechanisms of the hidden-charm pentaquarks with strangeness varying from 0 to -3 were also investigated within the same coupled-channel formalism [24–26], and the $P_{c\bar{c}}$'s were interpreted as the molecular states associated with the heavy mesons $\bar{D}(\bar{D}^*)$ and singly heavy baryons $\Sigma_c(\Sigma_c^*)$. The same theoretical framework was also employed to describe the $D_{s0}^*(2317)$ as a DK molecular state [27] and to investigate the doubly-charmed tetraquark state T_{cc} [28]. A merit of this off-shell coupled-channel formalism can be found in the fact that it incorporates relevant symmetries and energy scales across not only S waves but also higher partial-wave components.

In the present study, we investigate the dynamical generation of charmoniumlike states (χ_{cJ} and ψ_J states) with the hidden-charm $c\bar{c}$ content in the range of center-of-mass energies from 3.6 to 4.3 GeV. Figure 1 summarizes the results of this work in the phenomenologically dense 3.7–4.1 GeV interval, in comparison with the conventional $c\bar{c}$ states listed by the Particle Data Group (PDG) [29]. Since we mainly focus on the dynamical generation of exotic states, we intentionally exclude s -channel pole diagrams

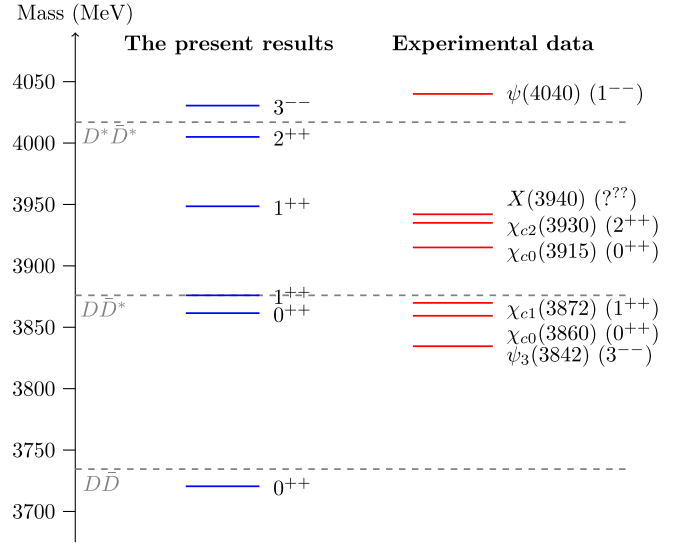


FIG. 1. Mass spectrum of hidden-charm states in the range of 3.7–4.1 GeV. The lines in the left column indicate the pole states obtained in the present work, whereas those in the right column denote the experimental data for the states with $c\bar{c}$ content [29].

from the kernel amplitudes to ensure that all resonances are dynamically generated solely from coupled-channel dynamics. Consequently, we identify six dynamically generated pole states by solving the coupled-channel integral equations. Three of these poles are consistent with the experimentally observed charmoniumlike structures, such as the $\chi_{c1}(3872)$ near the $D\bar{D}^*$ threshold. Furthermore, our results predict three additional poles that are dynamically generated through coupled-channel dynamics.

This paper is organized as follows. In Sec. II, we explain the general formalism for the coupled-channel BbS scheme in detail. The interactions between heavy mesons, charmonia, and light mesons are derived from effective Lagrangians governed by heavy-quark spin-flavor and chiral symmetries, which serve as the basis for constructing the kernel matrices. In Sec. III, we discuss the numerical results for each spin-parity assignment. By scanning the transition amplitudes, we extract pole positions and coupling strengths to various hadronic channels to elucidate their dynamical origins and properties. We will also discuss the physical origin of the agreement and discrepancy with the experimental data in detail. Section IV is devoted to a summary of our numerical analysis and concluding remarks.

II. GENERAL FORMALISM

The scattering amplitude of the $i \rightarrow f$ process is given by

$$S_{fi} = \delta_{fi} - i(2\pi)^4 \delta^{(4)}(P_f - P_i) \mathcal{T}_{fi}, \quad (1)$$

where P_i and P_f are the total four-momenta of the initial and final states, respectively. The transition amplitudes $\mathcal{T}_{fi}$

contain nontrivial information on how the resonances and bound states are generated dynamically. The $\mathcal{T}_{fi}$ are obtained by solving the BS equation with the two-body Feynman kernel amplitudes

$$\mathcal{T}_{fi}(p, p'; s) = \mathcal{V}_{fi}(p, p'; s) + \frac{1}{(2\pi)^4} \sum_k \int d^4 q_k \mathcal{V}_{fk}(p, q_k; s) \times G_k(q_k; s) \mathcal{T}_{ki}(q_k, p'; s), \quad (2)$$

where p and p' denote respectively the relative four-momenta of the initial and final states, and q_k is the off-shell four-momentum for intermediate states in the center-of-mass (CM) frame. The variable $s \equiv P_i^2 = P_f^2$ is the square of the total energy. The indices i, f , and k indicate the initial, final, and intermediate states, respectively. The index k runs over all relevant intermediate channels to isoscalar hidden-charm states. The schematic diagram for the coupled BS Eq. (2) is depicted in Fig. 2.

Since it is computationally demanding to solve the four-dimensional BS integral equations, we introduce a three-dimensional reduction approach. While there are several three-dimensional reduction schemes, we adopt the BbS formalism [30,31], where the two-body propagator is taken to be

$$G_k(q_k) = \frac{\pi}{\omega_1^k \omega_2^k} \delta\left(q_k^0 - \frac{\omega_1^k - \omega_2^k}{2}\right) \frac{\omega_1^k + \omega_2^k}{s - (\omega_1^k + \omega_2^k)^2 + i\epsilon}, \quad (3)$$

where $\omega_{1(2)}^k = \sqrt{\mathbf{q}_k^2 + (m_{1(2)}^k)^2}$ is the energy of the particle 1(2) in the channel k on the mass shell. We then insert G_k in Eq. (2) and perform the q_k^0 integration using the delta function, so that we obtain the three-dimensional coupled integral equation

$$\begin{aligned} T_{fi}(\mathbf{p}, \mathbf{p}'; s) &= V_{fi}(\mathbf{p}, \mathbf{p}'; s) \\ &+ \frac{1}{(2\pi)^3} \sum_k \int \frac{d^3 q_k}{2\omega_1^k \omega_2^k} \frac{\omega_1^k + \omega_2^k}{s - (\omega_1^k + \omega_2^k)^2 + i\epsilon} \\ &\times V_{fk}(\mathbf{p}, \mathbf{q}_k; s) T_{ki}(\mathbf{q}_k, \mathbf{p}'; s). \end{aligned} \quad (4)$$

To investigate dynamically generated resonances with specific quantum numbers, we need to perform the partial-wave decomposition of the transition amplitudes as follows:



FIG. 2. Schematic diagram for the two-body coupled integral equations.

$$\begin{aligned} T_{\lambda\lambda'}^{J(fi)}(\mathbf{p}, \mathbf{p}'; s) &= V_{\lambda\lambda'}^{J(fi)}(\mathbf{p}, \mathbf{p}'; s) \\ &+ \sum_{k, \{\lambda_k\}} \frac{1}{(2\pi)^3} \int_0^\infty dq_k \frac{\omega_1^k + \omega_2^k}{2\omega_1^k \omega_2^k} \\ &\times \frac{q_k^2 V_{\lambda'\lambda_k}^{J(fk)}(\mathbf{p}, \mathbf{q}_k; s) T_{\lambda_k\lambda}^{J(ki)}(\mathbf{q}_k, \mathbf{p}'; s)}{s - (\omega_1^k + \omega_2^k)^2 + i\epsilon}, \end{aligned} \quad (5)$$

where $\{\lambda_k\}$ stands for the helicity components for the particles involved in the intermediate hadron channel k . The partial-wave kernel and transition amplitudes in the helicity basis are derived as

$$V_{\lambda\lambda'}^{J(fi)}(\mathbf{p}, \mathbf{p}'; s) = 2\pi \int d\cos\theta V_{fi}(\mathbf{p}, \mathbf{p}', \theta; s) d_{\lambda\lambda'}^J(\theta), \quad (6)$$

$$T_{\lambda\lambda'}^{J(fi)}(\mathbf{p}, \mathbf{p}'; s) = 2\pi \int d\cos\theta T_{fi}(\mathbf{p}, \mathbf{p}', \theta; s) d_{\lambda\lambda'}^J(\theta), \quad (7)$$

where $d_{\lambda\lambda'}^J$ denotes the reduced Wigner function as a function of the scattering angle θ . $\lambda^{(i)}$ indicates the helicity difference between two particles in the initial (final) state. $\mathbf{p}$, $\mathbf{p}'$, and $\mathbf{q}_k$ are the magnitudes of three-momenta for initial, final, and intermediate states, respectively.

To classify the resulting amplitudes by orbital angular momentum ℓ and total two-particle spin s , we transform from the helicity basis of Eqs. (6) and (7) to the LSJ basis using the standard Jacob-Wick relation between helicity and LS -coupled two-particle states,

$$\begin{aligned} |JM\lambda_1\lambda_2\rangle &= \sum_{\ell s} \left(\frac{2\ell+1}{2J+1} \right)^{1/2} \\ &\times (\ell 0 s \lambda | J \lambda) (s_1 \lambda_1 s_2 - \lambda_2 | s \lambda) | JM \ell s \rangle, \end{aligned} \quad (8)$$

where $\lambda = \lambda_1 - \lambda_2$ is the helicity difference of the two particles, $s_{1,2}$ and $\lambda_{1,2}$ are their spins and helicities, and $(j_1 m_1 j_2 m_2 | j m)$ denotes the standard Clebsch-Gordan coefficient. Inverting Eq. (8) projects the helicity-basis amplitudes onto definite ℓ and s , which we then combine with parity, C -parity, and G -parity to obtain the $I^G(J^{PC})$ -classified amplitudes discussed in Sec. III. For channels built from identical or charge-conjugate meson pairs, this projection is carried out on the C -parity eigenstates of Eq. (41) rather than on a single charge ordering.

The partial-wave integral Eq. (5) contains a singularity originating from the two-body propagator. To manage it, we isolate the singular part of the integrand and treat it separately, so that the regularized integral equation is expressed as

$$\begin{aligned}
T_{\lambda'\lambda}^{J(fi)}(p, p'; s) &= V_{\lambda'\lambda}^{J(fi)}(p, p'; s) + \frac{1}{(2\pi)^3} \sum_{k, \{\lambda_k\}} \left[\int_0^\infty dq_k \frac{q_k(\omega_1^k + \omega_2^k)}{\omega_1^k \omega_2^k} \right. \\
&\quad \times \frac{\mathcal{F}(q_k) - \mathcal{F}(\tilde{q}_k)}{s - (\omega_1^k + \omega_2^k)^2} + \frac{1}{2\sqrt{s}} \left(\ln \left| \frac{\sqrt{s} - E_k^{\text{thr}}}{\sqrt{s} + E_k^{\text{thr}}} \right| - i\pi \right) \mathcal{F}(\tilde{q}_k) \Big], \quad (9)
\end{aligned}$$

with

$$\mathcal{F}(q_k) = \frac{1}{2} q_k V_{\lambda'\lambda_k}^{J(fk)}(p, q_k; s) T_{\lambda_k\lambda}^{J(ki)}(q_k, p'; s), \quad (10)$$

where $\tilde{q}_k$ is the on-shell momentum satisfying $\omega_1^k + \omega_2^k = \sqrt{s}$, and $E_k^{\text{thr}} = m_1^k + m_2^k$ is the threshold energy of channel k . The logarithmic term is real and present for all q_k , while the $-i\pi\mathcal{F}(\tilde{q}_k)$ piece, arising from the Sokhotski-Plemelj identity applied to the propagator in Eq. (5), is included only once $\sqrt{s}$ exceeds E_k^{thr} and supplies the correct unitarity cut for channel k as it opens. Because $\mathcal{F}(q_k) - \mathcal{F}(\tilde{q}_k)$ vanishes at $q_k = \tilde{q}_k$, the remaining integral in Eq. (9) is free of the propagator singularity, and the form factors already contained in V suppress the large- q_k region enough for it to converge numerically.

This same term also fixes our Riemann-sheet convention, though the log and $-i\pi$ pieces in Eq. (9) must be treated differently: they originate from the principal-value integral and its δ -function discontinuity, respectively, and only the latter is sheet dependent. The principal-value term is constructed by subtracting the singularity at the positive, physical root $\tilde{q}_k$ and integrating that piece in closed form, so it carries no sheet ambiguity and is left unchanged between sheets. The discontinuity coefficient, by contrast, evaluates via the Jacobian of $q_k \rightarrow \omega_1^k + \omega_2^k$ to a term proportional to $\tilde{q}_k VT$, odd under $\tilde{q}_k \rightarrow -\tilde{q}_k$: this is precisely the piece whose sign flips when the on-shell momentum $q_k(s)$, which has a square-root branch point at $s = (E_k^{\text{thr}})^2$, is continued once through the associated cut and $\text{Im } q_k$ changes sign. Reaching the second (unphysical) sheet for channel k therefore flips only the sign of the discontinuity piece, $-i\pi\mathcal{F}(\tilde{q}_k) \rightarrow +i\pi\mathcal{F}(\tilde{q}_k)$, the standard definition of an adjacent Riemann sheet [32], while the logarithmic term retains its physical-sheet value. For a closed channel there is no discontinuity term to flip in the first place, since the integral in Eq. (9) is then already real and free of any singularity on the real q_k axis; reaching that channel's second sheet instead requires continuing s directly into the complex plane. We write a full Riemann sheet as the string $(s_1, \dots, s_N)$ for N coupled channels, with $s_k = +$ denoting $\text{Im } q_k > 0$ (first, physical) and $s_k = -$ denoting $\text{Im } q_k < 0$ (second, unphysical), realized in Eq. (9) by the sign of the discontinuity term for open channels as described above. For real $\sqrt{s}$ between the n th and $(n+1)$ th thresholds, the physical amplitude $T(s+i0)$ connects continuously only

to the sheet with the n open channels continued to $s_k = -$ and the remaining channels left at $s_k = +$; we search for poles only on this proximal sheet in each energy window, since poles elsewhere are separated from the physical amplitude by additional cuts and are correspondingly screened [32,33], except within about a width of a threshold, where the shadow pole obtained by flipping that one channel's sign alone must also be checked [34,35]. All six poles reported in Sec. III are found on this proximal sheet; the $0^{++} D\bar{D}$ resonance at 3861.31 MeV, for instance, lies below the $J/\psi\omega$ threshold and is obtained with only the $D\bar{D}$ channel continued to $s_1 = -$, and it is this same sign that flips as the $0^{++} D\bar{D}$ pole crosses threshold and turns into the virtual state discussed in Sec. III D.

To solve the coupled integral Eq. (5), we first construct the kernel matrix V_{fi} based on the effective Lagrangians [36–38] with heavy-quark spin-flavor symmetry and chiral symmetry, given as

$$\begin{aligned}
\mathcal{L}_{\text{heavy}}^Q &= ig\text{Tr}[H_b^Q \gamma_\mu \gamma_5 A_{ba}^\mu \bar{H}_a^Q] + i\beta\text{Tr}[H_b^Q v_\mu (\mathcal{V} - \rho)_{ba}^\mu \bar{H}_a^Q] \\
&\quad + i\lambda\text{Tr}[H_b^Q \sigma_{\mu\nu} F_{ba}^{\mu\nu} \bar{H}_a^Q] + g_\sigma \bar{H}_a^Q H_a^Q \sigma, \quad (11)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\text{heavy}}^{\bar{Q}} &= ig\text{Tr}[\bar{H}_a^{\bar{Q}} \gamma_\mu \gamma_5 A_{ab}^\mu H_b^{\bar{Q}}] - i\beta\text{Tr}[\bar{H}_a^{\bar{Q}} v_\mu (\mathcal{V} - \rho)_{ab}^\mu H_b^{\bar{Q}}] \\
&\quad + i\lambda\text{Tr}[\bar{H}_a^{\bar{Q}} \sigma_{\mu\nu} F_{ab}^{\mu\nu} H_b^{\bar{Q}}] + g_\sigma \bar{H}_a^{\bar{Q}} H_a^{\bar{Q}} \sigma, \quad (12)
\end{aligned}$$

where H^Q and its conjugate field $\bar{H}^Q$ represent the heavy superfields that contain a heavy quark Q , and $H^{\bar{Q}}$ and $\bar{H}^{\bar{Q}}$ denote those for superfields with antiheavy quark $\bar{Q}$. The heavy-quark spin-flavor symmetry allows us to write down the field H in terms of the pseudoscalar and vector heavy-meson fields as follows:

$$\begin{aligned}
H_a^Q &= \frac{1 + \not{v}}{2} (P_a^{*Q\mu} \gamma_\mu - P_a^Q \gamma_5), \\
\bar{H}_a^Q &= \gamma_0 H_a^{Q\dagger} \gamma_0 = (P_a^{*Q\dagger\mu} \gamma_\mu + P_a^{Q\dagger} \gamma_5) \frac{1 + \not{v}}{2}, \quad (13)
\end{aligned}$$

$$\begin{aligned}
H_a^{\bar{Q}} &= (P_a^{*\bar{Q}\mu} \gamma_\mu - P_a^{\bar{Q}} \gamma_5) \frac{1 - \not{v}}{2}, \\
\bar{H}_a^{\bar{Q}} &= \gamma^0 H_a^{\bar{Q}\dagger} \gamma^0 = \frac{1 - \not{v}}{2} [P_a^{*\bar{Q}} + P_a^{\bar{Q}} \gamma_5], \quad (14)
\end{aligned}$$

where P^Q and P^{*Q} stand for the heavy meson antitriplet in flavor SU(3) symmetry with a heavy quark Q regarded as the static color source. These fields are normalized by

$$\langle 0 | P | D(B) \rangle = \sqrt{M_{D(B)}}, \quad (15)$$

$$\langle 0 | P^{*\mu} | D^*(B^*) \rangle = \epsilon^\mu \sqrt{M_{D^*(B^*)}}, \quad (16)$$

where ϵ^μ designates the polarization vector of the heavy vector meson. The SU(3) triplet field $P^{\bar{Q}}$ and $P^{*\bar{Q}}$ are also

defined in the same manner. The interactions of the pseudo-Nambu-Goldstone (pNG) bosons with the matter field H can be derived by the nonlinear realization of the pNG fields. The coset field $\xi(x)$, which arises from spontaneous breakdown of chiral symmetry, is parametrized in the coset space $SU(3)_L \otimes SU(3)_R/SU(3)_V$ as $\xi(x) = \exp(i\mathcal{M}/f_\pi)$ with the pion decay constant $f_\pi = 132$ MeV taken as a normalization factor. Here the $\mathcal{M}$ represents the matrix in flavor space that describes the $SU(3)$ pNG fields or the light pseudoscalar meson matrix

$$\mathcal{M} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \quad (17)$$

One can introduce the axial-vector and vector currents

$$\mathcal{A}^\mu = \frac{1}{2}(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger), \quad (18)$$

$$\mathcal{V}^\mu = \frac{1}{2}(\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger), \quad (19)$$

and the axial-vector current can be expanded as

$$\mathcal{A}^\mu = \frac{1}{2}(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger) = \frac{i}{f_\pi} \partial^\mu \mathcal{M} + \dots \quad (20)$$

This expansion shows that the derivative coupling is natural for the light pseudoscalar meson and heavy mesons on account of chiral symmetry and its spontaneous breakdown.

The effective Lagrangian for the vector mesons is constructed by means of the hidden local symmetry approach. Bando *et al.* [39] suggested the gauge equivalence between $SU(3)_L \otimes SU(3)_R/SU(3)_V$ and $[SU(3)_L \otimes SU(3)_R] \otimes [SU(3)_V]_{\text{local}}$, where $[SU(3)_V]_{\text{local}}$ stands for the hidden local symmetry and the dynamical gauge bosons appear as the composite fields. The invariance under the local symmetry requires the new interaction terms in the effective Lagrangian corresponding to the second and third term in Eq. (11). Assuming ideal ω - ϕ mixing, the physical vector meson nonet is then expressed as

$$\rho^\mu = i \frac{g_V}{\sqrt{2}} V^\mu, \quad V^\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}^\mu, \quad (21)$$

with the field strength tensor $F^{\mu\nu} = \partial^\mu \rho^\nu - \partial^\nu \rho^\mu + [\rho^\mu, \rho^\nu]$.

The expansion of the effective Lagrangian in Eqs. (11) and (12) represents the interaction terms for $P^Q(\bar{Q})$, $P^*Q(\bar{Q})$, $\mathcal{M}$, and V . The first term of Eq. (11) gives the effective Lagrangians for $PP^*\mathcal{M}$ and $P^*P^*\mathcal{M}$ vertices

$$\mathcal{L}_{P^Q P^* Q \mathcal{M}} = -\frac{2g}{f_\pi} P_b^{*Q\mu} \partial_\mu \mathcal{M}_{ba} P_a^Q + \text{H.c.}, \quad (22)$$

$$\mathcal{L}_{P^* Q P^* Q \mathcal{M}} = \frac{2ig}{f_\pi} P_b^{*Q\beta} \partial^\mu \mathcal{M}_{ba} v^\nu P_a^{*Q\alpha} \epsilon_{\alpha\beta\mu\nu} + \text{H.c.}, \quad (23)$$

$$\mathcal{L}_{P^{\bar{Q}} P^* \bar{Q} \mathcal{M}} = \frac{2g}{f_\pi} P_b^{*\bar{Q}\mu} \partial_\mu \mathcal{M}_{ba} P_a^{\bar{Q}} + \text{H.c.}, \quad (24)$$

$$\mathcal{L}_{P^{\bar{Q}} P^* \bar{Q} \mathcal{M}} = \frac{2ig}{f_\pi} P_b^{*\bar{Q}\beta} \partial^\mu \mathcal{M}_{ba} v^\nu P_a^{*\bar{Q}\alpha} \epsilon_{\alpha\beta\mu\nu} + \text{H.c.} \quad (25)$$

The interactions of PPV , PP^*V , and P^*P^*V then come from the second and third terms of Eq. (11) and (12)

$$\mathcal{L}_{P^Q P^Q V} = -2i\beta P^Q P^Q \partial^\mu v \cdot \rho = -\sqrt{2}\beta g_V P^Q P^Q \partial^\mu v \cdot V, \quad (26)$$

$$\begin{aligned} \mathcal{L}_{P^Q P^* Q V} &= -4i\lambda \epsilon_{\mu\nu\alpha\beta} (P^Q \partial^\mu P^{*Q\nu} + P^{Q\dagger} \partial^\mu P^{*Q\nu}) v^\alpha \rho^\beta \\ &= 2\sqrt{2}\lambda g_V \epsilon_{\mu\nu\alpha\beta} (P^Q \partial^\mu P^{*Q\nu} + P^{Q\dagger} \partial^\mu P^{*Q\nu}) v^\alpha V^\beta, \end{aligned} \quad (27)$$

$$\begin{aligned} \mathcal{L}_{P^* Q P^* Q V} &= -2i\beta P^{*Q\mu} P_\mu^{*Q\dagger} v \cdot \rho - 4\lambda (\partial_\mu P^{*Q\dagger} P_\nu^{*Q} \\ &\quad - \partial_\mu P^{*Q\mu} P_\nu^{*Q\dagger}) \rho^\nu \\ &= \sqrt{2}\beta g_V P^{*Q\mu} P_\mu^{*Q\dagger} v \cdot V - 2\sqrt{2}i\lambda g_V \\ &\quad \times (\partial_\mu P^{*Q\dagger} P_\nu^{*Q} - \partial_\mu P^{*Q\mu} P_\nu^{*Q\dagger}) V^\nu, \end{aligned} \quad (28)$$

$$\mathcal{L}_{P^{\bar{Q}} P^{\bar{Q}} V} = 2i\beta P^{\bar{Q}} P^{\bar{Q}} \partial^\mu v \cdot \rho = \sqrt{2}\beta g_V P^{\bar{Q}} P^{\bar{Q}} \partial^\mu v \cdot V, \quad (29)$$

$$\begin{aligned} \mathcal{L}_{P^{\bar{Q}} P^* \bar{Q} V} &= -4i\lambda \epsilon_{\mu\nu\alpha\beta} (P^{\bar{Q}} \partial^\mu P^{*\bar{Q}\nu} + P^{\bar{Q}\dagger} \partial^\mu P^{*\bar{Q}\nu}) v^\alpha \rho^\beta \\ &= 2\sqrt{2}\lambda g_V \epsilon_{\mu\nu\alpha\beta} (P^{\bar{Q}} \partial^\mu P^{*\bar{Q}\nu} + P^{\bar{Q}\dagger} \partial^\mu P^{*\bar{Q}\nu}) v^\alpha V^\beta, \end{aligned} \quad (30)$$

$$\begin{aligned} \mathcal{L}_{P^* \bar{Q} P^* \bar{Q} V} &= -2i\beta P^{*\bar{Q}\mu} P_\mu^{*\bar{Q}\dagger} v \cdot \rho + 4\lambda (\partial_\mu P^{*\bar{Q}\dagger} P_\nu^{*\bar{Q}} \\ &\quad - \partial_\mu P^{*\bar{Q}\mu} P_\nu^{*\bar{Q}\dagger}) \rho^\nu \\ &= \sqrt{2}\beta g_V P^{*\bar{Q}\mu} P_\mu^{*\bar{Q}\dagger} v \cdot V + 2\sqrt{2}i\lambda g_V \\ &\quad \times (\partial_\mu P^{*\bar{Q}\dagger} P_\nu^{*\bar{Q}} - \partial_\mu P^{*\bar{Q}\mu} P_\nu^{*\bar{Q}\dagger}) V^\nu. \end{aligned} \quad (31)$$

The effective Lagrangians for interaction with the scalar meson σ are obtained as

$$\mathcal{L}_{P^Q P^Q \sigma} = -2g_{P^Q P^Q \sigma} P_a^{Q\dagger} P_a^Q \sigma, \quad (32)$$

$$\mathcal{L}_{P^* Q P^* Q \sigma} = 2g_{P^* Q P^* Q \sigma} P_{a\mu}^{*Q\dagger} P_a^{*Q\mu} \sigma, \quad (33)$$

$$\mathcal{L}_{P^{\bar{Q}} P^{\bar{Q}} \sigma} = -2g_{P^{\bar{Q}} P^{\bar{Q}} \sigma} P_a^{\bar{Q}\dagger} P_a^{\bar{Q}} \sigma, \quad (34)$$

$$\mathcal{L}_{P^* \bar{Q} P^* \bar{Q} \sigma} = 2g_{P^* \bar{Q} P^* \bar{Q} \sigma} P_{a\mu}^{*\bar{Q}\dagger} P_a^{*\bar{Q}\mu} \sigma, \quad (35)$$

where the heavy quark velocity can be replaced by $i\vec{\partial}_\mu/(2\sqrt{MM'}) = i(\vec{\partial} - \vec{\partial}')/(2\sqrt{MM'})$, with $M^{(\prime)}$ denoting the mass of the heavy meson.

Channels containing charmonia and light unflavored mesons are kinematically relevant in the energy region where hidden-charm exotic mesons are dynamically generated. The corresponding effective Lagrangian is given by

$$\mathcal{L}_J = g_\psi \text{Tr}[J \bar{H}_a^{\bar{Q}} \gamma_\mu \partial^\mu \bar{H}_a^Q], \quad (36)$$

where J represents the charmonium superfield containing both pseudoscalar (η_c) and vector (J/ψ) components

$$J = \frac{1 + \not{v}}{2} [\psi^\mu \gamma_\mu - \eta_c \gamma_5] \frac{1 - \not{v}}{2}. \quad (37)$$

We get the corresponding effective Lagrangian for the pseudoscalar charmonium η_c :

$$\begin{aligned} \mathcal{L}_{\eta_c} = & -g_\psi \eta_c (P^{\bar{Q}\dagger} v \cdot P^{Q\dagger} + P^{Q\dagger} v \cdot P^{\bar{Q}\dagger}) \\ & - i g_\psi \eta_c \epsilon_{\beta\alpha\mu\nu} P^{*\bar{Q}\dagger} v^\beta P^{*Q\dagger\mu} v^\nu, \end{aligned} \quad (38)$$

and for the vector charmonium $J/\psi(\psi^\mu)$:

$$\begin{aligned} \mathcal{L}_{J/\psi} = & i g_\psi \epsilon^{\beta\alpha\mu\nu} v_\nu \psi_\alpha (P^{Q\dagger} v_\beta P^{*\bar{Q}\dagger} + P^{*\bar{Q}\dagger} v_\beta P^{Q\dagger}) \\ & + g_\psi \psi_\mu P^{*Q\dagger} v_\alpha P^{*\bar{Q}\dagger} (g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}) \\ & - g_\psi P^{Q\dagger} v \cdot \psi P^{\bar{Q}\dagger}. \end{aligned} \quad (39)$$

We now discuss how the values of the coupling constants in Eq. (11) and (12) can be determined. g is the axial coupling constant and its value is extracted to be $g = 0.59$ from the $D^* \rightarrow D\pi$ decay rate [40]

$$\Gamma(D^{*+} \rightarrow D^0 \pi^+) = \frac{g^2}{6\pi f_\pi^2} |\mathbf{p}_\pi|^3. \quad (40)$$

$\beta = 0.9$ is determined by the vector meson dominance [39]. The value of $\lambda = 0.56 \text{ GeV}^{-1}$ is taken from Ref. [41], which is the coupling constant for the third term given in Eq. (12). Note that it is related to the PP^*V vertex and P^*P^*V tensor coupling. The value of λ can be determined from the $B \rightarrow K^*$ vector transition form factor at very high q^2 based on the effective field theory approach. With the results from the light-cone QCD sum rules and lattice QCD matched, its value is set to be $\lambda = 0.56 \text{ GeV}^{-1}$. The σ state [also known as $f_0(500)$] in Eq. (35) is often regarded as a chiral partner of the pion, so that the following relation was suggested: $g_\sigma = g_\pi/(2\sqrt{6})$. In Ref. [42], however, the large width of the σ from dynamics of the correlated 2π exchange was considered and the values of $g_{DD\sigma} = 1.50$ and $g_{D^*D^*\sigma} = 5.21$ were extracted by using the dispersion relation. The $g_{DD\rho} = 1.65$ and $g_{D^*D^*\rho} = 6.47$ were also obtained

in the same manner by analyzing the vector-isovector 2π -correlated exchange. The $g_{DD^*\rho}$ coupling constant is fixed by using the Kawarabayashi–Suzuki–Riazuddin–Fayyazuddin (KSFR) relation [43,44]: $g_{DD^*\rho} = 5.8$. As with g_σ above, these directly extracted couplings supersede the vector-meson-dominance relation to g_V for the ρ -exchange vertices. Since ρ and ω enter through the same vector multiplet V^μ in Eq. (21), however, g_V is still needed separately for the ω -exchange vertices, for which we use the KSFR relation [43,44] directly, $g_V = m_\rho/f_\pi \approx 5.8$.

In this work we consider only the vector charmonium (J/ψ) coupling, since it is directly related to experiment; the extension to the pseudoscalar (η_c) state is straightforward under the heavy-quark spin symmetry assumed for the quarkonium sector as well [45]. Since there is no experimental data on the $J/\psi \rightarrow D\bar{D}$ decay, we adopt the estimate of Ref. [46], obtained by relating the coupling $g_{\phi K\bar{K}}$, fixed from the $\phi \rightarrow K\bar{K}$ decay width, to g_ψ under the assumption that the J/ψ decay proceeds analogously to that of ϕ apart from the meson masses involved: $g_\psi = 0.679 \text{ GeV}^{-3/2}$.

We now construct the kernel matrix for charmoniumlike states, χ_{cJ} and ψ_J , within the energy range of the $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$ thresholds. We are able to list the two-hadron states with $I = 0$ lying near these thresholds as follows:

- (i) $D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}^*$, $D_s\bar{D}_s$, $D_s\bar{D}_s^*$, $D_s^*\bar{D}_s^*$ ($G = \pm 1$),
- (ii) $J/\psi\omega$, $J/\psi\phi$, ($G = +1$)
- (iii) $\eta_c\omega$, $\eta_c\phi$, ($G = -1$).

To properly define the allowed hadronic channels for the χ_{cJ} and ψ_J families, we must classify the two-body states according to their exact quantum numbers. Since our scope is restricted to the isoscalar sector ($I = 0$), the G -parity is equivalent to the charge-conjugation parity, $G = C(-1)^I = C$. Furthermore, because the constituent mesons in the present study are either pseudoscalar ($J^P = 0^-$) or vector ($J^P = 1^-$), the intrinsic parity of any two-meson channel is always positive ($P_1 P_2 = (-1)^2 = +1$). Consequently, the total parity of the system is determined entirely by the relative orbital angular momentum L , yielding $P = (-1)^L$.

The assignment of C -parity requires careful treatment depending on the nature of the interacting mesons. For hidden-charm channels consisting of a charmonium and a light vector meson (such as $J/\psi\omega$ and $\eta_c\phi$), the total C -parity is simply the product of the intrinsic C -parities of the constituents, $C = C_1 C_2$, completely independent of the relative orbital angular momentum. For open-flavor channels composed of identical meson-antimeson pairs (such as $D\bar{D}$ and $D^*\bar{D}^*$), the C -parity depends on both the orbital angular momentum L and the total spin S of the two-meson system, given by $C = (-1)^{L+S}$.

On the other hand, nonidentical open-flavor meson pairs, such as $D\bar{D}^*$, are not C -eigenstates. The C -parity eigenstates are constructed via symmetric or antisymmetric

linear combinations

$$|D\bar{D}^*\rangle_{C=\pm} = \frac{1}{\sqrt{2}}(|D\bar{D}^*\rangle \mp |\bar{D}D^*\rangle), \quad (41)$$

with the phase convention $C|D\rangle = |\bar{D}\rangle$, $C|\bar{D}\rangle = |D\rangle$ and $C|D^*\rangle = -|\bar{D}^*\rangle$, $C|\bar{D}^*\rangle = -|D^*\rangle$. By imposing these exact selection rules, we systematically construct two independent kernel matrices composed of the relevant hadronic channels for the positive- G -parity (χ_{cJ}) and negative- G -parity (ψ_J) sectors.

The kernel matrix element is written as the sum of tree-level Feynman amplitudes for all allowed single-meson exchanges

$$\mathcal{V}_{i \rightarrow j} = \sum_A \mathcal{M}_{i \rightarrow j}^A, \quad (42)$$

where A labels the exchanged particle in the process $i \rightarrow j$. As mentioned in the Introduction, we have intentionally excluded any s -channel contributions in this study, since we focus on the resonances generated by the coupled-channel formalism. Figure 3 shows a generic tree-level diagram with two vertices and a meson propagator. The Feynman amplitudes for the exchange of A read

$$\begin{aligned} \mathcal{M}_{12 \rightarrow 34}^A(t) &= \text{IS} F_{A13}^2 \Gamma_{13}^A P^A \Gamma_{24}^A, \\ \mathcal{M}_{12 \rightarrow 34}^A(u) &= \text{IS} F_{A14}^2 \Gamma_{14}^A P^A \Gamma_{23}^A, \end{aligned} \quad (43)$$

where Γ_{ij}^A denotes the vertex function that couples the i and j legs to the exchanged particle A . The vertex functions are obtained straightforwardly from the effective Lagrangians in Eqs. (22)–(39). The quantity P^A is the propagator of the exchanged particle A . The propagators for spin-0 and spin-1 particles are given by

$$P^A(q) = \frac{1}{q^2 - m_A^2}, \quad P_{\mu\nu}^A(q) = \frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_A^2}}{q^2 - m_A^2}. \quad (44)$$

The factor IS denotes the isospin factor arising from the isospin projector and SU(3) Clebsch-Gordan coefficients.

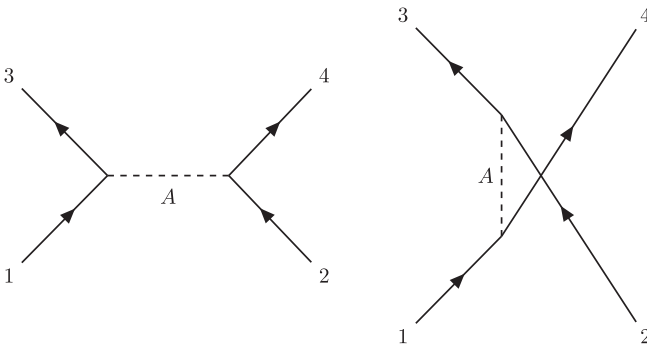


FIG. 3. t - and u -channel Feynman diagrams for $12 \rightarrow 34$.

We introduce a phenomenological vertex form factor to regularize the kernel amplitudes and model finite hadron size as follows [47]:

$$F(q^2) = \left(\frac{n\Lambda^2 - m_A^2}{n\Lambda^2 - q^2} \right)^n, \quad (45)$$

where the value of n depends on the momentum power present in the vertex function Γ . Since the BbS propagator of Eq. (3) already fixes the relative energy of the two-body intermediate state to its on-shell value in the center-of-mass frame, we consistently evaluate the exchanged-particle form factor with $q^0 = 0$, i.e., $q^2 \rightarrow -\mathbf{q}^2 = -(p^2 + p'^2 - 2pp' \cos \theta)$, where $\mathbf{q}$ is the three-momentum transfer between the initial and final states. Keeping the full q^0 dependence instead would introduce unphysical singularities inconsistent with this three-dimensional reduction.

There are merits of using the type of the form factors given in Eq. (45). The Bonn nucleon-nucleon potential employs the conventional form-factor definition without an explicit factor n multiplying Λ^2 . Nevertheless, when the dipole rather than the monopole form is fitted, the corresponding cutoff is found to be larger by approximately a factor of $\sqrt{2}$ [48]. This relation is incorporated directly into our definition through the combination $n\Lambda^2$: for $n = 2$, the effective pole scale is $\sqrt{2}\Lambda$, so the same underlying cutoff parameter Λ can be retained without a separate rescaling. This is a practical advantage of the present parametrization. The same $n\Lambda^2$ -type convention has been used in Ref. [49] and in more recent applications of the same coupled-channel formalism [50], including our own recent works [21–24, 26–28, 42, 51]. We also note that this n -dependent parametrization continuously connects the usual monopole and dipole forms to a Gaussian-type regulator in the limit $n \rightarrow \infty$. The role of n in this framework is to systematically suppress the increasingly severe high-momentum divergence of the vertex function associated with higher-spin exchange. A pseudoscalar-exchange vertex is comparatively mild at large q^2 , whereas vector- and higher-spin-exchange vertices carry extra powers of momentum from their Lorentz structure, so a higher power n is required to suppress the resulting ultraviolet enhancement.

Although there is no experimental information on the values of the cutoff masses Λ in Eq. (45), one can reduce the uncertainties associated with them by considering physical properties of the hadrons involved. Investigations of the electromagnetic form factors of singly heavy baryons [52, 53] have shown that sizes of heavy hadrons are more compact than those of lighter ones. This implies that higher cutoff masses are more suitable for heavy hadrons than for light ones, since the value of the cutoff mass is proportional to the inverse of the size of the corresponding hadron. Thus, we define the reduced cutoff mass as $\Lambda_0 := \Lambda - m_{\text{ex}}$. Interestingly, in Ref. [54], the cutoff mass is related to Λ_{QCD} via $\Lambda = m + \eta\Lambda_{\text{QCD}}$, where

TABLE I. Numerical inputs used in the calculation, with their sources. Meson masses are isospin-averaged PDG values [29].

Quantity	Value	Source
f_π	132 MeV	Input
g	0.59	$D^* \rightarrow D\pi$ [40]
β	0.9	Vector meson dominance [39]
λ	0.56 GeV^{-1}	$B \rightarrow K^*$ form factor [41]
g_ψ	$0.679 \text{ GeV}^{-3/2}$	$\phi \rightarrow K\bar{K}$ analogy [46]
g_V	5.8	KSRF relation, m_ρ/f_π [43,44]
$g_{DD\sigma}$	1.50	Dispersion relation [42]
$g_{D^*D^*\sigma}$	5.21	Dispersion relation [42]
$g_{DD\rho}$	1.65	Dispersion relation [42]
$g_{D^*D^*\rho}$	6.47	Dispersion relation [42]
$g_{DD^*\rho}$	5.8	KSRF relation [43,44]
m_σ	500 MeV	Input

TABLE II. The combined SU(3) and isospin factors for the transitions between open-flavor channels in the isoscalar ($I = 0$) sector. Λ_0 denotes the reduced cutoff mass defined as $\Lambda - m_{\text{ex}}$.

Reaction	Exchange	IS factor		Λ_0 [MeV]
		t -ch	u -ch	
$D\bar{D} \rightarrow D\bar{D}$	σ	1	-1	600
	ρ	3/2	-3/2	600
	ω	1/2	-1/2	600
$D\bar{D} \rightarrow D\bar{D}^*$	ρ	3/2	-3/2	600
	ω	1/2	-1/2	600
$D\bar{D} \rightarrow D_s\bar{D}_s$	K^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$D\bar{D} \rightarrow D_s\bar{D}_s^*$	K^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$D\bar{D} \rightarrow D^*\bar{D}^*$	π, ρ	3/2	-3/2	600
	η	1/6	-1/6	600
	ω	1/2	-1/2	600
$D\bar{D} \rightarrow D_s^*\bar{D}_s^*$	K, K^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$D\bar{D}^* \rightarrow D\bar{D}^*$	σ	1	$\dots$	600
	π	$\dots$	-3/2	600
	η	$\dots$	1/6	600
	ρ	3/2	-3/2	600
	ω	1/2	-1/2	600
$D\bar{D}^* \rightarrow D_s\bar{D}_s$	K^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$D\bar{D}^* \rightarrow D^*\bar{D}^*$	π, ρ	3/2	-3/2	600
	η	1/6	-1/6	600
	ω	1/2	-1/2	600
$D\bar{D}^* \rightarrow D_s\bar{D}_s^*$	K	$\dots$	$-\sqrt{2}$	600
	K^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$D\bar{D}^* \rightarrow D_s^*\bar{D}_s^*$	K, K^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$D_s\bar{D}_s \rightarrow D_s\bar{D}_s$	σ, ϕ	1	1	600
$D_s\bar{D}_s \rightarrow D^*\bar{D}^*$	K, K^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$D_s\bar{D}_s \rightarrow D_s^*\bar{D}_s^*$	ϕ	1	1	600
$D_s\bar{D}_s \rightarrow D_s^*\bar{D}_s^*$	η	2/3	2/3	600
	ϕ	1	1	600
$D^*\bar{D}^* \rightarrow D^*\bar{D}^*$	σ	1	-1	600
	π, ρ	3/2	-3/2	600
	η	1/6	-1/6	600
	ω	1/2	-1/2	600

(Table continued)

TABLE II. (Continued)

Reaction	Exchange	IS factor		Λ_0 [MeV]
		t -ch	u -ch	
$D^*\bar{D}^* \rightarrow D_s\bar{D}_s^*$	K, K^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$D^*\bar{D}^* \rightarrow D_s^*\bar{D}_s^*$	K, K^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$D_s\bar{D}_s \rightarrow D_s\bar{D}_s$	σ	1	$\dots$	600
	η	$\dots$	2/3	600
	ϕ	1	1	600
$D_s\bar{D}_s^* \rightarrow D_s^*\bar{D}_s^*$	η	2/3	2/3	600
	ϕ	1	1	600
$D_s^*\bar{D}_s^* \rightarrow D_s^*\bar{D}_s^*$	σ, ϕ	1	1	600
	η	2/3	2/3	600

η is of order unity. This method has been successfully applied in previous works [21–24,26–28,51]. In the present study, all central results in Secs. III A–C use a common reduced cutoff of $\Lambda_0 = 600$ MeV for every vertex; the sensitivity of this choice is assessed in Sec. III D by varying this common value over $\Lambda_0 = 400$ –800 MeV. The values of the isospin factors IS and the adopted central Λ_0 for the open-flavor and charmonium channels are listed in Tables II and III, respectively. It should be emphasized that no fitting procedure is employed in the present study; the same reduced cutoff mass is used for all vertices at any given Λ_0 , as listed in Tables II and III. For reproducibility, Table I collects all masses and couplings used in the numerical calculation, together with their sources. Isospin-averaged meson masses are used throughout; these reproduce the 3734.5, 3875.8, and 4017.1 MeV thresholds listed in Table IX.

TABLE III. The combined SU(3) and isospin factors for the transitions involving a charmonium channel in the isoscalar ($I = 0$) sector. Λ_0 denotes the reduced cutoff mass defined as $\Lambda - m_{\text{ex}}$.

Reaction	Exchange	IS factor		Λ_0 [MeV]
		t -ch	u -ch	
$\omega\eta_c \rightarrow D\bar{D}$	D^*	-1	-1	600
$\omega\eta_c \rightarrow D\bar{D}^*$	D, D^*	-1	-1	600
$\omega\eta_c \rightarrow D^*\bar{D}^*$	D, D^*	-1	-1	600
$\phi\eta_c \rightarrow D_s\bar{D}_s$	D_s^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$\phi\eta_c \rightarrow D_s\bar{D}_s^*$	D_s, D_s^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$\phi\eta_c \rightarrow D_s^*\bar{D}_s^*$	D_s, D_s^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$\omega J/\psi \rightarrow D\bar{D}$	D, D^*	-1	-1	600
$\omega J/\psi \rightarrow D\bar{D}^*$	D, D^*	-1	-1	600
$\omega J/\psi \rightarrow D^*\bar{D}^*$	D, D^*	-1	-1	600
$\phi J/\psi \rightarrow D_s\bar{D}_s$	D_s, D_s^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$\phi J/\psi \rightarrow D_s\bar{D}_s^*$	D_s, D_s^*	$-\sqrt{2}$	$-\sqrt{2}$	600
$\phi J/\psi \rightarrow D_s^*\bar{D}_s^*$	D_s, D_s^*	$-\sqrt{2}$	$-\sqrt{2}$	600

TABLE IV. Riemann sheet occupied by each dynamically generated pole at $\Lambda_0 = 600$ MeV, labeled by $(s_1, \dots, s_N)$ following the convention of Sec. II in the channel ordering of Tables II and III for the corresponding sector.

Pole	J^{PC}	Sheet	Type
$D\bar{D}$ bound state, 3720.35 MeV	0^{++}	(+, +, +, +, +, +)	Physical (bound)
$D\bar{D}$ resonance, 3861.31 MeV	0^{++}	(-, +, +, +, +, +)	Proximal, 1 open
$D^*\bar{D}^*$ resonance, 4005.26 MeV	2^{++}	(-, -, -, +, +, +)	Proximal, 3 open
$D\bar{D}^*$ bound state, 3874.26 MeV	1^{++}	(+, +, +, +, +, +)	Physical (bound)
$D\bar{D}^*$ resonance, 3961.40 MeV	1^{++}	(-, -, +, +, +, +)	Proximal, 2 open
$D^*\bar{D}^*$ resonance, 4030.48 MeV	3^{--}	(-, -, -, -, -, +)	Proximal, 5 open

III. RESULTS AND DISCUSSION

We are now in a position to present the numerical results and to discuss them. We focus on the dynamical generation of bound states and resonances in the transition amplitudes in the hidden-charm sector over 3.6 to 4.3 GeV. Since the partial-wave decomposition is most conveniently performed in the helicity basis using the Wigner d -functions, the transition amplitudes are first obtained by solving the coupled-channel integral Eq. (5) in the helicity basis and subsequently transformed to the LSJ basis, which allows us to analyze resonant behavior with definite spin, orbital, and total angular momentum. In addition to parity, we impose charge conjugation and G parity, and classify states by $I^G(J^{PC})$. In the charmonium sector, four families are in

principle relevant: χ_{cJ} with J^{++} , ψ_J with J^{--} , η_c with J^{--} , and h_c with J^{+-} . Since most experimentally reported excited $c\bar{c}$ states fall into the χ_{cJ} and ψ_J families, we restrict our analysis to these two sectors. We scan the transition amplitudes extended into the complex energy plane and search for poles with the quantum numbers $I^G(J^{PC})$ generated dynamically by the corresponding coupled-channel reactions. The coupling strength to each hadronic channel is then extracted from the residues of the poles, providing information on the nature of the bound or resonance states. Table IV summarizes, for the six poles discussed below, which Riemann sheet each is found on following the convention of Sec. II.

A. χ_{cJ} states: $I^G(J^{PC}) = 0^+(J^{++})$

1. χ_{c0} $J^{PC} = 0^{++}$

Figure 4 shows the diagonal transition amplitudes for χ_{c0} states in the 1S_0 partial wave near open-flavor thresholds. Note that charmonium channels do not appear as diagonal entries in Fig. 4, since their contribution to elastic scattering are so marginal in this partial wave. We find two pole structures in this channel. The first is a bound state just below the $D\bar{D}$ threshold, generated primarily by elastic $D\bar{D}$ scattering, as signaled by the sharp variation of the $D\bar{D}$ amplitude near $\sqrt{s} \approx 3720$ MeV in Fig. 4. When the six 0^{++} channels $D\bar{D}$, $D_s\bar{D}_s$, $D^*\bar{D}^*$, $D_s^*\bar{D}_s^*$, $J/\psi\omega$, and $J/\psi\phi$ are coupled, with the flavor factors of Tables II and III, the pole is found at $\sqrt{s_R} = 3720.35$ MeV, corresponding to a binding energy of 14 MeV. Experimentally, however, no bound state with $J^{PC} = 0^{++}$ below the $D\bar{D}$ threshold has been established. The second is a clear resonance between the $D\bar{D}$ and $J/\psi\omega$ thresholds, with pole position

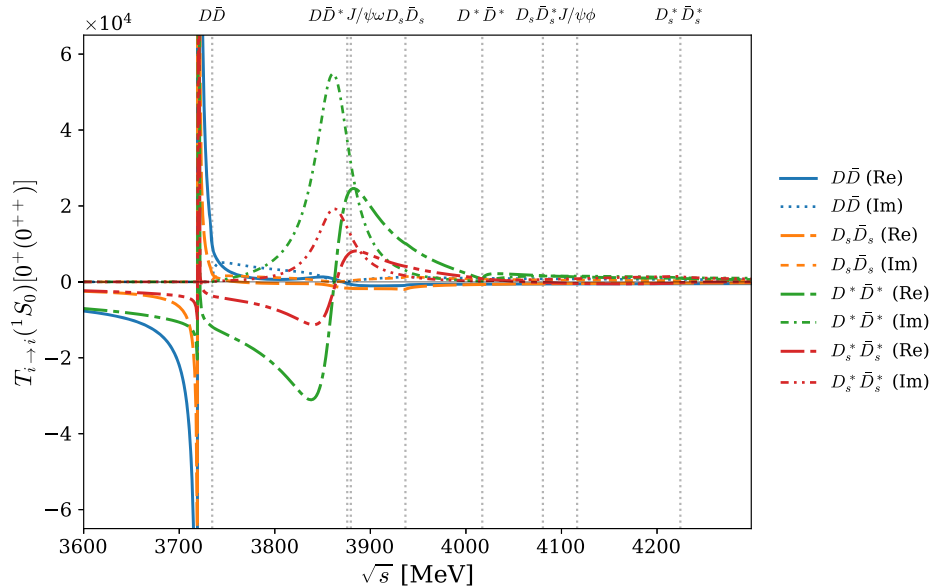


FIG. 4. Diagonal partial-wave transition amplitudes for the $I^G(J^{PC}) = 0^+(0^{++})$ channel as functions of $\sqrt{s}$.

$\sqrt{s_R} = (3861.31 - i22.5)$ MeV. As can be seen in Fig. 4, the most pronounced structure in the transition amplitude appears in the $D^*\bar{D}^*$ channel near $\sqrt{s} \approx 3900$ MeV, indicating that the dynamical generation of this resonance is driven primarily by the $D^*\bar{D}^*$ channel, even though the pole is positioned near the $J/\psi\omega$ threshold. We call this pole a $\chi_{c0}(3861)$ -like state. Above the $D_s\bar{D}_s$ threshold we do not find any further peak structure in this channel. Both the mass and the open-charm accessibility of this pole favor a comparison with $\chi_{c0}(3860)$, reported at $M_{\chi_{c0}(3860)} = 3862^{+50}_{-35}$ MeV and $\Gamma_{\chi_{c0}(3860)} = 200^{+180}_{-110}$ MeV, whose mass agrees well with our result, $M = 3861.31$ MeV, while our width, $\Gamma \simeq 45$ MeV, is smaller than the central value quoted for $\chi_{c0}(3860)$. We also quote $\chi_{c0}(3915)$, reported at $M_{\chi_{c0}(3915)} = 3922.1 \pm 1.8$ MeV and $\Gamma_{\chi_{c0}(3915)} = 20 \pm 4$ MeV: our width lies between the two reported values and is numerically closer to that of $\chi_{c0}(3915)$, although, as discussed in Sec. III C, this pole lies below the $J/\psi\omega$ threshold and therefore cannot reach that channel on shell, whereas $\chi_{c0}(3915)$ was originally established, and continues to be listed, in the $J/\psi\omega$ mode. This channel-accessibility mismatch disfavors a direct identification with $\chi_{c0}(3915)$.

For a quantitative analysis of coupled-channel effects, we extract the coupling strength to each channel from the pole-residue factorization of the transition matrix near the complex pole position $\sqrt{s_R}$, with $s_R \equiv (\sqrt{s_R})^2$,

$$T_{a \rightarrow b}(\sqrt{s}) \simeq 4\pi \frac{g_a g_b}{s - s_R}, \quad (46)$$

which we evaluate in the diagonal, $a = b$, case, $T_{a \rightarrow a}(\sqrt{s}) \simeq 4\pi g_a^2/(s - s_R)$, to extract $|g_a|$ channel by channel. The residues g_a carry mass dimension fixed by

TABLE V. Coupling strengths g_i for the 1S_0 states (in GeV). Higher partial wave contributions are omitted as they are negligible. Values in parentheses denote $|g_i|$.

$\sqrt{s_R}$ [MeV]	3720.35	3861.31 - i22.5
$g_{D\bar{D}}(^1S_0)$	14.13	$5.547 + i5.774(8.007)$
$g_{J/\psi\omega}(^1S_0)$	0.190	$0.561 - i0.099(0.570)$
$g_{D_s\bar{D}_s}(^1S_0)$	7.484	$2.272 + i4.924(5.423)$
$g_{D^*\bar{D}^*}(^1S_0)$	2.680	$36.41 - i16.022(39.78)$
$g_{J/\psi\phi}(^1S_0)$	0.178	$0.482 - i0.074(0.488)$
$g_{D_s^*\bar{D}_s^*}(^1S_0)$	2.228	$20.89 - i10.769(23.51)$

that of the corresponding partial-wave amplitude, are defined on the Riemann sheet on which the pole is found (Sec. II), and are determined only up to an overall phase common to all channels for a given pole; only relative phases between channels are physical. This pole-residue parametrization applies equally to bound, virtual, and resonance poles and should not be read as a Breit-Wigner lineshape, which is specific to resonances on the real axis. The results are listed in Table V. The bound state below the $D\bar{D}$ threshold couples dominantly to $D\bar{D}$, with a subleading coupling to $D_s\bar{D}_s$, both in S wave. The resonance at $(3861.31 - i22.5)$ MeV couples strongly to $D^*\bar{D}^*$, and the coupling to $D_s^*\bar{D}_s^*$ is also large, consistent with strong vector-meson exchange attraction in S wave. Indeed, this resonance originates as a deep bound state in the $D^*\bar{D}^*$ channel and is shifted upward by coupled-channel dynamics, driven predominantly by the $D_s^*\bar{D}_s^*$ channel.

2. $\chi_{c1} J^{PC} = 1^{++}$

Figure 5 shows the transition amplitudes for χ_{c1} states near open-charm thresholds in the $1^{++}(^3S_1)$ channel.

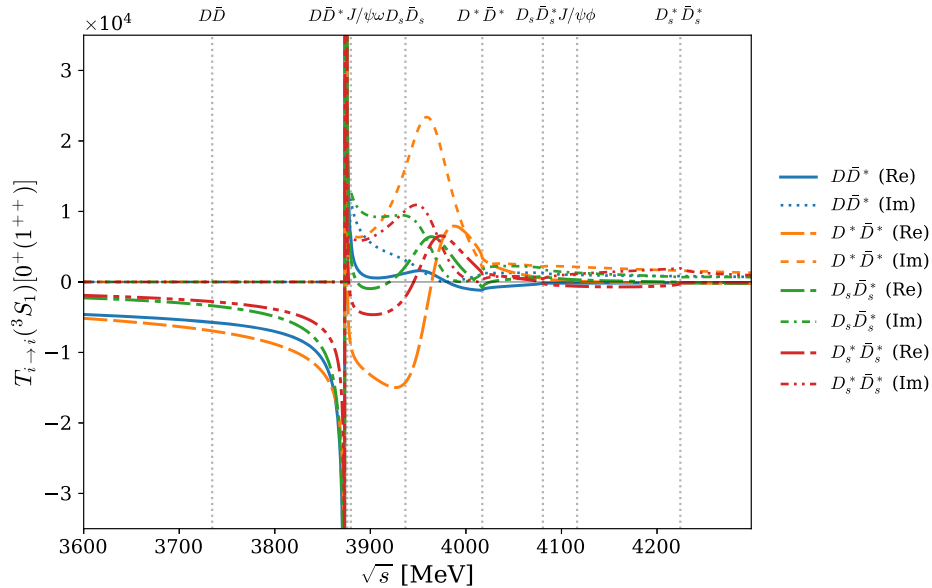


FIG. 5. Axial-vector transition amplitudes for diagonal elements as functions of $\sqrt{s}$ for $I^G(J^{PC}) = 0^+(1^{++})$ channel.

TABLE VI. Coupling strengths g_i for the 3S_1 and 3D_1 channels (in GeV). For the bound state at 3874.26 MeV, imaginary parts are omitted due to their negligible magnitude. Values in parentheses denote $|g_i|$.

$\sqrt{s_R}$ [MeV]	3874.26	3961.40 – $i32.25$
$g_{D\bar{D}^*}(^3S_1)$	7.125	3.712 + $i6.796(7.744)$
$g_{J/\psi\omega}(^3S_1)$	0.102	0.447 + $i0.060(0.451)$
$g_{D^*\bar{D}^*}(^3S_1)$	4.863	22.03 + $i1.355(22.07)$
$g_{D_s\bar{D}_s^*}(^3S_1)$	6.898	10.73 + $i8.910(13.95)$
$g_{J/\psi\phi}(^3S_1)$	0.018	0.023 + $i0.044(0.050)$
$g_{D_s^*\bar{D}_s}(^3S_1)$	4.657	14.33 + $i6.075(15.56)$
$g_{D\bar{D}^*}(^3D_1)$	0.001	0.179 – $i1.046(1.061)$
$g_{J/\psi\omega}(^3D_1)$	0.000	0.023 – $i0.039(0.045)$
$g_{D^*\bar{D}^*}(^3D_1)$	0.084	0.140 + $i0.093(0.168)$
$g_{D_s\bar{D}_s^*}(^3D_1)$	0.161	0.111 + $i0.158(0.193)$
$g_{J/\psi\phi}(^3D_1)$	0.001	0.000 + $i0.001(0.001)$
$g_{D_s^*\bar{D}_s}(^3D_1)$	0.193	0.415 + $i0.241(0.480)$

We find two pole structures in this channel. The first is a bound state at $\sqrt{s_R} = 3874.26$ MeV, lying 1.5 MeV below the isospin-averaged $D\bar{D}^*$ threshold. As can be seen in Fig. 5, the sharp variation of the $D\bar{D}^*$ amplitude near this energy confirms that this state is generated mainly by elastic $D\bar{D}^*$ scattering in S wave, naturally associated with the well-known $\chi_{c1}(3872)$; a precision description of its mass and line shape would require the charged-neutral mass splitting and the finite D^* width, both of which are omitted here. The second is a broader resonance above the $D\bar{D}^*$ threshold at $\sqrt{s_R} = (3961.40 - i32.25)$ MeV, whose extracted pole position suggests a possible identification with the $X(3940)$, observed in the $D\bar{D}^*$ decay mode but not in $D\bar{D}$ [29].

The channel couplings in Table VI clarify the dynamical origin of the two 1^{++} poles. For the bound state at $\sqrt{s_R} = 3874.26$ MeV, the largest coupling is to $D\bar{D}^*$ and $D_s\bar{D}_s^*$ in 3S_1 . Interestingly, the couplings to $D^*\bar{D}^*(^3S_1)$ and $D_s^*\bar{D}_s(^3S_1)$ are comparable to those for $D\bar{D}^*$ and $D_s\bar{D}_s^*$, indicating a nontrivial structure of the $\chi_{c1}(3872)$ beyond a pure $D\bar{D}^*$ molecular picture, even without invoking isospin breaking. Hidden-charm channels are small, and the D -wave components are essentially negligible. For the higher pole at $\sqrt{s_R} = (3961.40 - i32.25)$ MeV, the S -wave vector-meson pair channel $D^*\bar{D}^*(^3S_1)$ carries the dominant coupling. The $D_s\bar{D}_s^*(^3S_1)$ and $D\bar{D}^*(^3S_1)$ couplings are sizable but sub-leading, and the D -wave components are smaller by one to two orders of magnitude than the S -wave contributions. Thus this resonance, dominated by open-flavor S -wave channels, is consistent with the broad structure of the $X(3940)$.

3. $\chi_{c2} J^{PC} = 2^{++}$

Figure 6 shows the transition amplitudes for χ_{c2} states near open-charm thresholds in the 2^{++} channel. We find a

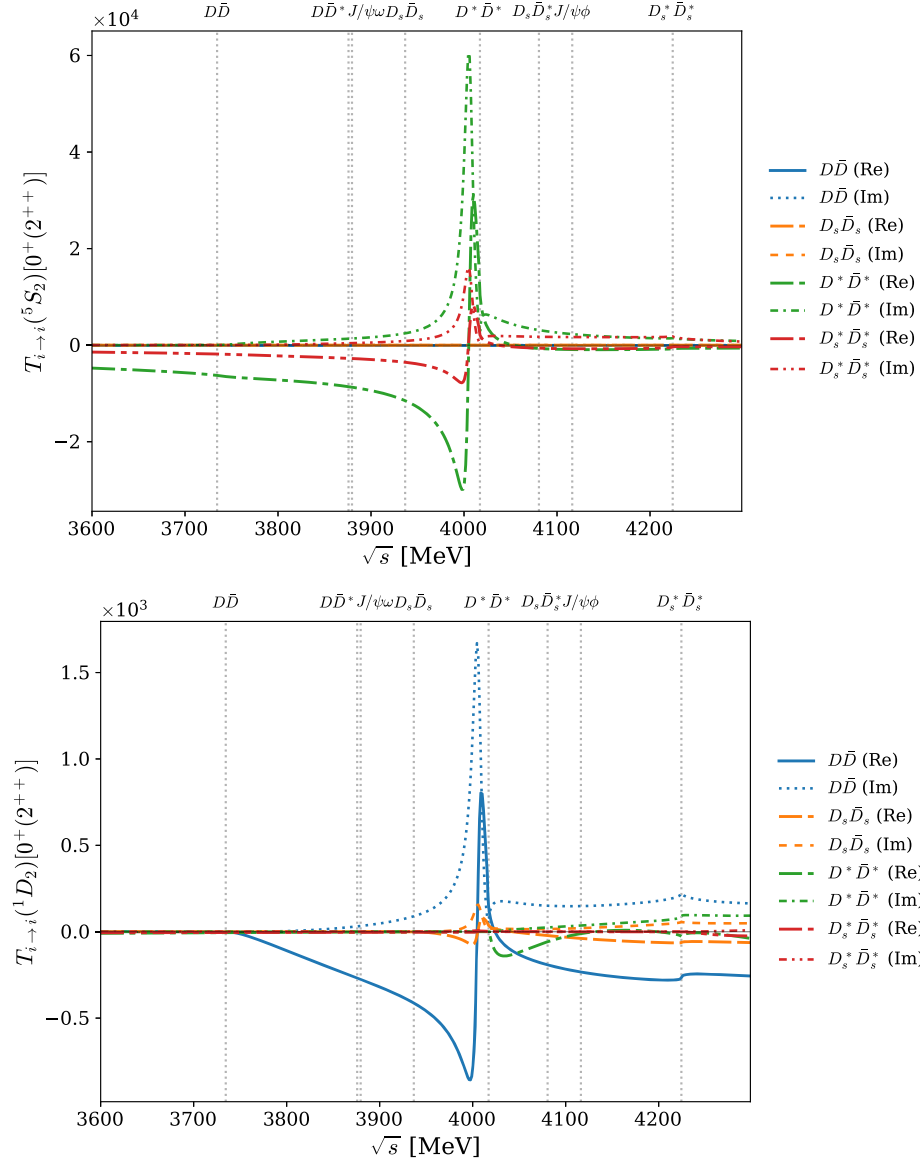
narrow pole at $\sqrt{s_R} = (4005.26 - i5.95)$ MeV, which is located between the $D_s\bar{D}_s$ and $D^*\bar{D}^*$ thresholds. This pole appears in both 5S_2 and 1D_2 partial waves, and the small imaginary part indicates a narrow width of about 12 MeV, suggesting only moderate coupled-channel effects. The residues in Table VII clarify the structure of this state. The largest coupling is to $D^*\bar{D}^*$ in S wave with $|g_{D^*\bar{D}^*}| \simeq 1.71$, and this pole also appears in the $D^*\bar{D}^*$ single-channel solution. As in the χ_{c0} and χ_{c1} cases, the S -wave contribution from $D_s^*\bar{D}_s^*$ is comparable to the dominant one, with $|g_{D_s^*\bar{D}_s^*}| \simeq 0.88$. The hidden-charm channel couplings are negligibly small. Among the D -wave contributions, the $D\bar{D}(^1D_2)$ component is the largest with $|g_{D\bar{D}}| \simeq 0.28$, consistent with the peak structure seen in the lower panel of Fig. 6. We note that the pole mass lies about 70 MeV above the typical values quoted for $\chi_{c2}(3930)$. Whether this state corresponds to a new tensor charmonium not yet observed experimentally remains to be seen.

B. ψ states: $I^G(J^{PC}) = 0^-(J^{--})$

Using the same coupled-channel framework, we analyze the transition amplitudes for the negative- C , negative-parity ψ_J sectors with several values of J . Within the energy range of interest, we do not find dynamically generated states for $J = 0$, $J = 1$, or $J = 2$. For $J = 1$, the well-known vector charmonium states in this sector, such as $\psi(4040)$ with $J^{PC} = 1^{--}$, can be regarded, based on quark models, as having a dominant $c\bar{c}$ core, suggesting that their description would require the inclusion of s -channel pole diagrams, which have been intentionally excluded in the present framework; $J^{PC} = 0^{--}$, by contrast, is not reachable by any conventional $q\bar{q}$ configuration, so the absence of a pole there is expected rather than a shortcoming of the present framework. As the current approach is designed to capture solely the states generated by coupled-channel dynamics, the description of the $J = 1$ and $J = 2$ sectors could be improved by incorporating s -channel contributions, which we leave for future work. In contrast, for $J = 3$ we find a pole above the $D^*\bar{D}^*$ threshold in the $J^{PC} = 3^{--}$ sector. Since we have intentionally excluded s -channel pole diagrams, this pole demonstrates that coupled-channel dynamics alone can generate a 3^{--} state near 4.03 GeV; discriminating this mechanism from a radially excited $3D_3$ charmonium core, such as that of $\psi_3(3842)$, would require a framework containing both bare $c\bar{c}$ poles and meson loops, which is beyond the scope of the present study. We discuss this state in detail below.

1. $\psi_3 J^{PC} = 3^{--}$

For the ψ_3 state with $J^{PC} = 3^{--}$, the selection rules require odd orbital angular momentum L and, for identical meson pairs, even total spin S . Consequently, the allowed partial waves are restricted to 1F_3 for $D\bar{D}$ and $D_s\bar{D}_s$, and

FIG. 6. Tensor transition amplitudes for diagonal elements as functions of $\sqrt{s}$ for $I^G(J^{PC}) = 0^+(2^{++})$ channel.TABLE VII. Coupling strengths g_i for the 5S_2 and 1D_2 channels (in GeV).

$\sqrt{s_R}$ [MeV]	4005.26 - i5.95	
	g_i	$ g_i $
$g_{J/\psi\omega}(^5S_2)$	0.003 - i0.002	0.004
$g_{D^*\bar{D}^*}(^5S_2)$	1.692 - i0.216	1.706
$g_{J/\psi\phi}(^5S_2)$	0.003 + i0.000	0.003
$g_{D_s^*\bar{D}_s^*}(^5S_2)$	0.877 - i0.066	0.879
$g_{D\bar{D}}(^1D_2)$	0.283 - i0.015	0.283
$g_{D_s\bar{D}_s}(^1D_2)$	0.086 - i0.018	0.088
$g_{D^*\bar{D}^*}(^1D_2)$	0.002 + i0.001	0.002
$g_{D_s^*\bar{D}_s^*}(^1D_2)$	0.019 - i0.001	0.019

1F_3 , 5P_3 , and 5F_3 for $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$. Nonidentical pairs and hidden-charm channels couple through the 3F_3 state. Figure 7 shows the diagonal transition amplitudes near open-charm thresholds over 3.6–4.3 GeV for $J^{PC} = 3^{--}$ in the dominant 5P_3 and 1F_3 partial waves. In the P -wave transition amplitude shown in the top panel of Fig. 7, we find a rapid threshold enhancement of the elastic $D^*\bar{D}^*$ scattering just above the $D^*\bar{D}^*$ threshold. The elastic $D\bar{D}$ transition amplitude in F wave also exhibits a clear peak structure just above the $D^*\bar{D}^*$ threshold. Scanning the complex energy plane of the transition matrix, we locate a pole at $\sqrt{s_R} = (4030.48 - i33.33)$ MeV.

The channel coupling strengths listed in Table VIII confirm the dynamical picture suggested by the line shapes.

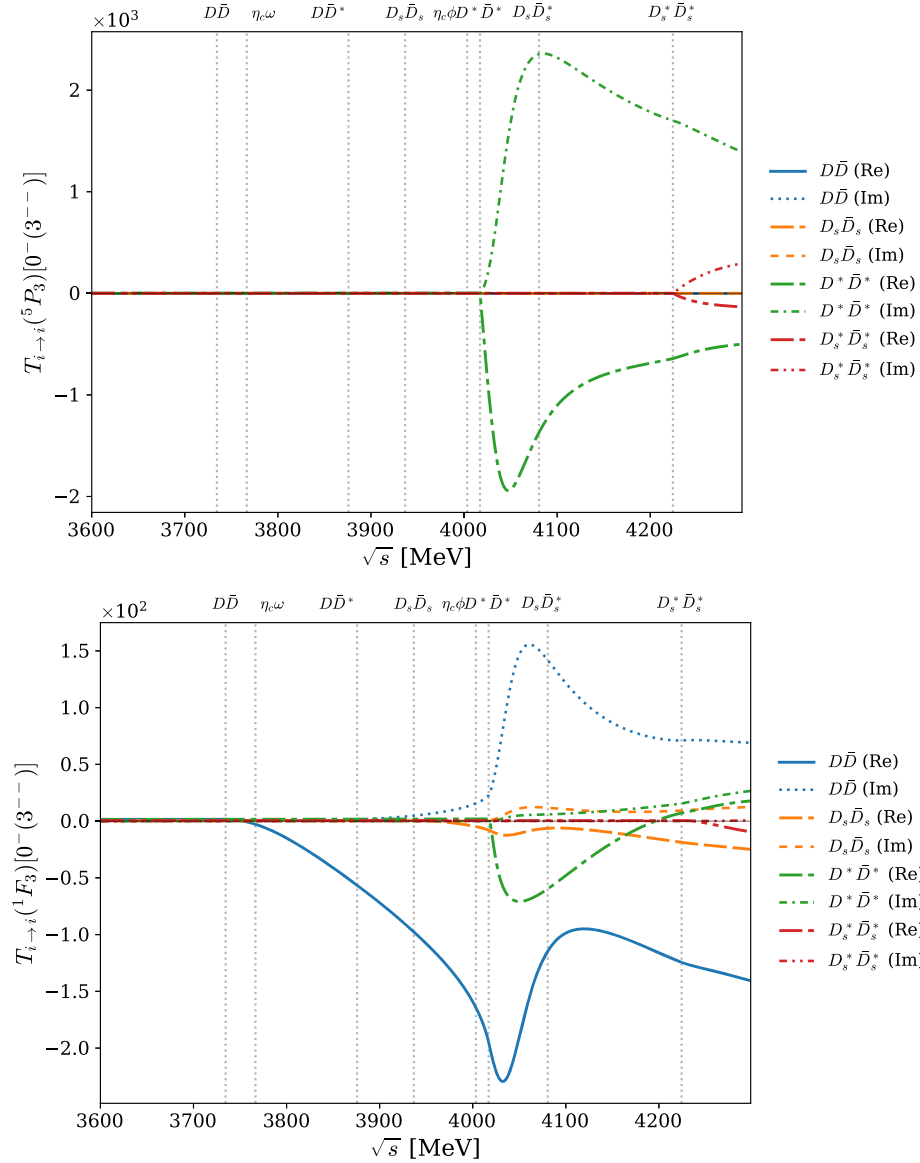


FIG. 7. Transition amplitudes for diagonal elements as functions of $\sqrt{s}$ for $I^G(J^{PC}) = 0^-(3^{--})$ channel in 5P_3 and 1F_3 partial waves.

The dominant coupling is to the $D^*\bar{D}^*(^5P_3)$ channel with $|g_{D^*\bar{D}^*}| \simeq 7.663$, consistent with the pronounced threshold enhancement seen in the top panel of Fig. 7. The $D\bar{D}(^1F_3)$ coupling is also sizable with $|g_{D\bar{D}}| \simeq 2.057$, in accordance with the peak structure in the lower panel. The remaining partial-wave contributions are considerably smaller. We regard this pole as a candidate for a new 3^{--} charmonium state, distinct from the experimentally known $\psi_3(3842)$ with $J^{PC} = 3^{--}$ [29], whose mass lies about 190 MeV lower.

C. Cross sections and experimental signatures

Having identified the poles and their couplings, we now examine how the poles manifest themselves in experimentally accessible line shapes. We plot the partial-wave cross section defined as

$$\sigma_{i \rightarrow f}^I(\sqrt{s}) = \frac{2J+1}{s} \frac{p_f}{p_i} \sum_{\lambda'\lambda} |T_{\lambda'\lambda}^{J(fi)}(\sqrt{s})|^2, \quad (47)$$

where p_i (p_f) is the cm momentum in the initial (final) channel. Figures 8 and 9 show the resulting open- and hidden-charm cross sections at $\Lambda_0 = 600$ MeV.

Figure 8 shows the open-charm elastic cross sections only above their respective thresholds, where σ is a physical observable; below threshold p_i becomes imaginary and no two-body cross section exists there. The two bound states discussed in Sec. III, the 0^{++} $D\bar{D}$ state at 3720.35 MeV and the 1^{++} $D\bar{D}^*$ state at 3874.26 MeV, therefore do not appear as cross-section features and are instead characterized by their pole positions. The 1^{++} state is accessible experimentally in $D\bar{D}^*\pi$, i.e. the $D\bar{D}^*$ channel with $D^* \rightarrow D\pi$,

TABLE VIII. Allowed coupling strengths g_i for the allowed partial waves of the ψ_3 state (in GeV); the $\eta_c\phi(^3F_3)$ coupling is numerically zero at this pole even though the channel is allowed. Partial waves forbidden by spin, parity, and C -parity conservation are omitted.

	g_i	$ g_i $
$g_{D^*\bar{D}^*}(^5P_3)$	$2.883 - i7.100$	7.663
$g_{D_s^*\bar{D}_s^*}(^5P_3)$	$0.004 - i0.002$	0.004
$g_{D\bar{D}}(^1F_3)$	$1.326 - i1.573$	2.057
$g_{D_s\bar{D}_s}(^1F_3)$	$0.351 - i0.486$	0.599
$g_{D^*\bar{D}^*}(^1F_3)$	$0.289 - i0.222$	0.364
$g_{D_s^*\bar{D}_s^*}(^1F_3)$	$2.661 \times 10^{-4} - i0.001$	1.228×10^{-3}
$g_{D\bar{D}}(^3F_3)$	$0.008 + i9.985 \times 10^{-5}$	0.008
$g_{\eta_c\omega}(^3F_3)$	$1.874 \times 10^{-4} - i8.331 \times 10^{-5}$	2.051×10^{-4}
$g_{D_s\bar{D}_s}(^3F_3)$	$2.152 \times 10^{-6} + i5.338 \times 10^{-4}$	5.338×10^{-4}
$g_{\eta_c\phi}(^3F_3)$	0	0
$g_{D^*\bar{D}^*}(^5F_3)$	$0.702 - i0.245$	0.744
$g_{D_s^*\bar{D}_s^*}(^5F_3)$	$8.869 \times 10^{-4} - i0.002$	0.002

which is precisely the mode in which $X(3872)$ is observed; a quantitative line shape in that channel would require the finite D^* width and three-body kinematics, so the present two-body treatment should be regarded as schematic in that respect. The χ_{c0} -like resonance at $(3861.31 - i22.5)$ MeV instead lies above the $D\bar{D}$ threshold and shows up directly in $D\bar{D}$, consistent with both $\chi_{c0}(3860)$ and $\chi_{c0}(3915)$ having been reported in that mode.

The hidden-charm channel cross sections are depicted in Fig. 9. The 1^{++} resonance at 3961.40 MeV and the 2^{++} state at 4005.26 MeV both lie above the $J/\psi\omega$ threshold at 3879.5 MeV and stand out as bumps. By contrast the 0^{++} and 3^{--} sectors are predicted to be visible in open charm only since the χ_{c0} -like resonance lies below the $J/\psi\omega$ threshold and cannot reach it on shell.

D. Cutoff dependence of the dynamically generated poles

We have examined the dependence of our results on the reduced cutoff mass by repeating the full coupled-channel

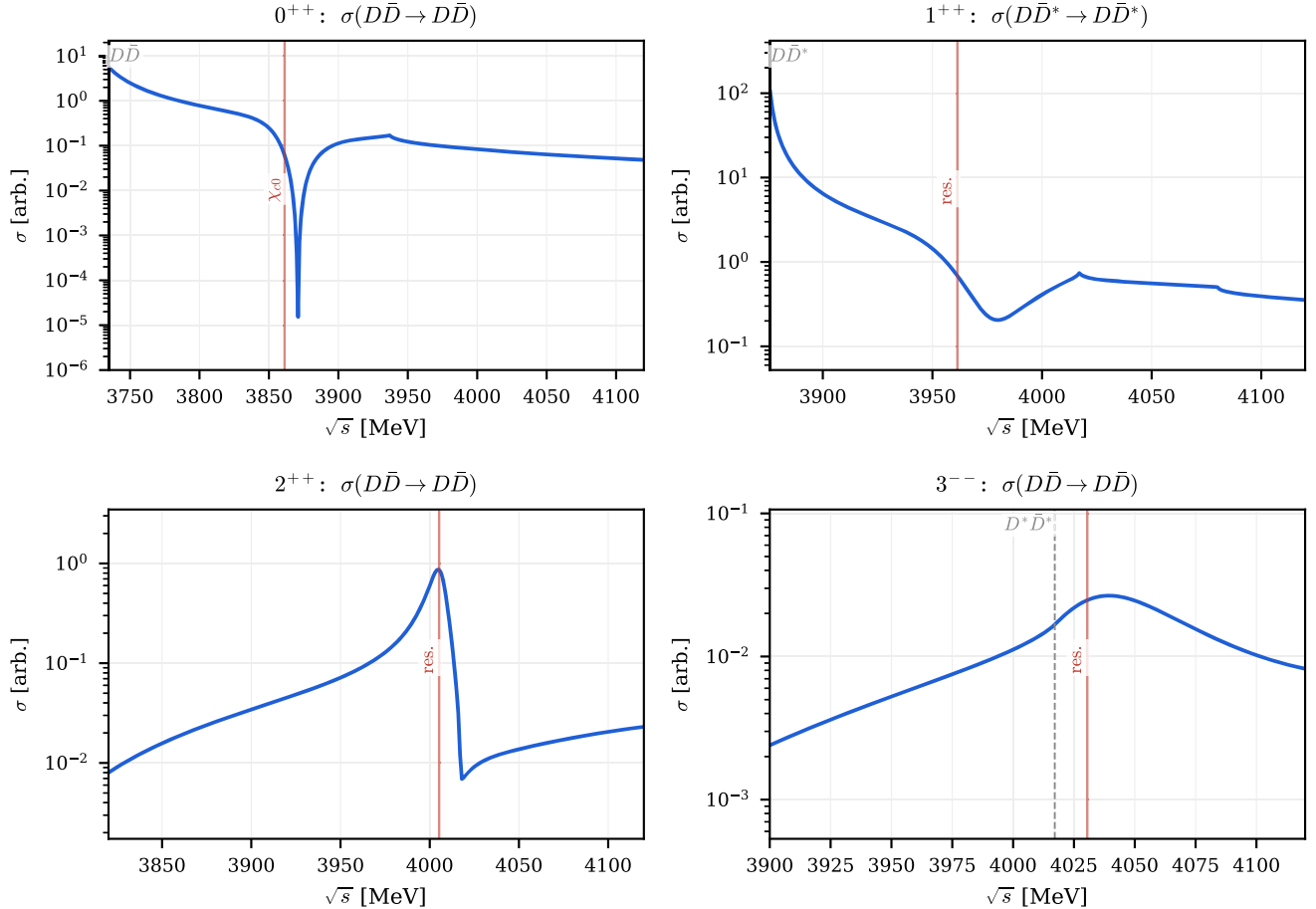


FIG. 8. Open-charm elastic cross sections at $\Lambda_0 = 600$ MeV (arbitrary units; only the line shapes are meaningful), restricted to the physical region above each channel threshold: $\sigma(D\bar{D} \rightarrow D\bar{D})$ for the 0^{++} , 2^{++} and 3^{--} states and $\sigma(D\bar{D}^* \rightarrow D\bar{D}^*)$ for the 1^{++} state. Gray dashed lines mark channel thresholds; red lines mark $\text{Re}\sqrt{s_R}$ of the resonances. The two bound states (0^{++} at 3720.35 MeV, 1^{++} at 3874.26 MeV) lie below threshold, where σ is not a physical observable, and are not shown here; see Sec. III for their pole positions.

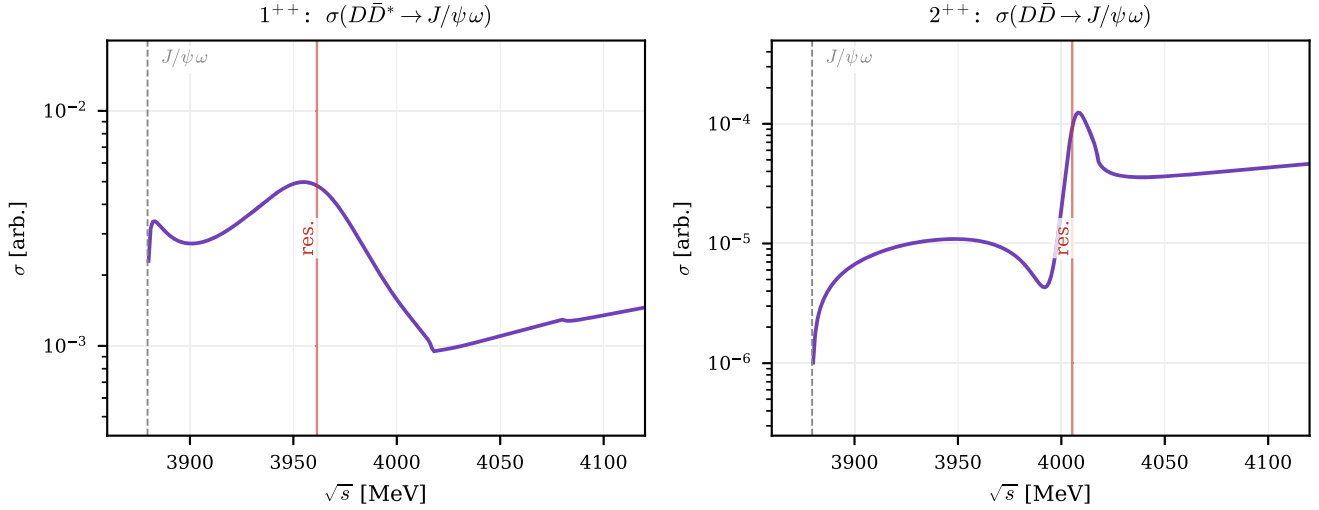


FIG. 9. Hidden-charm transition cross sections into $J/\psi\omega$ at $\Lambda_0 = 600$ MeV (arbitrary units): $\sigma(D\bar{D}^* \rightarrow J/\psi\omega)$ for the 1^{++} resonance and $\sigma(D\bar{D} \rightarrow J/\psi\omega)$ for the 2^{++} state. Gray dashed lines mark the $J/\psi\omega$ threshold; red lines mark $\text{Re}\sqrt{s_R}$ of the resonances.

calculation at $\Lambda_0 = 400, 500, 600, 700$ and 800 MeV and following each pole through the complex plane. Their trajectories are shown in Figs. 10 and 11 and the corresponding positions are listed in Table IX. All the positive-parity poles move monotonically toward lower $\text{Re}\sqrt{s_R}$ as the cutoff is increased so that they become progressively more strongly bound, broadening at the same time as they are pulled away from their thresholds and into the lighter open channels. The 2^{++} $D^*\bar{D}^*$ pole first appears near $\Lambda_0 \simeq 440$ MeV and the 0^{++} $D\bar{D}$ bound state near $\Lambda_0 \simeq 500$ MeV, so that every J^{PC} sector considered here supports at least one pole throughout the range $\Lambda_0 \gtrsim 500$ MeV. This is, however, not a simple statement that poles are gained but never lost: the 0^{++} $D\bar{D}$ resonance itself crosses its threshold and turns into a virtual state near $\Lambda_0 \simeq 690$ MeV, a qualitative change of sheet assignment and observability rather than the disappearance of the pole, and the 1^{++} bound-state pole is likewise virtual at $\Lambda_0 = 500$ MeV before becoming a genuine bound state at larger cutoffs. The number of poles, their sheet assignments, and their phenomenological identifications are therefore cutoff-sensitive, even though the qualitative picture, that every sector supports at least one dynamically generated pole for $\Lambda_0 \gtrsim 500$ MeV, is robust. The scan thus supports the qualitative importance of the coupled-channel interactions considered here but does not by itself establish cutoff-independent pole parameters.

Within this common trend, however, the individual S -wave poles behave rather differently in detail. The 0^{++} $D\bar{D}$ resonance is the most strongly affected: its mass falls steadily from 4011.4 MeV at $\Lambda_0 = 400$ MeV to 3861.31 MeV at 600 MeV and its width grows from $\Gamma = 7.8$ to 45 MeV. We find that between $\Lambda_0 = 675$ and 700 MeV the pole crosses the $D\bar{D}$ threshold and turns into a virtual state, staying on the unphysical sheet. The 0^{++} $D\bar{D}$

bound state, a distinct pole which emerges at the $D\bar{D}$ threshold at $\Lambda_0 \simeq 500$ MeV with small binding energy 0.3 MeV, deepens thereafter to binding energies of $14, 33.5$ and 44.5 MeV. In the 1^{++} sector the $D\bar{D}^*$ pole is virtual at $\Lambda_0 = 500$ MeV, binds by only 1.5 MeV at 600 MeV, and then by 42.6 and 128.3 MeV at 700 and 800 MeV. The 1^{++} $D\bar{D}^*$ resonance descends from 4016.8 to 3880.8 MeV, its width rising rapidly from 65 to 122 and then 134 MeV between $\Lambda_0 = 600, 700$ and 800 MeV. At the largest cutoff it is a very broad structure lying only 5 MeV above the $D\bar{D}^*$ threshold, but it remains a well-defined pole on the same Riemann sheet at every cutoff. The 2^{++} $D^*\bar{D}^*$ pole follows the same pattern, moving from 1.8 at $\Lambda_0 \simeq 500$ MeV to 65.6 MeV at 800 MeV below its threshold with the width growing from 4.2 to 34 MeV, and staying below $D^*\bar{D}^*$ at every cutoff. The 3^{--} $D^*\bar{D}^*$ pole behaves qualitatively differently. Built on the same threshold as the 2^{++} state but in odd partial waves, it does not follow the general trend toward stronger binding. Its real part rises from 4017.7 MeV at $\Lambda_0 = 400$ MeV to 4031.5 MeV at 700 MeV before turning back down to 4025.0 MeV at 800 MeV. The pole remains between 0.6 and 14.4 MeV above the $D^*\bar{D}^*$ threshold for every cutoff considered. Its width, in contrast to those of the S -wave poles, decreases with the cutoff, from $\Gamma \simeq 65$ MeV for $\Lambda_0 \lesssim 600$ MeV to $\Gamma \simeq 49$ MeV at 800 MeV.

E. Comparison with other theoretical estimates

This work predicts a 0^{++} $D\bar{D}$ bound state at 3720.35 MeV and a 2^{++} state at 4005.26 MeV. Table X compares these masses with related predictions from a heavy-quark spin-symmetry EFT [55], a coupled-channel dispersive analysis [56], and a lattice QCD calculation [57], which are also representative of a longer line of molecular-state predictions

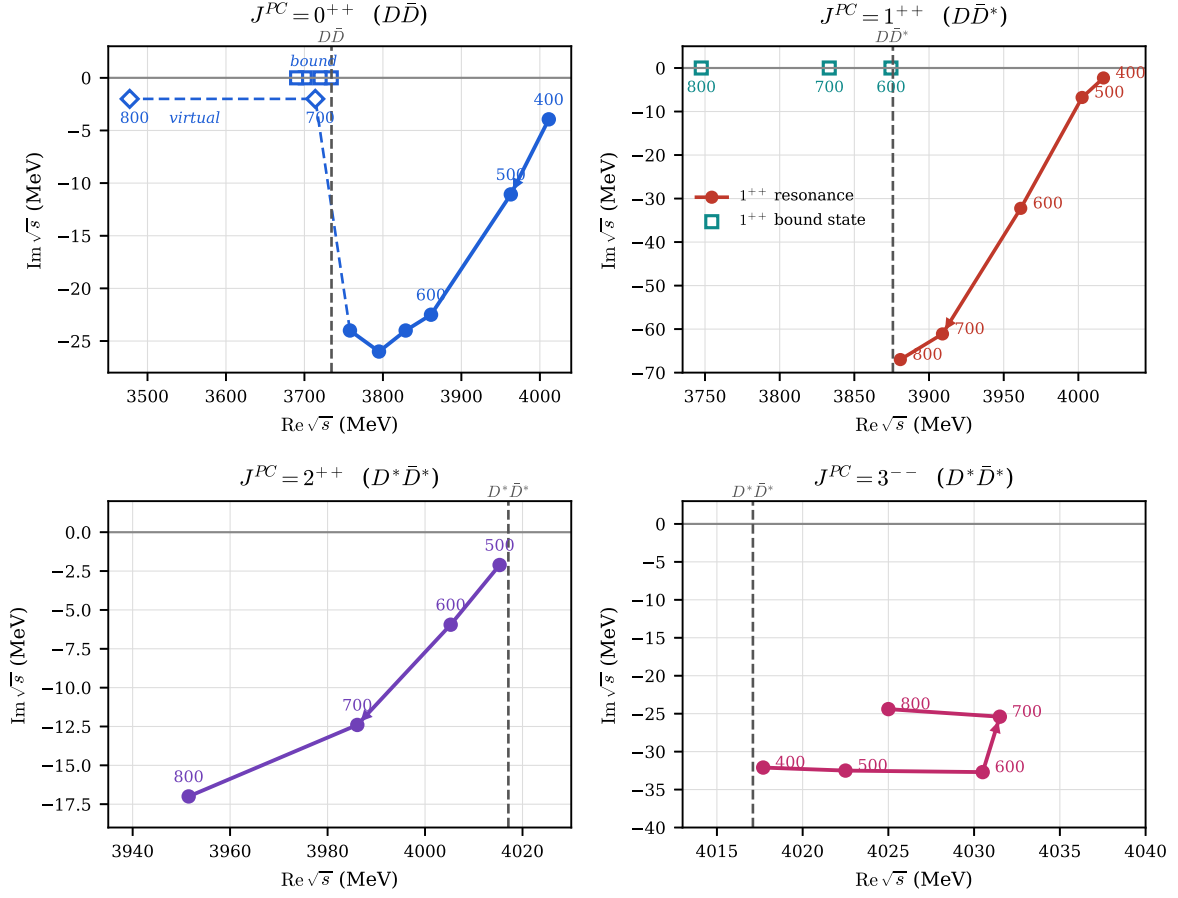


FIG. 10. Trajectories of the dynamically generated poles in the complex $\sqrt{s}$ plane as the reduced cutoff is varied over $\Lambda_0 = 400\text{--}800$ MeV, shown separately for each J^{PC} sector. Filled circles denote resonances, open squares bound states, and open diamonds virtual (second-sheet, subthreshold) states; the labels give Λ_0 in MeV and the dashed vertical lines mark the driver threshold of each sector. For the 0^{++} sector the resonance trajectory is shown on a finer Λ_0 grid (steps of 25 MeV between 600 and 700 MeV) to resolve its descent to the $D\bar{D}$ threshold and its continuation as a virtual state.

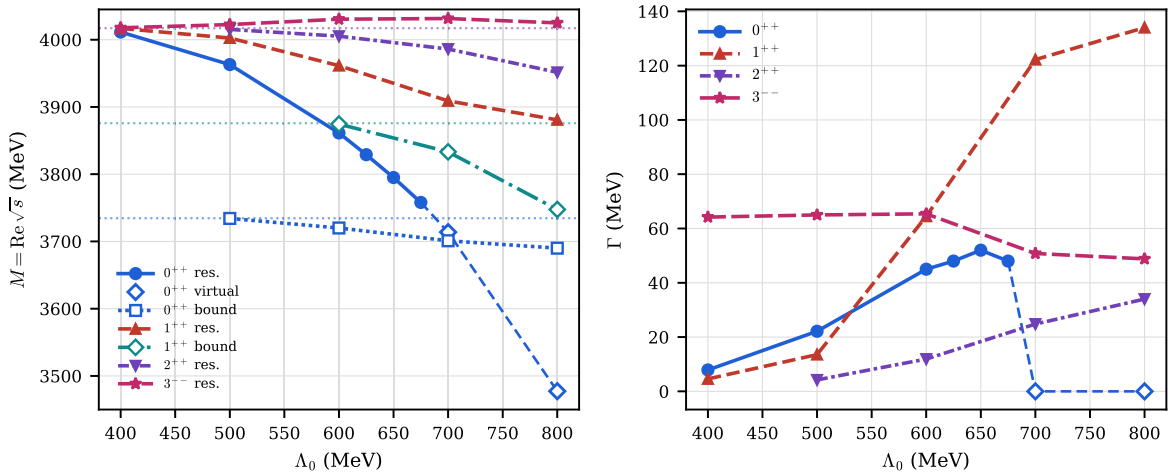


FIG. 11. Pole mass $M = \text{Re}\sqrt{s_R}$ (left) and total width Γ (right) as functions of the reduced cutoff Λ_0 . Dotted horizontal lines in the left panel indicate the relevant thresholds; open squares denote bound states and open diamonds the virtual 0^{++} state below the $D\bar{D}$ threshold. The collapse of the 0^{++} width in the right panel marks the resonance-to-virtual transition near $\Lambda_0 \simeq 690$ MeV.

TABLE IX. Pole positions at the five reduced cutoffs $\Lambda_0 = 400\text{--}800$ MeV. Bound states are listed as M (binding energy B) in MeV; resonances as $\sqrt{s_R} = M - i\Gamma/2$ in MeV. A dash denotes the absence of a pole in that sector, and “virtual” a pole lying below its lowest threshold on the second Riemann sheet. The χ_{c0} -like 0^{++} resonance descends to the $D\bar{D}$ threshold near $\Lambda_0 \simeq 690$ MeV and continues below it as a virtual state ($M \approx 3714$ and 3477 MeV at $\Lambda_0 = 700$ and 800 MeV, respectively). Superscripts give the Riemann sheet ($s_1, \dots, s_6$) in the channel ordering of Table IV for the corresponding sector: $a = (+, +, +, +, +, +)$; $b = (-, +, +, +, +, +)$; $c = (-, -, +, +, +, +)$; $d = (-, -, -, +, +, +)$; $e = (-, -, -, -, +, +)$.

Pole	J^{PC}	$\Lambda_0 = 400$	500	600	700	800
Bound states [M (B)]						
$D\bar{D}$	0^{++}	...	3734.2(0.3) ^a	3720.35(14) ^a	3701(33.5) ^a	3690(44.5) ^a
$D\bar{D}^*$	1^{++}	...	virtual ^b	3874.26(1.5) ^a	3833.2(42.6) ^a	3747.5(128.3) ^a
Resonances [$M - i\Gamma/2$]						
$D\bar{D}$	0^{++}	4011.4 – 3.9i ^d	3963.0 – 11.1i ^d	3861.31 – 22.5i ^b	virtual ^b	virtual ^b
$D\bar{D}^*$	1^{++}	4016.8 – 2.3i ^c	4002.5 – 6.8i ^c	3961.40 – 32.25i ^c	3909.0 – 61.1i ^c	3880.8 – 67.0i ^c
$D^*\bar{D}^*$	2^{++}	...	4015.3 – 2.1i ^d	4005.26 – 5.95i ^d	3986.1 – 12.4i ^d	3951.5 – 17.0i ^d
$D^*\bar{D}^*$	3^{--}	4017.7 – 32.1i ^e	4022.5 – 32.5i ^e	4030.48 – 33.33i ^e	4031.5 – 25.4i ^e	4025.0 – 24.4i ^e

TABLE X. Comparison of the predicted 0^{++} $D\bar{D}$ and 2^{++} $D^*\bar{D}^*$ pole positions with other theoretical estimates. Numerical bound-state and resonance masses are quoted in MeV, as a real mass and as $M - i\Gamma/2$ respectively; where a work reports only the qualitative pole character rather than a numerical mass, that character is quoted instead. A dash denotes that the channel was not computed in that work.

Reference	Method	0^{++} $D\bar{D}$	2^{++} $D^*\bar{D}^*$
This work	Coupled-channel BbS	3720.35	4005.26 – i5.95
Nieves and Valderrama [55]	HQSS EFT	3658–3720	4012–4015
Ji <i>et al.</i> [56]	Coupled-channel dispersive	3727.8	...
Prelovsek <i>et al.</i> [57]	Lattice QCD	Shallow bound state	...

[58–63] for states of this kind. The closest individual predictions, from Ref. [56] and the upper end of the range in Ref. [55], agree with our values to within about 10 MeV, though the full spread of schemes considered in Ref. [55] for the 0^{++} state extends to several tens of MeV. One difference is that our 2^{++} state acquires a small width, $\Gamma \simeq 12$ MeV, from its coupling to open-flavor channels such as $D\bar{D}$ that lie outside the restricted $D^*\bar{D}^*$ channel space of Ref. [55], whereas the corresponding $X(4012)$ state there remains an exactly stable bound state.

IV. SUMMARY AND CONCLUSIONS

In the present work, we have investigated the dynamical generation of charmoniumlike states with isospin $I = 0$ and spin-parity $J^{PC} = 0^{++}, 1^{++}, 2^{++}$, and 3^{--} in the mass range of 3.6 to 4.3 GeV, employing an off-shell coupled-channel formalism based on the Blankenbecler-Sugar reduction of the Bethe-Salpeter equation. The kernel amplitudes were constructed from effective Lagrangians respecting heavy-quark spin-flavor and chiral symmetries, including t - and u -channel meson exchanges while intentionally excluding s -channel pole diagrams, so that all identified poles are generated solely by coupled-channel dynamics. Having solved the coupled-channel integral

equations and scanned the transition amplitudes in the complex energy plane, we have identified six poles in total. Their properties are summarized as follows:

- (i) In the scalar (0^{++}) sector, two poles are identified. The first is a bound state at $\sqrt{s_R} = 3720.35$ MeV, lying 14 MeV below the $D\bar{D}$ threshold and generated primarily by elastic $D\bar{D}$ scattering; no experimental evidence has been established for this state. The second pole at $\sqrt{s_R} = (3861.31 - i22.5)$ MeV is dynamically generated by the $D^*\bar{D}^*$ channel; its mass and open-charm accessibility favor a comparison with $\chi_{c0}(3860)$, while $\chi_{c0}(3915)$ is comparable only in width, since the pole lies below the $J/\psi\omega$ threshold and cannot reach that channel on shell, in tension with the established $J/\psi\omega$ decay of $\chi_{c0}(3915)$.
- (ii) In the axial-vector (1^{++}) sector, we reproduce a narrow bound state at $\sqrt{s_R} = 3874.26$ MeV, lying 1.5 MeV below the isospin-averaged $D\bar{D}^*$ threshold and generated predominantly by elastic $D\bar{D}^*$ scattering. This state is naturally associated with the well-known $\chi_{c1}(3872)$, though a precision description of its mass and line shape requires the charged-neutral mass splitting and finite D^* width omitted here. The coupling analysis reveals a nontrivial

channel structure beyond a pure $D\bar{D}^*$ molecular picture, with sizable contributions from $D^*\bar{D}^*$ and $D_s\bar{D}_s^*$. A second, broader pole is found at $\sqrt{s_R} = (3961.40 - i32.25)$ MeV, dominated by the $D^*\bar{D}^*(^3S_1)$ channel, and is identified as a plausible candidate for the $X(3940)$.

- (iii) In the tensor (2^{++}) sector, we find a narrow pole at $\sqrt{s_R} = (4005.26 - i5.95)$ MeV, lying below the $D^*\bar{D}^*$ threshold and appearing in both 5S_2 and 1D_2 partial waves. The dominant coupling is to $D^*\bar{D}^*$ in S wave, and the pole also appears in the $D^*\bar{D}^*$ single-channel, indicating that coupled-channel effects are moderate. The pole mass lies about 70 MeV above the typical values quoted for $\chi_{c2}(3930)$. This state has not yet been observed experimentally.
- (iv) In the spin-3 (3^{--}) sector, we find a pole at $\sqrt{s_R} = (4030.48 - i33.33)$ MeV, above the $D^*\bar{D}^*$ threshold. This state is dominantly coupled to $D^*\bar{D}^*$ in the 5P_3 partial wave, with a sizable $D\bar{D}(^1F_3)$ contribution. We regard this as a candidate for a new 3^{--} charmonium state, distinct from $\psi_3(3842)$, whose mass lies about 190 MeV lower.

A common pattern emerging from the present analysis is the crucial role of the $D^*\bar{D}^*$ channel in driving the dynamical generation of the resonances across all spin-parity sectors investigated. In particular, the $D_s^*\bar{D}_s^*$ channel consistently provides a subleading but non-negligible contribution, reflecting the importance of hidden-strangeness components in the coupled-channel dynamics.

We have examined the robustness of these results in two independent ways. First, repeating the calculation from $\Lambda_0 = 400$ MeV to 800 MeV (Sec. III D) shows that every J^{PC} sector considered here supports at least one dynamically generated pole for $\Lambda_0 \gtrsim 500$ MeV, though the number of poles, their sheet assignments, and their phenomenological identifications are cutoff-sensitive: the 0^{++} resonance turns into a virtual state near

$\Lambda_0 \simeq 690$ MeV, and the 1^{++} and 2^{++} pole positions move substantially over the scanned range. The scan therefore supports the qualitative importance of these interactions without establishing cutoff-independent pole parameters. Second, for the two states without an established experimental counterpart, the 0^{++} $D\bar{D}$ bound state and the 2^{++} state, the mass of the 0^{++} state agrees to within about 10 MeV with the closest heavy-quark-symmetry EFT and coupled-channel dispersive estimates, while a lattice QCD calculation independently supports the qualitative presence of a shallow 0^{++} $D\bar{D}$ bound state without providing a directly comparable numerical mass (Sec. III E), lending further support to their dynamical origin.

We note that no dynamically generated poles are found in the ψ sectors with $J = 0, 1$, and 2. For $J = 1$, this may be related to the fact that the well-known vector charmonium states in this sector, such as $\psi(4040)$, are widely regarded as having a dominant $c\bar{c}$ core, whose description would require the inclusion of s -channel contributions not considered in the present framework; $J^{PC} = 0^{--}$ has no conventional $q\bar{q}$ assignment at all, so no pole is expected there regardless. A more complete treatment incorporating both s -channel pole diagrams and coupled-channel dynamics would be needed to describe the full charmonium spectrum, and we leave this for future work.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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