

Universal two-zero texture in SO(10): Implications of JUNO observations and realization from noninvertible symmetries

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We apply the universal two-zero texture (UTZT) to all quark and lepton mass matrices in the SO(10) grand unified framework. With charged fermion masses fixed at their best-fit values, this texture contains only seven free parameters to account for nine flavor observables, rendering it highly predictive. Motivated by the recent JUNO indication in favor of the normal ordering of light neutrino masses, we perform an updated analysis of the UTZT in SO(10). The texture remains fully compatible with all current flavor data. The Dirac phase is predicted mainly in two regions, one of which matches very well with current data. A meV-scale $m_{\beta\beta}$ is predicted, beyond the sensitivity bound of future neutrinoless double beta decay measurements. We further explore the origin of the UTZT from noninvertible symmetries, without introducing additional low-energy degrees of freedom. We show that the UTZT can be realized through noninvertible selection rules arising from the Z_3 gauging of Z_N , with a minimal realization corresponding to $N = 7$.

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I. INTRODUCTION

The texture-zero approach, after its proposal [1–3], remains an enduring solution to the flavor puzzle in particle physics. It assumes a few vanishing entries in the fermion mass matrices, which enforce correlations between fermion masses and mixing, and the number of free parameters in the flavor space is efficiently reduced. A widely discussed example is a type of two-zero texture where a fermion mass matrix takes the form

$$M_f \sim \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad (1)$$

in which “ $\times$ ” denotes a nonvanishing arbitrary value. The two-zeros correspond to the vanishing (1, 1) and (1, 3) entries in the matrix, and the (3, 1) entry is usually not considered to be independent based on the requirement of

matrix hermiticity or symmetry in specific models. It is extended from the three-zero Fritzsch texture [4,5], where M_f is Hermitian and (2, 2) entry vanishes. A prominent textured ansatz is the four-zero texture [6–8], wherein both the up- and down-type quark mass matrices are assumed to be Hermitian and take the above form. It takes advantage of its analytical simplicity [9,10] and solving the strong CP problem [11,12].

The two-zero texture is applied to the neutrino mass matrix M_ν in the basis where charged lepton mass matrix M_l is first considered as diagonal [13–15]—see [16–18] for discussions on how well this fits to new data. However, a leptonic version of four-zero texture, that both M_l and M_ν have two-zero textures, is proposed in [19], which allows the generalization to universal two-zero textures (UTZTs) for all lepton mass matrices, including Dirac and Majorana mass matrices for right-handed neutrinos (RHNs), in the seesaw framework. The UTZT includes enough parameter space to fit data and has been continuously studied in light of updated data from neutrino measurements [20–25].

Motivated by its phenomenological success, the UTZT for all quark and lepton masses has been embedded into SO(10) grand unified theories (GUTs) [26]. Below, we denote it as UTZT-SO(10). An important advantage of the GUT version is the substantial reduction of the dimensionality of the flavor parameter space. Since all quarks and leptons, including RHNs, are unified into the 16-dimensional spinorial representation, nontrivial correlations among

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fermion masses and mixing parameters naturally emerge. With the steadily improving precision of neutrino oscillation measurements, such quark-lepton correlations have become increasingly powerful probes of GUT scenarios. Extensive numerical analyses have shown a preference for the normal ordering (NO) for light neutrino masses in several compelling and realistic GUT scenarios [27–34], although inverted ordering is not fully ruled out [35–37]. This is further confirmed by a recent multidimensional numerical scan [38] taking account of JUNO’s first physical results [39]. This finding is in good agreement with the latest JUNO data on the indication of NO at 2.3σ [40].

Texture zeros can be realized in the discrete flavor symmetries, either via Abelian [41] or non-Abelian symmetries [42], however, both at the price of extra copies of Higgses. This approach is applied to the UTZT by employing a Z_6 symmetry in the seesaw framework [20]. Applying Abelian discrete symmetries to mass textures in the GUT framework has also been studied in [43–46]. A UTZT-SO(10) model is constructed in [26], where the contribution of extra Higgses must be carefully treated to avoid the Landau pole in the gauge coupling renormalization group running and to satisfy constraints from proton decay experiments. Modular symmetries provide an alternative possibility to realize texture zeros if the theory is supersymmetric [47,48].

Noninvertible symmetries (NISs) were implemented in particle physics in recent years [49–53]. A bottom-up approach of applying NIS to flavor model building has been proposed in [54]. In particular, the nearest-neighbor interaction texture [55], an asymmetric extension of the Fritzsch texture, is realized in the Z_2 gauging of Z_N . The NIS provides the most economical approach to texture zeros, as it requires no new particles to be introduced at low energies. A systematic study of texture zeros in the neutrino sector via the Z_2 gauging of Z_N in the minimal seesaw framework is performed in [56]. Textures in both charged lepton and neutrino sectors are further explored in the same symmetries in [57]. Three-zero textures for the down-quark masses are proposed in the GUT framework based on the symmetries [58]. In the charged lepton flavor basis, all two-zero textures for neutrino masses are achieved in the Z_3 gauging of Z_N [59].

In this work, we will discuss the up-to-date fitting of the UTZT-SO(10) and its realization in NISs. Compared with former works, the main novelties of this paper are addressed below:

- (i) The UTZT-SO(10) framework remains compatible with the new data of precision neutrino measurements.
- (ii) We adopt the numerical strategy of Ref. [38] to perform a more extensive scan of the parameter space. This scan reveals additional disconnected viable branches in the allowed ranges of δ^l , including

regions close to maximal leptonic CP violation and regions of $m_{\beta\beta}$, which were not discovered in Ref. [26].

- (iii) Since no new particles are introduced at low energies in the NIS framework, this approach automatically avoids the problems of gauge coupling renormalization group (RG) running and experimental constraints of proton decay.

The rest of this paper is organized as follows. In Sec. II, we review the fermion masses and mixing of the UTZT-SO(10), where the quark-lepton correlation induced by GUT is emphasized. The numerical analysis of the fermion masses and mixing fitting to the most recent JUNO data is performed in Sec. III. We consider only the NO case for neutrino masses, since NO is supported by recent JUNO results and no viable points in the inverted ordering case are found in former fits [26]. In Sec. IV, we discuss how the UTZT is realized via noninvertible selection rules. An explicit model based on the Z_3 gauging of Z_N will be presented, while realizations in other NISs will also be discussed. We conclude in Sec. V.

II. FERMION MASSES AND MIXING IN UTZT-SO(10)

In this section, we briefly revisit the UTZT-SO(10), where light neutrino masses are generated via the Type-I+II seesaw mechanism after spontaneous symmetry breaking. We follow the discussion in Ref. [26], where detailed derivations of flavor mixing in two-zero textures and a parametrization of quark-lepton correlations are provided. New contents of the present work include the updated fit and the more comprehensive parameter scan in Sec. III and the realization of the UTZT from noninvertible selection rules in Sec. IV.

We assume the following breaking chain of gauge symmetries in the nonsupersymmetric framework:

$$\begin{aligned}
 SO(10) &\xrightarrow[M_{54}]{M_{\text{GUT}}} G_{422}^C = SU(4)_c \times SU(2)_L \times SU(2)_R \times Z_2^C \\
 &\xrightarrow[M_{45}]{M_2} G_{3221} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \\
 &\xrightarrow[M_{126}]{M_1} G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y, \quad (2)
 \end{aligned}$$

where M_{GUT} and $M_{1,2}$ denote breaking scales of GUT and intermediate gauge symmetries, respectively, with the responsible Higgs multiplet indicated in the subscript. Z_2^C is the parity symmetry that relates left-handed particles to their right-handed charge-conjugate partners. This chain has been well studied in [60,61] and denoted as II3 in [61]. The CP symmetry is also imposed above the GUT scale, essential for the achievement of Hermitian Yukawa matrices at low energies [26]. Assuming the existence of the

UTZT at the GUT scale, we then discuss its analytical properties and reparametrize the fermion Yukawa matrices.

A. Quark-lepton correlation in SO(10) GUTs

In SO(10) GUTs, each generation of SM fermions, including RHNs, is embedded in a single $\mathbf{16}$ -dimensional spinor representation. The Higgs sector contains $\mathbf{10}_H$, $\mathbf{126}_H$, $\mathbf{120}_H$, $\mathbf{54}_H$, and $\mathbf{45}_H$. The multiplets $\mathbf{10}_H$, $\mathbf{126}_H$, and $\mathbf{120}_H$ couple to the fermion bilinear $\mathbf{16} \times \mathbf{16}$ and generate SO(10)-invariant Yukawa interactions. The $\mathbf{45}_H$ and $\mathbf{54}_H$ fields are introduced for gauge symmetry breaking, as presented in Eq. (2), while $\mathbf{126}_H$ also breaks $B - L$ and gives Majorana masses to RHNs. The Yukawa couplings of fermions are given by

$$-\mathcal{L}_Y = (A)_{\alpha\beta} \mathbf{16}_F^\alpha \mathbf{16}_F^\beta \mathbf{10}_H + (B)_{\alpha\beta} \mathbf{16}_F^\alpha \mathbf{16}_F^\beta \mathbf{126}_H + (C)_{\alpha\beta} \mathbf{16}_F^\alpha \mathbf{16}_F^\beta \mathbf{120}_H + \text{H.c.}, \quad (3)$$

assuming $\mathbf{10}_H$, $\mathbf{126}_H$, and $\mathbf{120}_H$ are all complex fields. There are additional terms

$$(A')_{\alpha\beta} \mathbf{16}_F^\alpha \mathbf{16}_F^\beta \mathbf{10}_H^* + (C')_{\alpha\beta} \mathbf{16}_F^\alpha \mathbf{16}_F^\beta \mathbf{120}_H^* + \text{H.c.}, \quad (4)$$

which can be forbidden by imposing a Peccei-Quinn $U(1)$ symmetry.

The decomposition of $\mathbf{16} \times \mathbf{16}$ into $\mathbf{10}_S$, $\mathbf{126}_S$, and $\mathbf{120}_A$ indicates that the Yukawa coupling matrices generated by $\mathbf{10}_H$ and $\mathbf{126}_H$, i.e., $(A)_{\alpha\beta}$ and $(B)_{\alpha\beta}$, are symmetric, while $(C)_{\alpha\beta}$ is antisymmetric. The structure of Yukawa couplings can be further constrained by additional symmetries. We impose the CP symmetry above the GUT breaking scale, leading to real coupling matrices A , B , and C [62]. In order to obtain CP violation in the mixing matrix, we consider spontaneous CP violation, as originally proposed in [63]. It is achieved by assuming real vacuum expectation values (VEVs) of $\mathbf{10}_H$ and $\mathbf{126}_H$ and a purely imaginary VEV of $\mathbf{120}_H$.

The resulting Yukawa couplings of up-type quarks, down-type quarks, charged leptons, and neutrinos are expressed as [64–67]

$$\begin{aligned} Y_u &= H + r_2 F + ir_3 G, \\ Y_d &= r_1 (H + F + iG), \\ Y_\nu &= H - 3r_2 F + ic_\nu G, \\ Y_e &= r_1 (H - 3F + ic_e G), \end{aligned} \quad (5)$$

where H , F , and G are 3×3 real matrices proportional to the coupling matrices A , B , and C , respectively, and the parameters r_1 , r_2 , r_3 , c_ν , and c_e are all real free parameters parametrizing the Higgs VEV contributions. Consequently, the Yukawa coupling matrices remain Hermitian. The light neutrino mass matrix is generated by the Type-(I + II)

seesaw mechanism, given by

$$M_\nu = m_L F - m_R Y_\nu F^{-1} Y_\nu^T, \quad (6)$$

where m_L and m_R are generated by VEVs of triplet Higgses of $SU(2)_L$ and $SU(2)_R$, respectively, both decomposed from $\mathbf{126}_H$ of SO(10). The RHN mass matrix is given by

$$M_R = \frac{v^2}{2m_R} F, \quad (7)$$

where v is the VEV of the SM Higgs doublet.

B. Fermion masses and mixing with UTZT-SO(10)

A characteristic feature of GUT theory is that all fermion mass matrices are correlated with each other, which efficiently reduces the number of free parameters in the model. The UTZT for all fermion mass matrices can further reduce the number of free parameters. The UTZT-SO(10) has been shown to be compatible with current experimental data, while providing predictions for neutrino Majorana masses that can be tested in future experiments. To illustrate the correlations between fermion masses and mixing, Yukawa matrices of charged leptons and neutrinos, as well as the parametrized matrix F , are expressed in terms of those of quarks as

$$\begin{aligned} Y_e &= -\frac{4r_1}{r_2 - 1} \text{Re} Y_u + \frac{r_2 + 3}{r_2 - 1} \text{Re} Y_d + ic_e \text{Im} Y_d, \\ Y_\nu &= -\frac{3r_2 + 1}{r_2 - 1} \text{Re} Y_u + \frac{4r_2}{r_1(r_2 - 1)} \text{Re} Y_d + i \frac{c_\nu}{r_1} \text{Im} Y_d, \\ F &= \frac{\text{Re} Y_u}{r_2 - 1} - \frac{\text{Re} Y_d}{r_1(r_2 - 1)}. \end{aligned} \quad (8)$$

From Eqs. (6) and (7), the light neutrino mass matrix M_ν and the RHN mass matrix M_R also depend on Y_u , Y_d , r_1 , r_2 , and c_ν . Assuming that the Yukawa couplings matrices A , B , and C have the two-zero texture, it is straightforward to show that all Yukawa coupling matrices Y_f , as well as M_ν , have the same two-zero texture, i.e., the UTZT is realized in this model.

To reduce the number of free parameters, the Yukawa coupling matrices are assumed to be Hermitian, i.e.,

$$Y_f = \zeta_f \begin{pmatrix} 0 & C_f & 0 \\ C_f^* & \tilde{B}_f & B_f \\ 0 & B_f^* & A_f \end{pmatrix}, \quad (9)$$

where $f = u, d, \nu, e$, the entries A_f and $\tilde{B}_f$ are real, while B_f and C_f are complex, $A_f > 0$, and $\zeta_f = \pm 1$. Y_u can be reparametrized in terms of the up-type quark Yukawa eigenvalues y_u , y_c , and y_t . ζ_u is set to +1 by a phase redefinition of $\mathbf{16}_F^\alpha$. The Hermitian Yukawa matrix Y_u is

transformed into a real and symmetric matrix by a phase rotation $P_u = \text{diag}(1, e^{i\phi_u}, e^{i\phi'_u})$, i.e.,

$$\bar{Y}_u \equiv P_u Y_u P_u^* = \begin{pmatrix} 0 & |C_u| & 0 \\ |C_u| & \tilde{B}_u & |B_u| \\ 0 & |B_u| & A_u \end{pmatrix}, \quad (10)$$

where $A_u > 0$ and $\tilde{B}_u$ can be either positive or negative. $\bar{Y}_u$ can be diagonalized by a real orthogonal matrix O_u ,

$$\bar{Y}_u = O_u \hat{Y}_u O_u^T, \quad (11)$$

with $\hat{Y}_u = \text{diag}(-\eta_u y_u, \eta_u y_c, y_t)$, where $\eta_u = \pm 1$ is an undetermined sign. We claim that the (3,3) entry of the texture is always smaller than the largest eigenvalue, i.e., $A_u < y_t$. This is proven below. Starting from the identity

$$\det(Y_u - A_u I) = A_u |B_u|^2 = \det(\hat{Y}_u - A_u I) \\ = (-\eta_u y_u - A_u)(\eta_u y_c - A_u)(y_t - A_u) > 0,$$

a direct analysis gives $y_c < A_u < y_t$ for $\eta_u = +1$, and $y_u < A_u < y_t$ for $\eta_u = -1$. O_u can be explicitly expressed in terms of A_u and y_u, y_c , and y_t as [9,12]

$$O_u = \begin{pmatrix} \sqrt{\frac{y_c y_t (A_u + \eta_u y_u)}{A_u (y_u + y_c) (\eta_u y_u + y_t)}} & \eta_u \sqrt{\frac{y_u y_t (A_u - \eta_u y_c)}{A_u (y_u + y_c) (y_t - \eta_u y_c)}} & \sqrt{\frac{y_u y_c (y_t - A_u)}{A_u (\eta_u y_u + y_t) (y_t - \eta_u y_c)}} \\ -\eta_u \sqrt{\frac{y_u (A_u + \eta_u y_u)}{(y_u + y_c) (\eta_u y_u + y_t)}} & \sqrt{\frac{y_c (A_u - \eta_u y_c)}{(y_u + y_c) (y_t - \eta_u y_c)}} & \sqrt{\frac{y_t (y_t - A_u)}{(\eta_u y_u + y_t) (y_t - \eta_u y_c)}} \\ \eta_u \sqrt{\frac{y_u (y_t - A_u) (A_u - \eta_u y_c)}{A_u (y_u + y_c) (\eta_u y_u + y_t)}} & -\sqrt{\frac{y_c (y_t - A_u) (A_u + \eta_u y_u)}{A_u (y_u + y_c) (y_t - \eta_u y_c)}} & \sqrt{\frac{y_t (A_u + \eta_u y_u) (A_u - \eta_u y_c)}{A_u (\eta_u y_u + y_t) (y_t - \eta_u y_c)}} \end{pmatrix}. \quad (12)$$

Then, we get the relations

$$\begin{aligned} \tilde{B}_u &= -\eta_u y_u + \eta_u y_c + y_t - A_u, \\ |B_u| &= \sqrt{(A_u + \eta_u y_u)(A_u - \eta_u y_c)(y_t - A_u)/A_u}, \\ |C_u| &= \sqrt{y_u y_c y_t / A_u}. \end{aligned} \quad (13)$$

The reparametrization of Y_d and Y_e follows the same procedure as that of Y_u , except for the sign ambiguity of ζ_d and ζ_e , i.e.,

$$Y_d = \zeta_d P_d^* O_d \hat{Y}_d O_d^T P_d, \quad Y_e = \zeta_e P_e^* O_e \hat{Y}_e O_e^T P_e, \quad (14)$$

where $\{\hat{Y}_d, O_d, P_d\}$ and $\{\hat{Y}_e, O_e, P_e\}$ have the same structure as $\{\hat{Y}_u, O_u, P_u\}$, under the substitution of $u \rightarrow d, c \rightarrow s, t \rightarrow b$ and $u \rightarrow e, c \rightarrow \mu, t \rightarrow \tau$, respectively. The Cabibbo–Kobayashi–Maskawa (CKM) matrix is represented as

$$V_{\text{CKM}} = O_u^T P_q O_d, \quad (15)$$

where $P_q = P_u P_d^* = \text{diag}(1, e^{i\phi_1}, e^{i\phi_2})$ with $\phi_1 = \phi_u - \phi_d$ and $\phi_2 = \phi'_u - \phi'_d$. We refer to [26] for an analytical approximation of the mixing angles in terms of quark masses, A_u, A_d , phases $\phi_{1,2}$, and signs $\eta_{u,d}$, which will not be repeated here.

In brief, there are four free parameters in the quark sector, $\{A_u, A_d, \phi_1, \phi_2\}$, with the ambiguity of signs $\{\zeta_d, \eta_u, \eta_d\}$.

In the lepton sector, the first relation of Eq. (8) provides four constraints on Y_e . Combining these with

$$\begin{aligned} \tilde{B}_e &= -\eta_e y_e + \eta_e y_\mu + y_\tau - A_e, \\ |B_e| &= \sqrt{(A_e + \eta_e y_e)(A_e - \eta_e y_\mu)(y_\tau - A_e)/A_e}, \\ |C_e| &= \sqrt{y_e y_\mu y_\tau / A_e}, \end{aligned} \quad (16)$$

we can solve for four parameters, A_e, c_e, r_1 , and r_2 , as well as the sign ζ_e , given η_e , eigenvalues of the charged lepton Yukawas and parameters in the quark sector. The remaining free parameters in the lepton sector are c_ν, m_L , and m_R , which will be further used to determine M_ν . The sign ζ_ν cancels out in Eq. (6) and thus does not represent a free parameter.

Given that F and Y_ν follow the two-zero textures, it has been proven that M_ν via the Type-I+II seesaw formula also takes the two-zero texture [19], however, it is not Hermitian but complex symmetric. Phases of nonvanishing entries of M_ν cannot be fully rotated away via phase rotation. In this case, one has to diagonalize this matrix numerically. Suppose M_ν has been diagonalized by a unitary matrix U_ν , i.e., $M_\nu = U_\nu \text{diag}\{m_1, m_2, m_3\} U_\nu^T$. Here, both M_ν and U_ν are determined by $\{m_L, m_R, c_\nu\}$. The Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix is then given by

$$U_{\text{PMNS}} = O_e^T P_e U_\nu. \quad (17)$$

To end this section, we summarize that, apart from the fermion Yukawa eigenvalues, there are seven free parameters in the model,

$$\{A_u, A_d, \phi_1, \phi_2, m_L, m_R, c_\nu\}, \quad (18)$$

with the sign ambiguities $\{\eta_u, \eta_d, \eta_e, \zeta_d\}$.

III. PARAMETER FITTING

Assuming the UTZT at the GUT scale, we perform a numerical analysis of fermion masses and flavor mixing, taking into account recent JUNO data. To reduce the

dimensionality of the parameter space, we fix the eigenvalues of the Hermitian fermion Yukawa matrices to their best-fit values at $M_{\text{GUT}} = 2 \times 10^{16}$ GeV, as provided in Ref. [28],

$$\begin{aligned} y_u^{\text{bf}} &= 2.54 \times 10^{-6}, & y_c^{\text{bf}} &= 1.37 \times 10^{-3}, & y_t^{\text{bf}} &= 0.428, \\ y_d^{\text{bf}} &= 6.56 \times 10^{-6}, & y_s^{\text{bf}} &= 1.24 \times 10^{-4}, & y_b^{\text{bf}} &= 5.7 \times 10^{-3}, \\ y_e^{\text{bf}} &= 2.70341 \times 10^{-6}, & y_\mu^{\text{bf}} &= 5.70705 \times 10^{-4}, & y_\tau^{\text{bf}} &= 9.7020 \times 10^{-3}. \end{aligned} \quad (19)$$

The best-fit values and 1σ uncertainties of the CKM mixing angles and the quark CP -violating phase are

$$\begin{aligned} \theta_{12}^q &= 13.028^\circ \pm 0.034^\circ, & \theta_{23}^q &= 2.783^\circ \pm 0.034^\circ, \\ \theta_{13}^q &= 0.2408^\circ \pm 0.0074^\circ, & \delta^q &= 69.16^\circ \pm 3.09^\circ. \end{aligned} \quad (20)$$

In the neutrino sector, as indicated by JUNO's most recent result [40], we will consider only NO for light neutrino masses and use the following three latest inputs:

$$\begin{aligned} \Delta m_{21}^2 &= (7.388 \pm 0.078) \times 10^{-5} \text{ eV}^2, \\ \Delta m_{31}^2 &= (2.509_{-0.025}^{+0.027}) \times 10^{-3} \text{ eV}^2, \\ \sin^2 \theta_{12}^l &= 0.3036 \pm 0.0064. \end{aligned} \quad (21)$$

Inputs for the remaining oscillation parameters are taken from NuFIT 6.1 in NO for the IC23 fit without Super-Kamiokande atmospheric data [68],

$$\sin^2 \theta_{23}^l = 0.470_{-0.014}^{+0.017}, \quad \sin^2 \theta_{13}^l = 0.02249 \pm 0.00057. \quad (22)$$

The numerical fit is performed in two stages. For both quark and lepton sectors, we define

$$\chi_\alpha^2 = \sum_{i=1}^{n_{\text{obs}}} \left(\frac{\mathcal{O}_i - \mathcal{O}_i^{\text{bf}}}{\sigma_{\mathcal{O}_i}} \right)^2, \quad \alpha = q, l, \quad (23)$$

where $\mathcal{O}_i$, $\mathcal{O}_i^{\text{bf}}$, and $\sigma_{\mathcal{O}_i}$ denote the theoretical prediction, experimental best-fit value, and 1σ uncertainty of the i th observable, respectively. The minimization of the χ_α^2 function is carried out with a differential evolution algorithm, following the setup tested in Ref. [38]. The fitting procedure is summarized as follows:

TABLE I. Model parameter counting.

7 free parameters
Quarks: $A_u/y_t, A_d/y_b \in (0, 1); \phi_1, \phi_2 \in (0, 2\pi)$,
Leptons: $c_\nu \in (10^{-2}, 15); m_L, m_R \in (10^{-1}, 10^2) \text{ eV}$

- (1) In the quark sector, the eigenvalues of Y_u and Y_d are fixed. For each point in the parameter space of Table I, the values of A_u and A_d ($A_u < y_t$ and $A_d < y_b$, as proven in the last section) determine O_u and O_d through Eq. (12) up to the signs $\eta_u, \eta_d = \pm 1$. Together with the phase differences $\phi_1 = \phi_u - \phi_d$ and $\phi_2 = \phi'_u - \phi'_d$, the CKM matrix is obtained from Eq. (15). For each choice of $\eta_u, \eta_d = \pm 1$, the four parameters $\{A_u, A_d, \phi_1, \phi_2\}$ are varied to fit $\{\theta_{12}^q, \theta_{13}^q, \theta_{23}^q, \delta^q\}$. Points satisfying $\chi_q^2 < 10$ are retained for the subsequent lepton sector fit.
- (2) In the lepton sector, as presented in the previous section, A_e , r_1 , r_2 , and c_e are determined via Eqs. (16) and (8) after signs η_e and ζ_d , eigenvalues of the charged lepton Yukawas, and parameters in quark sector are given. Once these parameters are fixed, Y_ν is only determined by c_ν via the second equation of Eq. (8). Consequently, M_ν is determined by three parameters $\{m_L, m_R, c_\nu\}$ via the Type-I+II seesaw formula in Eq. (5). We vary these parameters and minimize χ_l^2 by fitting $\{\theta_{12}^l, \theta_{13}^l, \theta_{23}^l, \Delta m_{21}^2, \Delta m_{31}^2\}$.
- (3) The final results are selected by requiring $\chi_{\text{tot}}^2 = \chi_q^2 + \chi_l^2 < 10$.

In summary, we randomly scan seven free model parameters, $\{A_u, A_d, \phi_1, \phi_2, m_L, m_R, c_\nu\}$, together with the four independent undetermined signs, $\{\eta_u, \eta_d, \eta_e, \zeta_d\}$. The fit is performed using nine independent observables,

$$\text{obs} \in \{\theta_{12}^q, \theta_{13}^q, \theta_{23}^q, \delta^q, \theta_{12}^l, \theta_{13}^l, \theta_{23}^l, \Delta m_{21}^2, \Delta m_{31}^2\}, \quad (24)$$

which contribute to the computation of χ_{tot}^2 .

Numerical results satisfying $\chi_{\text{tot}}^2 < 10$ are shown in Figs. 1 and 2. Points and crosses denote the scan points with $\eta_d = +1$ and $\eta_d = -1$, respectively. In the present fit, all viable points have $\eta_e = -1$ while no solutions with $\eta_e = +1$ are found.

In the upper panels of Fig. 1, the allowed values of c_ν span a wide range, $c_\nu \in (0.016, 15)$. Most solutions prefer a large- c_ν region. The viable points of the quark sector

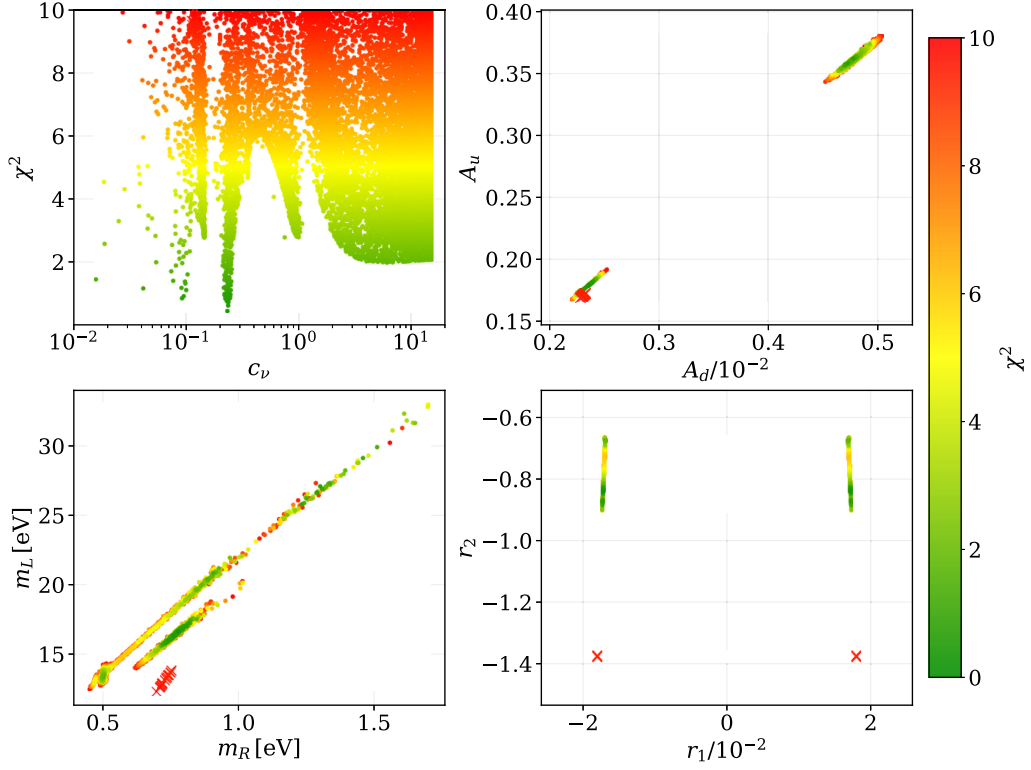


FIG. 1. Scan of model parameters with $\chi^2 \leq 10$. Points and crosses denote the scan points with $\eta_d = +1$ and $\eta_d = -1$, respectively.

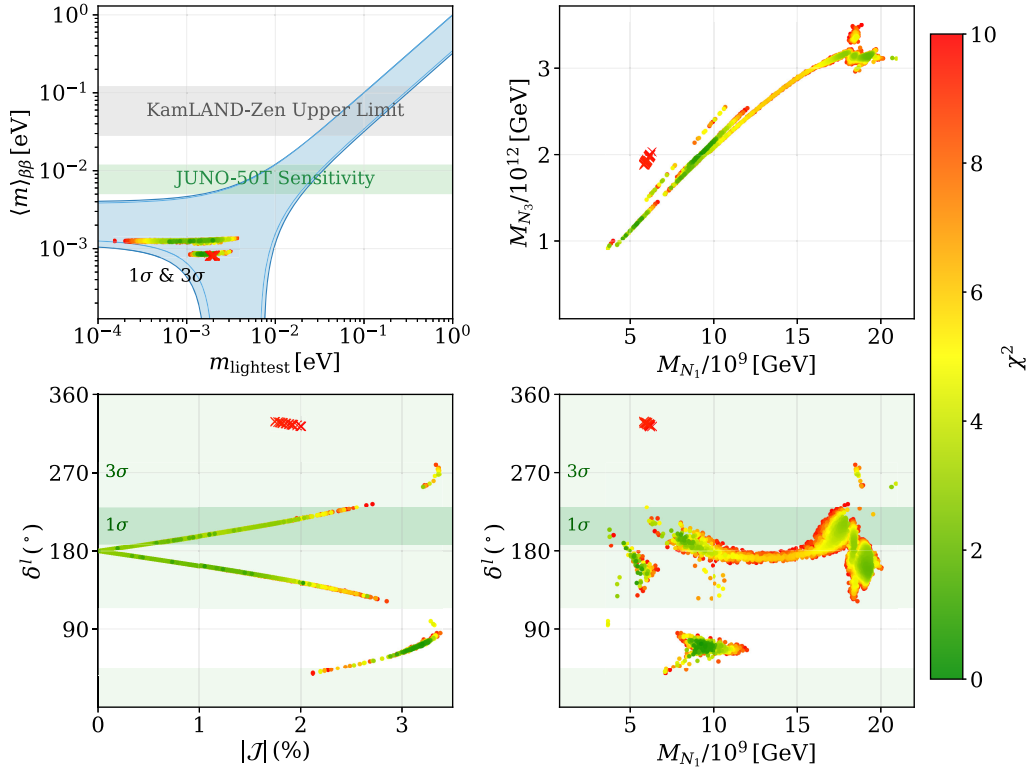


FIG. 2. The effective neutrino mass prediction (top left panel) and two-dimensional correlations between predicted observables. Points and crosses denote the scan points with $\eta_d = +1$ and $\eta_d = -1$, respectively.

parameters A_u and A_d form two narrow and approximately linear branches, centered around $\{A_u, A_d\} \sim (0.18, 2.3 \times 10^{-3})$ and $(0.36, 4.7 \times 10^{-3})$, respectively. The linear behavior can be understood from the parametrizations $A_u = y_t(r + \epsilon)$ and $A_d = y_b(r - \epsilon)$. Since the CKM constraints require $|\epsilon| \ll |r|$, both A_u/y_t and A_d/y_b are primarily controlled by the same parameter r , leading to the observed linear correlation. In the lower panels of Fig. 1, most viable points are found near $m_L \simeq (12, 33)$ eV and $m_R \simeq (0.45, 0.95)$ eV. We have not found any viable points with either m_L or m_R vanishing. Thus, both Type-I and Type-II seesaw are crucial to fit data in UTZT-SO(10). The parameters r_1 and r_2 are approximately linearly correlated, compatible with results in Ref. [26]. For the dominant $\eta_d = +1$ branch, the two signs of r_1 correspond to the two assignments $\zeta_d = \pm 1$. In addition, a few $\eta_d = -1$ points appear in a separate region with $r_2 \simeq -1.38$, which represents a new region of parameter space found in this paper. The viable points satisfy

$$A_d \ll A_u, \quad m_R \ll m_L, \quad |r_1| \ll |r_2|. \quad (25)$$

The predicted observables are shown in Fig. 2. The effective mass of neutrinoless double beta decay is predicted to fall into two narrow bands, $m_{\beta\beta} \in (0.80, 0.93)$ meV and $(1.14, 1.34)$ meV, well beyond the current KamLAND-Zen bound and the future JUNO-50T sensitivity. The lightest neutrino mass is predicted in the range $m_{\text{lightest}} \in (0.16, 3.7)$ meV.

We explain why $m_{\beta\beta}$ is confined to narrow bands and is insensitive to m_{lightest} . Firstly, we consider the constraints from the two-zero texture of M_ν , which is diagonalized by U_ν ,

$$U_\nu = U_e U_{\text{PMNS}} = P_e O_e P_{\text{un}} V_{\text{PMNS}} P_{\text{Maj}} \equiv P_{L\nu} V_\nu P_{R\nu}, \\ M_\nu = U_\nu \text{diag}(m_1, m_2, m_3) U_\nu^T \equiv P_{L\nu} \overline{M}_\nu P_{L\nu}, \quad (26)$$

where $P_{\text{un}} = \text{diag}(e^{i(\phi_{n1} + \phi_{n3})}, e^{i(\phi_{n2} + \phi_{n3})}, e^{i\phi_{n3}})$ contains the unphysical phases of the PMNS matrix, $P_{\text{Maj}} = \text{diag}(e^{i\rho}, e^{i\sigma}, 1)$ contains the Majorana phases, and the last equality indicates that U_ν is parametrized in the same way as the PMNS matrix. Due to the nontrivial complex diagonal matrix P_{un} , $P_{L\nu}$ depends on A_e , the phases contained in P_e , ϕ_{n1} , ϕ_{n2} , ϕ_{n3} , δ^l , and the observables of the lepton sector, while $V_\nu P_{R\nu}$ depends on A_e , ϕ_{n1} , ϕ_{n2} , δ^l , ρ , σ , and the observables. Given that the Hermitian Y_ν and the real symmetric F take the two-zero textures, Eq. (6) implies that

$$M_\nu = \begin{pmatrix} 0 & M_{12} & 0 \\ M_{12} & M_{22}e^{i(\phi_{n1} + 2\phi_{n2})} & M_{23}e^{i\phi_{n2}} \\ 0 & M_{23}e^{i\phi_{n2}} & M_{33} \end{pmatrix} \\ = P_{L\nu} \begin{pmatrix} 0 & M_{12} & 0 \\ M_{12} & M_{22}e^{i\phi_{n1}} & M_{23} \\ 0 & M_{23} & M_{33} \end{pmatrix} P_{L\nu}, \quad (27)$$

where M_{12} , M_{22} , M_{23} , and M_{33} are real and $P_{L\nu} = \text{diag}(e^{-i\phi_{n2}}, e^{i\phi_{n2}}, 1)$. The two constraints on $P_{L\nu}$ imply that ϕ_{n1} and ϕ_{n2} can be expressed in terms of ϕ_{n3} , δ^l , A_e , P_e , and the observables. The two vanishing elements, $(\overline{M}_\nu)_{11}$ and $(\overline{M}_\nu)_{13}$, provide four constraints on the parameters m_1 , δ^l , ρ , σ , ϕ_{n1} , ϕ_{n2} , A_e , and the lepton mixing observables. The parameters m_1 , δ^l , ρ , and σ can be solved in terms of A_e , P_e , ϕ_{n3} , and the observables. Applying these conditions to the expression $m_{\beta\beta} = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|$, we find that $m_{\beta\beta}$ depends on A_e , P_e , and the observables in the lepton sector (note that ϕ_{n3} is the overall phase of U_{PMNS} and thus has no effect on $m_{\beta\beta}$). As shown in Eq. (8), A_e and P_e are directly constrained by the quark sector and must be restricted to a narrow range. Compared with A_e and P_e , m_{lightest} is influenced by three additional free parameters from the lepton sector, $\{m_L, m_R, c_\nu\}$, and thus $m_{\beta\beta}$ is insensitive to m_{lightest} .

The RHN masses are predicted to lie in narrow regions with hierarchy. The lightest one $M_{N_1} \in (0.3, 2.0) \times 10^{10}$ GeV and the heaviest one $M_{N_3} \in (0.9, 3.6) \times 10^{12}$ GeV. These mass scales are potentially compatible with hierarchical thermal leptogenesis, for which successful N_1 -dominated leptogenesis typically satisfies the Davidson-Ibarra bound, $M_{N_1} \gtrsim 10^9$ GeV [69,70]. The lower panels of Fig. 2 show a multibranch distribution of δ^l . For the dominant $\eta_d = +1$ solutions, the main branch lies around $\delta^l \simeq (120^\circ, 235^\circ)$ and gives the upper $m_{\beta\beta}$ band. A smaller branch around $\delta^l \simeq (40^\circ, 85^\circ)$ gives the lower $m_{\beta\beta}$ band, which is not favored by global-fit data in the 3σ region. In addition, a few $\eta_d = -1$ points, absent from Ref. [26], appear around $\delta^l \simeq (320^\circ, 330^\circ)$ and $M_{N_1} \simeq (0.6, 0.7) \times 10^{10}$ GeV. However, their χ^2 values are relatively large. These points are shown only for completeness and should be regarded as marginal solutions. The Jarlskog invariant can reach the percent level, with $|\mathcal{J}| \lesssim 3.5\%$.

As a stability check, we further check the parameter space by imposing a tighter cut $\chi_{\text{tot}}^2 < 4$. The effective mass is restricted to two separated narrow intervals, $m_{\beta\beta} \in (0.84, 0.87)$ meV and $(1.16, 1.29)$ meV. The allowed RHN mass ranges become $M_{N_1} \in (0.4, 2.0) \times 10^{10}$ GeV and $M_{N_3} \in (1.0, 3.2) \times 10^{12}$ GeV. The allowed ranges of the CP phase are $\delta^l \in (54^\circ, 80^\circ)$ and $\delta^l \in (142^\circ, 218^\circ)$. The second interval remains allowed, but fewer points survive

the tighter cut. Compared with the $\chi^2_{\text{tot}} < 10$ case, the dominant $\eta_d = +1$ regions remain stable with only moderate narrowing, whereas the marginal solutions associated with the $\eta_d = -1$ branch disappear.

For comparison, Ref. [26], using NuFIT 5.3 inputs, found $c_\nu \lesssim 1.7$, a connected region $\delta^l \in (90^\circ, 230^\circ)$, a single band $m_{\beta\beta} \in (1.15, 1.28)$ meV, and a single $\{A_u, A_d\}$ branch around $(0.37, 4.8 \times 10^{-3})$. The present analysis differs from there in both updating the experimental inputs and applying the numerical search strategy. The more comprehensive scan retains the previous $\{A_u, A_d\}$ branch and reveals an additional lower branch around $(0.18, 2.3 \times 10^{-3})$, together with solutions in the $\zeta_d = +1$ branch and a small number of marginal $\eta_d = -1$ solutions. In the resulting viable parameter space, the allowed range extends to $c_\nu \in (0.016, 15)$, while δ^l becomes disconnected into two main intervals, $(40^\circ, 85^\circ)$ and $(120^\circ, 235^\circ)$. Here, the additional branch at $\delta^l \in (40^\circ, 85^\circ)$ was found in a more extensive parameter scan using the improved numerical algorithm and is not solely a result of the updated JUNO inputs. Correspondingly, the original $m_{\beta\beta}$ band is retained and slightly broadened to $(1.14, 1.34)$ meV, while an additional lower band emerges at $(0.80, 0.93)$ meV.

IV. UTZT FROM NONINVERTIBLE SYMMETRIES

Having discussed the phenomenological implications of the UTZT in the previous section, we now turn to its realization. Texture zeros realized in the canonical approach via either Abelian or non-Abelian discrete symmetries require more than one generation of Higgs fields [41,42]. These fields introduce a lot of new physical degrees of freedom above the electroweak scale and may greatly affect the RG running to the GUT scale. A detailed numerical analysis of the RG running and the consequent prediction of proton decay can be found in [26], where the influence of the repetition number of Higgs fields 10_H , $\overline{126}_H$, and 120_H is discussed in particular.

Texture zeros realized in NISs do not introduce any new particles below the GUT scale. In this work, we take the economical assumption that only one generation of Higgs fields is introduced, corresponding to the same breaking chain and same particle content as those appearing in the scenario S1 in [26] with Higgs repetition number $n_H = 1$. Therefore, the same RG behavior and prediction on the proton decay are expected and we do not repeat them here. We further note that the same breaking chain was discussed earlier in [60,61].¹ Although the particle contents at intermediate scales in these works are not exactly the same, the GUT scales are all predicted to roughly lie in the similar range from 10^{15} GeV to 10^{16} GeV. As mentioned

in the introduction, the simplest way to realize the UTZT is to impose a noninvertible selection rule of the Z_3 gauging of Z_7 . In this section, we will first discuss the multiplication rules for conjugacy classes in $Z_N \rtimes Z_3$ and then the model building in the Z_3 gauging of Z_7 . We will also introduce the generalized CP symmetry and its implications for the Yukawa couplings. Since NIS is a type of discrete symmetry, its spontaneous breaking does not generate additional light states and thus does not affect the RG running.

A. Noninvertible selection rules on Z_3 gauging of Z_N

In this subsection, we assume N is prime. Any nonprime N can be expressed as a product of prime factors, i.e., $Z_N \simeq Z_{N_1} \times Z_{N_2} \times \dots$, which we will comment on at the end of this section. The generators of $Z_N \rtimes Z_3$ are denoted as a and b , with the following relations:

$$a^N = e, \quad b^3 = e, \quad b^{-1}ab = a^m, \quad (28)$$

where e is the identity element. Evidently, Z_3 is an outer automorphism of Z_N with the mapping $a \rightarrow a^m$. The last relation, $ab = ba^m$, implies that

$$ab^3 = b^3 a^{(m^3)} \Rightarrow a = a^{(m^3)}. \quad (29)$$

It gives a strong constraint on m and N [59,71],

$$(m^3 - 1) = (m - 1)(m^2 + m + 1) = 0 \pmod{N}. \quad (30)$$

Except for the trivial case $m = 1$, the above relation is satisfied by $m^2 + m + 1 = 0 \pmod{N}$. The possible combinations of N and m are

$$\begin{aligned} N = 7, \quad m = 2, 4, \\ N = 13, \quad m = 3, 9 \dots \end{aligned} \quad (31)$$

Using the relation $ab = ba^m$ and the condition $m^3 \equiv 1 \pmod{N}$, the conjugacy classes of $Z_N \rtimes Z_3$ are given by

$$\begin{aligned} C_1 &= \{e\}, \\ C_3^1 &= \{a, a^m = b^{-1}ab, a^{m^2} = b^{-2}ab^2\}, \\ C_3^k &= \{a^k, a^{km}, a^{km^2}\}, \\ C_3^{(1,0)} &= \{b, ba^{m-1} = aba^{-1}, \dots, ba^{j(m-1)} = a^j ba^{-j}, \dots\}, \\ C_3^{(1,k)} &= \{ba^k, \dots, ba^{j(m-1)+k}, \dots\}, \\ C_3^{(2,0)} &= \{b^2, b^2 a^{2m-1} = ab^2 a^{-1}, \dots, b^2 a^{j(2m-1)} = a^j b^2 a^{-j}, \dots\}, \\ C_3^{(2,k)} &= \{b^2 a^k, \dots, b^2 a^{j(2m-1)+k}, \dots\}. \end{aligned} \quad (32)$$

The normal subgroup Z_N generated by a is decomposed into a few conjugacy classes, C_1 and C_3^k for $k = 1, 2, \dots, \frac{N-1}{3}$, whose fusion rules will be used in the

¹The breaking chain in Eq. (2) is denoted as III in [60] and II3 in [61], respectively.

following. The multiplication rules of conjugacy classes are given by

$$C_3^k C_3^l = C_3^l C_3^k = C_3^{k+l} + C_3^{k+ml} + C_3^{k+m^2l}. \quad (33)$$

In the following, we will treat NISs as a bottom-up approach to the texture zeros in the Yukawa coupling. Each field ϕ_k is assigned a conjugacy class C_3^k of $Z_N \rtimes Z_3$. A Yukawa coupling $\phi_1 \phi_2 \phi_3$ is allowed if the fusion product of the corresponding conjugacy classes contains the identity class C_1 , i.e., $C_{\phi_1} C_{\phi_2} C_{\phi_3} \supset C_1$.

For consistency with the kinetic terms, the conjugate field ϕ_k^* is labeled by the conjugacy class $[a^{-k}] \equiv C_3^{N-k}$. There is an additional Z_2 outer automorphism of the group $Z_N \rtimes Z_3$ with the mapping $a \rightarrow a^{-1}$, corresponding to the charge conjugation. Its transformation on the conjugacy classes of a group G is defined as

$$[g_i] \rightarrow X_{ij}^{CP} [g_j^{-1}], \quad (34)$$

where X^{CP} is a unitary matrix. In the case of the Z_2 gauging, a and a^{-1} belong to the same conjugacy class, making this a trivial case with respect to the generalized CP symmetry. Combining the parity transformation with the charge conjugation, we can define the generalized CP transformation on a field as [72]

$$\phi(x) \rightarrow X^{CP} \phi(x_P)^*, \quad (35)$$

with $x = (t, x)$ and $x_P = (t, -x)$. If there is a global discrete symmetry G_f , an additional consistency condition should be satisfied. The generalized CP transformation reduces to the canonical CP transformation discussed in Sec. II if $X^{CP} = I$.

We conclude this subsection with an example of the Z_3 gauging of Z_7 with $m = 2$. The conjugacy classes are

$$\begin{aligned} C_1 &= \{e\}, \\ C_3^1 &= \{a, a^2, a^4\}, \\ C_3^2 &= \{a^3, a^6, a^5\}, \end{aligned}$$

whose multiplication rules are listed in Table II. Under the generalized CP transformation, conjugacy classes transform as $C_1 \leftrightarrow C_1$ and $C_3^1 \leftrightarrow C_3^2$.

TABLE II. Multiplication rules for the conjugacy classes in the Z_3 gauging of Z_7 .

	C_1	C_3^1	C_3^2
C_1	C_1	C_3^1	C_3^2
C_3^1	C_3^1	$C_3^1 + 2C_3^2$	$3C_1 + C_3^1 + C_3^2$
C_3^2	C_3^2	$3C_1 + C_3^1 + C_3^2$	$2C_3^1 + C_3^2$

TABLE III. Particle content of SO(10) with the Z_3 gauging of Z_7 . Here, the superscript i labels three fermion generations, $i = 1, 2, 3$.

	Particles in SO(10)	Class assignment in Z_3 gauged Z_7
Fermions	16_F^i	(C_1, C_3^1, C_3^2)
	$10_H, 120_H$	C_3^2
Higgses	$\overline{126}_H$	C_3^2
	54_H	C_3^2
	45_H	C_3^2

B. Model building under noninvertible selection rules

To realize the UTZT, we assign different conjugacy classes of $Z_7 \rtimes Z_3$ to the Higgs fields and three generations of fermions, as shown in Table III. Given the fusion rules in Table II, it is straightforward to see that the three Yukawa matrices in Eq. (3) share the same two-zero texture. For example, the (1,3) element of A is zero because the fusion product of the corresponding conjugacy classes, $C_1 C_3^2 C_3^2$, does not contain the identity class C_1 . The same argument applies to the other elements in the Yukawa matrices. In the Yukawa sector, the class assignment for fermions and for $10_H, 120_H$, and $\overline{126}_H$ in Table III is unique up to the trivial interchange of C_3^1 and C_3^2 .

For completeness, we have also considered an example of the class assignments for the Higgs fields 54_H and 45_H , which are responsible for the spontaneous breaking of SO(10) to G_{422}^C and G_{422}^C to G_{3221} , respectively. Table II implies that arbitrary couplings involving three or more Higgs fields are not forbidden by the selection rules, provided all fields are assigned either C_3^1 or C_3^2 . Quadratic terms formed by fields with their conjugation, such as $(54_H 54_H^*)$, $(45_H 45_H^*)$, and $(\overline{126}_H \overline{126}_H^*)$, are allowed regardless of class assignments. The class assignments for 54_H and 45_H are not unique, but this does not affect the UTZT in flavor space, and we, therefore, do not discuss them in detail.

The same two-zero texture of the neutrino mass matrix was realized in an earlier work [59], where the authors assumed that neutrino masses are generated by either the Weinberg operator or Type-I seesaw mechanism without considering the origin of $B - L$ breaking. Since no charge can be assigned to coefficients of the Weinberg operator or Majorana mass parameters, they concluded that the Z_3 gauging of Z_{13} is the minimal symmetry that realizes the same two-zero texture, which differs from our hypotheses and results. As an example, consider the realization of texture zeros in the RHN mass term, $\frac{1}{2} (M_N)_{\alpha\beta} \overline{\nu_{R\alpha}^c} \nu_{R\beta}$. In our model, M_N arises from the VEV of a Higgs that carries both the $B - L$ and NIS charges. After this Higgs acquires a

VEV, both the $B - L$ symmetry and the NIS are spontaneously broken. By contrast, the mass matrix M_N in Ref. [59] carries no NIS charge and breaks $B - L$ explicitly, consequently, the NIS remains unbroken in their construction.

Finally, we briefly discuss alternative ways to realize the UTZT. In the case of one generation of Higgs fields, it is impossible to realize the UTZT with either $Z_N \rtimes Z_2$ (a brief notation for the Z_2 gauging of Z_N), $Z_{13} \rtimes Z_3$, $(Z_2 \times Z'_2 \times Z''_2) \rtimes Z_3$, or $(Z_3 \times Z'_3) \rtimes Z_3$. However, the Z_2 gauging of Z_N ($N = 6, 7, 8, \dots$) can realize the UTZT if two generations of Higgs fields are included. For example, in the case of the Z_2 gauging of Z_6 , one option is to label the first generation of Higgs fields with $[a^2]$, the second with $[a^3]$, and the three generations of fermions with $[e], [a^2], [a]$, where a is the generator of Z_6 and e is the identity element. We will not discuss the details of these alternative realizations due to the enormous freedom introduced by the additional Higgs generation.

V. CONCLUSIONS

Texture zeros remain one of the most enduring approaches to the flavor puzzle in particle physics. A particularly predictive pattern is the UTZT, in which all fermion mass matrices possess two vanishing (1, 1) and (1, 3) entries. The UTZT provides a compelling description of fermion masses and mixing due to its remarkable consistency with current experimental data. Embedding it into SO(10) GUT further enhances its predictive power by exploiting the quark-lepton correlation of GUT. In this work, we presented a state-of-the-art analysis of the UTZT-SO(10). We took into account the new data released from the JUNO reactor neutrino experiment where the normal ordering of light neutrino masses are indicated in 2.3σ . We showed that the UTZT-SO(10) fits very well with the precision data. In addition, we investigated the possible origin of the UTZT from noninvertible symmetries and discussed explicit symmetry realizations of the texture structure.

Fixing the Yukawa eigenvalues of six quarks and three charged leptons at their best-fit values reduces the number

of independent free parameters to seven. These parameters are then used to fit nine observables, namely, the four CKM mixing parameters, the three leptonic mixing angles, and the two neutrino mass-squared differences. Our numerical analysis demonstrates that the UTZT within the SO(10) framework provides an excellent description of the latest experimental data. The Dirac CP -violating phase, regarded as a prediction of the fitting, is restricted mainly in two regimes, $\delta^l \simeq (120^\circ, 235^\circ)$ and $(40^\circ, 85^\circ)$. The former region is in very good agreement with current global-fit results, whereas the latter is disfavored at the 3σ level. The effective Majorana mass governing neutrinoless double beta decay is predicted in very narrow bands, $m_{\beta\beta} \simeq (0.80, 0.93)$ meV and $(1.14, 1.34)$ meV, associated with the two disconnected δ^l regions. The RHN mass spectrum is also strongly constrained. The lightest one is roughly around 10^9 GeV and the heaviest one is of order 10^{12} GeV.

Although texture zeros can be generated through conventional Abelian or non-Abelian flavor symmetries, realizing them via noninvertible symmetries offers a unique advantage: no additional low-energy degrees of freedom are required. We demonstrated the UTZT via noninvertible selection rules on the Z_3 gauging of Z_N . In particular, we presented explicit class assignment based on the Z_3 gauging of Z_7 , which, so far, is the minimal symmetry to generate this type of two-zero texture. We further comment that the UTZT can also be realized in the Z_2 gauging of Z_N if more than one generation of Higgses are included in the model.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [73].

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