

Precise measurements of $D^0 \rightarrow K^- \ell^+ \nu_\ell$ and $D^+ \rightarrow \bar{K}^0 \ell^+ \nu_\ell$ decays

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Using e^+e^- collision data corresponding to an integrated luminosity of 20.3 fb^{-1} , collected at the center-of-mass energy of 3.773 GeV with the BESIII detector, we present precise measurements of $D^0 \rightarrow K^- \ell^+ \nu_\ell$ and $D^+ \rightarrow \bar{K}^0 \ell^+ \nu_\ell$ ($\ell = e, \mu$) decays. The branching fractions of $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ are measured to be $(3.548 \pm 0.006_{\text{stat}} \pm 0.017_{\text{syst}})\%$, $(3.445 \pm 0.007_{\text{stat}} \pm 0.017_{\text{syst}})\%$, $(8.928 \pm 0.025_{\text{stat}} \pm 0.050_{\text{syst}})\%$, and $(8.770 \pm 0.029_{\text{stat}} \pm 0.053_{\text{syst}})\%$, respectively. The partial decay rates of these four decays are measured with improved precision, and their forward-backward asymmetries are determined for the first time. By performing a simultaneous fit to the measured partial decay rates and the forward-backward asymmetries of $D \rightarrow \bar{K} \ell^+ \nu_\ell$, a search for a possible scalar current contribution in the $c \rightarrow s \ell^+ \nu_\ell$ transition is performed. The results are $\text{Re}(c_S^\mu) = 0.017 \pm 0.008_{\text{stat}} \pm 0.006_{\text{syst}}$ and $\text{Im}(c_S^\mu) = \pm(0.077 \pm 0.011_{\text{stat}} \pm 0.009_{\text{syst}})$, corresponding to a difference from the Standard Model (SM) with a significance of 2.3σ . The product of the form factor $f_+(0)$ and the modulus of the $c \rightarrow s$ Cabibbo-Kobayashi-Maskawa matrix element $|V_{cs}|$ is determined to be $f_+(0)|V_{cs}| = 0.7183 \pm 0.0007_{\text{stat}} \pm 0.0014_{\text{syst}}$. With the inputs $|V_{cs}| = 0.97349 \pm 0.00016$ from the SM global fit or $f_+(0) = 0.7452 \pm 0.0031$ from the lattice quantum chromodynamics calculation, we derive $f_+(0) = 0.7383 \pm 0.0007_{\text{stat}} \pm 0.0014_{\text{syst}}$ and $|V_{cs}| = 0.9639 \pm 0.0009_{\text{stat}} \pm 0.0018_{\text{syst}} \pm 0.0040_{\text{LQCD}}$. Combining with the results of the semimuonic and semielectronic decays, lepton flavor universality is tested via the ratios of the decay rates between semimuonic and semielectronic decays in the full and different momentum transfer ranges.

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I. INTRODUCTION

Precise measurements of semileptonic decays of charmed mesons are important for our understanding of the weak and strong interactions in the charm sector. Their decay dynamics are sensitive to the magnitude of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cs(d)}|$ as well as to the corresponding hadronic transition form factors. As the dominant semileptonic $D^{0(+)}$ decay modes, the $D \rightarrow \bar{K} \ell^+ \nu_\ell$ channels are of special interest to investigate both theoretically and experimentally. Calculations on the hadronic form factor at zero-momentum transfer $f_+(0)$ [1–9] have been performed in various theoretical approaches, including lattice quantum chromodynamics (LQCD) [1–4], QCD light-cone sum rules (LCSRs) [5], covariant light-front quark model (LFQM) [6], the covariant confined quark model (CCQM) [7], and the relativistic quark model

(RQM) [8]. A precisely measured $f_+(0)$ enables a more stringent test of the different theoretical predictions, and the accurately determined $|V_{cs}|$ is important for the test of the unitarity of the CKM matrix.

Lepton flavor universality (LFU), as predicted by the Standard Model (SM), requires identical couplings between the gauge bosons and the three lepton families. Any violation of LFU could reveal potential new physics (NP) beyond the SM. In recent years, several differences from LFU have been observed in the $b \rightarrow c \tau^+ \nu_\tau$ [10–14] and $b \rightarrow s \ell^+ \ell^-$ [15–19] transitions. While some of these violations disappear after further update measurements, some anomalies remain. Various NP scenarios, including two-Higgs-doublet models [20,21] and leptoquark models [22,23], as well as model-independent investigation [24], suggest that these differences may originate from scalar currents in the weak interaction. Therefore, it is important to study the $c \rightarrow s \ell^+ \nu_\ell$ transition in the (semi)leptonic decays of charmed mesons, as highlighted in Refs. [25–28]. With large branching fractions (BFs) and low backgrounds, the semileptonic decays $D \rightarrow \bar{K} \ell^+ \nu_\ell$ offer a unique platform to access potential scalar current contributions in the $c \rightarrow s \ell^+ \nu_\ell$ transition. The corresponding effective Lagrangian is written as [26]

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$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* \sum_{\ell=e,\mu,\tau} \sum_i c_i^\ell \mathcal{O}_i^\ell + \text{H.c.}, \quad (1)$$

where the only allowed operator in the SM is $\mathcal{O}_{\text{SM}}^\ell = (\bar{s}\gamma_\mu P_L c)(\bar{\nu}_\ell \gamma^\mu P_L \ell)$ with coefficient $c_{\text{SM}}^\ell = 1$. A potential right- (left-) handed scalar current is described by the NP operator $\mathcal{O}_{R(L)}^\ell = (\bar{s}P_{R(L)} c)(\bar{\nu}_\ell P_{R(L)} \ell)$ with complex Wilson coefficient $c_{R(L)}^\ell$. Here, γ_μ indicates the Dirac matrix, and $P_{L,R}$ are the chirality projection operators. Nonzero values of $c_{R(L)}^\ell$ will cause differences from the SM in several observables, such as full or partial decay rates and forward-backward asymmetries. Therefore, the precise measurements of these variables are important for searching for the scalar current in the $c \rightarrow s\ell^+\nu_\ell$ transition.

Experimentally, the branching fractions and dynamics of the $D \rightarrow \bar{K}\ell^+\nu_\ell$ decays have been extensively studied by BES [29–31], BABAR [32], Belle [33], CLEO [34–37], and BESIII [38–45]. However, experimental studies of the scalar current and the forward-backward asymmetry in the $D \rightarrow \bar{K}\ell^+\nu_\ell$ transition have not been reported to date. In this paper, we provide the complete experimental details and results for the precise measurements of the partial decay rates and forward-backward asymmetries in the $D \rightarrow \bar{K}\ell^+\nu_\ell$ transition, from which the first constraint on the scalar current [46] is derived. These analyses are based on e^+e^- collision data, which corresponds to a total integrated luminosity of 20.3 fb^{-1} collected with the BESIII detector at the center-of-mass energy $\sqrt{s} = 3.773 \text{ GeV}$ [47] during 2010–2011 and 2022–2024. In addition, we present updated measurements of the branching fractions of $D \rightarrow K\ell^+\nu_\ell$, the form factor $f_+(0)$, and the modulus of the CKM matrix element $|V_{cs}|$, as well as a test of LFU using the $D \rightarrow \bar{K}\ell^+\nu_\ell$ decays. Compared to our previous measurements [45], which were based on a 7.9 fb^{-1} data sample taken during 2010–2011 and 2022, all results reported here are obtained with improved precision and thereby supersede those in Ref. [45]. Throughout this paper, charge conjugated modes are always included.

II. BESIII DETECTOR AND MONTE CARLO SIMULATIONS

The BESIII detector [48] records symmetric e^+e^- collisions provided by the BEPCII storage ring [49] and has collected large data samples in the center-of-mass energy range from 1.84 to 4.95 GeV [50,51], with a peak luminosity of $1.1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ achieved at $\sqrt{s} = 3.773 \text{ GeV}$. The cylindrical core of the BESIII detector covers 93% of the full solid angle and consists of a helium-based multilayer drift chamber (MDC), a time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return

yoke with resistive-plate muon counters interleaved with steel. The charged-particle momentum resolution at 1 GeV/c is 0.5%, and the dE/dx resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end-cap) region. The time resolution in the plastic scintillator TOF barrel region is 68 ps, while that in the end-cap region was 110 ps. The end-cap TOF system was upgraded in 2015 using multigap resistive-plate chamber technology, providing a time resolution of 60 ps, which benefits 85% of the data used in this analysis [52–54].

Simulated samples produced with Geant4-based [55] Monte Carlo (MC) software, which includes the geometric description [56] of the BESIII detector and the detector response, are used to determine detection efficiencies and to estimate backgrounds. The simulation models the beam energy spread and initial state radiation (ISR) in the e^+e^- annihilations with the generator KKMC [57]. Signal MC samples of the decays $D \rightarrow \bar{K}\ell^+\nu_\ell$ are simulated with a specific two-parameter series expansion model [58], which is described in Sec. VII A. The background is studied using an inclusive MC sample that includes $D\bar{D}$ pairs originating from the decay of the $\psi(3770)$ with quantum coherence for the neutral D channels considered, the non- $D\bar{D}$ decays of the $\psi(3770)$, the ISR production of the charmonium states, and the continuum processes. These processes are also generated with KKMC. The known decay modes are modeled by EvtGen [59,60] with branching fractions taken from the Particle Data Group (PDG) [61], while the remaining unknown charmonium decays are modeled with Lundcharm [62,63]. Final-state radiation from charged final-state particles is incorporated using Photos version 2.15 [64].

III. DOUBLE-TAG METHOD

At $\sqrt{s} = 3.773 \text{ GeV}$, the D and $\bar{D}$ mesons are produced in pairs via the $e^+e^- \rightarrow \psi(3770) \rightarrow D\bar{D}$ process, where D stands for D^0 or D^+ . This property allows for the absolute branching fraction measurement with the well-established double-tag (DT) method [65]. In this method, the single-tag (ST) candidate events are selected by reconstructing a $\bar{D}^0$ in six hadronic decay modes $\bar{D}^0 \rightarrow K^+\pi^-, K^+\pi^-\pi^0, K^+\pi^-\pi^-\pi^+, K_S^0\pi^+\pi^-, K^+\pi^-\pi^0\pi^0$, and $K^+\pi^-\pi^-\pi^+\pi^0$ or a D^- in six hadronic decay modes $D^- \rightarrow K^+\pi^-\pi^-, K_S^0\pi^-, K^+\pi^-\pi^-\pi^0, K_S^0\pi^-\pi^0, K_S^0\pi^+\pi^-\pi^-,$ and $K^+K^-\pi^-$. These inclusively selected candidates are referred to as ST $\bar{D}$ mesons. In events containing an ST $\bar{D}$ meson, candidates for the signal decays $D \rightarrow \bar{K}\ell^+\nu_\ell$ are selected to form DT events, where $\bar{K}^0$ is reconstructed via $K_S^0 \rightarrow \pi^+\pi^-$ and the lepton ℓ is either a positron or a muon. The branching fraction of the signal decay is determined as

$$\mathcal{B}_{\text{sig}} = N_{\text{DT}} / (N_{\text{ST}}^{\text{tot}} \cdot \bar{\epsilon}_{\text{sig}} \cdot \mathcal{B}_{\text{sub}}), \quad (2)$$

where $N_{\text{ST}}^{\text{tot}} = \sum_i N_{\text{ST}}^i$ and N_{DT} are the total ST and DT yields summed over all tag modes. The weighted signal efficiency is defined as

$$\bar{\epsilon}_{\text{sig}} = \sum_i \frac{N_{\text{ST}}^i \epsilon_{\text{DT}}^i}{N_{\text{ST}}^{\text{tot}} \epsilon_{\text{ST}}^i} \quad (3)$$

where ϵ_{ST}^i and ϵ_{DT}^i are the ST and DT efficiencies for the i th tag mode, respectively. The factor $\mathcal{B}_{\text{sub}}$ accounts for the branching fractions of intermediate decays, with $\mathcal{B}_{\text{sub}} = \mathcal{B}(\bar{K}^0 \rightarrow K_S^0) \cdot \mathcal{B}(K_S^0 \rightarrow \pi^+ \pi^-)$ [61] for D^+ modes and $\mathcal{B}_{\text{sub}} = 1$ for D^0 modes.

IV. SELECTION OF SINGLE-TAG $\bar{D}$ MESONS

For each charged track other than those used for K_S^0 reconstruction, the distances of closest approach to the interaction point in the plane perpendicular to the z -axis, $|V_{xy}|$, and along the z -axis, $|V_z|$, are required to be within 1 and 10 cm, respectively, where the z -axis is defined as the symmetry axis of the MDC. The polar angle θ of the track with respect to the z -axis is required to satisfy $|\cos \theta| < 0.93$. Particle identification (PID) for charged tracks is performed by combining the dE/dx and TOF information to calculate the confidence levels for the pion and kaon hypotheses. Each charged track is assigned the particle type with the higher probability.

The K_S^0 candidates are reconstructed via $K_S^0 \rightarrow \pi^+ \pi^-$ from pairs of oppositely charged tracks, satisfying $|\cos \theta| < 0.93$ and $|V_z| < 20$ cm, with no requirement on $|V_{xy}|$ nor PID. The two tracks are constrained to originate from a common vertex, and the decay length must be greater than twice its resolution. The quality of the vertex fit is required to satisfy $\chi^2 < 100$, and the invariant mass of the $\pi^+ \pi^-$ system is required to fall within $(0.487, 0.511)$ GeV/ c^2 .

The π^0 candidates are reconstructed via the $\pi^0 \rightarrow \gamma\gamma$ decay using the EMC showers. For each shower, the EMC time must be within (0, 700) ns of the event start time. The deposited energy in the EMC is required to be greater than 25 MeV in the barrel region ($|\cos \theta| < 0.80$) and 50 MeV in the end-cap region ($0.86 < |\cos \theta| < 0.92$). The opening angle between the EMC shower and the nearest charged track must be greater than 10° . The invariant mass of the photon pair is required to fall within $(0.115, 0.150)$ GeV/ c^2 . A mass-constrained (1C) fit to the known π^0 mass [61] is imposed on the photon pair, and the corresponding χ^2 is required to be less than 50. The four-momentum of the π^0 updated by the 1C fit is used in the subsequent analysis.

For the two-body tag mode $\bar{D}^0 \rightarrow K^+ \pi^-$, the backgrounds originating from cosmic rays, Bhabha and dimuon events are suppressed using the procedure defined in Ref. [66]. The TOF time difference between

the two charged tracks is required to be less than 5 ns. In addition, each event must contain either at least one EMC shower with energy greater than 50 MeV or at least one additional good charged track detected in the MDC.

To identify the ST $\bar{D}$ mesons, two kinematic variables are defined: the energy difference $\Delta E \equiv E_{\bar{D}} - E_{\text{beam}}$ and the beam-constrained mass $M_{\text{BC}} \equiv \sqrt{E_{\text{beam}}^2/c^4 - |\vec{p}_{\bar{D}}|^2/c^2}$, where E_{beam} is the beam energy and $(E_{\bar{D}}, \vec{p}_{\bar{D}})$ is the four-momentum of the ST $\bar{D}$ meson in the e^+e^- center-of-mass frame. If multiple $\bar{D}$ candidates are reconstructed in the same tag mode, the one with the smallest $|\Delta E|$ is kept for further analysis. The multiple combination rates are less than 10% for the two- and three-body decays without π^0 , while they are higher for the other decay modes. Good consistency between data and MC simulations on the multiple combinations rates is confirmed. To suppress the combinatorial backgrounds in the M_{BC} distribution, tag-mode-dependent ΔE requirements are applied on the ST candidates. The ΔE requirements and the corresponding ST efficiencies estimated with the inclusive MC sample are summarized in Table I.

The ST yield of each tag mode is obtained from a binned maximum likelihood fit to the corresponding M_{BC} distribution. The signal is described by the MC-simulated shape, where only events with the opening angle between the generated and reconstructed momentum directions of ST daughter particles smaller than 15° are retained. The MC shape is then convolved with a double-Gaussian function to account for the resolution difference between data and MC simulation, and an additional single-Gaussian function is included to describe ISR effects. The background is modeled by an ARGUS function [67] with the end point fixed at the E_{beam} . The potential peaking backgrounds such as doubly Cabibbo-suppressed $\bar{D}^0$ decays are subtracted from the ST yields. The fit results for various tag modes are shown in

TABLE I. The ΔE requirements, the ST $\bar{D}$ yields in data (N_{ST}^i), and the ST efficiencies of the tag mode i (ϵ_{ST}^i), where the uncertainties are statistical only.

D	Tag mode	ΔE (MeV)	N_{ST}^i (10^3)	ϵ_{ST}^i (%)
D^0	$K^+ \pi^-$	$(-27, +27)$	3725.9 ± 2.0	65.09 ± 0.01
	$K^+ \pi^- \pi^0$	$(-62, +49)$	7420.0 ± 3.2	35.53 ± 0.01
	$K^+ \pi^- \pi^- \pi^+$	$(-26, +24)$	4987.2 ± 2.5	40.70 ± 0.01
	$K_S^0 \pi^+ \pi^-$	$(-24, +24)$	1168.7 ± 1.2	37.81 ± 0.01
	$K^+ \pi^- \pi^0 \pi^0$	$(-68, +53)$	1771.8 ± 2.2	15.11 ± 0.01
	$K^+ \pi^- \pi^- \pi^+ \pi^0$	$(-57, +51)$	1152.8 ± 1.8	16.19 ± 0.01
D^-	$K^+ \pi^- \pi^-$	$(-25, +24)$	5552.7 ± 2.5	51.01 ± 0.01
	$K_S^0 \pi^-$	$(-25, +26)$	656.4 ± 0.8	51.41 ± 0.02
	$K^+ \pi^- \pi^- \pi^0$	$(-57, +46)$	1723.7 ± 1.8	24.40 ± 0.01
	$K_S^0 \pi^- \pi^0$	$(-62, +49)$	1442.3 ± 1.5	26.42 ± 0.01
	$K_S^0 \pi^- \pi^- \pi^+$	$(-28, +27)$	790.6 ± 1.1	29.61 ± 0.01
	$K^+ K^- \pi^-$	$(-24, +23)$	481.2 ± 0.9	40.87 ± 0.01

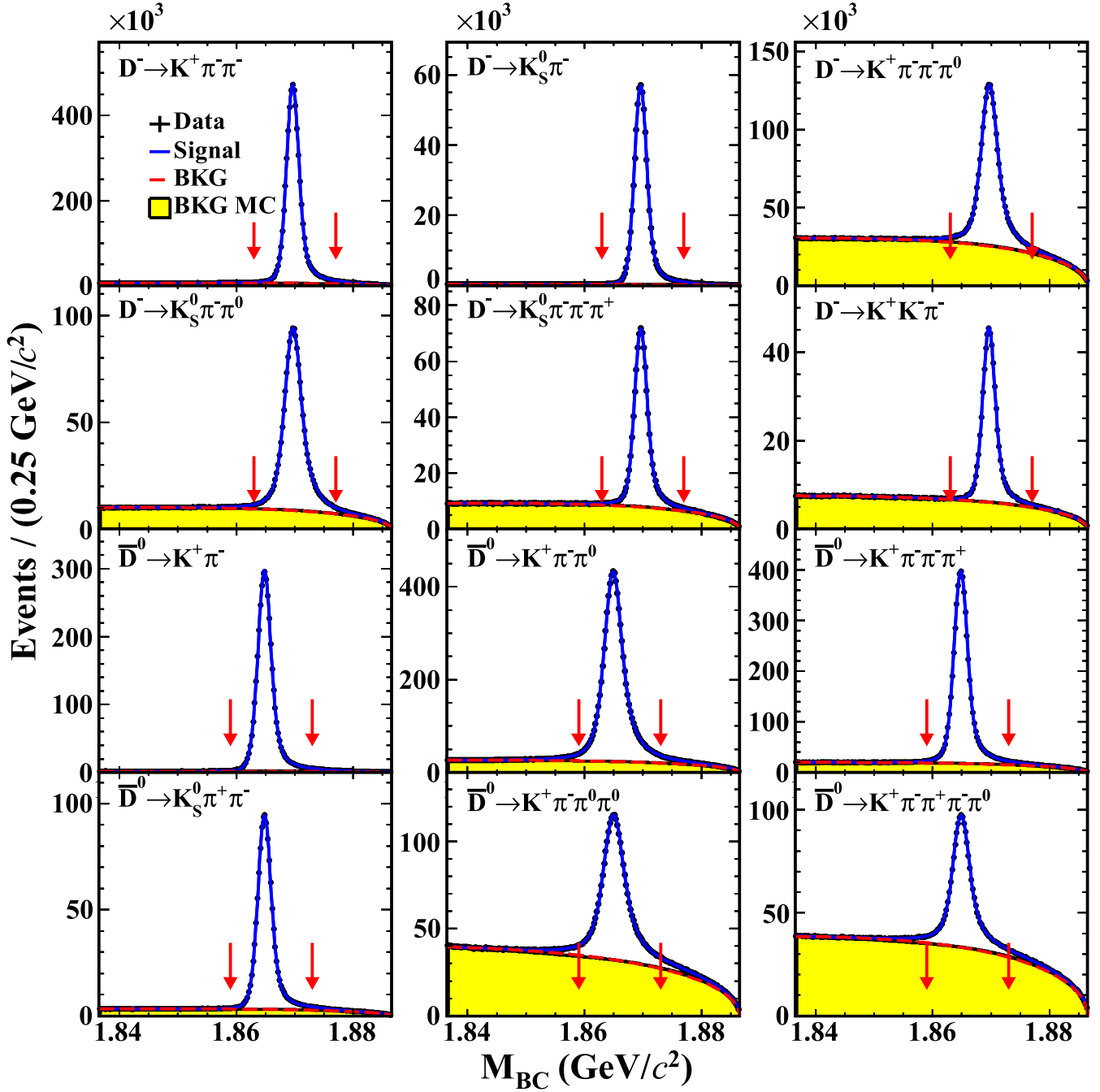


FIG. 1. The M_{BC} distributions of the ST $\bar{D}$ candidates in data, with fit results overlaid. The points with error bars are data, the blue curves are the best fits, and the red dashed curves are the fitted ARGUS functions. The pairs of red arrows show the M_{BC} signal windows, and the yellow filled histograms are the combinatorial background from the inclusive MC simulation.

Fig. 1. To further suppress the combinatorial backgrounds, M_{BC} is required to be within $(1.859, 1.873)$ GeV/c^2 for $\bar{D}^0$ tags and $(1.863, 1.877)$ GeV/c^2 for D^- tags. Summing over all the tag modes, the total ST yields of $\bar{D}^0$ and D^- mesons (N_{ST}^{tot}) are determined to be $(20, 229.8 \pm 5.5_{\text{stat}}) \times 10^3$ and $(10, 646.9 \pm 3.8_{\text{stat}}) \times 10^3$, respectively.

V. SELECTION OF DOUBLE-TAG EVENTS

The DT events for $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^- \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ are selected from the remaining tracks in the presence of the ST $\bar{D}$ candidates. The selection criteria for K^- and K_S^0 are the same as those used in the ST selection. The positron and muon candidates

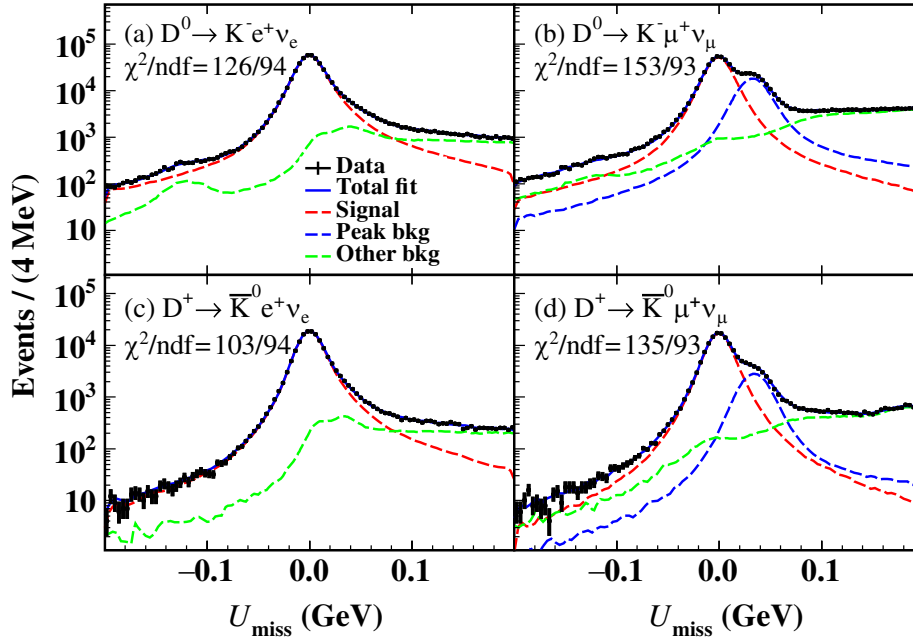


FIG. 2. U_{miss} distributions of the accepted candidate events for $D \rightarrow \bar{K} \ell^+ \nu_\ell$ in data, shown on a logarithmic scale with fit results overlaid. The points with error bars are data. The blue solid lines denote the total fits. The red, blue, and black dashed lines show the signal, peaking background, and combinatorial background contributions, respectively.

are identified using PID information based on the TOF, dE/dx , and EMC, from which the confidence levels $\mathcal{L}_e$, $\mathcal{L}_\mu$, $\mathcal{L}_K$, and $\mathcal{L}_\pi$ are calculated. The positron candidates must satisfy $\mathcal{L}_e > 0.8 \cdot (\mathcal{L}_e + \mathcal{L}_\pi + \mathcal{L}_K)$ and $\mathcal{L}_e > 0.001$. The muon candidates are required to satisfy $\mathcal{L}_\mu > \mathcal{L}_e$ and $\mathcal{L}_\mu > 0.001$, and the deposited energy of muon in the EMC is required to be within (0.1, 0.3) GeV.

The backgrounds due to hadronic D decays are suppressed by requiring no additional good charged tracks on the signal side ($N_{\text{extra}}^{\text{trk}} = 0$). The hadronic backgrounds with a π^0 are rejected by requiring the maximum energy of extra photons ($E_{\text{extra}\gamma}^{\text{max}}$) to be less than 0.25 GeV. To veto backgrounds associated with the misidentification between π^+ and ℓ^+ , the invariant mass of the $\bar{K} \ell^+$ system is required to be less than 1.83, 1.56, 1.84, and 1.59 GeV/ c^2 for $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, respectively.

Since the neutrino is not detectable by the BESIII detector, the kinematic variable $U_{\text{miss}} \equiv E_{\text{miss}} - |\vec{p}_{\text{miss}}|c$ is defined to determine the DT yields. The missing four-momentum ($E_{\text{miss}}, \vec{p}_{\text{miss}}$) in the e^+e^- center-of-mass frame is calculated as $E_{\text{miss}} \equiv E_{\text{beam}} - E_{\bar{K}} - E_{\ell^+}$ and $\vec{p}_{\text{miss}} \equiv \vec{p}_D - \vec{p}_{\bar{K}} - \vec{p}_{\ell^+}$. To improve the U_{miss} resolution, $\vec{p}_D$ is evaluated as $\vec{p}_D = -\hat{p}_{\bar{D}} \sqrt{E_{\text{beam}}^2/c^2 - m_{\bar{D}}^2 c^2}$, where $\hat{p}_{\bar{D}}$ is the unit vector along the momentum direction of the ST $\bar{D}$ meson, $m_{\bar{D}}$ is the known $\bar{D}$ mass [61], and $(E_{\bar{K}(\ell^+)}, \vec{p}_{\bar{K}(\ell^+)})$ is the measured four-momentum of the $\bar{K}(\ell^+)$ candidate.

VI. BRANCHING FRACTIONS

A. Branching fraction results

After applying all the selection criteria, the U_{miss} distributions of the accepted candidates for $D \rightarrow \bar{K} \ell^+ \nu_\ell$ in data are shown in Fig. 2. Based on the inclusive MC simulation, the remaining backgrounds are dominated by the misidentification of μ^+ or π^+ as e^+ for positron channels, misidentification of π^+ as μ^+ for muon channels, and events with a missing π^0 for all signal decays. The normalized yields and fractions of main background components are listed in Table II.

The DT signal yields (N_{DT}) in data are determined using binned maximum likelihood fits to the corresponding U_{miss} distributions. The signal is modeled using MC-simulated shape convolved with a double-Gaussian function with free parameters to account for resolution difference between data and MC simulation. For the muon channels, the dominant peaking backgrounds $D \rightarrow \bar{K} \pi^+ \pi^0$ are described using the corresponding MC-simulated shape convolved with the same Gaussian function as that for the signal. The other background components are combined into a combinatorial background, whose shape is obtained from the inclusive MC sample. The yields of the signal, the peaking background, and the combinatorial background are allowed to float in the fits.

The DT efficiencies are obtained with the signal MC samples. Table III summarizes the DT efficiencies and signal efficiencies for different signal decays in each tag mode and the weighted signal efficiencies ($\bar{\epsilon}_{\text{sig}}$). The data-MC differences on the $\bar{\epsilon}_{\text{sig}}$ associated with the tracking and

TABLE II. Main background sources for each signal decay mode, including the normalized yields (N_{bkg}) and the corresponding fractions (f_{bkg}) to the total background, as estimated from the inclusive MC sample.

Signal decay	Background source	N_{bkg}	$f_{\text{bkg}}(\%)$
$D^0 \rightarrow K^- e^+ \nu_e$	$D^0 \rightarrow K^{*-} e^+ \nu_e$	24816	43.4
	$D^0 \rightarrow K^- \mu^+ \nu_\mu$	14859	26.0
	$D^0 \rightarrow K^- \pi^+ \pi^0$	7740	13.5
$D^0 \rightarrow K^- \mu^+ \nu_\mu$	$D^0 \rightarrow K^- \pi^+ \pi^0$	194586	57.5
	$D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$	67142	19.8
	$D^0 \rightarrow K^{*-} \mu^+ \nu_\mu$	21061	6.2
$D^+ \rightarrow \bar{K}^0 e^+ \nu_e$	$D^+ \rightarrow \bar{K}^{*0} e^+ \nu_e$	7249	50.8
	$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$	4309	30.2
	$D^+ \rightarrow K_S^0 \pi^+ \pi^0$	740	5.2
$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$	$D^+ \rightarrow K_S^0 \pi^+ \pi^0$	28379	55.8
	$D^+ \rightarrow \bar{K}^{*0} \mu^+ \nu_\mu$	6514	12.8
	$D^+ \rightarrow K_S^0 \pi^+ \pi^0 \pi^0$	5179	10.2

PID have been corrected based on the studies of control samples, as described in Sec. VI B.

Combining the signal yields in data N_{DT} , the weighted signal efficiencies $\bar{\epsilon}_{\text{sig}}$, and the ST yields in data, the branching fractions of $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ are calculated using Eq. (2) and are summarized in Table IV. Input and output checks have been performed with MC simulation to ensure the analysis workflow are correct.

B. Systematic uncertainties on branching fractions

Table V summarizes the sources of the systematic uncertainties in the branching fraction measurements, the detailed estimations of which are described below. The uncertainties caused by ST $\bar{D}$ yields, tracking and PID of charged particles, K_S^0 reconstruction, $E_{\text{extra}\gamma}^{\text{max}}$ and $N_{\text{extra}}^{\text{trk}}$ requirements, and quoted branching fractions are correlated among the corresponding decay channels in the subsequent fits and calculations of LFU.

TABLE III. The DT efficiencies ϵ_{DT} , signal efficiencies ϵ for different signal decays in each tag mode, and the weighted signal efficiencies $\bar{\epsilon}_{\text{sig}}$. The listed efficiencies are all in units of percent. For the D^+ signal decays, the efficiencies also include the branching fraction of $\bar{K}^0 \rightarrow \pi^+ \pi^-$. The uncertainties are statistical only.

D^0 decay					D^+ decay				
Tag mode	$\epsilon_{\text{DT}, K^- e^+ \nu_e}$	$\epsilon_{K^- e^+ \nu_e}$	$\epsilon_{\text{DT}, K^- \mu^+ \nu_\mu}$	$\epsilon_{K^- \mu^+ \nu_\mu}$	Tag mode	$\epsilon_{\text{DT}, \bar{K}^0 e^+ \nu_e}$	$\epsilon_{\bar{K}^0 e^+ \nu_e}$	$\epsilon_{\text{DT}, \bar{K}^0 \mu^+ \nu_\mu}$	$\epsilon_{\bar{K}^0 \mu^+ \nu_\mu}$
$\bar{D}^0 \rightarrow K^+ \pi^-$	42.34 $\pm$ 0.02	65.04 $\pm$ 0.04	36.25 $\pm$ 0.02	55.69 $\pm$ 0.04	$D^- \rightarrow K^+ \pi^- \pi^-$	22.83 $\pm$ 0.01	44.75 $\pm$ 0.02	22.83 $\pm$ 0.01	44.75 $\pm$ 0.02
$\bar{D}^0 \rightarrow K^+ \pi^- \pi^0$	24.04 $\pm$ 0.01	67.67 $\pm$ 0.04	20.63 $\pm$ 0.01	58.07 $\pm$ 0.03	$D^- \rightarrow K_S^0 \pi^-$	23.09 $\pm$ 0.03	44.92 $\pm$ 0.06	23.09 $\pm$ 0.03	44.92 $\pm$ 0.06
$\bar{D}^0 \rightarrow K^+ \pi^+ \pi^- \pi^-$	26.34 $\pm$ 0.01	64.72 $\pm$ 0.04	22.15 $\pm$ 0.01	54.42 $\pm$ 0.04	$D^- \rightarrow K^+ \pi^- \pi^- \pi^0$	10.78 $\pm$ 0.01	44.17 $\pm$ 0.04	10.78 $\pm$ 0.01	44.17 $\pm$ 0.04
$\bar{D}^0 \rightarrow K_S^0 \pi^+ \pi^-$	24.48 $\pm$ 0.03	64.73 $\pm$ 0.08	20.67 $\pm$ 0.03	54.68 $\pm$ 0.08	$D^- \rightarrow K_S^0 \pi^- \pi^0$	11.72 $\pm$ 0.01	44.37 $\pm$ 0.05	11.72 $\pm$ 0.01	44.37 $\pm$ 0.05
$\bar{D}^0 \rightarrow K^+ \pi^- \pi^0 \pi^0$	10.93 $\pm$ 0.01	72.35 $\pm$ 0.09	9.42 $\pm$ 0.01	62.33 $\pm$ 0.08	$D^- \rightarrow K_S^0 \pi^- \pi^- \pi^+$	12.60 $\pm$ 0.02	42.55 $\pm$ 0.06	12.60 $\pm$ 0.02	42.55 $\pm$ 0.06
$\bar{D}^0 \rightarrow K^+ \pi^+ \pi^- \pi^- \pi^0$	11.46 $\pm$ 0.01	70.81 $\pm$ 0.10	9.77 $\pm$ 0.01	60.32 $\pm$ 0.09	$D^- \rightarrow K^+ K^- \pi^-$	18.11 $\pm$ 0.03	44.30 $\pm$ 0.07	18.11 $\pm$ 0.03	44.30 $\pm$ 0.07
$\bar{\epsilon}_{\text{sig}}$		68.25 $\pm$ 0.02		57.96 $\pm$ 0.02	$\bar{\epsilon}_{\text{sig}}$		45.39 $\pm$ 0.02		38.74 $\pm$ 0.02

TABLE IV. The signal yields in data N_{DT} , the weighted signal efficiency $\bar{\epsilon}_{\text{sig}}$, and the branching fractions $\mathcal{B}_{\text{sig}}$ for the four signal decay modes. For $\mathcal{B}_{\text{sig}}$, the first and second uncertainties are statistical and systematic, respectively. For other quantities, the uncertainties are statistical only.

Signal decay	N_{DT}	$\bar{\epsilon}_{\text{sig}}(\%)$	$\mathcal{B}_{\text{sig}}(\%)$
$D^0 \rightarrow K^- e^+ \nu_e$	48 9811 $\pm$ 770	68.25 $\pm$ 0.02	3.548 $\pm$ 0.006 $\pm$ 0.017
$D^0 \rightarrow K^- \mu^+ \nu_\mu$	403 893 $\pm$ 830	57.96 $\pm$ 0.02	3.445 $\pm$ 0.007 $\pm$ 0.017
$D^+ \rightarrow \bar{K}^0 e^+ \nu_e$	149 286 $\pm$ 413	45.39 $\pm$ 0.02	8.928 $\pm$ 0.025 $\pm$ 0.050
$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$	125 156 $\pm$ 409	38.74 $\pm$ 0.02	8.770 $\pm$ 0.029 $\pm$ 0.053

TABLE V. Relative systematic uncertainties (in percent) in the measurements of the branching fractions. The symbol “...” indicates “not applicable.”

Source	$D^0 \rightarrow K^- e^+ \nu_e$	$D^0 \rightarrow K^- \mu^+ \nu_\mu$	$D^+ \rightarrow \bar{K}^0 e^+ \nu_e$	$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$
$N_{\text{ST}}^{\text{tot}}$	0.30	0.30	0.30	0.30
K^- tracking	0.10	0.10	...	...
K^- PID	0.10	0.10	...	...
K_S^0	...	...	0.41	0.45
reconstruction				
ℓ^+ tracking	0.10	0.10	0.10	0.10
ℓ^+ PID	0.10	0.10	0.10	0.10
MC model	0.12	0.15	0.04	0.15
$M_{\bar{K}\ell}$	...	...	...	...
requirement				
$E_{\text{extra}\gamma}^{\text{max}}$ and $N_{\text{extra}}^{\text{trk}}$	0.10	0.10	0.10	0.10
requirement				
U_{miss} fit	0.27	0.29	0.08	0.09
MC statistics	0.03	0.03	0.04	0.04
Quoted	...	...	0.07	0.07
branching fractions				
Total	0.47	0.51	0.56	0.60

- (a) *ST $\bar{D}$ yields:* The systematic uncertainty of ST $\bar{D}$ yields is estimated by varying the signal and background shapes in the fits to the M_{BC} spectra. The alternative signal shape is obtained by varying the parameters of Gaussian functions, while the alternative background

shape is obtained by varying the end point by ± 0.1 MeV. The quadratic sum of these two sources results in a 0.3% variation, which is taken as the systematic uncertainty on N_{ST} .

- (b) *K^- tracking and PID*: The tracking and PID efficiencies of the K^- are studied with a control sample of hadronic $D\bar{D}$ events, where D^0 decays into $K^-\pi^+$, $K^-\pi^+\pi^-$, D^+ decays into $K^-\pi^+\pi^+$, and $\bar{D}$ decays into the corresponding charge conjugated channels. The correction factors are 0.997 ± 0.001 and 1.001 ± 0.001 for the tracking and PID, respectively. Based on these values, the signal efficiencies are corrected, and the uncertainties of correction factors are taken as a source of systematic uncertainty.
- (c) *K_S^0 reconstruction*: The uncertainty in the reconstruction efficiency of $K_S^0 \rightarrow \pi^+\pi^-$ includes two sources. The tracking efficiencies of the $\pi^\pm$ are studied with the same control sample as for the K^- , with the correction factor determined to be 0.996 ± 0.001 . Based on this value, the signal efficiencies are corrected, and the uncertainties of the correction factor are taken as a source of the systematic uncertainty. The efficiencies due to the K_S^0 mass window and K_S^0 decay vertex fit are investigated using the hadronic $D\bar{D}$ events, where D^0 decays into $K_S^0\pi^+\pi^-$, $K_S^0\pi^+\pi^-\pi^0$, and $K_S^0\pi^0$, and D^+ decays into $K_S^0\pi^+$, $K_S^0\pi^+\pi^0$, and $K_S^0\pi^+\pi^+\pi^-$. The $\bar{D}$ is reconstructed via the tag modes $\bar{D}^0 \rightarrow K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^-\pi^+$, and $D^- \rightarrow K^+\pi^-\pi^-$. The correction factors are determined to be 0.41% for $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ and 0.45% for $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, respectively, which are taken as the systematic uncertainties.
- (d) *ℓ^+ tracking and PID*: The tracking and PID efficiencies of e^+ and μ^+ are studied by using the control samples of $e^+e^- \rightarrow \gamma e^+e^-$ and $e^+e^- \rightarrow \gamma \mu^+\mu^-$, respectively. The correction factors are obtained to be 0.999 ± 0.001 for e^+ tracking and 0.983 ± 0.001 for e^+ PID, as well as 1.001 ± 0.001 for μ^+ tracking and 0.985 ± 0.001 for μ^+ PID. The signal efficiencies are corrected by these factors, and their uncertainties are assigned as sources of systematic uncertainty.
- (e) *MC model*: The uncertainty due to the hadronic transition form factors used in the MC generation is estimated by varying corresponding parameters by $\pm 1\sigma$.
- (f) *$M_{K\ell^+}$ requirement*: The requirements on $M_{K\ell^+}$ result in negligible efficiency loss for the positron channels, and thereby their corresponding uncertainties are ignored. For muon channels, the reliability of MC modeling on the $M_{K\ell}$ requirement is estimated with positron channels as a control sample. By applying the same $M_{K\ell}$ requirements of muon channels to the positron ones, no significant variation in the measured branching fractions is observed. Therefore, this source of uncertainty is neglected.

- (g) *$E_{\text{extra}}^{\max}$ and $N_{\text{extra}}^{\text{trk}}$ requirements*: The uncertainties associated with the $E_{\text{extra}}^{\max}$ and $N_{\text{extra}}^{\text{trk}}$ requirements are studied with the control samples of hadronic $D\bar{D}$ events, where both D and $\bar{D}$ decay into one of the ST hadronic final states used in this analysis. The efficiency differences between data and MC simulation are assigned as the systematic uncertainty.
- (h) *U_{miss} fit*: The systematic uncertainty due to the U_{miss} fit includes two sources. Given the resolution difference between data and MC simulation has been taken into account by convolving a Gaussian function to the simulated signal shapes, this contribution is neglected. The systematic uncertainty due to the background shape is estimated by varying the relative fractions of backgrounds from $e^+e^- \rightarrow q\bar{q}$ and the dominant background channels in the inclusive MC sample within the uncertainties of their input branching fractions. The variations of measured branching fractions are taken as the corresponding systematic uncertainties.
- (i) *MC statistics*: The relative uncertainties on the signal efficiencies are assigned as the systematic uncertainties due to the limited statistics of the MC samples.
- (j) *Quoted branching fractions*: For the $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ decays, the uncertainty due to the quoted branching fraction of $K_S^0 \rightarrow \pi^+\pi^-$ is 0.07% [61].

VII. HADRONIC TRANSITION FORM FACTORS

A. Theoretical formula

The differential decay width $\frac{d\Gamma_\ell}{dq^2}$ of the semileptonic decay $D \rightarrow \bar{K} \ell^+ \nu_\ell$ can be expressed as [8,26]

$$\frac{d\Gamma_\ell}{dq^2} = \mathcal{N}(q^2) \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) |h_0(q^2)|^2 + \frac{3m_\ell^2}{2q^2} |h_t(q^2)|^2 \right], \quad (4)$$

where $\mathcal{N}(q^2) = \frac{G_F^2 |V_{cs}|^2 |\mathbf{q}| q^2}{96\pi^3 m_D^2}$ contains the Fermi coupling constant G_F , the modulus of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cs}|$, and the mass of D meson m_D [61]; q is the four-momentum of the $\ell \nu_\ell$ system in the D rest frame and $|\mathbf{q}|$ is the magnitude of the three-momentum of q ; m_ℓ is the mass of the lepton. The hadronic helicity amplitudes $h_{0(t)}(q^2)$ are written as

$$h_0(q^2) = \frac{2m_D |\mathbf{q}|}{\sqrt{q^2}} f_+(q^2),$$

$$h_t(q^2) = \frac{m_D^2 - m_K^2}{\sqrt{q^2}} f_0(q^2), \quad (5)$$

where $f_+(q^2)$ and $f_0(q^2)$ are the vector and scalar form factors, respectively.

The series expansion [58] approach is applied to describe the hadronic transition form factor in this paper, which takes the form

$$f_{+,0}(q^2) = \frac{1}{P(q^2)\Phi(q^2)} \sum_{k=0}^{\infty} a_k(t_0)[z(q^2, t_0)]^k. \quad (6)$$

Here, $a_k(t_0)$ are the real coefficients, and $P(q^2) = z(q^2, m_{D_s^{*+}}^2)$, where $z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$. The function Φ is given by

$$\begin{aligned} \Phi(q^2) = & \sqrt{\frac{1}{24\pi\chi_V}} \left(\frac{t_+ - q^2}{t_+ - t_0} \right)^{1/4} \left(\sqrt{t_+ - q^2} + \sqrt{t_+} \right)^{-5} \\ & \times \left(\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0} \right) \\ & \times \left(\sqrt{t_+ - q^2} + \sqrt{t_+ - t_-} \right)^{3/2} (t_+ - q^2)^{3/4}, \end{aligned} \quad (7)$$

where $t_{\pm} = (m_D \pm m_K)^2$; $t_0 = t_+(1 - \sqrt{1 - t_-/t_+})$; m_D and m_K are the masses of D and K , respectively. The pole mass of the vector form factor $m_{D_s^{*+}} = 2112.2 \text{ MeV}/c^2$ is identified with the mass of the lowest lying $c\bar{s}$ vector meson D_s^{*+} [61]. The parameter $\chi_V = \frac{3}{32\pi^2 m_c^2}$ is obtained from dispersion relations using perturbative QCD [68], where $m_c = 1.27 \text{ GeV}/c^2$ is the $\overline{\text{MS}}$ mass of the c -quark at the energy scale $\mu = m_c$ [61].

Following Ref. [45], the vector form factor $f_+(q^2)$ is parametrized using a two-parameter series expansion as

$$f_+(q^2) = \frac{1}{P(q^2)\Phi(q^2)} [a_0(t_0) + a_1(t_0)z(q^2, t_0)]. \quad (8)$$

By defining $r_1(t_0) = a_1(t_0)/a_0(t_0)$, $f_+(q^2)$ is rewritten as

$$\begin{aligned} f_+(q^2) = & \frac{1}{P(q^2)\Phi(q^2)} \frac{f_+(0)P(0)\Phi(0)}{1 + r_1(t_0)z(0, t_0)} \\ & \times (1 + r_1(t_0)[z(q^2, t_0)]). \end{aligned} \quad (9)$$

The scalar form factor $f_0(q^2)$ is described using a one-parameter series expansion as

$$f_0(q^2) = \frac{1}{P(q^2)\Phi(q^2)} f_0(0)P(0)\Phi(0), \quad (10)$$

where the pole mass $m_{D_{s0}^*} = 2317.8 \text{ MeV}/c^2$ is assigned as the mass of $D_{s0}^*(2317)^+$ [61]. The usual kinematic constraint $f_+(0) = f_0(0)$ [26] is applied in this paper.

B. Partial decay rates in data

The form factors are determined by performing a fit to the q^2 -binned partial decay rates, which are defined as

$$\Delta\Gamma_i = \int_{q_{\min(i)}^2}^{q_{\max(i)}^2} \frac{d\Gamma}{dq^2} dq^2 \quad (11)$$

and determined with $\Delta\Gamma_i = N_{\text{prd}}^i / (\tau_D \cdot N_{\text{ST}}^{\text{tot}})$. Here, N_{prd}^i is the number of events produced in the i th q^2 interval ($q_{\min(i)}^2, q_{\max(i)}^2$) and τ_D is the D -meson lifetime [61]. The four-momentum of neutrino used in both q^2 and further $\cos\theta_W$ calculation is defined as $p_\nu = (E_{\text{miss}}, E_{\text{miss}} \times \hat{p}_{\text{miss}})$, where E_{miss} is the missing energy and $\hat{p}_{\text{miss}}$ is the unit vector along the missing momentum direction defined in Sec. V.

The number of events produced in the i th q^2 interval of the data sample is calculated as

$$N_{\text{prd}}^i = \sum_j^{N_{\text{intervals}}} (\epsilon^{-1})_{ij} N_{\text{DT}}^j, \quad (12)$$

where $(\epsilon^{-1})_{ij}$ is the element of the inverse efficiency matrix, obtained by analyzing the signal MC events. The statistical uncertainty of N_{prd}^i is calculated as

$$\sigma^2(N_{\text{prd}}^i) = \sum_j^{N_{\text{intervals}}} (\epsilon^{-1})_{ij}^2 \sigma_{\text{stat}}^2(N_{\text{DT}}^j), \quad (13)$$

where $\sigma_{\text{stat}}(N_{\text{DT}}^j)$ is the statistical uncertainty of N_{DT}^j . The efficiency matrix element ϵ_{ij}^α of the tag mode α is given as

$$\epsilon_{ij}^\alpha = \frac{N_{ij}^{\text{rec}}}{N_j^{\text{gen}}} \cdot \frac{1}{\epsilon_{\text{ST}}^\alpha}, \quad (14)$$

where N_{ij}^{rec} is the number of events generated in the j th q^2 interval and reconstructed in the i th q^2 interval; N_j^{gen} is the number of generated events in the j th q^2 interval; $\epsilon_{\text{ST}}^\alpha$ is the ST efficiency for the tag mode α . The efficiency matrix elements are further weighted by the ST yields in data as

$$\epsilon_{ij} = \sum_{\alpha=1}^6 \frac{N_{\text{ST}}^\alpha \epsilon_{ij}^\alpha}{N_{\text{ST}}^{\text{tot}}} \quad (15)$$

and corrected for the data-MC difference due to tracking and PID. The detailed efficiency matrices are shown in Tables XVI–XIX in the Appendix.

For each signal decay, the DT yield observed in the j th reconstructed q^2 interval N_{DT}^j is obtained by fitting the corresponding U_{miss} distribution. The fitting method is the same as that described in Sec. VI A. The fit results for the

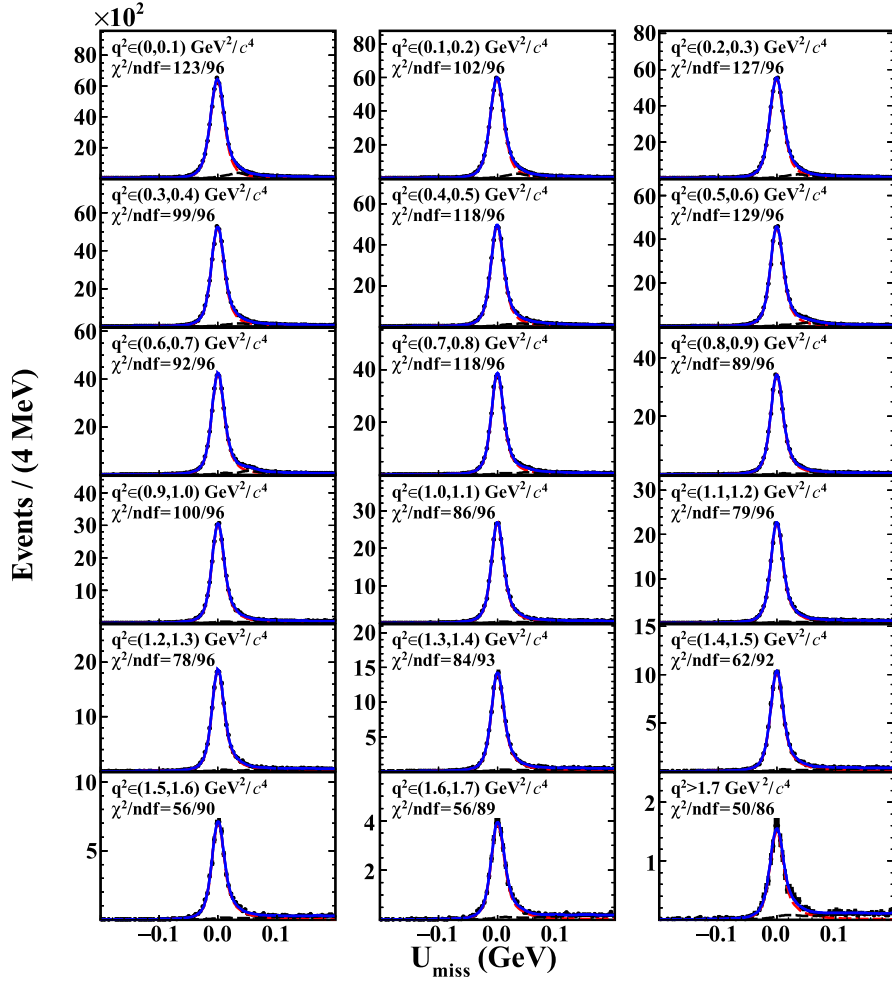


FIG. 3. The U_{miss} distributions of the accepted candidate events for $D^0 \rightarrow K^- e^+ \nu_e$ in individual q^2 intervals in data, with fit results overlaid. The points with error bars are data, the blue solid curves are the fit results, the red dashed curves are the signal shapes, and the black dashed curves are the fitted combinatorial background shapes.

U_{miss} distributions are shown in Fig. 3 for the signal decay $D^0 \rightarrow K^- e^+ \nu_e$, while those for the other three decays are presented in Figs. 12–14 of the Appendix.

Table VI summarizes the q^2 ranges, the fitted DT yields N_{DT} , the corresponding produced yields N_{prd} , and the q^2 -binned decay rates $\Delta\Gamma$ for the decay $D^0 \rightarrow K^- e^+ \nu_e$. The results for $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ decays are presented in Tables XX–XXII in the Appendix.

C. Fit to the partial decay rates

A fit to the q^2 -binned partial decay rates $\Delta\Gamma$ is performed using the least χ^2 method, with the objective function constructed as

$$\chi^2 = \sum_{i,j=1}^{N_{\text{intervals}}} (\Delta\Gamma_i^{\text{msr}} - \Delta\Gamma_i^{\text{fit}}) (C^{-1})_{ij} (\Delta\Gamma_j^{\text{msr}} - \Delta\Gamma_j^{\text{fit}}). \quad (16)$$

Here, $\Delta\Gamma_i^{\text{msr}}$ is the measured $\Delta\Gamma_i$ and $\Delta\Gamma_i^{\text{fit}}$ is the fitted decay rate obtained by integrating Eq. (4) in the i th interval. The covariance matrix is defined as $C_{ij} = C_{ij}^{\text{stat}} + C_{ij}^{\text{syst}}$, which includes both the statistical and systematic uncertainties and takes the correlations between different q^2 intervals into consideration. The elements of the statistical covariance matrix are defined as

$$C_{ij}^{\text{stat}} = \left(\frac{1}{\tau_D N_{\text{ST}}^{\text{tot}}} \right)^2 \sum_{\alpha} (\epsilon^{-1})_{i\alpha} (\epsilon^{-1})_{j\alpha} (\sigma(N_{\text{DT}}^{\alpha}))^2, \quad (17)$$

where $\sigma(N_{\text{DT}}^{\alpha})$ is the statistical uncertainty of the signal yield observed in the α th interval. The detailed statistical covariance matrices of $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ decays are listed in Tables XXIII–XXVI of the Appendix.

TABLE VI. The fitted DT yields N_{DT}^i , the produced yields N_{prd}^i , and the q^2 -binned partial decay rates $\Delta\Gamma$ of $D^0 \rightarrow K^- e^+ \nu_e$ in different q^2 intervals of data, where the uncertainties are statistical only.

$q^2(\text{GeV}^2/c^4)$	N_{DT}^i	N_{prd}^i	$\Delta\Gamma(\text{ns}^{-1})$
(0.0, 0.1)	55232 ± 258	76678 ± 383	9.240 ± 0.046
(0.1, 0.2)	51026 ± 248	72259 ± 401	8.707 ± 0.048
(0.2, 0.3)	47871 ± 241	68637 ± 403	8.271 ± 0.049
(0.3, 0.4)	44255 ± 229	63851 ± 393	7.694 ± 0.047
(0.4, 0.5)	41687 ± 222	60472 ± 386	7.287 ± 0.046
(0.5, 0.6)	38575 ± 214	56279 ± 376	6.782 ± 0.045
(0.6, 0.7)	35282 ± 204	51461 ± 360	6.201 ± 0.043
(0.7, 0.8)	31878 ± 193	46750 ± 343	5.633 ± 0.041
(0.8, 0.9)	28543 ± 180	42021 ± 321	5.063 ± 0.039
(0.9, 1.0)	25497 ± 170	37895 ± 304	4.566 ± 0.037
(1.0, 1.1)	22333 ± 158	33297 ± 283	4.012 ± 0.034
(1.1, 1.2)	18934 ± 146	28601 ± 263	3.446 ± 0.032
(1.2, 1.3)	15810 ± 133	24352 ± 243	2.934 ± 0.029
(1.3, 1.4)	12230 ± 118	19229 ± 218	2.317 ± 0.026
(1.4, 1.5)	8939 ± 101	14485 ± 192	1.745 ± 0.023
(1.5, 1.6)	6152 ± 84	10471 ± 164	1.262 ± 0.020
(1.6, 1.7)	3563 ± 65	6541 ± 135	0.788 ± 0.016
>1.7	1425 ± 43	3471 ± 115	0.418 ± 0.014

D. Systematic uncertainties on partial decay rates

The sources of systematic uncertainties in the measurement of q^2 -binned decay rates are discussed below. Table VII summarizes the systematic uncertainties of $\Delta\Gamma$ in different q^2 intervals for $D^0 \rightarrow K^- e^+ \nu_e$, while those for $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ are presented in Tables XXVII–XXIX of the Appendix. The systematic covariance matrices for all signal decays are also available in Tables XXX–XXXIII of the Appendix.

- (a) *ST $\bar{D}$ yields*: The systematic uncertainties due to the ST yields are fully correlated across all the q^2 intervals. The corresponding systematic covariance matrix is calculated as

$$C_{ij}^{\text{syst}}(N_{\text{ST}}) = \Delta\Gamma_i \Delta\Gamma_j \left(\frac{\sigma(N_{\text{ST}})}{N_{\text{ST}}} \right)^2, \quad (18)$$

where $\sigma(N_{\text{ST}})/N_{\text{ST}}$ is the relative uncertainty of ST yield.

- (b) *D lifetime*: The systematic uncertainties associated with the D -meson lifetime are fully correlated across all the q^2 intervals. The corresponding systematic covariance matrix is calculated as

$$C_{ij}^{\text{syst}}(\tau_D) = \sigma(\Delta\Gamma_i) \sigma(\Delta\Gamma_j), \quad (19)$$

where $\sigma(\Delta\Gamma_i) = \sigma\tau_D \cdot \Delta\Gamma_i$ and $\sigma\tau_D$ is the uncertainty of the D lifetime [61].

- (c) *MC statistics*: The systematic uncertainties of MC statistics are described with the covariance matrix as

$$C_{ij}^{\text{syst}}(\text{MC}^{\text{stat}}) = \left(\frac{1}{\tau_D N_{\text{ST}}} \right)^2 \times \sum_{\alpha\beta} N_{\text{DT}}^\alpha N_{\text{DT}}^\beta \text{Cov}((\epsilon^{-1})_{i\alpha}, (\epsilon^{-1})_{j\beta}), \quad (20)$$

where $N_{\text{DT}}^{\alpha(\beta)}$ is the DT yield observed in the interval $\alpha(\beta)$, and the covariances of the inverse efficiency matrix elements are given by

$$\begin{aligned} &\text{Cov}((\epsilon^{-1})_{i\alpha}, (\epsilon^{-1})_{j\beta}) \\ &= \sum_{mn} ((\epsilon^{-1})_{im} (\epsilon^{-1})_{jn}) [\sigma(\epsilon_{mn})]^2 ((\epsilon^{-1})_{an} (\epsilon^{-1})_{bn}). \end{aligned} \quad (21)$$

- (d) *MC model*: The systematic uncertainty from the MC model is estimated by varying the parameters of the two-parameter series expansion model by $\pm 1\sigma$. Alternative partial decay rates are calculated based on the updated efficiency matrix, and their differences from

TABLE VII. The systematic uncertainties (in percent) of the measured decay rates of $D^0 \rightarrow K^- e^+ \nu_e$ in different q^2 bins.

i th q^2 bin	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
N_{tag}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
D^0 lifetime	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
MC stat.	0.08	0.09	0.09	0.09	0.10	0.10	0.10	0.11	0.12	0.12	0.13	0.14	0.16	0.18	0.21	0.25	0.34	0.56
$E_{\text{extray}}^{\text{max}}$ cut	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
U_{miss} fit	0.19	0.18	0.20	0.17	0.17	0.18	0.18	0.20	0.15	0.14	0.14	0.13	0.15	0.18	0.18	0.17	0.25	0.45
K tracking	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.14	0.27
K PID	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
e tracking	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
e PID	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
MC model	0.08	0.21	0.37	0.29	0.23	0.38	0.25	0.06	0.31	0.75	0.12	0.19	0.17	0.44	0.06	0.70	0.27	0.83
Total	0.50	0.53	0.62	0.57	0.54	0.62	0.55	0.50	0.58	0.89	0.50	0.52	0.53	0.68	0.53	0.88	0.68	1.21

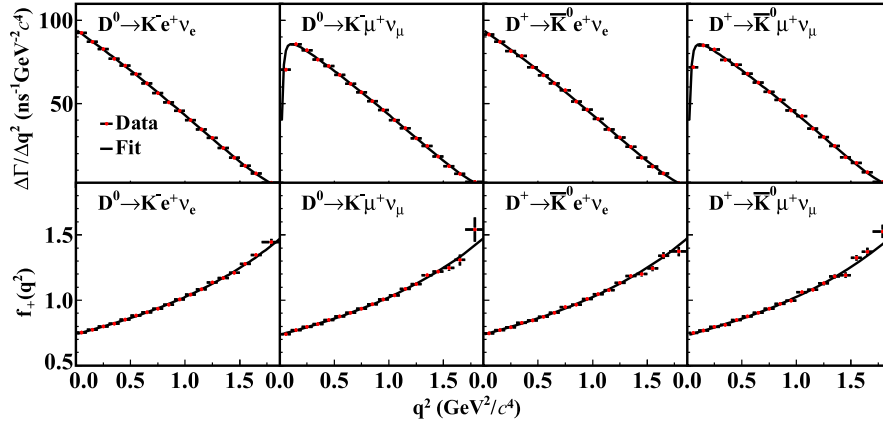


FIG. 4. Fits to the partial decay rates of $D \rightarrow \bar{K} \ell^+ \nu_\ell$ and projections of the form factor as functions of q^2 , where the red points with error bars are the measured partial decay rates and the solid curves are the best fits. The fit qualities are available in Table IX.

the nominal ones are taken as the systematic uncertainties. The corresponding covariance matrix is assigned as

$$C_{ij}^{\text{syst}}(\text{MC model}) = \delta(\Delta\Gamma_i)\delta(\Delta\Gamma_j), \quad (22)$$

where $\delta(\Delta\Gamma_i)$ denotes the change of the partial decay rate in the i th q^2 interval.

- (e) *Tracking and PID*: The systematic uncertainties associated with the ℓ^+ or K^- tracking and PID efficiencies are estimated by varying the corresponding correction factors within $\pm 1\sigma$. With the new efficiency matrices, alternative partial decay rates are obtained and their differences from the nominal ones are taken as the systematic uncertainties. The related covariance matrix is calculated as

$$C_{ij}^{\text{syst}}(\text{Tracking, PID}) = \delta(\Delta\Gamma_i)\delta(\Delta\Gamma_j), \quad (23)$$

where $\delta(\Delta\Gamma_i)$ denotes the change of the partial decay rate in the i th q^2 interval.

- (f) *U_{miss} fit*: The systematic uncertainties arising from the fit to the U_{miss} distributions in the interval α , $\sigma_\alpha^{\text{Fit}}$, are estimated in the same approach as described in Sec. VI B. The corresponding covariance matrix is assigned as

$$C_{ij}^{\text{syst}}(U_{\text{miss}} \text{ fit}) = \left(\frac{1}{\tau_D N_{\text{ST}}^{\text{tot}}} \right)^2 \sum_{\alpha} \varepsilon_{i\alpha}^{-1} \varepsilon_{j\alpha}^{-1} (\sigma_\alpha^{\text{Fit}})^2. \quad (24)$$

- (g) *Remaining uncertainties*: The remaining systematic uncertainties, including the $E_{\text{extra}\gamma}^{\text{max}}$ and $N_{\text{extra}}^{\text{trk}}$ requirements, K_S^0 reconstruction, and quoted branching fractions, are assumed to be fully correlated across q^2 intervals, and the corresponding systematic covariance matrix is calculated as

$$C_{ij}^{\text{syst}} = \sigma(\Delta\Gamma_i)\sigma(\Delta\Gamma_j), \quad (25)$$

where $\sigma(\Delta\Gamma_i) = \sigma_{\text{syst}} \cdot \Delta\Gamma_i$.

E. Results based on individual fits

Individual fits are performed to the q^2 -binned partial decay rates of $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ to determine the fit parameters $f_+(0)|V_{cs}|$ and $r_1(t_0)$ defined in Sec. VII A. The statistical uncertainties of the fitted parameters are obtained from the fits with only statistical covariance matrix included. The systematic uncertainties of the fitted parameters are assigned as the quadratic differences between the uncertainties obtained from the fits with the statistical covariance matrix alone and those with both the statistical and systematic covariance matrices included. The fit projections for all the decays are shown in Fig. 4 and the fitted parameters are listed in Table IX.

Furthermore, the projection of form factor f_+ in each q^2 interval is obtained as

$$f_+^{\text{data}}(q_i^2) = \sqrt{\frac{(\Delta\Gamma_i^{\text{measured}} - B) \cdot |f_+(q_i^2)|^2}{A}}, \quad (26)$$

where

$$A = \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} \mathcal{N}(q^2) \left(1 - \frac{m_\ell^2}{q^2} \right)^2 \times \left[\left(1 + \frac{m_\ell^2}{2q^2} \right) |h_0(q^2)|^2 \right] dq^2, \quad (27)$$

and

$$B = \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} \mathcal{N}(q^2) \left(1 - \frac{m_\ell^2}{q^2} \right)^2 \left[\frac{3m_\ell^2}{2q^2} |h_t(q^2)|^2 \right] dq^2, \quad (28)$$

TABLE VIII. The determined $f_+^{\text{data}}(q_i^2)$ values in different q^2 bins of $D \rightarrow \bar{K}\ell^+\nu_\ell$, where the first uncertainties are statistical and the second are systematic.

q^2 (GeV/c ⁴)	$D^0 \rightarrow K^- e^+ \nu_e$	$D^0 \rightarrow K^- \mu^+ \nu_\mu$	$D^+ \rightarrow \bar{K}^0 e^+ \nu_e$	$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$
(0.0, 0.1)	$0.754 \pm 0.002 \pm 0.002$	$0.741 \pm 0.004 \pm 0.003$	$0.748 \pm 0.003 \pm 0.002$	$0.749 \pm 0.006 \pm 0.004$
(0.1, 0.2)	$0.774 \pm 0.002 \pm 0.002$	$0.772 \pm 0.003 \pm 0.003$	$0.772 \pm 0.004 \pm 0.003$	$0.767 \pm 0.005 \pm 0.003$
(0.2, 0.3)	$0.800 \pm 0.002 \pm 0.002$	$0.796 \pm 0.003 \pm 0.003$	$0.792 \pm 0.004 \pm 0.003$	$0.797 \pm 0.005 \pm 0.003$
(0.3, 0.4)	$0.821 \pm 0.003 \pm 0.002$	$0.817 \pm 0.003 \pm 0.002$	$0.826 \pm 0.004 \pm 0.003$	$0.812 \pm 0.005 \pm 0.003$
(0.4, 0.5)	$0.852 \pm 0.003 \pm 0.002$	$0.848 \pm 0.003 \pm 0.002$	$0.844 \pm 0.005 \pm 0.003$	$0.849 \pm 0.005 \pm 0.003$
(0.5, 0.6)	$0.881 \pm 0.003 \pm 0.003$	$0.875 \pm 0.004 \pm 0.003$	$0.873 \pm 0.005 \pm 0.003$	$0.877 \pm 0.006 \pm 0.003$
(0.6, 0.7)	$0.907 \pm 0.003 \pm 0.003$	$0.903 \pm 0.004 \pm 0.003$	$0.905 \pm 0.006 \pm 0.004$	$0.904 \pm 0.006 \pm 0.004$
(0.7, 0.8)	$0.936 \pm 0.003 \pm 0.002$	$0.935 \pm 0.004 \pm 0.003$	$0.946 \pm 0.006 \pm 0.004$	$0.933 \pm 0.007 \pm 0.004$
(0.8, 0.9)	$0.966 \pm 0.004 \pm 0.003$	$0.970 \pm 0.004 \pm 0.003$	$0.966 \pm 0.007 \pm 0.005$	$0.971 \pm 0.007 \pm 0.004$
(0.9, 1.0)	$1.006 \pm 0.004 \pm 0.004$	$1.006 \pm 0.005 \pm 0.003$	$1.010 \pm 0.007 \pm 0.006$	$0.997 \pm 0.008 \pm 0.005$
(1.0, 1.1)	$1.044 \pm 0.004 \pm 0.003$	$1.036 \pm 0.005 \pm 0.003$	$1.046 \pm 0.008 \pm 0.006$	$1.062 \pm 0.009 \pm 0.006$
(1.1, 1.2)	$1.084 \pm 0.005 \pm 0.003$	$1.086 \pm 0.006 \pm 0.004$	$1.077 \pm 0.009 \pm 0.006$	$1.079 \pm 0.011 \pm 0.006$
(1.2, 1.3)	$1.136 \pm 0.006 \pm 0.003$	$1.125 \pm 0.007 \pm 0.004$	$1.134 \pm 0.010 \pm 0.006$	$1.130 \pm 0.013 \pm 0.008$
(1.3, 1.4)	$1.170 \pm 0.007 \pm 0.004$	$1.191 \pm 0.009 \pm 0.003$	$1.184 \pm 0.012 \pm 0.007$	$1.182 \pm 0.015 \pm 0.008$
(1.4, 1.5)	$1.209 \pm 0.008 \pm 0.003$	$1.220 \pm 0.012 \pm 0.007$	$1.200 \pm 0.014 \pm 0.009$	$1.191 \pm 0.019 \pm 0.010$
(1.5, 1.6)	$1.277 \pm 0.010 \pm 0.006$	$1.246 \pm 0.012 \pm 0.018$	$1.243 \pm 0.018 \pm 0.011$	$1.325 \pm 0.018 \pm 0.011$
(1.6, 1.7)	$1.346 \pm 0.014 \pm 0.005$	$1.308 \pm 0.019 \pm 0.037$	$1.341 \pm 0.023 \pm 0.014$	$1.370 \pm 0.026 \pm 0.015$
> 1.7	$1.444 \pm 0.024 \pm 0.009$	$1.542 \pm 0.038 \pm 0.085$	$1.373 \pm 0.035 \pm 0.012$	$1.526 \pm 0.044 \pm 0.027$

TABLE IX. The parameters $[f_+(0)|V_{cs}|, r_1(t_0)]$ determined from the fits to the partial decay rates of the $D \rightarrow \bar{K}\ell^+\nu_\ell$ decays, where the first and second uncertainties are statistical and systematic, respectively. The column labeled ρ gives the correlation coefficients of the two parameters, and ndf denotes the number of degrees of freedom.

Case	Signal decay	$f_+(0) V_{cs} $	$r_1(t_0)$	ρ	χ^2/ndf
Individual fit	$D^0 \rightarrow K^- e^+ \nu_e$	$0.7217 \pm 0.0010 \pm 0.0017$	$-2.22 \pm 0.04 \pm 0.02$	0.38	14.7/16
	$D^0 \rightarrow K^- \mu^+ \nu_\mu$	$0.7159 \pm 0.0013 \pm 0.0019$	$-2.33 \pm 0.05 \pm 0.02$	0.47	13.1/16
	$D^+ \rightarrow \bar{K}^0 e^+ \nu_e$	$0.7187 \pm 0.0017 \pm 0.0024$	$-2.30 \pm 0.06 \pm 0.06$	0.27	14.1/16
	$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$	$0.7132 \pm 0.0021 \pm 0.0025$	$-2.43 \pm 0.07 \pm 0.06$	0.41	18.1/16
Simultaneous fit	$D \rightarrow \bar{K}\ell^+\nu_\ell$	$0.7183 \pm 0.0007 \pm 0.0014$	$-2.28 \pm 0.02 \pm 0.02$	0.30	72.6/70

where $q_{\min(i)}^2$ and $q_{\max(i)}^2$ are the low and high boundaries of the i th q^2 bin. The numerical results are summarized in Table VIII.

F. Results based on a simultaneous fit

Considering the correlations between the systematic uncertainties of the fitted parameters from individual fits, a simultaneous fit is performed to the q^2 -binned partial decay rates of $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ to determine the combined $f_+(0)|V_{cs}|$ and $r_1(t_0)$. In the simultaneous fit, the parameters $f_+(0)|V_{cs}|$ and $r_1(t_0)$ are shared among the four signal decays. The corresponding χ^2 function is defined in the same way as in Eq. (16), where the indices i, j sum over all the 72 q^2 intervals. The covariance matrix C_{ij} is redefined as $C_{ij} = C_{ij}^{\text{stat}} + C_{ij}^{\text{csyst}} + C_{ij}^{\text{usyst}}$, where C_{ij}^{stat} is the element of statistical covariance matrix, which is block diagonal, i.e.

$$C^{\text{stat}} = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & D \end{pmatrix}. \quad (29)$$

Here A, B, C , and D are the statistical covariance matrices for each signal channel. The elements of the correlated systematic covariance matrix are defined as

$$C_{ij}^{\text{csyst}} = \delta(\Delta\Gamma_i)\delta(\Delta\Gamma_j). \quad (30)$$

The uncorrelated systematic covariance matrix is also defined in a block-diagonal form as

$$C^{\text{usyst}} = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}, \quad (31)$$

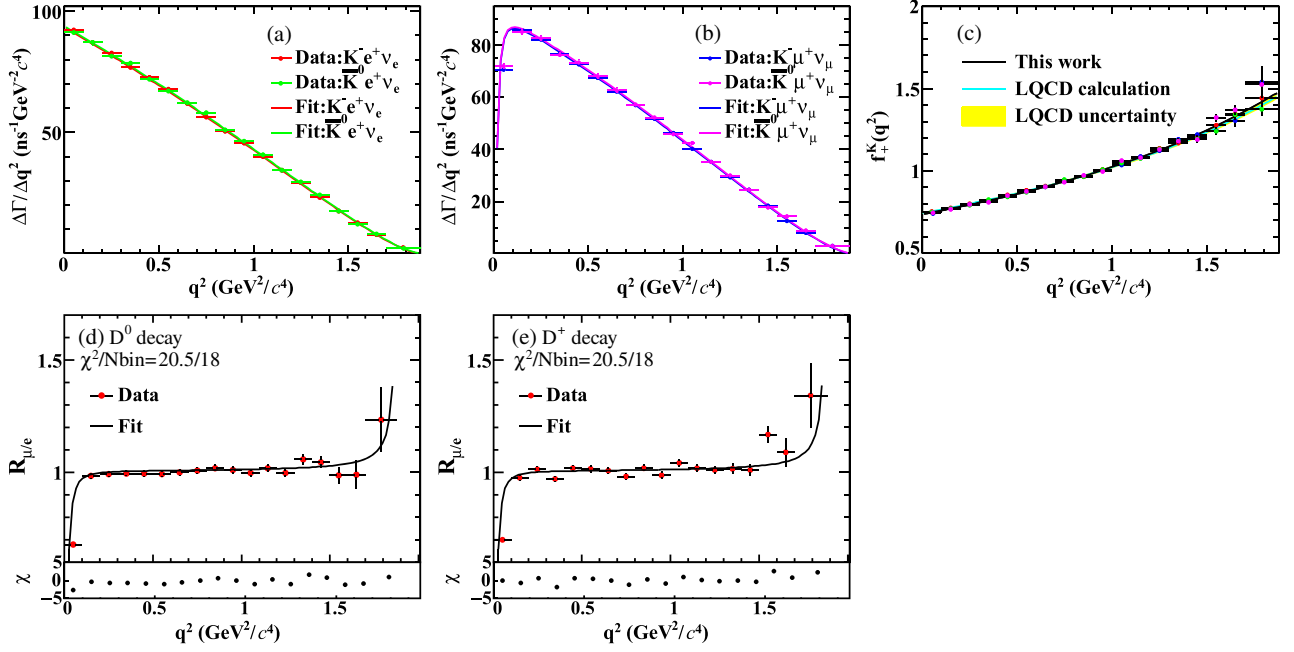


FIG. 5. (a),(b) Simultaneous fit to the partial decay rates of $D^0(D^+) \rightarrow \bar{K} \ell^+ \nu_\ell$. (c) Hadronic form factor $f_+(q^2)$ as a function of q^2 for the four signal modes. (d),(e) Ratios of differential decay rates of $D^0 \rightarrow K^- \mu^+ \nu_\mu$ over $D^0 \rightarrow K^- e^+ \nu_e$ and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ over $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ in each q^2 bin, respectively. The dots with error bars are data, and the solid lines are the results with the parameters of the simultaneous fit. The colors red, green, blue, and purple are for $D^0 \rightarrow K^- e^+ \nu_e$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, respectively. The fit quality is available in Table IX.

where a , b , c , and d are the uncorrelated systematic covariance matrices obtained from each signal channel. The detailed covariance matrices for the simultaneous fit are presented in Tables XXXIV–XXXVII of the Appendix.

The fit projections of the simultaneous fit are shown in Fig. 5. The parameters obtained are $f_+(0)|V_{cs}| = 0.7183 \pm 0.0007_{\text{stat}} \pm 0.0014_{\text{syst}}$ and $r_1(t_0) = -2.28 \pm 0.02_{\text{stat}} \pm 0.02_{\text{syst}}$.

VIII. FORWARD-BACKWARD ASYMMETRIES

A. Theoretical formula

In addition to the partial decay rate, the angular distribution of the semileptonic decay $D \rightarrow \bar{K} \ell^+ \nu_\ell$ is of particular interest because of its sensitivity to the potential scalar current contribution in the $c \rightarrow s \ell^+ \nu_\ell$ transition. The angular observable forward-backward asymmetry is defined as

$$A_{\text{FB}}(q^2) = \frac{d\Gamma^\ell(\cos \theta_W > 0) - d\Gamma^\ell(\cos \theta_W < 0)}{d\Gamma^\ell(\cos \theta_W > 0) + d\Gamma^\ell(\cos \theta_W < 0)}. \quad (32)$$

Here, θ_W is the angle between the lepton momentum and the direction opposite to the D -meson momentum in the $\ell \nu_\ell$ rest frame. The theoretical expression of $A_{\text{FB}}(q^2)$ is written as [8,26]

$$A_{\text{FB}}(q^2) = \frac{3\mathcal{N}(q^2)}{2} \frac{1}{d\Gamma^\ell/dq^2} \times \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \frac{m_\ell^2}{q^2} \Re(h_0(q^2)h_t^*(q^2)), \quad (33)$$

where the definitions of variables are exactly the same as those in Sec. VII A. In the SM, this asymmetry is proportional to the squared lepton mass m_ℓ^2 , resulting in a trivial distribution $A_{\text{FB}}^\ell(q^2) = 0$ for the positron channels in the full q^2 range. For the muon channels, the overall asymmetry is predicted to be -0.053 [8] and -0.055 ± 0.002 [26].

With the scalar currents included, the hadronic helicity amplitude $h_t(q^2)$ shown in Eq. (5) is modified as

$$h_t(q^2) = \left(1 + c_s^\ell \frac{q^2}{m_\ell(m_s - m_c)}\right) \frac{m_D^2 - m_K^2}{\sqrt{q^2}} f_0(q^2), \quad (34)$$

where $c_s^\ell = c_R^\ell + c_L^\ell$ is the scalar combination of Wilson coefficients defined in Eq. (1). Hence, any deviation of A_{FB} from the SM prediction may suggest potential scalar current contribution in the decay $D \rightarrow \bar{K} \ell^+ \nu_\ell$. In this paper, both the overall and q^2 -binned asymmetries are determined.

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TABLE XIII. The fitted signal yields for $D \rightarrow \bar{K} \ell^+ \nu_\ell$, and the overall forward-backward asymmetry $\langle A_{\text{FB}} \rangle$, where the first uncertainties are statistical and the second are systematic.

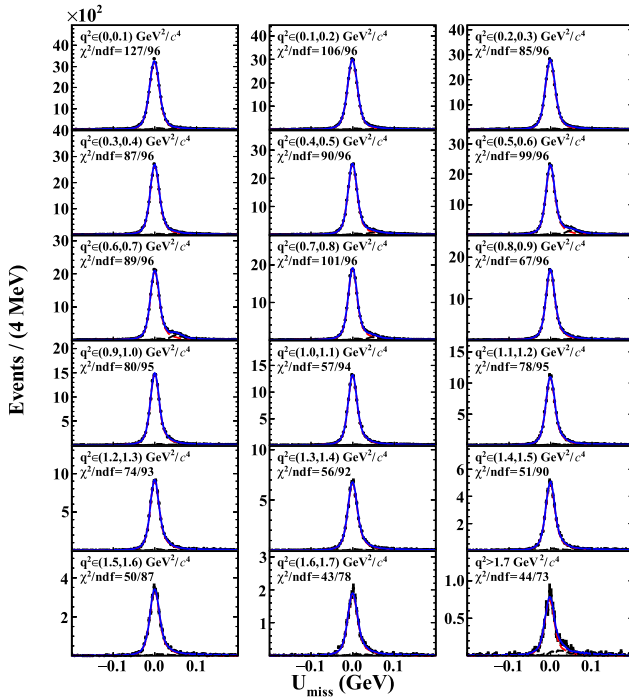
Signal decay	N_{DT}	$\langle A_{\text{FB}} \rangle (\times 10^{-3})$
$D^0 \rightarrow K^- e^+ \nu_e$		
$\cos \theta_l > 0$	244569 ± 550	$+0.2 \pm 1.6 \pm 1.6$
$\cos \theta_l < 0$	245113 ± 545	
$D^0 \rightarrow K^- \mu^+ \nu_\mu$		
$\cos \theta_l > 0$	178328 ± 539	$-59.2 \pm 2.2 \pm 2.2$
$\cos \theta_l < 0$	225896 ± 696	
$*D^+ \rightarrow \bar{K}^0 e^+ \nu_e$		
$\cos \theta_l > 0$	75066 ± 293	$+0.0 \pm 2.9 \pm 1.5$
$\cos \theta_l < 0$	74297 ± 294	
$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$		
$\cos \theta_l > 0$	57122 ± 281	$-53.1 \pm 3.5 \pm 0.7$
$\cos \theta_l < 0$	67591 ± 329	

Here, the statistical uncertainties are obtained with $C_{\alpha\beta} = C_{\alpha\beta}^{\text{stat}}$, while the systematic ones are assigned as the quadratic difference between the uncertainties obtained with $C_{\alpha\beta} = C_{\alpha\beta}^{\text{stat}} + C_{\alpha\beta}^{\text{sys}}$ and $C_{\alpha\beta} = C_{\alpha\beta}^{\text{stat}}$. The results are summarized in Table XIII. No significant deviation from the theoretical predictions [8,26] is observed.

C. q^2 -binned forward-backward asymmetries

The forward-backward asymmetry in the i th q^2 interval is defined as

$$A_{\text{FB},i} = \int_{q_{\text{min}}^{(i)}}^{q_{\text{max}}^{(i)}} A_{\text{FB}}(q^2) \frac{d\Gamma}{dq^2} dq^2 \bigg/ \int_{q_{\text{min}}^{(i)}}^{q_{\text{max}}^{(i)}} \frac{d\Gamma}{dq^2} dq^2, \quad (39)$$



where the q^2 intervals are the same as that of Sec. VII B. Similar to the measurement of the overall asymmetry, the number of produced events in the i th q^2 interval is obtained with

$$N_{\text{prd}}^{i,\alpha} = \sum_{(j,\beta)} (\epsilon^{-1})_{(i,\alpha)(j,\beta)} N_{\text{DT}}^{j,\beta}, \quad (40)$$

where N^{prd} is the number of produced events, and the indices α and β indicate forward and backward categories, respectively. The results of the fits to the U_{miss} distributions of $D^0 \rightarrow K^- e^+ \nu_e$ are shown in Fig. 7 with DT yields N_{DT} summarized in Table XIV. The results for the other three decay modes are provided in Figs. 15–17 and Tables XXXVIII–XL in the Appendix. The systematic uncertainties are estimated in the same approach as those of overall asymmetries.

Table XIV summarizes the measured q^2 -binned forward-backward asymmetries of $D^0 \rightarrow K^- e^+ \nu_e$, and those of the other three channels are listed in Tables XXXVIII–XL of the Appendix. Figure 8 shows the measured q^2 -binned forward-backward asymmetries, which are found to be in good agreement with the theoretical predictions.

IX. CONSTRAINT ON SCALAR CURRENT IN $c \rightarrow s \ell^+ \nu_\ell$ TRANSITION

As shown in Eq. (34), the presence of potential scalar current in the $c \rightarrow s \ell^+ \nu_\ell$ transition results in a modified

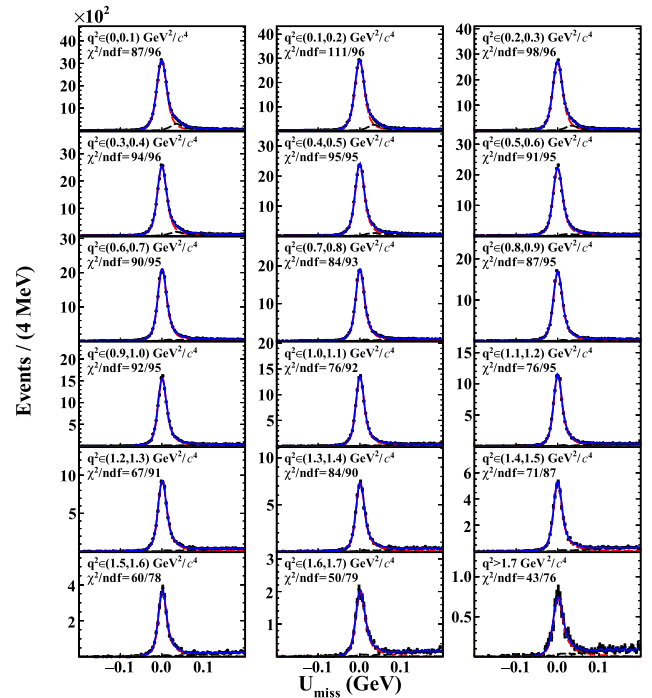


FIG. 7. The U_{miss} distributions of the accepted forward (left) and backward (right) candidate events in different q^2 bins for $D^0 \rightarrow K^- e^+ \nu_e$ in data, with fit results overlaid as the blue solid curves. The red dashed curves are the signal shapes, and the black dashed curves are the fitted combinatorial background shapes.

TABLE XIV. The numbers of observed forward/backward events N_{DT} in different q^2 intervals and the q^2 -binned forward-backward asymmetries of $D^0 \rightarrow K^- e^+ \nu_e$, where the first uncertainties are statistical and the second are systematic.

$q^2(\text{GeV}^2/c^4)$	$N_{\text{DT}}^{\text{Forward}}$	$N_{\text{DT}}^{\text{Backward}}$	A_{FB}
(0.0, 0.1)	27827 ± 178	27482 ± 186	$0.006 \pm 0.005 \pm 0.002$
(0.1, 0.2)	25812 ± 171	25316 ± 178	$0.004 \pm 0.006 \pm 0.002$
(0.2, 0.3)	23978 ± 166	23913 ± 173	$-0.008 \pm 0.006 \pm 0.002$
(0.3, 0.4)	22356 ± 161	21894 ± 163	$0.007 \pm 0.006 \pm 0.002$
(0.4, 0.5)	20925 ± 158	20703 ± 157	$0.004 \pm 0.007 \pm 0.002$
(0.5, 0.6)	19054 ± 151	19473 ± 151	$-0.009 \pm 0.007 \pm 0.002$
(0.6, 0.7)	17350 ± 144	17868 ± 143	$-0.012 \pm 0.007 \pm 0.002$
(0.7, 0.8)	15833 ± 139	15992 ± 135	$0.005 \pm 0.008 \pm 0.002$
(0.8, 0.9)	14155 ± 131	14357 ± 127	$0.006 \pm 0.008 \pm 0.002$
(0.9, 1.0)	12414 ± 121	13074 ± 121	$-0.016 \pm 0.008 \pm 0.002$
(1.0, 1.1)	11160 ± 114	11210 ± 112	$0.015 \pm 0.009 \pm 0.002$
(1.1, 1.2)	9303 ± 105	9593 ± 104	$-0.000 \pm 0.010 \pm 0.002$
(1.2, 1.3)	7846 ± 96	7957 ± 95	$0.008 \pm 0.011 \pm 0.002$
(1.3, 1.4)	6011 ± 84	6233 ± 84	$-0.005 \pm 0.012 \pm 0.003$
(1.4, 1.5)	4381 ± 72	4546 ± 72	$-0.005 \pm 0.014 \pm 0.003$
(1.5, 1.6)	3055 ± 61	3090 ± 60	$0.012 \pm 0.017 \pm 0.004$
(1.6, 1.7)	1727 ± 46	1850 ± 47	$-0.029 \pm 0.023 \pm 0.005$
> 1.7	677 ± 31	732 ± 30	$-0.011 \pm 0.039 \pm 0.008$

$h_i(q^2)$. Hence, a simultaneous fit to the q^2 -binned partial decay rates and forward-backward asymmetries of $D \rightarrow \bar{K} \ell^+ \nu_\ell$ is performed for a stringent constraint on the parameter space of c_s^ℓ . This fit is performed using the least χ^2 method with the objective function constructed as

$$\chi^2 = \chi_{\Delta\Gamma}^2 + \sum \chi_{A_{\text{FB}, D \rightarrow \bar{K} \ell^+ \nu_\ell}}^2. \quad (41)$$

Here, the decay rate part $\chi_{\Delta\Gamma}^2$ is identical to that defined in Sec. VII F. The forward-backward asymmetry part $\sum \chi_{A_{\text{FB}, D \rightarrow \bar{K} \ell^+ \nu_\ell}}^2$ sums over all four decay channels, with the $\chi_{A_{\text{FB}}}^2$ defined as

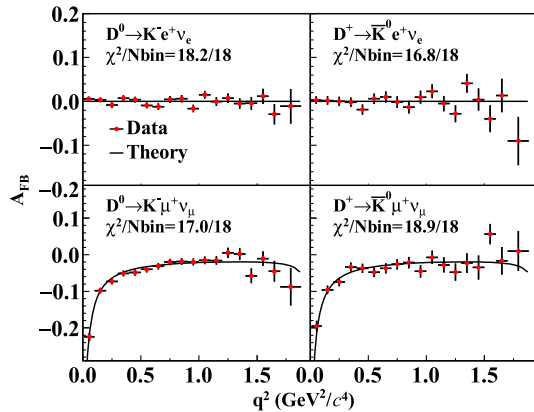


FIG. 8. The measured q^2 -binned forward and backward asymmetries. The red points with error bars are data. The black solid lines denote the theoretical curves.

$$\sum_{i,j} (A_{\text{FB},i}^{\text{msr}} - A_{\text{FB},i}^{\text{fit}}) (C_{\text{FB}}^{-1})_{ij} (A_{\text{FB},j}^{\text{msr}} - A_{\text{FB},j}^{\text{fit}}), \quad (42)$$

where the small correlations between different decay channels due to the correlated systematic uncertainties are neglected.

In this fit, in addition to the parameters $f_+(0)|V_{cs}|$ and $r_1(t_0)$, the real and imaginary parts of the complex coefficient c_s^μ of the muon channels are also treated as free parameters. For the positron channels, the partial decay rate is only sensitive to the modulus $|c_s^e|$, while the forward-backward asymmetry is insensitive to the scalar current contribution. Therefore, only a single parameter $|c_s^e|$ is included in the fit. The different sensitivities of positron and muon channels can be understood as follows. Under the limitation $m_\ell \rightarrow 0$ for the positron channels, the forward-backward asymmetries in Eq. (33) are expected to be always zero and hence cannot constrain the real part of c_s^e . For the decay rates, the $|h_i(q^2)|^2$ in Eq. (4) is proportional to $|c_s^e|^2$, so only the $|c_s^e|$ can be determined from the fit.

First, two different fits are performed with and without the scalar current contribution, with the c_s^ℓ shown in Eq. (34) allowed to float and fixed to zero, respectively. Corresponding fit projections are shown in Fig. 9, and the fitted parameters are summarized in Table XV. No significant deviation from the SM is found in the positron channels with the fitted $|c_s^e|$ consistent with zero. For the muon channels, c_s^μ is found to deviate from zero as shown in Fig. 10(a), especially on its imaginary part.

Since LFU may not hold for the scalar current, the coupling to muons may be significantly larger than that to positrons. Therefore, to estimate the significance of non-zero c_s^μ alone, a third fit is performed in which c_s^μ is float while $|c_s^e| = 0$ is fixed to zero; this fit serves as the alternative hypothesis and yields $\chi^2/\text{ndf} = 135.9/140$. Comparing this with the null hypothesis, which assumes no scalar current and gives $\chi^2/\text{ndf} = 143.5/142$, we obtained $\Delta\chi^2 = 7.6$ and $\Delta\text{ndf} = 2$, corresponding to a significance of 2.3σ based on the Wilks theorem.

Further constraints on the right- and left-handed components of the scalar current are obtained by combining

TABLE XV. Fitted parameters and overall fit qualities of fits with and without scalar current contribution shown in Fig. 9, where the first uncertainties are statistical and the second are systematic. The symbol “...” indicates “not applicable.”

Variable	With SC	Without SC
$f_+(0) V_{cs} $	$0.7193 \pm 0.0008 \pm 0.0014$	$0.7183 \pm 0.0007 \pm 0.0014$
r_1	$-2.23 \pm 0.04 \pm 0.02$	$-2.28 \pm 0.02 \pm 0.02$
$ c_s^e $	$0.02 \pm 0.02 \pm 0.02$	...
$\text{Re}(c_s^\mu)$	$0.017 \pm 0.008 \pm 0.006$	...
$\text{Im}(c_s^\mu)$	$\pm(0.077 \pm 0.011 \pm 0.009)$	...
χ^2/ndf	135.8/139	143.5/142

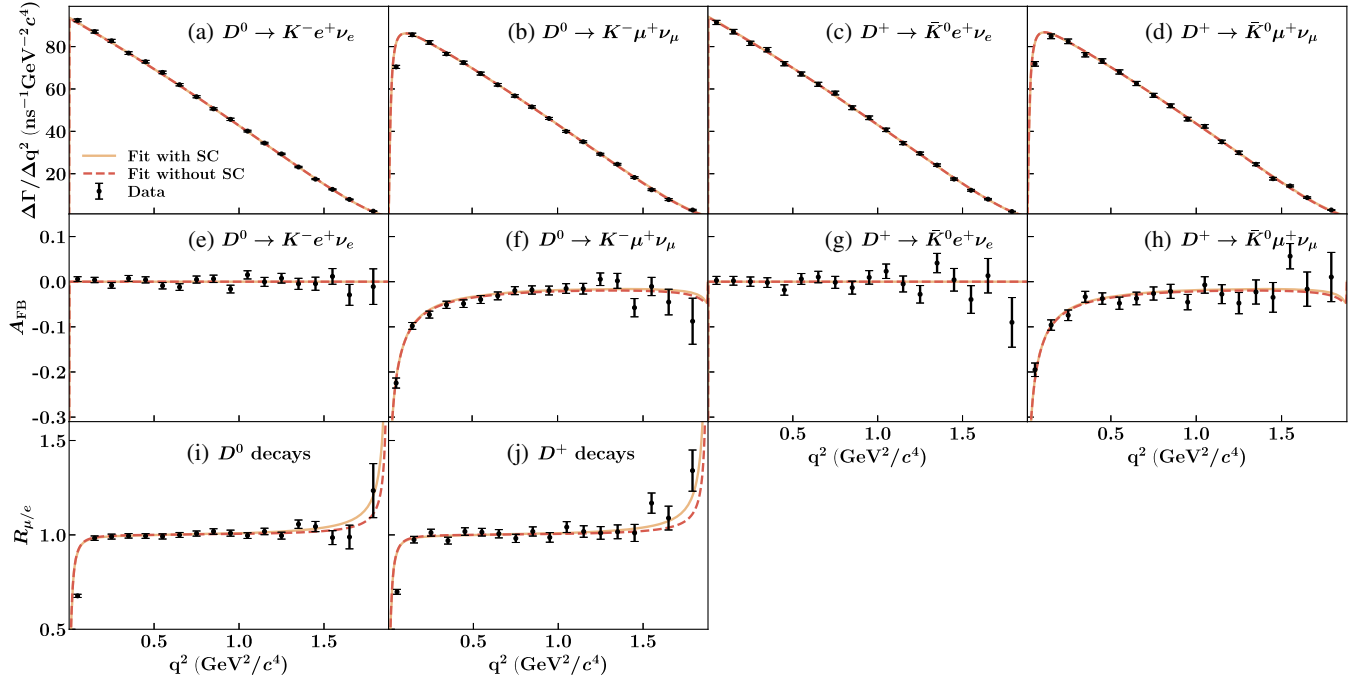


FIG. 9. The simultaneous fit to the measured partial decay rates (a)–(d) and forward-backward asymmetries (e)–(h) of $D \rightarrow \bar{K} \ell^+ \nu_\ell$ and the fit projections on the ratios of differential decay rates $\mathcal{R}_{\mu/\ell}$ (i),(j). The black points with red error bars are data, the solid lines are the fit projections with scalar current (SC) contribution included, and the dashed lines are the fit projections without scalar current contribution. The fit qualities are available in Table XV.

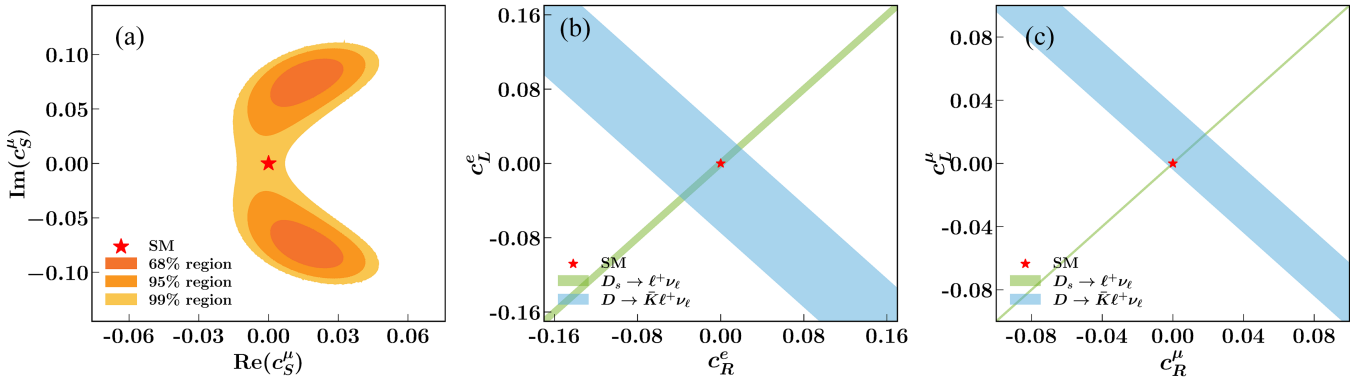


FIG. 10. The confidence regions of (a) the scalar combination of complex Wilson coefficients c_S^μ with probabilities of 68%, 95%, and 99% from $D \rightarrow \bar{K} \ell^+ \nu_\ell$, as well as (b),(c) the right- and left-handed real Wilson coefficients c_R^ℓ and c_L^ℓ with probabilities of 95% by combining with $D_s^+ \rightarrow \ell^+ \nu_\ell$. The red dot indicates the SM value.

with the world average rates of $D_s^+ \rightarrow \ell^+ \nu_\ell$, which are sensitive to the pseudoscalar combination of the Wilson coefficients $c_P^\ell = c_R^\ell - c_L^\ell$ as [26]

$$\mathcal{B}(D_s^+ \rightarrow \ell^+ \nu_\ell) = \tau_{D_s} \frac{m_{D_s}}{8\pi} f_{D_s}^2 \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2 G_F^2 \times |V_{cs}|^2 m_\ell^2 \left|1 - c_P^\ell \frac{m_{D_s}^2}{(m_c + m_s)m_\ell}\right|^2. \quad (43)$$

Here, the measured branching fractions $\mathcal{B}(D_s^+ \rightarrow \ell^+ \nu_\ell)$ are taken from the PDG [61], and the inputs of $f_{D_s} = (249.9 \pm 0.5)$ MeV from LQCD calculations [69] and $|V_{cs}| = 0.97349 \pm 0.00016$ from the global SM fit [61] are used. Since the $D_s^+ \rightarrow \ell^+ \nu_\ell$ decay rates cannot constrain the real and imaginary parts of c_P^ℓ simultaneously, the Wilson coefficient $c_{R(L)}^\ell$ is assumed to be real with T-parity conserved here. As shown in Figs. 10(b) and 10(c), both the right- and left-handed Wilson coefficients c_R^ℓ and c_L^ℓ are consistent with zero.

X. SUMMARY

In summary, using e^+e^- collision data corresponding to an integrated luminosity of 20.3 fb^{-1} , collected at $\sqrt{s} = 3.773 \text{ GeV}$ with the BESIII detector, we perform precise measurements of the semileptonic decays $D \rightarrow \bar{K} \ell^+ \nu_\ell$. The absolute branching fractions of $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ are determined to be $(3.548 \pm 0.006_{\text{stat}} \pm 0.017_{\text{syst}})\%$, $(3.445 \pm 0.007_{\text{stat}} \pm 0.017_{\text{syst}})\%$, $(8.928 \pm 0.025_{\text{stat}} \pm 0.050_{\text{syst}})\%$, and $(8.770 \pm 0.029_{\text{stat}} \pm 0.053_{\text{syst}})\%$, respectively. The ratios of the BF's between semimuonic and semielectronic D decays are determined to be $\mathcal{R}_{\mu/e}^{D^0} = 0.971 \pm 0.003_{\text{stat}} \pm 0.004_{\text{syst}}$ and $\mathcal{R}_{\mu/e}^{D^+} = 0.982 \pm 0.004_{\text{stat}} \pm 0.002_{\text{syst}}$, which exhibit a slight difference with the LFU prediction of 0.975 ± 0.001 [70] up to 1.5σ .

Based on the simultaneous fit to the partial decay rates of $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, the product of the hadronic form factor $f_+(0)$ and the modulus of the CKM matrix element $|V_{cs}|$ is determined to be $f_+(0)|V_{cs}| = 0.7183 \pm 0.0007_{\text{stat}} \pm 0.0014_{\text{syst}}$. Taking the value of $|V_{cs}| = 0.97349 \pm 0.00016$ given by the PDG [61] as input, the hadronic form factor is obtained to be $f_+(0) = 0.7383 \pm 0.0007_{\text{stat}} \pm 0.0014_{\text{syst}}$. A comparison of $f_+(0)$ values measured in different experiments and calculated in various theoretical approaches is shown in Fig. 11. This work yields improved precision compared to the previous BESIII measurements [39–41,43,45], providing an important benchmark for testing different theoretical calculations. Conversely, taking the LQCD determination $f_+(0) = 0.7452 \pm 0.0031$ [4], the CKM matrix

element is determined to be $|V_{cs}| = 0.9639 \pm 0.0009_{\text{stat}} \pm 0.0018_{\text{syst}} \pm 0.0040_{\text{LQCD}}$, which enables a stringent test of CKM unitarity via direct measurement.

Furthermore, the overall and q^2 -binned angular observables of forward-backward asymmetries A_{FB} are measured for $D \rightarrow \bar{K} \ell^+ \nu_\ell$ for the first time. Through a simultaneous fit to the q^2 -binned partial decay rates and forward-backward asymmetries, the scalar current contribution in the $c \rightarrow s \ell^+ \nu_\ell$ transition is searched for. The first experimental constraint on the scalar combination of complex Wilson coefficients is obtained to be $\text{Re}(c_S^\mu) = 0.017 \pm 0.008_{\text{stat}} \pm 0.006_{\text{syst}}$ and $\text{Im}(c_S^\mu) = \pm(0.077 \pm 0.011_{\text{stat}} \pm 0.009_{\text{syst}})$, corresponding to a deviation from the SM with a significance of 2.3σ . This result provides an important constraint on the scalar current contribution in the semileptonic $D^{0(+)}$ decays, offers new insights into the anomalies observed in the beauty sector [10–19], and strengthens the search for new scalar bosons beyond the SM in various new-physics scenarios.

ACKNOWLEDGMENTS

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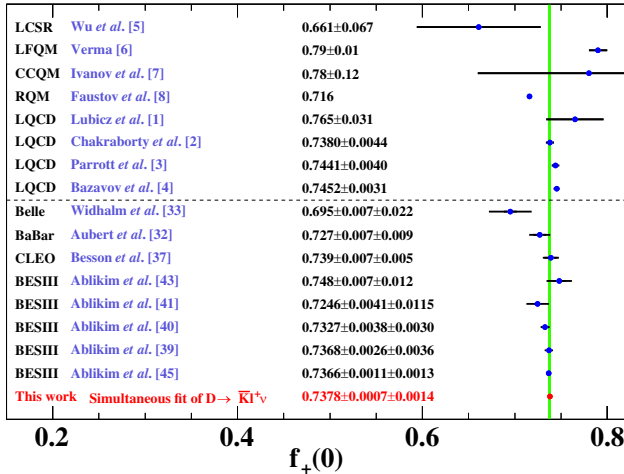


FIG. 11. Comparison of the form factor $f_+(0)$ measured in this work with the theoretical and experimental calculations. The first and second uncertainties are statistical and systematic, respectively. The green band corresponds to the $\pm 1\sigma$ limit of the form factor calculated in this work.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: FURTHER TABLES AND PLOTS

Tables XVI–XIX report the elements of the weighted efficiency matrices for $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, respectively.

Figures 12–14 show the results of the fits to the U_{miss} distributions in the reconstructed q^2 intervals for $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, respectively.

TABLE XVI. The weighted efficiency matrix ε_{ij} (in percent) for $D^0 \rightarrow K^- e^+ \nu_e$, where the indices i and j denote the reconstructed and the produced q^2 intervals, respectively.

ε_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	67.76	4.08	0.33	0.12	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	2.60	62.57	5.03	0.41	0.14	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.09	3.26	60.63	5.45	0.43	0.12	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.03	0.13	3.65	59.30	5.68	0.44	0.12	0.03	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.01	0.04	0.16	3.89	58.72	5.75	0.46	0.11	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.01	0.02	0.06	0.19	4.08	58.14	5.70	0.44	0.10	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00
7	0.01	0.01	0.03	0.06	0.22	4.26	57.89	5.67	0.42	0.10	0.04	0.02	0.01	0.00	0.00	0.00	0.00	0.00
8	0.01	0.01	0.01	0.03	0.07	0.24	4.27	57.51	5.64	0.40	0.11	0.04	0.02	0.01	0.00	0.00	0.00	0.00
9	0.00	0.01	0.01	0.02	0.03	0.08	0.26	4.26	57.38	5.39	0.40	0.10	0.04	0.01	0.00	0.00	0.00	0.00
10	0.00	0.00	0.01	0.01	0.02	0.03	0.08	0.27	4.30	57.00	5.24	0.37	0.09	0.03	0.01	0.01	0.00	0.00
11	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.08	0.28	4.18	57.04	5.09	0.35	0.09	0.03	0.01	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.09	0.31	4.06	56.46	4.79	0.32	0.09	0.03	0.00	0.00
13	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.04	0.08	0.31	3.95	55.87	4.51	0.28	0.06	0.01	0.00
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.03	0.08	0.32	3.72	54.89	4.19	0.21	0.03	0.01
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.07	0.30	3.51	53.46	3.86	0.18	0.02
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.06	0.24	3.12	51.56	3.48	0.10
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.16	2.58	48.40	2.76
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	1.84	37.29

TABLE XVII. The weighted efficiency matrix for $D^0 \rightarrow K^- \mu^+ \nu_\mu$, defined as in Table XVI.

ε_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	44.06	1.39	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	1.86	43.59	2.13	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.06	2.05	45.33	2.72	0.08	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.02	0.08	2.58	47.81	3.15	0.11	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.01	0.03	0.11	3.09	50.24	3.51	0.15	0.05	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.01	0.01	0.04	0.14	3.44	52.79	3.76	0.18	0.06	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.01	0.01	0.02	0.05	0.17	3.80	55.00	4.00	0.21	0.07	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.01	0.02	0.06	0.21	4.03	56.58	4.05	0.22	0.08	0.04	0.02	0.01	0.00	0.00	0.00	0.00
9	0.00	0.00	0.01	0.01	0.03	0.07	0.23	4.13	57.57	4.13	0.23	0.08	0.03	0.01	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.01	0.01	0.03	0.08	0.25	4.17	57.95	4.00	0.26	0.08	0.03	0.01	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.08	0.27	4.11	57.81	3.83	0.25	0.09	0.03	0.01	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.09	0.29	3.91	57.46	3.75	0.24	0.07	0.02	0.00	0.00
13	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.03	0.08	0.27	3.77	56.58	3.49	0.23	0.06	0.01	0.00
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.29	3.56	56.00	3.33	0.18	0.03	0.01
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.07	0.28	3.35	54.79	3.11	0.13	0.02
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.05	0.20	2.95	52.39	2.85	0.07
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.16	2.45	48.92	2.11
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.06	1.77	37.01

TABLE XVIII. The weighted efficiency matrix for $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, defined as in Table XVI.

ε_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	48.49	2.69	0.19	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	1.49	44.44	3.34	0.22	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.02	1.85	42.32	3.62	0.21	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.03	2.04	41.02	3.69	0.21	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.01	0.05	2.16	39.99	3.72	0.19	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.01	0.06	2.24	39.11	3.68	0.18	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.01	0.06	2.30	38.44	3.64	0.15	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.01	0.06	2.35	37.80	3.51	0.14	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.01	0.07	2.35	37.26	3.38	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.07	2.33	36.87	3.18	0.09	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	2.28	36.43	3.02	0.08	0.00	0.00	0.00	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	2.26	36.15	2.88	0.06	0.00	0.00	0.00	0.00
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	2.19	35.78	2.68	0.05	0.00	0.00	0.00
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	2.12	35.31	2.50	0.03	0.00	0.00
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	1.93	35.06	2.33	0.02	0.00
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	1.78	34.70	2.13	0.02
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.06	1.55	34.27	1.70
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.05	1.24	32.54

TABLE XIX. The weighted efficiency matrix for $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, defined as in Table XVI.

ε_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	33.03	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	1.15	32.04	1.45	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
3	0.01	1.24	32.64	1.80	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01
4	0.00	0.02	1.52	33.71	2.06	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01
5	0.00	0.00	0.02	1.76	34.89	2.28	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.01
6	0.00	0.00	0.00	0.04	1.92	35.93	2.43	0.03	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.01	0.01
7	0.00	0.00	0.00	0.01	0.04	2.10	36.77	2.48	0.03	0.01	0.00	0.00	0.01	0.01	0.01	0.01	0.02	0.01
8	0.00	0.00	0.00	0.00	0.01	0.05	2.19	37.18	2.52	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
9	0.00	0.00	0.00	0.00	0.00	0.01	0.06	2.25	37.28	2.46	0.03	0.01	0.01	0.00	0.01	0.01	0.01	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.07	2.26	36.87	2.40	0.03	0.01	0.01	0.01	0.01	0.01	0.00
11	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.07	2.20	36.54	2.28	0.03	0.01	0.01	0.01	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.02	0.08	2.14	36.33	2.19	0.02	0.01	0.01	0.00	0.00
13	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.02	0.07	2.08	35.81	2.05	0.02	0.00	0.00	0.00
14	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.02	0.07	1.98	35.69	1.90	0.00	0.00	0.00
15	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.02	0.07	1.84	35.09	1.88	0.01	0.00
16	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.07	1.70	35.08	1.68	0.00
17	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.05	1.46	34.20	1.28
18	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	1.17	32.74

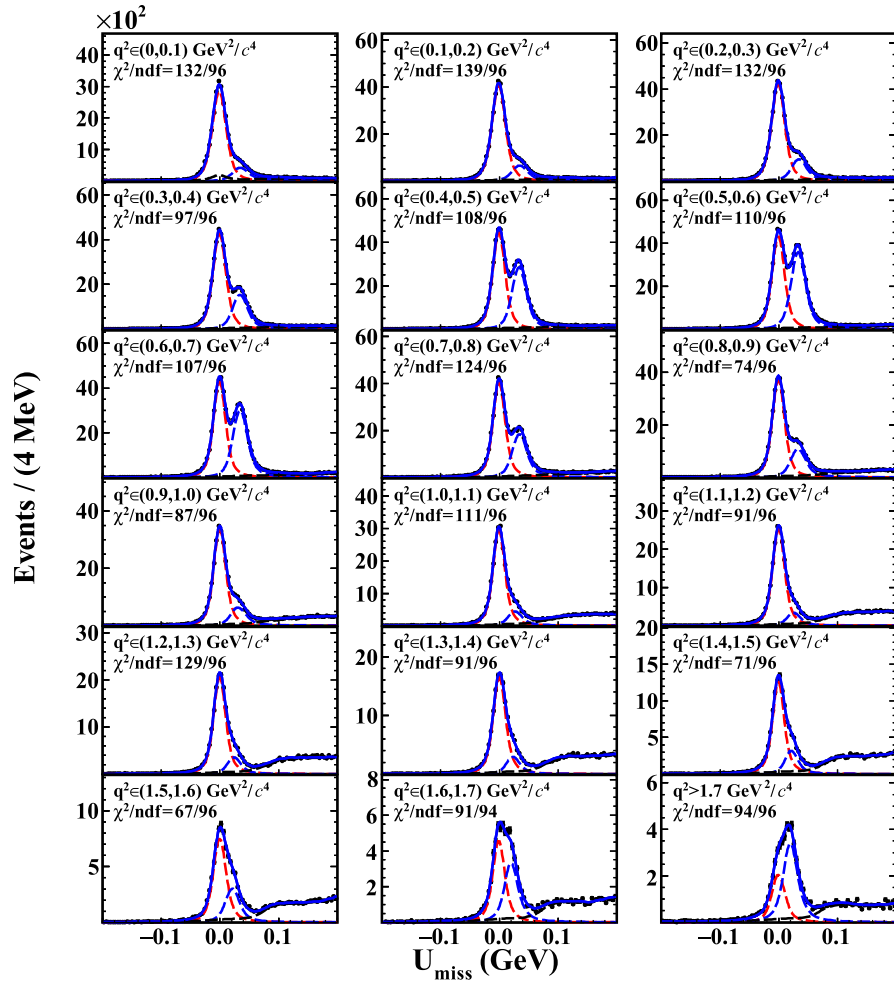


FIG. 12. The U_{miss} distributions of the accepted candidate events for $D^0 \rightarrow K^- \mu^+ \nu_\mu$ in individual q^2 intervals in data, with fit results overlaid. The points with error bars are data. The blue solid curves are the fit results. The violet dotted curves are the signal shapes. The black dash-dotted curves are the peaking backgrounds. The red dashed curves are the fitted combinatorial background shapes.

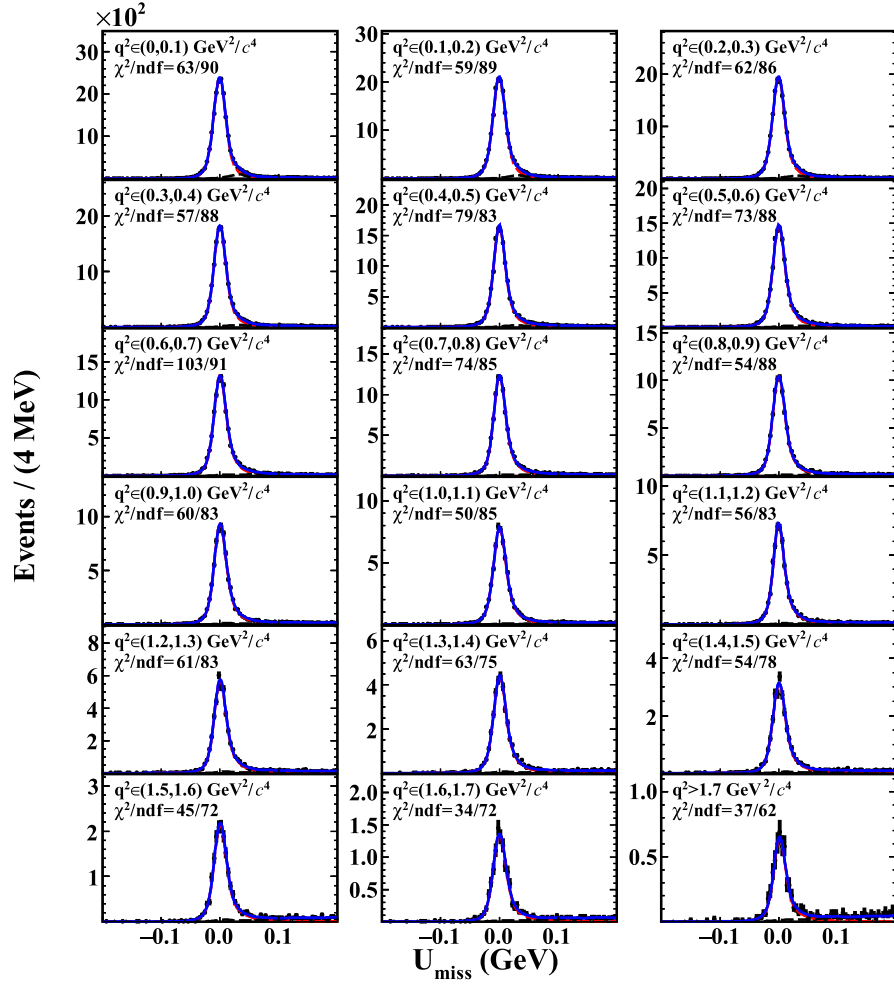


FIG. 13. The U_{miss} distributions of the accepted candidate events for $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ in individual q^2 intervals in data. The curves and symbols are defined the same as in Fig. 12.

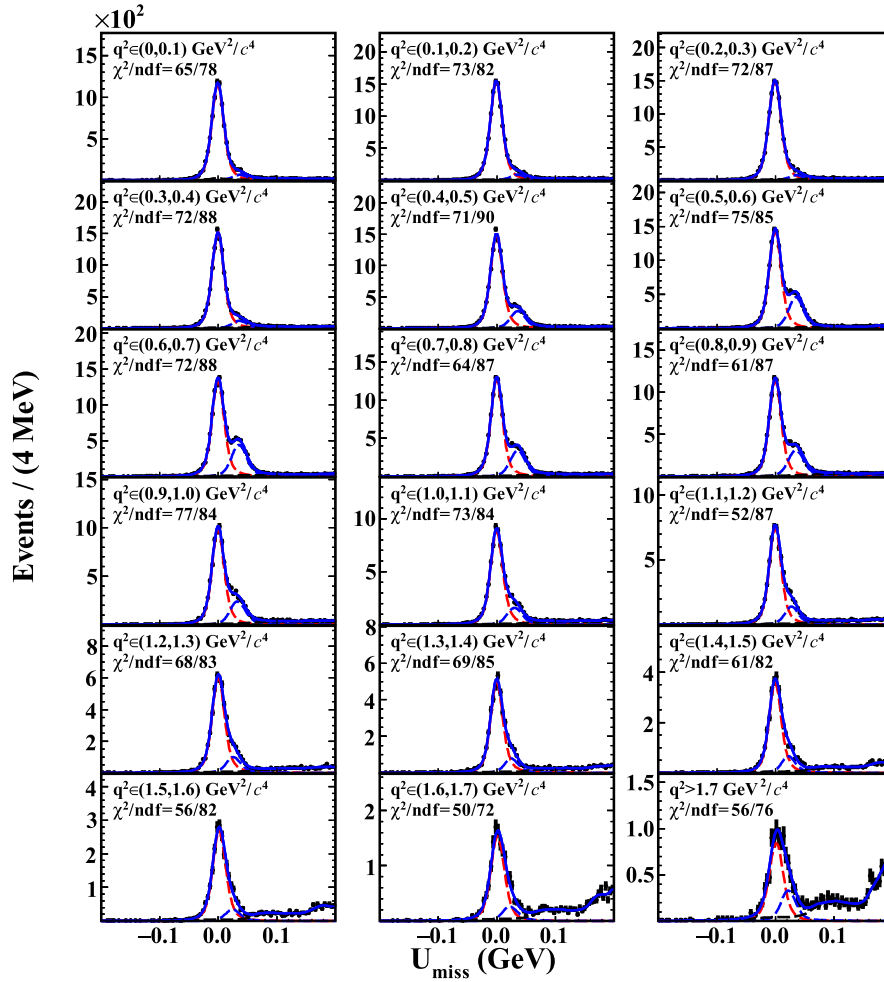


FIG. 14. The U_{miss} distributions of the accepted candidate events for $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ in individual q^2 intervals in data. The curves and symbols are defined the same as in Fig. 12.

Tables XX–XXII list the q^2 ranges, the fitted DT yields in data (N_{DT}), and the numbers of produced events (N_{prd}) calculated by the weighted efficiency matrices and the decay rates ($\Delta\Gamma$) of $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ in individual q^2 intervals.

Tables XXIII–XXVI give the elements of the statistical covariance matrices for $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, respectively.

Tables XXVII–XXIX summarize the systematic uncertainties σ_{syst} of $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ in different q^2 intervals.

Tables XXX–XXXIII present the elements of the systematic covariance matrices for $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, respectively.

Tables XXXIV–XXXVII provide the elements of the covariance matrix ρ_{ij} ($i, j \in [1, 72]$) for the simultaneous fit.

TABLE XX. The fitted DT yields (N_{DT}^i), the produced yields (N_{prd}^i), and the determined partial decay rates ($\Delta\Gamma$) of $D^0 \rightarrow K^-\mu^+\nu_\mu$ in different q^2 intervals of data, where the uncertainties are statistical only.

q^2 (GeV ² /c ⁴)	N_{DT}^i	N_{prd}^i	$\Delta\Gamma$ (ns ⁻¹)
(0.0, 0.1)	23870 ± 183	51906 ± 417	6.255 ± 0.050
(0.1, 0.2)	33402 ± 209	71004 ± 481	8.556 ± 0.058
(0.2, 0.3)	34109 ± 214	68012 ± 475	8.195 ± 0.057
(0.3, 0.4)	34171 ± 220	63532 ± 464	7.655 ± 0.056
(0.4, 0.5)	34333 ± 230	60112 ± 463	7.243 ± 0.056
(0.5, 0.6)	33721 ± 236	55816 ± 453	6.726 ± 0.055
(0.6, 0.7)	32588 ± 224	51439 ± 413	6.198 ± 0.050
(0.7, 0.8)	30747 ± 212	47083 ± 380	5.673 ± 0.046
(0.8, 0.9)	28447 ± 202	42776 ± 356	5.154 ± 0.043
(0.9, 1.0)	25562 ± 193	38241 ± 338	4.608 ± 0.041
(1.0, 1.1)	22134 ± 186	33171 ± 327	3.997 ± 0.039
(1.1, 1.2)	19180 ± 182	29121 ± 320	3.509 ± 0.039
(1.2, 1.3)	15714 ± 174	24253 ± 312	2.922 ± 0.038
(1.3, 1.4)	12893 ± 174	20318 ± 314	2.448 ± 0.038
(1.4, 1.5)	9396 ± 154	15124 ± 284	1.822 ± 0.034
(1.5, 1.6)	6102 ± 99	10320 ± 190	1.243 ± 0.023
(1.6, 1.7)	3540 ± 85	6467 ± 176	0.779 ± 0.021
>1.7	1710 ± 71	4285 ± 194	0.516 ± 0.023

TABLE XXI. Same as Table XX, but for $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$.

q^2 (GeV ² /c ⁴)	N_{DT}^i	N_{prd}^i	$\Delta\Gamma$ (ns ⁻¹)
(0.0, 0.1)	17837 ± 144	100514 ± 859	9.139 ± 0.078
(0.1, 0.2)	16363 ± 138	95738 ± 906	8.705 ± 0.082
(0.2, 0.3)	14902 ± 131	89666 ± 906	8.153 ± 0.082
(0.3, 0.4)	13977 ± 127	86390 ± 906	7.855 ± 0.082
(0.4, 0.5)	12609 ± 120	79104 ± 879	7.192 ± 0.080
(0.5, 0.6)	11532 ± 114	73772 ± 857	6.708 ± 0.078
(0.6, 0.7)	10540 ± 109	68410 ± 835	6.220 ± 0.076
(0.7, 0.8)	9637 ± 104	63872 ± 810	5.807 ± 0.074
(0.8, 0.9)	8403 ± 97	56229 ± 764	5.113 ± 0.069
(0.9, 1.0)	7485 ± 91	51026 ± 726	4.639 ± 0.066
(1.0, 1.1)	6466 ± 85	44755 ± 681	4.069 ± 0.062
(1.1, 1.2)	5432 ± 78	37865 ± 630	3.443 ± 0.057
(1.2, 1.3)	4577 ± 71	32532 ± 585	2.958 ± 0.053
(1.3, 1.4)	3652 ± 64	26470 ± 527	2.407 ± 0.048
(1.4, 1.5)	2626 ± 54	19210 ± 450	1.747 ± 0.041
(1.5, 1.6)	1801 ± 45	13404 ± 378	1.219 ± 0.034
(1.6, 1.7)	1146 ± 36	8800 ± 305	0.800 ± 0.028
>1.7	528 ± 25	4330 ± 218	0.394 ± 0.020

TABLE XXII. Same as Table XX, but for $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$.

q^2 (GeV ² /c ⁴)	N_{DT}^i	N_{prd}^i	$\Delta\Gamma(\text{ns}^{-1})$
(0.0, 0.1)	8335 ± 97	70192 ± 852	6.382 ± 0.078
(0.1, 0.2)	11083 ± 112	93318 ± 1012	8.485 ± 0.092
(0.2, 0.3)	11176 ± 113	90718 ± 1007	8.248 ± 0.092
(0.3, 0.4)	10832 ± 112	83751 ± 965	7.615 ± 0.088
(0.4, 0.5)	10835 ± 114	80508 ± 958	7.320 ± 0.087
(0.5, 0.6)	10436 ± 117	74820 ± 949	6.803 ± 0.086
(0.6, 0.7)	9864 ± 114	68826 ± 905	6.258 ± 0.082
(0.7, 0.8)	9113 ± 108	62691 ± 850	5.700 ± 0.077
(0.8, 0.9)	8327 ± 105	57250 ± 822	5.205 ± 0.075
(0.9, 1.0)	7283 ± 100	50350 ± 791	4.578 ± 0.072
(1.0, 1.1)	6606 ± 96	46606 ± 772	4.238 ± 0.070
(1.1, 1.2)	5465 ± 93	38543 ± 747	3.504 ± 0.068
(1.2, 1.3)	4561 ± 89	32869 ± 727	2.989 ± 0.066
(1.3, 1.4)	3696 ± 81	26902 ± 661	2.446 ± 0.060
(1.4, 1.5)	2650 ± 72	19408 ± 597	1.765 ± 0.054
(1.5, 1.6)	2092 ± 51	15661 ± 420	1.424 ± 0.038
(1.6, 1.7)	1253 ± 40	9583 ± 344	0.871 ± 0.031
>1.7	714 ± 34	5804 ± 304	0.528 ± 0.028

TABLE XXIII. The statistical covariance matrix for $D^0 \rightarrow K^- e^+ \nu_e$. The indices i and j denote the q^2 intervals.

ρ_{ij}^{stat}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1.000	-0.102	0.004	-0.002	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	0.000	-0.000	-0.000	-0.000	0.000
2	-0.102	1.000	-0.133	0.007	-0.002	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000
3	0.004	-0.133	1.000	-0.149	0.009	-0.002	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	-0.000	0.000	-0.000
4	-0.002	0.007	-0.149	1.000	-0.159	0.010	-0.002	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000	0.000	0.000	-0.000	0.000
5	-0.000	-0.002	0.009	-0.159	1.000	-0.165	0.011	-0.002	-0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	-0.000	0.000
6	-0.000	-0.000	-0.002	0.010	-0.165	1.000	-0.168	0.011	-0.002	-0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	-0.000	0.000
7	-0.000	-0.000	-0.000	-0.002	0.011	-0.168	1.000	-0.168	0.011	-0.002	-0.001	-0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.000
8	-0.000	-0.000	-0.000	-0.000	-0.002	0.011	-0.168	1.000	-0.168	0.011	-0.003	-0.001	-0.000	-0.000	0.000	0.000	0.000	-0.000
9	0.000	-0.000	-0.000	-0.000	-0.000	-0.002	0.011	-0.168	1.000	-0.166	0.010	-0.002	-0.001	-0.000	0.000	0.000	-0.000	0.000
10	-0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.002	0.011	-0.166	1.000	-0.162	0.008	-0.002	-0.001	-0.000	-0.000	0.000	-0.000
11	0.000	-0.000	0.000	-0.000	-0.000	-0.000	-0.001	-0.003	0.010	-0.162	1.000	-0.158	0.007	-0.002	-0.000	-0.000	0.000	-0.000
12	0.000	0.000	-0.000	0.000	0.000	-0.000	-0.000	-0.001	-0.002	0.008	-0.158	1.000	-0.152	0.006	-0.002	-0.000	0.000	-0.000
13	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	-0.000	-0.001	-0.002	0.007	-0.152	1.000	-0.146	0.005	-0.002	-0.000	0.000
14	0.000	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	-0.000	-0.001	-0.002	0.006	-0.146	1.000	-0.140	0.005	-0.001	0.000
15	-0.000	-0.000	0.000	0.000	0.000	0.000	-0.000	0.000	0.000	-0.000	-0.000	-0.002	0.005	-0.140	1.000	-0.131	0.005	-0.001
16	-0.000	0.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.002	0.005	-0.131	1.000	-0.119	0.005
17	-0.000	-0.000	0.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	-0.000	-0.001	0.005	-0.119	1.000	-0.106
18	0.000	0.000	-0.000	0.000	0.000	0.000	-0.000	-0.000	0.000	-0.000	-0.000	-0.000	0.000	0.000	-0.001	0.005	-0.106	1.000

TABLE XXIV. Same as Table XXIII, but for $D^0 \rightarrow K^- \mu^+ \nu_\mu$.

ρ_{ij}^{stat}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1.000	-0.073	0.003	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	0.000
2	-0.073	1.000	-0.094	0.005	-0.001	-0.000	-0.000	0.000	0.000	-0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	0.000
3	0.003	-0.094	1.000	-0.113	0.007	-0.001	-0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000
4	-0.000	0.005	-0.113	1.000	-0.127	0.008	-0.002	-0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	-0.000	0.000	-0.000
5	-0.000	-0.001	0.007	-0.127	1.000	-0.134	0.009	-0.002	-0.001	-0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.000
6	-0.000	-0.000	-0.001	0.008	-0.134	1.000	-0.139	0.008	-0.002	-0.001	-0.000	-0.000	0.000	-0.000	0.000	0.000	-0.000	0.000
7	-0.000	-0.000	-0.000	-0.002	0.009	-0.139	1.000	-0.143	0.008	-0.002	-0.000	-0.001	-0.000	0.000	0.000	0.000	-0.000	-0.000
8	0.000	0.000	-0.000	-0.000	-0.002	0.008	-0.143	1.000	-0.142	0.008	-0.002	-0.001	-0.000	-0.000	0.000	0.000	0.000	0.000
9	-0.000	0.000	-0.000	-0.000	-0.001	-0.002	0.008	-0.142	1.000	-0.142	0.007	-0.002	-0.001	-0.000	0.000	-0.000	0.000	0.000
10	0.000	-0.000	0.000	-0.000	-0.000	-0.001	-0.002	0.008	-0.142	1.000	-0.138	0.005	-0.002	-0.001	-0.000	0.000	-0.000	0.000
11	0.000	0.000	-0.000	0.000	-0.000	-0.000	-0.000	-0.002	0.007	-0.138	1.000	-0.133	0.005	-0.002	-0.000	-0.000	0.000	0.000
12	0.000	0.000	0.000	-0.000	0.000	-0.000	-0.001	-0.001	-0.002	0.005	-0.133	1.000	-0.130	0.003	-0.002	-0.000	0.000	-0.000
13	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	-0.000	-0.001	-0.002	0.005	-0.130	1.000	-0.124	0.003	-0.001	-0.000	0.000
14	-0.000	-0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	-0.000	-0.001	-0.002	0.003	-0.124	1.000	-0.120	0.004	-0.001	-0.000
15	-0.000	0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	0.000	-0.000	-0.000	-0.002	0.003	-0.120	1.000	-0.121	0.002	-0.000
16	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	-0.000	-0.001	0.004	-0.121	1.000	-0.103	0.004
17	-0.000	0.000	-0.000	0.000	-0.000	-0.000	-0.000	0.000	0.000	-0.000	0.000	0.000	-0.000	-0.001	0.002	-0.103	1.000	-0.091
18	0.000	0.000	0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	-0.000	0.004	-0.091	1.000

TABLE XXV. Same as Table XXIII, but for $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$.

ρ_{ij}^{stat}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1.000	-0.090	0.004	-0.001	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000
2	-0.090	1.000	-0.118	0.007	-0.002	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000
3	0.004	-0.118	1.000	-0.134	0.009	-0.002	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000
4	-0.001	0.007	-0.134	1.000	-0.142	0.010	-0.001	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000
5	0.000	-0.002	0.009	-0.142	1.000	-0.148	0.011	-0.001	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000
6	0.000	0.000	-0.002	0.010	-0.148	1.000	-0.152	0.012	-0.001	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000
7	-0.000	0.000	0.000	-0.001	0.011	-0.152	1.000	-0.155	0.013	-0.001	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000
8	0.000	0.000	-0.000	0.000	-0.001	0.012	-0.155	1.000	-0.153	0.012	-0.001	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000
9	-0.000	-0.000	0.000	-0.000	0.000	-0.001	0.013	-0.153	1.000	-0.152	0.012	-0.001	0.000	-0.000	0.000	-0.000	0.000	-0.000
10	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001	0.012	-0.152	1.000	-0.147	0.012	-0.001	0.000	-0.000	0.000	-0.000	0.000
11	-0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001	0.012	-0.147	1.000	-0.143	0.011	-0.001	0.000	-0.000	0.000	-0.000
12	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001	0.012	-0.143	1.000	-0.139	0.010	-0.001	0.000	-0.000	0.000
13	-0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001	0.011	-0.139	1.000	-0.133	0.009	-0.001	0.000	-0.000
14	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001	0.010	-0.133	1.000	-0.124	0.007	-0.001	0.000
15	-0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001	0.009	-0.124	1.000	-0.116	0.006	-0.001
16	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001	0.007	-0.116	1.000	-0.105	0.004
17	-0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001	0.006	-0.105	1.000	-0.088
18	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001	0.004	-0.088	1.000

TABLE XXVI. Same as Table XXIII, but for $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$.

ρ_{ij}^{stat}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1.000	-0.065	0.004	-0.000	0.000	-0.000	0.000	-0.000	-0.000	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	0.000
2	-0.065	1.000	-0.083	0.005	-0.000	0.000	-0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.001
3	0.004	-0.083	1.000	-0.100	0.007	-0.000	0.000	-0.000	0.000	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001
4	-0.000	0.005	-0.100	1.000	-0.111	0.008	-0.001	0.000	-0.000	0.000	-0.000	-0.000	-0.000	0.000	0.000	-0.000	-0.001	-0.001
5	0.000	-0.000	0.007	-0.111	1.000	-0.118	0.009	-0.001	0.000	-0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001
6	-0.000	0.000	-0.000	0.008	-0.118	1.000	-0.124	0.010	-0.001	0.000	-0.000	-0.000	-0.000	0.000	-0.000	-0.001	0.000	-0.001
7	0.000	-0.000	0.000	-0.001	0.009	-0.124	1.000	-0.126	0.010	-0.001	-0.000	0.000	-0.001	-0.000	-0.000	-0.000	-0.001	0.000
8	-0.000	0.000	-0.000	0.000	-0.001	0.010	-0.126	1.000	-0.127	0.010	-0.001	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.000
9	-0.000	0.000	0.000	-0.000	0.000	-0.001	0.010	-0.127	1.000	-0.127	0.009	-0.001	-0.000	-0.000	-0.000	-0.000	-0.001	0.000
10	0.000	-0.000	0.000	0.000	-0.000	0.000	-0.001	0.010	-0.127	1.000	-0.125	0.008	-0.001	-0.000	-0.000	-0.000	0.000	0.000
11	0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.000	-0.001	0.009	-0.125	1.000	-0.121	0.008	-0.001	-0.000	-0.001	0.000	-0.000
12	0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.001	0.008	-0.121	1.000	-0.118	0.008	-0.001	-0.000	0.000	0.000
13	0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.001	-0.000	-0.000	-0.001	0.008	-0.118	1.000	-0.113	0.006	-0.001	0.000	-0.000
14	-0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.001	0.008	-0.113	1.000	-0.106	0.006	-0.001	0.000
15	0.000	-0.000	0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	0.006	-0.106	1.000	-0.106	0.004	-0.001
16	-0.000	0.000	-0.000	-0.000	-0.000	-0.001	-0.000	-0.000	-0.000	-0.000	-0.001	-0.000	-0.001	0.006	-0.106	1.000	-0.091	0.003
17	0.000	-0.000	0.000	-0.001	0.000	0.000	-0.001	-0.001	-0.001	0.000	0.000	0.000	-0.001	0.004	-0.091	1.000	-0.073	-0.073
18	0.000	-0.001	-0.001	-0.001	-0.001	-0.001	0.000	-0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.001	0.003	-0.073	1.000

TABLE XXVII. Systematic uncertainties (in percent) of the partial decay rates for $D^0 \rightarrow K^- \mu^+ \nu_\mu$ in individual q^2 intervals.

i th q^2 bin	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
N_{tag}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
D^0 lifetime	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
MC stat.	0.13	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.13	0.14	0.16	0.18	0.20	0.25	0.33	0.54
$E_{\text{extra}\gamma}^{\text{max}}$ cut	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
U_{miss} fit	0.42	0.13	0.12	0.12	0.14	0.17	0.16	0.18	0.18	0.18	0.17	0.19	0.17	0.15	0.18	2.59	5.12	9.89
K tracking	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.14	0.27
K PID	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
μ tracking	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
μ PID	0.12	0.12	0.11	0.11	0.11	0.11	0.10	0.10	0.10	0.11	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.15
MC model	0.12	0.40	0.59	0.07	0.20	0.39	0.24	0.38	0.47	0.27	0.34	0.49	0.44	0.12	0.92	0.61	1.23	1.83
Total	0.64	0.63	0.76	0.48	0.52	0.63	0.55	0.62	0.68	0.56	0.60	0.70	0.67	0.52	1.06	2.71	5.30	10.08

TABLE XXVIII. Same as Table XXVII, but for $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$.

i th q^2 bin	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
N_{tag}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
D^+ lifetime	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48
MC stat.	0.10	0.11	0.11	0.12	0.12	0.13	0.13	0.14	0.15	0.16	0.17	0.18	0.20	0.23	0.26	0.31	0.40	0.58
$E_{\text{extra}\gamma}^{\text{max}}$ cut	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
U_{miss} fit	0.07	0.07	0.06	0.06	0.06	0.06	0.06	0.05	0.07	0.05	0.04	0.06	0.07	0.05	0.08	0.08	0.09	0.09
e^+ tracking	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
e^+ PID	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
K_S^0 reconstruction	0.15	0.17	0.19	0.22	0.24	0.31	0.39	0.46	0.54	0.63	0.75	0.83	0.88	0.95	1.04	1.13	1.17	1.37
π tracking	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.12	0.12	0.13	0.13	0.13	0.14	0.14	0.14	0.15	0.16	0.19
Quoted BF	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
MC model	0.15	0.40	0.27	0.42	0.41	0.21	0.54	0.29	0.66	0.74	0.69	0.47	0.14	0.33	0.87	1.24	1.52	0.54
Total	0.65	0.76	0.70	0.78	0.78	0.73	0.91	0.83	1.06	1.16	1.20	1.15	1.10	1.20	1.51	1.82	2.06	1.71

TABLE XXIX. Same as Table XXVII, but for $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$.

i th q^2 bin	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
N_{tag}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
D^+ lifetime	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48
MC stat.	0.15	0.13	0.13	0.13	0.13	0.13	0.14	0.14	0.15	0.16	0.17	0.18	0.20	0.23	0.26	0.31	0.40	0.55
$E_{\text{extray}}^{\text{max}}$ cut	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
U_{miss} fit	0.04	0.03	0.04	0.05	0.04	0.06	0.06	0.06	0.06	0.07	0.07	0.06	0.09	0.08	0.11	0.84	1.14	2.52
μ^+ tracking	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
μ^+ PID	0.10	0.11	0.11	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.15
K_S^0 reconstruction	0.55	0.17	0.20	0.22	0.24	0.31	0.39	0.46	0.54	0.63	0.75	0.83	0.88	0.95	1.04	1.13	1.18	1.40
π tracking	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.12	0.12	0.13	0.13	0.13	0.14	0.14	0.14	0.15	0.16	0.19
Quoted BF	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
MC model	0.33	0.37	0.30	0.07	0.40	0.16	0.42	0.42	0.10	0.28	0.38	0.21	0.67	0.50	1.11	0.36	1.11	1.22
Total	0.90	0.74	0.72	0.66	0.78	0.72	0.85	0.89	0.84	0.94	1.06	1.07	1.29	1.26	1.67	1.61	2.12	3.25

TABLE XXX. The systematic covariance matrix for $D^0 \rightarrow K^- e^+ \nu_e$. The indices i and j denote the q^2 intervals.

ρ_{ij}^{syst}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1.000	0.764	0.716	0.757	0.770	0.709	0.754	0.775	0.750	0.569	0.806	0.779	0.776	0.674	0.720	0.558	0.608	0.442
2	0.764	1.000	0.776	0.821	0.809	0.798	0.801	0.745	0.819	0.726	0.799	0.803	0.793	0.775	0.692	0.701	0.649	0.567
3	0.716	0.776	1.000	0.807	0.801	0.834	0.799	0.673	0.836	0.830	0.746	0.778	0.762	0.825	0.625	0.793	0.650	0.650
4	0.757	0.821	0.807	1.000	0.789	0.833	0.812	0.716	0.842	0.796	0.782	0.802	0.788	0.816	0.665	0.764	0.660	0.623
5	0.770	0.809	0.801	0.789	1.000	0.770	0.799	0.730	0.817	0.736	0.788	0.795	0.785	0.778	0.679	0.710	0.646	0.575
6	0.709	0.798	0.834	0.833	0.770	1.000	0.768	0.669	0.837	0.839	0.741	0.776	0.760	0.829	0.619	0.801	0.650	0.657
7	0.754	0.801	0.799	0.812	0.799	0.768	1.000	0.677	0.816	0.746	0.774	0.786	0.774	0.780	0.664	0.718	0.641	0.583
8	0.775	0.745	0.673	0.716	0.730	0.669	0.677	1.000	0.675	0.525	0.771	0.742	0.739	0.632	0.692	0.515	0.576	0.407
9	0.750	0.819	0.836	0.842	0.817	0.837	0.816	0.675	1.000	0.795	0.778	0.799	0.786	0.824	0.658	0.777	0.661	0.634
10	0.569	0.726	0.830	0.796	0.736	0.839	0.746	0.525	0.795	1.000	0.603	0.694	0.670	0.850	0.486	0.884	0.611	0.735
11	0.806	0.799	0.746	0.782	0.788	0.741	0.774	0.771	0.778	0.603	1.000	0.761	0.788	0.709	0.717	0.605	0.625	0.483
12	0.779	0.803	0.778	0.802	0.795	0.776	0.786	0.742	0.799	0.694	0.761	1.000	0.749	0.753	0.689	0.671	0.636	0.541
13	0.776	0.793	0.762	0.788	0.785	0.760	0.774	0.739	0.786	0.670	0.788	0.749	1.000	0.705	0.688	0.649	0.627	0.523
14	0.674	0.775	0.825	0.816	0.778	0.829	0.780	0.632	0.824	0.850	0.709	0.753	0.705	1.000	0.552	0.810	0.635	0.666
15	0.720	0.692	0.625	0.665	0.679	0.619	0.664	0.692	0.658	0.486	0.717	0.689	0.688	0.552	1.000	0.446	0.537	0.377
16	0.558	0.701	0.793	0.764	0.710	0.801	0.718	0.515	0.777	0.884	0.605	0.671	0.649	0.810	0.446	1.000	0.554	0.697
17	0.608	0.649	0.650	0.660	0.646	0.650	0.641	0.576	0.661	0.611	0.625	0.636	0.627	0.635	0.537	0.554	1.000	0.438
18	0.442	0.567	0.650	0.623	0.575	0.657	0.583	0.407	0.634	0.735	0.483	0.541	0.523	0.666	0.377	0.697	0.438	1.000

TABLE XXXI. Same as Table XXX, but for $D^0 \rightarrow K^- \mu^+ \nu_\mu$.

ρ_{ij}^{syst}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1.000	0.579	0.542	0.643	0.640	0.587	0.621	0.585	0.557	0.605	0.591	0.546	0.563	0.603	0.440	0.137	0.091	0.062
2	0.579	1.000	0.904	0.733	0.833	0.885	0.843	0.876	0.883	0.838	0.862	0.875	0.874	0.725	0.840	0.232	0.187	0.140
3	0.542	0.904	1.000	0.630	0.781	0.881	0.805	0.870	0.895	0.806	0.847	0.891	0.880	0.653	0.904	0.240	0.205	0.157
4	0.643	0.733	0.630	1.000	0.799	0.721	0.785	0.719	0.672	0.762	0.733	0.657	0.684	0.786	0.494	0.161	0.099	0.065
5	0.640	0.833	0.781	0.799	1.000	0.794	0.834	0.809	0.786	0.816	0.811	0.774	0.788	0.779	0.665	0.197	0.142	0.101
6	0.587	0.885	0.881	0.721	0.794	1.000	0.801	0.853	0.856	0.816	0.838	0.848	0.848	0.711	0.810	0.224	0.180	0.134
7	0.621	0.843	0.805	0.785	0.834	0.801	1.000	0.794	0.803	0.811	0.814	0.791	0.801	0.754	0.704	0.204	0.152	0.110
8	0.585	0.876	0.870	0.719	0.809	0.853	0.794	1.000	0.826	0.812	0.831	0.839	0.839	0.708	0.798	0.221	0.177	0.132
9	0.557	0.883	0.895	0.672	0.786	0.856	0.803	0.826	1.000	0.775	0.830	0.854	0.849	0.673	0.840	0.228	0.189	0.142
10	0.605	0.838	0.806	0.762	0.816	0.816	0.811	0.812	0.775	1.000	0.779	0.789	0.796	0.735	0.711	0.204	0.155	0.112
11	0.591	0.862	0.847	0.733	0.811	0.838	0.814	0.831	0.830	0.779	1.000	0.799	0.825	0.716	0.767	0.215	0.169	0.125
12	0.546	0.875	0.891	0.657	0.774	0.848	0.791	0.839	0.854	0.789	0.799	1.000	0.822	0.661	0.839	0.227	0.189	0.143
13	0.563	0.874	0.880	0.684	0.788	0.848	0.801	0.839	0.849	0.796	0.825	0.822	1.000	0.655	0.819	0.222	0.183	0.137
14	0.603	0.725	0.653	0.786	0.779	0.711	0.754	0.708	0.673	0.735	0.716	0.661	0.655	1.000	0.509	0.166	0.108	0.073
15	0.440	0.840	0.904	0.494	0.665	0.810	0.704	0.798	0.840	0.711	0.767	0.839	0.819	0.509	1.000	0.126	0.215	0.161
16	0.137	0.232	0.240	0.161	0.197	0.224	0.204	0.221	0.228	0.204	0.215	0.227	0.222	0.166	0.126	1.000	-0.048	0.045
17	0.091	0.187	0.205	0.099	0.142	0.180	0.152	0.177	0.189	0.155	0.169	0.189	0.183	0.108	0.215	-0.048	1.000	-0.050
18	0.062	0.140	0.157	0.065	0.101	0.134	0.110	0.132	0.142	0.112	0.125	0.143	0.137	0.073	0.161	0.045	-0.050	1.000

TABLE XXXII. Same as Table XXX, but for $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$.

ρ_{ij}^{syst}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1.000	0.749	0.774	0.745	0.739	0.742	0.697	0.706	0.653	0.629	0.616	0.606	0.570	0.558	0.528	0.486	0.445	0.371
2	0.749	1.000	0.758	0.804	0.791	0.730	0.777	0.717	0.752	0.737	0.709	0.653	0.544	0.572	0.628	0.619	0.592	0.403
3	0.774	0.758	1.000	0.737	0.761	0.731	0.737	0.713	0.707	0.690	0.672	0.641	0.569	0.578	0.591	0.566	0.531	0.401
4	0.745	0.804	0.737	1.000	0.748	0.723	0.773	0.715	0.754	0.742	0.717	0.664	0.557	0.586	0.641	0.633	0.605	0.418
5	0.739	0.791	0.761	0.748	1.000	0.680	0.770	0.716	0.750	0.739	0.718	0.669	0.568	0.595	0.644	0.632	0.603	0.425
6	0.742	0.730	0.731	0.723	0.680	1.000	0.671	0.720	0.687	0.675	0.672	0.665	0.623	0.620	0.602	0.564	0.520	0.438
7	0.697	0.777	0.737	0.773	0.770	0.671	1.000	0.693	0.790	0.785	0.769	0.718	0.606	0.644	0.708	0.701	0.670	0.477
8	0.706	0.717	0.713	0.715	0.716	0.720	0.693	1.000	0.691	0.725	0.725	0.718	0.669	0.675	0.667	0.632	0.586	0.492
9	0.653	0.752	0.707	0.754	0.750	0.687	0.790	0.691	1.000	0.775	0.795	0.741	0.624	0.668	0.742	0.740	0.709	0.506
10	0.629	0.737	0.690	0.742	0.739	0.675	0.785	0.725	0.775	1.000	0.773	0.752	0.630	0.677	0.756	0.755	0.724	0.518
11	0.616	0.709	0.672	0.717	0.718	0.672	0.769	0.725	0.795	0.773	1.000	0.731	0.663	0.700	0.760	0.752	0.715	0.537
12	0.606	0.653	0.641	0.664	0.669	0.665	0.718	0.718	0.741	0.752	0.731	1.000	0.660	0.719	0.726	0.699	0.653	0.545
13	0.570	0.544	0.569	0.557	0.568	0.623	0.606	0.669	0.624	0.630	0.663	0.660	1.000	0.652	0.633	0.576	0.519	0.518
14	0.558	0.572	0.578	0.586	0.595	0.620	0.644	0.675	0.668	0.677	0.700	0.719	0.652	1.000	0.637	0.635	0.581	0.529
15	0.528	0.628	0.591	0.641	0.644	0.602	0.708	0.667	0.742	0.756	0.760	0.726	0.633	0.637	1.000	0.696	0.694	0.525
16	0.486	0.619	0.566	0.633	0.632	0.564	0.701	0.632	0.740	0.755	0.752	0.699	0.576	0.635	0.696	1.000	0.680	0.503
17	0.445	0.592	0.531	0.605	0.603	0.520	0.670	0.586	0.709	0.724	0.715	0.653	0.519	0.581	0.694	0.680	1.000	0.427
18	0.371	0.403	0.401	0.418	0.425	0.438	0.477	0.492	0.506	0.518	0.537	0.545	0.518	0.529	0.525	0.503	0.427	1.000

TABLE XXXIII. Same as Table XXX, but for $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$.

ρ_{ij}^{syst}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1.000	0.688	0.708	0.661	0.725	0.717	0.762	0.770	0.730	0.762	0.770	0.738	0.754	0.736	0.708	0.586	0.555	0.410
2	0.688	1.000	0.727	0.664	0.756	0.692	0.738	0.724	0.618	0.650	0.643	0.578	0.639	0.593	0.615	0.437	0.468	0.335
3	0.708	0.727	1.000	0.649	0.744	0.701	0.729	0.718	0.640	0.660	0.650	0.597	0.632	0.597	0.595	0.449	0.455	0.326
4	0.661	0.664	0.649	1.000	0.624	0.694	0.650	0.643	0.661	0.634	0.608	0.597	0.545	0.545	0.466	0.434	0.361	0.259
5	0.725	0.756	0.744	0.624	1.000	0.663	0.753	0.741	0.637	0.677	0.675	0.612	0.675	0.630	0.652	0.472	0.500	0.361
6	0.717	0.692	0.701	0.694	0.663	1.000	0.671	0.703	0.691	0.688	0.674	0.652	0.630	0.621	0.565	0.492	0.440	0.319
7	0.762	0.738	0.729	0.650	0.753	0.671	1.000	0.736	0.680	0.722	0.728	0.674	0.728	0.691	0.700	0.531	0.542	0.396
8	0.770	0.724	0.718	0.643	0.741	0.703	0.736	1.000	0.657	0.736	0.743	0.694	0.742	0.709	0.713	0.551	0.555	0.407
9	0.730	0.618	0.640	0.661	0.637	0.691	0.680	0.657	1.000	0.685	0.715	0.715	0.660	0.671	0.582	0.555	0.462	0.342
10	0.762	0.650	0.660	0.634	0.677	0.688	0.722	0.736	0.685	1.000	0.720	0.734	0.728	0.720	0.675	0.583	0.533	0.396
11	0.770	0.643	0.650	0.608	0.675	0.674	0.728	0.743	0.715	0.720	1.000	0.712	0.756	0.742	0.712	0.600	0.563	0.419
12	0.738	0.578	0.597	0.597	0.612	0.652	0.674	0.694	0.715	0.734	0.712	1.000	0.681	0.719	0.646	0.595	0.517	0.387
13	0.754	0.639	0.632	0.545	0.675	0.630	0.728	0.742	0.660	0.728	0.756	0.681	1.000	0.716	0.762	0.588	0.598	0.445
14	0.736	0.593	0.597	0.545	0.630	0.621	0.691	0.709	0.671	0.720	0.742	0.719	0.716	1.000	0.685	0.592	0.563	0.422
15	0.708	0.615	0.595	0.466	0.652	0.565	0.700	0.713	0.582	0.675	0.712	0.646	0.762	0.685	1.000	0.514	0.608	0.450
16	0.586	0.437	0.449	0.434	0.472	0.492	0.531	0.551	0.555	0.583	0.600	0.595	0.588	0.592	0.514	1.000	0.393	0.333
17	0.555	0.468	0.455	0.361	0.500	0.440	0.542	0.555	0.462	0.533	0.563	0.517	0.598	0.563	0.608	0.393	1.000	0.309
18	0.410	0.335	0.326	0.259	0.361	0.319	0.396	0.407	0.342	0.396	0.419	0.387	0.445	0.422	0.450	0.333	0.309	1.000

TABLE XXXIV. Elements of the covariance matrix ρ_{ij} ($i, j \in [1, 36]$) for the simultaneous fit.

ρ_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	1.000	0.331	0.376	0.369	0.361	0.349	0.341	0.320	0.327	0.303	0.296	0.282	0.264	0.249	0.200	0.199	0.145	0.112	0.038	0.034	0.033	0.031	0.030	0.029	0.027	0.027	0.023	0.022	0.021	0.020	0.019	0.017	0.014	0.012	0.010	0.008
2	0.331	1.000	0.335	0.400	0.374	0.388	0.357	0.303	0.352	0.381	0.290	0.287	0.267	0.282	0.190	0.246	0.153	0.142	0.034	0.031	0.030	0.028	0.027	0.024	0.024	0.021	0.020	0.019	0.018	0.017	0.016	0.013	0.011	0.009	0.007	
3	0.376	0.335	1.000	0.333	0.393	0.423	0.372	0.286	0.376	0.455	0.283	0.291	0.268	0.314	0.179	0.292	0.160	0.170	0.031	0.028	0.027	0.026	0.024	0.022	0.022	0.019	0.018	0.017	0.016	0.016	0.014	0.012	0.010	0.008	0.006	
4	0.369	0.400	0.333	1.000	0.271	0.403	0.353	0.285	0.355	0.409	0.278	0.281	0.260	0.291	0.179	0.263	0.152	0.153	0.032	0.028	0.028	0.026	0.025	0.022	0.022	0.020	0.019	0.018	0.017	0.016	0.014	0.012	0.010	0.008	0.006	
5	0.361	0.374	0.393	0.271	1.000	0.263	0.342	0.278	0.331	0.363	0.269	0.268	0.248	0.267	0.175	0.235	0.143	0.136	0.032	0.028	0.028	0.026	0.025	0.022	0.022	0.020	0.020	0.018	0.017	0.016	0.014	0.012	0.010	0.008	0.006	
6	0.349	0.388	0.423	0.403	0.263	1.000	0.245	0.275	0.355	0.435	0.265	0.274	0.253	0.299	0.168	0.278	0.151	0.163	0.029	0.026	0.025	0.024	0.023	0.020	0.020	0.018	0.017	0.016	0.015	0.015	0.013	0.011	0.009	0.008	0.006	
7	0.341	0.357	0.372	0.353	0.342	0.245	1.000	0.143	0.325	0.353	0.254	0.254	0.236	0.257	0.165	0.229	0.137	0.132	0.030	0.026	0.026	0.024	0.023	0.021	0.021	0.018	0.017	0.016	0.016	0.015	0.014	0.011	0.009	0.008	0.006	
8	0.320	0.303	0.286	0.285	0.278	0.275	0.143	1.000	0.132	0.234	0.229	0.219	0.206	0.190	0.157	0.150	0.112	0.084	0.030	0.026	0.026	0.024	0.023	0.021	0.021	0.019	0.017	0.016	0.016	0.015	0.014	0.011	0.009	0.008	0.006	
9	0.327	0.352	0.376	0.355	0.331	0.355	0.325	0.132	1.000	0.277	0.253	0.248	0.230	0.262	0.158	0.239	0.136	0.139	0.028	0.025	0.024	0.023	0.022	0.022	0.020	0.020	0.017	0.016	0.015	0.015	0.014	0.013	0.011	0.009	0.007	0.006
10	0.303	0.381	0.455	0.409	0.363	0.435	0.353	0.234	0.277	1.000	0.141	0.269	0.239	0.329	0.142	0.331	0.153	0.196	0.022	0.020	0.019	0.018	0.017	0.017	0.016	0.016	0.014	0.013	0.012	0.012	0.011	0.010	0.009	0.007	0.006	0.004
11	0.296	0.290	0.283	0.278	0.269	0.265	0.254	0.229	0.253	0.141	1.000	0.084	0.201	0.189	0.145	0.157	0.108	0.089	0.027	0.024	0.024	0.022	0.021	0.021	0.019	0.019	0.017	0.016	0.015	0.015	0.014	0.012	0.010	0.009	0.007	0.005
12	0.282	0.287	0.291	0.281	0.268	0.274	0.254	0.219	0.248	0.269	0.084	1.000	0.068	0.203	0.136	0.171	0.109	0.089	0.025	0.022	0.022	0.021	0.020	0.020	0.018	0.018	0.016	0.015	0.014	0.013	0.013	0.011	0.010	0.008	0.007	0.005
13	0.264	0.267	0.268	0.260	0.248	0.253	0.236	0.206	0.230	0.239	0.201	0.068	1.000	0.066	0.133	0.155	0.101	0.089	0.024	0.021	0.021	0.019	0.018	0.017	0.017	0.015	0.014	0.013	0.012	0.012	0.011	0.009	0.008	0.006	0.005	
14	0.249	0.282	0.314	0.291	0.267	0.299	0.257	0.190	0.262	0.329	0.189	0.203	0.066	1.000	0.002	0.215	0.110	0.123	0.020	0.018	0.018	0.017	0.016	0.014	0.014	0.013	0.012	0.011	0.011	0.010	0.009	0.008	0.006	0.005	0.004	
15	0.200	0.190	0.179	0.179	0.175	0.168	0.165	0.157	0.158	0.142	0.145	0.136	0.133	0.002	1.000	-0.017	0.075	0.052	0.019	0.017	0.016	0.015	0.015	0.013	0.013	0.012	0.011	0.010	0.010	0.009	0.009	0.007	0.006	0.005	0.004	
16	0.199	0.246	0.292	0.263	0.235	0.278	0.229	0.150	0.239	0.331	0.157	0.171	0.155	0.215	-0.017	1.000	-0.004	0.129	0.015	0.013	0.013	0.012	0.012	0.010	0.010	0.009	0.009	0.008	0.008	0.007	0.007	0.006	0.005	0.004	0.003	
17	0.145	0.153	0.160	0.152	0.143	0.151	0.137	0.112	0.136	0.153	0.108	0.109	0.101	0.110	0.075	-0.004	1.000	-0.041	0.012	0.011	0.011	0.010	0.010	0.009	0.009	0.009	0.008	0.007	0.007	0.006	0.006	0.005	0.004	0.003	0.002	
18	0.112	0.142	0.170	0.153	0.136	0.163	0.132	0.084	0.139	0.196	0.089	0.098	0.089	0.123	0.052	0.129	-0.041	1.000	0.008	0.007	0.007	0.006	0.006	0.006	0.005	0.005	0.004	0.004	0.004	0.004	0.003	0.003	0.002	0.002	0.002	
19	0.038	0.034	0.031	0.032	0.032	0.029	0.030	0.030	0.028	0.022	0.027	0.025	0.024	0.020	0.019	0.015	0.012	0.008	1.000	0.265	0.309	0.305	0.295	0.280	0.287	0.273	0.274	0.270	0.260	0.240	0.213	0.208	0.202	0.186	0.164	0.106
20	0.034	0.031	0.028	0.028	0.028	0.026	0.026	0.026	0.025	0.020	0.024	0.022	0.021	0.018	0.017	0.013	0.011	0.007	0.265	1.000	0.238	0.342	0.323	0.282	0.328	0.284	0.324	0.324	0.307	0.265	0.209	0.218	0.246	0.243	0.224	0.118
21	0.033	0.030	0.027	0.028	0.028	0.025	0.026	0.024	0.019	0.024	0.022	0.021	0.018	0.016	0.016	0.013	0.011	0.007	0.309	0.238	1.000	0.209	0.297	0.264	0.292	0.265	0.286	0.284	0.273	0.244	0.205	0.207	0.217	0.208	0.188	0.110
22	0.031	0.028	0.026	0.026	0.024	0.024	0.024	0.024	0.023	0.018	0.022	0.021	0.019	0.017	0.015	0.012	0.010	0.006	0.305	0.342	0.209	1.000	0.212	0.277	0.315	0.275	0.315	0.316	0.301	0.262	0.207	0.217	0.244	0.240	0.221	0.118
23	0.030	0.027	0.024	0.025	0.025	0.023	0.023	0.023	0.022	0.017	0.021	0.020	0.019	0.016	0.015	0.012	0.010	0.006	0.295	0.323	0.297	0.212	1.000	0.155	0.314	0.267	0.305	0.307	0.293	0.257	0.206	0.214	0.239	0.234	0.215	0.117
24	0.029	0.027	0.024	0.025	0.025	0.023	0.023	0.023	0.022	0.017	0.021	0.020	0.018	0.016	0.015	0.012	0.010	0.006	0.280	0.282	0.264	0.277	0.155	1.000	0.158	0.262	0.263	0.265	0.259	0.241	0.213	0.211	0.197	0.175	0.114	
25	0.027	0.024	0.022	0.022	0.020	0.020	0.021	0.021	0.020	0.016	0.019	0.018	0.017	0.014	0.013	0.010	0.009	0.005	0.287	0.328	0.292	0.315	0.314	1.000	0.173	0.339	0.335	0.324	0.284	0.227	0.239	0.270	0.267	0.247	0.135	
26	0.027	0.024	0.022	0.022	0.020	0.020	0.021	0.021	0.020	0.016	0.019	0.018	0.017	0.014	0.013	0.010	0.009	0.005	0.273	0.284	0.265	0.275	0.267	0.262	1.000	0.180	0.299	0.286	0.267	0.235	0.239	0.226	0.202	0.131		
27	0.023	0.021	0.019	0.020	0.020	0.018	0.018	0.019	0.017	0.014	0.017	0.016	0.015	0.013	0.012	0.009	0.008	0.005	0.274	0.324	0.286	0.315	0.305	0.263	0.339	0.180	1.000	0.253	0.349	0.299	0.238	0.253	0.290	0.288	0.266	0.147
28	0.022	0.020	0.018	0.019	0.018	0.017	0.017	0.017	0.016	0.013	0.016	0.015	0.014	0.012	0.011	0.009	0.007	0.004	0.270	0.324	0.284	0.316	0.307	0.265	0.335	0.299	0.253	1.000	0.257	0.317	0.245	0.262	0.301	0.300	0.277	0.153
29	0.021	0.019	0.017	0.018	0.017	0.016	0.016	0.016	0.015	0.012	0.015	0.014	0.013	0.011	0.010	0.008	0.007	0.004	0.260	0.307	0.273	0.301	0.293	0.259	0.324	0.286	0.349	0.257	1.000	0.211	0.261	0.266	0.298	0.294	0.270	0.157
30	0.020	0.018	0.016	0.017	0.017	0.015	0.016	0.016	0.015	0.012	0.014	0.013	0.012	0.011	0.010	0.008	0.006	0.004	0.240	0.265	0.244	0.262	0.257	0.241	0.284	0.267	0.299	0.317	0.211	1.000	0.149	0.264	0.267	0.257	0.231	0.149
31	0.019	0.017	0.016	0.016	0.016	0.015	0.015	0.014	0.011	0.014	0.013	0.012	0.010	0.009	0.007	0.006	0.004	0.004	0.213	0.209	0.205	0.207	0.206	0.213	0.227	0.235	0.238	0.245	0.261	0.149	1.000	0.132	0.226	0.199	0.174	0.134
32	0.017	0.016	0.014	0.014	0.014	0.013	0.014	0.014	0.013	0.010	0.012	0.011	0.011	0.009	0.009	0.007	0.006	0.003	0.208	0.218	0.207	0.217	0.214	0.211	0.239	0.235	0.253	0.262	0.266	0.264	0.132	1.000	0.139	0.224	0.193	0.136
33	0.014	0.013	0.012	0.012	0.012	0.011	0.011	0.011	0.011	0.009	0.010	0.010	0.009	0.008	0.007	0.006	0.005	0.003</																		

TABLE XXXV. Elements of the covariance matrix ρ_{ij} ($i \in [1, 36]$ and $j \in [37, 72]$) for the simultaneous fit.

ρ_{ij}	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
1	0.244	0.274	0.246	0.289	0.271	0.246	0.259	0.247	0.233	0.240	0.218	0.192	0.174	0.155	0.117	0.071	0.040	0.023	0.009	0.010	0.010	0.010	0.009	0.009	0.009	0.008	0.008	0.007	0.007	0.006	0.005	0.005	0.004	0.004	0.003	0.002
2	0.223	0.250	0.224	0.263	0.247	0.224	0.236	0.225	0.213	0.218	0.199	0.175	0.158	0.141	0.106	0.065	0.036	0.021	0.008	0.009	0.009	0.009	0.008	0.008	0.008	0.008	0.007	0.007	0.006	0.006	0.005	0.004	0.004	0.004	0.003	0.002
3	0.201	0.225	0.202	0.237	0.223	0.202	0.213	0.203	0.192	0.197	0.179	0.158	0.143	0.127	0.096	0.059	0.033	0.019	0.007	0.008	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.005	0.004	0.004	0.003	0.003	0.002	
4	0.205	0.230	0.206	0.242	0.227	0.206	0.217	0.207	0.196	0.201	0.183	0.161	0.146	0.130	0.098	0.060	0.033	0.020	0.008	0.009	0.009	0.008	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.005	0.004	0.004	0.003	0.003	0.002	
5	0.204	0.229	0.206	0.242	0.227	0.206	0.217	0.207	0.195	0.201	0.182	0.161	0.145	0.129	0.098	0.060	0.033	0.020	0.008	0.009	0.009	0.008	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.005	0.004	0.004	0.003	0.003	0.002	
6	0.187	0.210	0.188	0.221	0.208	0.188	0.199	0.189	0.179	0.184	0.167	0.147	0.133	0.119	0.089	0.055	0.030	0.018	0.007	0.008	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.002	0.002	
7	0.191	0.215	0.192	0.226	0.213	0.193	0.203	0.193	0.183	0.188	0.171	0.151	0.136	0.121	0.091	0.056	0.031	0.018	0.007	0.008	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.003	0.002	
8	0.193	0.216	0.194	0.227	0.214	0.194	0.204	0.195	0.184	0.189	0.172	0.152	0.137	0.122	0.092	0.056	0.031	0.018	0.007	0.008	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.003	0.002	
9	0.180	0.201	0.181	0.212	0.199	0.181	0.191	0.182	0.172	0.176	0.160	0.141	0.128	0.114	0.086	0.052	0.029	0.017	0.007	0.008	0.007	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.003	0.002	0.002	
10	0.144	0.161	0.145	0.170	0.160	0.145	0.153	0.145	0.137	0.141	0.128	0.113	0.102	0.091	0.069	0.042	0.023	0.014	0.005	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.003	0.002	0.002	0.001	0.001	
11	0.175	0.196	0.176	0.207	0.194	0.176	0.186	0.177	0.167	0.171	0.156	0.138	0.124	0.111	0.083	0.051	0.028	0.017	0.006	0.007	0.007	0.007	0.007	0.006	0.006	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.002	0.002	0.001	
12	0.162	0.182	0.163	0.192	0.180	0.163	0.172	0.164	0.155	0.159	0.145	0.128	0.115	0.103	0.077	0.047	0.026	0.016	0.006	0.007	0.007	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.003	0.002	0.001	
13	0.153	0.171	0.154	0.180	0.170	0.154	0.162	0.154	0.146	0.150	0.136	0.120	0.109	0.097	0.073	0.044	0.025	0.015	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.003	0.002	0.002	0.001		
14	0.131	0.147	0.132	0.155	0.145	0.132	0.139	0.132	0.125	0.128	0.117	0.103	0.093	0.083	0.062	0.038	0.021	0.013	0.005	0.006	0.005	0.005	0.005	0.004	0.004	0.004	0.003	0.003	0.003	0.002	0.002	0.002	0.001	0.001		
15	0.121	0.135	0.121	0.143	0.134	0.122	0.128	0.122	0.115	0.118	0.108	0.095	0.086	0.076	0.068	0.061	0.046	0.028	0.016	0.009	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.001	0.001		
16	0.096	0.108	0.097	0.114	0.107	0.097	0.102	0.097	0.092	0.094	0.086	0.076	0.068	0.061	0.046	0.028	0.016	0.009	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.001	0.001	0.001		
17	0.081	0.091	0.082	0.096	0.090	0.082	0.086	0.082	0.078	0.080	0.072	0.064	0.058	0.051	0.039	0.024	0.013	0.008	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.001	0.001	0.001	0.001		
18	0.054	0.060	0.054	0.063	0.060	0.054	0.057	0.054	0.051	0.053	0.048	0.042	0.038	0.034	0.026	0.016	0.009	0.006	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.000		
19	0.009	0.010	0.009	0.010	0.010	0.009	0.009	0.009	0.008	0.009	0.008	0.007	0.006	0.005	0.004	0.003	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
20	0.008	0.009	0.008	0.009	0.008	0.008	0.008	0.007	0.008	0.007	0.007	0.006	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
21	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.006	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
22	0.007	0.008	0.007	0.008	0.007	0.007	0.007	0.007	0.007	0.007	0.006	0.006	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
23	0.007	0.008	0.007	0.008	0.007	0.007	0.007	0.007	0.007	0.007	0.006	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
24	0.007	0.008	0.007	0.008	0.007	0.007	0.007	0.006	0.006	0.007	0.006	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
25	0.006	0.007	0.006	0.007	0.006	0.006	0.006	0.006	0.006	0.006	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
26	0.006	0.007	0.006	0.007	0.006	0.006	0.006	0.006	0.006	0.006	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
27	0.005	0.006	0.005	0.006	0.005	0.005	0.005	0.005	0.005	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
28	0.005	0.006	0.005	0.006	0.005	0.005	0.005	0.005	0.005	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
29	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.004	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
30	0.005	0.005	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
31	0.004	0.005	0.004	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
32	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.003	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
33	0.003	0.004	0.003	0.004	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
34	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.001	0.001	0.001																					

y_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
37	0.244	0.223	0.201	0.205	0.204	0.187	0.191	0.193	0.180	0.144	0.175	0.162	0.153	0.131	0.121	0.096	0.081	0.054	0.009	0.008	0.008	0.007	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.002	0.002	
38	0.274	0.250	0.225	0.230	0.229	0.210	0.212	0.216	0.201	0.161	0.196	0.182	0.171	0.147	0.135	0.108	0.098	0.060	0.010	0.009	0.008	0.008	0.008	0.008	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.002	
39	0.246	0.224	0.202	0.206	0.206	0.188	0.192	0.194	0.181	0.145	0.176	0.163	0.154	0.132	0.121	0.097	0.082	0.054	0.009	0.008	0.008	0.007	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.002	0.002	
40	0.289	0.263	0.237	0.242	0.221	0.226	0.227	0.212	0.170	0.207	0.193	0.182	0.155	0.143	0.114	0.096	0.063	0.010	0.009	0.009	0.008	0.008	0.008	0.008	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.004	0.003	0.003	0.002	0.002	
41	0.271	0.247	0.223	0.227	0.227	0.208	0.213	0.214	0.199	0.160	0.194	0.180	0.170	0.145	0.134	0.107	0.090	0.060	0.010	0.009	0.008	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.002	
42	0.246	0.224	0.202	0.206	0.206	0.188	0.193	0.194	0.181	0.145	0.176	0.163	0.154	0.132	0.122	0.097	0.082	0.054	0.009	0.008	0.008	0.007	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.002	0.002	
43	0.259	0.236	0.213	0.217	0.217	0.199	0.203	0.204	0.191	0.153	0.186	0.172	0.162	0.139	0.128	0.102	0.086	0.057	0.009	0.008	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.005	0.004	0.004	0.003	0.002	0.002	
44	0.247	0.225	0.203	0.207	0.207	0.189	0.193	0.195	0.182	0.145	0.177	0.164	0.154	0.132	0.122	0.097	0.082	0.054	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.002	0.002
45	0.233	0.213	0.192	0.196	0.195	0.179	0.183	0.184	0.172	0.137	0.167	0.155	0.146	0.125	0.115	0.092	0.078	0.051	0.008	0.007	0.007	0.007	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.002	0.002	
46	0.240	0.218	0.197	0.201	0.201	0.184	0.188	0.189	0.176	0.141	0.171	0.159	0.150	0.128	0.118	0.094	0.080	0.053	0.009	0.008	0.007	0.007	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.002	0.002	
47	0.218	0.199	0.179	0.183	0.182	0.167	0.171	0.172	0.160	0.128	0.156																										

η_{ij}	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
37	1.000	0.204	0.251	0.222	0.225	0.226	0.220	0.225	0.223	0.206	0.195	0.187	0.164	0.123	0.138	0.073	0.052	0.036	0.012	0.014	0.014	0.014	0.013	0.013	0.012	0.011	0.011	0.010	0.010	0.008	0.007	0.005	0.006	0.004	0.003	
38	0.204	1.000	0.402	0.276	0.315	0.367	0.323	0.364	0.381	0.308	0.307	0.323	0.275	0.160	0.283	0.134	0.115	0.087	0.014	0.016	0.016	0.016	0.015	0.014	0.013	0.013	0.013	0.011	0.011	0.009	0.008	0.008	0.006	0.007	0.005	0.003
39	0.251	0.402	1.000	0.189	0.324	0.395	0.334	0.391	0.419	0.321	0.327	0.356	0.300	0.156	0.330	0.151	0.137	0.106	0.012	0.014	0.014	0.014	0.013	0.013	0.012	0.011	0.010	0.010	0.008	0.007	0.005	0.006	0.004	0.003	0.003	
40	0.222	0.276	0.189	1.000	0.158	0.248	0.243	0.242	0.236	0.227	0.212	0.196	0.174	0.140	0.135	0.076	0.049	0.033	0.015	0.017	0.017	0.016	0.015	0.015	0.014	0.013	0.012	0.011	0.010	0.009	0.008	0.006	0.007	0.005	0.004	
41	0.225	0.315	0.324	0.158	1.000	0.184	0.270	0.276	0.280	0.248	0.238	0.236	0.205	0.142	0.185	0.095	0.072	0.052	0.014	0.016	0.016	0.015	0.015	0.014	0.013	0.013	0.012	0.011	0.011	0.009	0.008	0.006	0.007	0.005	0.003	
42	0.226	0.367	0.395	0.248	0.184	1.000	0.186	0.325	0.333	0.271	0.269	0.283	0.241	0.142	0.247	0.118	0.100	0.076	0.012	0.014	0.014	0.014	0.013	0.013	0.012	0.012	0.011	0.010	0.010	0.008	0.007	0.005	0.006	0.004	0.003	
43	0.220	0.323	0.334	0.243	0.270	0.186	1.000	0.182	0.295	0.247	0.242	0.243	0.210	0.139	0.198	0.099	0.078	0.057	0.013	0.015	0.015	0.015	0.014	0.014	0.013	0.012	0.012	0.011	0.010	0.009	0.008	0.007	0.006	0.005	0.003	
44	0.225	0.364	0.391	0.242	0.276	0.325	0.182	1.000	0.236	0.275	0.266	0.279	0.239	0.141	0.243	0.116	0.099	0.074	0.012	0.014	0.014	0.014	0.013	0.013	0.012	0.012	0.011	0.010	0.009	0.007	0.005	0.006	0.004	0.003		
45	0.223	0.381	0.419	0.236	0.280	0.333	0.295	0.236	1.000	0.176	0.283	0.295	0.251	0.140	0.267	0.125	0.110	0.084	0.012	0.014	0.013	0.013	0.012	0.012	0.011	0.011	0.011	0.010	0.009	0.007	0.006	0.005	0.004	0.003	0.003	
46	0.206	0.308	0.321	0.227	0.248	0.271	0.247	0.275	0.176	1.000	0.123	0.237	0.199	0.129	0.192	0.095	0.076	0.056	0.012	0.014	0.014	0.014	0.013	0.013	0.012	0.011	0.011	0.010	0.009	0.008	0.007	0.005	0.006	0.004	0.003	
47	0.195	0.307	0.327	0.212	0.238	0.269	0.242	0.266	0.283	0.123	1.000	0.134	0.205	0.121	0.200	0.097	0.081	0.061	0.011	0.013	0.013															

Figures 15–17 provide the results of the fits to the U_{miss} distributions of forward/backward events in the reconstructed q^2 intervals of $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, respectively.

Tables XXXVIII–XL exhibit the fitted forward/backward DT yields in data and the determined q^2 -binned forward-backward asymmetries of $D^0 \rightarrow K^- \mu^+ \nu_\mu$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$, respectively.

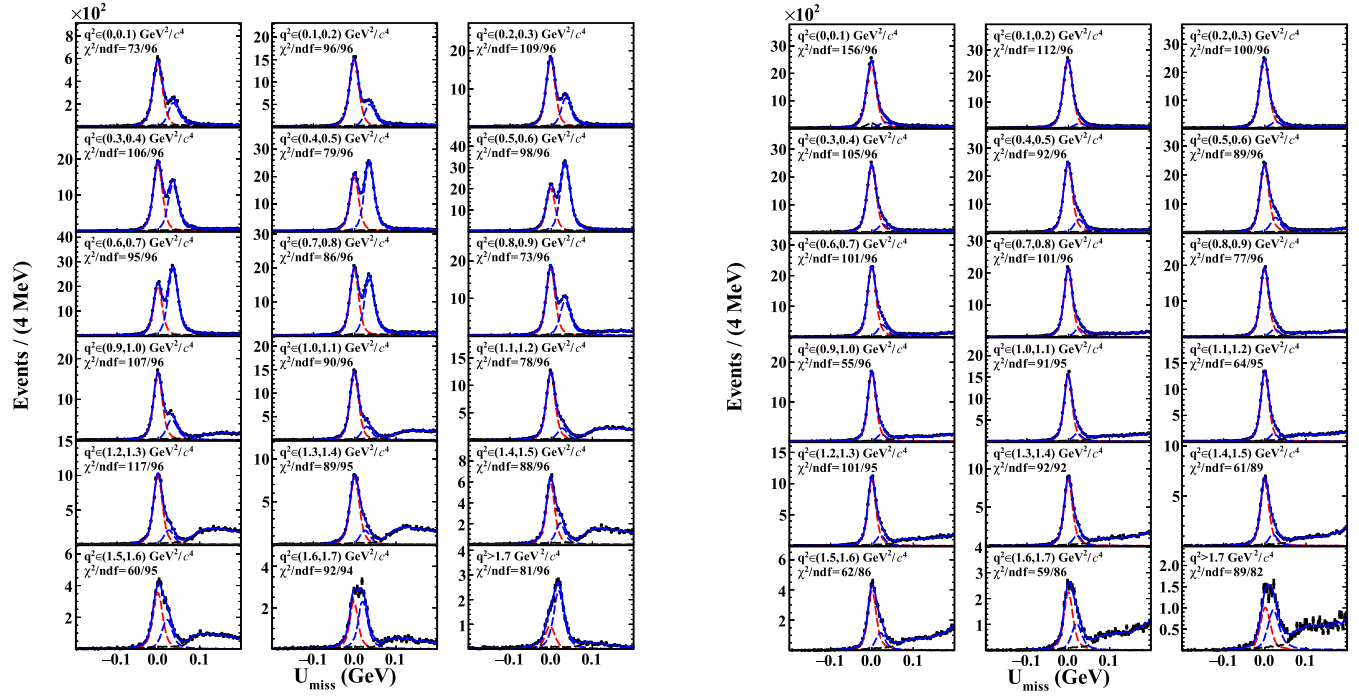


FIG. 15. The U_{miss} distributions of the accepted forward (left) and backward (right) candidate events in individual q^2 intervals for $D^0 \rightarrow K^- \mu^+ \nu_\mu$ in data, with fit results overlaid. The blue solid curves are the fit results. The red dashed curves are the signal shapes, and the black dashed curves are the fitted combinatorial background shapes.

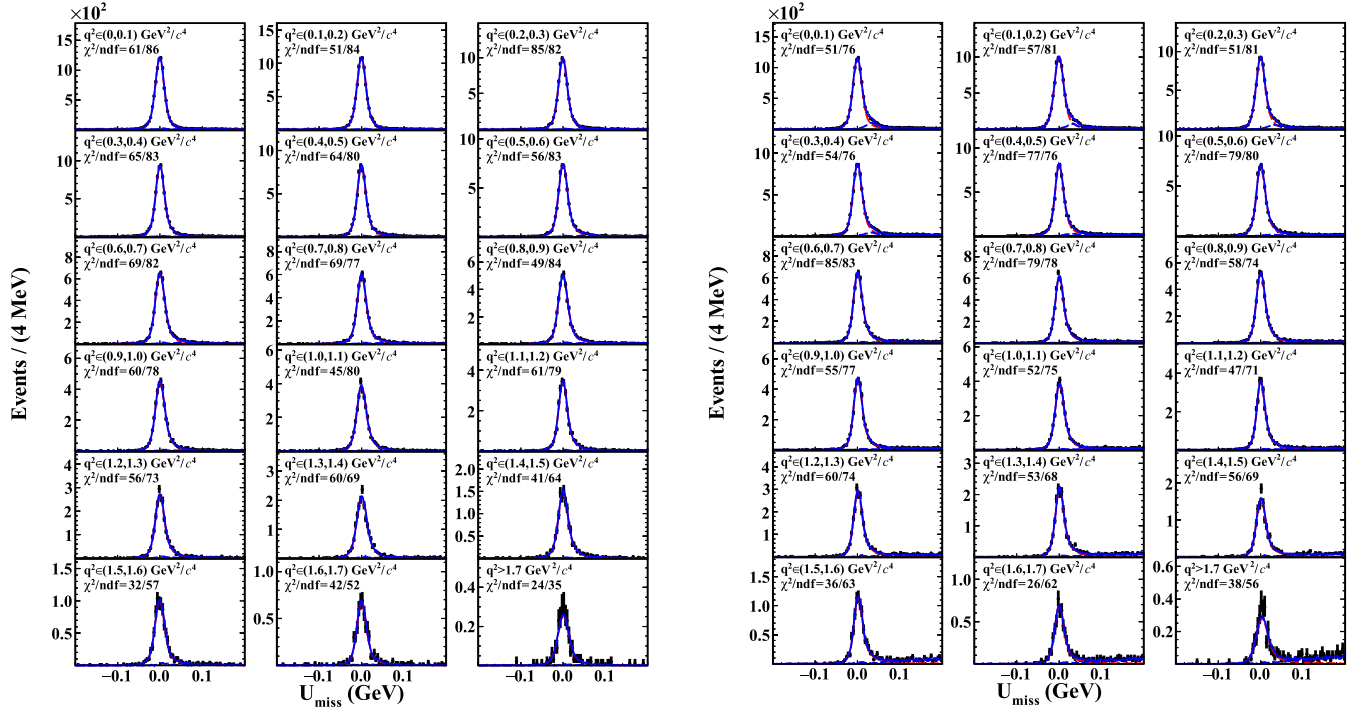


FIG. 16. The U_{miss} distributions of the forward (left) and backward (right) events for $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$. The curves and symbols are defined the same as in Fig. 15.

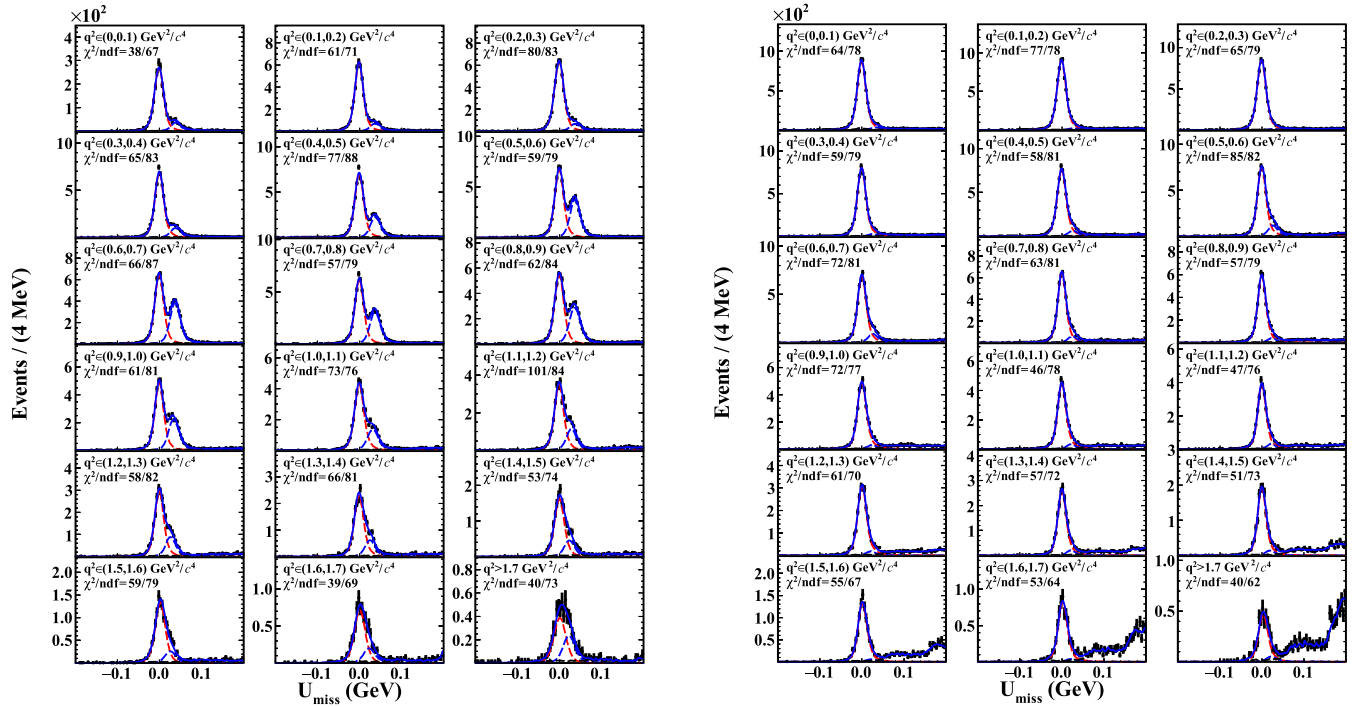


FIG. 17. The U_{miss} distributions of the forward (left) and backward (right) events for $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$. The curves and symbols are defined the same as in Fig. 15.

TABLE XXXVIII. The fitted forward/backward DT yields $N_{\text{DT}}^{\text{Forward/Backward}}$ in individual q^2 intervals and the resulting forward-backward asymmetries (A_{FB}) for $D^0 \rightarrow K^- \mu^+ \nu_\mu$, where the first uncertainties are statistical and the second are systematic.

$q^2(\text{GeV}/c^4)$	$N_{\text{DT}}^{\text{Forward}}$	$N_{\text{DT}}^{\text{Backward}}$	A_{FB}
(0.0, 0.1)	4704 ± 80	19188 ± 166	$-0.225 \pm 0.011 \pm 0.003$
(0.1, 0.2)	12133 ± 127	21254 ± 174	$-0.098 \pm 0.007 \pm 0.002$
(0.2, 0.3)	13941 ± 137	20150 ± 175	$-0.073 \pm 0.008 \pm 0.002$
(0.3, 0.4)	14802 ± 146	19411 ± 176	$-0.051 \pm 0.008 \pm 0.002$
(0.4, 0.5)	15226 ± 155	18994 ± 182	$-0.049 \pm 0.008 \pm 0.002$
(0.5, 0.6)	15493 ± 161	18332 ± 186	$-0.039 \pm 0.009 \pm 0.002$
(0.6, 0.7)	15398 ± 155	17276 ± 174	$-0.031 \pm 0.009 \pm 0.002$
(0.7, 0.8)	14857 ± 150	15806 ± 162	$-0.020 \pm 0.009 \pm 0.002$
(0.8, 0.9)	13990 ± 143	14353 ± 157	$-0.018 \pm 0.009 \pm 0.002$
(0.9, 1.0)	12614 ± 136	12822 ± 150	$-0.020 \pm 0.010 \pm 0.002$
(1.0, 1.1)	10953 ± 127	11090 ± 147	$-0.015 \pm 0.011 \pm 0.002$
(1.1, 1.2)	9444 ± 124	9668 ± 140	$-0.016 \pm 0.012 \pm 0.002$
(1.2, 1.3)	7865 ± 120	7771 ± 134	$0.005 \pm 0.014 \pm 0.002$
(1.3, 1.4)	6449 ± 121	6425 ± 130	$0.002 \pm 0.016 \pm 0.003$
(1.4, 1.5)	4489 ± 100	4934 ± 126	$-0.058 \pm 0.020 \pm 0.003$
(1.5, 1.6)	3024 ± 72	3070 ± 67	$-0.011 \pm 0.019 \pm 0.006$
(1.6, 1.7)	1735 ± 60	1826 ± 57	$-0.045 \pm 0.028 \pm 0.007$
>1.7	782 ± 54	906 ± 47	$-0.088 \pm 0.050 \pm 0.010$

TABLE XXXIX. Same as Table XXXVIII, but for $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$.

$q^2(\text{GeV}/c^4)$	$N_{\text{DT}}^{\text{Forward}}$	$N_{\text{DT}}^{\text{Backward}}$	A_{FB}
(0.0, 0.1)	9023 ± 99	8839 ± 103	$0.003 \pm 0.009 \pm 0.001$
(0.1, 0.2)	8300 ± 95	8041 ± 99	$0.002 \pm 0.010 \pm 0.001$
(0.2, 0.3)	7581 ± 91	7347 ± 94	$0.000 \pm 0.010 \pm 0.001$
(0.3, 0.4)	7076 ± 89	6909 ± 90	$-0.002 \pm 0.011 \pm 0.001$
(0.4, 0.5)	6288 ± 85	6331 ± 85	$-0.018 \pm 0.011 \pm 0.002$
(0.5, 0.6)	5842 ± 82	5698 ± 80	$0.006 \pm 0.012 \pm 0.002$
(0.6, 0.7)	5343 ± 79	5185 ± 76	$0.010 \pm 0.013 \pm 0.002$
(0.7, 0.8)	4808 ± 76	4825 ± 73	$-0.001 \pm 0.013 \pm 0.002$
(0.8, 0.9)	4134 ± 70	4245 ± 68	$-0.013 \pm 0.014 \pm 0.002$
(0.9, 1.0)	3749 ± 67	3735 ± 64	$0.010 \pm 0.015 \pm 0.002$
(1.0, 1.1)	3270 ± 62	3214 ± 59	$0.023 \pm 0.016 \pm 0.002$
(1.1, 1.2)	2687 ± 56	2750 ± 55	$-0.005 \pm 0.018 \pm 0.002$
(1.2, 1.3)	2207 ± 52	2358 ± 51	$-0.028 \pm 0.019 \pm 0.003$
(1.3, 1.4)	1859 ± 47	1796 ± 45	$0.041 \pm 0.021 \pm 0.003$
(1.4, 1.5)	1310 ± 39	1337 ± 39	$0.004 \pm 0.025 \pm 0.003$
(1.5, 1.6)	857 ± 32	948 ± 33	$-0.039 \pm 0.030 \pm 0.004$
(1.6, 1.7)	567 ± 26	586 ± 26	$0.013 \pm 0.038 \pm 0.005$
>1.7	243 ± 16	302 ± 19	$-0.090 \pm 0.055 \pm 0.008$

TABLE XL. Same as Table XXXVIII, but for $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$.

$q^2 (\text{GeV}/c^4)$	$N_{\text{DT}}^{\text{Forward}}$	$N_{\text{DT}}^{\text{Backward}}$	A_{FB}
(0.0, 0.1)	2013 ± 47	6324 ± 85	$-0.195 \pm 0.015 \pm 0.002$
(0.1, 0.2)	4389 ± 70	6695 ± 89	$-0.096 \pm 0.011 \pm 0.002$
(0.2, 0.3)	4857 ± 75	6324 ± 87	$-0.074 \pm 0.012 \pm 0.002$
(0.3, 0.4)	5043 ± 76	5777 ± 85	$-0.034 \pm 0.012 \pm 0.002$
(0.4, 0.5)	5118 ± 78	5694 ± 89	$-0.037 \pm 0.013 \pm 0.002$
(0.5, 0.6)	4934 ± 80	5432 ± 90	$-0.048 \pm 0.013 \pm 0.002$
(0.6, 0.7)	4778 ± 79	5072 ± 87	$-0.037 \pm 0.014 \pm 0.002$
(0.7, 0.8)	4530 ± 76	4556 ± 83	$-0.026 \pm 0.014 \pm 0.002$
(0.8, 0.9)	4164 ± 76	4136 ± 80	$-0.021 \pm 0.015 \pm 0.002$
(0.9, 1.0)	3576 ± 70	3709 ± 79	$-0.045 \pm 0.017 \pm 0.002$
(1.0, 1.1)	3311 ± 70	3260 ± 76	$-0.007 \pm 0.018 \pm 0.002$
(1.1, 1.2)	2660 ± 68	2759 ± 69	$-0.028 \pm 0.021 \pm 0.002$
(1.2, 1.3)	2189 ± 63	2337 ± 66	$-0.047 \pm 0.023 \pm 0.003$
(1.3, 1.4)	1789 ± 62	1876 ± 57	$-0.023 \pm 0.027 \pm 0.003$
(1.4, 1.5)	1298 ± 55	1378 ± 50	$-0.035 \pm 0.033 \pm 0.003$
(1.5, 1.6)	1088 ± 38	1015 ± 34	$0.056 \pm 0.028 \pm 0.004$
(1.6, 1.7)	626 ± 30	641 ± 27	$-0.016 \pm 0.038 \pm 0.005$
>1.7	373 ± 27	375 ± 22	$0.010 \pm 0.054 \pm 0.008$

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