

Internal kinematics of dwarf galaxies orbitally moving in ultralight dark matter

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For dwarf galaxies modeled as deformed Plummer spheres and orbitally moving in ultralight dark matter halo of the Milky Way, the torque induced by the dynamical friction force is determined and obtained to be much smaller than the torque produced by the gravitational force of the Milky Way. As to the tidal torque, it is found that its magnitude differs significantly for the Plummer and Gaussian density profiles of ultralight dark matter. Possible oscillations of dwarf galaxies caused by the tidal torque and misalignment of dwarf galaxies with respect to their equilibrium position are investigated. It is suggested that more accurate measurements of dwarf-galaxy rotation velocities and improved modeling of dark matter distributions could provide useful constraints on the nature and properties of dark matter.

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I. INTRODUCTION

Dwarf galaxies are small galaxies, typically containing a few million to a few billion stars, compared to hundreds of billions in large galaxies like the Milky Way. The mass-to-light ratio in dwarf galaxies can exceed 100 (solar units) compared to 10–20 in normal galaxies. In addition, the velocity dispersion of stars in dwarf galaxies is larger than expected based on the visible matter alone. This suggests that dwarfs are strongly dark matter dominated structures, which makes them excellent laboratories for the study of dark matter and its properties [1–3].

Dwarf galaxies are mainly observed in the Local Group orbiting the Milky Way and Andromeda [4]. There are also 61 dwarf galaxies confirmed to be within 420 kpc of the Milky Way [5]. The orbits of these dwarf galaxies lie far from the Milky Way center, where dark matter begins to dominate over baryonic matter as the Milky Way rotation curve implies (see, e.g., [6]). Indeed, while the Milky Way diameter is around 27 kpc, cosmological simulations suggest that the edge of the Milky Way halo is situated at 292 kpc [7]. The visible disk of the Milky Way Galaxy is thought to be embedded in a much larger, roughly spherical halo of dark matter whose density decreases with distance from the Galactic Center. This means that, for dwarf

galaxies orbiting the Milky Way, the dynamical friction force due to dark matter should dominate compared to the contribution due to baryonic matter.

Models of ultralight dark matter (ULDM) with boson particle masses in the range of $10^{-23} - 10^{-21}$ eV attract considerable interest due to their unique phenomenological features and have been the subject of extensive research (for a review, see Refs. [8–10]). These models reproduce the large-scale structure of the Universe similar to the cold dark matter (CDM) model and, in view of the ULDM kiloparsec scale of the de Broglie wavelength, could address several issues that CDM faces on the galactic scale.

Dynamical friction experienced by moving stars, globular clusters, and dwarf galaxies is a classical subject [11] which is very important in the study of the kinetics of astronomical bodies. In ULDM models, the dynamical friction was studied for pointlike probes in the fuzzy ultralight dark matter (self-interactions among dark matter particles are absent) in [12–15]. The role of ULDM self-interaction for the frictional force acting on stellar bodies was investigated in [16–20]. Note that, using the radial distribution of stars in six ultra-faint dwarfs measured by the *Hubble* Space Telescope, it was recently argued [21] that collisions among dark matter particles, i.e., self-interaction is necessary to explain the observed stellar distribution.

Globular clusters and dwarf galaxies are spatially extended objects that are often modeled as Plummer spheres. This motivated the study of the dynamical friction force acting on circularly moving Plummer spheres performed in [22,23], where analytic expressions for both radial and tangential components of the dynamical friction

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force were found. In this paper, we study the role of dynamical friction torque, as well as the torque produced by the gravitational force of the Milky Way, on the internal kinematics of dwarf galaxies. In this connection, we would like to mention that the problem of the impact of dynamical friction on nonpoint objects indeed draws the attention of researchers [24,25].

In ULDM models, the dark matter profile in galaxies is characterized by the presence of a dense solitonic core in the state of a Bose-Einstein condensate, surrounded by a diffuse halo. Since dwarf galaxies are dark matter dominated, the baryonic matter (stars and gas) distribution follows that of dark matter. It was shown in [26] that the stellar density distribution (and the dark matter distribution as well) can be reasonably well approximated within the core by a Plummer profile. We use this profile in our study here to model the matter distribution in dwarf galaxies aiming to address their internal kinematics caused by the dynamical friction and tidal forces.

We note that, for ULDM particles with mass $\sim 10^{-22}$ eV, the de Broglie wavelength $\lambda_{\text{dB}} = \hbar/(mv) \sim 1$ kpc is comparable to the characteristic size of dwarf galaxies. Strictly speaking, the microscopic gravitational potential in ULDM exhibits spatial wave interference patterns. However, on timescales relevant for global orbital dynamics and internal kinematics, the time-averaged density distribution of the solitonic core and stars in dwarf galaxies is well approximated by smooth classical profiles, such as the Plummer profile [26]. Similarly, the background Milky Way potential at large radii $r_0 \gg \lambda_{\text{dB}}$ is well described by an effective macroscopic Navarro-Frenk-White (NFW) profile. Here, we adopt these smooth profiles as a zero-order approximation, neglecting fine-scale wave granularity [12,27].

An important parameter in the description of ultralight dark matter far from the solitonic core is an effective temperature produced by gravitational cooling and violent relaxation, which explains how effectively collisionless self-gravitating systems acquire an effective temperature [27–32]. Adding phenomenological temperature to the Gross-Pitaevskii equation as a parameter in a logarithmic

nonlinearity (accounting for an “effective thermal pressure”) produces ULDM halos with central solitonic cores and outer isothermal atmospheres [27,32]. The latter provides the corresponding environment where dwarf galaxies orbit the Milky Way in ULDM models.

The paper is organized as follows. Our model is described and the dynamical friction force for an orbiting deformed Plummer sphere is derived in Sec. II. The torque induced by the dynamical friction force is calculated in Sec. III. The torque due to tidal forces acting on orbiting dwarf galaxies is determined in Sec. IV. Numerical results for torques are presented in Sec. V. Oscillations of dwarf galaxies with respect to their equilibrium position are studied in Sec. VI. The Gaussian profile of ULDM density in dwarf galaxies is considered in Sec. VII and oscillations due to tidal torque are studied. The results are discussed and summarized in Sec. VIII.

II. MODEL AND DYNAMICAL FRICTION FORCE

Let us consider a dwarf galaxy of mass M that moves on a circular orbit of radius r_0 with constant angular velocity Ω in the steady-state regime. Although dwarf galaxies are often modeled [4,33–35] as spherically symmetric Plummer spheres, the tidal and dynamical friction forces may deform dwarf galaxies during their orbital motion. In fact, ultrafaint dwarf galaxies are rather elongated systems [36,37] with a mean ellipticity of about $\varepsilon = 0.4$.

Without loss of generality, we could assume that the circular orbital motion of a dwarf galaxy takes place in a plane with radius r_0 so that the radius vector of its center of mass is given by

$$\mathbf{r}_{\text{c.m.}}(t) = (r_0 \cos(\Omega t), r_0 \sin(\Omega t), 0).$$

As for the deformed Plummer sphere that we use to model the dwarf galaxy, a natural assumption is to consider it as an ellipsoid elongated along $\mathbf{r}_{\text{c.m.}}(t)$ due to tidal forces with the largest axis of the ellipsoid rotated by angle θ with respect to $\mathbf{r}_{\text{c.m.}}(t)$. Then, the deformed Plummer sphere density profile is defined by

$$\rho(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) = \frac{3M}{4\pi l_p^3 ab} \frac{1}{\left(1 + \frac{R_z^2}{l_p^2} + \frac{(R_{\parallel} \cos \theta + R_t \sin \theta)^2}{a^2 l_p^2} + \frac{(-R_{\parallel} \sin \theta + R_t \cos \theta)^2}{b^2 l_p^2}\right)^{5/2}}, \quad (1)$$

where a and b are the deformation parameters. Notice that, for $\theta = 0$ and $a = b = 1$, the density profile (1) reduces to the standard Plummer sphere profile of radius l_p . In addition, we assume that $a > 1$ and $b \leq 1$ so that dwarfs are elongated mostly along $\mathbf{r}_{\text{c.m.}}(t)$ due to tidal forces with small inclination in the tangential direction and are flattened in the tangential direction due to the dynamical friction force. Further, $R_z \equiv z$, $R_{\parallel}(t) = (\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t))_{\parallel}$ and

$R_t(t) = (\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t))_t$ are the projections of $\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)$ onto $\mathbf{r}_{\text{c.m.}}(t)$ and the tangential direction $\dot{\mathbf{r}}_{\text{c.m.}}(t)/|\dot{\mathbf{r}}_{\text{c.m.}}(t)|$, respectively. Finally, we introduced the factor $1/ab$ in Eq. (1) so that the deformed Plummer sphere has the same mass M as nondeformed Plummer sphere. Geometrically, the considered deformed Plummer sphere is a triaxial ellipsoid rotated by angle θ with respect to $\mathbf{r}_{\text{c.m.}}(t)$ (see Fig. 1). Although we treat dwarfs as rigid bodies,

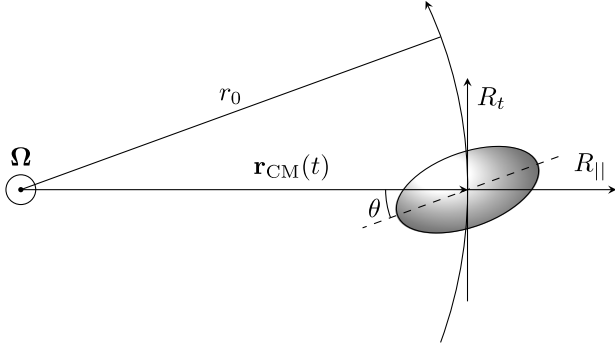


FIG. 1. Schematics of the plane of an orbiting dwarf galaxy modeled as a deformed Plummer sphere.

which rotate as solid lumps, this definitely is only a first step in the study of internal kinematics of dwarf galaxies, which hopefully will be useful for subsequent investigations of this problem.

In our derivation of the dynamical friction force, we follow the setup developed in [22] based on the approach considered in [16,19]. A deformed Plummer sphere moving in homogeneous ultralight dark matter with density profile $n_0(r)$ composed of ultralight bosonic particles of mass m perturbs due to gravitational interaction the ULDM density $n_{\text{DM}}(t, \mathbf{r}) = n_0(r)(1 + \alpha(t, \mathbf{r}))$. In the linear response approach, the corresponding density inhomogeneity $\alpha(t, \mathbf{r})$ is defined by the equation [38]

$$\partial_t^2 \alpha - c_s^2 \nabla_{\mathbf{r}}^2 \alpha + \frac{\nabla_{\mathbf{r}}^4 \alpha}{4m^2} + \xi \partial_t \alpha = 4\pi G \rho(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)). \quad (2)$$

Here $c_s^2 = \partial P / \partial n_0$ is the sound velocity squared connected with the ULDM self-interaction and ξ is a parameter related to the damping term in the generalized Gross-Pitaevskii equation, which ensures that the system relaxes to equilibrium [27]. The value of ξ was estimated in [27] via a generalized Einstein relation which expresses the fluctuation-dissipation theorem

$$\xi = \frac{2T}{\hbar}, \quad (3)$$

where T is the effective temperature of the isothermal halo (we set the Boltzmann constant to unity). The role of the damping term, whose magnitude is defined by ξ for the dynamical friction force experienced by moving globular clusters in the Fornax dwarf galaxy was recently studied in [38].

In momentum space, for the considered circular steady-state motion, we find

$$\begin{aligned} & \left(-\omega^2 + c_s^2 \mathbf{k}^2 + \frac{\mathbf{k}^4}{4m^2} - i\xi\omega \right) \alpha(\omega, \mathbf{k}) \\ &= 4\pi G \int_{-\infty}^{+\infty} dt \rho(\mathbf{k}, t) e^{i\omega t - i\mathbf{k} \cdot \mathbf{r}_{\text{c.m.}}(t)}, \end{aligned} \quad (4)$$

where $\rho(\mathbf{k}, t)$ is the Fourier transform of the mass density profile of the deformed Plummer sphere given by

$$\rho(\mathbf{k}, t) = \rho(k_d(\mathbf{k}, t)) = M k_d(\mathbf{k}, t) l_p K_1(k_d(\mathbf{k}, t) l_p). \quad (5)$$

Here $K_1(x)$ is the modified Bessel function of the second kind,

$$k_d(\mathbf{k}, t) = \sqrt{k_z^2 + a^2(k_{\parallel} \cos \theta + k_t \sin \theta)^2 + b^2(-k_{\parallel} \sin \theta + k_t \cos \theta)^2}, \quad (6)$$

and $k_{\parallel}$ and k_t are projections of $\mathbf{k}$ onto $\mathbf{r}_{\text{c.m.}}(t)$ and the tangential direction $\dot{\mathbf{r}}_{\text{c.m.}}(t)/|\dot{\mathbf{r}}_{\text{c.m.}}(t)|$, respectively.

According to the Poisson equation, the density inhomogeneity $\alpha(t, \mathbf{r})$ and the deformed Plummer sphere density $\rho(t, \mathbf{r})$ source a perturbation ϕ of the gravitational potential

$$\nabla_{\mathbf{r}}^2 \phi(t, \mathbf{r}) = 4\pi G n_0(r) \alpha(t, \mathbf{r}) + 4\pi G \rho(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)). \quad (7)$$

Solving Eq. (4) for $\alpha(\omega, \mathbf{k})$, Eq. (7) gives the following gravitational potential due to perturbed ULDM density by moving the deformed Plummer sphere (the complete perturbed gravitational potential is obviously $\phi = \phi_{\alpha} + \phi_{dPl}$, where ϕ_{dPl} is the Newtonian potential of the deformed Plummer sphere):

$$\phi_{\alpha}(t, \mathbf{r}) = -4\pi G n_0 \int \frac{d\omega d\mathbf{k}}{(2\pi)^4} \frac{\alpha(\omega, \mathbf{k})}{k^2} e^{-i\omega t + i\mathbf{k} \cdot \mathbf{r}}. \quad (8)$$

Then the local dynamical friction force density equals

$$\begin{aligned} \mathbf{f}_{\text{fr}}(t, \mathbf{r}) &= -\rho(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \nabla_{\mathbf{r}} \phi_{\alpha}(t, \mathbf{r}) = 4\pi G n_0 \rho(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \int \frac{d\omega d\mathbf{k}}{(2\pi)^4} \frac{i\mathbf{k}}{k^2} \alpha(\omega, \mathbf{k}) e^{-i\omega t + i\mathbf{k} \cdot \mathbf{r}} \\ &= \frac{G^2 n_0}{\pi^2} \rho(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \int_{-\infty}^{+\infty} dt' \int d\omega d\mathbf{k} \frac{i\mathbf{k} \rho(\mathbf{k}, t') e^{-i\omega(t-t') + i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t'))}}{-\omega^2 - i\xi\omega + c_s^2 k^2 + \frac{k^4}{4m^2}}. \end{aligned} \quad (9)$$

It is convenient to change the variable $\tau = t - t'$ and then take into account that the integral over ω vanishes for $\tau < 0$. We obtain

$$\mathbf{f}_{\text{fr}}(t, \mathbf{r}) = \frac{G^2 n_0}{\pi^2} \rho(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \int_0^{+\infty} d\tau \int d\omega d\mathbf{k} \frac{i\mathbf{k} \rho(\mathbf{k}, t - \tau) e^{-i\omega\tau + i\mathbf{k}(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t - \tau))}}{k^2 (-\omega^2 - i\xi\omega + c_s^2 k^2 + \frac{k^4}{4m^2})}. \quad (10)$$

Clearly, this local dynamical force is different at different positions $\mathbf{r}$ within the deformed Plummer sphere producing a torque.

In the next section, we calculate the torque induced by the dynamical friction force.

III. TORQUE CAUSED BY DYNAMICAL FRICTION FORCE

Using Eq. (10), we find that the total torque due to the dynamical friction force acting on a moving dwarf galaxy equals

$$\begin{aligned} \boldsymbol{\tau} &= \int d\mathbf{r} \mathbf{r} \times \mathbf{f}_{\text{fr}}(t, \mathbf{r}) \\ &\equiv \int d\mathbf{r} (\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \mathbf{f}_{\text{fr}}(t, \mathbf{r}) + \mathbf{r}_{\text{c.m.}}(t) \times \mathbf{F}_{\text{fr}}(t), \end{aligned}$$

where the last term $\mathbf{r}_{\text{c.m.}}(t) \times \mathbf{F}_{\text{fr}}(t)$ is the torque due to the total dynamical friction force which describes the loss of the angular momentum of the deformed Plummer sphere in its orbital motion. The other term in the above equation defines the inner torque $\boldsymbol{\tau}_{\text{inner}}$, which does not have an analog in the case of a point probe. It may cause rotation of the deformed Plummer sphere as a whole and, thus, is important for the study of internal kinematics of the modeled dwarf galaxies. This torque equals

$$\begin{aligned} \boldsymbol{\tau}_{\text{inner}}(t) &= \frac{G^2 n_0}{\pi^2} \int d\mathbf{r} \rho(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \int_0^{+\infty} d\tau \\ &\quad \times \int d\omega \frac{d\mathbf{k}}{k^2} i(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \\ &\quad \times \mathbf{k} \frac{\rho(\mathbf{k}, t - \tau) e^{-i\omega\tau + i\mathbf{k}\mathbf{r} - i\mathbf{k}\mathbf{r}_{\text{c.m.}}(t - \tau)}}{-\omega^2 - i\xi\omega + c_s^2 k^2 + \frac{k^4}{4m^2}}. \quad (11) \end{aligned}$$

Making the change of variable $\mathbf{r} = \mathbf{R} + \mathbf{r}_{\text{c.m.}}(t)$, we obtain

$$\begin{aligned} \boldsymbol{\tau}_{\text{inner}}(t) &= \frac{G^2 n_0}{\pi^2} \int_0^{+\infty} d\tau \\ &\quad \times \int d\omega \frac{d\mathbf{k}}{k^2} \rho(\mathbf{k}, t - \tau) e^{-i\omega\tau + i\mathbf{k}\mathbf{r}_{\text{c.m.}} - i\mathbf{k}\mathbf{r}_{\text{c.m.}}(t - \tau)} \\ &\quad \times \int d\mathbf{R} e^{i\mathbf{k}\mathbf{R}} \rho(\mathbf{R}, t) i\mathbf{R} \times \mathbf{k}. \quad (12) \end{aligned}$$

The last integral in (12) can be rewritten in more simple form using

$$\int d\mathbf{R} e^{i\mathbf{k}\mathbf{R}} \rho(\mathbf{R}, t) i\mathbf{R} = \nabla_{\mathbf{k}} \int d\mathbf{R} e^{i\mathbf{k}\mathbf{R}} \rho(\mathbf{R}, t) = \nabla_{\mathbf{k}} \rho(-\mathbf{k}, t). \quad (13)$$

In view of Eqs. (5) and (6), $\rho(-\mathbf{k}, t) = \rho(\mathbf{k}, t)$. Further, we find in cylindrical coordinates

$$\nabla_{\mathbf{k}} \rho \times \mathbf{k} = -\mathbf{e}_z \partial_{\phi} \rho + \hat{\mathbf{k}}_p \frac{k_z}{k_p} \partial_{\phi} \rho + \hat{\boldsymbol{\phi}} (k_p \partial_{k_z} \rho - k_z \partial_{k_p} \rho).$$

Since ρ depends on k_z^2 , the last two terms in the cross product above are odd functions of k_z . Therefore, their contribution vanishes when integrating over k_z in the expression for the inner torque. Thus, we conclude that, as expected, the inner torque has only one nonzero component in the direction perpendicular to the plane of orbital motion

$$\begin{aligned} \tau_{\text{inner}}(t) &= -\mathbf{e}_z \frac{G^2 n_0}{\pi^2} \int_0^{+\infty} d\tau \int d\omega \frac{d\mathbf{k}}{k^2} e^{-i\omega\tau + i\mathbf{k}\mathbf{r}_{\text{c.m.}}(t) - i\mathbf{k}\mathbf{r}_{\text{c.m.}}(t - \tau)} \\ &\quad \times \frac{\rho(\mathbf{k}, t - \tau) \partial_{\phi} \rho(\mathbf{k}, t)}{-\omega^2 - i\xi\omega + c_s^2 k^2 + \frac{k^4}{4m^2}}. \quad (14) \end{aligned}$$

The poles of the integrand in Eq. (14) are situated in the lower half-plane of the complex plane

$$\omega_{1,2} = -\frac{i\xi}{2} \pm D(k), \quad D(k) = \sqrt{-\frac{\xi^2}{4} + c_s^2 k^2 + \frac{k^4}{4m^2}}. \quad (15)$$

For $\tau > 0$, closing the contour over ω in the lower half-plane and integrating over ω gives

$$\begin{aligned} \tau_{\text{inner}}(t) &= -\frac{2G^2 n_0}{\pi} \int_0^{+\infty} d\tau \int \frac{d\mathbf{k}}{k^2} e^{i\mathbf{k}\mathbf{r}_{\text{c.m.}}(t) - i\mathbf{k}\mathbf{r}_{\text{c.m.}}(t - \tau)} \\ &\quad \times \frac{e^{-\xi\tau/2}}{D(k)} \sin(\tau D(k)) \rho(\mathbf{k}, t - \tau) \partial_{\phi} \rho(\mathbf{k}, t). \quad (16) \end{aligned}$$

Further, using

$$\begin{aligned} \mathbf{r}_{\text{c.m.}}(t) &= r_0(\cos(\Omega t), \sin(\Omega t), 0), \\ \mathbf{k} &= (k_p \cos \phi, k_p \sin \phi, k_z), \end{aligned}$$

we find

$$\begin{aligned} \mathbf{k}\mathbf{r}_{\text{c.m.}}(t) - \mathbf{k}\mathbf{r}_{\text{c.m.}}(t - \tau) &= k_p r_0 (\cos(\Omega t - \phi) - \cos(\Omega(t - \tau) - \phi)), \\ k_{\parallel}(t) &= \mathbf{k} \cdot \frac{\mathbf{r}_{\text{c.m.}}(t)}{|\mathbf{r}_{\text{c.m.}}(t)|} = k_p \cos(\Omega t - \phi), \quad k_t(t) = \mathbf{k} \cdot \frac{\dot{\mathbf{r}}_{\text{c.m.}}(t)}{|\dot{\mathbf{r}}_{\text{c.m.}}(t)|} = -k_p \sin(\Omega t - \phi). \end{aligned} \quad (17)$$

Then Eq. (16) takes the form

$$\begin{aligned} \tau_{\text{inner}}(t) &= -\frac{2G^2 n_0}{\pi} \int_0^{+\infty} d\tau \int_{-\infty}^{+\infty} dk_z \int_0^{\infty} dk_p \frac{k_p}{k^2} \frac{e^{-\xi\tau/2}}{D(k)} \sin(\tau D(k)) \\ &\quad \times \int_0^{2\pi} d\phi e^{ik_p r_0 (\cos(\Omega t - \phi) - \cos(\Omega(t - \tau) - \phi))} \rho(\mathbf{k}, t - \tau) \partial_{\phi} \rho(\mathbf{k}, t), \end{aligned} \quad (18)$$

where $\rho(\mathbf{k}, t)$ is defined in Eq. (5) and

$$k_d(\mathbf{k}, t) = \sqrt{k_z^2 + a^2 k_p^2 \cos^2(\phi - \Omega t - \theta) + b^2 k_p^2 \sin^2(\phi - \Omega t - \theta)}.$$

Changing the variable $\phi \rightarrow \phi + \Omega t$ in the last integral, we obtain

$$\int_{-\Omega t}^{2\pi - \Omega t} d\phi e^{ik_p r_0 (\cos \phi - \cos(\phi + \Omega\tau))} \rho(k_d(\mathbf{k}, -\tau)) \partial_{\phi} \rho(k_d(\mathbf{k}, 0)). \quad (19)$$

Since the integral with respect to ϕ in Eq. (19) is over the full period 2π , this integral and hence the inner torque do not depend on t as expected for the case of steady-state motion under consideration. Therefore, we set $t = 0$ and the inner torque equals

$$\begin{aligned} \tau_{\text{inner}} &= -\frac{2G^2 n_0}{\pi} \int_0^{+\infty} d\tau \int_{-\infty}^{+\infty} dk_z \int_0^{\infty} dk_p \frac{k_p}{k^2} \frac{e^{-\xi\tau/2}}{D(k)} \sin(\tau D(k)) \\ &\quad \times \int_0^{2\pi} d\phi e^{ik_p r_0 (\cos \phi - \cos(\phi + \Omega\tau))} \rho(k_d(\mathbf{k}, -\tau)) \partial_{\phi} \rho(k_d(\mathbf{k}, 0)). \end{aligned} \quad (20)$$

Let us rewrite the inner torque (20) in an explicitly real form. For this, we split the integral over ϕ in two equal parts $I_{\phi} = I_{\phi}/2 + I_{\phi}/2$ and make the change of variable $\phi \rightarrow \phi + \pi$ in the second term, which gives $\cos \phi - \cos(\phi + \Omega\tau) \rightarrow -(\cos \phi - \cos(\phi + \Omega\tau))$. Taking into account that we integrate over the full period, the limit of integration in the second term can be kept unchanged. Then Eq. (20) takes the explicitly real form

$$\begin{aligned} \tau_{\text{inner}} &= -\frac{2G^2 n_0}{\pi} \int_0^{+\infty} d\tau \int_{-\infty}^{+\infty} dk_z \int_0^{\infty} dk_p \frac{k_p}{k^2} \frac{e^{-\xi\tau/2}}{D(k)} \sin(\tau D(k)) \\ &\quad \times \int_0^{2\pi} d\phi \cos[k_p r_0 (\cos \phi - \cos(\phi + \Omega\tau))] \rho(k_d(\mathbf{k}, -\tau)) \partial_{\phi} \rho(k_d(\mathbf{k}, 0)). \end{aligned} \quad (21)$$

Using

$$\partial_{\phi} \rho(k_d) = \partial_{\phi} [M k_d l_p K_1(k_d l_p)] = -M k_d l_p^2 K_0(k_d l_p) \partial_{\phi} k_d, \quad (22)$$

where

$$\partial_{\phi} k_d(\mathbf{k}, 0) = \frac{k_p^2}{2k_d(\mathbf{k}, 0)} (b^2 - a^2) \sin(2\phi - 2\theta),$$

the inner torque takes the following final form:

$$\begin{aligned} \tau_{\text{inner}} &= (b^2 - a^2) \frac{G^2 M^2 l_p^3 n_0}{\pi} \int_0^{+\infty} d\tau \int_{-\infty}^{+\infty} dk_z \int_0^{\infty} dk_p \frac{k_p^3}{k^2} \frac{e^{-\xi\tau/2}}{D(k)} \sin(\tau D(k)) \\ &\quad \times \int_0^{2\pi} d\phi \cos[k_p r_0 (\cos \phi - \cos(\phi + \Omega\tau))] k_d(\mathbf{k}, -\tau) \times K_1(l_p k_d(\mathbf{k}, -\tau)) K_0(l_p k_d(\mathbf{k}, 0)) \sin(2\phi - 2\theta). \end{aligned} \quad (23)$$

Obviously, this torque vanishes for $a = b$ when the dwarf galaxy is an ellipsoid of revolution with respect to the axis z .

If this inner torque is not compensated, it will induce rotation of the dwarf galaxy under consideration. Further, for dwarf galaxies orbiting the Milky Way, there exists a torque due to tidal bulges displaced by angle θ from the axis defined by $\mathbf{r}_{\text{c.m.}}(t)$ as we assumed in the setup of our model. This means that we should calculate and take into account the tidal torque.

IV. TORQUE DUE TO TIDAL FORCES

The tidal torque is defined by the equation

$$\boldsymbol{\tau}_g = \int d\mathbf{r} (\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \times \mathbf{f}_g(t, \mathbf{r}), \quad (24)$$

where $\mathbf{f}_g(t, \mathbf{r})$ is the local gravitational force density acting on a dwarf galaxy orbiting the Milky Way

$$\mathbf{f}_g(t, \mathbf{r}) = -\rho(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \nabla_{\mathbf{r}} \phi_{\text{MW}}(t, \mathbf{r}).$$

Gravitational potential for a spherical density distribution $n_0(r)$ is given by [39]

$$\phi_{\text{MW}}(t, \mathbf{r}) = -\frac{GM_{\text{MW}}(r)}{r} - G \int_r^\infty \frac{dM_{\text{MW}}(r')}{r'}, \quad (25)$$

where

$$M_{\text{MW}}(r) = 4\pi \int_0^r dr' r'^2 n_0(r').$$

Equation (24) gives

$$\begin{aligned} \boldsymbol{\tau}_g &= -G \int d\mathbf{r} \rho(\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \frac{1}{r^3} M_{\text{MW}}(r) (\mathbf{r} - \mathbf{r}_{\text{c.m.}}(t)) \times \mathbf{r} \\ &= -G \int d\mathbf{R} \rho(\mathbf{R}) \frac{1}{r^3} M_{\text{MW}}(r) \mathbf{R} \times \mathbf{r}_{\text{c.m.}}(t), \end{aligned} \quad (26)$$

where $r = |\mathbf{R} + \mathbf{r}_{\text{c.m.}}(t)| = \sqrt{(R_{\parallel} + r_{\text{c.m.}}(t))^2 + R_t^2 + R_z^2}$.

Since $\mathbf{r}_{\text{c.m.}}(t)$ lies in the equatorial plane and $\rho(\mathbf{R})$ is even in R_z [see Eq. (1)], torque (26) reduces to

$$\begin{aligned} \boldsymbol{\tau}_g &= -G \int dR_z dR_{\parallel} dR_t \rho(\mathbf{R}) \frac{1}{r^3} M_{\text{MW}}(r) \mathbf{R}_t \times \mathbf{r}_{\text{c.m.}}(t) \\ &= \mathbf{e}_z G r_0 \int dR_z dR_{\parallel} dR_t \rho(\mathbf{R}) R_t \frac{1}{r^3} M_{\text{MW}}(r), \end{aligned} \quad (27)$$

which is like the inner torque τ_{inner} given by Eq. (14) is directed perpendicular to the plane of orbital motion. Finally, note that if angle θ in Eq. (1) is zero, then the deformed Plummer sphere density profile $\rho(\mathbf{R})$ is even in R_t and, therefore, torque $\boldsymbol{\tau}_g$ vanishes when integrated over R_t .

Using the Navarro-Frenk-White profile for the ULDM density in the vicinity of the Milky Way

$$n_0(r) = \frac{n_s}{(r/r_s)(1 + r/r_s)^2}, \quad (28)$$

with $r_s = 16$ kpc and $n_s = 8.4868 \times 10^6 M_{\odot}/\text{kpc}^3$ [40], we obtain

$$M_{\text{MW}}(r) = 4\pi n_s r_s^3 \left[\ln \left(1 + \frac{r}{r_s} \right) - \frac{r}{r_s + r} \right], \quad (29)$$

where $r < r_{\text{vir}} = 245$ kpc.

Since, $r_0 \gg l_p$, then $M_{\text{MW}}(r)$ can be approximated with $M_{\text{MW}}(r) \approx M_{\text{MW}}(r_0)$.

In dimensionless variables $X = R_{\parallel}/l_p$, $Y = R_t/l_p$, and $Z = R_z/l_p$, which are useful for numerical studies, the density profile (1) takes the form

$$\rho(\mathbf{R}) = \frac{3M}{4\pi l_p^3 ab} \frac{1}{(Z^2 + A^2)^{5/2}}, \quad (30)$$

where

$$A^2 = 1 + \frac{(X \cos \theta + Y \sin \theta)^2}{a^2} + \frac{(-X \sin \theta + Y \cos \theta)^2}{b^2},$$

and the tidal torque equals

$$\begin{aligned} \boldsymbol{\tau}_g &= \mathbf{e}_z r_0 \frac{3GMM_{\text{MW}}(r_0)}{4\pi l_p^2 ab} \\ &\times \int dX dY \int_{-\infty}^{\infty} \frac{dZ}{(Z^2 + B^2)^{3/2} (Z^2 + A^2)^{5/2}} \end{aligned} \quad (31)$$

with $B^2 = Y^2 + (X + r_0/l_p)^2$.

The integral over Z in Eq. (31) can be calculated analytically. Changing the variable $Z = A \tan \beta$, we find for the integral over Z

$$I_z = \frac{2}{A^4 B^3} \int_0^{\pi/2} \frac{\cos^6 \beta d\beta}{\left(1 - \frac{B^2 - A^2}{B^2} \sin^2 \beta\right)^{3/2}}. \quad (32)$$

Denoting $(B^2 - A^2)/B^2 \equiv k^2$ and once again changing the integration variable $\sin \beta = x$, we obtain

$$\begin{aligned} I_z &= \frac{2}{A^4 B^3} \int_0^1 \frac{(1 - x^2)^{5/2} dx}{(1 - k^2 x^2)^{3/2}} \\ &= \frac{2((9k^4 - 17k^2 + 8)K(k^2) - (3k^4 - 13k^2 + 8)E(k^2))}{3A^4 B^3 k^6}, \end{aligned} \quad (33)$$

where $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind [41].

Finally, we find the following tidal torque:

$$\tau_g = \frac{3GMM_{\text{MW}}(r_0)}{4\pi l_p} T_g, \quad (34)$$

where T_g is the dimensionless tidal torque

$$T_g = \frac{1}{abL_p} \int \int dXdYI_z(X, Y) \quad (35)$$

with $L_p = l_p/r_0$.

V. NUMERICAL RESULTS FOR TORQUES OF DWARF GALAXIES ORBITING THE MILKY WAY

To proceed with numerical analysis, it is convenient to rewrite the inner torque (23) also in dimensionless variables

$K_z = k_z r_0$, $K_p = k_p r_0$, and $\mathcal{T} = \tau c_s / r_0$. Other dimensionless parameters are the Mach number $\mathcal{M} = r_0 \Omega / c_s$, $A = m r_0 c_s$, and $\Xi = \xi r_0 / c_s$. Then

$$D(k) = \frac{\tilde{D}(K)}{2mr_0^2}, \quad \tilde{D}(K) = \sqrt{K^4 + 4A^2 K^2 - A^2 \Xi^2}$$

and the inner torque (23) takes the following form in dimensionless variables:

$$\tau_{\text{inner}} = \mathbf{e}_z \frac{4\pi G^2 M^2 n_0 r_0}{c_s^2} T_{\text{inner}}, \quad (36)$$

$$T_{\text{inner}} = (b^2 - a^2) \frac{2AL_p^3}{\pi^2} \int_0^{+\infty} d\mathcal{T} \int_0^{+\infty} dK_z \int_0^\infty dK_p \frac{K_p^3 e^{-\Xi \mathcal{T}/2}}{K^2 \tilde{D}(K)} \sin\left(\frac{\mathcal{T} \tilde{D}(K)}{2A}\right) \times \int_0^\pi d\phi \cos\left[K_p (\cos\phi - \cos(\phi + \mathcal{M}\mathcal{T}))\right] K_d(\mathbf{K}, -\mathcal{T}) K_1(L_p K_d(\mathbf{K}, -\mathcal{T})) K_0(L_p K_d(\mathbf{K}, 0)) \sin(2\phi - 2\theta). \quad (37)$$

Since the inner torque (37) is proportional to $a^2 - b^2$, its value is largest for dwarf galaxies with considerable ellipticity $\varepsilon = 1 - b/a$. Further, our model is developed for dwarf galaxies moving around the Milky Way on circular orbits. Hence, we choose for our analysis dwarf galaxies listed in Table I whose orbits have low eccentricity e . The three columns before the last of this table define the values of the Mach number $\mathcal{M} = v_{\text{tan}}/c_s$ and parameters $A = m r_0 c_s / \hbar$ and $\Xi = \xi r_0 / c_s$.

The ULDM pressure $P = K n_0^2 + n_0 k_B T / m$ in the isothermal halo [27] is dominated by the thermal term $P = n_0 k_B T / m$ with the sound velocity given by $c_s^2 = \partial P / \partial n_0 = k_B T / m$. For the ULDM effective temperature in the Milky Way halo $T = 5 \times 10^{-25}$ K and $m = 3 \times 10^{-22}$ eV [44], we obtain $c_s \approx 114$ km s⁻¹. Note that the Mach number for the dwarf galaxies under consideration is $\mathcal{M} > 1$, i.e., the orbital motion of the dwarf galaxies proceeds in the supersonic regime. For the

damping term, we have

$$\xi = \frac{2T}{\hbar} = 1.3 \times 10^{-13} \text{ s}^{-1}. \quad (38)$$

We plot in Fig. 2 the dimensionless inner torque (37) as a function of angle θ . One can see that this dependence is nonmonotonous and changes sign. In addition, the torque vanishes for different values of θ for different dwarf galaxies.

Calculating the dimensionless tidal torque T_g given by Eq. (35) numerically, we depict it as a function of θ in the left panel of Fig. 3. As for the dependence of T_g on r_0/l_p in a log-log plot, it is well approximated by a linear function and is shown in the right panel of Fig. 3, i.e., $T_g \approx 10^{1.289} (r_0/l_p)^{-2.818}$. One can see from the left panel in Fig. 3 that the tidal torque is negative for $\theta \in (0, \pi/2)$, i.e., it is directed opposite to the z axis and tends to decrease

TABLE I. Galactocentric distance r_0 , galactocentric tangential velocity v_{tan} , and eccentricity e of dwarf galaxies orbiting the Milky Way in the pseudo-NFW model [42] with ellipticity $\varepsilon = 1 - b/a$ in the Plummer model [43] with radius l_p .

Dwarf galaxy	r_0 (kpc)	v_{tan} (km s ⁻¹)	e	ε	l_p (pc)	r_0/l_p	$\mathcal{M}$	A	Ξ	$M_{\text{MW}}(r_0)$ ($10^{11} M_\odot$)
CarI	107.6	192.2	0.3	0.36	308	349	1.69	1912	3823	5.13
FnxI	141.0	126.9	0.26	0.29	838	168	1.12	2505	5010	6.05
ScII	86.0	162.7	0.36	0.33	280	307	1.43	1528	3056	4.41
UMaI	102.0	133.3	0.25	0.59	234	436	1.17	1812	3624	4.95
UMiI	77.8	147.6	0.37	0.55	407	191	1.30	1382	2764	4.10
Will	49.3	159.2	0.25	0.47	27.7	1780	1.40	876	1752	2.85

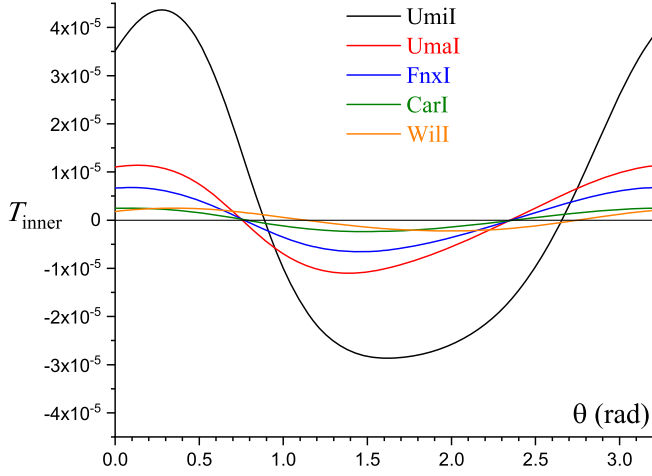


FIG. 2. Dimensionless inner torque T_{inner} as a function of angle θ for dwarf galaxies UmiI, UmaI, FnxI, CarI, and Will.

the angle θ . T_g reaches its maximum absolute value for $\theta \approx 36^\circ$ and vanishes at $\theta = \pi/2$. However, the latter position is unstable because a small change $\delta\theta$ leads to the appearance of a torque that further increases $\delta\theta$. For $\theta \in (\pi/2, \pi)$, the tidal torque is positive and tends to increase θ up to π .

Having determined dimensionless torques, we compare the inner torque (36) and the tidal torque (34) by calculating their ratio

$$\frac{\tau_{\text{inner}}}{\tau_g} = k \frac{T_{\text{inner}}}{T_g}, \quad k = \frac{(4\pi)^2 G M n_0(r_0) r_0 l_p}{3 M_{\text{MW}}(r_0) c_s^2}. \quad (39)$$

Using the ULDM density profile $n_0(r)$ given in Eq. (28) and values of $M_{\text{MW}}(r_0)$ for the corresponding dwarfs given in the last column of Table I, we easily find

$$k = \frac{2.3783 M [M_\odot] l_p [\text{kpc}]}{(1 + r_0 [\text{kpc}] / 16)^2 M_{\text{MW}}(r_0) [M_\odot]}. \quad (40)$$

Hence, k/M lies in the range 1.39×10^{-14} for Will to 6.87×10^{-14} for UmiI. Using the estimates of the total (baryonic plus dark matter) mass of dwarf galaxies $M \sim 10^8 M_\odot$, we conclude that the ratio of the inner torque to the tidal torque is negligible. However, this ratio can be substantial for objects of similar mass rotating around their common center of mass or in the process of merger. Then the inner torque should be taken into account in the analysis, although a nonlinear approach may be needed in this case.

In addition, we studied the dependence of the inner torque on boson mass m and temperature T . It turned out that the results are quite sensitive to m . The absolute value of the torque decreases notably as mass increases to $m = 8 \times 10^{-22}$ eV and, for large mass, the torque can even change sign. We performed calculations for two additional values of temperature. The absolute value of the torque is slightly smaller at $T = 3 \times 10^{-25}$ K and $T = 8 \times 10^{-25}$ K compared to $T = 5 \times 10^{-25}$ K. Thus, although the inner torque depends on boson mass and temperature, this dependence does not change our main conclusion that the ratio of the inner torque to the tidal torque is very small.

VI. MOMENT OF INERTIA AND PERIOD OF OSCILLATIONS

The form of the tidal torque in the left panel of Fig. 3 suggests the possibility of oscillations of dwarf galaxies around their equilibrium position. The start of oscillations or, in general, inner rotation is related to the problem of the initial conditions for dwarf galaxies orbiting the Milky Way. These initial conditions are set by early structure formation and depend on when and how they were accreted to the Milky Way's halo. To describe these

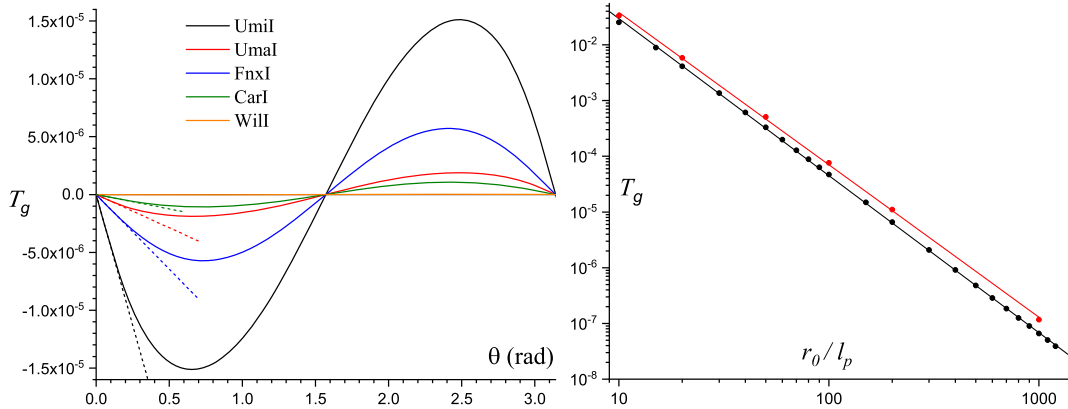


FIG. 3. Left: dimensionless tidal torque T_g as a function of θ for dwarf galaxies UmiI, UmaI, FnxI, CarI, and Will. Dashed lines correspond to the linear approximation $T_g = -\alpha\theta$ with values of α listed in Table II. Right: dimensionless tidal torque T as a function of r_0/l_p for $a = 2$, $b = 0.8$, $\theta = 0.2$ rad (black solid line) and $\theta = 0.5$ rad (red dashed line) with points defined numerically using Eq. (35) and the straight lines are linear approximations.

TABLE II. The values of parameter α in the linear approximation for the tidal torque $T_g = -\alpha\theta$, the frequency of oscillations ω , the period of oscillations T , the critical kinematic velocity $v_{cr} = \omega_{cr}l_p$, and the tangential velocity of internal rotation V_T (Ref. [45]).

Dwarf galaxy	$\alpha \times 10^5$	ω (Gyr $^{-1}$)	T (Gyr)	$v_{cr} = \omega_{cr}l_p$ (km/s)	V_T (km/s)
CarI	0.248	2.98	2.11	0.858	-9.56 ± 4.50
FnxI	1.29	1.65	3.82	1.303	-2.77 ± 1.30
ScII	0.296	3.48	1.80	0.922	-3.00 ± 1.02
UMaI	0.575	6.73	0.933	1.298	...
UMiI	4.53	7.50	0.838	2.531	0.03 ± 2.28
Will	4.59×10^{-3}	11.2	0.561	0.284	...

oscillations, we need to determine the moment of inertia of dwarf galaxies. To proceed and obtain an estimate of the moment of inertia, we assume that oscillations or full-scale rotation do not affect the matter distribution profile. Of course, to check this assumption, a coupled system of equations that describe dwarf-galaxy matter and the perturbed Milky Way halo should be analyzed and solved. We leave this problem for future studies. Then, taking this assumption into account, the relevant for us z component is given by the standard expression

$$I_z = \iiint \rho(\mathbf{r})(x^2 + y^2) d\mathbf{r}, \quad (41)$$

where

$$\rho(\mathbf{r}) = \frac{3M}{4\pi l_p^3 ab} \frac{1}{\left(1 + \frac{z^2}{l_p^2} + \frac{x^2}{a^2 l_p^2} + \frac{y^2}{b^2 l_p^2}\right)^{5/2}}.$$

Integrating over x and y gives a logarithmically divergent integral

$$I_z = M l_p^2 \frac{a^2 + b^2}{2} \int_{-\infty}^{\infty} \frac{dz}{\sqrt{1 + z^2}}. \quad (42)$$

This divergence is regulated by a certain physical scale related to the size of dwarf galaxies. Since the logarithmic dependence is quite mild, the exact value of this physical scale is not important and could approximate the moment of inertia as $I_z = C M l_p^2$, where $C \sim O(1)$. Since the tidal torque near the equilibrium position $\theta = 0$ is given by $T_g = -\alpha\theta$, harmonic oscillations for small deviations θ are realized with frequency

$$\begin{aligned} \omega &= \sqrt{\frac{\tau_g/\theta}{I_z}} = \sqrt{\frac{3GM_{\text{MW}}(r_0)\alpha}{4\pi C l_p^3}} \\ &= \sqrt{\frac{3 \times 4.30091 \times 10^{-6} [\text{kpc}(\text{km/s})^2/M_\odot] M_{\text{MW}}[M_\odot] \alpha}{4\pi C l_p [\text{kpc}] (l_p [\text{km}])^2}}. \end{aligned} \quad (43)$$

For the dwarf galaxy UmiI with $l_p = 407$ pc and $C = 5$, we find $\omega \approx 7.5$ Gyr $^{-1}$ and the period of oscillations $T = 2\pi/\omega \approx 0.838$ Gyr.

For other dwarf galaxies, the frequency and period of their oscillations are given in the third and fourth columns of Table II. Note that the oscillation frequency is proportional to the square root of the $M_{\text{MW}}(r_0)$ rather than the square root of the Milky Way mass M_{MW} . Further, $M_{\text{MW}}(r_0)$ depends on the dwarf's orbit radius r_0 and differs substantially if the dwarf orbit radii r_0 are different, e.g., it equals $2.85 \times 10^{11} M_\odot$ for Will and $6.05 \times 10^{11} M_\odot$ for FnxI (see the last column in Table I).

Integrating the tidal torque τ_g given by Eq. (34) with respect to angle θ over the range $[\pi/2, 0]$, we obtain the maximal potential energy connected with the work done by the tidal torque on a dwarf,

$$U_{\text{max}} = \frac{3GM_{\text{MW}}(r_0)}{4\pi l_p} I_T, \quad (44)$$

where $I_T = -\int_0^{\pi/2} T_g d\theta$ and T_g is the dimensionless tidal torque (see Fig. 3). To estimate the maximum value of angular velocity reached at $\theta = 0$, which a dwarf galaxy can acquire during oscillations induced by tidal torque, we use the relation $I_z \omega_{cr}^2/2 = U_{\text{max}}$, which yields

$$\omega_{cr} = \sqrt{\frac{3GM_{\text{MW}}(r_0)I_T}{2\pi l_p^3 C}}. \quad (45)$$

The values of corresponding kinematic velocities at the Plummer radius $v_{cr} = \omega_{cr}l_p$ for the considered dwarfs are listed in the fifth column of Table II. Clearly, if the observed tangential velocity V_T exceeds v_{cr} , then the corresponding dwarf galaxy is in a rotational state. Otherwise, it may undergo oscillations driven by tidal torque in the case where the plane of oscillations is perpendicular to the line of sight. Comparing v_{cr} with V_T in Table II, we conclude that, while the dwarf galaxy UMiI could undergo oscillations, other dwarfs CarI, FnxI, and ScII rotate.

VII. GAUSSIAN PROFILE OF ULDM DENSITY

It was argued in [46] that the ULDM distribution in a dwarf galaxy is well approximated with the Gaussian function

$$\rho(R) = \frac{M}{(\pi L^2)^{3/2}} \exp\left(-\frac{R^2}{L^2}\right), \quad (46)$$

where L defines the length scale of the ULDM halo.

Substituting deformed distribution

$$\rho(R) = \frac{M}{ab(\pi L^2)^{3/2}} \exp\left(-\frac{(R_{\parallel} \cos \theta + R_t \sin \theta)^2}{a^2 L^2} - \frac{(-R_{\parallel} \sin \theta + R_t \cos \theta)^2}{L^2 b^2} - \frac{R_z^2}{L^2}\right) \quad (47)$$

into Eq. (27), we obtain the following expression for the tidal torque:

$$\begin{aligned} \tau_g^{(G)} &= \mathbf{e}_z G r_0 M_{\text{MW}}(r_0) \\ &\times \int dR_z dR_{\parallel} dR_t \frac{R_t \rho(R)}{\left((R_{\parallel} + r_{\text{c.m.}}(t))^2 + R_t^2 + R_z^2\right)^{3/2}}. \end{aligned} \quad (48)$$

In dimensionless variables $X = R_{\parallel}/L$, $Y = R_t/L$, and $Z = R_z/L$, we have

$$\begin{aligned} \tau_g^{(G)} &= G r_0 M_{\text{MW}}(r_0) \frac{M}{ab\pi^{3/2}L^2} \\ &\times \int dX dY dZ \frac{Y \exp(-A_1^2) \exp(-Z^2)}{(B_1^2 + Z^2)^{3/2}}, \end{aligned} \quad (49)$$

where $B_1^2 = Y^2 + (X + r_0/L)^2$ and $A_1^2 = (X \cos \theta + Y \sin \theta)^2/a^2 + (-X \sin \theta + Y \cos \theta)^2/b^2$. Integration over Z could be performed analytically. Using

$$\begin{aligned} I_2 &= \int_{-\infty}^{\infty} dZ \frac{e^{-Z^2}}{(B_1^2 + Z^2)^{3/2}} \\ &= e^{B_1^2/2} [K_1(B_1^2/2) - K_0(B_1^2/2)], \end{aligned} \quad (50)$$

we obtain

$$\begin{aligned} \tau_g^{(G)} &= \frac{GM_{\text{MW}}(r_0)M}{\pi^{3/2}L} T_g^{(G)}, \\ T_g^{(G)} &= \frac{r_0}{abL} \int dX dY Y \exp(-A_1^2) I_2(X, Y), \end{aligned} \quad (51)$$

where $T_g^{(G)}$ is dimensionless tidal torque.

Let us proceed now to numerical results. The best fit radii of the ULDM halos in the case of attractive ULDM in the presence of baryonic gravitational potential gives values $L = 787$ pc for Carina and $L = 753$ pc for Fornax [46].

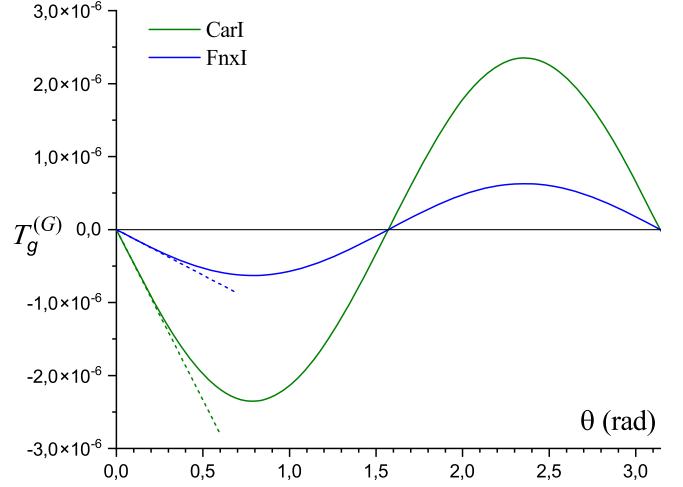


FIG. 4. The dimensionless tidal torque $T_g^{(G)}$ defined in Eq. (51) as a function of θ for dwarf galaxies FnxI and CarI. Dashed lines correspond to the linear approximation $T_g^{(G)} = -\alpha_2 \theta$ with values of α_2 listed in Table III.

We plot the dimensionless tidal torque $T_g^{(G)}$ given by Eq. (51) as a function of θ for these dwarf galaxies in Fig. 4.

Comparing the angular dependence of $T_g^{(G)}$ with the dimensionless tidal torque T_g for the Plummer mass density distribution of these dwarf galaxies considered in Fig. 3, we conclude that these angular dependencies are similar. However, the absolute value of the torque for CarI is now few times larger than for FnxI, unlike in Fig. 3 where the situation is opposite. This result can be explained by the fact that the Fornax Plummer radius l_p is almost 3 times larger compared to the Carina radius (see Table I). On the other hand, the ULDM halo of Carina in the case of the Gaussian profile is almost the same as for Fornax. In addition, Carina has higher ellipticity that in the end results in higher tidal torque. Thus, we conclude that the ULDM profile plays a crucial role in determining the tidal torque for dwarf galaxies.

The moment of inertia (41) for the Gaussian density distribution (46) can be found analytically,

$$I_z^{(G)} = \frac{ab}{2} (a^2 + b^2) M L^2, \quad (52)$$

and then the frequency of oscillations driven by tidal torque equals

$$\begin{aligned} \omega &= \sqrt{\frac{\tau_g^{(G)}/\theta}{I_z^{(G)}}} = \sqrt{\frac{2GM_{\text{MW}}\alpha_2}{\pi^{3/2}ab(a^2 + b^2)L^3}} \\ &= \sqrt{\frac{2 \times 4.3009 \times 10^{-6} [\text{kpc}(\text{km/s})^2/M_{\odot}] M_{\text{MW}}[M_{\odot}] \alpha_2}{\pi^{3/2}ab(a^2 + b^2)L[\text{kpc}](L[\text{km}])^2}}. \end{aligned} \quad (53)$$

TABLE III. The values of parameter α_2 in the linear approximation for the tidal torque $T_g^{(G)} = -\alpha_2\theta$, the frequency of oscillations ω , the period of oscillations T , the critical kinematic velocity $v_{cr} = \omega_{cr}L$, and the observed tangential velocity of internal rotation V_T [45].

Dwarf galaxy	$\alpha_2 \times 10^6$	ω (Gyr ⁻¹)	T (Gyr)	$v_{cr} = \omega_{cr}L$ (km/s)	V_T (km/s)
CarI	4.67	1.18	5.30	0.938	-9.56 ± 4.50
FnxI	1.25	0.803	7.83	0.607	-2.77 ± 1.30

Comparing the oscillation frequencies for CarI and FnxI in Table III with those in Table II, we see that they are approximately 2 times smaller in the case of the Gaussian ULDM density profile. Thus, we conclude that improving the knowledge of observationally deduced tangential velocity of rotation V_T for dwarf galaxies as well as improving the theoretical analysis of dark matter distribution in dwarf galaxies could provide important information on dark matter and its physical properties.

VIII. DISCUSSION AND CONCLUSIONS

Modeling dwarf galaxies as deformed Plummer spheres, the dynamical friction force and the corresponding induced torque for dwarf galaxies orbiting the Milky Way are determined in ultralight dark matter models. In addition, the torque due to the tidal force of the Milky Way is calculated. Numerically solving the obtained analytic formulas, both torques are found for CarI, FnxI, UMaI, UMiI, and Will dwarf galaxies. One of our main findings is that for all considered dwarf galaxies the torque produced by the dynamical friction force is suppressed by 3 or 4 orders of magnitude compared to the tidal torque. Thus, to study the internal kinematics of dwarf galaxies orbiting much more massive galaxies, it suffices to retain only the tidal torque in the corresponding analysis. However, the inner torque can be substantial for objects of similar mass rotating around their common center of mass or in the process of merger. Then this torque should be included in the analysis.

As for astrophysical observations of internal kinematics of dwarf galaxies, they show that many of the Milky Way's classical dwarf spheroidals are dispersion dominated, i.e., ($|V_{rot}|/\sigma_v \leq 1$) with the Carina dwarf spheroidal being the exception [45]. According to the study of dwarf satellites of MV/M31-like hosts from the IllustrisTNG50 simulation [47], internal rotation tends to be stronger in less evolved dwarfs or those that are less tidally stripped. As satellites pass close to the Milky Way center (pericenters), tidal forces tend to reduce their internal ordered rotation, converting it into random motions in agreement with simulations [34]. Gaia (especially its EDR3) has enabled for the first time, to the best of our knowledge, detection of

proper motions of stars within dwarf galaxies, which allows measurement of sky-plane rotation (not just line-of-sight velocity component) [45].

According to our study, the tidal torque is characterized by a nonmonotonous dependence on angle θ which defines misalignment of the largest axis of a dwarf galaxy with respect to the radius vector of its center-of-mass position. For small θ , it could be approximated by a linear dependence. For misaligned dwarf galaxies, this implies the possibility of oscillations around their equilibrium position. We found that the period of oscillations lies in a large interval from 0.561 Gyr for Will to 3.82 Gyr for FnxI. Comparing the critical kinematic velocity at Plummer radius v_{cr} with the observed tangential velocity V_T (see Table II), we conclude that, while the dwarf galaxy UMiI could undergo oscillations, other dwarfs CarI, FnxI, and ScII rotate.

Obviously, our assumption of a solid body rotation may be unrealistic for dwarf galaxies. Therefore, its validity should be checked in future studies of the internal kinematics of dwarf galaxies orbiting the Milky Way. In addition, it would be definitely interesting to extend our study of internal kinematics, restricted here to the analysis of small oscillations of dwarf galaxies, to the case of full-scale rotation that is astrophysically observed in dwarf spiral and irregular galaxies [48–53].

We found that, although the angular dependence of the tidal torque is similar for the Plummer and Gaussian density profiles of ULDM, the torque magnitude differs significantly. In the Gaussian profile case, the Carina dwarf galaxy exhibits a higher tidal torque than Fornax and this ratio is opposite for the Plummer density profile. These results show that the ULDM density profile significantly influences the tidal torque of dwarf galaxies. Furthermore, oscillation frequencies are approximately 2 times lower for the Gaussian profile than for the Plummer profile for both these dwarf galaxies. Therefore, more accurate measurements of dwarf-galaxy rotation velocities and improved modeling of dark matter distributions are essential for constraining the nature and properties of dark matter.

Finally, we briefly comment on the applicability of smooth density profiles (Plummer, Gaussian, and NFW) in ULDM models. While time-averaged macroscopic profiles and corresponding potentials capture the global friction and tidal torques τ_{inner} and τ_g , full Schrödinger-Poisson numerical simulations would be necessary to resolve the impact of fine-scale wave fluctuations on individual stellar trajectories. Incorporating such full wave dynamics constitutes a promising direction for future research.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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