

Nucleon axial form factors in generalized nonlocal chiral effective theory

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In this work, we first investigate the chiral transformation properties of the nonlocal pion field operator and construct a generalized nonlocal chiral Lagrangian that is invariant under $SU(2)$ chiral symmetry transformations. As a simple application, we then calculate the leading order and next-to-leading order pion one loop corrections to the nucleon axial form factors within nonlocal chiral perturbation theory. Then, we fit the next-to-leading order and next-to-next-to-leading order low energy coupling constants (LECs) to lattice QCD data. The fitted results show that the nonlocal LECs are comparatively smaller than their local counterparts. Finally, using these LECs, we compute the Q^2 dependence of the nucleon axial form factors. The numerical results indicate that the nonlocal nucleon axial form factors are consistent with lattice QCD data in a wide Q^2 range, up to $Q^2 = 1 \text{ GeV}^2$.

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I. INTRODUCTION

One of the main issues in quantum field theories is that, in most cases, the loop corrections to the physical observables are ultraviolet (UV) divergent. In renormalizable quantum field theories, divergences can be removed by introducing corresponding counterterms for these parameters such as mass, charge, and coupling constants. To renormalize a given theory, first, it is necessary to replace the original loop integral with a corresponding convergent one by modifying the integral limit, dimension, or the propagator and then calculate the loop integral. Such a mathematical process is usually referred to as regularization. Traditionally, dimensional and Pauli-Villars regularization methods have been widely applied to regularize the divergence of the theories. Nevertheless, the dimensional regularization cannot be applied to string field theory owing to nonlocal properties of strings [1–4]. In this case, nonlocal regularization has been introduced to cure the UV infinities of string loop integrals and extended to the gravity and other aspects of field theory [5–13]. Compared with other conventional regularization methods, the main advantage of nonlocal regularization is that it can be implemented in the Lagrangian level. More specifically, one can obtain a UV finite loop result by using the nonlocal propagators or vertices, which can be systematically generated from the nonlocal Lagrangian in a more

self-consistent way. More importantly, the nonlocal Lagrangian preserves the symmetry of the underlying theory.

In fact, the UV divergence stems from the problematic nature of interaction between the particles, in which it was assumed that particles interact with each other at the same space-time point [14]. However, for the composite particles such as baryons and mesons, this assumption breaks down due to the non-negligible finite size of particles in question. Therefore it is necessary to explore a fundamental theory that is compatible with the nonpoint nature of particle. Conventionally, as an important theoretical tool of describing the interactions of the baryons and mesons, the basic framework of chiral effective field theory and its renormalization have been extensively investigated. Although chiral perturbation theory successfully explained the experimental data for the nucleon magnetic moments and effective charge radius, the Q^2 dependence of electromagnetic and axial vector form factors in the large Q^2 region is incompatible with data [15–17]. On the other hand, in Refs. [12,18–20], some preliminary studies have been performed and constructed the basic theoretical framework of nonlocal regularization of electromagnetic interaction. In addition to this, in Refs. [13,21–26], it was shown that the nonlocally regularized pion and nucleon electromagnetic form factors successfully describe the data in the $0 \leq Q^2 \leq 1 \text{ GeV}^2$ region. This sparked the interest of exploring the feasibility of integrating the nonlocal regularization with chiral perturbation theory. Although there are some preliminary studies on nonlocal renormalization of chiral meson loop corrections to electromagnetic form factors and parton distribution functions that are only $U(1)$ gauge invariant, the systematic nonlocal renormalization of chiral effective theory is not yet well understood especially one considers the full $SU(2)$ or $SU(3)$ chiral symmetry of the

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nonlocal meson field. More importantly, although the Q^2 dependence of nucleon electromagnetic form factors in the large momentum transfer region was improved by the nonlocal framework, the nonlocal effects to the nucleon axial structure are not well defined. This prompted our interest to investigate the nucleon axial form factors in the nonlocal framework.

In this paper, we will develop a more generalized nonlocal regularization method for chiral meson loops in a more systematic way. Just like dimensional renormalization, the meson loop results from such nonlocal Lagrangian are ultraviolet finite and Lorentz invariant. More importantly, in contrast to previous formulations of nonlocal chiral theory, this method appears to be more efficient when we take into account the nonlocal interaction between the nucleon axial vector current and axial vector gauge field, since the newly constructed nonlocal Lagrangian is invariant under SU(2) chiral transformation. In addition, the generalized nonlocal chiral Lagrangian systematically yields regularized meson loop contributions at all orders without manually modifying the propagator or loop integral as in Pauli-Villars and sharp cutoff regularization.

It is necessary to mention that in this work we employ a nonlocal or finite-range regularization method to improve the convergence of loop contributions. However, as demonstrated in Ref. [27], the traditional power-counting rule, which is a fundamental ingredient of chiral perturbation theory, cannot be directly applied to nonlocal framework, since loop contributions in such scheme explicitly involve a regularization parameter.

This paper is organized as follows: In Sec. II we will shortly review the local chiral transformation properties of meson and fermion fields and analyze the chiral transformation of nonlocal meson fields and explicit parametrization of gauge link operators, then construct nonlocal chiral Lagrangians for pion-nucleon interactions. In Sec. III, to explicitly illustrate the nonlocal effect on nucleon axial form factors, we calculate pion one-loop corrections to the nucleon axial form factors within the nonlocal framework. In Sec. IV, we first fit low energy coupling constants (LECs) to the lattice data, then numerically compute the Q^2 dependence of axial form factors and discuss the axial charge as well as axial mean square radius.

II. NONLOCAL CHIRAL EFFECTIVE LAGRANGIAN

In this Section, we develop a more general nonlocal regularization scheme for chiral meson loops. As mentioned in Sec. I, nonlocal regularization of ultraviolet divergence of pion loops can be encoded into the Lagrangian without violating the underlying symmetry of the theories. The key step to obtain a nonlocally regularized Lagrangian is to, first of all, delocalize the field that is associated with loop contributions, then construct a nonlocal Lagrangian by replacing the local

field operator with its corresponding delocalized version and corresponding gauge link operators, which guarantees the chiral symmetry of the nonlocal Lagrangian. Actually, such a procedure can be implemented by delocalizing the field operators in both kinetic and interaction terms, which can regularize the ultraviolet divergences of loop contributions in the same manner. Therefore, we have a choice of where to locate the delocalized field operator in the Lagrangian. According to the location of the nonlocal field operator, the nonlocally regularized Lagrangian can be constructed in the inhomogeneous and homogeneous fashion. In the inhomogeneous nonlocal field theory, the delocalized field operator is located in the interaction Lagrangian while the kinetic term keeps its local form, and they can be transformed into each other by redefining the field operator [28]. In other words, in the inhomogeneous nonlocal theory the propagator for the field that is associated with loop corrections is the same as the local one and the interaction vertex introduces an extra regulator function that increases the superficial dimension of denominator of the loop integral. Consequently, the loop result is UV convergent. However, the price for this is that the symmetry of the theory is violated. To restore symmetry, it is necessary to introduce corresponding gauge link operators based on the symmetry transformation of the nonlocal field. To this end, we first will review the SU(2) chiral symmetry of local chiral perturbation theory under the investigation. For the local pion-nucleon interactions, as we have learned, it is governed by the chiral Lagrangians, which fulfill the SU(2) chiral symmetry. Generally, fundamental components of SU(2) chiral perturbation theory are the nucleon and pion fields. Under chiral transformation, they transform as,

$$U'(x) = R(x)U(x)L^\dagger(x), \quad N'(x) = K(x)N(x), \quad (1)$$

where $U(x)$ is the exponential representation of pion and is defined as $U(x) = \text{Exp}[i\frac{\phi(x)}{f_\pi}]$ with the SU(2) pion field matrix $\phi(x)$, $N(x)$ denotes nucleon fields, $R(x)$ and $L(x)$ denote the elements of the SU(2) chiral group and are parametrized in terms of exponential representations $R(x) = \exp[-i\theta_R(x)]$ and $L(x) = \exp[-i\theta_L(x)]$, where matrices $\theta_R(x)$ and $\theta_L(x)$ are defined as $\theta_R(x) = \sum_a \frac{1}{2}\theta_R^a(x)\tau^a$ and $\theta_L(x) = \sum_a \frac{1}{2}\theta_L^a(x)\tau^a$, with the τ^a ($a = 1, 2, 3$) denoting the Pauli matrices, and $K(x)$ denotes the element of $\text{SU}_V(2)$.

Our investigation now proceeds to examine the delocalization of pion fields and their chiral transformations. In fact, a nonlocal pion field operator can readily be obtained by replacing the local meson field operator $\phi(x)$ with the corresponding nonlocal field operator $\int d^4a H(a)\phi(x+a)$. Nevertheless, the chiral transformation rule for the nonlocal pion field $\int d^4a H(a)\phi(x+a)$ cannot be well defined according to chiral transformations in Eq. (1). In fact,

the chiral transformations include vector and axial vector transformations of pion and nucleon, since $SU(2)_L \times SU(2)_R \cong SU(2)_V \times SU(2)_A$. For the sake of convenience, we may initially consider the coupling between the external vector gauge field and the chiral vector current, where the local meson field $U(x)$ and nucleon field obey explicit $SU(2)_V$ transformation rules. Thus one is able to construct $SU(2)_V$ invariant nonlocal Lagrangian by including the nonlocal pion field in the theory and introducing the corresponding gauge link operator. However, such a

method is only valid when we construct a $SU(2)_V$ invariant nonlocal Lagrangian. As for axial vector couplings, since the local meson field lacks an explicit $SU(2)_A$ transformation behavior, one cannot properly define the symmetry properties of nonlocal interaction between the external axial vector field and axial vector current. Therefore, we investigate a more general method in which the nonlocal pion field exhibits approximate $SU(2)$ chiral transformation behavior just like its local counterpart. For that purpose, we define a nonlocal pion field operator as,

$$\hat{U}(x) \equiv \text{Exp} \left\{ \frac{1}{c_0} \int d^4a H(a) \text{Log}[G_R(x, x+a) U(x+a) G_L^\dagger(x, x+a)] \right\}, \quad (2)$$

where c_0 is a normalization constant and satisfies $c_0 \equiv \int da H(a)$, $G_{L(R)}(x, x+a)$ are the gauge link operators, $H(a)$ is the meson regulator function in space-time, $G_R(x, x+a)$ and $G_L(x, x+a)$ are right- and left-handed gauge link operators. The regulator function $H(a)$ has some properties and satisfies certain constraints, which are discussed in more detail in [5]. It is obvious that, according to Eq. (1), the pion field $U(x+a)$ transforms as,

$$U'(x+a) = R(x+a) U(x+a) L^\dagger(x+a). \quad (3)$$

Indeed, such a transformation gives rise to a chiral phase factor at two different space-time points. To cancel this, the gauge link operators have to transform,

$$G'_{L[R]}(x, y) = L(x) [R(x)] G_{L[R]}(x, y) L^\dagger(y) [R^\dagger(y)]. \quad (4)$$

Using these transformation properties in Eqs. (3) and (4), the nonlocal pion field operator $\hat{U}(x)$ then transforms as,

$$\hat{U}'(x) = \text{Exp} \left\{ \frac{1}{c_0} \int da H(a) \text{Log}[R(x) G_R(x, x+a) U(x+a) G_L^\dagger(x, x+a) L^\dagger(x)] \right\}. \quad (5)$$

Notice that left- and right-handed chiral phase factors $L(x)$ and $R(x)$ are now in the same space-time point. This allows us to define explicit chiral transformations of nonlocal field operator $\hat{U}(x)$. Actually, the expression in Eq. (5) can be further simplified using the Baker-Campbell-Hausdorff formula, which states that product of two matrix exponential $e^X e^Y$ can be expressed as $Z = \text{Log}(e^X e^Y)$ and Z is given in terms of a series of commutators of X and Y up to third order as,

$$Z = X + Y + \frac{1}{2} [X, Y] + \frac{1}{12} [X, [X, Y]] - \frac{1}{12} [Y, [X, Y]] + \dots \quad (6)$$

On the other hand, the operator $G_L(x, x+a) U(x+a) G_R^\dagger(x, x+a)$ is independent of local phase factors and by virtue of Baker-Campbell Hausdorff formula, it can be notationally written as,

$$G_R(x, x+a) U(x+a) G_L^\dagger(x, x+a) \equiv e^{iX(x,a)}, \quad (7)$$

where $X(x, a)$ is a nominal operator that we do not need to know [keep it mind that $G_L(x, x+a)$ and $G_R(x, x+a)$ will be parametrized in terms of right- and left-handed external vector fields $l_\mu(x)$ and $r_\mu(x)$]. Applying the Baker-Campbell-Hausdorff formula again, Eq. (5) is written as

$$\hat{U}'(x) = \text{Exp} \left\{ -i\theta_R(x) + i\theta_L(x) + \frac{i}{c_0} \int da H(a) X(x, a) + \frac{1}{2c_0} [\theta_L(x) + \theta_R(x), \int da H(a) X(x, a)] + \dots \right\}, \quad (8)$$

where the ellipsis represents the higher order Baker-Campbell Hausdorff series. Neglecting higher order series in the exponent, the chiral transformation of nonlocal meson field $\hat{U}'(x)$ finally can be expressed as,

$$\hat{U}'(x) = R(x)\hat{U}(x)L^\dagger(x). \quad (9)$$

Obviously, chiral transformation of nonlocal meson field $\hat{U}(x)$ is the same as its local counterpart $U(x)$. Furthermore, in the local limit $H(a) = c_0\delta^4(a)$ the nonlocal pion exactly recovers to the local case $\hat{U}(x) = U(x)$. In addition, taking $\phi(x+a) = 0$, the ground state of $\hat{U}$ is $\hat{U} = 1$, which is gauge invariant under $SU(2)_V$ transformation (for gauge link operators, take $r_\mu = l_\mu = v_\mu$). Finally, these right- and left-handed gauge link operators can easily be parametrized in terms of left- and right-handed external vector fields by analogy with the Wilson line operators in QCD [29],

$$G_R(x, x+a) = \mathcal{P}\text{Exp}\left[-i \int_x^{x+a} dz^\mu r_\mu(z)\right], \quad G_L(x, x+a) = \mathcal{P}\text{Exp}\left[-i \int_x^{x+a} dz^\mu l_\mu(z)\right], \quad (10)$$

where $\mathcal{P}$ denotes the path ordering operator, and $r_\mu(x)$ and $l_\mu(x)$ are the right- and left-handed gauge fields and whose chiral transformation properties are defined as,

$$l'_\mu(x) = L(x)l_\mu(x)L^\dagger(x) + iL(x)\partial_\mu L^\dagger(x), \quad r'_\mu(x) = Rr_\mu(x)R^\dagger + iR\partial_\mu R^\dagger. \quad (11)$$

Now let us consider the strategy of constructing a nonlocal effective Lagrangian for the pion-nucleon interaction. As shown in Eq. (9), with the help of the gauge link operators in Eq. (10), the nonlocal pion field $\hat{U}(x)$ exhibits chiral transformation behavior as its local counterpart $U(x)$. Thus, it is easy to obtain any order nonlocal chiral Lagrangian that will produce nonlocally regularized chiral meson loops. For example, the leading order (LO), next-to-leading order (NLO), and next-to-next-to-leading order (NNLO) $SU(2)$ chiral Lagrangians for nonlocal nucleon-pion interactions are just given by replacing the local meson field operator $U(x)$ with the nonlocal one $\hat{U}(x)$ in the local Lagrangian [30,31],

$$\begin{aligned} \mathcal{L}_{\pi N}^{(1)} &= \bar{N} \left\{ i\not{D} - m + \frac{g_A}{2} \gamma^\mu \gamma_5 \hat{u}_\mu \right\} N, \\ \mathcal{L}_{\pi N}^{(2)} &= \bar{N} \left\{ c_1 \langle \chi_+ \rangle - \frac{c_2}{8m^2} (\langle \hat{u}_\mu \hat{u}_\nu \rangle \{ \bar{D}^\mu, \bar{D}^\nu \} + \{ \bar{D}^\mu, \bar{D}^\nu \}) + \frac{c_3}{2} \langle \hat{u}_\mu \hat{u}^\mu \rangle + \frac{ic_4}{4} \sigma^{\mu\nu} [\hat{u}_\mu, \hat{u}_\nu] \right\} N, \\ \mathcal{L}_{\pi N}^{(3)} &= \bar{N} \left\{ \frac{d_{16}}{2} \gamma^\mu \gamma_5 \langle \chi_+ \rangle \hat{u}_\mu + \frac{id_{18}}{2} \gamma^\mu \gamma_5 [D_\mu, \chi_-] + \frac{d_{22}}{2} \gamma^\mu \gamma_5 [D_\nu, F_{\mu\nu}^-] \right\} N + \dots, \end{aligned} \quad (12)$$

where N denotes the nucleon field, which includes the proton and neutron via the definition $N = (p, n)^T$, m is the nucleon mass in the chiral limit, g_A , $c_i (i = 1 \dots 4)$, d_{16} , d_{18} , and d_{22} are the LO, NLO, and NNLO low energy coupling constants in the chiral limit, the covariant derivative D_μ , and the nonlocal vector and axial vector connections $\hat{\Gamma}_\mu$ and $\hat{u}_\mu$ are defined as,

$$\begin{aligned} D_\mu &= \partial_\mu + \hat{\Gamma}_\mu, \\ \hat{\Gamma}_\mu &= \frac{1}{2} [\hat{u}^\dagger (\partial_\mu - ir_\mu) \hat{u} + \hat{u} (\partial_\mu - il_\mu) \hat{u}^\dagger], \\ \hat{u}_\mu &= i[\hat{u}^\dagger (\partial_\mu - ir_\mu) \hat{u} - \hat{u} (\partial_\mu - il_\mu) \hat{u}^\dagger], \\ \chi_\pm &= \hat{u}^\dagger \chi \hat{u}^\dagger \pm \hat{u} \chi^\dagger \hat{u}, \\ F_{\mu\nu}^\pm &= \hat{u}^\dagger F_{\mu\nu}^R \hat{u} \pm \hat{u} F_{\mu\nu}^L \hat{u}^\dagger, \\ F_{\mu\nu}^L &= \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu], \\ F_{\mu\nu}^R &= \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu], \end{aligned} \quad (13)$$

where the nonlocal $\hat{u}$ is defined as $\hat{u} = \sqrt{\hat{U}}$, $\hat{u}^\dagger = \sqrt{\hat{U}^\dagger}$, and χ symbolically represents the explicit chiral symmetry breaking term and is defined in terms of liner combination of external scalar and pseudoscalar field $\chi = 2B_0[s(x) + ip(x)]$. According to the chiral transformation property of the nonlocal meson field $\hat{U}(x)$, the nonlocal $\hat{u}$ transforms as $\hat{u}' = R\hat{u}K^\dagger = K\hat{u}L^\dagger$. Under this notation, nonlocal gauge connections $\hat{\Gamma}_\mu$ and $\hat{u}_\mu$ transform as,

$$\begin{aligned} \hat{\Gamma}'_\mu &= K\hat{\Gamma}_\mu K^\dagger + iK^\dagger \partial_\mu K, \\ \hat{u}'_\mu &= K\hat{u}_\mu K^\dagger. \end{aligned} \quad (14)$$

Employing the generalized $SU(2)$ nonlocal Lagrangians, as an example, it is necessary to discuss the consistency between two results from the previous $U_V(1)$ invariant nonlocal pion-nucleon interactions and $SU(2)$ nonlocal chiral Lagrangians in Eq. (12). To be specific, expanding Eq. (12) in terms of pion and gauge fields gives rise to pion-nucleon interactions as,

$$\begin{aligned}
\mathcal{L}_{\pi N}^{(1)} = & -\frac{g_A}{2f_\pi c_0} \int d^4a H(a) \bar{N}(x) \gamma^\mu \gamma^5 \partial_\mu \phi(x+a) N(x) + \frac{i}{8f_\pi^2 c_0^2} \int d^4a_1 H(a_1) \int d^4a_2 H(a_2) \\
& \times \bar{N}(x) \gamma^\mu [\phi(x+a_1) \partial_\mu \phi(x+a_1) - \partial_\mu \phi(x+a_1) \phi(x+a_2)] N(x) + \bar{N}(x) \gamma^\mu V_\mu(x) N(x) \\
& + i \frac{g_A}{2f_\pi c_0} \int da H(a) \bar{N}(x) \gamma^\mu \gamma^5 [V_\mu(x+a) \phi(x+a) - \phi(x+a) V_\mu(x+a)] N(x) \\
& + i \frac{g_A}{2f_\pi c_0} \int da H(a) \bar{N}(x) \gamma^\mu \gamma^5 [\partial_\mu \phi(x+a) \int_{x+a}^x dz^\nu V_\nu(z) - \int_{x+a}^x dz^\nu V_\nu(z) \partial_\mu \phi(x+a)] N(x) \\
& + \frac{g_A}{c_0} \int da H(a) \bar{N}(x) \gamma^\mu \gamma^5 A_\mu(x+a) N(x) + i \frac{1}{4f_\pi c_0^2} \int da_1 H(a_1) \int da_2 H(a_2) \bar{N}(x) \gamma^\mu \\
& \times \{ [A_\mu(x+a_2) + A_\mu(x)] \phi(x+a_1) - \phi(x+a_1) [A_\mu(x+a_2) + A_\mu(x)] \} N(x) \\
& + i \frac{1}{4f_\pi c_0^2} \int da_1 H(a_1) \int da_2 H(a_2) \bar{N}(x) \gamma^\mu \left\{ \int_{x+a_2}^x dz^\nu A_\nu(z) \partial_\mu \phi(x+a_1) \right. \\
& \left. - \partial_\mu \phi(x+a_1) \int_{x+a_2}^x dz^\nu A_\nu(z) \right\} N(x) + \dots
\end{aligned} \tag{15}$$

where the ellipsis denotes higher-order pion derivative terms. From Eq. (15) we can see that interactions that contain the vector gauge field $V(x)$ are identical to the previous nonlocal result that was constructed based on the $U_V(1)$ vector gauge symmetry of nonlocal meson-nucleon interactions [19,21]. In addition, it is worth mentioning that the pion field and regulator function $H(a)$ are in one-to-one correspondence. In other words, every pion field can be regularized by a corresponding regulator function so that pion loop integrals are convergent at any chiral order. More importantly, in generalized SU(2) nonlocal framework, the nucleon axial vector current couples to a nonlocal axial vector gauge field. Consequently, the nucleon axial form factor at the tree level is proportional to the nonlocal regulator function, which has a non-negligible effect on the physical observables such as nucleon axial charge and axial mean square radius.

III. NUCLEON AXIAL FORM FACTORS

A. Definition of axial form factors

Theoretically, by analogy with electromagnetic form factors, the matrix elements of nucleon isovector axial vector currents can be parametrized in terms of nucleon axial form factors as [17,32],

$$\begin{aligned}
\langle N(p', s') | A^{\mu, a} | N(p, s) \rangle \\
= \bar{u}(p') \left[\gamma^\mu G_A(q^2) + \frac{q_\mu}{2m} G_P(q^2) \right] \frac{\tau^a}{2} \gamma_5 u(p), \tag{16}
\end{aligned}$$

where $u(p)$ and $u(p')$ are the nucleon spinors for initial and final states, τ^a are the Pauli isospin matrices, q is the four momentum transfer $q = p' - p$ and $Q^2 = -q^2$, and scalar functions $G_A(q^2)$ and $G_P(q^2)$ denote the nucleon axial-vector and induced pseudoscalar form factors. To describe the axial charge distribution, it is necessary to introduce the mean square of the axial charge radius, which is defined by

the slope of $G_A(Q^2)$ at zero momentum transfer. To this end, the nucleon axial vector form factor $G_A(Q^2)$ can be expanded around the small Q^2 as,

$$G_A(Q^2) = g_A \left[1 - \frac{1}{6} \langle r_A^2 \rangle Q^2 + \dots \right], \tag{17}$$

where g_A and $\langle r_A^2 \rangle$ are the renormalized nucleon axial charge and squared axial charge radius, respectively. Furthermore, the nucleon induced pseudoscalar coupling constant is defined as,

$$g_P \equiv \frac{m_\mu}{2m_N} G_P(-0.88m_\mu^2), \tag{18}$$

where m_μ denotes the muon mass. Experimentally, the induced pseudoscalar coupling constant g_P is determined from the muon-Hydrogen capture experiment. Similarly, the nucleon pseudoscalar form factor is defined as,

$$m_q \langle N(p') | P^a(0) | N(p) \rangle = \frac{m_\pi^2 f_\pi}{m_\pi^2 - q^2} G_{\pi N}(q^2) i \bar{u}(p') \gamma_5 \tau^a u(p) \tag{19}$$

where m_π represents the pion mass, f_π is the pion decay constant, and m_q is the quark mass under the isospin-symmetric limit $m_q = m_u = m_d$. Using the partial conservation of the axial current (PCAC) relation, the nucleon axial form factors $G_A(q^2)$ and $G_P(q^2)$ can be related to the nucleon pseudoscalar form factor $G_{\pi N}(q^2)$ via,

$$2m_N G_A(q^2) + \frac{q^2}{2m_N} G_P(q^2) = 2 \frac{m_\pi^2 f_\pi}{m_\pi^2 - q^2} G_{\pi N}(q^2). \tag{20}$$

In general, the pion-nucleon coupling constant $g_{\pi N}$ is defined as the limit of $G_{\pi N}(q^2)$ as $q^2 \rightarrow m_\pi^2$. Phenomenologically, the nucleon-pion coupling constant $g_{\pi N}$ is well defined



FIG. 1. LO (a) and NLO (b) pion one loop corrections to the nucleon self energy, dashed and solid lines denote the pion and nucleon fields, respectively.

by the pion-nucleon scattering length and takes the value $g_{\pi N} = 13.12 \pm 0.10$ [33,34]. In this work, we will use this value to fit the NNLO low-energy coupling constant d_{18} .

B. One loop contributions to the nucleon self energy

To obtain enormalized axial form factors $G_A(q^2)$ and $G_P(q^2)$, the loop results should be multiplied by the wave function renormalization constant Z . Generally, wave function renormalization constant Z and nucleon physical mass m_N can be obtained from the corresponding nucleon self-energy according to the definition,

$$m_N = m - 4m_\pi^2 c_1 + \Sigma(\not{p}, m), \quad (21)$$

$$Z = 1 + \left. \frac{d\Sigma(\not{p}, m)}{d\not{p}} \right|_{\not{p}=m}, \quad (22)$$

where the nucleon self energy $\Sigma(\not{p}, m)$ includes LO and NLO pion one loop contributions in Fig. 1 and they explicitly can be expressed as,

$$\begin{aligned} \Sigma_a(\not{p}, m) &= i \frac{3g_A^2}{4f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{k\gamma_5(\not{p}' - k + m)k\gamma_5}{D_\phi(k)D_N(p-k)} \tilde{F}^2(k), \\ \Sigma_b(\not{p}, m) &= -i \frac{1}{f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{-6c_1 m_\pi^2 + 3\frac{c_2}{m_\pi^2}(p.k)^2 + 3c_3 k^2}{D_\phi(k)} \\ &\quad \times \tilde{F}^2(k), \end{aligned} \quad (23)$$

where $\tilde{F}(k)$ is the Fourier transform of the regulator function $H(a)$ and is defined as $\tilde{F}(k) = \frac{1}{c_0} \int d^4 a H(a) e^{-ik.a}$, the $D_N(k)$ and $D_\pi(k)$ are defined as $D_\pi(k) = k^2 - m_\pi^2 + i\epsilon$, and $D_N(k) = k^2 - m^2 + i\epsilon$.

C. One loop contributions to the axial form factors

In this Subsection, we analytically calculate the pion one loop corrections to the nucleon axial form factors within the nonlocal framework. Compared to the local case, as shown in Eq. (15), the tree level coupling between the nucleon axial vector current and axial vector gauge field is proportional to a nonlocal regulator. Consequently, such a factor modifies the shape of Q^2 distribution as well as the mean square radius even at tree level. Therefore, as a simple example, we explore the effects of non locality on the distribution of nucleon axial form factors. Using the nonlocal interactions and axial vector currents, in Fig. 2 we display down relevant LO, NLO, and NNLO Feynman diagrams that will contribute to the nucleon axial form factors. From Fig. 2(a) one can easily write down the axial vertex operator for the tree diagram as,

$$\Gamma_a^\mu(p, q) = g_A \tilde{F}(q) \bar{u}(p') \gamma^\mu \gamma_5 u(p). \quad (24)$$

From Eq. (24) we can obtain that at tree level, the nucleon axial vector form factor is given by $G_{A,\text{tree}}(q^2) = g_A \tilde{F}(q)$. Notice that in the nonlocal case the tree level axial form factor explicitly depends on regulator $\tilde{F}(q)$. This is totally different from the local case, where Q^2 dependence only comes from the loop contributions. Moreover, this is also a direct evidence for the nonpoint nature of the pion since the factor $\tilde{F}(q)$ has a nonzero contribution to the pion axial charge radius. In the same way, the vertex operator for Fig. 2(b) is given by,

$$\Gamma_b^\mu(p, q) = -i \frac{g_A^3}{4f_\pi^2} \int \frac{dk^4}{(2\pi)^4} \bar{u}(p') k\gamma_5 \frac{\not{p}' - k + m}{D_N(p' - k)} \gamma^\mu \gamma_5 \frac{\not{p} - k + m}{D_N(p - k)} k\gamma_5 \frac{1}{D_\pi(k)} u(p) \quad (25)$$

$$\tilde{F}^2(k) \tilde{F}(q). \quad (26)$$

Note that the contribution of Fig. 2(b), apart from $\tilde{F}(q)$, is proportional to regulator function $\tilde{F}^2(k)$, which arises from the nonlocal pion field. More importantly, the loop integral is UV convergent if $\tilde{F}^2(k)$ is properly parametrized to increase the superficial dimension of the loop integrand. In the same manner, the loop contribution of Figs. 2(c) and 2(d) is expressed as,

$$\Gamma_{c+d}^\mu(p, q) = -i \frac{g_A}{2f_\pi^2} \int \frac{dk^4}{(2\pi)^4} \bar{u}(p') \left[k\gamma_5 \frac{\not{p}' - k + m}{D_N(p' - k)} \gamma^\mu + \gamma^\mu \frac{\not{p} - k + m}{D_N(p - k)} k\gamma_5 \right] \frac{1}{D_\pi(k)} u(p) \tilde{F}^2(k) [1 + \tilde{F}(q)]. \quad (27)$$

As mentioned in the previous Section, the nonlocal Lagrangian includes additional gauge link interactions, which generate additional Feynman diagrams like Figs. 2(e) and 2(f). Correspondingly, the loop contribution is given by,

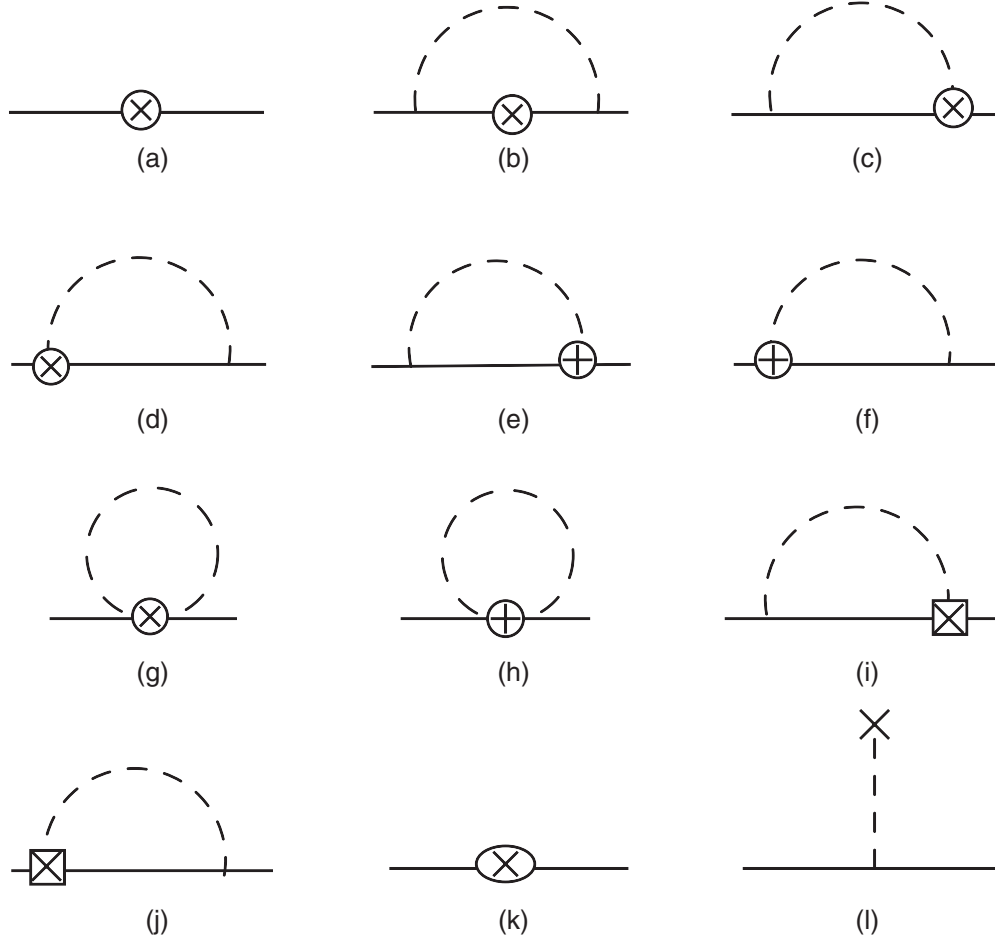


FIG. 2. Pion one loop corrections to the nucleon axial form factors are shown, where the crossed circle, the crossed square, and the crossed ellipse represent LO, NLO, and NNLO axial vector currents, the circled plus is the LO additional gauge link axial vector current, and the cross with a dashed line is the pion pole dominant diagram.

$$\Gamma_{e+f}^\mu(p, q) = i \frac{g_A}{2f_\pi^2} \int \frac{dk^4}{(2\pi)^4} \bar{u}(p') \left[k\gamma^5 \frac{\not{p}' - k + m}{D_N(p' - k)} k - k \frac{\not{p}' - k + m}{D_N(p - k)} k\gamma^5 \right] \frac{1}{D_\pi(k)} u(p) \tilde{F}^2(k) [1 - \tilde{F}(q)] \frac{1}{q^\mu}. \quad (28)$$

Obviously, the loop contribution from such a diagram vanishes in the local limit $\tilde{F}(q) = 1$. In addition to this, the tadpole diagram [Fig. 2(g)] that arises from the leading order Lagrangian is given by,

$$\Gamma_g^\mu(p, q) = -i \frac{g_A}{3f_\pi^2} \int \frac{dk^4}{(2\pi)^4} \bar{u}(p') \frac{1}{D_\pi(k)} \gamma^\mu \gamma^5 u(p) [\tilde{F}^2(k) \tilde{F}(q) + 2F^2(k)]. \quad (29)$$

Similarly, in the nonlocal case additional gauge link interaction yields a tadpolelike diagram [Fig. 2(h)] and the corresponding contribution can be written as,

$$\Gamma_h^\mu(p, q) = 0. \quad (30)$$

Note that the tadpolelike additional diagram does not contribute to axial form factors due to the oddity of the loop integrand. As for the NLO contribution, it only includes Figs. 2(i) and 2(j) and the loop contributions are given by,

$$\begin{aligned} \Gamma_{i+j}^\mu(p, q) = i \frac{g_A}{f_\pi^2} \int \frac{dk^4}{(2\pi)^4} \bar{u}(p') & \left[k\gamma^5 \frac{\not{p}' - k + m}{D_N(p' - k)} (2c_3 k^\mu + c_4 [\gamma^\mu, k]) \right. \\ & \left. + (2c_3 k^\mu - c_4 [\gamma^\mu, k]) \frac{\not{p}' - k + m}{D_N(p - k)} k\gamma^5 \right] \frac{1}{D_\pi(k)} u(p) \tilde{F}^2(k) \tilde{F}(q), \end{aligned} \quad (31)$$

where the c_3 and c_4 are the NLO effective coupling constants. At NNLO level, we only take into account the tree diagram Fig. 2(k) and the vertex operator reads,

$$\Gamma_k^\mu(p, q) = \bar{u}(p')\gamma^\mu\gamma^5 u(p)(4m_\pi^2 d_{16} + d_{22}q^2)\tilde{F}(q) + (-2md_{22})\bar{u}(p')q^\mu\gamma^5 u(p)\tilde{F}(q). \quad (32)$$

Obviously, NNLO contribution modifies the Q^2 distribution of the nucleon axial form factors as well as the axial charge radius of the nucleon. Finally, the external axial vector field couples with the nucleon by a pion intermediate state. Correspondingly, the vertex operator is given by,

$$\Gamma_l^\mu(p, q) = -\frac{2mg_A}{q^2 - m_\pi^2}\bar{u}(p)q^\mu\gamma^5 u(p)\tilde{F}(q) - \frac{(8d_{16} - 4d_{18})mm_\pi^2}{q^2 - m_\pi^2}\bar{u}(p)q^\mu\gamma^5 u(p)\tilde{F}(q). \quad (33)$$

From Eq. (33) we can see that the pion pole dominant diagram plays a crucial role in the improvement of the Q^2 dependence of the pseudoscalar form factor. By collecting each loop contribution listed above, the renormalized nucleon axial vector and axial scalar form factors are obtained as,

$$\begin{aligned} G_A(Q^2) &= Z \sum G_A^{(i)}(Q^2), \\ G_P(Q^2) &= Z \sum G_P^{(i)}(Q^2), \end{aligned} \quad (34)$$

where $G_A^{(i)}(Q^2)$ and $G_P^{(i)}(Q^2)$ ($i = a, b, c, \dots$) represent the loop contribution from each Feynman diagram.

IV. NUMERICAL RESULTS

A. Fitting of LECs

To numerically evaluate the Q^2 distribution of the axial form factors and axial charge radius of the nucleon, first of all, we need to parametrize the explicit expression of the regulator function $\tilde{F}(k)$. There are various forms of regulator functions, such as monopole, dipole, and Gaussian types. For convenience's sake, in this work we use the dipole form of the regulator function as [35,36],

$$\tilde{F}(k) = \frac{(m_\pi^2 - \Lambda^2)^2}{(k^2 - \Lambda^2 + i\epsilon)^2}, \quad (35)$$

where the Λ represents the nonlocal cutoff parameter, which was fitted from the differential cross section for inclusive hadron production and takes the value $\Lambda = (1.0 \pm 0.1)$ GeV [37]. In addition, as input parameters we also have to decide values of the NLO and NNLO low energy coupling constants c_3 , c_4 , d_{16} , and d_{22} . In the standard phenomenological analysis, these parameters can be extracted from the pion-nucleon scattering. However, in such a method, one has to take into account loop corrections to the scattering length. From this point of view, values of LECs depend on renormalization schemes. Therefore values of locally fitted LECs differ from those of the nonlocal framework and cannot be applied in this work. If we are seeking to extract them from the pion-

nucleon scattering data within the nonlocal framework, we have to handle massive loop contributions. To this end, in a straightforward manner we will refit them to the lattice data for the pion mass dependence of nucleon mass. As seen from Eq. (22), up to next-to-next-to-leading order nucleon mass m_N is proportional to LECs c_1 , c_2 , c_3 . Using the m_N as a fitting function, we have performed least χ^2 analysis on two groups of lattice data and the best fit result is displayed in Fig. 3, where the error bars represent the lattice data while the error bands represent the best fit model result. Correspondingly, the best fit values for c_1 , c_2 , c_3 are listed in Table I. From Table I, we can observe that best fit values for next-to-leading order LECs c_1 , c_2 , and c_3 are $c_1 = (-0.300 \pm 0.213)$ GeV⁻¹, $c_2 = (0.526 \pm 0.106)$ GeV⁻¹, $c_3 = (0.131 \pm 0.423)$ GeV⁻¹, $c_1 = (-0.211 \pm 0.020)$ GeV⁻¹, $c_2 = (-0.221 \pm 0.021)$ GeV⁻¹, $c_3 = (-0.055 \pm 0.084)$ GeV⁻¹, and $c_1 = (-0.119 \pm 0.036)$ GeV⁻¹, $c_2 = (-0.755 \pm 0.035)$ GeV⁻¹, $c_3 = (-0.189 \pm 0.140)$ GeV⁻¹ with corresponding best fit parameters $\chi^2/\text{d.o.f.} = 0.23$, $\chi^2/\text{d.o.f.} = 0.18$ and $\chi^2/\text{d.o.f.} = 0.37$, respectively. It is worth mentioning that in the nonlocal case the LECs are comparatively smaller than their local counterpart. In

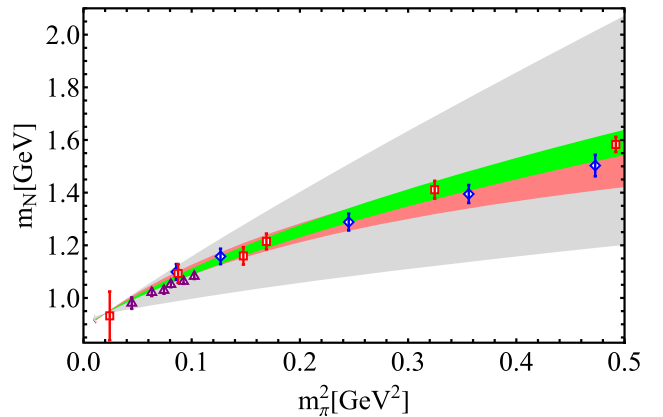


FIG. 3. Nucleon mass as a function of pion mass, where the purple triangles, blue diamonds, and red squares are the lattice data points from [38,39] and [40], and the gray, pink, and green error bands represent the theoretical uncertainty that originates from the uncertainty of c_1 , c_2 , and c_3 .

TABLE I. NLO low energy coupling constants c_1 , c_2 , and c_3 are determined by fitting to the m_N lattice data from [38–40]. The nonlocally renormalized nucleon mass can be obtained from Eq. (22).

$m(\text{GeV})$	$c_1(\text{GeV}^{-1})$	$c_2(\text{GeV}^{-1})$	$c_3(\text{GeV}^{-1})$	$\chi^2/\text{d.o.f.}$	Lattice data
1.045 ± 0.173	-0.300 ± 0.213	0.526 ± 0.106	0.131 ± 0.423	0.23	[38]
1.184 ± 0.033	-0.211 ± 0.020	-0.221 ± 0.021	-0.055 ± 0.084	0.18	[39]
1.285 ± 0.050	-0.119 ± 0.036	-0.755 ± 0.035	-0.189 ± 0.140	0.37	[40]

TABLE II. NLO and NNLO low-energy constants c_4 and d_{16} are fitted to the lattice data for pion-mass dependence of the renormalized axial charge g_A^R of the nucleon.

g_A	$d_{16}(\text{GeV}^{-2})$	$c_4(\text{GeV}^{-1})$	$\chi^2/\text{d.o.f.}$	Lattice data
1.156 ± 0.043	1.078 ± 0.164	0.948 ± 0.219	1.095	[43]
1.107 ± 0.028	0.757 ± 0.105	1.095 ± 0.128	0.735	[44]

addition to this, the nucleon chiral mass m is larger than in the local case.

In a similar manner, NLO and NNLO low energy coupling constants c_4 and d_{16} can be fitted to the lattice data for the pion mass dependence of nucleon axial charge g_A . The best values of these parameters are listed in Table II. Notice that bare axial charge g_A and d_{16} are apparently smaller than the local one [41]. The corresponding fitting curves are displayed in the left panel of Fig. 4. Eventually, the NNLO coupling constant d_{22} can be fitted to the pion mass dependence of axial charge radius $\langle r_A^2 \rangle$. The fitting result shows that best values of d_{22} are $d_{22} = (-1.533 \pm 0.216) \text{ GeV}^{-2}$ and $d_{22} = (-1.687 \pm 0.037) \text{ GeV}^{-2}$ with best fit parameters $\chi^2/\text{d.o.f.} = 1.57$ and $\chi^2/\text{d.o.f.} = 1.14$.

B. Q^2 dependence of axial form factors

Having determined these LECs, we are now in a position to numerically compute the Q^2 distributions of

the nucleon axial form factors $G_A(Q^2)$ and $G_P(Q^2)$. In addition to these parameters, in the numerical analysis we will use physical values of pion and nucleon mass as well as the pion decay constant as $m_\pi = 139.57 \text{ MeV}$, $m_N = 938.92 \text{ MeV}$, $f_\pi = 92.42 \text{ MeV}$. The numerical results for the Q^2 dependence of the nucleon axial vector form factor $G_A(Q^2)$ are displayed in the left panel of Fig. 5. It is obvious that the nonlocal model predictions are in good agreement with lattice data in the wide momentum transfer region $0 \leq Q^2 \leq 1 \text{ GeV}^2$. This implies that the nonlocal model significantly improves the Q^2 dependence of axial form factors in the large- Q^2 region compared to the local case, in which the standard (local) chiral perturbation theory can only describe the experimental data in the region $0 \leq Q^2 \leq 0.4 \text{ GeV}^2$ [17]. This is an important feature of nonlocal theory, in which the high-energy effects that were originally poorly convergent are suppressed due to the large mass scale Λ . In addition to this, the calculations yield a nucleon axial charge of $g_A = (1.212 \pm 0.071)$. Correspondingly, the mean

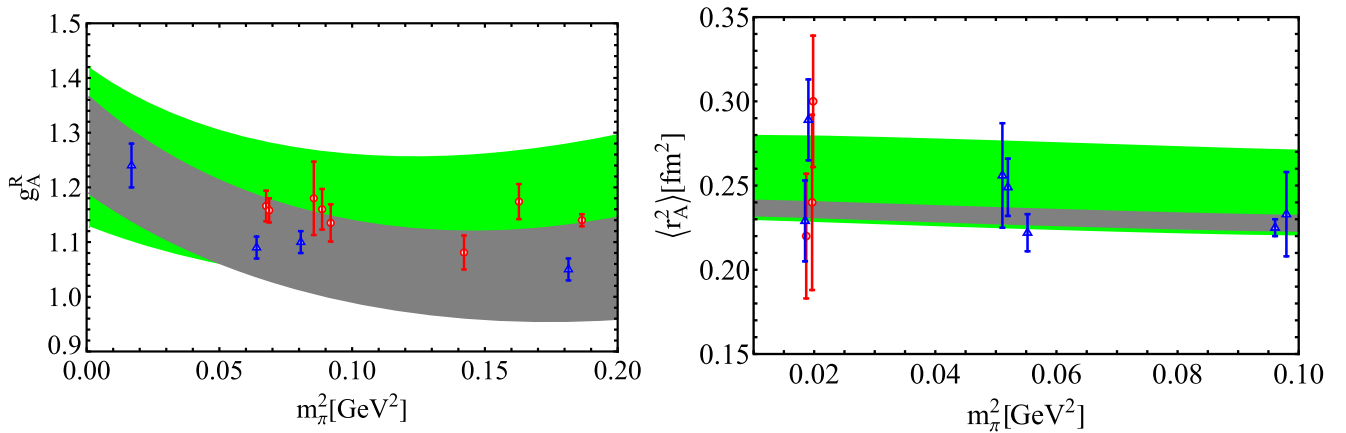


FIG. 4. Nucleon axial charge g_A^R (left panel) and squared axial charge radius $\langle r_A^2 \rangle$ (right panel) as a function of pion mass, where the red circles and blue triangles are the lattice data from [42–45], and the green and pink error bands represent the theoretical uncertainty arising from the uncertainty of c_4 , d_{16} , d_{22} .

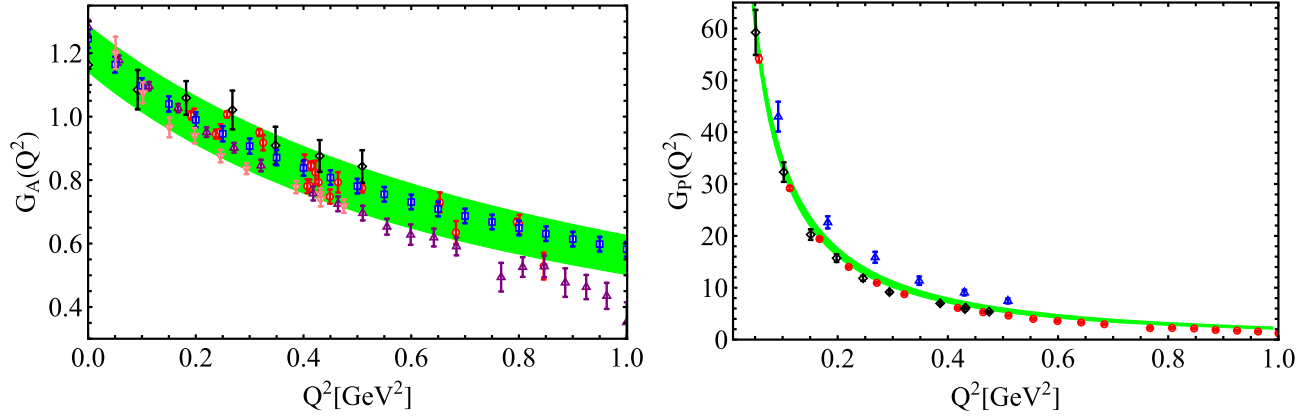


FIG. 5. Q^2 dependence of the nucleon axial form factors $G_A(Q^2)$ (left) and $G_P(Q^2)$ (right) is compared with lattice QCD data, where the error bars with different symbols denote lattice data from Refs. [45,47–49,54], while the green error bands represent the theoretical uncertainties arising from the LECs and the regulator cutoff Λ .

square radius of the nucleon axial charge is obtained as $\langle r_A^2 \rangle = (0.254 \pm 0.025) \text{ fm}^2$. These values are consistent with recent lattice QCD results, the MuCap experimental measurement, and the generalized parton distribution analysis, within their respective uncertainty ranges [45–50]. As for the induced pseudoscalar form factor $G_P(Q^2)$, in addition to NLO and NNLO chiral low energy coupling constants c_3 , c_4 , d_{16} and d_{22} , it also depends on the undefined chiral coupling constants d_{18} . On the other hand, according to the relation in Eq. (20) and the definition of $g_{\pi N}$, the parameter d_{18} can be expressed in terms of $g_{\pi N}$. Using the experimental value of pion-nucleon coupling constant $g_{\pi N} = 13.12 \pm 10$, the low-energy coupling constant d_{18} is fitted as $d_{18} = -19.58 \pm 0.38$. Note that this value is significantly larger than its local counterpart. We assume that such discrepancy arises from the neglecting axial vector-meson and NLO pure mesonic contributions in the present analysis [17,51]. Furthermore, according to the definition in Eq. (18), the nucleon induced pseudoscalar coupling constant is found to be $g_p = 8.57 \pm 0.26$, which agrees with the lattice

simulation values [45,47,48,52] and MuCap measurement value $g_p = 8.06 \pm 0.55$ [53]. The resulting Q^2 dependence of $G_P(Q^2)$ is shown in the right panel of Fig. 5 and the nonlocal model result is in good agreement with lattice data in the wide four-momentum transform region $0 \leq Q^2 \leq 1 \text{ GeV}^2$.

In Fig. 6, we have plotted the loop and tree contributions to the axial form factors. As seen from the left panel of Fig. 6, in the nonlocal case the tree contributions to the $G_A(Q^2)$ are no longer a constant and strongly improve the Q^2 dependence. This in some sense explains the nonpoint nature of the nucleon since at tree level the nucleon axial charge radius is not zero. This is contrary to local results in which the axial charge radius is defined in terms of the slope of loop contribution. As for the pseudoscalar form factors, the pion-pole dominant diagram accounts for most of the total contribution, while the loop contribution is substantially suppressed. Such conclusion also stands for the previous result for nucleon electromagnetic form factors from the local chiral perturbation theory [21]. To explain

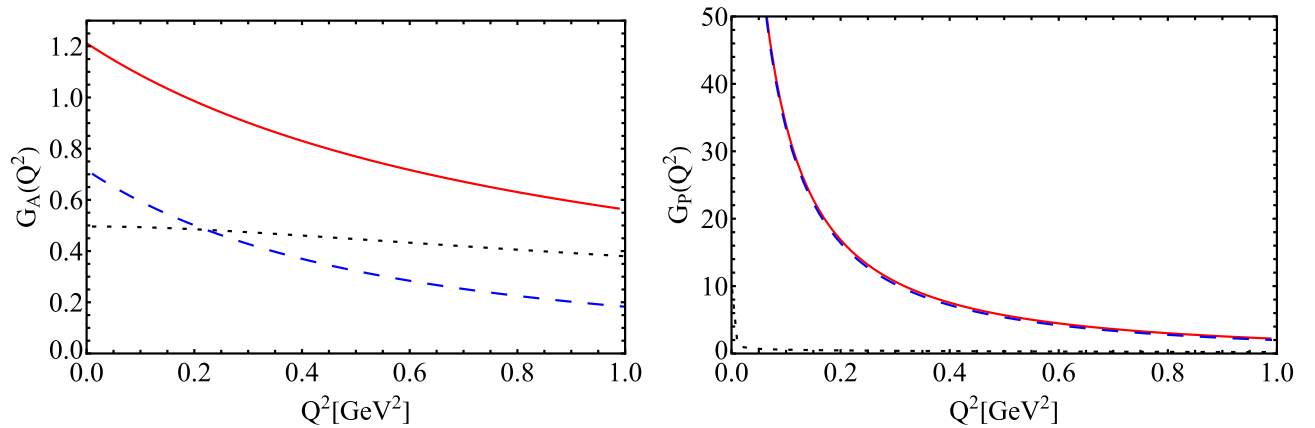


FIG. 6. Tree-level and loop contributions to the nucleon axial form factor (left panel) and to the induced pseudoscalar form factor (right panel) are shown. The solid red curves denote the total contributions, the blue dashed curves correspond to the tree-level contributions, and the black dotted curves represent the loop contributions.

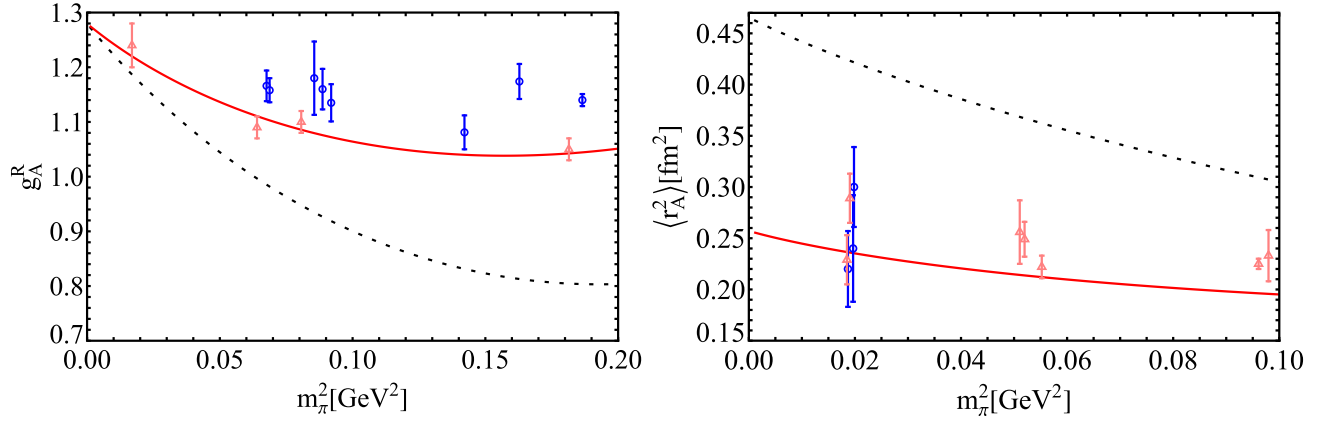


FIG. 7. Pion mass dependence of the nucleon axial charge (left panel) and the axial charge radius (right panel) at NLO (dashed line) and NNLO solid line) orders is displayed. The pink triangles and blue circles denote the lattice data points from [42–45].

the convergence of different chiral orders, in Fig. 7, we illustrate the pion mass dependence of nucleon axial charge g_A and the axial charge radius $\langle r_A^2 \rangle$ at NLO and NNLO order, where blue circles and pink triangles denote lattice simulation data. As shown in the left panel of Fig. 7, the NLO prediction for the nucleon axial charge is consistently lower than the lattice data. Specifically, the discrepancy becomes more obvious in the large pion mass region, while the deviation between the NNLO result and the lattice data is significantly smaller. With the inclusion of NNLO corrections, the theoretical prediction gradually converges toward the lattice data, demonstrating the improved convergence of the perturbative expansion in the nonlocal framework. For the squared axial charge radius $\langle r_A^2 \rangle$, the nonlocal prediction at NLO yields a relatively large value of $\langle r_A^2 \rangle = 0.46 \text{ fm}^2$ and decreases as the pion mass increases. By considering the NNLO corrections, the result from the nonlocal model prediction becomes consistent with the

lattice data, and the pion mass dependence of $\langle r_A^2 \rangle$ becomes significantly weaker. Similarly, a comparison of the induced pseudoscalar coupling constant g_P at NLO and NNLO orders is presented in Fig. 8. Note that the nonlocal model predictions at NLO are lower than the Lattice QCD data. By taking into account the NNLO corrections, the pion mass dependence of g_P is significantly improved, and the nonlocal result then successfully describes the lattice data at the physical pion mass. We have to mention that in the present analysis, the low-energy constant d_{18} was fitted from the experimental value of the pion-nucleon coupling constant $g_{\pi N}$, which leads to a rather large value. Consequently, the discrepancy between the model prediction and lattice data becomes worse for large pion mass. Conversely, one may alternatively fit d_{18} to the pion mass dependence of g_P . However, we find that this approach yields a g_P value smaller than the MuCap experimental measurement. As a result, the Q^2 dependence of $G_P(Q^2)$ fails to match the lattice and experimental data.

V. CONCLUSION

In the conventional field theory, the UV divergence intrinsically is connected with the point particle assumption. On the other hand, the nonlocal field theory reflects the nonpointlike nature of particles that need to be investigated. From this viewpoint, nonlocal theory gives rise to UV convergent loop contributions by introducing nonlocal field operators and corresponding gauge link operators with which the underlying symmetry of the theory is still preserved. In chiral perturbation theory, where the chiral symmetry serves as a fundamental symmetry of the underlying theory, the SU(2) or SU(3) chiral symmetry must be fulfilled when the nonlocal pion field is included.

In this work, we first investigate a generalized nonlocal regularization method to regularize the pion loop contribution. Our analysis shows that the novel nonlocal

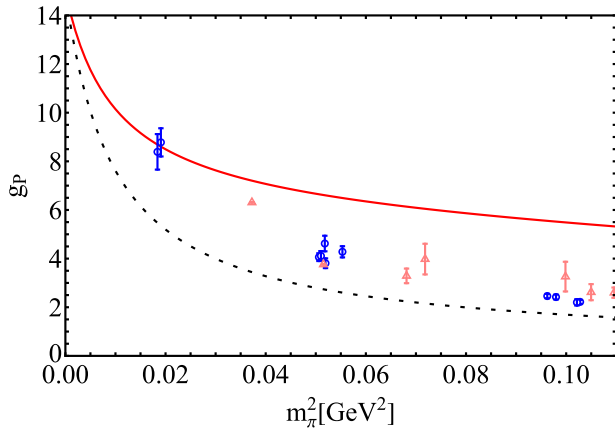


FIG. 8. Pion mass dependence of induced pseudoscalar coupling constant is shown, where the solid and dashed lines represent the NLO and NNLO contributions, and the pink triangles and blue circles denote the lattice data points from [47,54].

chiral Lagrangians are still SU(2) chiral invariant and can regularize the pion loop contributions up to any chiral order. An obvious feature of the nonlocal chiral interaction is that the coupling between the nucleon axial current and the axial vector field introduces an additional regulator function $\tilde{F}(q)$ that modifies the Q^2 dependence of axial form factors and their slopes. Then, as an application of nonlocal regularization, we have calculated the pion one loop correction to the nucleon axial form factors. To numerically evaluate the axial form factors, we have fit NLO and NNLO low-energy coupling constants to the QCD lattice data. The results show that the fitting values of these constants are apparently smaller than their local counterparts. Finally, using these parameters, we have calculated the Q^2 dependence of axial form factors $G_A(Q^2)$ and $G_P(Q^2)$. Our analysis shows that the nonlocal model result for axial vector form factor $G_A(Q^2)$ is consistent with the QCD lattice data over a wide momentum-transfer region $0 \leq Q^2 \leq 1 \text{ GeV}^2$.

Correspondingly, the nucleon axial charge and mean square radius are obtained as $g_A = (1.212 \pm 0.071)$, $\langle r_A^2 \rangle = (0.254 \pm 0.025) \text{ fm}^2$. Moreover, the induced pseudoscalar form factor $G_P(Q^2)$ is consistent with lattice data within the momentum-transfer region $0 \leq Q^2 \leq 1 \text{ GeV}^2$. Correspondingly, the pseudoscalar coupling constant is extracted to be $g_P = 8.57 \pm 0.26$. This value is in good agreement with both lattice and experimental data from the MuCap experiment.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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- [1] D. Evens, J. W. Moffat, G. Kleppe, and R. P. Woodard, *Phys. Rev. D* **43**, 499 (1991).
 - [2] L. Buoninfante, G. Lambiase, and M. Yamaguchi, *Phys. Rev. D* **100**, 026019 (2019).
 - [3] J. W. Moffat, *Phys. Rev. D* **41**, 1177 (1990).
 - [4] A. Basu and S. D. Joglekar, *Eur. Phys. J. Direct* **3**, 4 (2001).
 - [5] G. V. Efimov, *Commun. Math. Phys.* **5**, 42 (1967).
 - [6] V. A. Alebastrov and G. V. Efimov, *Commun. Math. Phys.* **31**, 1 (1973).
 - [7] E. T. Tomboulis, *Phys. Rev. D* **92**, 125037 (2015).
 - [8] G. Kleppe and R. P. Woodard, *Nucl. Phys.* **B388**, 81 (1992).
 - [9] J. Boos, J. Pinedo Soto, and V. P. Frolov, *Phys. Rev. D* **101**, 124065 (2020).
 - [10] A. Mazumdar and G. Stettinger, *Nucl. Phys.* **B956**, 115024 (2020).
 - [11] J. Terning, *Phys. Rev. D* **44**, 887 (1991).
 - [12] K. Ohta, *Phys. Rev. C* **41**, 1213 (1990).
 - [13] I. V. Anikin, M. A. Ivanov, N. B. Kulimanova, and V. E. Lyubovitskij, *Z. Phys. C* **65**, 681 (1995).
 - [14] E. Di Grezia, G. Esposito, and G. Miele, *Int. J. Geom. Methods Mod. Phys.* **06**, 201 (2009).
 - [15] T. Fuchs, J. Gegelia, and S. Scherer, *J. Phys. G* **30**, 1407 (2004).
 - [16] B. Kubis and U. G. Meissner, *Nucl. Phys.* **A679**, 698 (2001).
 - [17] M. R. Schindler, T. Fuchs, J. Gegelia, and S. Scherer, *Phys. Rev. C* **75**, 025202 (2007).
 - [18] B. Holdom, J. Terning, and K. Verbeek, *Phys. Lett. B* **232**, 351 (1989).
 - [19] Y. Salamu, C.-R. Ji, W. Melnitchouk, A. W. Thomas, and P. Wang, *Phys. Rev. D* **99**, 014041 (2019).
 - [20] Y. Salamu, C.-R. Ji, W. Melnitchouk, A. W. Thomas, P. Wang, and X. G. Wang, *Phys. Rev. D* **100**, 094026 (2019).
 - [21] F. He and P. Wang, *Phys. Rev. D* **97**, 036007 (2018).
 - [22] F. He and P. Wang, *Phys. Rev. D* **98**, 036007 (2018).
 - [23] F. He and P. Wang, *Phys. Rev. D* **100**, 074032 (2019).
 - [24] F. He, C.-R. Ji, W. Melnitchouk, Y. Salamu, A. W. Thomas, P. Wang, and X. G. Wang, *Phys. Rev. D* **105**, 094007 (2022).
 - [25] P. Wang, F. He, C.-R. Ji, and W. Melnitchouk, *Prog. Part. Nucl. Phys.* **129**, 104017 (2023).
 - [26] M. A. Ivanov, M. P. Locher, and V. E. Lyubovitskij, *Few-Body Syst.* **21**, 131 (1996).
 - [27] J. F. Donoghue, B. R. Holstein, and B. Borasoy, *Phys. Rev. D* **59**, 036002 (1999).
 - [28] L. Buoninfante, G. Lambiase, and A. Mazumdar, *Nucl. Phys.* **B944**, 114646 (2019).
 - [29] H. Cheng and E. C. Tsai, *Phys. Rev. D* **36**, 3196 (1987).
 - [30] N. Fettes, U. G. Meissner, M. Mojzis, and S. Steininger, *Ann. Phys. (N.Y.)* **283**, 273 (2000).
 - [31] N. Fettes, U. G. Meissner, and S. Steininger, *Nucl. Phys.* **A640**, 199 (1998).
 - [32] V. Bernard, N. Kaiser, and U. G. Meissner, *Phys. Rev. D* **50**, 6899 (1994).
 - [33] V. Baru, C. Hanhart, M. Hoferichter, B. Kubis, A. Nogga, and D. R. Phillips, *Phys. Lett. B* **694**, 473 (2011).
 - [34] V. Baru *et al.*, *Nucl. Phys.* **A872**, 69 (2011).
 - [35] H. Forkel, M. Nielsen, X. m. Jin, and T. D. Cohen, *Phys. Rev. C* **50**, 3108 (1994).
 - [36] M. J. Musolf and M. Burkardt, *Z. Phys. C* **61**, 433 (1994).
 - [37] Y. Salamu, C.-R. Ji, W. Melnitchouk, A. W. Thomas, P. Wang, and X. G. Wang, *Phys. Rev. D* **100**, 094026 (2019).
 - [38] B. Hu, H. Du, X. Jiang, K. F. Liu, P. Sun, and Y. B. Yang, *arXiv:2411.18402*.
 - [39] A. Walker-Loud *et al.*, *Phys. Rev. D* **79**, 054502 (2009).

- [40] S. Aoki *et al.* (PACS-CS Collaboration), *Phys. Rev. D* **79**, 034503 (2009).
- [41] D. L. Yao, L. Alvarez-Ruso, and M. J. Vicente-Vacas, *Phys. Rev. D* **96**, 116022 (2017).
- [42] B. Jäger *et al.*, *Proc. Sci. LATTICE2013* (2014) 272 [arXiv:1311.5804].
- [43] R. G. Edwards, G. T. Fleming, Ph. Hägler, J. W. Negele, K. Orginos, A. V. Pochinsky, D. B. Renner, D. G. Richards, and W. Schroers (LHPC Collaboration), *Phys. Rev. Lett.* **96**, 052001 (2006).
- [44] R. Horsley, Y. Nakamura, A. Nobile, P. E. L. Rakow, G. Schierholz, and J. M. Zanotti, *Phys. Lett. B* **732**, 41 (2014).
- [45] R. Gupta, Y.-C. Jang, H.-W. Lin, B. Yoon, and T. Bhattacharya, *Phys. Rev. D* **96**, 114503 (2017).
- [46] D. Djukanovic, G. von Hippel, J. Koponen, H. B. Meyer, K. Ottnad, T. Schulz, and H. Wittig, *Phys. Rev. D* **106**, 074503 (2022).
- [47] S. Capitani, M. Della Morte, D. Djukanovic, G. M. von Hippel, J. Hua, B. Jäger, P. M. Junnarkar, H. B. Meyer, T. D. Rae, and H. Wittig, *Int. J. Mod. Phys. A* **34**, 1950009 (2019).
- [48] C. Alexandrou, S. Bacchio, M. Constantinou, J. Finkenrath, R. Frezzotti, B. Kostrzewa, G. Koutsou, G. Spanoudes, and C. Urbach (Extended Twisted Mass Collaboration), *Phys. Rev. D* **109**, 034503 (2024).
- [49] C. Alexandrou *et al.*, *Phys. Rev. D* **103**, 034509 (2021).
- [50] F. Irani, M. Goharipour, H. Hashamipour, and K. Azizi, *Phys. Rev. D* **108**, 074018 (2023).
- [51] J. M. Chen, Z. R. Liang, and D. L. Yao, *Front. Phys. (Beijing)* **19**, 64202 (2024).
- [52] T. Yamazaki, Y. Aoki, T. Blum, H.-W. Lin, S. Ohta, S. Sasaki, R. Tweedie, and J. Zanotti, *Phys. Rev. D* **79**, 114505 (2009).
- [53] V. A. Andreev *et al.* (MuCap Collaboration), *Phys. Rev. Lett.* **110**, 012504 (2013).
- [54] Y. C. Jang *et al.* (PNDME Collaboration), *Phys. Rev. D* **109**, 014503 (2024).