

Chiral electromagnetic surface waves on Chern-Simons interfaces

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We show that Maxwell theory with a codimension-1 Chern-Simons interface may support chiral electromagnetic surface waves on the interface, even when the bulk theory on both sides is conventional vacuum electrodynamics in infinite space. Solving the exact boundary value problem, we find that the Chern-Simons interaction acts with opposite sign on the two helicities oriented along the interface, giving rise to one normalizable mode localized on the interface. This mode is a gapless chiral surface photon with linear dispersion and a frequency-independent index of refraction set by the Chern-Simons coefficient. This mode exists despite the absence of bulk ambient material response or geometric confinement.

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Topological terms localized to interfaces can induce qualitatively new phenomena in conventional bulk theories. Chern-Simons couplings in electromagnetism provide a particularly simple example. These terms do not modify local propagation in regions where their coefficient is constant. However spatial variations of the coefficient introduce nontrivial boundary effects that act directly on photon polarization and mode structure, as first emphasized in axion electrodynamics [1]. Such interface-borne terms arise in a variety of effective field theories after integrating out heavy degrees of freedom and are most naturally understood as topological response phenomena whose physical consequences are localized at interfaces [2]. Electromagnetic response effects associated with such θ -terms and their interfaces have been extensively discussed in the literature [3]. However, an explicit field-theoretic realization of propagating single-helicity electromagnetic modes localized on Chern-Simons interfaces has not been identified in a purely field-theoretic setting. In this Letter, we show that Maxwell theory with a Chern-Simons interface does support an intrinsic chiral surface electromagnetic excitation. This mode arises from the interplay of Maxwell dynamics and the Chern-Simons boundary conditions, without a need for bulk ambient material.

Below, we consider Maxwell theory in the presence of a codimension-1 Chern-Simons interface. Our motivation comes from recalling that Chern-Simons terms are inherently chiral, coupling with opposite sign to different photon

helicities at the interface. This acts as an impulsive momentum transfer whose sign flips between those two helicities. Thus, for bulk modes, the interface boundary conditions are equivalent to a codimension-1 δ -function potential, with an effective interaction that is repulsive for one helicity and attractive for the other. This results in a normalizable, surface-localized mode for only one helicity parallel with the interface, whose localization weakens at low frequency. To look for modes bound to the interface, we analyze the spectrum of the electromagnetic field focusing on excitations supported at the Chern-Simons interface.

Starting from the ensuing equations of motion, we formulate the exact boundary value problem for electric and magnetic fields of the modes propagating along the interface. Diagonalization in the 2D helicity eigenbasis along the interface reveals that one helicity indeed experiences an attractive δ -function interaction, confirming our intuition. Allowing physical momentum to be parallel to the interface then produces a surface-bound photon with a definite helicity and with dispersion relation fixed completely by the Chern-Simons coefficient. This chiral surface photon realizes an intrinsic electromagnetic waveguide, in which confinement arises from topology rather than geometry or response of the ambient material.

We couple Maxwell theory to a pseudoscalar $\theta(x)$ through the Chern-Simons density,

$$S = -\frac{1}{4} \int d^4x F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \int d^4x \theta(x) F^{\mu\nu} {}^*F_{\mu\nu}, \quad (1)$$

where we use the Hodge dual of the electromagnetic field strength ${}^*F^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma}$ and the mostly minus metric. In the regions where θ is constant, (1) reduces to conventional electrodynamics. In those regions the bulk photon propagation is not modified [4]. We include the interface by

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specializing to configurations in which θ is piecewise constant, with a discontinuous jump across a codimension-1 hypersurface. Writing $\theta(z) = \theta_- + \Delta\theta\Theta(z)$, the bulk term is a total derivative for $z \neq 0$, while the jump $\Delta\theta$ at $z = 0$ induces a Chern-Simons interaction on the interface, a construction that appears naturally in both field-theoretic and condensed matter realizations of θ electrodynamics [5,6]. We assume that the interface is approximately planar on the scale of the electromagnetic wavelength. Thus, translational invariance parallel to the surface is a good approximation, and propagation along the interface is well defined. As a result, we can identify the wall with the $z = 0$ plane.

Treating $\theta(x)$ as a fixed given background field and varying the action with respect to the gauge field A_ν yields the modified Maxwell equations

$$\partial_\mu(F^{\mu\nu} + \theta(x)^*F^{\mu\nu}) = 0, \quad (2)$$

where external electric charges and currents vanish. Where θ is constant, electromagnetic dynamics reduces locally to ordinary Maxwell theory, since the additional term in (2) is proportional to $\epsilon^{\mu\nu\lambda\sigma}\partial_\mu F_{\lambda\sigma}$, which vanishes identically as a consequence of the Bianchi identity $\partial_{[\mu}F_{\lambda\sigma]} = 0$ in Maxwell theory without magnetic monopoles. Nontrivial effects of the Chern-Simons coupling arise only from spacetime variations of $\theta(x)$. We consider configurations in which θ is piecewise constant, with a discrete jump across the interface at $z = 0$,

$$\theta(z) = \theta_- + \Delta\theta\Theta(z), \quad (3)$$

where $\Theta(z)$ is the Heaviside step function. In this case $\partial_z\theta = \Delta\theta\delta(z)$, and the Chern-Simons interaction is localized on the interface, as claimed.

To circumvent issues with gauge fixing, we work directly with the field strengths $E^i = F^{0i}$ and $B^i = \frac{1}{2}\epsilon^{ijk}F_{jk}$, which are automatically gauge invariant and physical. The field equations (2) and the Bianchi identity $\partial_{[\mu}F_{\lambda\sigma]} = 0$ then yield [7]

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= -\Delta\theta\delta(z)B_z, & \vec{\nabla} \times \vec{B} - \partial_t\vec{E} &= \Delta\theta\delta(z)\hat{z} \times \vec{E}, \\ \vec{\nabla} \cdot \vec{B} &= 0, & \vec{\nabla} \times \vec{E} + \partial_t\vec{B} &= 0, \end{aligned} \quad (4)$$

where we substituted the wall profile (3), and $\hat{z}$ is the unit vector normal to the interface. As a quick check, we see that away from the interface Eq. (4) reduce to the conventional vacuum Maxwell equations.

The physical effect of the Chern-Simons interaction on the interface is enforced by the matching conditions encoded in Eq. (4). Integrating the scalar equations in (4) over an infinitesimal pillbox whose faces are parallel to the interface and the vector equations around an infinitesimal rectangular loop perpendicular to the interface, we find [7]

$$\begin{aligned} E_z(0^+) - E_z(0^-) &= -\Delta\theta B_z(0), \\ \hat{z} \times (\vec{B}_\parallel(0^+) - \vec{B}_\parallel(0^-)) &= \Delta\theta \hat{z} \times \vec{E}_\parallel(0), \\ B_z(0^+) - B_z(0^-) &= 0, \\ \hat{z} \times (\vec{E}_\parallel(0^+) - \vec{E}_\parallel(0^-)) &= 0. \end{aligned} \quad (5)$$

The first equation shows that the normal component of the electric field is discontinuous at the wall whenever the magnetic field has a nonvanishing normal component there. The second equation shows that the tangential components of the magnetic field are also discontinuous when the tangential electric field does not vanish at the interface. The remaining two equations enforce continuity of B_z and of the tangential electric field. After these Gaussian pillbox integrations of (5), we find the complete set of boundary conditions governing electromagnetic fields at a Chern-Simons interface.

Since we are looking for solutions representing waves propagating along the interface and localized to it, exponentially diminishing away in the normal direction, we take the *Ansätze*

$$\vec{E} = \vec{\mathcal{E}}(z)e^{i(\vec{k}_\parallel \cdot \vec{x}_\parallel - \omega t)}, \quad \vec{B} = \vec{\mathcal{B}}(z)e^{i(\vec{k}_\parallel \cdot \vec{x}_\parallel - \omega t)}, \quad (6)$$

where $\vec{x}_\parallel = (x, y)$ are coordinates parallel to the wall and $\vec{k}_\parallel$ is the conserved momentum along the interface. Substituting into the bulk Eq. (4) and combining the results yields equations for the bulk wave profiles on each side ($\pm$), which have the same form

$$(\partial_z^2 + \omega^2 - k_\parallel^2)\vec{\mathcal{E}}_\pm(z) = 0, \quad (\partial_z^2 + \omega^2 - k_\parallel^2)\vec{\mathcal{B}}_\pm(z) = 0, \quad (7)$$

where $k_\parallel^2 = \vec{k}_\parallel^2$, and follow the same bulk dispersion relation away from the interface, because the bulk on either side satisfies standard vacuum Maxwell equations. The electromagnetic fields on different sides of the interface are not the same, since the boundary conditions come from the Chern-Simons term. In particular, these fields are not invariant under parity reflection across the interface.

For $\kappa^2 = k_\parallel^2 - \omega^2 > 0$, Eq. (6) may yield exponentially decaying solutions on both sides of the interface, if they simultaneously satisfy the boundary conditions

$$\vec{\mathcal{E}}_\pm(z) = \vec{\mathcal{E}}_\pm(0)e^{-\kappa|z|}, \quad \vec{\mathcal{B}}_\pm(z) = \vec{\mathcal{B}}_\pm(0)e^{-\kappa|z|}. \quad (8)$$

Those are the solutions we seek. We are thus isolating the surface-localized branch of the spectrum, distinct from the continuum of propagating bulk modes.

The integration constants $\vec{\mathcal{E}}_\pm(0)$ and $\vec{\mathcal{B}}_\pm(0)$ are not all independent. In the bulk, the first order Maxwell equations relate $\vec{\mathcal{B}}$ algebraically to $\vec{\mathcal{E}}$, while the divergence constraints relate the normal components of both fields to their tangential components. As a result, a complete set of

independent integration constants may be chosen to be the tangential components of the electric field on each side of the interface. These amplitudes are fixed by the boundary conditions (5), up to an overall normalization.

To proceed, away from the interface we use the *Ansätze* (6), which means that spatial derivatives acting on the bulk profiles reduce to $\vec{\nabla} \rightarrow i\vec{k}_{\parallel} \mp \kappa\hat{z}$, where the upper (lower) sign corresponds to the region $z > 0$ ($z < 0$). As above, $\kappa^2 = k_{\parallel}^2 - \omega^2 > 0$. The bulk Maxwell equations then yield linear algebraic relations between $\vec{\mathcal{E}}_{\pm}$ and $\vec{\mathcal{B}}_{\pm}$. Specifically, Faraday's law $\vec{\nabla} \times \vec{E} + \partial_t \vec{B} = 0$ after straightforward algebra yields

$$\vec{\mathcal{B}}_{\pm} = \frac{1}{\omega} (\vec{k}_{\parallel} \times \vec{\mathcal{E}}_{\pm} \pm i\kappa \hat{z} \times \vec{\mathcal{E}}_{\pm}). \quad (9)$$

Similarly, Ampère's law in vacuum $\vec{\nabla} \times \vec{B} - \partial_t \vec{E} = 0$ yields $(i\vec{k}_{\parallel} \pm \kappa\hat{z}) \times \vec{\mathcal{B}}_{\pm} = -i\omega\vec{\mathcal{E}}_{\pm}$. However, this equation is completely equivalent to (9), and so redundant, when the bulk on-shell condition $\omega^2 = k_{\parallel}^2 - \kappa^2$ holds. Thus, (9) is the only independent relationship between integration constants that follows from vectorial Maxwell equations.

Additional relations come from the scalar Maxwell equations involving the divergence operator, which restrict the electric and magnetic field components normal to the interface. In the absence of sources, $\vec{\nabla} \cdot \vec{E} = 0$ and $\vec{\nabla} \cdot \vec{B} = 0$ imply

$$\vec{k}_{\parallel} \cdot \vec{\mathcal{E}}_{\pm} \pm i\kappa\mathcal{E}_{z,\pm} = 0, \quad \vec{k}_{\parallel} \cdot \vec{\mathcal{B}}_{\pm} \pm i\kappa\mathcal{B}_{z,\pm} = 0. \quad (10)$$

Thus, the normal components are fixed by the longitudinal projections of the tangential components. Together with Eq. (9), these relations show that, in each bulk region, an evanescent mode is completely specified by the two tangential components of the electric field.

The boundary conditions (5) include the operator $\hat{z} \times$, which rotates tangential vectors. To disentangle those modes we switch to two-dimensional helicity eigenstates parallel to the interface since they are rotation eigenvectors, $\vec{e}_{L,R} = (\hat{x} \pm i\hat{y})/\sqrt{2}$, so that $\hat{z} \times \vec{e}_L = -i\vec{e}_L$ and $\hat{z} \times \vec{e}_R = +i\vec{e}_R$. Then for any vector $\vec{V}_{\parallel} = (V_x, V_y)$, the components in this basis are $V^{L,R} = \vec{e}_{L,R}^* \cdot \vec{V}$, or

$$V^L \equiv \frac{1}{\sqrt{2}}(V_x - iV_y), \quad V^R \equiv \frac{1}{\sqrt{2}}(V_x + iV_y). \quad (11)$$

The last line of Eq. (5), obtained by integrating Faraday's law across the interface, implies that the normal component of the magnetic field and the tangential component of the electric field are continuous on the interface: $\mathcal{B}_{z,+}(0) = \mathcal{B}_{z,-}(0)$ and $\vec{\mathcal{E}}_{\parallel,+}(0) = \vec{\mathcal{E}}_{\parallel,-}(0)$. In the 2D helicity eigenbasis,

$$\begin{aligned} \mathcal{B}_{z,+}(0) &= \mathcal{B}_{z,-}(0) = \mathcal{B}_z(0), \\ \mathcal{E}_{+}^L(0) &= \mathcal{E}_{-}^L(0) = \mathcal{E}^L(0), \\ \mathcal{E}_{+}^R(0) &= \mathcal{E}_{-}^R(0) = \mathcal{E}^R(0). \end{aligned} \quad (12)$$

The tangential boundary condition in (5) is manifestly chiral since it includes the rotation generator $\hat{z} \times$. Expanding the vector fields in the 2D helicity eigenstate basis on each side of the interface and bearing in mind that $\vec{\mathcal{E}}_{\parallel}$ is continuous on the interface then yields

$$\mathcal{B}_{+}^L(0) - \mathcal{B}_{-}^L(0) = \Delta\theta\mathcal{E}^L(0), \quad \mathcal{B}_{+}^R(0) - \mathcal{B}_{-}^R(0) = \Delta\theta\mathcal{E}^R(0). \quad (13)$$

The final boundary condition, the first equation in (5), involves only the normal components of the electric and magnetic fields, and hence simply carries over to the helicity eigenbasis,

$$\mathcal{E}_{z,+}(0) - \mathcal{E}_{z,-}(0) = -\Delta\theta\mathcal{B}_z(0). \quad (14)$$

This completes the reformulation of the boundary conditions in the helicity eigenbasis.

Finally, we combine the boundary conditions in Eqs. (12)–(14) and the relations we obtained from bulk Maxwell equations. Away from the interface, the bulk solutions satisfy (9). Projecting that equation onto the interface-parallel helicity eigenstates,

$$\begin{aligned} \mathcal{B}_{\pm}^L(0) &= \frac{1}{\omega} [(\vec{k}_{\parallel} \times \vec{\mathcal{E}})^L \pm \kappa\mathcal{E}^L(0)], \\ \mathcal{B}_{\pm}^R(0) &= \frac{1}{\omega} [(\vec{k}_{\parallel} \times \vec{\mathcal{E}})^R \mp \kappa\mathcal{E}^R(0)]. \end{aligned} \quad (15)$$

Here we have used $\vec{\mathcal{E}}_{+}^{L,R}(0) = \vec{\mathcal{E}}_{-}^{L,R}(0)$, and $\hat{z} \times \vec{\mathcal{E}}^L = -i\vec{\mathcal{E}}^L$, $\hat{z} \times \vec{\mathcal{E}}^R = +i\vec{\mathcal{E}}^R$, which follow from the continuity of components of $\vec{\mathcal{E}}_{\parallel}$ in the helicity eigenbasis [Eq. (12)].

Next, the terms $\propto \vec{k}_{\parallel} \times \vec{\mathcal{E}}$ cancel in the difference of the magnetic fields across the interface,

$$\mathcal{B}_{+}^L(0) - \mathcal{B}_{-}^L(0) = \frac{2\kappa}{\omega}\mathcal{E}^L(0), \quad \mathcal{B}_{+}^R(0) - \mathcal{B}_{-}^R(0) = -\frac{2\kappa}{\omega}\mathcal{E}^R(0). \quad (16)$$

Substituting Eq. (16) into the helicity eigenbasis boundary conditions (13) yields

$$\frac{2\kappa}{\omega}\mathcal{E}^L(0) = \Delta\theta\mathcal{E}^L(0), \quad \frac{2\kappa}{\omega}\mathcal{E}^R(0) = -\Delta\theta\mathcal{E}^R(0). \quad (17)$$

These equations provide the crucial information controlling the existence of the solutions which satisfy our initial *Ansätze*. For nontrivial solutions $\mathcal{E}^{L,R}(0) \neq 0$, these equations require

$$\kappa = \frac{\Delta\theta\omega}{2} \quad (\text{left handed}) \quad \text{or} \quad \kappa = -\frac{\Delta\theta\omega}{2} \quad (\text{right handed}). \quad (18)$$

Normalizability of the evanescent bulk profiles requires $\kappa > 0$, so for $\omega > 0$ exactly one helicity sector admits a localized solution, depending on the sign of $\Delta\theta$. Without any loss of generality, we can take $\Delta\theta > 0$, which selects the left-handed polarization for the localized mode. For $\Delta\theta < 0$, the roles of the two helicities are reversed. Importantly, (18) reveals that the “decay constant” κ is not a mass, since $\kappa \propto \omega$. Hence, the spectrum of the localized mode is gapless. Note that (18) implies that after we fix the integration constants to solve the boundary conditions for the selected helicity, no additional normalizable solutions exist.

To complete the explicit construction of the localized mode, we can therefore focus on the left-handed modes. The tangential electric field continuity (12) yields $\mathcal{E}_+^L(0) = \mathcal{E}_-^L(0) \equiv \mathcal{E}_0$, where $\mathcal{E}_0$ is an arbitrary constant, and so

$$\vec{\mathcal{E}}_{\parallel,\pm}(z) = \mathcal{E}_0 \vec{e}_L e^{-\kappa|z|}. \quad (19)$$

The normal component of the electric field is fixed by Gauss’s law in the bulk, $\vec{\nabla} \cdot \vec{E} = 0$, which implies $\vec{k}_{\parallel} \cdot \vec{\mathcal{E}}_{\pm} \pm i\kappa \mathcal{E}_{z,\pm} = 0$. Solving for $\mathcal{E}_{z,\pm}$ yields

$$\mathcal{E}_{z,+}(0) = \frac{i}{\kappa} (\vec{k}_{\parallel} \cdot \vec{e}_L) \mathcal{E}_0, \quad \mathcal{E}_{z,-}(0) = -\frac{i}{\kappa} (\vec{k}_{\parallel} \cdot \vec{e}_L) \mathcal{E}_0. \quad (20)$$

Thus, the normal component of the electric field is odd across the interface, as we noted above; it jumps by $\mathcal{E}_{z,+}(0) - \mathcal{E}_{z,-}(0) = \frac{2i}{\kappa} (\vec{k}_{\parallel} \cdot \vec{e}_L) \mathcal{E}_0$. To find $\mathcal{B}_{z,\pm}(0)$, we substitute the solution $\vec{\mathcal{E}}_{\pm}(0) = \mathcal{E}_0 \vec{e}_L$ into Eq. (9), and bear in mind that $(\hat{z} \times \vec{e}_L)_z = 0$ —so that we can drop the subscripts $\pm$. We find $\mathcal{B}_z(0) = \frac{\mathcal{E}_0}{\omega} (\vec{k}_{\parallel} \times \vec{e}_L)_z$. Since both $\vec{k}_{\parallel}$ and $\vec{e}_L$ lie in the interface plane, $(\vec{k}_{\parallel} \times \vec{e}_L)_z = -i(\vec{k}_{\parallel} \cdot \vec{e}_L)$, so that $\mathcal{B}_z(0) = -\frac{i}{\omega} (\vec{k}_{\parallel} \cdot \vec{e}_L) \mathcal{E}_0$. Substituting into (14) yields

$$\Delta\theta \mathcal{B}_z(0) = -\frac{i\Delta\theta}{\omega} (\vec{k}_{\parallel} \cdot \vec{e}_L) \mathcal{E}_0, \quad (21)$$

which independently matches the jump in $\mathcal{E}_z$ provided

$$\kappa = \frac{2\omega}{\Delta\theta} \quad (\text{left handed}). \quad (22)$$

This yields the second condition on the left-handed localized mode on an infinitely thin wall, which in this limit must be simultaneously satisfied with the first of Eq. (18). Combining them finally yields

$$\Delta\theta = \pm 2 \quad (\text{left handed}). \quad (23)$$

Similar analysis of the right-handed modes reproduces once again (23). Thus, the special class of chiral electromagnetic waves localized on infinitely thin, stiff Chern-Simons interfaces satisfies $\Delta\theta = \pm 2$.

However, this issue is subtle. The boundary condition (14) for an infinitely thin wall follows from the axion-modified Gauss law $\vec{\nabla} \cdot (\vec{E} + \theta \vec{B}) = 0$, obtained from the first of Eq. (4) using $\vec{\nabla} \cdot \vec{B} = 0$. Here we have imposed the idealized wall profile (3) by hand, treating the wall as an infinitely thin, stiff defect. Now, the corresponding Gauss law constraint $\Psi \equiv \vec{\nabla} \cdot (\vec{E} + \theta \vec{B}) = 0$ is an integral of motion, $\partial_t \Psi \equiv 0$. Therefore, it constrains the initial data rather than the field evolution. The additional condition (22) thus reflects compatibility of the initial data with the idealized thin-wall profile (3). For the source-free localized modes considered here, this is achieved for $\Delta\theta = \pm 2$. For a smooth-wall profile [8], the singular matching condition (14) is replaced by the continuous constraint equation throughout the wall. The implementation of the Gauss law constraint is then spread across the finite wall smoothly, rather than encoded in a separate distributional boundary condition at a sharp interface. The scattering analysis [8] reveals the modes that only satisfy (18). Hence, we retain both the localized solutions satisfying only Eq. (18) and the special subclass of infinitely thin, stiff-wall solutions satisfying the stronger condition (18) and (23).

To get the wave profiles for both solutions—the localized one and the non-normalizable counterpart—we combine the tangential and normal field strengths. For the localized solutions, the electric field is

$$\vec{\mathcal{E}}_{\pm}(z) = \mathcal{E}_0 \left[\vec{e}_L \pm \frac{i}{\kappa} (\vec{k}_{\parallel} \cdot \vec{e}_L) \hat{z} \right] e^{-\kappa|z|}. \quad (24)$$

The magnetic field follows from Faraday’s law (9). Substituting (24) into (9) gives

$$\vec{\mathcal{B}}_{\pm}(z) = \frac{\mathcal{E}_0}{\omega} [\vec{k}_{\parallel} \times \vec{e}_L \pm \kappa \vec{e}_L] e^{-\kappa|z|}. \quad (25)$$

The term $\vec{k}_{\parallel} \times \vec{e}_L$ is normal to the interface and provides the entire B_z component, while the term proportional to $\vec{e}_L$ is tangential. The overall normalization $\mathcal{E}_0$ remains arbitrary, to be set by the initial conditions. For the localized mode, $\kappa > 0$ (18); the other mode corresponds to $L \rightarrow R$ and the associated sign flip of κ . Upon $\omega \leftrightarrow -\omega$, these solutions flip around.

The wave function decay constant κ is not an independent parameter. From the *Ansätze* (8) and (18), for the general family of solutions

$$\kappa^2 = k_{\parallel}^2 - \omega^2, \quad \kappa^2 = \frac{(\Delta\theta)^2 \omega^2}{4}, \quad (26)$$

where $k_{\parallel} = |\vec{k}_{\parallel}|$ is the conserved momentum along the interface. Combining these,

$$\omega^2 = \frac{k_{\parallel}^2}{1 + \frac{\Delta\theta^2}{4}}. \quad (27)$$

We note that our normalizable mode can be identified with a pole in the analytically continued electromagnetic transmission amplitude at the interface [8]. This shows that the chiral surface photon arises from the same scattering dynamics that governs polarization rotation and reflection in axion electrodynamics. For the special family of modes on thin walls, we have $\omega^2 = \frac{1}{2} k_{\parallel}^2$. The localized mode does not hybridize with propagating bulk photons because its dispersion lies in the evanescent regime $k_{\parallel}^2 > \omega^2$, where no on-shell bulk photon state exists with the same conserved frequency and interface-parallel momentum.

The dispersion relation (27) is unexpected—given the context where it appears—in that it is linear at all scales below the UV cutoff, with a coefficient smaller than unity and independent of scale. As a result, both the phase velocity and the group velocity of the surface photon are

$$v_{\text{ph}} = v_g = \frac{1}{\sqrt{1 + \frac{1}{4}(\Delta\theta)^2}} < 1, \quad (28)$$

and so are strictly less than the speed of light in the vacuum bulk and independent of frequency. The reduction in propagation speed is set by the Chern-Simons jump $\Delta\theta$ and does not depend on any microscopic details of the interface. This result is robust all the way to the cutoff of the effective field theory, where the structure of the interface is microscopically resolved. This roughly occurs at the scale given by the inverse thickness of the interface. The step-function description is reliable when the electromagnetic wavelength exceeds the wall thickness L and begins to break down once the interface is resolved microscopically, with the corrections proportional to powers of ωL .

Below this cutoff, the localized mode propagates as light in a homogeneous medium with a constant refractive index $n_{\text{eff}} = \sqrt{1 + \frac{1}{4}(\Delta\theta)^2}$. It is nondispersive, in the sense that $v_{\text{ph}} = v_g$, although the bulk on either side of the interface is the vacuum without any ambient material response present. The Chern-Simons interface behaves as a thin, frequency-independent optical medium whose refractive properties are set mainly by topology. It is effectively trapping an electromagnetic wave without any assistance by bulk ambient material degrees of freedom. This sharply contrasts with conventional surface plasmons and polaritons [9], whose existence relies on collective charge oscillations and material response in the surrounding medium. Here

both bulk regions are vacuum, and the localization arises entirely from the Chern-Simons interaction supported on the interface. Note that since the decay constant $\kappa \propto \omega$, Eq. (26), the lowest frequencies are the least localized ones. Consequently, the factor $1/\kappa$ appearing in Eq. (20) simply reflects the growth of the localization length. Conversely, in the stationary field limit $\omega \rightarrow 0$, localized modes disappear as $\kappa \rightarrow 0$, in agreement with [10].

This behavior sharply distinguishes our Chern-Simons waveguide from conventional ones. In systems such as parallel conducting plates, transverse confinement introduces a frequency gap, and at high frequency the modes behave as free photons in the bulk so that confinement effectively ceases [11]. By contrast, the surface mode supported by a Chern-Simons interface is gapless and remains slow at all frequencies below the cutoff. There is no regime in which it merges with a bulk photon or forgets that it is interface supported.

In addition, our localized mode is not equivalent to the photon of intrinsic $(2+1)$ -dimensional Maxwell theory [4], although it propagates a single degree of freedom on a $(2+1)$ -dimensional interface. The intrinsic $2+1$ Maxwell mode is parity and time-reversal invariant and is dual to a scalar field. In contrast our interface mode is manifestly chiral, with its polarization fixed by the sign of the Chern-Simons discontinuity. As a result, the theory of the chiral interface mode is not invariant under time reversal, since it maps the allowed excitation into a mode of opposite helicity, which is not supported by the interface.

Finally, as a check of the efficiency of mode localization to the interface, we compute the Poynting vector of the localized wave. Since the solution is monochromatic, it suffices to consider the time-averaged energy flux, which for harmonic fields $\vec{E} = \Re[\vec{\mathcal{E}}(z)e^{i(\vec{k}_{\parallel} \cdot \vec{x}_{\parallel} - \omega t)}]$ ($\Re$ denotes the real part of a complex number) and $\vec{B} = \Re[\vec{\mathcal{B}}(z)e^{i(\vec{k}_{\parallel} \cdot \vec{x}_{\parallel} - \omega t)}]$ is given by

$$\langle \vec{S} \rangle = \frac{1}{2\mu_0} \Re(\vec{\mathcal{E}} \times \vec{\mathcal{B}}^*). \quad (29)$$

The time-averaged electromagnetic energy density is $\langle u \rangle = \frac{1}{4}(\epsilon_0 |\vec{\mathcal{E}}|^2 + \frac{1}{\mu_0} |\vec{\mathcal{B}}|^2)$, which is manifestly non-negative and inherits the same exponential factor $e^{-2\kappa|z|}$ as the energy flux. Substituting the explicit localized solution obtained in Eqs. (24) and (25) shows that all contributions are proportional to $e^{-2\kappa|z|}$, reflecting the exponential localization of the energy flux near the interface. The normal component vanishes, $\langle S_z \rangle = 0$, and the energy flux is purely tangential, pointing along $\vec{k}_{\parallel}$,

$$\langle \vec{S}(z) \rangle = \frac{|\mathcal{E}_0|^2}{2\mu_0 \omega} \vec{k}_{\parallel} e^{-2\kappa|z|}. \quad (30)$$

This result is fully consistent with the interface matching conditions (5): although individual field components are

discontinuous across the wall, the net energy flux normal to the interface vanishes identically. The wall-localized Chern-Simons term changes the boundary conditions but does not carry electromagnetic energy of its own. Equation (30) provides a complete description of the energy transport carried by the surface mode. Energy density is manifestly non-negative by construction. The absence of any normal component of the Poynting vector confirms that the mode is bound to the interface and does not radiate into the surrounding bulk.

We can recap our results here. We have shown that Maxwell electrodynamics with a codimension-1 Chern-Simons interface necessarily supports an interface-localized electromagnetic mode of definite helicity. This mode does not rely on any modification of bulk photon dynamics. Instead, it comes about because the interface-borne electromagnetic Chern-Simons interaction acts with opposite sign on the two photon helicities. As a result, one polarization experiences an attractive δ -function interaction and forms a localized state on the interface. This surface mode is therefore strictly chiral, with its handedness fixed by the sign of the Chern-Simons jump $\Delta\theta$. In the infinitely thin, stiff-wall limit, the mode is restricted to require $\Delta\theta = \pm 2$, which is, however, relaxed for thick axionic walls [8].

While the resulting mode carries a single propagating degree of freedom, it is not equivalent to Maxwell theory intrinsic to $(2+1)$ dimensions. Our interface-localized excitation is fundamentally chiral and so not time-reversal invariant, reflecting helicity selection enforced by its four-dimensional origin.

The localized chiral photon propagates with a universal, linear dispersion relation, featuring a constant index of

refraction. The index of refraction is fixed by $\Delta\theta$ for all momenta below the cutoff of the interface effective field theory. Both the phase and group velocities are reduced relative to the speed of light in the bulk. We find it remarkable that the propagation of the surface photon is identical in form to that of light traveling in a homogeneous medium with constant refractive index, even though the bulk on either side of the interface is vacuum and no ambient material degrees of freedom are present. In this sense, the dispersion relation (27) is structural rather than dynamical, closely paralleling the geometric origin of the Pancharatnam phase in polarization rotation across Chern-Simons interfaces [12]. This persists all the way up to the ultraviolet cutoff of the effective theory, set roughly by the inverse thickness of the interface, separating two different θ vacua. Clearly, near the cutoff, the corrections will appear and force a more detailed description that requires a UV completion of the theory (1).

In closing, our results demonstrate that Maxwell theory admits intrinsic chiral surface excitations when supplemented with a Chern-Simons interface. The surface photon is not a modified bulk mode nor a medium-induced fluctuation. Instead, it is a boundary degree of freedom supported by the interface topology. Thus, our results point at a novel regime of electromagnetic boundary physics and provide new insight on the interplay between topology and electromagnetic field dynamics.

Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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