

Discovery prospects of a singly charged scalar at μ TRISTAN

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In this article, we study the associated production of a singly charged (Δ^+) scalar along with a W^+ boson in the newly proposed $\mu^+\mu^+$ collider (also known as μ TRISTAN) at $\sqrt{s} = 2$ TeV. Such a singly charged scalar is naturally accommodated in an extremely well-motivated neutrino mass model, namely, the Type-II seesaw model. This model, besides providing a viable explanation of neutrino mass generation, also allows for lepton flavor violating (LFV) processes. Since LFV processes are not allowed in the standard model (SM), we focus on the discovery prospect of the singly charged scalar in the Type-II seesaw model at μ TRISTAN through a LFV process, owing to the advantage of this process being free of any SM background. Additionally, this article also proposes a method to indicate if the underlying theory follows a normal or an inverted hierarchy depending on the distribution of lepton flavors in the final state.

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I. INTRODUCTION

The standard model (SM), till date, is the most successful theory of nature, which attained its peak with the discovery of the Higgs boson in 2012 at ATLAS and CMS [1,2]. Despite such massive success of the SM, it cannot explain various natural phenomena, such as (i) the neutrino mass generation mechanism [3,4], (ii) Higgs mass instability in the electroweak sector [5–7], (iii) matter-antimatter asymmetry [8], (iv) the strong CP problem [9–11], and several others. Such drawbacks of the SM motivate the existence of beyond standard model (BSM) scenarios.

An extremely well-motivated BSM scenario is the Type-II seesaw scenario [12–15], which not only provides a valid explanation for the neutrino mass generation mechanism but also allows for lepton flavor violating (LFV) processes, which the SM does not accommodate. Therefore, it is highly lucrative to look for the exotic BSM particles, which can vouch for the existence of the Type-II

seesaw model through a LFV process since such a process will have no relevant SM background and hence will be extremely clean. All existing collider constraints on the Type-II seesaw model can be found in Refs. [16,17]. References [18,19] summarize the LFV constraints.

Lepton colliders are extremely advantageous for looking into LFV processes, as they provide a much cleaner environment than hadron colliders. Additionally, since leptons are fundamental particles, the entire beam energy is available for the hard collision. Whereas, in a hadron collider such as the LHC, only a fraction of the proton-beam energy that is carried by the colliding partons is available for collision. Therefore, a higher physics reach is obtained in lepton colliders. Lepton colliders are constructed by colliding electrons or muons. However, since muons are heavier than electrons, colliding muons leads to much smaller loss of energy due to synchrotron radiations, as compared to colliding electrons. Thanks to the huge development in technology we can now solve several technical difficulties that had been hindering the operation of a $\mu^+\mu^-$ collider, thereby, making the $\mu^+\mu^-$ collider gain huge interest in the community in recent years [20–24]. Numerous studies have been performed showing the huge prospect of the $\mu^+\mu^-$ collider in search of new physics [25].

However, in this article we concentrate on a slightly different setup, namely, the μ TRISTAN [26], which plans on operating as a μ^+e^- collider at $\sqrt{s} = 346$ GeV and a $\mu^+\mu^+$ collider at $\sqrt{s} = 2$ TeV and higher [27]. We study

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our signature of choice in the $\mu^+\mu^+$ mode of the μ TRISTAN. Apparently, the $\mu^+\mu^+$ collider is quite convenient to build, thanks to the cooling technology developed at J-PARC [28]. Several studies have been performed showing the discovery prospects of new physics in μ TRISTAN [29–39]. In this article we intend to look for the singly charged scalar that can be naturally accommodated in the Type-II seesaw model via its associated production along with a W^+ boson in the $\mu^+\mu^+$ collider at $\sqrt{s} = 2$ TeV, followed by its decay in channels such that our signature of interest is a LFV process. Following the discovery prospects of the singly charged scalar, we also propose a technique to identify whether the underlying theory respects normal hierarchy or inverted hierarchy depending on the flavors of the lepton in the final state.

The rest of the article is organized as follows. In Sec. II, we discuss in detail the Type-II seesaw model. In Sec. III, we present our signature of interest and motivation behind choosing this signal along with the benchmark (bm) points. Section IV discusses the signal analysis and mass reach of the singly charged scalar, while Sec. V presents the technique to distinguish between the normal and inverted hierarchies. Finally, in Sec. VI we provide the summary and outlook.

II. TYPE-II SEESAW MODEL

In the Type-II seesaw scenario [12–15] the SM particle spectrum is augmented with a $SU(2)_L$ triplet scalar $\Delta = (\Delta^{++}, \Delta^+, \Delta^0)$ with hypercharge $Y_\Delta = 1$. Therefore, the scalar sector of this BSM scenario consists of the following:

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad (2.1)$$

where Φ is the SM Higgs doublet.

This additional scalar Δ , being charged under $SU(2)_L$, interacts with the SM gauge sector and also has Yukawa interaction terms with the SM lepton doublets. Therefore, the Lagrangian terms that arise due to this additional scalar beside the SM Lagrangian are

$$\mathcal{L}_{\text{BSM}} = \text{Tr}[(D_\mu \Delta)^\dagger D^\mu \Delta] + (Y_{\alpha\beta} \bar{L}^{\alpha c} (i\sigma_2) \Delta L^\beta + \text{h.c.}) - V(\Delta, \Phi), \quad (2.2)$$

where the first term with covariant derivative D_μ denotes the interaction between Δ and the SM gauge sector, where D_μ is given by

$$D_\mu \Delta = \partial_\mu \Delta - \frac{i}{2} g \left(\sum_{k=1,2,3} W_\mu^k [\sigma_k \Delta - \Delta \sigma_k] \right) - ig' y_\Delta B_\mu \Delta, \quad (2.3)$$

where σ_k are the Pauli spin matrices, g and g' are the gauge couplings of Δ with $SU(2)_L$ and $U(1)_Y$ SM gauge bosons, respectively, and y_Δ is the hypercharge of Δ , which is postulated to be 1.

The second term in Eq. (2.2) denotes the Yukawa interaction between Δ and the SM left-handed weak lepton doublets denoted by L^α and L^β , where α and β are the flavor indices. The generic form of the SM left-handed weak lepton doublet is given by

$$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L. \quad (2.4)$$

The term $V(\Delta, \Phi)$ in Eq. (2.2) is the scalar potential, which can be written as

$$\begin{aligned} V(\Delta, \Phi) = & -m_\Phi^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 + M_\Delta^2 \text{Tr}[\Delta^\dagger \Delta] \\ & + \lambda_1 [\text{Tr} \Delta^\dagger \Delta]^2 + \lambda_2 \text{Tr}[\Delta^\dagger \Delta]^2 \\ & + [\mu \Phi^T i \sigma_2 \Delta^\dagger \Phi + \text{h.c.}] \\ & + \lambda_3 (\Phi^\dagger \Phi) \text{Tr}[\Delta^\dagger \Delta] + \lambda_4 \Phi^\dagger \Delta \Delta^\dagger \Phi, \end{aligned} \quad (2.5)$$

where μ and λ_i ($i = 1, 2, 3, 4$) are the relevant couplings among the SM and BSM scalars.

On minimizing the scalar potential in Eq. (2.5) one can obtain a Vacuum Expectation Value (VEV) of Δ^0 (denoted by v_Δ) in Eq. (2.2), and with the SM Higgs VEV (denoted by v), one can obtain the neutrino mass as

$$m_\nu = \sqrt{2} Y v_\Delta, \quad (2.6)$$

where $v_\Delta \sim \mu v^2 / (\sqrt{2} M_\Delta^2)$.

The neutrino mass matrix can be diagonalized using the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix denoted by U_{PMNS} as follows:

$$m_\nu^{\text{diag}} = \text{Diag}[m_{\nu_1}, m_{\nu_2}, m_{\nu_3}] = U_{\text{PMNS}}^* m_\nu U_{\text{PMNS}}. \quad (2.7)$$

The neutrino mass hierarchy can either respect normal ordering (i.e., ν_1 is the lightest) or inverted ordering (i.e., ν_3 is the lightest). Here, we study both cases and propose a method to distinguish between these two cases depending on the phenomenological manifestation at the $\mu^+\mu^+$ collider.

After the electroweak symmetry breaking, this BSM scenario yields seven physical states of definite mass in the scalar sector. These states are denoted by $H^{\pm\pm}$, $H^\pm$, H^0 , A^0 , and h , where h is the SM Higgs boson, A^0 is a pseudoscalar, H^0 is the electrically neutral heavy scalar, and $H^{\pm\pm}$ and $H^\pm$ are electrically charged heavy scalars.

From Eq. (2.7), one can deduce that in order to obtain a small mass for the neutrino, as supported by experiments, v_Δ must be quite small. In the simplest Type-II seesaw model [12–15], v_Δ is bounded by the electroweak ρ parameter constraining $v_\Delta \lesssim 3$ GeV [40]. In the limit $v_\Delta \ll v$, the mass eigenstates of the scalar triplet appear as

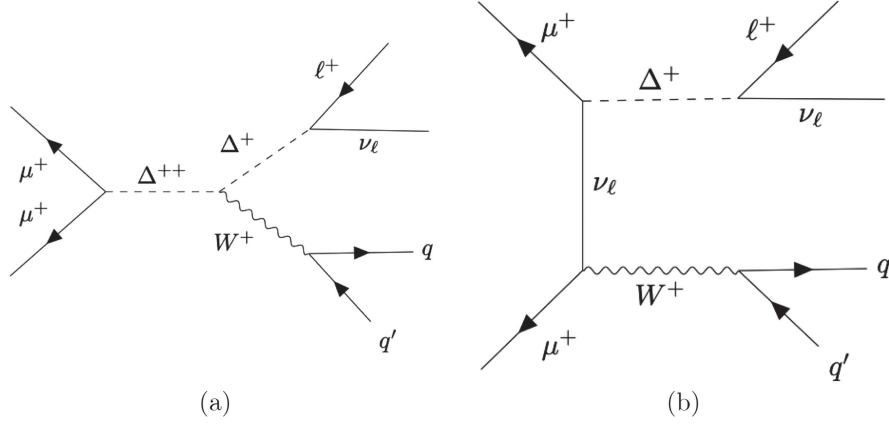


FIG. 1. Generic Feynman diagram for the associated production of the singly charged scalar Δ^+ and W^+ boson mediated by (a) a virtual Δ^{++} in s-channel and (b) exchange of a ν_ℓ in t-channel, in the $\mu^+\mu^+$ mode of the μ TRISTAN, followed by hadronic decay of the W^+ boson and leptonic decay of Δ^+ .

$$\begin{aligned}
 m_{H^{++}} &\sim m_{\Delta^{++}}, & m_{H^+} &\sim m_{\Delta^+}, \\
 m_{H^0} &\sim \text{Re}[m_{\Delta^0}], & m_{A^0} &\sim \text{Im}[m_{\Delta^0}] \\
 m_{H^0}^2 &\approx m_{A^0}^2 \approx m_{H^+}^2 + \frac{\lambda_4}{4} v^2 \approx m_{H^{++}}^2 + \frac{\lambda_4}{2} v^2. \quad (2.8)
 \end{aligned}$$

Depending on the value and sign of λ_4 , the mass ordering of the scalar triplets can be as follows:

- (i) $\lambda_4 = 0$: $m_{H^0}^2 = m_{A^0}^2 = m_{H^+}^2 = m_{H^{++}}^2$.
- (ii) $\lambda_4 < 0$: $m_{H^0}^2 = m_{A^0}^2 < m_{H^+}^2 < m_{H^{++}}^2$.
- (iii) $\lambda_4 > 0$: $m_{H^0}^2 = m_{A^0}^2 > m_{H^+}^2 > m_{H^{++}}^2$.

In this article, we choose the second case with the mass hierarchy $m_{H^0}^2 = m_{A^0}^2 < m_{H^+}^2 < m_{H^{++}}^2$ with the mass difference of $\sim 1\text{--}2$ GeV. In the rest of the article, we denote the physical states $H^{\pm\pm}$, $H^\pm$, and H^0 by $\Delta^{\pm\pm}$, $\Delta^\pm$, and Δ^0 , respectively.

III. SIGNATURE OF INTEREST AND BENCHMARK POINTS

Several studies have previously been performed to look for the doubly charged scalar Δ^{++} and electrically neutral scalar Δ^0 in the muon collider [25,41] as well as the $\mu^+\mu^+$ mode of the μ TRISTAN [33–36]. A comparative study of the doubly charged scalar Δ^{++} at the muon collider and the $\mu^+\mu^+$ mode of the μ TRISTAN for the chosen benchmark points has been shown in Appendixes A and B. However, the discovery prospects of the singly charged scalar Δ^+ have not yet received much deserved attention. Therefore, here in this article we study the associated production of Δ^+ along with the W^+ boson in the $\mu^+\mu^+$ mode of the μ TRISTAN, followed by hadronic decay of the W^+ boson and leptonic decay of Δ^+ . This associated production of Δ^+ and W^+ involves two types of contribution: (i) mediated by a virtual Δ^{++} in s-channel and (ii) mediated by exchange of a ν_ℓ in t-channel. The generic Feynman diagrams of these processes are shown in Figs. 1(a) and 1(b), respectively.

If the lepton arising from the decay of the Δ^+ is a μ^+ , then the process in Fig. 1 has a sizeable SM background

≈ 0.92 pb. However, if one considers the final-state lepton to be e^+ or τ^+ , then there is no relevant SM background, as in order to generate such a process, lepton flavor violation is needed, which the SM does not accommodate. Hence, finally, our chosen signature of interest consists of only e^+ or τ^+ as the final-state charged leptons, as shown in Fig. 2. This choice helps to get rid of any SM background, as discussed above.

Benchmark points: As benchmark points, we consider two values of the lightest neutrino mass: 0.05 and 0.001 eV. The VEV of Δ^0 , denoted as v_Δ , is fixed at 10^{-9} GeV, as at such a small value of v_Δ , Δ^+ primarily decays to leptons. The analysis is performed by varying the mass of Δ^+ in between 101 and 1901 GeV at an interval of 100 GeV followed by a more detailed scan in the high-mass region, i.e., 1700–1900 GeV at an interval of 25 GeV for both normal and inverted hierarchies at $\sqrt{s} = 2$ TeV.¹ The benchmark points for $m_{\Delta^+} = 101, 501, 1001, 1501, 1701, 1801, 1851, \text{ and } 1901$ GeV and the lightest neutrino mass, 0.05 and 0.001 eV, are shown in Tables I and II, respectively. We emphasize on benchmark points with $m_{\Delta^+} \geq 1.5$ TeV following the current and future LHC limit on $m_{\Delta^{++}}$, as found in Refs. [43,44,50] and [51], respectively. The production cross section for our signal of interest is shown in Fig. 3, which exhibits a fall and rise behavior with increase in mass of Δ^+ . This typical behavior of the cross section for our signal of interest is due to the contribution from Fig. 2(b), as discussed in detail in Appendix C. The solid green (dashed purple) vertical line in Fig. 3 corresponds to the current (future) LHC limit on

¹The mass limits on Δ^+ and the couplings have been carefully chosen motivated by several experimental constraints [42–49] and the reach of the μ TRISTAN collider used in this analysis. Furthermore, the couplings can be changed by changing v_Δ and we have checked that the cross section of the signature of our interest is directly proportional to v_Δ^2 . Therefore, one can easily scale the cross section for different values of v_Δ and hence different couplings.

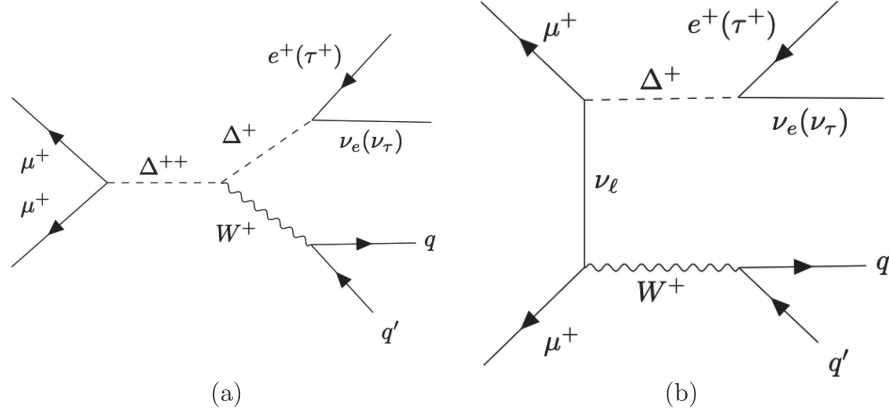


FIG. 2. Feynman diagram for the associated production of the singly charged scalar Δ^+ and W^+ boson mediated by (a) a virtual Δ^{++} in s-channel and (b) exchange of a ν_l in t-channel, in the $\mu^+\mu^+$ mode of the μ TRISTAN, followed by hadronic decay of the W^+ boson and leptonic (e^+ or τ^+) decay of Δ^+ . Such specific Lepton Flavor violating (LFV) decay channels of Δ^+ have been chosen to get rid of any SM background.

$m_{\Delta^{++}}$ obtained from Refs. [43,44,50,51], while the green (purple) shaded region denotes the parameter space excluded by the current (projected) reach of the LHC. The branching ratios of Δ^+ in different channels are shown

for normal and inverted hierarchies in Figs. 4(a) and 4(b), respectively. The branching ratios of Δ^+ do not change with change in its mass, as can be seen in Fig. 4. Although for normal hierarchy the branching ratio for Δ^+ to decay into

TABLE I. Benchmark points for normal and inverted hierarchies at $v_\Delta = 10^{-9}$ GeV and lightest neutrino mass = 0.05 eV. The cross sections listed here are for the processes before and after decay of Δ^+ and W^+ (as in Fig. 2) at $\sqrt{s} = 2$ TeV. The significance listed here is at integrated luminosity $\mathcal{L} = 30 \text{ fb}^{-1}$. All the masses and decay widths (Γ) are in units of GeV and the cross sections $\sigma_{\text{Production}}$ and σ are in units of fb.

Masses [GeV]			Normal hierarchy						Inverted hierarchy					
m_{Δ^0}	m_{Δ^+}	$m_{\Delta^{++}}$	Γ_{Δ^0}	Γ_{Δ^+}	$\Gamma_{\Delta^{++}}$	$\sigma_{\text{Production}}$	σ	$S/\sqrt{S}$	Γ_{Δ^0}	Γ_{Δ^+}	$\Gamma_{\Delta^{++}}$	$\sigma_{\text{Production}}$	σ	$S/\sqrt{S}$
100	101	103	0.02004	0.02024	0.02063	14.74	5.500	12.85	0.02472	0.02496	0.02545	13.24	5.802	13.19
500	501	503	0.1002	0.1004	0.1008	10.38	3.876	10.78	0.1236	0.1239	0.1243	9.332	4.087	11.07
1000	1001	1003	0.2004	0.2006	0.2010	10.47	3.911	10.83	0.2472	0.2475	0.2480	9.415	4.123	11.12
1500	1501	1503	0.3006	0.3008	0.3012	15.26	5.7	13.08	0.3708	0.3711	0.3716	13.71	6.0	13.42
1700	1701	1703	0.3407	0.3409	0.3413	18.82	7.008	14.50	0.4203	0.4205	0.4210	16.92	7.394	14.89
1800	1801	1803	0.3607	0.3609	0.3613	18.90	7.030	14.52	0.4450	0.4452	0.4457	16.99	7.412	14.91
1850	1851	1853	0.3708	0.3710	0.3714	15.84	5.874	13.28	0.4573	0.4576	0.4581	14.23	6.195	13.63
1900	1901	1903	0.3808	0.3810	0.3814	5.63	2.1	7.94	0.4697	0.4700	0.4704	5.06	2.2	8.12

TABLE II. Benchmark points for normal and inverted hierarchies at $v_\Delta = 10^{-9}$ GeV and lightest neutrino mass = 0.001 eV. The cross sections listed here are for the processes before and after decay of Δ^+ and W^+ (as in Fig. 2) at $\sqrt{s} = 2$ TeV. The significance listed here is at integrated luminosity $\mathcal{L} = 30 \text{ fb}^{-1}$. All the masses and decay widths (Γ) are in units of GeV and the cross sections $\sigma_{\text{Production}}$ and σ are in units of fb.

Masses [GeV]			Normal hierarchy						Inverted hierarchy					
m_{Δ^0}	m_{Δ^+}	$m_{\Delta^{++}}$	Γ_{Δ^0}	Γ_{Δ^+}	$\Gamma_{\Delta^{++}}$	$\sigma_{\text{Production}}$	σ	$S/\sqrt{S}$	Γ_{Δ^0}	Γ_{Δ^+}	$\Gamma_{\Delta^{++}}$	$\sigma_{\text{Production}}$	σ	$S/\sqrt{S}$
100	101	103	0.005127	0.005176	0.005278	3.614	0.9744	5.407	0.009806	0.009902	0.01010	1.833	0.8835	5.149
500	501	503	0.02563	0.02568	0.02579	2.546	0.6863	4.538	0.04903	0.04913	0.04932	1.291	0.6223	4.320
1000	1001	1003	0.05127	0.05132	0.05142	2.568	0.6922	4.559	0.09806	0.09816	0.09835	1.303	0.6279	4.339
1500	1501	1503	0.07690	0.07695	0.07705	3.742	1.007	5.495	0.1471	0.1472	0.1474	1.898	0.9135	5.234
1700	1701	1703	0.08715	0.08720	0.08731	4.616	1.240	6.099	0.1667	0.1668	0.1670	2.342	1.125	5.809
1800	1801	1803	0.09228	0.09233	0.09243	4.634	1.244	6.109	0.1765	0.1766	0.1768	2.351	1.128	5.817
1850	1851	1853	0.09484	0.09489	0.09499	3.884	1.040	5.586	0.1814	0.1815	0.1817	1.970	0.9435	5.319
1900	1901	1903	0.09740	0.09746	0.09756	1.381	0.3692	3.328	0.1863	0.1864	0.1866	0.7004	0.3350	3.171

μ^+ is higher than that for decaying into e^+ or τ^+ for a particular value of lightest neutrino mass, as evident from Fig. 4(a), we consider the decay of Δ^+ to e^+ and τ^+ only, as

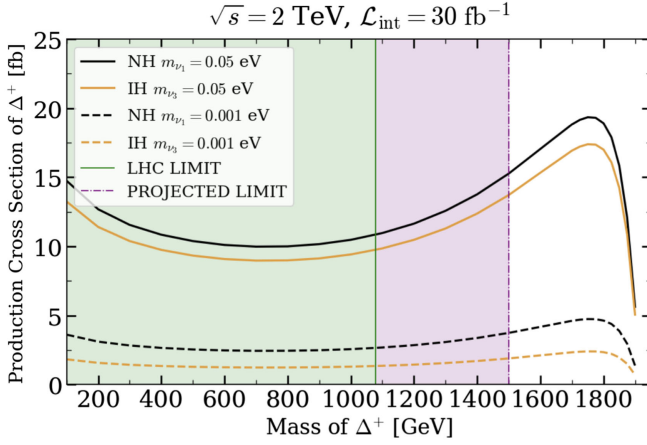


FIG. 3. Cross section for the process $\mu^+\mu^+ \rightarrow \Delta^+W^+$ for masses of Δ^+ varying between 101 and 1901 GeV.

then the process is absolutely free of SM background, as mentioned earlier, although at a cost of losing a reasonable number of signal events. But since the surviving events do not have any SM background, those small number of events are sufficient to yield 5σ significance up to a certain mass of Δ^+ , which will be discussed in detail in Sec. IV. For inverted hierarchy, however, the branching ratio for Δ^+ to decay into μ^+ is smaller than that for decaying into e^+ or τ^+ . Therefore, for inverted hierarchy, selecting the decay of Δ^+ to e^+ and τ^+ only makes the process free of SM background without much loss of signal events. Although the branching ratios of Δ^+ do not change with its mass, they do change with change in the mass of the lightest neutrino, as evident from Fig. 5.

IV. SIGNAL ANALYSIS AND MASS REACH OF Δ^+

All benchmark points have been generated using the UFO model [52] and simulated using MadGraph [53] followed by showering in Pythia [54] and detector-level analysis in Delphes [55] using the default muon collider delphes card. Since the

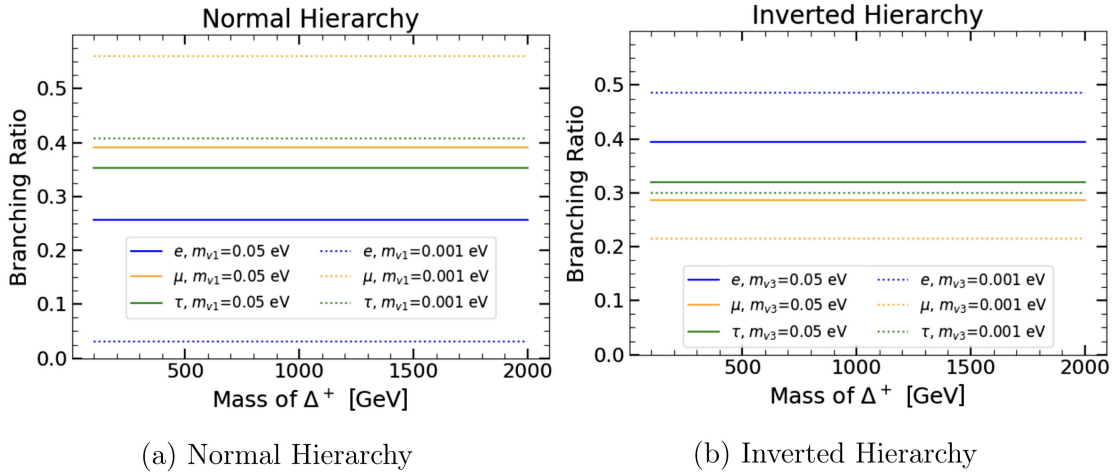


FIG. 4. Branching ratio of Δ^+ for masses of Δ^+ varying between 101 and 1901 GeV.

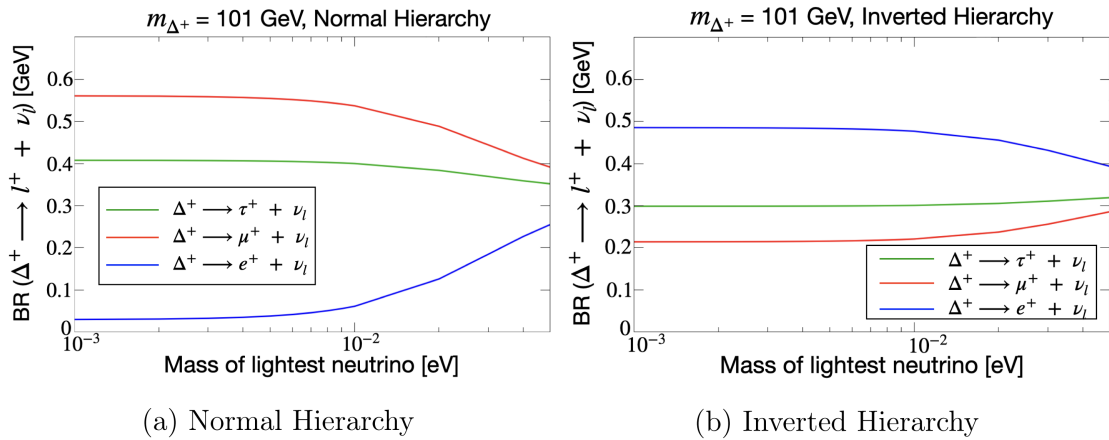


FIG. 5. Branching ratio of Δ^+ for masses of the lightest neutrino varying between 0.05 and 0.001 eV for a fixed $m_{\Delta^+} = 101$ GeV.

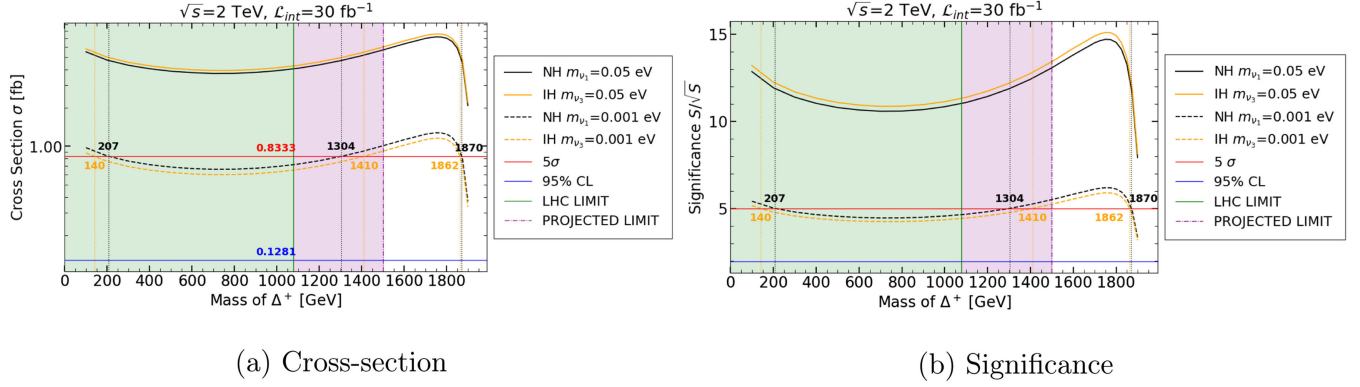


FIG. 6. Plot of (a) cross section and (b) significance of the signal process (shown in Fig. 2) vs m_{Δ^+} showing the 5σ and 95% C.L. by the horizontal red and blue lines, respectively.

μ TRISTAN collider has not yet been validated by full simulation, therefore, the default muon collider delphes card is a hybrid between FCC-hh [56] and the CLIC [57] colliders, as stated by the authors of the default muon collider delphes card. Therefore, the default muon collider delphes card has the provision of using several different algorithms to cluster and identify the final-state particles. Out of these available algorithms, we have used the default muon collider delphes card with the following specifications:

- (i) The jets are clustered using the Valencia algorithm [58] with $R = 1.5$ and $p_T(\text{jet}) > 20$ GeV with the requirement that all the hadronic particles are clustered into exactly two jets, as that is what is expected in the final state, as stated by the Feynman diagrams in Fig. 2. We have used this particular setup for identifying and clustering the jets, as this setup yielded the invariant mass of the two jets to be closest to the mass of a $W^\pm$ boson.
- (ii) The b-jets are tagged based on the CLICdp-Note-2014-002 [59]. Although, the flavor of the jets does not play any significant role in the analysis since the focus is on the decay of Δ^+ , which is forced to decay leptonically. Therefore, using any other b-tagging algorithm would yield similar results.
- (iii) The τ lepton is tagged with 80% efficiency if it has $p_T > 10.0$ GeV, $\eta < 2.5$, and if it lies within $\Delta R = 0.5$ of the jet axis.
- (iv) The e lepton is isolated following the criterion that the ratio of the sum of transverse momenta greater than 0.5 GeV of all particles that lie within a cone of $\Delta R = 0.1$ around the e lepton to the transverse momentum of that e lepton is less than 0.2 and is tagged only if the e lepton has $p_T > 3.0$ GeV and $\eta < 2.5$.

Since our signature of interest involves lepton number violation and lepton flavor violation, therefore, it is, in principle, free of any relevant SM background. Hence, at the ROOT analysis level, no other cuts, besides the detector efficiency at the Delphes level mentioned above,

have been employed to extract this rather clean signature. The total cross section of the full process, as in Fig. 2, is calculated at $\sqrt{s} = 2$ TeV and shown in Fig. 6(a). Since the process involves lepton flavor violation, it is a noble signature with absolutely zero SM background. Therefore, the significance for Fig. 2 can be calculated as $\text{significance} = S/\sqrt{S}$,² where S is the number of signal events, which is calculated as

$$S = \sigma \times \mathcal{L}_{\text{int}}, \quad (4.1)$$

where σ is the cross section, which is shown in Fig. 6(a), and $\mathcal{L}_{\text{int}}$ is the integrated luminosity, which has been considered to be 30 fb^{-1} .

The significance for all the benchmark points has been shown in Fig. 6(b). In Figs. 6(a) and 6(b), the solid curves are for the points with mass of lightest neutrino = 0.05 eV and the dashed curves are for the points with mass of lightest neutrino = 0.001 eV. The black curves denote normal hierarchy and the orange curves denote inverted hierarchy. The solid green (dashed purple) vertical line and shaded region hold the same meaning as in Fig. 3. The horizontal red and blue lines in Figs. 6(a) and 6(b) denote significance

²For simplicity, Gaussian distribution has been used here to calculate the significance. In experimental analyses, Poisson statistics should be used to compute the statistical significance more accurately, particularly in regimes with limited background events. At the limit $B \rightarrow 0$, Poisson statistics will yield the significance $\sim \sqrt{2S \ln(1/B)}$, where B is the number of background events and S is the number of signal events. Although theoretically, one would expect absolutely no background events for the signature considered here, in practice, a nonzero background will arise experimentally due to uncertainties. Such a background can then be incorporated into this expression to obtain a finite and physically meaningful significance. This approach generally yields a higher significance than that obtained here using Gaussian approximations. Consequently, our calculation is conservative: an experimental determination of the significance is expected to be equal to or greater than our result, but not smaller.

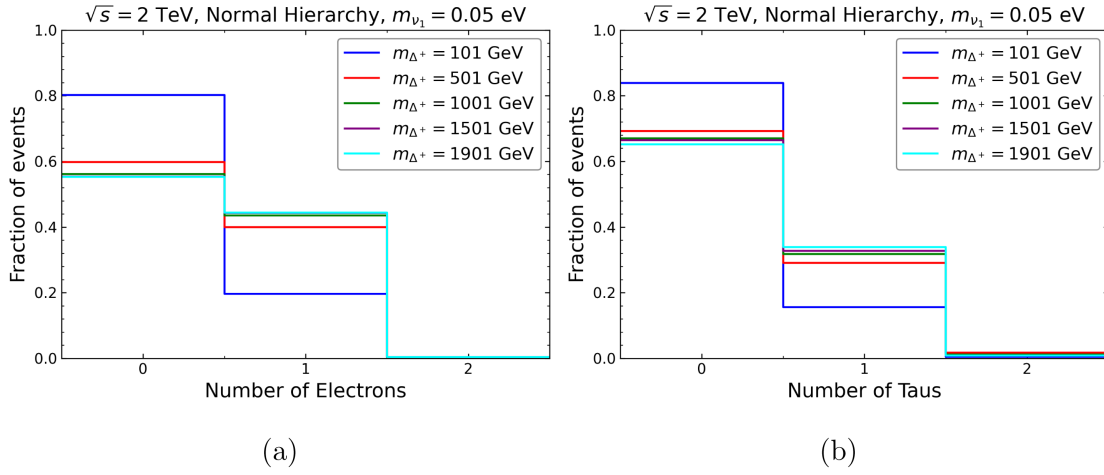


FIG. 7. Distribution for (a) electrons in the final state and (b) taus in the final state for $m_{\nu_1} = 0.05$ eV assuming normal hierarchy.

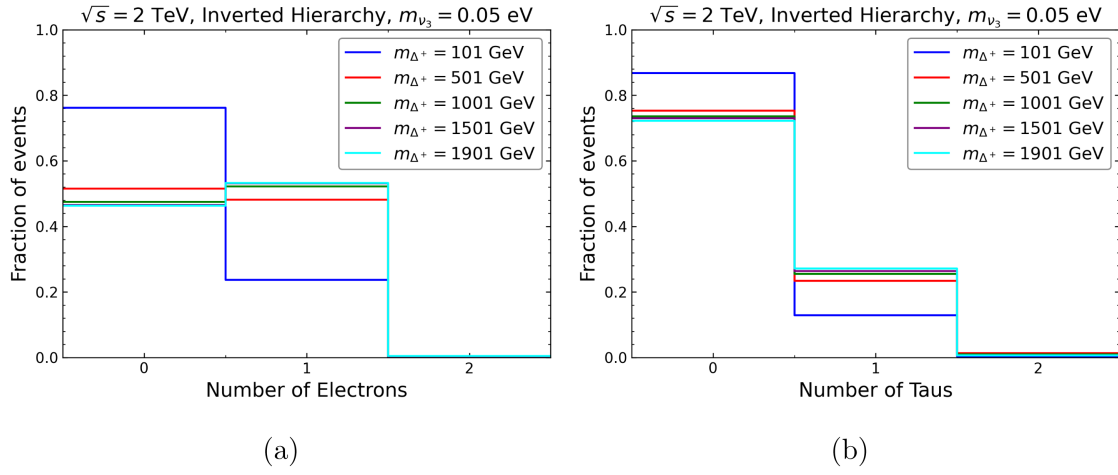
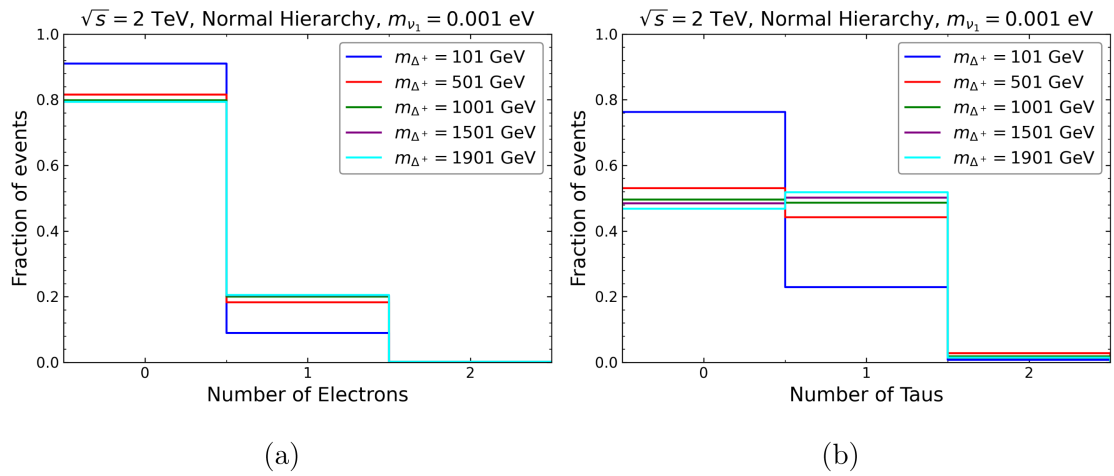
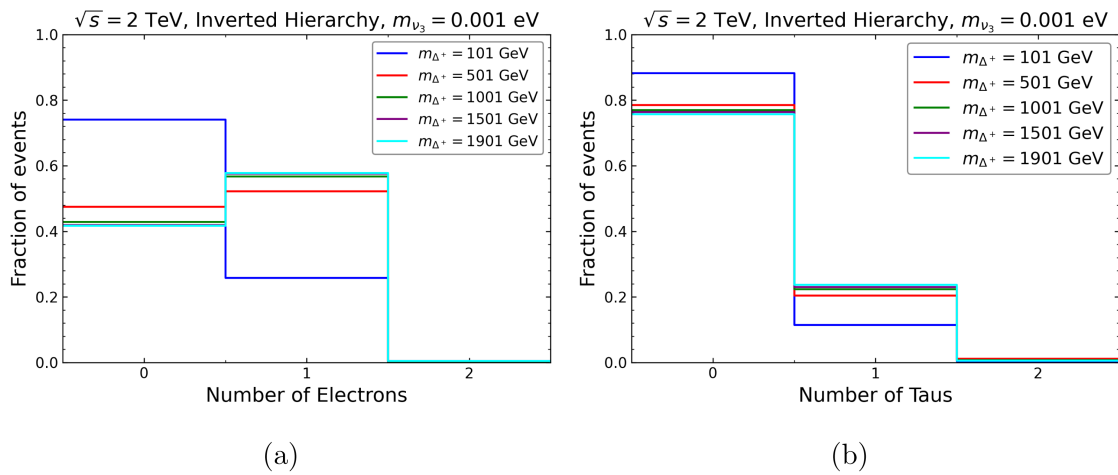
of 5σ and 95% C.L., respectively. As can be noted from Fig. 6, all the points for $m_{\nu_{\text{lightest}}} = 0.05$ eV and for both normal and inverted hierarchies have significance above 5σ , while for $m_{\nu_{\text{lightest}}} = 0.001$ eV, all the points have significance above 95% C.L. but not all of them have significance above 5σ . Normal hierarchy points with $m_{\nu_{\text{lightest}}} = 0.001$ eV, $m_{\Delta^+} \lesssim 207$ GeV, and $1304 \lesssim m_{\Delta^+} \lesssim 1870$ GeV have significance above 5σ , and inverted hierarchy points with $m_{\nu_{\text{lightest}}} = 0.001$ eV, $m_{\Delta^+} \lesssim 140$ GeV, and $1410 \lesssim m_{\Delta^+} \lesssim 1862$ GeV have significance above 5σ , as evident from Fig. 6. However, owing to the current (future) LHC limits on $m_{\Delta^{++}}$, one must concentrate on the signal significance in the parameter space only to the right of the solid green (dashed purple) vertical line. One should note that apart from a slight difference in the values of cross section and significance in between normal and inverted hierarchies, there is not much difference between these two cases. The shapes of the curves, as shown in Fig. 6, are the same for both normal and inverted hierarchies. Therefore, using these curves one cannot identify whether the underlying mechanism is normal or inverted hierarchy. However, the flavors in the final-state leptons will vary a lot for these two cases and hence can be used to differentiate between normal and inverted hierarchies, as discussed in detail in the next section.

V. DIFFERENTIATING NORMAL HIERARCHY FROM INVERTED HIERARCHY

As discussed in Sec. IV, it is possible to obtain a significant amount of cross section for our signature of interest for both normal and inverted hierarchies in the $\mu^+\mu^+$ collider, enough to confirm discovery of a singly charged scalar Δ^+ . However, the results obtained in Fig. 6 are not sufficient to identify whether the underlying theory respects normal hierarchy or inverted hierarchy. This important fact can be ensured by the distribution of final-state electrons and taus originating from the decay of Δ^+ .

The distribution of electrons and taus in the final state for normal and inverted hierarchies are shown in Figs. 7 and 8, respectively, for benchmark points with the lightest neutrino mass $m_{\nu_{\text{lightest}}} = 0.05$ eV and $m_{\Delta^+} = 101, 501, 1001, 1501, \text{ and } 1901$ GeV. Similar plots for benchmark points with the lightest neutrino mass $m_{\nu_{\text{lightest}}} = 0.001$ eV and $m_{\Delta^+} = 101, 501, 1001, 1501, \text{ and } 1901$ GeV are shown in Figs. 9 and 10 for normal and inverted hierarchies, respectively. In this section, the primary goal is to present the nature of the change in the distribution of electrons and taus depending on m_{Δ^+} , $m_{\nu_{\text{lightest}}}$, and the underlying hierarchy, albeit some of the chosen benchmark points have been already excluded by the LHC. Therefore, one can easily predict the distribution of electrons and taus for normal and inverted hierarchies for all of the allowed benchmark points. Although the signature of interest should ideally yield exactly one charged lepton (either e^+ or τ^+), there are many events with no e^+ or τ^+ in the final state, as evident in Figs. 7–10. This is because for a huge number of events, the final-state charged lepton escapes detection for lying outside the range of pseudorapidity that the detector can cover and hence is highly dependent on the detector design. However, this situation improves as m_{Δ^+} increases and more and more events yield detectable charged leptons in the final state. One might notice there are some events that have two charged electrons/taus in the final state, but these events arise due to misidentification of a jet as a lepton. However, one must note that these events are negligibly small in number.

In order to identify whether the theory respects normal or inverted hierarchy, one must compare the distribution of final-state electrons and taus. We compare the distribution of electrons and taus in the final state for normal and inverted hierarchies in Figs. 11(a) and 11(b), respectively, for a particular benchmark point characterized with $m_{\nu_{\text{lightest}}} = 0.05$ eV and $m_{\Delta^+} = 101$ GeV. Similar plots for two other masses of $\Delta^+ = 1001$ GeV and 1901 GeV have been shown in Figs. 12 and 13, respectively. While

FIG. 8. Distribution for (a) electrons in the final state and (b) taus in the final state for $m_{\nu_3} = 0.05$ eV assuming inverted hierarchy.FIG. 9. Distribution for (a) electrons in the final state and (b) taus in the final state for $m_{\nu_1} = 0.001$ eV assuming normal hierarchy.FIG. 10. Distribution for (a) electrons in the final state and (b) taus in the final state for $m_{\nu_3} = 0.001$ eV assuming inverted hierarchy.

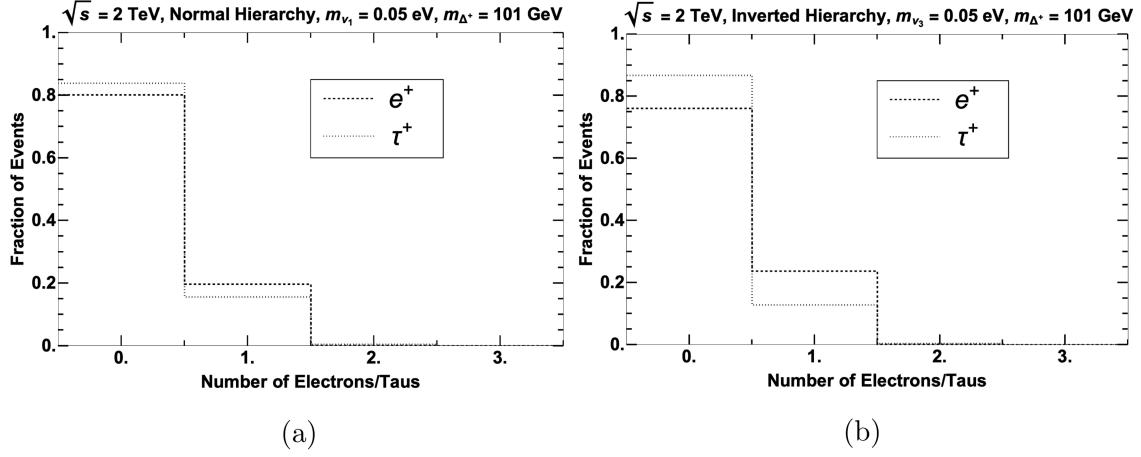


FIG. 11. Distributions of electrons and taus for $m_{\nu_{\text{lightest}}} = 0.05$ eV and $m_{\Delta^+} = 101$ GeV assuming (a) normal hierarchy and (b) inverted hierarchy.

one compares the electron and tau distribution for a particular benchmark point with a characteristic hierarchy, one must be careful to compare the relevant events, i.e., the events that have exactly one charged lepton in the final state. Therefore, comparing such relevant events in Figs. 11–13, one would expect that in case of normal hierarchy there would be more events with taus in the final state than with final-state electrons, while for inverted hierarchy there would be more events with electrons in the final state than with final-state taus, as evident from Fig. 4. However, for bm points with lightest neutrino mass = 0.05 eV, there are always more events with electrons than taus in the final state irrespective of the underlying hierarchy. The difference between the hierarchies are not observable for bm points with lightest neutrino mass = 0.05 eV because for such high mass of the lightest neutrino the difference in the branching ratios of Δ^+ in different leptonic channels is quite small, as evident from Fig. 5. The difference between the hierarchies for massive lightest

neutrino can be improved by modifying the detector structure and taking into consideration several other factors, such as uncertainties, response functions, etc. However, these factors are not yet known and hence beyond the scope of this work. Therefore, given the detector design, as in the default muon collider delphes card, we find the limit at what mass of the lightest neutrino the difference between the hierarchies could be observed from the flavor of the final-state leptons. We find that for lightest neutrino mass ≤ 0.02 eV, one starts to differentiate between normal and inverted hierarchies. We compare the distribution of electrons and taus for normal and inverted hierarchies for $m_{\nu_{\text{lightest}}} = 0.02$ eV and $m_{\Delta^+} = 101, 1001$, and 1901 GeV, respectively, in Figs. 14–16, where it is evident that indeed if the underlying theory respects normal hierarchy, then there will be more events with taus in the final state, and if the underlying theory respects inverted hierarchy then there will be more events with electrons in the final state. The same conclusion holds true for lightest neutrino mass

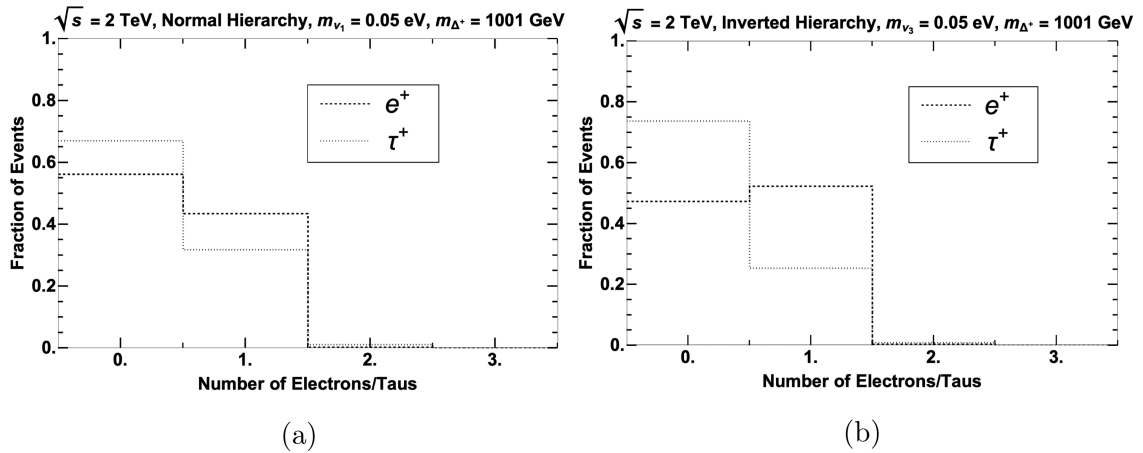


FIG. 12. Distributions of electrons and taus for $m_{\nu_{\text{lightest}}} = 0.05$ eV and $m_{\Delta^+} = 1001$ GeV assuming (a) normal hierarchy and (b) inverted hierarchy.

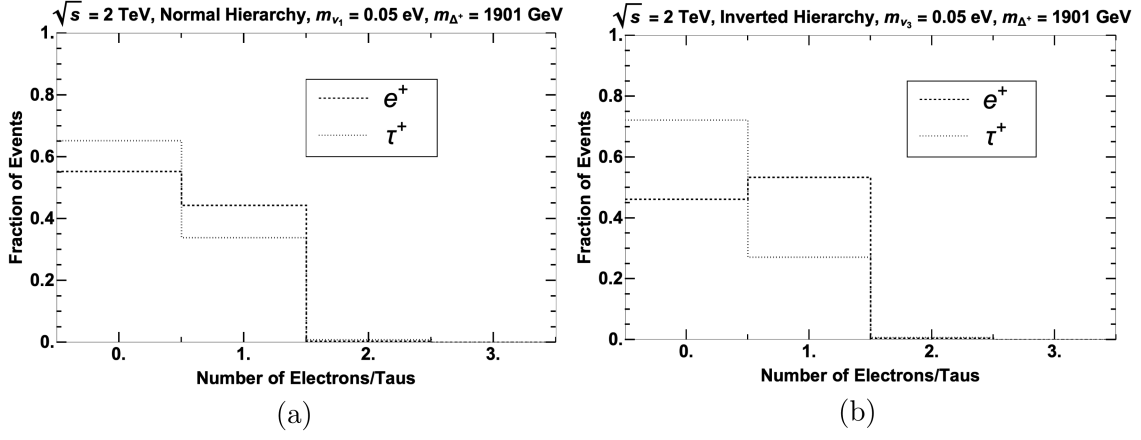


FIG. 13. Distributions of electrons and taus for $m_{\nu_{\text{lightest}}} = 0.05$ eV and $m_{\Delta^+} = 1901$ GeV assuming (a) normal hierarchy and (b) inverted hierarchy.

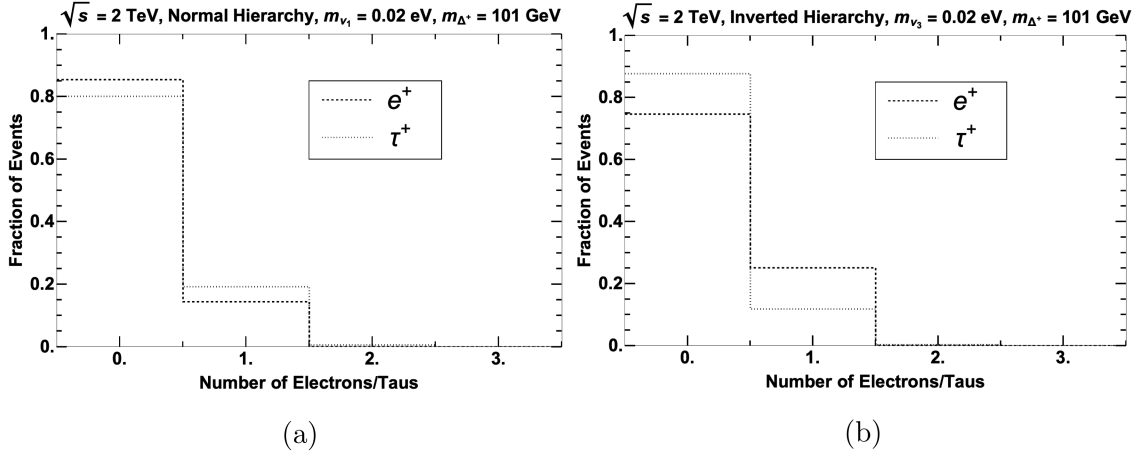


FIG. 14. Distributions of electrons and taus for $m_{\nu_{\text{lightest}}} = 0.02$ eV and $m_{\Delta^+} = 101$ GeV assuming (a) normal hierarchy and (b) inverted hierarchy.

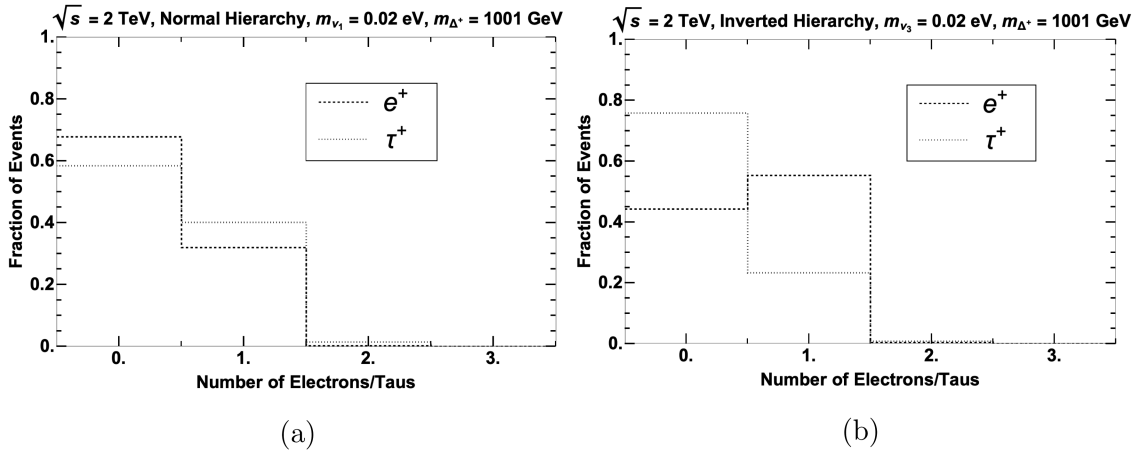


FIG. 15. Distributions of electrons and taus for $m_{\nu_{\text{lightest}}} = 0.02$ eV and $m_{\Delta^+} = 1001$ GeV assuming (a) normal hierarchy and (b) inverted hierarchy.

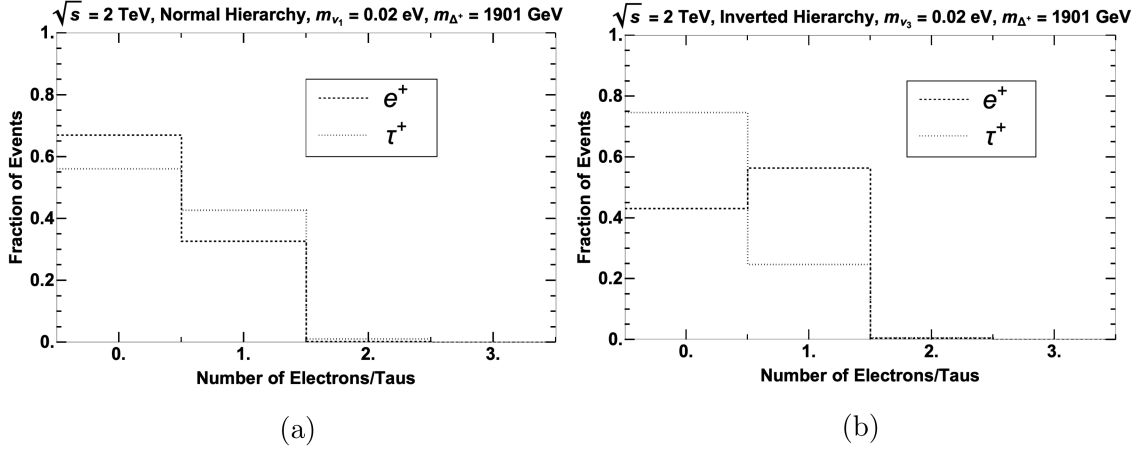


FIG. 16. Distributions of electrons and taus for $m_{\nu_{\text{lightest}}} = 0.02$ eV and $m_{\Delta^+} = 1901$ GeV assuming (a) normal hierarchy and (b) inverted hierarchy.

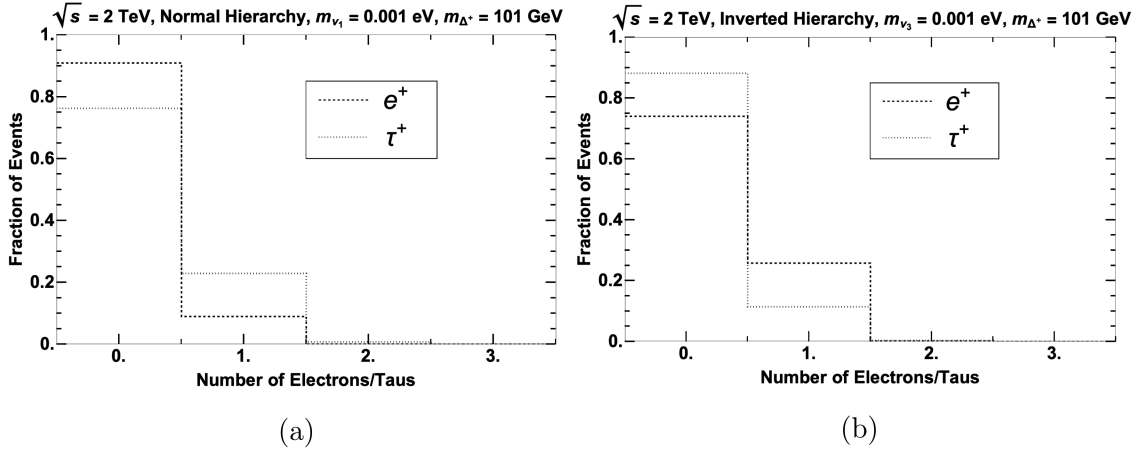


FIG. 17. Distributions of electrons and taus for $m_{\nu_{\text{lightest}}} = 0.001$ eV and $m_{\Delta^+} = 101$ GeV assuming (a) normal hierarchy and (b) inverted hierarchy.

$= 0.001$ eV, as it is evident from Figs. 17–19, which compare the distribution of electrons and taus for normal and inverted hierarchies for $m_{\nu_{\text{lightest}}} = 0.001$ eV and $m_{\Delta^+} = 101, 1001, \text{ and } 1901$ GeV, respectively.

VI. SUMMARY AND OUTLOOK

Although hadron colliders, such as the LHC, are extremely advantageous in looking for new BSM particles produced via direct production, owing to much higher center-of-mass (c.o.m.) energy, as compared to the c.o.m. energy in lepton colliders, a lepton collider enjoys a much cleaner environment with reduced SM background and is, therefore, more advantageous in looking for new BSM particles/processes involving electroweak interactions only. Additionally, in lepton colliders, the full c.o.m. energy is used, as a lepton, unlike a hadron, is a fundamental particle, and hence lepton colliders offer a higher physics reach than hadron colliders for a given c.o.m. energy. Several lepton

colliders, such as e^+e^- collider, $\mu^+\mu^-$ collider, $\mu^+\mu^+$ collider, and μ^+e^- collider, have been proposed to build. However, in this article we focus on the $\mu^+\mu^+$ mode of the μ TRISTAN [26] that is planned to operate at $\sqrt{s} = 2$ TeV. The reasons behind choosing this particular setup among several proposed lepton colliders are twofold: (i) muon colliders lead to smaller loss of energy due to synchrotron radiations than electron colliders, as muons are heavier than electrons, and (ii) building a $\mu^+\mu^+$ collider appears to be more convenient than building a $\mu^+\mu^-$ collider due to the cooling technology developed at J-PARC [28], which can apparently yield μ^+ more efficiently than μ^- such that they are suitable for performing collision at the desired c.o.m. energy. While several studies have been performed in the $\mu^+\mu^+$ collider searching for doubly charged scalar Δ^{++} and electrically neutral heavy scalar Δ^0 [33–36], both of which naturally occur in several BSM scenarios, such as [12,13,15,60–77], there has not been any study in search of the singly charged scalar in the $\mu^+\mu^+$ collider, which also

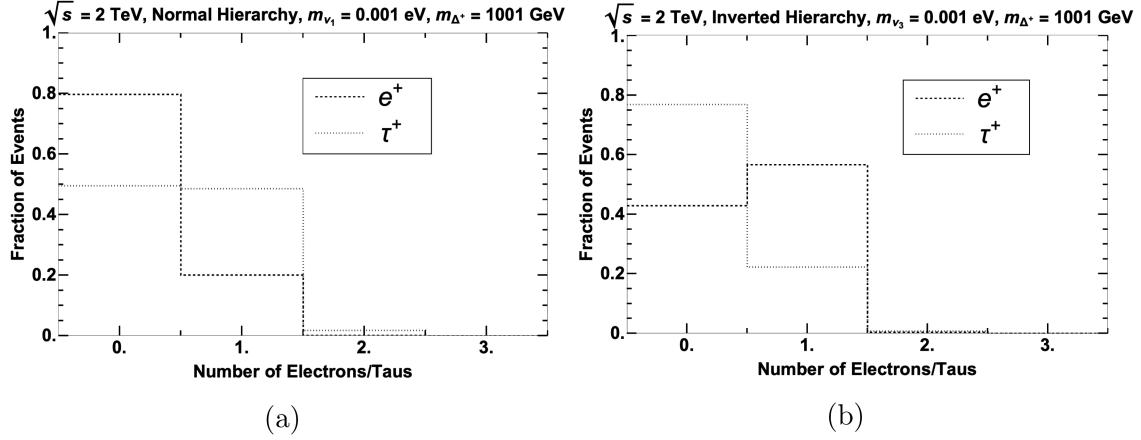


FIG. 18. Distributions of electrons and taus for $m_{\nu_{\text{lightest}}} = 0.001$ eV and $m_{\Delta^+} = 1001$ GeV assuming (a) normal hierarchy and (b) inverted hierarchy.

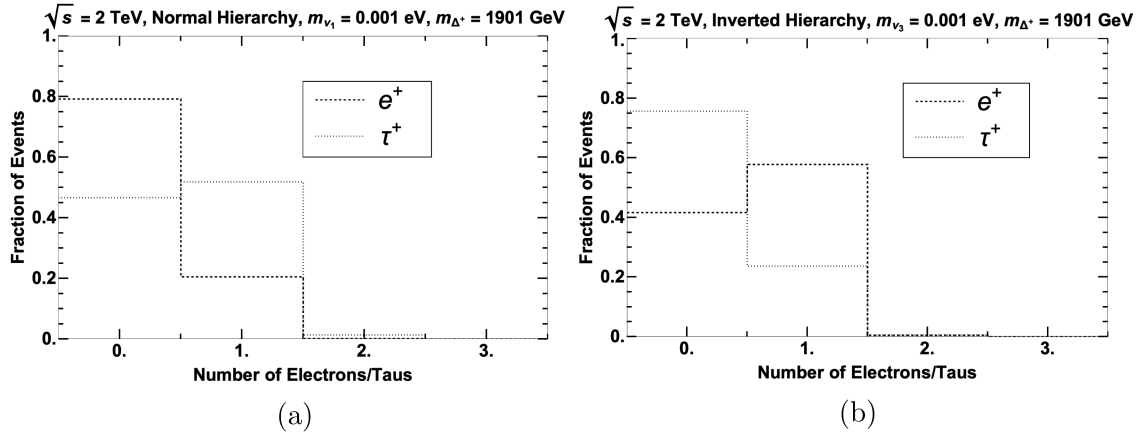


FIG. 19. Distributions of electrons and taus for $m_{\nu_{\text{lightest}}} = 0.001$ eV and $m_{\Delta^+} = 1901$ GeV assuming (a) normal hierarchy and (b) inverted hierarchy.

naturally occurs in the above-mentioned BSM scenarios. In this article we look into the discovery prospect of the singly charged scalar at the $\mu^+\mu^+$ collider at $\sqrt{s} = 2$ TeV in one of the above-mentioned BSM scenarios, namely, the simplest Type-II seesaw scenario [12–15], which is an extremely well-motivated BSM scenario, as it provides a viable mechanism for neutrino mass generation. We study the associated production of Δ^+ along with a W boson followed by the former decaying leptonically and the latter decaying hadronically. Additionally, since the Type-II seesaw scenario allows for LFV processes, which the SM does not, we take into consideration Δ^+ decaying into e^+ or τ^+ only, as this constraint gets rid of the SM background completely, which would otherwise be huge if Δ^+ decaying to μ^+ would also have been considered. On studying the signature of our choice (Fig. 2) for two benchmark points with a mass of the lightest neutrino = 0.05 eV and 0.001 eV, and m_{Δ^+} varying between 101 and 1901 GeV, we find that the $\mu^+\mu^+$ collider operating at $\sqrt{s} = 2$ TeV is capable of yielding a cross section with

significance above 5σ for the entire mass range of Δ^+ for $m_{\nu_{\text{lightest}}} = 0.05$ eV assuming normal or inverted hierarchy, while for $m_{\nu_{\text{lightest}}} = 0.001$ eV, assuming normal hierarchy $m_{\Delta^+} \lesssim 207$ GeV and $1304 \lesssim m_{\Delta^+} \lesssim 1870$ GeV have significance above 5σ and assuming inverted hierarchy points with $m_{\nu_{\text{lightest}}} = 0.001$ eV and $m_{\Delta^+} \lesssim 140$ GeV and $1410 \lesssim m_{\Delta^+} \lesssim 1862$ GeV have significance above 5σ . Both the benchmark points with $m_{\nu_{\text{lightest}}} = 0.05$ and 0.001 eV have cross section above 95% C.L. over the complete mass range of Δ^+ from 101 to 1901 GeV.³ However, the cross section or the significance plot is not sufficient to indicate whether the underlying theory respects normal or inverted hierarchy since both the cases yield curves of similar shape, as can be seen in

³Although here we show the cross section and significance for $101 \text{ GeV} \leq m_{\Delta^+} \leq 1901 \text{ GeV}$, one should focus on the results obtained for $m_{\Delta^+} \gtrsim 1.0(1.5)$ TeV, owing to the current (future) limit on $m_{\Delta^{++}}$ obtained from the LHC.

Fig. 6. The negligibly small difference between the curves for normal and inverted hierarchies, as in Fig. 6, might vanish subject to experimental uncertainties. Therefore, in order to identify the hierarchy one must look into the distribution of the charged leptons in the final state originating from the decay of Δ^+ . As discussed in Sec. V, normal hierarchy leads Δ^+ to primarily decay to τ^+ , while with inverted hierarchy Δ^+ primarily decays to e^+ . However, as we see in Sec. V, with the given detector design the distinction between the underlying hierarchies can be done only if $m_{\nu_{\text{lightest}}} \leq 0.02$ eV. With $m_{\nu_{\text{lightest}}} > 0.02$ eV, the distinction between the underlying hierarchies could be possible with some modifications in the detector design and considering other factors, such as uncertainties, response function, etc., which will be referred to in a related future work. Hence, to conclude, the $\mu^+\mu^+$ collider running at $\sqrt{s} = 2$ TeV is an excellent tool for discovering a singly charged scalar, which can, in turn, hint toward the existence of the Type-II seesaw BSM scenario. This experimental setup can also be used to distinguish between normal and inverted hierarchies, leading to a highly important deduction regarding the underlying theory.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: DOUBLY CHARGED SCALAR (Δ^{++}) IN MUON COLLIDER

Here, we present a comparative study for pair production of the doubly charged scalar in the muon collider at $\sqrt{s} = 3$ and 10 TeV.⁴ The cross sections for the lightest neutrino

⁴Since the LHC predicts a projected limit on $m_{\Delta^{++}} \sim 1.5$ TeV [51], therefore, we present results for $\sqrt{s} = 10$ TeV, as a 3 TeV muon collider is not sufficient for pair production of the doubly charged scalar with $m_{\Delta^{++}} \gtrsim 1.5$ TeV.

TABLE III. Benchmark points for normal and inverted hierarchies at $v_\Delta = 10^{-9}$ GeV and lightest neutrino mass = 0.05 eV. The cross section listed here is for the process $\mu^+\mu^- \rightarrow \Delta^{++}\Delta^{--}$ at $\sqrt{s} = 3$ TeV. All the masses and decay widths (Γ) are in units of GeV and the cross sections σ are in units of fb.

Masses [GeV]			Normal hierarchy	Inverted hierarchy
m_{Δ^0}	m_{Δ^+}	$m_{\Delta^{++}}$	σ	σ
100	101	103	12.3	12.36
500	501	503	10.25	10.32
1000	1001	1003	5.0	5.022

TABLE IV. Benchmark points for normal and inverted hierarchies at $v_\Delta = 10^{-9}$ GeV and lightest neutrino mass = 0.05 eV. The cross section listed here is for the process $\mu^+\mu^- \rightarrow \Delta^{++}\Delta^{--}$ at $\sqrt{s} = 10$ TeV. All the masses and decay widths (Γ) are in units of GeV and the cross sections σ are in units of fb.

Masses [GeV]			Normal hierarchy	Inverted hierarchy
m_{Δ^0}	m_{Δ^+}	$m_{\Delta^{++}}$	σ	σ
500	501	503	1.1	1.1
1000	1001	1003	1.04	1.05
1500	1501	1503	0.96	0.96
1700	1701	1703	0.92	0.922
1900	1901	1903	0.87	0.88
2000	2001	2003	0.85	0.852
3000	3001	3003	0.56	0.563
4000	4001	4003	0.23	0.235
4900	4901	4903	8.1×10^{-3}	8.2×10^{-3}

TABLE V. Benchmark points for normal and inverted hierarchies at $v_\Delta = 10^{-9}$ GeV and lightest neutrino mass = 0.001 eV. The cross section listed here is for the process $\mu^+\mu^- \rightarrow \Delta^{++}\Delta^{--}$ at $\sqrt{s} = 3$ TeV. All the masses and decay widths (Γ) are in units of GeV and the cross sections σ are in units of fb.

Masses [GeV]			Normal hierarchy	Inverted hierarchy
m_{Δ^0}	m_{Δ^+}	$m_{\Delta^{++}}$	σ	σ
100	101	103	12.75	12.84
500	501	503	10.71	10.78
1000	1001	1003	5.246	5.289

mass = 0.05 eV for both normal and inverted hierarchies are presented in Tables III and IV at $\sqrt{s} = 3$ and 10 TeV, respectively. Similar results for lightest neutrino mass = 0.001 eV are presented in Tables V and VI.

APPENDIX B: DOUBLY CHARGED SCALAR (Δ^{++}) IN $\mu^+\mu^+$ MODE OF μ TRISTAN

This appendix shows another comparative study for production of the doubly charged scalar in the μ TRISTAN at $\sqrt{s} = 2$ TeV. The cross sections for the lightest neutrino mass = 0.05 eV for both normal and inverted hierarchies are presented in Table VII. Similar results for lightest neutrino mass = 0.001 eV are presented in Table VIII.

APPENDIX C: ANALYTICAL FORMULA FOR THE SIGNAL PROCESS

This appendix explains the fall and rise nature of the curve obtained in Fig. 3 for the cross section of our signature of interest. For the processes (a) and (b) shown in Fig. 2, the corresponding helicity amplitudes $\mathcal{M}_{(a)}(s_1, s_2, s_3)$ and $\mathcal{M}_{(b)}(s_1, s_2, s_3)$ are given as follows. Here, $s_1 = \pm$, $s_2 = \pm$ are the helicities of the initial antimuons, and $s_3 = \pm, 0$ is the helicity of the final-state W boson,

$$\begin{aligned} \mathcal{M}_{(a)}(-, -, 0)/Y_{\mu\mu}^* &= -\mathcal{M}_{(a)}(+, +, 0)/Y_{\mu\mu}^* \\ &= -\frac{\sqrt{2}\sqrt{s}\sqrt{(s-m_\Delta^2-m_W^2)^2-4m_\Delta^2m_W^2}}{m_W(s-m_\Delta^2)}, \end{aligned} \quad (C1)$$

$$\begin{aligned} \mathcal{M}_{(b)}(-, -, \mp)/Y_{\mu\mu}^* &= -\mathcal{M}_{(b)}(+, +, \pm)/Y_{\mu\mu}^* \\ &= \frac{\sin\theta}{2} \frac{s-m_W^2+m_\Delta^2 \pm \sqrt{(s-m_\Delta^2-m_W^2)^2-4m_\Delta^2m_W^2}}{s-m_W^2-m_\Delta^2-\cos\theta\sqrt{(s-m_\Delta^2-m_W^2)^2-4m_\Delta^2m_W^2}} \\ &\quad - \frac{\sin\theta}{2} \frac{s-m_W^2+m_\Delta^2 \pm \sqrt{(s-m_\Delta^2-m_W^2)^2-4m_\Delta^2m_W^2}}{s-m_W^2-m_\Delta^2+\cos\theta\sqrt{(s-m_\Delta^2-m_W^2)^2-4m_\Delta^2m_W^2}}, \end{aligned} \quad (C2)$$

$$\begin{aligned} \mathcal{M}_{(b)}(-, -, 0)/Y_{\mu\mu}^* &= -\mathcal{M}_{(b)}(+, +, 0)/Y_{\mu\mu}^* \\ &= \frac{\sqrt{s}(\sqrt{(s+m_W^2-m_\Delta^2)^2-4m_\Delta^2m_W^2}-\cos\theta(s-m_\Delta^2-m_W^2))}{\sqrt{2}m_W(s-m_\Delta^2-m_W^2-\cos\theta\sqrt{(s-m_\Delta^2-m_W^2)^2-4m_\Delta^2m_W^2})} \\ &\quad + \frac{\sqrt{s}(\sqrt{(s+m_W^2-m_\Delta^2)^2-4m_\Delta^2m_W^2}+\cos\theta(s-m_\Delta^2-m_W^2))}{\sqrt{2}m_W(s-m_\Delta^2-m_W^2+\cos\theta\sqrt{(s-m_\Delta^2-m_W^2)^2-4m_\Delta^2m_W^2})}. \end{aligned} \quad (C3)$$

TABLE VII. Benchmark points for normal and inverted hierarchies at $v_\Delta = 10^{-9}$ GeV and lightest neutrino mass = 0.05 eV. The cross section listed here is for the process $\mu^+\mu^+ \rightarrow \Delta^{++} \rightarrow \mu^+\mu^+$ at $\sqrt{s} = 2$ TeV. All the masses and decay widths (Γ) are in units of GeV and the cross sections σ are in units of fb.

Masses [GeV]			Normal hierarchy				Inverted hierarchy			
m_{Δ^0}	m_{Δ^+}	$m_{\Delta^{++}}$	Γ_{Δ^0}	Γ_{Δ^+}	$\Gamma_{\Delta^{++}}$	σ	Γ_{Δ^0}	Γ_{Δ^+}	$\Gamma_{\Delta^{++}}$	σ
100	101	103	0.02	0.0202	0.0206	1.419×10^{-2}	0.0247	0.0249	0.0254	1.146×10^{-2}
300	301	303	0.0601	0.0603	0.0607	1.478×10^{-2}	0.0742	0.0744	0.0749	1.194×10^{-2}
500	501	503	0.1002	0.1004	0.1008	1.608×10^{-2}	0.1236	0.1239	0.1243	1.299×10^{-2}
1000	1001	1003	0.2004	0.2006	0.201	2.519×10^{-2}	0.2472	0.2475	0.248	2.035×10^{-2}
1500	1501	1503	0.3006	0.3008	0.3012	7.448×10^{-2}	0.3708	0.3711	0.3716	6.018×10^{-2}
1900	1901	1903	0.3808	0.381	0.3814	1.575	0.4697	0.47	0.4704	1.273

TABLE VI. Benchmark points for normal and inverted hierarchies at $v_\Delta = 10^{-9}$ GeV and lightest neutrino mass = 0.001 eV. The cross section listed here is for the process $\mu^+\mu^- \rightarrow \Delta^{++}\Delta^{--}$ at $\sqrt{s} = 10$ TeV. All the masses and decay widths (Γ) are in units of GeV and the cross sections σ are in units of fb.

Masses [GeV]			Normal hierarchy	Inverted hierarchy
m_{Δ^0}	m_{Δ^+}	$m_{\Delta^{++}}$	σ	σ
500	501	503	1.14	1.144
1000	1001	1003	1.085	1.092
1500	1501	1503	1.0	1.01
1700	1701	1703	0.96	0.965
1900	1901	1903	0.911	0.92
2000	2001	2003	0.87	0.9
3000	3001	3003	0.59	0.6
4000	4001	4003	0.246	0.25
4900	4901	4903	8.6×10^{-3}	8.7×10^{-3}

TABLE VIII. Benchmark points for normal and inverted hierarchies at $v_\Delta = 10^{-9}$ GeV and lightest neutrino mass = 0.001 eV. The cross section listed here is for the process $\mu^+\mu^+ \rightarrow \Delta^{++} \rightarrow \mu^+\mu^+$ at $\sqrt{s} = 2$ TeV. All the masses and decay widths (Γ) are in units of GeV and the cross sections σ are in units of fb.

Masses [GeV]			Normal hierarchy				Inverted hierarchy			
m_{Δ^0}	m_{Δ^+}	$m_{\Delta^{++}}$	Γ_{Δ^0}	Γ_{Δ^+}	$\Gamma_{\Delta^{++}}$	σ	Γ_{Δ^0}	Γ_{Δ^+}	$\Gamma_{\Delta^{++}}$	σ
100	101	103	5.13×10^{-3}	5.18×10^{-3}	5.28×10^{-3}	8.531×10^{-4}	9.806×10^{-3}	9.902×10^{-3}	1.01×10^{-2}	2.195×10^{-4}
300	301	303	1.538×10^{-2}	1.543×10^{-2}	1.553×10^{-2}	8.889×10^{-4}	2.942×10^{-2}	2.952×10^{-2}	2.971×10^{-2}	2.287×10^{-4}
500	501	503	2.563×10^{-2}	2.568×10^{-2}	2.58×10^{-2}	9.67×10^{-4}	4.903×10^{-2}	4.913×10^{-2}	4.932×10^{-2}	2.488×10^{-4}
1000	1001	1003	5.126×10^{-2}	5.132×10^{-2}	5.142×10^{-2}	1.515×10^{-3}	9.806×10^{-2}	9.816×10^{-2}	9.836×10^{-2}	3.897×10^{-4}
1500	1501	1503	7.69×10^{-2}	7.695×10^{-2}	7.705×10^{-2}	4.479×10^{-3}	1.471×10^{-1}	1.472×10^{-1}	1.474×10^{-1}	1.153×10^{-3}
1900	1901	1903	9.741×10^{-2}	9.746×10^{-2}	9.756×10^{-2}	9.472×10^{-2}	1.863×10^{-1}	1.864×10^{-1}	1.866×10^{-1}	2.437×10^{-2}

Other helicity combinations are all zero. For the process (b), we have neglected the ν_ℓ masses. The cross-term between $\mathcal{M}_{(a)}(\pm, \pm, 0)$ and $\mathcal{M}_{(b)}(\pm, \pm, 0)$ yields destructive interference that first decreases and then increases

with increasing m_Δ . This fall and rise behavior of the destructive interference term explains the typical fall and rise nature of the curves with increasing m_Δ , as seen in Figs. 3 and 6.

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