

Classification of Higgs sectors from group theoretical properties of UV gauge theories

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Extended Higgs sectors are often introduced to explain phenomena beyond the standard model (BSM). The existence of multiple scalar fields may cause the Landau pole below the Planck scale. In this case, the low-energy theory may be replaced by an asymptotic-free gauge theory. In this paper, we consider an SU(2) gauge theory with confinement as such an ultraviolet theory of the extended Higgs sectors. We investigate the relation between scalar particle contents at the low energy and group theoretical properties of fundamental fermions of the gauge theory. We find that particle contents of various extended Higgs sectors previously proposed to explain the BSM problems are deduced by each charge assignment of flavor symmetry of the fundamental fermions of the SU(2) gauge symmetry. Our findings may provide a new picture for the ultraviolet completion of the extended Higgs sectors.

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I. INTRODUCTION

Although the Higgs boson was discovered at the Large Hadron Collider [1,2], the true shape of the Higgs sector, the number of the scalar fields, their representations under the gauge symmetries, dynamics behind, and so on, remain unknown. While the discovered Higgs boson is consistent with the one in the standard model (SM) under the theoretical and experimental uncertainties [3,4], there is no theoretical guiding principle for the Higgs sector of the SM. Therefore, the possibility of nonstandard Higgs sectors has been discussed in various physics contexts. In fact, there remain phenomena which the SM cannot explain, such as tiny neutrino masses, dark matter, and baryon asymmetry of the Universe. Some of them might be related to physics of nonstandard (extended) Higgs sectors.

From experiments, several strong constraints on the shape of the Higgs sector are indicated, such that the

electroweak rho parameter [5–7] is very close to unity [8–10], and that the flavor changing neutral currents are strongly suppressed [11–13]. The LHC data [3,4] also indicate that an extended Higgs sector, if it exists, has to show the SM-like behavior at the electroweak scale at least approximately. Future experiments may detect evidence of the extended Higgs sectors.

Extended Higgs models can explain the tiny neutrino masses by the type-II seesaw mechanism [14–17] or by quantum effects [18–21]. By imposing an unbroken Z_2 symmetry to nonminimal Higgs sectors, weakly interacting dark matter candidates can be provided [22]. Tiny neutrino masses and dark matter simultaneously are explained by the combination of these mechanisms in various models [23–26]. Extended Higgs sectors can also make the electroweak phase transition strongly first order which cannot be realized in the SM Higgs sector [27–35]. As extended Higgs sectors naturally contain additional CP violating phases, the scenario of electroweak baryogenesis [36] is possible. Many models with extended Higgs sectors have been considered for electroweak baryogenesis [37–50]. Furthermore, combination of these ideas leads to TeV scale models which can explain all the three problems simultaneously [51–53].

Each model mentioned above has specific particle contents for the Higgs sector. While some of such models are phenomenologically successful, a specific pattern of additional

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scalar fields is introduced in an *ad hoc* way. However, such a specific scalar sector should be deduced as a low-energy effective theory of a more fundamental theory at high energies.

In the SM with the 125 GeV Higgs boson, the quartic coupling of the Higgs boson falls down to be negative below the Planck scale due to the large mass of the top quark, and the vacuum becomes metastable [54–57]. On the contrary, in the extended Higgs sectors, the ultraviolet (UV) behavior of the couplings can be changed by quantum effects of additional scalar bosons [58–68]. In general, at the one-loop level, additional scalar bosons give positive contributions to the beta functions of the scalar quartic couplings. If the contributions are large, then the running coupling constants may blow up at the Landau pole below the Planck scale. Such extended Higgs models should then have a cutoff scale, above which they are replaced by a more fundamental theory.

In this paper, we consider a possibility of an asymptotic-free gauge theory with confinement as a candidate of the fundamental theory above the cutoff scale. We here adopt SU(2) (hereafter we denote SU(2)_H) as such a gauge symmetry as the simplest example [69–75]. We assume that the scalar bosons in the extended Higgs sectors appear at the low energy as hadronlike bound states of the fundamental fermions due to the strong interaction of SU(2)_H. Particle contents and their symmetry properties in the extended Higgs sectors will be inherited from the nature of flavor symmetry of the fundamental fermions.

Many scenarios, in which the electroweak symmetry breaking sector is a low-energy effective theory of a fundamental theory above the cutoff scale, have been discussed in different contexts, such as technicolor models [80–76]], composite Higgs models [81–84], models for confinement with supersymmetry [70,85], and so on.

In our scenario, the SM gauge group SU(2)_L ⊗ U(1)_Y is embedded into the flavor symmetry of the SU(2)_H gauge theory. Unlike the technicolor theory, spontaneous symmetry breaking (SSB) of the SM gauge group is assumed to be caused by the scalar bosons within the IR effective theory. We do not specify those scalar bosons as the Nambu-Goldstone (NG) bosons of SSB of the flavor symmetries. As a consequence of the confinement of the SU(2)_H gauge theory, the extended Higgs sectors for various phenomenological models are classified by the number of the fundamental fermions and their nature under the flavor symmetry and the SM gauge group.

This paper is organized as follows. In Sec. II, we give general discussions on the SU(2)_H gauge theory. In Sec. III, we classify extended Higgs models in terms of the flavor number and the charge assignment in the SU(2)_H gauge theory. The discussions and the conclusions are given in Secs. IV and V.

II. SU(2)_H GAUGE THEORY

In this section, we discuss the SU(2)_H gauge theory with n fundamental fermions. The theory is asymptotic-free or

conformal, when $n \lesssim 10$ [86,87]. We assume that, at the energy scale below a scale Λ , the fermions are confined due to the strong interaction of SU(2)_H. It is known that $n \lesssim 4.7$ is needed for the massless fermions to be confined [87,88].

A. Flavor symmetry

We first discuss properties of a fundamental fermion ψ under SU(2)_H and its Dirac conjugate $\bar{\psi}$. They are written by

$$\psi_h = \begin{pmatrix} \chi_{ah} \\ \eta_{\dot{a}h} \end{pmatrix}, \quad \bar{\psi}^h = (\eta^{*ah}, \chi_{\dot{a}}^{*h}), \quad (1)$$

where χ_α and $\eta^{*\alpha}$ (χ_α^* and $\eta^{\dot{\alpha}}$) are the **2** (**2**^{*}) spinors of SL(2, C). The spinor index is raised or lowered by the antisymmetric tensors $\epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} = \epsilon_{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}}$ ($\epsilon^{12} = +1$). The subscript (superscript) h represents **2** (**2**^{*}) of SU(2)_H, and they can be connected by the antisymmetric tensors $\epsilon_{Hh_1h_2} = \epsilon_H^{h_1h_2}$ ($\epsilon_{H12} = +1$). A proper charge conjugation for ψ is defined as

$$\tilde{\psi}_{h_1} \equiv \epsilon_{Hh_1h_2} (\psi^C)^{h_2} = \begin{pmatrix} \eta_{ah_1}^* \\ \chi_{h_1}^{*\dot{a}} \end{pmatrix}. \quad (2)$$

It is notable that, for any index $I = \{\alpha, \dot{\alpha}, h\}$, $a^I b_I = -a_I b^I$ is satisfied.

Let us see the flavor symmetry in the kinetic term of the SU(2)_H sector. With n flavor fermions ψ_i ($i = 1, \dots, n$), it is given by

$$\mathcal{L}_{\text{kin}} = \bar{\psi}_i i \gamma^\mu D_\mu \psi_i, \quad (3)$$

where D_μ is the covariant derivative of the SU(2)_H gauge field, and the summation of the flavor index i is implicit. In terms of χ and η , it is subsequently rewritten by

$$\begin{aligned} \mathcal{L}_{\text{kin}} &= \chi_{i\dot{\alpha}}^{*h_1} (i\bar{\sigma}^\mu)^{\dot{\alpha}\beta} (D_\mu)_{h_1}^{h_2} \chi_{i\beta h_2} + \eta_i^{*ah_1} (i\sigma^\mu)_{a\dot{\beta}} (D_\mu)_{h_1}^{h_2} \eta_{ih_2}^{\dot{\beta}} \\ &= (\tilde{T})_{\dot{\alpha}}^{h_1} (i\bar{\sigma}^\mu)^{\dot{\alpha}\beta} (D_\mu)_{h_1}^{h_2} (T)_{ah_2}, \end{aligned} \quad (4)$$

where the sign conventions for the Pauli matrices $\sigma^\mu = (\mathbb{I}_2, -\sigma^a)$ and $\bar{\sigma}^\mu = (\mathbb{I}_2, \sigma^a)$ ($a = 1, 2, 3$) are taken. We have defined the flavor multiplet T as

$$T \equiv (\chi_1, \eta_1^*, \dots, \chi_n, \eta_n^*)^T \equiv (T_1, \dots, T_n)^T, \quad (5)$$

In this kinetic term, there is SU(2n) symmetry.

Without loss of generality, the flavor index can be taken for the basis where the Dirac mass is diagonalized: the mass term of the SU(2)_H sector is, in general, given by

$$\begin{aligned}\mathcal{L}_{\text{mass}} &= -m_i \bar{\psi}_i \psi_i - \mathcal{M}_{ij} (\bar{\tilde{\psi}}_i \psi_j - \bar{\psi}_i \tilde{\psi}_j) \\ &= -\frac{1}{2} (\mathbf{T}^\dagger)^{ah} \mathbf{m}(\mathbf{T})_{ah} - \frac{1}{2} (\mathbf{T}^\dagger)^{ah} \mathbf{M}(\mathbf{T})_{ah} + \text{H.c.},\end{aligned}\quad (6)$$

where m_i and $\mathcal{M}_{ij}$ are the Dirac and Majorana masses, respectively. We assume all of the masses are real. Defining $M_{ij} \equiv \mathcal{M}_{ij} - \mathcal{M}_{ji}$, the mass matrices are given by

$$\begin{aligned}\mathbf{m} &= \mathbf{m}_{n \times n} \otimes -i\sigma_2, \\ \mathbf{M} &= \mathbf{M}_{n \times n} \otimes \mathbb{I}_2,\end{aligned}\quad (7)$$

where

$$\mathbf{m}_{n \times n} = \text{diag}(m_1, \dots, m_n),$$

$$\mathbf{M}_{n \times n} = \begin{pmatrix} 0 & M_{12} & \cdots & \cdots & M_{1n} \\ -M_{12} & 0 & \cdots & \cdots & M_{2n} \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & 0 & M_{n-1n} \\ -M_{1n} & -M_{2n} & \cdots & -M_{n-1n} & 0 \end{pmatrix}. \quad (8)$$

Depending on the form of the mass term, various explicit breakings of the $\text{SU}(2n)$ flavor symmetry are realized. For example, when we have $m_i = m$ and $M_{ij} = 0$, the explicit symmetry breaking is $\text{SU}(2n) \rightarrow \text{Sp}(2n)$. When we have $M_{ij} = 0$ ($j = 1, \dots, n$) for the i th fermion T_i , we can see an $\text{SU}(2)$ invariance $T_i \rightarrow UT_i$ ($U \in \text{SU}(2)_i$). Therefore, if $M_{ij} = 0$ for all i and j , we have the $\text{SU}(2)^n (= \otimes_{i=1}^n \text{SU}(2)_i)$ symmetry in this Lagrangian. We note that, the $\text{U}(1)_i$ symmetries are realized as the subgroup of the $\text{SU}(2)_i$ symmetries, under which the flavor multiplets transform as $T_i \rightarrow \exp(i\sigma_3\theta_i)T_i$. This symmetry corresponds to the global $\text{U}(1)_V$ symmetry for ψ_i ; the Lagrangian is invariant under $\psi_i \rightarrow \exp(i\theta_i)\psi_i$. By contrast, the $\text{U}(1)_A$ symmetries ($\psi_i \rightarrow \exp(i\theta_i\gamma^5)\psi_i$) are explicitly broken by the Dirac mass terms.

When $\chi_i = \eta_i^*$ is satisfied, the corresponding i th fermion ψ_i satisfies the Majorana condition, $\psi_i = \tilde{\psi}_i$. In this paper, we do not discuss cases where such a condition is imposed.

B. Discrete symmetries

First, we define the parity transformation for the multiplets $\mathbf{T}$, which are used to classify bound states. The usual parity transformation defined in terms of ψ_i deduces

$$\chi_{iah}(x) \xrightarrow{P} \eta_{ih}^{\dot{\alpha}}(x_P), \quad \eta_{ih}^{\dot{\alpha}}(x) \xrightarrow{P} \chi_{iah}(x_P), \quad (9)$$

where $x = (t, \mathbf{x})$ and $x_P = (t, -\mathbf{x})$. However, this transformation is not invariant under the flavor symmetry. Following Ref. [75], we define a *proper* parity

transformation Q as

$$\chi_{iah}(x) \xrightarrow{Q} \beta \eta_{ih}^{\dot{\alpha}}(x_P), \quad \eta_{ih}^{\dot{\alpha}}(x) \xrightarrow{Q} \beta \chi_{iah}(x_P), \quad (10)$$

where β is an arbitrary phase.

In terms of $\mathbf{T}$, the transformation is given by

$$(\mathbf{T})_{ah_1}^A \xrightarrow{Q} Q \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon_{Hh_1h_2} (\mathbf{T}^*)_{\dot{\beta}}^{\bar{A}h_2} = Q (\mathbf{T}^*)_{h_1}^{\bar{A}\dot{\alpha}}, \quad (11)$$

where

$$Q = \mathbb{I}_n \otimes \begin{pmatrix} 0 & \beta \\ \beta^* & 0 \end{pmatrix}. \quad (12)$$

We here have omitted the space-time variables. The index of the (complex conjugate) representation of the flavor symmetry is denoted by A ($\bar{A}$). Since the multiplets transform under the flavor symmetry as $\mathbf{T}^A \rightarrow U^{AB} \mathbf{T}^B$ or $\mathbf{T}^{*\bar{A}} \rightarrow U^{\bar{A}\bar{B}} \mathbf{T}^{*\bar{B}}$, we need to satisfy the conditions,

$$QU^* = UQ, \quad (13)$$

so that $Q\mathbf{T}Q^{-1}$ transforms the same as $\mathbf{T}$. We cannot satisfy these conditions if the flavor symmetry is maximal, i.e., $U \in \text{SU}(2n)$. However, in the case of $\text{Sp}(2n)$, the solution is $\beta = \pm i$, by taking which Q is proportional to

$$J^{A\bar{B}} = (\mathbb{I}_n \otimes i\sigma_2)^{A\bar{B}}. \quad (14)$$

For any of $U \in \text{Sp}(2n)$, $U^\dagger U = J$ is satisfied. In the following discussions, we consider the explicit breaking of $\text{SU}(2n)$ by the mass term, and thus the maximal symmetry is $\text{Sp}(2n)$. Therefore, taking $\beta = +i$ is enough to discuss the bound states. As a result, the parity transformation is given by

$$(\mathbf{T})_{ah}^A \xrightarrow{Q} iJ^{A\bar{B}} (\mathbf{T}^*)_{h_1}^{\bar{B}\dot{\alpha}} = i(\mathbf{T}^*)_{h_1}^{\bar{A}\dot{\alpha}}. \quad (15)$$

For later discussions, we here consider a discrete symmetry $Z_N = \otimes_{i=1}^n Z_N^i$ of the multiplet $\mathbf{T}$. We consider the Z_N transformation commuting to the transformations under the remaining flavor symmetry and the parity. The Z_N transformation is defined as

$$\mathbf{T} \xrightarrow{Z_N} \begin{pmatrix} \omega_1 & & & \\ & \omega_1^* & & \\ & & \ddots & \\ & & & \omega_n \\ & & & & \omega_n^* \end{pmatrix} \mathbf{T} \equiv Z\mathbf{T}, \quad (16)$$

where $\omega_i \equiv \exp(2\pi i/N_i)$ ($N_i \in \mathbb{Z}$). The Dirac mass term is invariant under this transformation.

When the $\text{Sp}(2n)$ symmetry exists in the Lagrangian, it must be satisfied that

$$ZU = UZ, \quad (17)$$

where $U \in \text{Sp}(2n)$. From Schur's Lemma, all of ω_i must have the same Z_2 charge as $\omega_i = \omega = \pm 1$. When the $\text{SU}(2)^n$ symmetry is present, the Z_2 charges for each flavor can be different: i.e., $\omega_i = \omega_i^* = \pm 1$ is allowed. By taking ω_i in these ways, the matrix can be written as $Z = \text{diag}(\omega_1, \dots, \omega_n) \otimes \mathbb{I}_2$, which commutes with the parity transformation matrix $Q = iJ = i(\mathbb{I}_n \otimes i\sigma_2)$.

C. Bound states

The scalar bosons, which are bound states of two fermions, are classified by irreducible representations of the flavor symmetry in the UV Lagrangian. Letting T^A be an irrep. of a flavor symmetry, the scalar bound states are either

$$(T)^{Aah}(T)_{ah}^B \quad \text{or} \quad (T^*)^{A\dot{a}h}(T^*)_{\dot{a}h}^B. \quad (18)$$

We here have assumed that the irrep. is real or pseudoreal. From the discussions given in Sec. II B, it is shown that the states written by

$$S_{\pm}^{AB} = (T)^{Aah}(T)_{ah}^B \pm (T^*)^{A\dot{a}h}(T^*)_{\dot{a}h}^B, \quad (19)$$

are parity even and odd for $+$ and $-$, respectively.

We note that totally symmetric representations with respect to the flavor indices cannot be the scalar bosons due to the anticommuting property of the fermions. Such a symmetric representation can appear as the vectorlike boson, e.g.,

$$V^{\mu AB} = (T)^{Aah}\sigma_{a\dot{b}}^{\mu}(T^*)_{\dot{h}}^{B\dot{b}} + (A \leftrightarrow B). \quad (20)$$

III. EXTENDED HIGGS MODELS

In this section, we discuss extended Higgs models deduced by the $\text{SU}(2)_H$ theory with n flavor fermions ψ_i ($i = 1, \dots, n$). In our setup, the SM quarks and leptons are singlets under $\text{SU}(2)_H$, and the scalar sectors of various extended Higgs models are realized by the confinement of the fundamental fermions of $\text{SU}(2)_H$.

As mentioned in the Introduction, we do not specify the scalar bosons as the NG boson, and therefore we do not focus on the spontaneous breaking of the flavor symmetry induced by the confinement.¹ The chiral symmetry $\text{SU}(2n)$ is explicitly broken by the fermion mass terms. The SM gauge group is embedded in the flavor symmetry of the

$\text{SU}(2)_H$ Lagrangian, which is assumed to be spontaneously broken in the context of extended Higgs models around the electroweak scale $v \simeq 246$ GeV.

Our focus is not on the concrete mass spectrum in the extended Higgs models, but rather on revealing how those models are classified into the flavor symmetry of the UV Lagrangian and the charge assignment of the SM gauge group.

In the next subsection, we present examples of the extended Higgs models derived from the n flavor $\text{SU}(2)_H$ gauge theory, for n up to 4. We then discuss the Yukawa interaction in the extended Higgs models.

A. $n = 1$ case

We suppose the $\text{SU}(2)_H$ theory with a fermion $T_1 = (\chi_1, \eta_1^*)^T$. The flavor symmetry is $\text{Sp}(2) \simeq \text{SU}(2)$ regardless of the Dirac mass m_1 . This $\text{SU}(2)$ can be regarded as the $\text{SU}(2)_L$ gauge symmetry by introducing the minimal interaction [84]

$$\mathcal{L} = (\overline{T_1})_{\dot{\alpha}}^h (i\bar{\sigma}^{\mu})^{\dot{\alpha}\beta} \left(-ig \frac{\sigma}{2} \cdot W_{\mu} \right) (T_1)_{\beta h}, \quad (21)$$

where $W_{\mu} = (W_{\mu}^1, W_{\mu}^2, W_{\mu}^3)^T$ and g are the $\text{SU}(2)_L$ gauge bosons and coupling, respectively. However, the possible representation of the scalar is only the singlet state of $\text{SU}(2)_L$. In addition, the $\text{U}(1)_Y$ interaction cannot be embedded into the flavor symmetry. Therefore, the SM Higgs doublet is not realized in this case.

B. $n = 2$ case

The smallest flavor number including the SM Higgs is $n = 2$. The $\text{SU}(2)_H$ sector is composed of two fermions, $T_1 = (\chi_1, \eta_1^*)^T$ and $T_2 = (\chi_2, \eta_2^*)^T$. The mass parameters are m_1 , m_2 , and M_{12} .

1. Class 1: Real singlet extension of the SM

We first consider the case that the Majorana mass is zero, $M_{12} = 0$. In the $\text{SU}(2)_H$ Lagrangian, in general, we have $\text{SU}(2)_1 \otimes \text{SU}(2)_2$ symmetry for T_1 and T_2 , respectively. If $m_1 = m_2$, the symmetry is expanded into $\text{Sp}(4)$.

We regard $\text{SU}(2)_1$ and $\text{U}(1)_2 \subset \text{SU}(2)_2$ as the gauged $\text{SU}(2)_L$ and $\text{U}(1)_Y$ symmetries. The $\text{U}(1)_Y$ gauge interaction with the gauge coupling g' and the hypercharge $Y = y_2$ is given by

$$\mathcal{L} = (\overline{T_2})_{\dot{\alpha}}^h (i\bar{\sigma}^{\mu})^{\dot{\alpha}\beta} \left(-ig' 2y_2 \frac{\sigma_3}{2} B_{\mu} \right) (T_2)_{\beta h}. \quad (22)$$

To realize the SM Higgs doublet, we set $y_2 = 1/2$.

The latter $\text{SU}(2)_2$ can be regarded as a global $\text{SU}(2)_R$ symmetry of the SM. This $\text{SU}(2)_R$ symmetry guarantees the rho parameter to be unity at the tree level [89], which is explicitly broken by the $\text{U}(1)_Y$ interaction and the Yukawa interaction, which will be explained in Sec. III E.

¹For the case of $n = 2$, lattice simulations have shown that spontaneous breaking $\text{SU}(4) \rightarrow \text{Sp}(4)$ indeed occurs [69,73–75].

By denoting a_1 and b_1 (a_2 and b_2) as the indices of $SU(2)_L$ ($SU(2)_R$), the scalar bosons are given by

$$\phi_1^\pm \equiv S_{a_1 b_1}^\pm \epsilon_L^{a_1 b_1}, \quad \phi_2^\pm \equiv S_{a_2 b_2}^\pm \epsilon_R^{a_2 b_2}, \quad \mathbb{M}_{a_1 a_2}^\pm \equiv S_{a_1 a_2}^\pm, \quad (23)$$

where ϵ_L and ϵ_R are the antisymmetric tensors for each $SU(2)_L$ and $SU(2)_R$, respectively, and the same notation as that for $SU(2)_H$ is used. The sign represents the eigenvalue of the parity transformation.

The former two are the singlet states under $SU(2)_L \otimes SU(2)_R$. In addition, they are not charged by $U(1)_Y$. By taking the Hermitian conjugate, it is shown that

$$(\phi_i^\pm)^\dagger = \mp \phi_i^\pm, \quad (i = 1, 2) \quad (24)$$

which indicates that $i\phi_i^+$ and ϕ_i^- are parity-even and parity-odd Hermitian scalar fields, respectively. In general, these fields are independent, since the $U(1)_A$ transformation acting on ψ_i , which can be regarded as an $SO(2)$ transformation on $(i\phi_i^+, \phi_i^-)^T$, is explicitly broken by m_1 and m_2 .

The last one is the bidoublet $(\mathbf{2}, \mathbf{2})$ state of two $SU(2)$. For the bidoublet,

$$(\mathbb{M}_{a_1 a_2}^\pm)^\dagger = \pm \mathbb{M}_{b_1 b_2}^\pm \epsilon_L^{b_1 a_1} \epsilon_R^{b_2 a_2} = \pm \mathbb{M}^{\pm a_1 a_2}, \quad (25)$$

is shown. Therefore, the pairs of $(a_1, a_2) = (1, 1), (2, 2)$ and $(1, 2), (2, 1)$ components are connected by the Hermitian conjugate. When we define

$$\mathbb{M}_{a_1 a_2} \equiv -(\mathbb{M}_{a_1 a_2}^+ + i\mathbb{M}_{a_1 a_2}^-), \quad (26)$$

each component of the matrix can be identified with the complex scalars due to the $U(1)_i \subset SU(2)_i$ symmetry. We then obtain the $(\mathbf{2}, \mathbf{2}^*)$ matrix corresponding to $\mathbb{M}_{\text{SM}} = (i\sigma_2 \Phi_{\text{SM}}^*, \Phi_{\text{SM}}) = (\tilde{\Phi}_{\text{SM}}, \Phi_{\text{SM}})$ as

$$\mathbb{M}_{\text{SM}} \propto \mathbb{M}_{a_1}^{a_2} = \begin{pmatrix} \mathbb{M}_{12}^+ + i\mathbb{M}_{12}^- & -(\mathbb{M}_{11}^+ + i\mathbb{M}_{11}^-) \\ (\mathbb{M}_{11}^+ + i\mathbb{M}_{11}^-)^\dagger & (\mathbb{M}_{12}^+ + i\mathbb{M}_{12}^-)^\dagger \end{pmatrix}_{a_1}^{a_2}. \quad (27)$$

The particle contents in class 1 are summarized in Table I. This class leads to the real singlet extension of the SM (xSM) [90–92]. In this table, we have omitted the

TABLE I. Scalar particle contents below Λ in the $n = 2$ case. The value of the hypercharge is the highest weight of $SU(2)_R$.

	$\mathbb{M}$	$\phi_1^\pm$	$\phi_2^\pm$
Y	$1/2$	0	0
L	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
R	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$

flavor indices, and the value of the hypercharge is the highest weight of $SU(2)_R$.

In the case of $m_1 = m_2$ in the limit of neglecting the breaking effects of $Sp(4)$, $\mathbb{M}_{a_1 a_2}$ and $\phi_i^\pm$ are unified into the antisymmetric representations $\mathbf{5} + \mathbf{1}$.

We note that, if the Majorana mass M_{12} is nonzero but $m_1 = m_2 = 0$, there is an $SU(2)_1 \otimes SU(2)_2$ symmetry for the two multiplets $(\chi_1, \chi_2)^T$ and (η_1^*, η_2^*) . By replacing the label of the particles as $\eta_1^* \leftrightarrow \chi_2$, we obtain the same $SU(2)_H$ Lagrangian as that with $m_1 = m_2$ and $M_{12} = 0$. Therefore, the scalar bound states are $\mathbf{5} + \mathbf{1}$ of $Sp(4)$.

C. $n = 3$ case

In this subsection, we show the extended Higgs models deduced by the $SU(2)_H$ theory with the three flavor fermions T_1, T_2 , and T_3 . In the $n = 3$ case, the rich flavor symmetry allows for various extended Higgs models, including the 2HDM as well as models addressing dark matter and neutrino masses.

We consider a case where $m_1, m_2, m_3 \neq 0$ and $M_{ij} = 0$ are taken. Following similar arguments as in the $n = 2$ case, we have the $SU(2)_1 \otimes SU(2)_2 \otimes SU(2)_3$ symmetry for T_1, T_2 and T_3 , respectively.

In order to embed the SM gauge group, $SU(2)_1 (SU(2)_L)$ and the third component of $SU(2)_2 (SU(2)_{R_1})$ are gauged in the same way as in the $n = 2$ case, i.e., with $y_2 = 1/2$. The remaining $SU(2)_3 (SU(2)_{R_2})$ can likewise be gauged by assigning the hypercharge $Y = y_3$ to T_3 . We assign a Z_2 charge $\omega_3 = \pm 1$ for T_3 .

With a similar discussion for the $n = 2$ case, which we do not repeat here, we obtain scalar bound states that are eigenstates of the parity. There are three bidoublet fields and three singlet states under the flavor symmetry. The $SU(2)_L \otimes SU(2)_{R_1}$ bidoublet $\mathbb{M}_{12}$, which is composed of T_1 and T_2 , corresponds to the SM Higgs bidoublet $\mathbb{M}_{\text{SM}}$. The $\mathbb{M}_{13} (\mathbb{M}_{23})$ field is the $SU(2)_L \otimes SU(2)_{R_2} (SU(2)_{R_1} \otimes SU(2)_{R_2})$ bidoublet, which is the tensor product of T_1 and T_3 (T_2 and T_3). The $\phi_i^\pm$ ($i = 1, 2, 3$) fields are totally antisymmetric singlet representations of each $SU(2)$. Their hypercharge is determined by $Y = (T_{R_1})_3 + 2y_3(T_{R_2})_3$, where $(T_{R_1})_3$ and $(T_{R_2})_3$ denote the representation matrices of the third components of $SU(2)_{R_1}$ and $SU(2)_{R_2}$, respectively.

A summary of the particle contents in the $n = 3$ case is presented in Table II. As before, flavor indices are omitted. The hypercharge values correspond to the highest weights of $SU(2)_{R_1} \otimes SU(2)_{R_2}$.

The symmetries marked by the spade suit $\spadesuit$ have to be always exact. As we have noted, since the $U(1)_Y$ interactions violate $SU(2)_{R_1}$, it is an approximate symmetry (marked by diamond $\diamond$). On the contrary, when we set $y_3 = 0$, the $SU(2)_{R_2}$ symmetry can be exact. When we have a nonzero hypercharge of T_3 or other interactions breaking

TABLE II. Scalar particle contents below Λ in the $n = 3$ case. In class 2 (class 3), $y_3 = 1/2$ and $\omega_3 = +1$ ($\omega_3 = -1$) are taken. In class 4, $y_3 = 3/2$ and $\omega_3 = +1$ are taken. The hypercharge values correspond to the highest weights of $SU(2)_{R_1} \otimes SU(2)_{R_2}$.

	$\mathbb{M}_{12}$	$\mathbb{M}_{13}$	$\mathbb{M}_{23}$	$\phi_1^\pm$	$\phi_2^\pm$	$\phi_3^\pm$
Z_2	+	ω_3	ω_3	+	+	+
$\spadesuit Y$	$1/2$	y_3	$1/2 + y_3$	0	0	0
$\spadesuit L$	2	2	1	1	1	1
$\diamond R_1$	2	1	2	1	1	1
$\heartsuit R_2$	1	2	2	1	1	1

$SU(2)_{R_2}$ (e.g., the Yukawa interaction), it becomes not exact. This kind of symmetry, which can be either exact or not exact, is marked by the heart suit $\heartsuit$.

Up to now, the charge assignment of y_3 and ω_3 has been left unspecified. In the following three classes of the charge assignment, we find that the $n = 3$ case effectively encompasses the two Higgs doublet model (2HDM) [93,94], the Inert doublet model (IDM) [22], the Tao-Ma model [23,24], the complex singlet extension of the SM (CxSM) [33], the scalar singlet dark matter model (SDM) [95–97], the Zee model [18,19], and the Zee-Babu model [20,21].

1. Class 2: 2HDM, Zee model, and CxSM

As a charge assignment, we take $y_3 = 1/2$ and $\omega_3 = +1$. In this class, $\mathbb{M}_{12}$ and $\mathbb{M}_{13}$ form part of the 2HDM scalar potential, which is composed of two $SU(2)_L$ doublets Φ_1 and Φ_2 with $Y = 1/2$. Making the $SU(2)_{R_1}$ and $SU(2)_{R_2}$ indices raised, we denote $(\mathbf{2}, \mathbf{2}^*, \mathbf{1})$ and $(\mathbf{2}, \mathbf{1}, \mathbf{2}^*)$ as $\mathbb{M}_1 = (\tilde{\Phi}_1, \Phi_1)$ and $\mathbb{M}_2 = (\tilde{\Phi}_2, \Phi_2)$, respectively.

The transformation law under the three $SU(2)$ is given by

$$\mathbb{M}_1 \rightarrow L\mathbb{M}_1 R_1^\dagger, \quad \mathbb{M}_2 \rightarrow L\mathbb{M}_2 R_2^\dagger, \quad (28)$$

where $L \in SU(2)_L$, $R_1 \in SU(2)_{R_1}$, and $R_2 \in SU(2)_{R_2}$. The $U(1)_Y$ transformation is given by

$$\mathbb{M}_{1,2} \rightarrow \mathbb{M}_{1,2} \exp\left(-ig' \frac{\sigma^3}{2}\right). \quad (29)$$

There are four invariants under $SU(2)_L \otimes U(1)_Y$,

$$\text{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_1], \quad \text{Tr}[\mathbb{M}_2^\dagger \mathbb{M}_2], \quad \text{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_2], \quad \text{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_2 \sigma_3], \quad (30)$$

and the former two are also invariant under $SU(2)_{R_1} \otimes SU(2)_{R_2}$. Therefore, if we assume that couplings violating $SU(2)_{R_1}$ or $SU(2)_{R_2}$ are only g' or the Yukawa couplings, the potential terms written by the latter two invariants are expected to be small [84].

More concretely, the potential which is invariant under $SU(2)_{R_1} \otimes SU(2)_{R_2}$ is written up to dimension four as

$$V_{R_1 R_2}^{(d \leq 4)}(\Phi_1, \Phi_2) = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2. \quad (31)$$

The potential terms violating $SU(2)_{R_1} \otimes SU(2)_{R_2}$ written by

$$V_{\hat{R}_1 \hat{R}_2}^{(d \leq 4)}(\Phi_1, \Phi_2) = \mu_3^2 (\Phi_1^\dagger \Phi_2 + \text{H.c.}) + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \left(\frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2) + \lambda_6 |\Phi_1|^2 + \lambda_7 |\Phi_2|^2 \right) \times (\Phi_1^\dagger \Phi_2) + \text{H.c.}, \quad (32)$$

should be suppressed by the effects violating $SU(2)_{R_1} \otimes SU(2)_{R_2}$. We here assume all couplings are real. We also have higher dimensional operators in the potential, which are suppressed by the cutoff scale ($\sim \Lambda$) of this model. Since the significant one-loop corrections to the rho parameter in the 2HDM come from effects violating the custodial symmetry, this potential does not cause such a large correction. If only Φ_1 acquires the vacuum expectation value, while Φ_2 does not, or if the condition $\lambda_1 \simeq \lambda_2 \simeq \lambda_3$ is also satisfied,² the mixing among the neutral scalar bosons is also suppressed [99], which satisfies the current LHC measurements [3,4]. This class of the 2HDM has been discussed in Refs. [98,100].

If we discard the $SU(2)_{R_1} \otimes SU(2)_{R_2}$ symmetry by introducing a nonzero Majorana mass M_{23} , those symmetry breaking terms in the potential can be large.³ However, in such a case, significant deviations of the rho parameter and the Higgs couplings from the SM prediction are expected.

Since this class also includes the $SU(2)_L$ singlets with $Y = 1$ in $\mathbb{M}_{23}$, the particle contents of the original Zee model are contained. When we denote the charged singlet scalar by $h^\pm$, the relevant Lagrangian of the Zee-model is given by

$$-\mathcal{L}_{\text{Zee}} = f_L \bar{\ell}_L^c \ell_L h^+ + \mu \tilde{\Phi}_1^\dagger \Phi_2 h^- + \text{H.c.}, \quad (34)$$

where ℓ_L (e_R) is the left-handed (right-handed) SM lepton doublet (singlet), and f_L is antisymmetric with respect to interchanging the lepton flavors. In the Zee-model, the active neutrino masses are generated at the one-loop level as

$$m_\nu \sim \frac{f_L \zeta \mu m_\ell^2}{(16\pi^2) M^2} \mathcal{I}_{\text{Zee}}, \quad (35)$$

²This parametrization can be realized as a consequence of an $SO(5)$ symmetry in the potential [98–102].

³In this case, the kinetic mixing terms

$$\text{Tr}[(D_\mu \mathbb{M}_1)^\dagger D^\mu \mathbb{M}_2], \quad \text{Tr}[(D_\mu \mathbb{M}_1)^\dagger D^\mu \mathbb{M}_2 \sigma_3], \quad (33)$$

which are invariant under $SU(2)_L \otimes U(1)_Y$ are induced. By using $GL(2, \mathbb{C})/U(2)$ transformation with respect to Φ_1 and Φ_2 , the canonical basis can be taken [103].

where $\mathcal{I}$ is a dimensionless factor as a function of masses, and we denote a representative scale of new physics by $M(< \Lambda)$. The Yukawa coupling associated with Φ_2 is denoted by ζ . We note that the three-point scalar coupling μ should be suppressed due to the $SU(2)_{R_1} \otimes SU(2)_{R_2}$ symmetry. Since it is known that the observed data cannot be explained by the original Zee model [104,105], some extensions have been discussed [106,107].

In addition, there is the $SU(2)_L$ singlet with $Y = 0$ contained in $\mathbb{M}_{23}$. Although this field has vanishing hypercharge, it can be identified with a complex scalar field if the global $U(1)_3$ symmetry is exact. As long as such breaking effects vanish or can be neglected, this class effectively includes the CxSM.

2. Class 3: IDM, Tao-Ma model, and SDM

By taking $y_3 = 1/2$ and $\omega_3 = -1$, the bidoublets $\mathbb{M}_{13}$ and $\mathbb{M}_{23}$ can be Z_2 -odd. The Z_2 parity of T_3 is the only difference between class 2 and class 3. In this class, the shape of the potential made by the two $Y = 1/2$ bidoublets $\mathbb{M}_1$ (Z_2 -even) and $\mathbb{M}_2$ (Z_2 -odd) is similar to Eqs. (31), (32). However, μ_3^2 , λ_6 and λ_7 terms are not allowed due to the exact Z_2 symmetry. The λ_4 and λ_5 terms are allowed but should be suppressed due to the approximate $SU(2)_{R_1} \otimes SU(2)_{R_2}$ symmetry. This potential is that of the IDM. In this model, the Z_2 -even $\mathbb{M}_1$ should be regarded as the SM Higgs bidoublet, and the Z_2 -odd $\mathbb{M}_2$ includes the scalar dark matter candidates.

When we additionally have Z_2 -odd right-handed neutrinos (RHNS) N with the Majorana masses M_N , the particle contents include that of the Tao-Ma model. The relevant Lagrangian is given by

$$-\mathcal{L}_{\text{Tao-Ma}} = \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + h \overline{\ell}_L \Phi_2 N + \text{H.c.}, \quad (36)$$

and the neutrino masses are given as

$$m_\nu \sim \frac{h^\dagger h \lambda_5 v^2}{(16\pi^2) M_N} \mathcal{I}_{\text{Tao-Ma}}. \quad (37)$$

We note that, in this class, the Z_2 -odd $SU(2)_L$ singlet scalar with $Y = 0$ is also contained in $\mathbb{M}_{23}$. Therefore, this class also includes the SDM, in which the Z_2 -odd singlet can serve as another dark matter candidate as the singlet scalar dark matter.

3. Class 4: Zee-Babu model

By taking $y_3 = 3/2$ and $\omega_3 = +1$, in addition to the SM Higgs doublet, we have an additional $SU(2)_L$ doublet with $Y = 3/2$ in $\mathbb{M}_{13}$. We also have $Y = 1$ and $Y = 2$ $SU(2)_L$ singlets in $\mathbb{M}_{23}$, which are denoted by h^+ and k^{++} , respectively.

This class realizes the particle contents of the Zee-Babu model, in which the neutrino masses are generated at the

two-loop level. As in the Zee model, the relevant Lagrangian of the Zee-Babu model is given by

$$-\mathcal{L}_{\text{Zee-Babu}} = f_L \overline{\ell}_L^c \ell_L h^+ + f_R \overline{e}_R^c e_R k^{++} + \text{H.c.} \\ + \sigma h^- h^- k^{++} + \text{H.c.}, \quad (38)$$

where f_R is symmetric under changing lepton flavor indices. The σ term should be suppressed by the $SU(2)_{R_1} \otimes SU(2)_{R_2}$ symmetry. The neutrino masses are represented by

$$m_\nu \sim \frac{f_L^\dagger f_R f_L \sigma m_\ell m_{\ell'}}{(16\pi^2)^2 M^2} \mathcal{I}_{\text{Zee-Babu}}. \quad (39)$$

D. $n = 4$ case

In this subsection, we discuss the $n = 4$ case, in which T_1 , T_2 , T_3 , and T_4 are involved. In the following, we consider two ways to embed the $SU(2)_L$ gauge symmetry. The first one is a straightforward extension of the previous $n = 3$ case, where $SU(2)_L$ is embedded in $SU(2)$ of T_1 . In the other way, it is embedded into a subgroup of $Sp(6)$ in $(T_1, T_2, T_3)^\dagger$.

The former way realizes several models generating the neutrino masses at the loop level, such as the Aoki-Kanemura-Yagyu (AKY) model [108], the Gustafsson-No-Rivera (GNR) model [25], the Krauss-Nasri-Trodden (KNT) model [26], and the Aoki-Kanemura-Seto (AKS) model [51]. The next-to-minimal 2HDM (N2HDM) [109] is also included. In the latter way, the $SU(2)_L$ triplet model, which is represented by the Georgi-Machacek (GM) model [110,111], is realized.

E. Models for the neutrino mass

In addition to the $n = 3$ case, we introduce T_4 , which has the hypercharge y_4 and the Z_2 charge ω_4 . By assuming that different Dirac masses m_1, m_2, m_3 , and m_4 and $M_{ij} = 0$, we additionally have $SU(2)_{R_3}$ in the T_4 sector. There are new bidoublet and singlet fields, $\mathbb{M}_{14}$, $\mathbb{M}_{24}$, $\mathbb{M}_{34}$, and $\phi_4^\pm$. The summary of the particle contents is given in Table III.

1. Class 5: AKY model, GNR model, and KNT model

In this class, we assign the hypercharge and the Z_2 charge as $y_3 = 1/2$, $y_4 = 3/2$, and $\omega_3 = \omega_4 = -1$. In addition to the SM Higgs doublet ($\Phi_1 \in \mathbb{M}_{12}$), we have the Z_2 -odd additional $SU(2)_L$ doublets with $Y = 1/2$ ($\Phi_2 \in \mathbb{M}_{13}$) and $Y = 3/2$ ($\Phi_{3/2} \in \mathbb{M}_{14}$).

When a Z_2 -odd vectorlike fermion ψ with the Dirac mass m_ψ is added in this class, the AKY model, where the neutrino masses are generated at the one-loop level, is realized.⁴ With the relevant interactions given by

⁴To reproduce the experimental data for the neutrino, at least two vectorlike fermions are needed [108].

TABLE III. Scalar particle contents below Λ in the $n = 4$ case for models generating the neutrino masses. In class 5 and class 6, the charge assignments $(y_3, \omega_3, y_4, \omega_4) = (1/2, -1, 3/2, -1)$ and $(1/2, +1, 1/2, -1)$ are taken, respectively. The hypercharge values shown here are the highest weights of $SU(2)_{R_1} \otimes SU(2)_{R_2} \otimes SU(2)_{R_3}$.

	$\mathbb{M}_{12}$	$\mathbb{M}_{13}$	$\mathbb{M}_{14}$	$\mathbb{M}_{23}$	$\mathbb{M}_{24}$	$\mathbb{M}_{34}$	$\phi_1^\pm$	$\phi_2^\pm$	$\phi_3^\pm$	$\phi_4^\pm$
$\spadesuit Z_2$	+	ω_3	ω_4	ω_3	ω_4	$\omega_3\omega_4$	+	+	+	+
$\spadesuit Y$	$1/2$	y_3	y_4	$1/2 + y_3$	$1/2 + y_4$	$y_3 + y_4$	0	0	0	0
$\spadesuit L$	2	2	2	1	1	1	1	1	1	1
$\diamond R_1$	2	1	1	2	2	1	1	1	1	1
$\heartsuit R_2$	1	2	1	2	1	2	1	1	1	1
$\heartsuit R_3$	1	1	2	1	2	2	1	1	1	1

$$-\mathcal{L}_{\text{AKY}} = \lambda_{3/2}(\Phi_{3/2}^\dagger \Phi_1)(\tilde{\Phi}_2^\dagger \Phi_1) + \text{H.c.} + f_{\psi_L} \bar{\ell}_L^c \psi_L \Phi_{3/2} + f_{\psi_R} \bar{\ell}_L^c \psi_R \Phi_2 + \text{H.c.}, \quad (40)$$

the mass formula of the active neutrino is written as

$$m_\nu \sim \frac{f_{\psi_L} f_{\psi_R} \lambda_{3/2} v^2}{(16\pi^2) m_\psi} \mathcal{I}_{\text{AKY}}. \quad (41)$$

The $\lambda_{3/2}$ coupling should be suppressed by the $SU(2)_{R_1} \otimes SU(2)_{R_2} \otimes SU(2)_{R_3}$ symmetry. In this model, the lightest Z_2 -odd particle can be the dark matter candidate.

This class also includes the particle contents of the GNR model: there are Z_2 -odd $SU(2)_L$ doublet Φ_2 ($Y = 1/2$), Z_2 -odd singlet $S^\pm$ ($Y = 1$), and Z_2 -even singlet $k^{\pm\pm}$ ($Y = 2$) in $\mathbb{M}_{13}$, $\mathbb{M}_{24}$ and $\mathbb{M}_{34}$, respectively. The relevant Lagrangian of the GNR model is written by

$$-\mathcal{L}_{\text{GNR}} = \frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_2)^2 + \mu \tilde{\Phi}_1^\dagger \Phi_2 S^- + \sigma S^- S^- k^{++} + f_R \bar{e}_R^c e_R k^{++} + \xi \tilde{\Phi}_2^\dagger \Phi_1 S^+ k^{--} + \text{H.c.} \quad (42)$$

The neutrino mass is generated via the three-loop diagram as

$$m_\nu \sim \frac{f_R m_\ell m_{\ell'} \sin 2\beta}{(16\pi^2)^3 M} \mathcal{I}_{\text{GNR}}, \quad (43)$$

where β is the mixing angle between $S^\pm$ and the charged component of Φ_2 . The loop function $\mathcal{I}_{\text{GNR}}$ is approximately proportional to λ_5 and μ with a dependence of σ and ξ . These couplings should be suppressed, if the flavor symmetry is respected. The lightest Z_2 -odd neutral scalars in Φ_2 can be the dark matter candidate.

If the Z_2 -odd RHNs are additionally introduced in this class, the neutrino masses can be generated by the mechanism proposed in the KNT model. There are Z_2 -even singlet $h^\pm$ ($Y = 1$) and the Z_2 -odd singlet $S^\pm$ ($Y = 1$) in $\mathbb{M}_{34}$ and $\mathbb{M}_{23}$ (or $\mathbb{M}_{24}$), respectively. Therefore, by the Lagrangian given by

$$-\mathcal{L}_{\text{KNT}} = f_L \bar{\ell}_L^c \ell_L h^+ + f_N \bar{N}^c e_R S^+ + \lambda_S (h^+ S^-)^2 + \text{H.c.}, \quad (44)$$

the neutrino masses are calculated as

$$m_\nu \sim \frac{\lambda_S f_L^2 f_N^2 m_\ell m_{\ell'}}{(16\pi^2)^3 M} \mathcal{I}_{\text{KNT}}. \quad (45)$$

This model also has the dark matter candidates in the Z_2 -odd sector. The λ_S coupling can also be suppressed by the flavor symmetry.

2. Class 6: AKS model and N2HDM

We take $y_3 = y_4 = 1/2$, $\omega_3 = +1$, and $\omega_4 = -1$. The particle contents in this class contain the Z_2 -even additional $SU(2)_L$ doublet Φ_2 in $\mathbb{M}_{13}$ and the Z_2 -odd singlets with $Y = 0$ and $Y = 1$ (η and $S^\pm$) in $\mathbb{M}_{24}$ or $\mathbb{M}_{34}$. When the Z_2 -odd RHNs exist, this class contains the particle contents of the AKS model [51,53]. Since the softly broken Z_2 symmetry is not imposed here to forbid the FCNC couplings, which was discussed in the original AKS model, the structure of the Yukawa interaction is similar to that in Ref. [52].

In the AKS model, the neutrino mass is induced by the following Lagrangian,

$$-\mathcal{L}_{\text{AKS}} = f_N \bar{N}^c e_R S^+ + \kappa \tilde{\Phi}_2^\dagger \Phi_1 S^- \eta + \text{H.c.}, \quad (46)$$

which deduces the neutrino masses at the three-loop level as

$$m_\nu \sim \frac{\kappa^2 f_N^2 \zeta^2 m_\ell m_{\ell'}}{(16\pi^2)^3 M} \mathcal{I}_{\text{AKS}}. \quad (47)$$

In this AKS model, the problems of the neutrino mass, the dark matter, and the baryon asymmetry of the Universe can be solved simultaneously by the TeV scale physics.

We note that, since the Z_2 -even doublet Φ_2 and the Z_2 -odd singlet η are present, this class also encompasses the structure of the N2HDM. The N2HDM has been extensively studied in the contexts of dark matter and collider phenomenology [109,112–115].

F. Models for $SU(2)_L$ triplet scalar

1. Class 7: GM model

We consider the degenerated three flavor fermions $(T_1, T_2, T_3)^T$ with the Dirac mass m and one additional flavor T_4 with $m_4 \neq m$. The Majorana masses are taken to be zero, $M_{ij} = 0$. In this case, the multiplet $(T_1, T_2, T_3)^T$, which is the part of $\mathbf{T}$, behaves as $\mathbf{6}$ of $\text{Sp}(6)$.

From the branching rule of $\mathfrak{sp}(6) \supset \mathfrak{su}(2) \oplus \mathfrak{su}(2)$, $\mathbf{6}$ can be decomposed into $(\mathbf{2}, \mathbf{3})$, namely

$$(T_1, T_2, T_3)^T \sim \mathbf{2} \otimes \mathbf{3}. \quad (48)$$

TABLE IV. Scalar particle contents below Λ in the $n = 4$ case for the triplet models (class 7). The hypercharges shown here are for $SU(2)_{R_2}$ components.

	$(\mathbb{M}_{\Phi_{3/2}}, \mathbb{M}_{\Phi_{1/2}}, \mathbb{M}_{\Phi_{-1/2}})^T$	$(\chi^*, \xi, \chi)^T$	$(S^{++}, S^+, S^0, S^-, S^{--})^T$	$\phi_1^\pm$	$\phi_2^\pm$
$\spadesuit Y$	$3/2, \pm 1/2$	$\pm 1, 0$	$\pm 2, \pm 1, 0$	0	0
$\spadesuit L$	2	3	1	1	1
$\diamond R_1$	2	1	1	1	1
$\diamond R_2$	3	3	5	1	1

We regard the former $SU(2)$ as $SU(2)_L$ and the latter one as $SU(2)_{R_2}$. We also have T_4 , which is 2 of the other $SU(2)_{R_1}$. Therefore, there are the fundamental fermions represented by $(2, 1, 3)$ and $(1, 2, 1)$ under $SU(2)_L \otimes SU(2)_{R_1} \otimes SU(2)_{R_2}$.

By assuming $SU(2)_L$ and the third component of $SU(2)_{R_1}$ and $SU(2)_{R_2}$ are gauged to give the hypercharge, we obtain the following scalar bound states as totally antisymmetric representations:

$$\begin{aligned}
 (2, 2, 3) &\sim (\mathbb{M}_{\Phi_{3/2}}, \mathbb{M}_{\Phi_{1/2}}, \mathbb{M}_{\Phi_{-1/2}})^T, \\
 (3, 1, 3) &\sim (\chi^*, \xi, \chi)^T, \\
 (1, 1, 5) &\sim (S^{++}, S^+, S^0, S^-, S^{--})^T \\
 (1, 1, 1) &\sim \phi_1^\pm, \phi_2^\pm.
 \end{aligned} \tag{49}$$

The fields shown here correspond to the components of $SU(2)_{R_2}$.

We summarize the particle contents in this class in Table IV. The hypercharges shown here are for $SU(2)_{R_2}$ components.

The three bidoublet states $\mathbb{M}_{\Phi_{3/2}}, \mathbb{M}_{\Phi_{1/2}}$, and $\mathbb{M}_{\Phi_{-1/2}}$ making 3 of $SU(2)_{R_2}$ have the hypercharge $Y = 3/2, 1/2$, and $-1/2$, respectively. The bidoublet field $\mathbb{M}_{\Phi_{1/2}}$ (or $\mathbb{M}_{\Phi_{-1/2}}$) corresponds to the SM Higgs doublet. The bitriplet field under $SU(2)_L \otimes SU(2)_{R_2}$ contains $Y = 1$ triplet $\chi = (\chi^{++}, \chi^+, \chi^0)$ and $Y = 0$ triplet $\xi = (\xi^+, \xi^0, \xi^-)$. There are the quintuplet field of the $SU(2)_{R_2}$, which contains doubly, singly, and neutrally charged $SU(2)_L$ singlet scalars. Similar to the previous models, we also have the $Y = 0$ parity-even and odd scalars $\phi_1^\pm$ and $\phi_2^\pm$, which are the singlet states under any of the flavor symmetry.

This class corresponds to the GM model, in which $SU(2)_L$ triplet scalars with $Y = 0$ and $Y = 1$ are involved. When the electroweak symmetry breaking occurs as $\langle \mathbb{M}_{SM} \rangle \propto a \mathbb{I}_2$ and $\langle \chi^0 \rangle = \langle \xi^0 \rangle \propto b$ satisfying $\sqrt{a^2 + 8b^2} = v$, the rho parameter is unity at the tree level, as in the GM model.

G. Yukawa interaction

In this subsection, we discuss the Yukawa interaction based on a similar mechanism to the partial compositeness [78,83,84]. The Yukawa interaction in the low-energy

effective theory is provided by the confinement in a part of higher dimensional operators. In the UV sector, a massive scalar is introduced to violate the chiral symmetry of the SM quarks and leptons.

Letting ψ be a SM quark or lepton, or a right-handed neutrino, we consider a case where the dimension four Yukawa interaction

$$-\mathcal{L}_Y^{(d=4)} = \sum_S \sum_{\phi_S \in S} \sum_{(\psi_L, \psi_R)} y_{(\psi_L, \psi_R)}^{\phi_S} \bar{\psi}_L \psi_R \phi_S + \text{H.c.}, \tag{50}$$

is deduced in an extended Higgs model. We denote S as a set of $SU(2)_H$ singlet scalar bosons ϕ_S , all of which have the same quantum charges under the SM gauge group. The Yukawa matrices, in which a combination of ψ_L, ψ_R , and ϕ_S are involved, are represented by $y_{(\psi_L, \psi_R)}^{\phi_S}$. The flavor indices of ψ are omitted.

To obtain the Yukawa interaction, we introduce a scalar field Φ_S with a mass $M_{\Phi_S} (\gtrsim \Lambda)$, which has the quantum charges of ϕ_S . By denoting $(TT)_{\phi_S}$ as a $SU(2)_H$ singlet state reproducing ϕ_S after the confinement, namely $(TT)_{\phi_S} \sim \Lambda^2 \phi_S$, we have the following UV Lagrangian:

$$\begin{aligned}
 -\mathcal{L}_{UV} &= \sum_S \sum_{(\psi_L, \psi_R)} \mathcal{Y}_{(\psi_L, \psi_R)} \bar{\psi}_L \psi_R \Phi_S + \text{H.c.} \\
 &+ \sum_S \sum_{\phi_S \in S} \mathcal{H}^{\phi_S}(TT)_{\phi_S} \Phi_S^\dagger + \text{H.c.}
 \end{aligned} \tag{51}$$

By integrating out Φ_S , we effectively obtain the four-Fermi operators as written by

$$-\mathcal{L}_{\text{eff}} = \sum_S \sum_{\phi_S \in S} \sum_{(\psi_L, \psi_R)} \frac{\mathcal{H}^{\phi_S} \mathcal{Y}_{(\psi_L, \psi_R)}}{M_{\Phi_S}^2} \bar{\psi}_L \psi_R (TT)_{\phi_S}. \tag{52}$$

After the confinement, we obtain the Yukawa interaction as Eq. (50), namely $y_{(\psi_L, \psi_R)}^{\phi_S} \sim \mathcal{H}^{\phi_S} \mathcal{Y}_{(\psi_L, \psi_R)} \Lambda^2 / M_{\Phi_S}^2$.

Since only $\mathcal{Y}_{(\psi_L, \psi_R)}$ has the flavor indices of ψ , the multiple Yukawa matrices $y_{(\psi_L, \psi_R)}^{\phi_S}$ have the same flavor structure. For example, in the 2HDM, the Yukawa interaction of the up-type quarks (u, c, t) is obtained as

$$-\mathcal{L}_{Y, 2\text{HDM}}^{(d=4)} = \text{diag}(y_u, y_c, y_t) \bar{Q}_L u_R (\tilde{\Phi}_1 + \zeta \tilde{\Phi}_2), \tag{53}$$

where $Q_L (u_R)$ is the left-handed (right-handed) SM quark doublet (singlet), and $\zeta = \mathcal{H}^{\Phi_2} / \mathcal{H}^{\Phi_1}$. The additional Yukawa interactions for the down-type quarks and the leptons are also controlled by the same factor ζ . Therefore, in our setup, the minimal flavor violating Yukawa interaction (so-called Yukawa alignment [116]) is realized. The Yukawa interactions for different models, such as a model with a charged $SU(2)_L$ singlet, can also be obtained by introducing other heavy scalars Φ_S .

TABLE V. Summary of classification of the extended Higgs models.

Class	n	Charge assignment of the UV model	Higgs sectors as a part of the low-energy effective theory
1	2	$y_2 = 1/2$	xSM
2	3	$(y_3, \omega_3) = (1/2, +1)$	xSM, 2HDM, Zee, CxSM
3	3	$(y_3, \omega_3) = (1/2, -1)$	xSM, IDM (Tao-Ma), SDM
4	3	$(y_3, \omega_3) = (3/2, +1)$	xSM, Zee-Babu
5	4	$(y_3, \omega_3, y_4, \omega_4) = (1/2, -1, 3/2, -1)$	xSM, IDM (Tao-Ma), SDM, Zee-Babu, AKY, KNT, GNR
6	4	$(y_3, \omega_3, y_4, \omega_4) = (1/2, +1, 1/2, -1)$	xSM, Zee, CxSM, IDM (Tao-Ma), SDM, KNT, AKS, N2HDM
7	4	$(T_1, T_2, T_3)^T \sim (2, 1, 3), T_4 \sim (1, 2, 1)$	xSM, 2HDM, Zee, Zee-Babu, GM

Models	xSM	2HDM	Zee	CxSM	IDM (Tao-Ma)	SDM	Zee-Babu	AKY
(Y, L, Z_2)	$(0, 1, +) \in \mathbb{R}$	$(1/2, 2, +)$	$(1/2, 2, +), (1, 1, +)$	$(0, 1, +) \in \mathbb{C}$	$(1/2, 2, -)$	$(0, 1, -)$	$(2, 1, +), (1, 1, +)$	$(1/2, 2, -), (3/2, 2, -)$
Class	1, 2, 3, 4, 5, 6, 7	2, 6, 7	2, 6, 7	2, 6	3, 5, 6	3, 5, 6	4, 5, 7	5

Models	KNT	GNR	AKS	N2HDM	GM
(Y, L, Z_2)	$(1, 1, -), (1, 1, +)$	$(1/2, 2, -), (1, 1, -), (2, 1, +)$	$(1/2, 2, +), (0, 1, -), (1, 1, +)$	$(1/2, 2, +), (0, 1, -)$	$(1, 3, +), (0, 3, +)$
Class	5, 6	5	6	6	7

IV. DISCUSSIONS

In Table V, we summarize the list of the extended Higgs models as a part of the low-energy effective theory, which are classified by the flavor number and the charge assignment of the $SU(2)_H$ theory.

In this paper, we discussed the relation between the scalar particle contents in the low-energy effective theory and the flavor number and the charge assignment in the UV gauge theory, assuming implicitly the existence of a specific energy scale like the Landau pole, where the low-energy theory is replaced by the UV theory. In this sense, the deduced extended Higgs models should be regarded as valid only for energy scales below the confinement scale Λ . Moreover, the Landau-pole scale, which is estimated in the literature for the 2HDM [62–64], IDM [65–67], GM model [68], etc., is the result of extrapolating the low-energy theory toward the UV. We expect that such apparent pathological behavior of the IR effective theory around that scale is cured once the description is replaced by the asymptotically free $SU(2)_H$ gauge theory. However, whether the Landau pole really exists or not strongly depends on the parameters of the model. It is nontrivial whether our discussion can be applied to the case where the Landau pole does not exist below the Planck scale. As it is strongly parameter-dependent, we do not discuss this issue in this paper.

In this paper, we have not pursued the question of how to reproduce the 125 GeV Higgs boson. The mass spectrum of the bound states is expected to be linked to the non-perturbative dynamics of the $SU(2)_H$ gauge theory at the scale Λ , which lies beyond the scope of our analysis. We briefly mention several possibilities for the mechanism to generate a light Higgs boson in the context of strong gauge dynamics. One possibility is the pseudo-Nambu-Goldstone boson (pNGB) scenario, in which the Higgs boson arises as a pNGB associated with the spontaneous breaking of a

global symmetry, such as $SU(4) \rightarrow Sp(4) \supset SU(2) \otimes SU(2)$ in class 1 [69,71–75], triggered by the confinement of the underlying gauge theory. Another possibility is due to the renormalization group running of the Higgs mass parameters below the confinement scale Λ [58,117]. Even if only small mass differences among the bound states are generated at the scale Λ , the differences can be enhanced by the running due to the steep change of the mass parameters and the quartic couplings at around Λ . While the present study does not commit to any specific mechanism, investigating the mechanism to reproduce the 125 GeV Higgs boson should be an important future work.

Connecting the UV theory and the low-energy effective theory has been done in the context of supersymmetric (SUSY) models [70,85], in which the low-energy effective potential can be written down by using the UV/IR duality [118]. A SUSY gauge theory with n flavors is described by the n pairs of chiral superfields, $(\hat{T}_{Li}, \hat{T}_{Ri}^c) = (\tilde{T}_{Li} + \sqrt{2}\theta\chi_i + \theta^2 F_{Li}, \tilde{T}_{Ri}^* + \sqrt{2}\theta\eta_i^* + \theta^2 F_{Ri}^*)$. Here, the bound states $\hat{M}_{LR}^{ij} \sim \hat{T}_{Li} \hat{T}_{Rj}^c$ ($\hat{M}_{LL}^{ij} \sim \hat{T}_{Li} \hat{T}_{Lj}$) is an independent super field of $\hat{M}_{LR}^{ji} \sim \hat{T}_{Ri}^c \hat{T}_{Lj}$ ($\hat{M}_{RR}^{ji} \sim \hat{T}_{Ri}^c \hat{T}_{Rj}^c$), in contrast to the non-SUSY case with n fundamental Dirac fermions, where the bound state $M_{\eta\chi}^{ij} \sim \eta_i^* \chi_j$ ($M^{ij} \sim \eta_i \eta_j$) is related to $M_{\eta\chi}^{ji} \sim \chi_i \eta_j^*$ ($M_{\eta\eta}^{ij} \sim \eta_i^* \eta_j^*$) by charge conjugation. Thus, the number of scalar degrees of freedom below Λ_H in the SUSY gauge theory is doubled from that of the non-SUSY one. It is actually shown in the comparison between the SUSY $SU(2)_H$ theory with $n = 3$ studied in Refs. [70,85] and the model discussed in Sec. III C.

In Sec. III E, we discussed the Yukawa interactions in our framework and presented a natural realization of Yukawa alignment in the 2HDM at the scale Λ as an example. However, we do not address the origin of the quark and lepton flavor structures of the SM. Although this question is beyond the scope of this paper, there exist

various mechanisms in the literature that aim to explain the observed flavor hierarchies and mixings (e.g., introducing flavor symmetries [119–121], invoking strong dynamics [122–124], radiatively generating fermion masses [125,126], and so on). We note that additional flavor mixing will be generated by the renormalization group running below Λ . In addition, the extra scalar contribution, such as a charged Higgs boson exchange, can provide deviations from the SM predictions for flavor observables, even if the minimal flavor violation structure is kept at the electroweak scale.

In this paper, we have considered an $SU(2)_H$ gauge theory as a UV completion of various extended Higgs models. However, it is nontrivial whether this picture can be consistently maintained all the way up to the Planck scale. Above the confinement scale, the gauge group is $G_{\text{SM}} \otimes SU(2)_H$, and one may consider a grand unification of these gauge couplings at some higher scale. Representative examples of unified gauge groups embedding the SM gauge groups are $G_{\text{GUT}} = SU(5)$ [127], $SO(10)$ [128,129], etc., and a straightforward extension is to consider a GUT based on $G_{\text{GUT}} \otimes SU(2)_H$. In such a framework, the vectorlike fundamental fermions of $SU(2)_H$ can be charged under G_{GUT} without introducing additional gauge anomalies. One may also consider embedding $G_{\text{SM}} \otimes SU(2)_H$ into a larger group in which $SU(2)_H$ appears as a subgroup. However, a fully realistic implementation of gauge coupling unification involving $SU(2)_H$ is left for future work [130].

Finally, in this paper, a specific shape of the extended Higgs models protected by symmetries are naturally derived. To obtain more general Higgs potentials, it is necessary to take into account the breaking term of the flavor symmetries. For example, in the case of the 2HDM, the exact flavor symmetry forbids λ_4 , λ_5 , λ_6 , λ_7 , and μ_3 . These parameters emerge as effects of the symmetry breaking. Naively, all these couplings are expected to appear at the similar order. However, this discussion holds at the confinement scale and can change at low energy due

to various effects. It is not obvious whether the experimental constraints on the 2HDM are properly satisfied by such a more general 2HDMs. At this stage, it is difficult to conclude whether our framework can truly reproduce various existing Higgs models at the low energy. Regardless of the question whether the Higgs models predicted by our framework at the low energy coincide with the existing ones or not, it is an extremely interesting question whether our Higgs models deduced from the UV theory can explain the problems beyond the SM. In this paper, we do not pursue this issue further, and this will be examined elsewhere [130].

V. CONCLUSIONS

In this paper, we have considered the $SU(2)_H$ gauge theory with confinement as a UV theory of the extended Higgs sectors. We have investigated the relation between the scalar particle contents at the low energy and flavor number and the charge assignment of the $SU(2)_H$ theory. We have found that particle contents of various extended Higgs sectors previously proposed to explain the beyond the SM problems are deduced by the $SU(2)_H$ theory, as a part of the low-energy theory. Our findings may provide a new picture for the UV completion of the extended Higgs sectors.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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