

Ultralight scalar dark matter with off-diagonal flavor couplings

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Ultralight dark matter can behave as a coherent background field and induce time-dependent modifications of Standard Model parameters. We study a scenario in which a real ultralight scalar ϕ couples off-diagonally to down-type quarks, linking ultralight dark sectors to flavor physics. Working within an effective field theory, we diagonalize the quark mass matrix in a coherent ϕ background and derive analytic expressions for oscillatory shifts in down-type quark masses and Cabibbo-Kobayashi-Maskawa parameters. These effects lead to signatures in both the classical regime, where ϕ acts as a background field, and the quantum (particle) regime, where it contributes through on-shell production or off-shell mediation. Using precision flavor measurements, nuclear β decays, atomic-clock data, compiled NANOGrav-derived limits, and meson observables, we derive constraints on the flavor-violating couplings λ_{ij} for $m_\phi \sim 10^{-24}$ – 10^{-12} eV, highlighting the complementarity of time-domain and flavor probes of ultralight dark sectors.

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I. INTRODUCTION

Dark matter accounts for about 85% of the matter content of the Universe, yet its microscopic nature remains a central open problem in particle physics and cosmology [1,2]. Ultralight dark matter (ULDM), often modeled as a weakly coupled oscillating real scalar field with masses spanning $m_\phi \sim 10^{-24}$ eV to $\mathcal{O}(\text{eV})$, is a well-motivated framework [3]. Owing to its macroscopic de Broglie wavelength, ULDM can behave coherently on astrophysical scales and is probed most efficiently through precision measurements rather than conventional heavy-dark-matter searches. In particular, even feeble couplings of ULDM to Standard Model (SM) fields can induce time-dependent variations in high-precision observables, such as atomic transition frequencies and nuclear decay rates, providing sensitive probes of ULDM interactions with the SM [4–15].

Existing literature on scalar ULDM predominantly explores flavor-diagonal portals—often termed dilatonlike couplings—where ϕ interacts with SM fermion mass terms. These interactions manifest as oscillations in fermion masses and related fundamental constants [16–20], and are stringently constrained by atomic-clock comparisons, interferometry, and tests of the equivalence principle [21–27]. For a representative scalar interaction $\mathcal{L}_{\text{int}} \sim \phi \sum_f \frac{m_f}{M_f} \bar{f}f$, the differential nucleon-electron scale $M_{\text{eff}} \equiv (M^{-1} - M_e^{-1})^{-1}$ is already pushed to $M_{\text{eff}}/M_{\text{Pl}} \gtrsim 10^5$ (taking $M_p = M_n \equiv M$) across much of the ULDM mass range [21]. In contrast, the quark flavor sector remains relatively unexplored. In generic UV completions, couplings defined in the weak basis are not guaranteed to remain diagonal following electroweak symmetry breaking and the subsequent rotation to the mass basis. Consequently, off-diagonal ϕ -quark interactions arise naturally, necessitating a systematic investigation of flavor-violating ULDM portals. This work focuses on the down-type sector, where such interactions directly modulate the Cabibbo-Kobayashi-Maskawa (CKM) matrix structure, subjecting ULDM to a diverse array of laboratory constraints.

We begin with an effective Lagrangian containing off-diagonal ULDM couplings to down-type quarks. In Sec. II, we demonstrate that a coherent ULDM background induces three key effects: (i) shifts in down-type quark masses, (ii) time-dependent modifications of the CKM matrix, and (iii) derivative-induced kinetic mixings. We also provide the complementary particle interpretation relevant for

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decay and force searches. In Sec. III, by treating ϕ as a classical condensate, we translate these effects into constraints derived from direct CKM determinations, time-resolved nuclear β -decay data, tritium decay measurements, atomic-clock comparisons, compiled NANOGrav-derived bounds, and meson mixing. Sec. IV considers ϕ as an on-shell particle or an off-shell mediator, deriving bounds from quark-flavor-violating meson decays, meson oscillations, and loop-induced fifth-force effects. Additional astrophysical considerations are summarized in Sec. V, and we conclude in Sec. VI.

II. A FLAVOR-VIOLATING ULDM MODEL

In this section, we present a framework for studying flavor-violating interactions between ULDM and SM quarks. We concentrate on a real scalar ULDM field, represented by ϕ , which connects to quark generations through off-diagonal interactions. Due to its extremely small mass, ULDM can be described both as a coherent classical field and as particles. We develop these two perspectives in parallel and demonstrate that they yield identical physical consequences.

The effective Lagrangian in the SM quark mass eigenbasis is given by

$$\mathcal{L} \supset \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 - \phi \sum_{i=1}^3 \sum_{j=1, j \neq i}^3 \lambda_{ij} \bar{d}_i d_j, \quad (1)$$

where m_ϕ represents the mass of the ULDM field ϕ , d_i denotes the i th generation of down-type quarks in the SM mass eigenbasis, and λ_{ij} are the flavor-violating couplings

between generations. For simplicity, we restrict our analysis to off-diagonal couplings by assuming $\lambda_{ii} = 0$ (i.e., no diagonal interactions). This approach allows us to isolate genuinely flavor-changing effects, which have received comparatively less attention in the literature. Diagonal flavor-conserving interactions have been thoroughly studied and face more stringent constraints, thus we exclude them from our consideration.

To connect the above low-energy effective interaction to a UV completion, we start by considering a simple extension of the SM, where the interaction of ϕ with quarks arises from higher-dimensional operators. Before electroweak symmetry breaking (EWSB), the relevant Lagrangian is

$$-\mathcal{L} \supset y_{u,ij} \bar{Q}_{L,i}^{u,0} \tilde{H} u_{R,j}^0 + \left(y_{d,ij} + \frac{\lambda'_{ij} \phi}{\Lambda'} \right) \bar{Q}_{L,i}^{d,0} H d_{R,j}^0 + \text{H.c.} \quad (2)$$

where $y_{u/d}$ are the SM Yukawa coupling matrices, $Q_L^{u/d,0}$ are the left-handed $SU(2)_L$ quark doublets, u_R^0 and d_R^0 are the right-handed up and down quark singlets, and H is the Higgs doublet. The term $\frac{\lambda'_{ij} \phi}{\Lambda'}$ parametrizes a higher-dimensional operator, which can arise from integrating out some heavier fermions; here λ'_{ij} are the coupling constants of the new interaction, and Λ' is the scale of new physics responsible for generating this interaction.

After EWSB, the down-type Yukawa sector induces a ϕ -dependent contribution to the mass matrix. In the SM down-quark mass eigenstate basis, this can be written as

$$-\mathcal{L} \supset \left(1 + \frac{h}{v} \right) (\bar{d}_L, \bar{s}_L, \bar{b}_L) \left[\text{diag}(m_d, m_s, m_b) + \frac{v\phi}{\sqrt{2}\Lambda'} V_d^{L\dagger} \begin{pmatrix} \lambda'_{11} & \lambda'_{12} & \lambda'_{13} \\ \lambda'_{21} & \lambda'_{22} & \lambda'_{23} \\ \lambda'_{31} & \lambda'_{32} & \lambda'_{33} \end{pmatrix} V_d^R \right] \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} + \text{H.c.} \quad (3)$$

where $V_d^{L,R}$ are the unitary matrices that rotate to the down-quark mass basis. The coupling term $\frac{v}{\sqrt{2}\Lambda'} V_d^{L\dagger} \lambda' V_d^R$ is generically nondiagonal. To isolate the novel phenomenology of flavor-violating ULDM, we assume this mass-basis coupling is symmetric with vanishing diagonal entries. Defining the off-diagonal components as λ_{ij} , the effective Lagrangian becomes

$$\begin{aligned} -\mathcal{L} &\supset \left(1 + \frac{h}{v} \right) (\bar{d}_L, \bar{s}_L, \bar{b}_L) \left[\text{diag}(m_d, m_s, m_b) + \phi \begin{pmatrix} 0 & \lambda_{12} & \lambda_{13} \\ \lambda_{12} & 0 & \lambda_{23} \\ \lambda_{13} & \lambda_{23} & 0 \end{pmatrix} \right] \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} + \text{H.c.}, \\ &\equiv \left(1 + \frac{h}{v} \right) \bar{\mathbf{d}}_L \mathbf{M} d_R + \text{H.c.} \end{aligned} \quad (4)$$

This is the effective Lagrangian in the down-quark mass basis. Here $\mathbf{d} = (d, s, b)^T$ denotes the down-type quark fields. Consequently, for the remainder of this paper, unless otherwise specified, we work with the effective interaction in Eq. (1) and do not return to Eq. (2).

A. Classical interpretation

In the classical (coherent-field or wave) interpretation, the ULDM field in a volume V can be written as

$$\phi(x) = \sum_{\mathbf{p}} \sqrt{\frac{2N_{\mathbf{p}}}{V\omega_{\mathbf{p}}}} \cos(\omega_{\mathbf{p}}t - \mathbf{p} \cdot \mathbf{x} + \phi_{\mathbf{p}}), \quad (5)$$

where the phases $\phi_{\mathbf{p}}$ are random variables uniformly distributed in $[0, 2\pi]$. The occupation number for each mode $\mathbf{p}$ is determined by the local dark matter energy density ρ_{DM} and the velocity distribution $f(\mathbf{p})$ as $N_{\mathbf{p}} = \rho_{\text{DM}} V f(\mathbf{p}) (\Delta p)^3 / \omega_{\mathbf{p}}$, where $f(\mathbf{p})$ follows the standard Maxwell-Boltzmann distribution of the standard halo model [28], normalized such that $\int d^3\mathbf{p} f(\mathbf{p}) = 1$. Locally, within a coherence time, the field can be approximated as spatially homogeneous, with a time dependence given by

$$\phi(t) = \bar{\phi} \cos(m_{\phi}t + \phi_0) = \frac{\sqrt{2\rho_{\text{DM}}}}{m_{\phi}} \cos(m_{\phi}t + \phi_0), \quad (6)$$

where ϕ_0 is a random initial phase and we take the local DM energy density to be $\rho_{\text{DM}} = 0.325 \text{ GeV cm}^{-3}$ [29].

In this background, the interaction between ϕ and down-type quarks induces time-dependent corrections to the quark mass matrix M . Including both the SM masses and the ULDM-induced off-diagonal terms, the mass matrix can be diagonalized order by order. In the perturbative regime $|\lambda_{ij}\phi| \ll |m_i - m_j|$ (equivalently $|\lambda_{ij}\phi/(m_i - m_j)| \ll 1$, shown in Fig. 4), the mass matrix can be diagonalized by an orthogonal transformation to first order in $\lambda_{ij}\phi$, leading to

$$U \simeq \begin{pmatrix} 1 & \frac{\lambda_{12}\phi}{m_s - m_d} & \frac{\lambda_{13}\phi}{m_b - m_d} \\ -\frac{\lambda_{12}\phi}{m_s - m_d} & 1 & \frac{\lambda_{23}\phi}{m_b - m_s} \\ -\frac{\lambda_{13}\phi}{m_b - m_d} & -\frac{\lambda_{23}\phi}{m_b - m_s} & 1 \end{pmatrix}, \quad (7)$$

$$U^T M U = \text{diag}(m_d, m_s, m_b) + \mathcal{O}((\lambda\phi)^2).$$

This leads to the mass eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U^T \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (8)$$

At second order in ϕ , the down-type quark masses receive corrections given by

$$\begin{aligned} m'_d &\simeq m_d + \phi^2 \left(\frac{\lambda_{12}^2}{m_d - m_s} + \frac{\lambda_{13}^2}{m_d - m_b} \right), \\ m'_s &\simeq m_s + \phi^2 \left(\frac{\lambda_{12}^2}{m_s - m_d} + \frac{\lambda_{23}^2}{m_s - m_b} \right), \\ m'_b &\simeq m_b + \phi^2 \left(\frac{\lambda_{13}^2}{m_b - m_d} + \frac{\lambda_{23}^2}{m_b - m_s} \right). \end{aligned} \quad (9)$$

These shifts are of order $\lambda_{ij}^2 \phi^2$, arising from a double insertion of the off-diagonal interaction $\lambda_{ij}\phi \bar{d}_i d_j$. They can be encoded in an effective interaction,

$$\begin{aligned} \delta\mathcal{L} &\supset - \sum_{i,j=1,j \neq i}^3 \frac{\lambda_{ij}^2}{(m_i - m_j)} \phi^2 \bar{d}'_i d'_i \\ &= - \sum_{i=1}^3 \frac{1}{\Lambda_i^2} m_i \phi^2 \bar{d}'_i d'_i. \end{aligned} \quad (10)$$

Here Λ_i labels the effective ULDM coupling to the i th down-type quark and satisfies $\Lambda_i^{-2} = \sum_{j=1,j \neq i}^3 \frac{\lambda_{ij}^2}{m_i(m_i - m_j)}$. With this, the effective scalar–nucleon interaction can be expressed as

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{\Lambda_p} \phi^2 \bar{p} p - \frac{1}{\Lambda_n} \phi^2 \bar{n} n, \quad (11)$$

where the coupling strengths are derived following Refs. [30,31] as

$$\begin{aligned} \frac{1}{\Lambda_p} &= \frac{\sigma_d^p}{\Lambda_d^2} + \frac{\sigma_s^p}{\Lambda_s^2} + \frac{2}{27} \frac{m_p - \sigma_u^p - \sigma_d^p - \sigma_s^p}{\Lambda_b^2}, \\ \frac{1}{\Lambda_n} &= \frac{\sigma_d^n}{\Lambda_d^2} + \frac{\sigma_s^n}{\Lambda_s^2} + \frac{2}{27} \frac{m_n - \sigma_u^n - \sigma_d^n - \sigma_s^n}{\Lambda_b^2}, \end{aligned} \quad (12)$$

with $\sigma_u^p = 17 \pm 5 \text{ MeV}$, $\sigma_d^p = 32 \pm 10 \text{ MeV}$, $\sigma_u^n = 15 \pm 5 \text{ MeV}$, $\sigma_d^n = 36 \pm 10 \text{ MeV}$, $\sigma_s^{p/n} = 41.3 \pm 7.7 \text{ MeV}$, $m_p = 938.27 \text{ MeV}$, and $m_n = 939.57 \text{ MeV}$ [32–40].

In the SM quark mass basis, the quark kinetic terms and electroweak interactions read

$$\begin{aligned} \mathcal{L} &\supset \bar{u}_L i\gamma^\mu \partial_\mu u_L + \bar{d}_L i\gamma^\mu \partial_\mu d_L + \bar{u}_R i\gamma^\mu \partial_\mu u_R + \bar{d}_R i\gamma^\mu \partial_\mu d_R - \frac{g}{\sqrt{2}} \bar{u}_L V_{\text{CKM}} \gamma^\mu W_\mu^+ d_L + \text{H.c.} \\ &- \frac{g}{c_w} Z_\mu \left[\left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) \bar{u}_L \gamma^\mu u_L - \frac{2}{3} \sin^2 \theta_w \bar{u}_R \gamma^\mu u_R + \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w \right) \bar{d}_L \gamma^\mu d_L + \frac{1}{3} \sin^2 \theta_w \bar{d}_R \gamma^\mu d_R \right] \\ &- \frac{2}{3} e A_\mu \bar{u} \gamma^\mu u + \frac{1}{3} e A_\mu \bar{d} \gamma^\mu d, \end{aligned} \quad (13)$$

where V_{CKM} is the CKM matrix [41,42]. After the ϕ -induced mixing, the kinetic term and the charged-current interaction receive corrections. The corrected Lagrangian can be written as

$$\mathcal{L} \supset \bar{\mathbf{d}}' i \gamma^\mu (U^T \partial_\mu U + \partial_\mu) \mathbf{d}' + \left[-\frac{g}{\sqrt{2}} \bar{\mathbf{u}}'_L V_{\text{CKM}} U \gamma^\mu W_\mu^+ \mathbf{d}'_L + \text{H.c.} \right], \quad (14)$$

where the up-type quarks remain unchanged, $u'_L = u_L$. The kinetic correction induces a derivative interaction among different down-type quarks. To leading order in $\lambda_{ij}\phi$, one finds

$$\mathcal{L}_{\text{kin}} \supset i g_{ij} (\partial_\mu \phi) \bar{d}'_i \gamma^\mu d'_j, \quad (15)$$

with

$$V'_{\text{CKM}} \simeq \begin{pmatrix} V_{ud} - \frac{V_{us}\phi\lambda_{12}}{-m_d+m_s} - \frac{V_{ub}\phi\lambda_{13}}{m_b-m_d} & V_{us} - \frac{V_{ud}\phi\lambda_{12}}{m_d-m_s} - \frac{V_{ub}\phi\lambda_{23}}{m_b-m_s} & V_{ub} - \frac{V_{ud}\phi\lambda_{13}}{-m_b+m_d} - \frac{V_{us}\phi\lambda_{23}}{-m_b+m_s} \\ V_{cd} - \frac{V_{cs}\phi\lambda_{12}}{-m_d+m_s} - \frac{V_{cb}\phi\lambda_{13}}{m_b-m_d} & V_{cs} - \frac{V_{cd}\phi\lambda_{12}}{m_d-m_s} - \frac{V_{cb}\phi\lambda_{23}}{m_b-m_s} & V_{cb} - \frac{V_{cd}\phi\lambda_{13}}{-m_b+m_d} - \frac{V_{cs}\phi\lambda_{23}}{-m_b+m_s} \\ V_{td} - \frac{V_{ts}\phi\lambda_{12}}{-m_d+m_s} - \frac{V_{tb}\phi\lambda_{13}}{m_b-m_d} & V_{ts} - \frac{V_{td}\phi\lambda_{12}}{m_d-m_s} - \frac{V_{tb}\phi\lambda_{23}}{m_b-m_s} & V_{tb} - \frac{V_{td}\phi\lambda_{13}}{-m_b+m_d} - \frac{V_{ts}\phi\lambda_{23}}{-m_b+m_s} \end{pmatrix} \quad (17)$$

All CKM elements therefore receive $\mathcal{O}(\lambda_{ij}\phi)$ corrections in the coherent-field limit, potentially inducing observable flavor-violating effects.

These results can be corroborated by a direct calculation in the SM mass basis. To establish this equivalence explicitly, we consider the self-energy and vertex corrections shown in Fig. 1. In the nonrelativistic limit where $m_\phi \ll m_i$ and the field momentum is negligible ($m_\phi |\vec{v}_\phi| \ll m_i$), the amplitude in the left panel reduces via the Dirac equation to

$$\begin{aligned} -i\lambda_{ij}^2 \phi^2 \bar{d}_i(p) \frac{1}{\not{p} - m_j} d_i(p) &= -i\lambda_{ij}^2 \phi^2 \bar{d}_i(p) \frac{\not{p} + m_j}{m_i^2 - m_j^2} d_i(p) \\ &\simeq -i\phi^2 \frac{\lambda_{ij}^2}{m_i - m_j} \bar{d}_i(p) d_i(p). \end{aligned} \quad (18)$$

Similarly, for the charged-current amplitude shown in the right panel, one finds

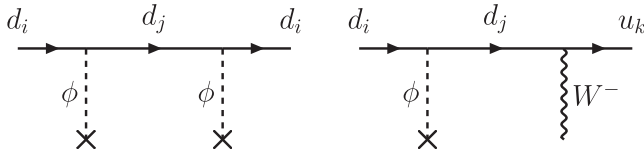


FIG. 1. Two equivalent viewpoints: rediagonalization in the presence of a ULDM background (mass mixing), and perturbation theory in the SM mass basis.

$$g_{ij} = \begin{pmatrix} 0 & -\frac{\lambda_{12}}{m_s-m_d} & -\frac{\lambda_{13}}{m_b-m_d} \\ \frac{\lambda_{12}}{m_s-m_d} & 0 & -\frac{\lambda_{23}}{m_b-m_s} \\ \frac{\lambda_{13}}{m_b-m_d} & \frac{\lambda_{23}}{m_b-m_s} & 0 \end{pmatrix}. \quad (16)$$

Since ϕ is a ULDM background field, $\partial_\mu \phi$ is characterized by a typical four-momentum $k_\mu \sim (m_\phi, m_\phi \vec{v}_\phi)$ with $|\vec{v}_\phi| \sim 10^{-3}$. The derivative interaction in Eq. (15) is therefore additionally suppressed by m_ϕ and is subdominant compared with the mass-mixing and CKM effects; we neglect it following Ref. [43].

For the charged-current interaction, we define the modified CKM matrix as $V'_{\text{CKM}} = V_{\text{CKM}} U$, which can be expressed explicitly as

$$\begin{aligned} &-i\lambda_{ij} V_{\text{CKM},kj} \bar{u}_{L,k}(p_k) \gamma^\mu \epsilon_\mu^-(p_W) \frac{1}{\not{p}_i - m_j} d_{L,i}(p_i) \\ &\simeq -i\lambda_{ij} V_{\text{CKM},kj} \bar{u}_{L,k}(p_k) \gamma^\mu \epsilon_\mu^-(p_W) \frac{\not{p}_i + m_j}{m_i^2 - m_j^2} d_{L,i}(p_i) \\ &= \frac{-i\lambda_{ij}}{m_i - m_j} \cdot V_{\text{CKM},kj} \bar{u}_{L,k}(p_k) \gamma^\mu \epsilon_\mu^-(p_W) d_{L,i}(p_i). \end{aligned} \quad (19)$$

The agreement between Eqs. (9), [(17)], (18), and [(19)] confirms that the perturbative rotation and the diagrammatic expansion are consistent at leading order. For the derivative term in Eq. (15), integrating by parts and using the Dirac equation gives

$$\begin{aligned} \mathcal{L}_{\text{kin}} \supset i g_{ij} (\partial_\mu \phi) \bar{d}'_i \gamma^\mu d'_j &= -i g_{ij} \phi \partial_\mu (\bar{d}'_i \gamma^\mu d'_j) \\ &= -g_{ij} \phi (m'_j - m'_i) \bar{d}'_i d'_j \simeq -\lambda_{ij} \phi \bar{d}_i d_j, \end{aligned} \quad (20)$$

where the last step keeps only the leading-order contribution. This clarifies the physical meaning of the derivative term as a rewriting of the original Yukawa interaction at leading order.

In summary, we have established a quark flavor-violating ULDM setup in the coherent-field limit. The dominant effects are the induced mass mixing in the down sector and the resulting time-dependent corrections to the CKM matrix, which can be probed by current and upcoming experiments.

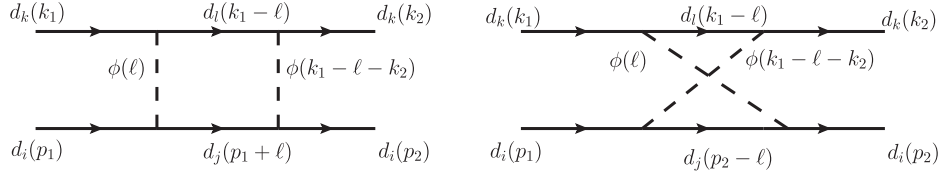


FIG. 2. Representative one-loop diagram for the induced fifth force: $d_i(p_1)d_k(k_1) \rightarrow d_i(p_2)d_k(k_2)$ via exchange of two virtual mediators ϕ , with d_i the i th generation of down-type quarks and p_i, k_i, ℓ the momenta.

B. Quantum interpretation

In the previous subsection, the ULDM field ϕ was treated as a coherent classical background. In this subsection, we present the quantum-field interpretation, in which ϕ is treated as a dynamical quantum field, and its effects are computed perturbatively in the SM mass eigenbasis.

In this framework, no field redefinition or re-diagonalization of the quark mass matrix is required. Instead, all physical effects arise from insertions of the flavor-violating interaction

$$\mathcal{L}_{\text{int}} = -\phi \sum_{i,j=1, i \neq j}^3 \lambda_{ij} \bar{d}_i d_j, \quad (21)$$

written directly in the SM mass eigenstates. At the level of a single insertion, this operator mediates flavor-changing transitions between down-type quarks. At the parton level, the corresponding decay width is

$$\Gamma(d_i \rightarrow d_j \phi) = \frac{N_c \lambda_{ij}^2 (m_i - m_j)(m_i + m_j)^3}{16\pi m_i^3} \simeq \frac{N_c \lambda_{ij}^2 m_i}{16\pi}, \quad (22)$$

where $N_c = 3$ is the color factor and the approximation holds for $m_i \gg m_j$, neglecting the ULDM mass m_ϕ .

For the exotic fifth force, the absence of diagonal couplings implies that the leading contribution arises at one loop. A representative diagram is shown in Fig. 2. To compute the induced fifth force between fermions d_i and d_k , one may evaluate the one-loop scattering amplitude with two off-shell scalars ϕ and two virtual fermions d_j and d_ℓ ,

$$\begin{aligned} i\mathcal{M} = & \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{(\ell^2 - m_\phi^2)((k_1 - \ell - k_2)^2 - m_\phi^2)} \\ & \cdot \left[\bar{u}_{d_i}(p_2, m_i) \lambda_{ij} \frac{i}{\not{p}_1 + \not{\ell} - m_j} \lambda_{ji} u_{d_i}(p_1, m_i) \times \bar{u}_{d_k}(k_2, m_k) \lambda_{k\ell} \frac{i}{\not{k}_1 - \not{\ell} - m_\ell} \lambda_{\ell k} u_{d_k}(k_1, m_k) \right. \\ & \left. + \bar{u}_{d_i}(p_2, m_i) \lambda_{ij} \frac{i}{\not{p}_2 - \not{\ell} - m_j} \lambda_{ji} u_{d_i}(p_1, m_i) \times \bar{u}_{d_k}(k_2, m_k) \lambda_{k\ell} \frac{i}{\not{k}_1 - \not{\ell} - m_\ell} \lambda_{\ell k} u_{d_k}(k_1, m_k) \right]. \quad (23) \end{aligned}$$

In the nonrelativistic limit relevant for fifth-force experiments, the external fermions are nearly on shell. The long-range potential is dominated by loop momenta $\ell \sim m_\phi$, while contributions from $\ell \gg m_\phi$ reduce to contact interactions. Using the Dirac equation and expanding in the small momentum transfer, one finds

$$\begin{aligned} i\mathcal{M} \simeq & \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{(\ell^2 - m_\phi^2)((k_1 - \ell - k_2)^2 - m_\phi^2)} \cdot \left[\frac{\lambda_{ij} \lambda_{ji}}{m_i - m_j} \bar{u}_{d_i}(p_2, m_i) u_{d_i}(p_1, m_i) \times \frac{\lambda_{k\ell} \lambda_{\ell k}}{m_k - m_\ell} \bar{u}_{d_k}(k_2, m_k) u_{d_k}(k_1, m_k) \right. \\ & \left. + \frac{\lambda_{ij} \lambda_{ji}}{m_i - m_j} \bar{u}_{d_i}(p_2, m_i) u_{d_i}(p_1, m_i) \times \frac{\lambda_{k\ell} \lambda_{\ell k}}{m_k - m_\ell} \bar{u}_{d_k}(k_2, m_k) u_{d_k}(k_1, m_k) \right] + \mathcal{O}\left(\frac{\ell}{(m_{i/k} - m_{j/\ell})^2}\right) \\ = & \int \frac{d^4 \ell}{(2\pi)^4} \frac{2}{(\ell^2 - m_\phi^2)((k_1 - \ell - k_2)^2 - m_\phi^2)} \\ & \times \left[\frac{\lambda_{ij} \lambda_{ji}}{m_i - m_j} \bar{u}_{d_i}(p_2, m_i) u_{d_i}(p_1, m_i) \cdot \frac{\lambda_{k\ell} \lambda_{\ell k}}{m_k - m_\ell} \bar{u}_{d_k}(k_2, m_k) u_{d_k}(k_1, m_k) \right] + \mathcal{O}\left(\frac{\ell}{(m_{i/k} - m_{j/\ell})^2}\right), \quad (24) \end{aligned}$$

where the limit $|\ell| \ll m_i$ has been applied.

The derivation is straightforward. For example, for the d_i line,

$$\begin{aligned} \bar{u}_{d_i}(p_2, m_i) \lambda_{ij} \frac{i}{\not{p}_1 + \not{\ell} - m_j} \lambda_{ji} u_{d_i}(p_1, m_i) &= i \lambda_{ij} \lambda_{ji} \bar{u}_{d_i}(p_2, m_i) \frac{\not{p}_1 + \not{\ell} + m_j}{(p_1 + \ell)^2 - m_j^2} u_{d_i}(p_1, m_i) \\ &\simeq \frac{i \lambda_{ij} \lambda_{ji}}{m_i - m_j} \bar{u}_{d_i}(p_2, m_i) u_{d_i}(p_1, m_i) + \mathcal{O}\left(\frac{\ell}{(m_i - m_j)^2}\right). \end{aligned} \quad (25)$$

It is therefore convenient to integrate out the virtual fermions and combine the two Yukawa vertices into a higher-dimensional effective operator. At leading order, the resulting interaction is

$$\mathcal{L}_{\text{eff}} \simeq - \sum_{i,j=1,j \neq i}^3 \frac{\lambda_{ij} \lambda_{ji}}{(m_i - m_j)} \bar{d}_i d_i \phi^2 = - \sum_{i=1}^3 \frac{1}{\Lambda_i^2} m_i \phi^2 \bar{d}_i d_i, \quad (26)$$

This interaction has the same structure as the quark mass corrections in Eq. (9) and provides a convenient starting

point for computing the induced fifth force between nuclei. As an intermediate step, we match onto scalar-nucleon interactions,

$$\mathcal{L}_{\text{eff}} \supset - \frac{1}{\Lambda_p} \phi^2 \bar{p} p - \frac{1}{\Lambda_n} \phi^2 \bar{n} n, \quad (27)$$

where the coupling strengths, following Refs. [30,31], take the same form as Eq. (12). With these interactions, the exotic fifth force between two atoms mediated by quadratic ϕ^2 exchange can be written as [44,45]

$$F_{12}(r) = \left(N_{1,p} \frac{1}{\Lambda_p} + N_{1,n} \frac{1}{\Lambda_n} \right) \left(N_{2,p} \frac{1}{\Lambda_p} + N_{2,n} \frac{1}{\Lambda_n} \right) \frac{m_\phi}{4\pi^3 r^3} \left(\frac{3}{2} K_1(2m_\phi r) + m_\phi r K_0(2m_\phi r) \right), \quad (28)$$

where $N_{i,p}$ and $N_{i,n}$ are the numbers of protons and neutrons in the i th atom, r is the separation between the two atoms, and K_n denotes the n th modified Bessel function of the second kind. Note that the force scales as r^{-3} rather than r^{-2} , since it arises from quadratic exchange (ϕ^2) rather than single- ϕ exchange. These expressions will be used in Sec. IV to derive experimental constraints on the model.

III. CONSTRAINTS ON ULDM AS A COHERENT BACKGROUND FIELD

As discussed above, the ULDM field ϕ can be treated as a coherently oscillating classical background. Such a background induces flavor mixing among down-type quarks, leading to periodic modulations of the quark masses and of the CKM matrix elements. In this section, we study these two characteristic signatures using direct CKM measurements, nuclear decays, atomic clock comparisons, and other related precision observables.

A. Direct measurement of CKM matrix

As shown in Eq. (17), in the presence of a ULDM background the CKM elements receive an oscillatory correction. In this subsection, we give only an order-of-magnitude sensitivity estimate by requiring that the ULDM-induced modulation does not exceed the current experimental uncertainty. We stress that a dedicated constraint would require the experiment-specific time binning, correlations, and the full global-fit machinery, which is beyond the scope of this subsection.

To obtain a meaningful constraint, the characteristic experimental time resolution $\Delta t_{\text{exp}}^{\text{res}}$ is crucial. Only when $\Delta t_{\text{exp}}^{\text{res}}$ is shorter than the ULDM oscillation period $\tau_\phi = 2\pi/m_\phi$, and the total running time is sufficiently long, can the measurements be sensitive to an oscillatory signal of this type. Using the PDG global-fit CKM magnitudes and their quoted 1σ uncertainties [46], which we denote as $|V_{ij}| = \bar{V}_{ij} \pm \delta V_{ij}^{\text{exp}}$, we take

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22501 \pm 0.00068 & 0.003732^{+0.000090}_{-0.000085} \\ 0.22487 \pm 0.00068 & 0.97349 \pm 0.00016 & 0.04183^{+0.00079}_{-0.00069} \\ 0.00858^{+0.00019}_{-0.00017} & 0.04111^{+0.00077}_{-0.00068} & 0.999118^{+0.000029}_{-0.000034} \end{pmatrix}. \quad (29)$$

In the absence of a time-resolved global fit, we adopt a conservative order-of-magnitude criterion and impose the element-wise requirement $|\delta V_{ij}^{\text{ULDM}}| \lesssim \delta V_{ij}^{\text{exp}}$, where $\delta V_{ij}^{\text{ULDM}}$ denotes the model-dependent amplitude of the ULDM-induced oscillatory correction to $|V_{ij}|$, while $\delta V_{ij}^{\text{exp}}$ is the corresponding PDG uncertainty. This criterion is intended only as a conservative estimate and does not account for correlations or experimental time-binning. Solving the resulting linear inequalities for the three independent couplings, we obtain indicative central-value bounds on the combinations $\lambda_{ij}\phi$: $\lambda_{12}\phi \lesssim 6 \times 10^{-5}$ GeV, $\lambda_{13}\phi \lesssim 4 \times 10^{-4}$ GeV, and $\lambda_{23}\phi \lesssim 2 \times 10^{-3}$ GeV.

Finally, treating ULDM as a coherently oscillating classical field with amplitude $\tilde{\phi} \simeq \frac{\sqrt{2\rho_{\text{DM}}}}{m_\phi}$ [see Eq. (6)], we translate these limits into indicative constraints on λ_{ij}

$$\begin{aligned}\lambda_{12} &\lesssim \mathcal{O}(10^{-11}) \left(\frac{m_\phi}{10^{-18} \text{ eV}} \right), \\ \lambda_{13} &\lesssim \mathcal{O}(10^{-10}) \left(\frac{m_\phi}{10^{-18} \text{ eV}} \right), \\ \lambda_{23} &\lesssim \mathcal{O}(10^{-9}) \left(\frac{m_\phi}{10^{-18} \text{ eV}} \right).\end{aligned}\quad (30)$$

These bounds are just order-of-magnitude estimates and should be regarded as indicative only, since they do not incorporate the experiment-dependent time resolution, sampling strategy, or the full structure of the CKM global fit.

B. Nuclear beta decay

ULDM behaves as a coherently oscillating classical field and can induce time-dependent variations in the SM parameters to which it couples, including CKM matrix elements. We focus on the element V_{ud} , whose most precise determinations rely on nuclear beta decay. Unlike the previous case, where the time information was left unspecified, time-resolved decay measurements enable a more targeted search for ULDM-induced oscillatory signatures.

Nuclear beta decay is useful for time-domain searches because many beta-unstable nuclei have macroscopic lifetimes and their decay curves can be measured with high precision. Since beta-decay widths contain the factor $G_F^2 |V_{ud}|^2$, a time-dependent modulation of V_{ud} would induce a corresponding modulation of the decay rate, $\delta\Gamma/\Gamma \simeq 2\delta V_{ud}/V_{ud}$, if the phase-space and nuclear-structure inputs are held fixed. The most precise extractions of V_{ud} conventionally come from corrected $\mathcal{F}t$ values of superallowed $0^+ \rightarrow 0^+$ beta decays [47]. In the present ^{37}K analysis, however, we use the available time-resolved decay data of Ref. [48] as an illustrative probe of a possible decay-rate modulation, rather than as a complete time-dependent global extraction of V_{ud} .

In the presence of ULDM, we parametrize the time dependence of V_{ud} as

$$V_{ud}(t) = V_{ud} + \delta V_{ud} \cos(m_\phi t), \quad (31)$$

with

$$\delta V_{ud} = - \left(\frac{V_{us}\lambda_{12}}{m_s - m_d} + \frac{V_{ub}\lambda_{13}}{m_b - m_d} \right) \frac{\sqrt{2\rho_{\text{DM}}}}{m_\phi}. \quad (32)$$

The ULDM-induced modulation of V_{ud} leads to a time-dependent nuclear decay width,

$$\begin{aligned}\Gamma(t) &= \Gamma_0 \left[1 + \frac{2\delta V_{ud}}{V_{ud}} \cos(m_\phi t) + \left(\frac{\delta V_{ud}}{V_{ud}} \right)^2 \cos^2(m_\phi t) \right] \\ &\simeq \Gamma_0 \left[1 + \frac{2\delta V_{ud}}{V_{ud}} \cos(m_\phi t) + \mathcal{O}\left(\frac{\delta V_{ud}^2}{V_{ud}^2} \right) \right],\end{aligned}\quad (33)$$

where Γ_0 denotes the SM decay width, while the additional terms arise from the ULDM-induced correction and correspond to the linear and quadratic contributions in $\delta V_{ud}/V_{ud}$. For $|\delta V_{ud}| \ll |V_{ud}|$, the linear term provides the leading modulation. This contribution can be resolved provided that the experimental time resolution is sufficient compared with the ULDM oscillation period. By contrast, if the ULDM period is much shorter than the experimental time resolution, the linear modulation averages to zero, and the quadratic correction becomes the leading residual effect.

In the present analysis, we only retain the modulation induced via V_{ud} as an illustrative example to demonstrate the methodological feasibility of the time-dependent approach. We emphasize that this is not intended as a fully rigorous treatment, given the subtleties involved in the Q -value analysis. In particular, ULDM-induced shifts in the proton and neutron masses can also modify the decay Q -value of ^{37}K and thus the phase-space factor governing the lifetime. This would introduce an additional oscillatory contribution to the decay width, typically at frequency $2m_\phi$ in the coherent-field regime. A complete and consistent treatment would require modeling the response of nuclear binding energies to the ULDM background, which lies beyond the scope of the present work and is deferred to future study.

Solving the decay equation $dN/dt = -\Gamma(t)N$, the nuclear population evolves as

$$\begin{aligned}N(t) &= N_0 \exp(-\Gamma_0 t) \exp \left[-\frac{\Gamma_0}{m_\phi} \frac{2\delta V_{ud}}{V_{ud}} \sin(m_\phi t) \right] \\ &\simeq N_0 \exp(-\Gamma_0 t) \left[1 - \frac{\Gamma_0}{m_\phi} \frac{2\delta V_{ud}}{V_{ud}} \sin(m_\phi t) \right],\end{aligned}\quad (34)$$

where N_0 is the initial number of nuclei at the start of the experiment, and the second line holds for $\frac{\Gamma_0}{m_\phi} \frac{2\delta V_{ud}}{V_{ud}} \ll 1$.

Consequently, the particle counts exhibit a small deviation from the standard exponential decay law, which can be quantified as

$$\frac{\delta N(t)}{N_0 \exp(-\Gamma_0 t)} \equiv \frac{\delta N(t)}{N(t)} \simeq -\frac{\Gamma_0}{m_\phi} \frac{2\delta V_{ud}}{V_{ud}} \sin(m_\phi t) = A_N \sin(m_\phi t). \quad (35)$$

Here A_N denotes the oscillatory component at the nonzero signal frequency. We do not use any constant offset in the decay curve as part of the search statistic: such a component would be absorbed into the fitted normalization or average decay rate and, in the numerical analysis below, the mean is subtracted before the FFT. The constraint is therefore derived from the frequency-domain amplitude of the residual time series. The oscillation amplitude A_N directly encodes the ULDM-induced modulation of V_{ud} . Denoting by $\mathcal{A}_{\text{det,sto}}^{95}(m_\phi)$ the final 95% CL amplitude limit obtained below, including the look-elsewhere treatment and the stochastic correction, we obtain

$$|\delta V_{ud}| < \frac{m_\phi}{2\Gamma_0} \mathcal{A}_{\text{det,sto}}^{95}(m_\phi) |V_{ud}|, \quad (36)$$

which is then translated into bounds on the couplings λ_{ij} via Eq. (17).

Existing high-statistics nuclear decay measurements can be directly applied to search for ULDM-induced oscillations. For each trial frequency, we obtain a 95% CL upper limit on the fractional-count modulation amplitude A_N appearing in Eq. (35). Following the frequency-domain statistical procedure of Ref. [49], the noise-only MC fixes the false-positive threshold, while the signal-injection MC fixes the amplitude whose false-negative probability is 5%. The steps used to obtain this limit are as follows.

For the ^{37}K dataset, we denote the measured residual series by

$$r_{\text{meas}}(t_n) \equiv \frac{\delta N(t_n)}{\sqrt{N(t_n)}}. \quad (37)$$

We use the first 10 s of data, sampled at $\Delta t = 50$ ms. This gives $N_{\text{samp}} = 200$ samples, sampling frequency $f_s = 20$ Hz, frequency spacing $\Delta f = 1/(N_{\text{samp}} \Delta t) = 0.1$ Hz, and Nyquist frequency $f_{\text{Nyq}} = 10$ Hz. The measured decay curve is fitted by $N_{\text{fit}}(t) = N_{\text{fit},0} \exp(-t/\tau_{\text{fit}})$, and $\sqrt{N_{\text{fit}}(t_n)}$ is evaluated on the same time grid. The unfiltered residuals $r_{\text{meas}}(t_n)$ determine the noise width $\sigma_r = 0.904$ in this normalization, which is used to generate noise-only MC residuals

$$r_{\text{MC},i}(t_n) \sim \mathcal{N}(0, \sigma_r). \quad (38)$$

The raw residual series has sizable low-frequency fluctuations. We therefore use a fourth-order Butterworth

high-pass filter with cutoff $f_c = 1$ Hz and zero-phase filtering. This filter is applied in the same way to the measured residuals, the noise-only MC residuals, and the signal-injection MC residuals. For any residual series $r_X(t_n)$, with $X = \text{meas}, \text{MC}, \text{MC} + \text{sig}$, we define

$$r_{X,\text{filt}} = \mathcal{F}_{\text{HP}}[r_X], \quad y_{X,n} \equiv \frac{\delta N_X(t_n)}{N(t_n)} = \frac{r_{X,\text{filt}}(t_n)}{\sqrt{N_{\text{fit}}(t_n)}}. \quad (39)$$

After subtracting the mean, the one-sided FFT amplitude is

$$\mathcal{A}_X(f_k) = \frac{2}{N_{\text{samp}}} \left| \sum_{n=0}^{N_{\text{samp}}-1} (y_{X,n} - \bar{y}_X) \exp(-2\pi i k n / N_{\text{samp}}) \right|, \quad f_k = \frac{k}{N_{\text{samp}} \Delta t}. \quad (40)$$

With this convention, a pure sinusoidal modulation $y(t) = A_N \cos(2\pi f_k t + \varphi)$ at an FFT bin has amplitude A_N .

The Fourier bins immediately above the high-pass cutoff are in the transition band of the filter. We therefore define the search window before constructing the test statistic as $1.3 \text{ Hz} \leq f \leq 9.9 \text{ Hz}$. Under the null hypothesis, we generate 10^4 noise-only MC series $r_{\text{MC},i}$. The ensemble of $\mathcal{A}_{\text{MC},i}(f_k)$ values gives the noise-only distribution at each frequency. We impose a global false-positive target $\alpha_{\text{global}} = 0.05$ over the $N_f = 87$ Fourier bins in this search window, using the Bonferroni local threshold

$$\alpha_{\text{local}} = \frac{\alpha_{\text{global}}}{N_f}. \quad (41)$$

The false-positive noise threshold $\mathcal{A}_{\text{noise}}^{95}(f_k)$ is the local $(1 - \alpha_{\text{local}})$ quantile of the noise-only distribution. As shown in Fig. 3, the experimental dataset does not exceed this threshold at any frequency, indicating consistency with the null hypothesis.

To obtain the corresponding signal-amplitude limit, we repeat the MC procedure with an injected sinusoid at a specified trial frequency f . The trial amplitude $A_{\text{sig}}(f)$ is defined in the fractional-count domain $\delta N/N$. Therefore the signal is added before filtering in the residual domain as

$$r_{\text{MC+sig},i}(t_n; f, A_{\text{sig}}) = r_{\text{MC},i}(t_n) + A_{\text{sig}}(f) \sqrt{N_{\text{fit}}(t_n)} \cos(2\pi f t_n + \varphi_i), \quad (42)$$

with random phase φ_i . The combined series is then passed through the same high-pass filter, converted to $y_{\text{MC+sig},i,n}$, and Fourier transformed. For fixed A_{sig} , this gives the distribution of the FFT amplitude $\mathcal{A}_{\text{MC+sig}}(f)$ at the injected frequency. For each trial frequency we use 5×10^4 signal-injection realizations and determine the deterministic 95% CL signal-amplitude limit $\mathcal{A}_{\text{det}}^{95}(f)$ by requiring a 5% false-negative probability,

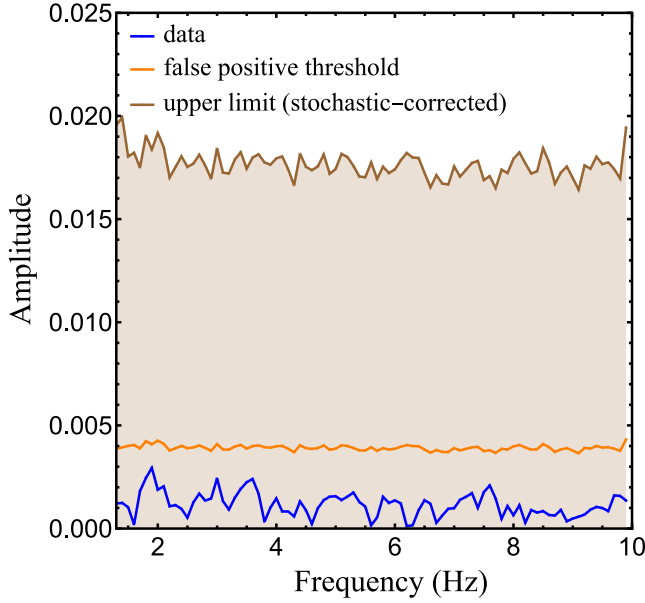


FIG. 3. Periodogram corresponding to the ^{37}K dataset in the predefined $f \geq 1.3$ Hz search window, together with the MC-derived false-positive noise threshold and the final stochastic-corrected upper-limit curve. The data are compatible with the null hypothesis.

$$P[\mathcal{A}_{\text{MC+sig}}(f) < \mathcal{A}_{\text{noise}}^{95}(f) | \mathcal{A}_{\text{sig}} = \mathcal{A}_{\text{det}}^{95}(f)] = \beta = 0.05. \quad (43)$$

Thus, $\alpha_{\text{global}} = 0.05$ controls the false-positive probability of the noise-only search, while $\beta = 0.05$ controls the false-negative probability in the signal-injection test. In the regime $t_{\text{exp}} \ll \tau_{\text{coh}}$, where $t_{\text{exp}} = 10$ s is the duration of the analyzed time series and τ_{coh} is the ULDM coherence time, we apply the stochastic-to-deterministic correction used in Refs. [49,50]; the final amplitude limit used for the

coupling bound is $\mathcal{A}_{\text{det,sto}}^{95}(f) = 3\mathcal{A}_{\text{det}}^{95}(f)$. This stochastic-corrected upper-limit curve is also shown in Fig. 3.

The search window corresponds to $5.37 \times 10^{-15} \text{ eV} \leq m_\phi = 2\pi\hbar f \leq 4.09 \times 10^{-14} \text{ eV}$. The resulting limits on λ_{12} and λ_{13} are included in Fig. 4; the strongest displayed points give approximately $\lambda_{12} \lesssim 6.3 \times 10^{-5}$ and $\lambda_{13} \lesssim 3.0 \times 10^{-3}$ near $m_\phi = 5.37 \times 10^{-15} \text{ eV}$.

C. Tritium decays

Long-term measurements of tritium decay provide a complementary constraint on our model, in addition to the ^{37}K decay bound. Null searches in a 12-year tritium dataset have already been used to constrain ULDM couplings [49,53]. In the present model, the ULDM field induces coherent oscillations in the down-quark Yukawa sector. These oscillations shift the neutron–proton mass difference, the relevant nuclear binding energies, and hence the tritium decay rate [49,58].

According to the calculation in Ref. [59], the total β -decay rate is

$$\Gamma = \frac{1}{2\pi^3} m_e (G_\beta m_e^2)^2 (B_F(^3\text{H}) + B_{\text{GT}}(^3\text{H})) I^\beta(^3\text{H}), \quad (44)$$

where $G_\beta \propto V_{ud}$ and I^β is the phase-space integral. Allowing both I^β and V_{ud} to vary in this rate formula gives, to leading order,

$$I_0 \equiv \frac{\delta\Gamma}{\Gamma} = \frac{\delta I^\beta}{I^\beta} + 2 \frac{\delta V_{ud}}{V_{ud}}. \quad (45)$$

Following Dai and Li [58], we evaluate the phase-space contribution $\frac{\delta I^\beta}{I^\beta}$ through the ULDM-induced shift of nuclear binding energies. The calculation proceeds in two steps. We first keep the dominant pion-mediated contribution and

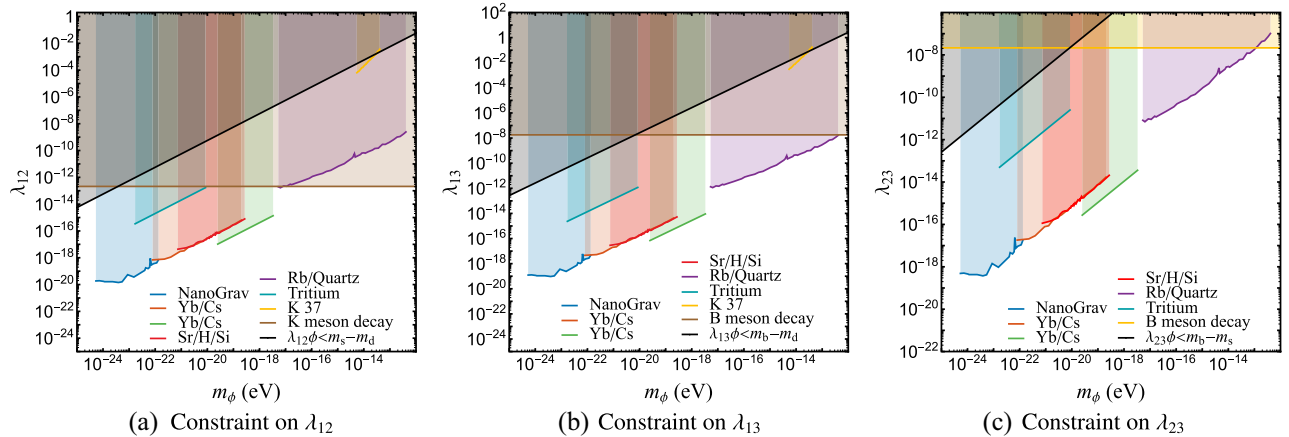


FIG. 4. Comparison of the constraints on ULDM derived from the NANOGrav-derived effective mass-ratio curve compiled in Ref. [21] and translated through the m_p response, Yb/Cs clocks [17,51], Sr/H/Si clocks [52], tritium beta decay [53], ^{37}K decay [48], meson decay [54–56], and Rb/Quartz clock [57]. The black line represents $|\lambda_{ij}\tilde{\phi}/(m_i - m_j)|$.

compute the ϕ -dependent deuteron binding-energy correction $\Delta B_2(\phi)$, using the deuteron calculation of Ref. [60]. We then convert this two-body correction into the ϕ -dependent averaged binding energy of the $A = 3$ system through the empirical binding-energy scaling relation introduced below. The numerical response coefficients quoted in this subsection are obtained by inserting central

values into the approximate nuclear-response model and are rounded to two significant digits; the last quoted digits should not be interpreted as physical uncertainties.

The effect of ULDM on the standard nuclear interactions can be described using the pion–nucleon chiral Lagrangian at next-to-leading order [61]. The ULDM-induced correction to the proton–neutron mass difference [cf. Eq. (11)] is

$$(m_n - m_p)(\phi) = 1.30 \text{ MeV} + [-7.2 \times 10^{-9}(\lambda_{12}\phi)^2 - 1.5 \times 10^{-10}(\lambda_{13}\phi)^2 + 1.3 \times 10^{-15}(\lambda_{23}\phi)^2] \text{ eV}^{-1}. \quad (46)$$

In the one-pion-exchange (OPE) model, the interaction between two nucleons is mediated by a pion of mass m_π , with the schematic form $V_\pi(r) \propto -\frac{g_{\pi NN}^2}{4\pi} \frac{e^{-m_\pi r}}{r}$. Including the spin and isospin factors, the potential is [62]

$$V_\pi(r) = (2S(S+1) - 3)(2I(I+1) - 3) \frac{g_{\pi NN}^2}{4\pi} \frac{1}{12} \left(\frac{m_\pi}{m_N}\right)^2 \frac{e^{-m_\pi r}}{r}, \quad (47)$$

where S and I are the total spin and total isospin, respectively. The ϕ -dependent pion and nucleon masses are parametrized as [62,63]

$$m_\pi^2(\phi) = B_0(m_u + m_d + \Delta m_d(\phi)) + B_s \Delta m_s, \quad m_N(\phi) = m_0 - 4c_1 m_\pi^2(\phi) - \frac{3g_A^2 [m_\pi^2(\phi)]^{3/2}}{32\pi F_\pi^2}. \quad (48)$$

Here $B_0 = 2.8 \text{ GeV}$ is the light-quark condensate parameter; the MILC SU(3) chiral analysis gives the closely related two-flavor chiral-limit value 2.87 GeV [64]. For the strange-quark response we use the representative estimate $(\partial m_\pi / \partial m_s)_{\text{phys}} \simeq 0.09$, within the range suggested by the MILC SU(3) chiral-fit parameters [64], giving $B_s \simeq 2m_\pi (\partial m_\pi / \partial m_s)_{\text{phys}} \simeq 25.1 \text{ MeV}$. The remaining hadronic inputs are $c_1 = -1.1 \text{ GeV}^{-1}$, $g_A = 1.27$, $F_\pi = 92.2 \text{ MeV}$, and $m_0 \approx 865 \text{ MeV}$ [62]. The corresponding pion-nucleon coupling is [62,65]

$$g_{\pi NN}(\phi) = \frac{g_A m_N(\phi)}{F_\pi} \left(1 - \frac{2m_\pi^2(\phi) \bar{d}_{18}}{g_A}\right), \quad (49)$$

where $\bar{d}_{18} = -0.47 \text{ GeV}^{-2}$ [62]. The deuteron binding-energy correction is computed from first-order perturbation

theory,

$$\Delta B_2(\phi) = -\langle \psi^{(0)} | V_\pi(\phi) - V_\pi(\phi = 0) | \psi^{(0)} \rangle, \quad (50)$$

where $\psi^{(0)}$ denotes the unperturbed approximate wave function. To simplify the calculation, we adopt the approach of Ref. [60] and approximate the ground-state wave function by the square-well form

$$\psi^{(0)}(r) = \begin{cases} A \frac{\sin kr}{r}, & r < R \\ B \frac{e^{-\rho r}}{r}, & r > R \end{cases} \quad (51)$$

with $R = 8.62 \times 10^{-3} \text{ MeV}^{-1}$, $k = 212 \text{ MeV}$, $\rho = 46.4 \text{ MeV}$, $A = 2.31 \text{ MeV}^{1/2}$, and $B = 1.44A$. The expectation value of the pion potential is then

$$\begin{aligned} \langle \psi^{(0)} | V_\pi(\phi) | \psi^{(0)} \rangle &= (2I(I+1) - 3)(2S(S+1) - 3) \frac{g_{\pi NN}^2}{4\pi} \frac{1}{12} \left(\frac{m_\pi}{m_N}\right)^2 \int d\Omega \left[\int_0^R dr A^2 \sin^2(kr) \frac{e^{-m_\pi r}}{r} + \int_R^\infty dr B^2 \frac{e^{-2\rho r - m_\pi r}}{r} \right] \\ &= 4\pi (2I(I+1) - 3)(2S(S+1) - 3) \frac{g_{\pi NN}^2}{4\pi} \frac{1}{12} \left(\frac{m_\pi}{m_N}\right)^2 \\ &\quad \times \left[A^2 \left(\frac{1}{4} \ln \left(1 + \frac{4k^2}{m_\pi^2} \right) - \frac{1}{2} E_1(m_\pi R) + \frac{1}{2} \text{Re}(E_1((m_\pi - 2ik)R)) \right) + B^2 E_1((m_\pi + 2\rho)R) \right], \end{aligned} \quad (52)$$

where $S = 1$, $I = 0$, $m_\pi = 139.57 \text{ MeV}$, $m_N = 938.92 \text{ MeV}$, and $g_{\pi NN}^2/4\pi = 13.7$ [62]. Here $E_1(kR) \equiv \int_R^\infty \frac{e^{-kx}}{x} dx$ is the exponential integral. This gives

$$\begin{aligned}
B_2(\phi) &= B_2 + \Delta B_2(\phi) \\
&= 2.22 \text{ MeV} - (3.1 \times 10^{-9}(\lambda_{12}\phi)^2 + 6.7 \times 10^{-11}(\lambda_{13}\phi)^2 + 6.2 \times 10^{-13}(\lambda_{23}\phi)^2) \text{ eV}^{-1}.
\end{aligned} \tag{53}$$

The reference value $B_2 \approx 2.22 \text{ MeV}$ is obtained by solving the radial Schrödinger equation for the two-nucleon system with $\phi = 0$ [62]. We next relate this result to the averaged binding energy of the $A = 3$ nuclear system. Following the empirical relation used in Refs. [58,66],

$$(\bar{B}_n(\phi)/\bar{B}_4)^{1/4} - (\bar{B}_2(\phi)/\bar{B}_4)^{1/4} = (\bar{B}_n/\bar{B}_4)^{1/4} - (\bar{B}_2/\bar{B}_4)^{1/4}, \tag{54}$$

where $n = 3, 4$. Taking $n = 3$ gives

$$\bar{B}_3(\phi) = (\bar{B}_2(\phi)^{1/4} + \bar{B}_3^{1/4} - \bar{B}_2^{1/4})^4. \tag{55}$$

Here $\bar{B}_2 \approx 1.03 \text{ MeV}$ is obtained by averaging over the physical deuteron and the spin-singlet channel. As in Ref. [58], we use the same perturbative shift for the averaged two-body binding energy,

$$\Delta \bar{B}_2(\phi) \equiv \bar{B}_2(\phi) - \bar{B}_2 = \Delta B_2(\phi). \tag{56}$$

Thus

$$\bar{B}_2(\phi) \simeq 1.03 \text{ MeV} - (3.1 \times 10^{-9}(\lambda_{12}\phi)^2 + 6.7 \times 10^{-11}(\lambda_{13}\phi)^2 + 6.2 \times 10^{-13}(\lambda_{23}\phi)^2) \text{ eV}^{-1}. \tag{57}$$

The value $\bar{B}_3 \approx 8.1 \text{ MeV}$ is obtained by averaging over the physical ^3H and ^3He states [49]. The ϕ -dependent averaged binding energy of the $A = 3$ system is then

$$\bar{B}_3(\phi) \approx 8.1 \text{ MeV} + (-1.5 \times 10^{-8}(\lambda_{12}\phi)^2 - 3.2 \times 10^{-10}(\lambda_{13}\phi)^2 - 2.9 \times 10^{-12}(\lambda_{23}\phi)^2) \frac{1}{\text{eV}}. \tag{58}$$

The dominant decay-rate modification enters through the mass difference between the initial and final nuclear states,

$$M_i(\phi) - M_f(\phi) = (m_n - m_p)(\phi) + B_f(\phi) - B_i(\phi), \tag{59}$$

where M_i and M_f denote the masses of the initial ^3H and final ^3He nuclei, respectively, and B_i and B_f are their binding energies. Only the binding-energy difference is needed. In the numerical evaluation, we take the SM value $B_f - B_i \approx -0.76 \text{ MeV}$ from Refs. [49,58] and scale this difference by the fractional response of $\bar{B}_3(\phi)$. Combining this contribution with the neutron-proton mass shift gives

$$\delta M_i(\phi) - \delta M_f(\phi) \approx (-5.8 \times 10^{-9}(\lambda_{12}\phi)^2 - 1.2 \times 10^{-10}(\lambda_{13}\phi)^2 + 2.7 \times 10^{-13}(\lambda_{23}\phi)^2) \frac{1}{\text{eV}}. \tag{60}$$

Following numerical results from [58],

$$\frac{\delta I^\beta}{I^\beta} = 0.18 \times \frac{\delta M_i - \delta M_f}{\text{keV}} = [-1.1 \times 10^{-12}(\lambda_{12}\phi)^2 - 2.3 \times 10^{-14}(\lambda_{13}\phi)^2 + 5.0 \times 10^{-17}(\lambda_{23}\phi)^2] \text{ eV}^{-2}. \tag{61}$$

Combining with Eq. (45), we obtain

$$\begin{aligned}
I_0 &= [-1.1 \times 10^{-12}(\lambda_{12}\phi)^2 - 2.3 \times 10^{-14}(\lambda_{13}\phi)^2 + 5.0 \times 10^{-17}(\lambda_{23}\phi)^2] \text{ eV}^{-2} \\
&\quad + [-5.2 \times 10^{-9}\lambda_{12}\phi - 1.8 \times 10^{-12}\lambda_{13}\phi] \text{ eV}^{-1}.
\end{aligned} \tag{62}$$

Here I_0 denotes the fractional deviation of the tritium β -decay rate from its standard value; Γ_0 is the decay rate in the absence of the ULDM field. This oscillatory behavior is central to identifying ULDM through its impact on nuclear processes such as tritium decay.

To derive the tritium constraint, we follow the Fourier-space treatment of Refs. [49,58]. We denote the physical ULDM mass by m_ϕ and write the local field as

$$\phi(t; m_\phi) = \Phi_0(m_\phi) \cos(m_\phi t + \theta), \quad \bar{\phi}(m_\phi) = \frac{\sqrt{2\rho_{\text{DM}}}}{m_\phi}. \quad (63)$$

For a uniformly sampled time series $d_n = d(t_n)$ with $n = 0, \dots, N-1$, we use the unnormalized DFT convention

$$\tilde{d}_p = \sum_{n=0}^{N-1} d_n e^{-2\pi i p n / N}, \quad \omega_p = \frac{2\pi p}{N\Delta t}. \quad (64)$$

At a trial DFT frequency bin p , the measured complex Fourier coefficient is modeled as

$$\tilde{d}_p = \tilde{n}_p + \tilde{I}_p, \quad (65)$$

where $\tilde{n}_p$ is complex Gaussian noise with variance $\langle |\tilde{n}_p|^2 \rangle = N\sigma^2$, and $\tilde{I}_p$ is the DFT of the ULDM-induced modulation. With this convention, a bin-matched sinusoidal modulation $I_n = a \cos(2\pi p n / N + \theta)$ gives $|\tilde{I}_p| = Na/2$, and therefore $|\tilde{I}_p|/\sqrt{N\sigma^2} = a\sqrt{N}/(2\sigma)$. After marginalizing over the unknown phase, the normalized modulus

$$A \equiv \frac{|\tilde{d}_p|}{\sqrt{N\sigma^2}}, \quad A_s \equiv \frac{|\tilde{I}_p|}{\sqrt{N\sigma^2}}, \quad (66)$$

follows the Rician distribution

$$\mathcal{L}(A|A_s) = 2Ae^{-A^2 - A_s^2} \mathcal{I}_0(2AA_s), \quad (67)$$

where $\mathcal{I}_0$ is the modified Bessel function. The field amplitude is treated as a stochastic variable with Rayleigh distribution

$$p(\Phi_0) d\Phi_0 = \frac{2\Phi_0}{\bar{\phi}^2(m_\phi)} e^{-\Phi_0^2/\bar{\phi}^2(m_\phi)} d\Phi_0. \quad (68)$$

The likelihood is therefore marginalized over Φ_0

$$\mathcal{L}_{\text{marg}}(A|\bar{A}_s) = \int_0^\infty \mathcal{L}(A|A_s(\Phi_0)) \frac{2\Phi_0}{\bar{\phi}^2(m_\phi)} e^{-\Phi_0^2/\bar{\phi}^2(m_\phi)} d\Phi_0. \quad (69)$$

Here $\bar{A}_s$ denotes the deterministic signal strength obtained

from $A_s(\Phi_0)$ by taking the reference field amplitude $\Phi_0 = \bar{\phi}(m_\phi)$; the integral above then marginalizes this signal over the stochastic realization of the ULDM field amplitude.

For the quadratic response, it is convenient to integrate over $X \equiv \Phi_0^2$, whose distribution is exponential:

$$\mathcal{L}_{\text{marg}}(A|\bar{A}_s) = \int_0^\infty \mathcal{L}(A|A_s(\Phi_0^2)) \frac{1}{\bar{\phi}^2(m_\phi)} e^{-\Phi_0^2/\bar{\phi}^2(m_\phi)} d(\Phi_0^2). \quad (70)$$

The 95% detection threshold A^{th} is determined by the false-positive rate $\alpha = 0.05$. In the absence of a signal, $A_s = 0$ and

$$\mathcal{L}(A|0) = 2Ae^{-A^2}. \quad (71)$$

Thus

$$\alpha = \int_{A^{\text{th}}}^\infty 2Ae^{-A^2} dA = e^{-(A^{\text{th}})^2}, \quad A^{\text{th}} = \sqrt{-\ln \alpha} = 1.73. \quad (72)$$

For a given signal strength $\bar{A}_s$, the miss probability, namely the probability that the measured amplitude remains below the detection threshold, is

$$\beta(\bar{A}_s) = \int_0^{A^{\text{th}}} \mathcal{L}_{\text{marg}}(A|\bar{A}_s) dA. \quad (73)$$

The sensitivity threshold is fixed by the equal-error criterion at $\alpha = 0.05$, i.e., we solve $\alpha = \beta(\bar{A}_s)$ for the signal strength. Larger signal strengths give $\beta \leq \alpha$ and are therefore excluded at this level. The linear and quadratic pieces in Eq. (62) enter different Fourier bins. The linear term oscillates at $\omega_{\text{FFT}} = m_\phi$, whereas the quadratic term contains

$$\cos^2(m_\phi t + \theta) = \frac{1}{2} [1 + \cos(2m_\phi t + 2\theta)], \quad (74)$$

and therefore contributes to the nonzero-frequency bin $\omega_{\text{FFT}} = 2m_\phi$. Thus, when the constraint is quoted as a bound on $\lambda_{ij}(m_\phi)$, the corresponding DFT frequency for the quadratic term is twice the field mass. The signal strengths are

$$\begin{aligned} A_s^{(2)}(m_\phi) &= |-5.4 \times 10^{-13}(\lambda_{12})^2 - 1.1 \times 10^{-14}(\lambda_{13})^2 + 2.5 \times 10^{-17}(\lambda_{23})^2| \Phi_0^2(m_\phi) \frac{\sqrt{N}}{2\sigma} \text{eV}^{-2}, \\ \omega_{\text{FFT}} &= 2m_\phi, \\ A_s^{(1)}(m_\phi) &= |-5.2 \times 10^{-9}\lambda_{12} - 1.8 \times 10^{-12}\lambda_{13}| \Phi_0(m_\phi) \frac{\sqrt{N}}{2\sigma} \text{eV}^{-1}, \quad \omega_{\text{FFT}} = m_\phi. \end{aligned} \quad (75)$$

Substituting these signal strengths into $\beta(\bar{A}_s)$ and imposing $\alpha = \beta$, we numerically determine the boundary of the excluded region. Equivalently, the excluded parameter values satisfy $\beta \leq \alpha$. For the quadratic contribution,

$$\alpha = \int_{1.73}^{\infty} dA 2A e^{-A^2} \geq \beta = \int_0^{1.73} dA \int_0^{\infty} dA_s 2A e^{-A^2 - A_s^2} \mathcal{I}_0(2AA_s) b_1 e^{-b_1 A_s}, \quad (76)$$

where

$$b_1 = \frac{eV^2}{|-5.4 \times 10^{-13}(\lambda_{12})^2 - 1.1 \times 10^{-14}(\lambda_{13})^2 + 2.5 \times 10^{-17}(\lambda_{23})^2| \frac{\sqrt{N}}{2\sigma} \bar{\phi}^2(m_\phi)}. \quad (77)$$

Here $N = 50000$ [49] and $\sigma = 0.4\%$ [53]. For the linear contribution,

$$\alpha = \int_{1.73}^{\infty} dA 2A e^{-A^2} \geq \beta = \int_0^{1.73} dA \int_0^{\infty} dA_s 2A e^{-A^2 - A_s^2} \mathcal{I}_0(2AA_s) b_2 A_s e^{-b_2 A_s^2/2}, \quad (78)$$

where

$$b_2 = \frac{4m_\phi^2 \sigma^2 \text{ eV}^2}{|-5.2 \times 10^{-9}\lambda_{12} - 1.8 \times 10^{-12}\lambda_{13}|^2 \rho_{\text{DM}} N}. \quad (79)$$

The equal-error thresholds are $b_1^* = 0.0331$ and $b_2^* = 0.0348$; signals with $b_1 \leq b_1^*$ or $b_2 \leq b_2^*$ are excluded. Therefore the excluded regions satisfy

$$\begin{aligned} |-5.4 \times 10^{-13}(\lambda_{12})^2 - 1.1 \times 10^{-14}(\lambda_{13})^2 + 2.5 \times 10^{-17}(\lambda_{23})^2| &\geq m_\phi^2 \frac{1}{2\rho_{\text{DM}}} \frac{2\sigma}{\sqrt{N}} \frac{1}{0.0331} \text{ eV}^2, \\ |-5.2 \times 10^{-9}\lambda_{12} - 1.8 \times 10^{-12}\lambda_{13}| &\geq \frac{1}{\sqrt{0.0348}} \frac{2m_\phi \sigma}{\sqrt{\rho_{\text{DM}} \sqrt{N}}} \text{ eV}. \end{aligned} \quad (80)$$

Because the quadratic signal is observed at $\omega_{\text{FFT}} = 2m_\phi$, the corresponding physical mass range is half of the Fourier-frequency range. We therefore use $m_\phi \in [1.7 \times 10^{-23}, 8.5 \times 10^{-21}]$ eV for the quadratic constraint and $m_\phi \in [3.4 \times 10^{-23}, 1.7 \times 10^{-20}]$ eV for the linear constraint. The quadratic constraint is stronger than the linear one, with the strongest bounds set at the lower edge of the quadratic mass range, $m_\phi = 1.7 \times 10^{-23}$ eV. Taking one coupling at a time, the strongest bounds are $\lambda_{12} \lesssim 3.4 \times 10^{-16}$, $\lambda_{13} \lesssim 2.3 \times 10^{-15}$, and $\lambda_{23} \lesssim 5.0 \times 10^{-14}$. The resulting bounds are shown in Fig. 4.

D. Constraints from quark mass measurements

As shown in Eq. (9), the ULDM background shifts the down-type quark masses by an amount proportional to ϕ^2 . We therefore interpret the experimental uncertainties on quark mass determinations as upper bounds on any additional DM-induced contributions.

We adopt the PDG $\overline{\text{MS}}$ masses and quoted 1σ uncertainties [46]: $m_d = (4.70 \pm 0.07)$ MeV, $m_s = (93.5 \pm 0.8)$ MeV, $m_b = (4.183 \pm 0.007)$ GeV, and require that the ULDM-induced shift does not exceed these uncertainties.

For an oscillating ULDM condensate, using Eq. (9), the time-averaged mass shift $\delta m_q \equiv \langle m'_q - m_q \rangle \propto \langle \phi^2 \rangle$ is quadratic in the couplings. In the one-coupling-at-a-time approximation, we derive the strongest bound by requiring that the corresponding ULDM-induced mass shift not exceed the quoted 1σ uncertainty of the most sensitive quark-mass determination $[\sigma(m_q)]$. This gives

$$\lambda_{ij} \lesssim m_\phi \left(\frac{\sigma(m_q) |m_i - m_j|}{\rho_{\text{DM}}} \right)^{1/2}, \quad (81)$$

where $q = d$ for λ_{12} and λ_{13} , while $q = s$ for λ_{23} . With $\rho_{\text{DM}} = 0.325 \text{ GeV cm}^{-3}$, we obtain

$$\begin{aligned} \lambda_{12} &\lesssim 1.6 \times 10^{-5} \left(\frac{m_\phi}{10^{-14} \text{ eV}} \right) \left(\frac{0.325 \text{ GeV cm}^{-3}}{\rho_{\text{DM}}} \right)^{1/2}, \\ \lambda_{13} &\lesssim 1.1 \times 10^{-4} \left(\frac{m_\phi}{10^{-14} \text{ eV}} \right) \left(\frac{0.325 \text{ GeV cm}^{-3}}{\rho_{\text{DM}}} \right)^{1/2}, \\ \lambda_{23} &\lesssim 3.6 \times 10^{-4} \left(\frac{m_\phi}{10^{-14} \text{ eV}} \right) \left(\frac{0.325 \text{ GeV cm}^{-3}}{\rho_{\text{DM}}} \right)^{1/2}. \end{aligned} \quad (82)$$

E. Atomic clocks

The ULDM background induces oscillating quark-mass shifts through Eq. (9). These shifts propagate to hadron masses and nuclear magnetic moments, and can therefore be constrained by precision searches for oscillating frequency ratios. A generic clock comparison may be written as

$$\frac{\delta(f_A/f_B)}{f_A/f_B} = \sum_X K_X^{A/B} \frac{\delta X}{X}, \quad (83)$$

where $K_X^{A/B}$ is the sensitivity of the ratio f_A/f_B to the constant or hadronic quantity X [67]. In the present work we use two kinds of input data. The first input consists of the effective electron-proton mass-ratio limits compiled in Ref. [21], including the Yb/Cs, Sr/H/Si, and NANOGrav-derived curves. The clock curves are based on the measurements of Refs. [17,51,52], while the NANOGrav-derived curve is based on the 15-year pulsar-timing dataset of Ref. [68]. Following the notation of Ref. [21], the constrained quantity is $Y(m_{\text{sig}}) \equiv \frac{M_{\text{eff}}(m_{\text{sig}})}{M_{\text{Pl}}}$, where $M_{\text{eff}}^{-1} = M^{-1} - M_e^{-1}$, with $M \equiv M_p = M_n$. We use these published curves as effective limits and do not reconstruct the individual clock or PTA analyses. Since the present interaction does not shift m_e , the constrained mass-ratio oscillation is translated into a fractional proton-mass oscillation, $\delta m_p/m_p$, and is then matched to the quadratic proton-mass response predicted by our model. A full clock-level reanalysis would require the transition-specific sensitivities to nuclear g -factors and other hadronic quantities, which is beyond the effective-limit translation performed here. The second input is the Rb/Quartz oscillation-amplitude limit of Ref. [57], for which we carry out the clock-specific response matching explicitly. The Rb nuclear magnetic-moment dependence on quark masses is taken from Ref. [69].

Because the response is quadratic in the ULDM field, the coherent field defined in Eq. (6) gives

$$[\phi(t)\lambda_{ij}]^2 = \frac{1}{2} \bar{\phi}^2(m_\phi) \lambda_{ij}^2 [1 + \cos(2m_\phi t + 2\phi_0)]. \quad (84)$$

The static component does not contribute to the oscillation search, so the quantity matched to the experimental limits is the $2m_\phi$ component with amplitude $\bar{\phi}^2(m_\phi) \lambda_{ij}^2/2$.

For the constraints digitized from Ref. [21], the published vertical axis is $Y(m_{\text{sig}}) \equiv M_{\text{eff}}/M_{\text{Pl}}$, with $M_{\text{Pl}} = 2.4 \times 10^{18}$ GeV. Here M_{eff} parametrizes the differential scalar-induced shifts of m_p and m_e , and hence the variation of $\mu \equiv m_p/m_e$. In the benchmark notation of Ref. [21], $M_{\text{eff}}^{-1} = M^{-1} - M_e^{-1}$, with $M \equiv M_p = M_n$. Since the present model does not shift m_e , the linear-benchmark mass-ratio amplitude is matched to the fractional proton-mass oscillation as

$$\left| \frac{\delta\mu}{\mu} \right|_{\text{lin}} = \left| \frac{\delta m_p}{m_p} \right|_{\text{lin}} = \frac{\bar{\phi}(m_{\text{sig}})}{Y(m_{\text{sig}})M_{\text{Pl}}}. \quad (85)$$

In our quadratic model the same signal frequency corresponds to $m_{\text{sig}} = 2m_\phi$. Using the quark-mass shifts in Eq. (9) together with the scalar-nucleon matching in Eq. (11), the induced proton-mass response can be written as

$$\delta m_p(t) = -\sum_{ij} C_{ij}^p [\phi(t)\lambda_{ij}]^2, \quad (86)$$

where the matching coefficients are central-value estimates rounded to two significant digits

$$\begin{aligned} C_{12}^p &= 7.2 \times 10^{-8} \text{ eV}^{-1}, \\ C_{13}^p &= 1.6 \times 10^{-9} \text{ eV}^{-1}, \\ C_{23}^p &= 1.0 \times 10^{-10} \text{ eV}^{-1}. \end{aligned} \quad (87)$$

Only the oscillation amplitude is relevant for the upper bound. The matching condition for each individual coupling is therefore

$$\frac{1}{2} C_{ij}^p \bar{\phi}^2(m_{\text{sig}}/2) \lambda_{ij}^2 = \bar{\phi}(m_{\text{sig}}) \frac{m_p}{Y(m_{\text{sig}})M_{\text{Pl}}}, \quad (88)$$

which gives

$$\lambda_{ij}(m_{\text{sig}}/2) = \left[\frac{2\bar{\phi}(m_{\text{sig}})m_p}{C_{ij}^p \bar{\phi}^2(m_{\text{sig}}/2)Y(m_{\text{sig}})M_{\text{Pl}}} \right]^{1/2}. \quad (89)$$

Thus the horizontal axis of these limits is $m_\phi = m_{\text{sig}}/2$. Although the NANOGrav-derived curve is not an atomic-clock measurement, it is included in the same effective mass-ratio compilation of Ref. [21]; we therefore translate its m_p -channel amplitude with the same matching relation.

The Rb/Quartz input is different: Ref. [57] directly constrains the oscillation amplitude $A_{\text{Rb/Q}}(f)$ of a fractional frequency ratio at ordinary frequency f in Hz. From the sensitivity formula and Table I of Ref. [57], the Rb/Quartz observable is

$$\frac{\delta f}{f} = \frac{\delta g_{\text{Rb}}}{g_{\text{Rb}}} - \frac{1}{2} \frac{\delta m_p}{m_p}. \quad (90)$$

The nuclear g -factor dependence is taken from Ref. [69],

$$\frac{\delta g_{\text{Rb}}}{g_{\text{Rb}}} = -0.064 \frac{\delta m_q}{m_q} - 0.010 \frac{\delta m_s}{m_s}, \quad (91)$$

where $m_q = (m_u + m_d)/2$. Since the present interaction shifts m_d but not m_u , we use

$$\frac{\delta m_q}{m_q} = \frac{\delta m_d}{m_u + m_d}. \quad (92)$$

Combining the quark-mass shifts in Eq. (9) with the proton-mass and nuclear g -factor responses gives

$$\frac{\delta f}{f} = \sum_{ij} C_{ij}^{\text{Rb/Q}} [\phi(t) \lambda_{ij}]^2, \quad (93)$$

where the resulting response coefficients are central-value estimates rounded to two significant digits

$$\begin{aligned} C_{12}^{\text{Rb/Q}} &= 1.4 \times 10^{-16} \text{ eV}^{-2}, \\ C_{13}^{\text{Rb/Q}} &= 3.1 \times 10^{-18} \text{ eV}^{-2}, \\ C_{23}^{\text{Rb/Q}} &= 8.2 \times 10^{-20} \text{ eV}^{-2}. \end{aligned} \quad (94)$$

The Rb/Quartz data are tabulated against the ordinary signal frequency f . Since the signal is produced by ϕ^2 , the field mass is

$$m_\phi = \frac{hf}{2}, \quad h = 4.1357 \times 10^{-15} \text{ eV Hz}^{-1}. \quad (95)$$

The amplitude matching is therefore

$$A_{\text{Rb/Q}}(f) = \frac{1}{2} C_{ij}^{\text{Rb/Q}} \bar{\phi}^2 (hf/2) \lambda_{ij}^2, \quad (96)$$

which gives

$$\lambda_{ij}^{\text{Rb/Q}}(hf/2) = \left[\frac{2A_{\text{Rb/Q}}(f)}{C_{ij}^{\text{Rb/Q}} \bar{\phi}^2(hf/2)} \right]^{1/2}. \quad (97)$$

In Fig. 4, we present the 95% CL upper bounds derived from the Yb/Cs, Sr/H/Si, and Rb/Quartz clock comparisons. The three panels correspond to λ_{12} , λ_{13} , and λ_{23} , with colors denoting the different input data sets. We also show the NANOGrav-derived m_p -channel bound from the same compilation. The strongest NANOGrav-derived constraints are approximately $\lambda_{12} < 1.4 \times 10^{-20}$, $\lambda_{13} < 9.5 \times 10^{-20}$, and $\lambda_{23} < 3.7 \times 10^{-19}$ near $m_\phi \simeq 4 \times 10^{-24}$ eV. For heavier dark-matter masses, the strongest Rb/Quartz bounds are approximately $\lambda_{12} < 1.7 \times 10^{-13}$, $\lambda_{13} < 1.1 \times 10^{-12}$, and $\lambda_{23} < 7.0 \times 10^{-12}$ near $m_\phi \simeq 6 \times 10^{-18}$ eV.

Beyond the clock systems considered here, other platforms, including single-ion Al^+ and Hg^+ optical clocks [70,71], Yb⁺ octupole and quadrupole clocks [72], Dy-based optical platforms [5], and the proposed ^{229}Th nuclear clock [73], also merit further study. These systems can be analyzed with analogous matching procedures once the relevant sensitivity coefficients are specified, and we therefore do not discuss them in detail here.

F. Meson mixing from a ULDM background field

In the presence of a ULDM background, the charged-current interaction is governed by the time-dependent CKM matrix V'_{CKM} in Eq. (17). As a consequence, neutral-meson mass splittings acquire a periodic modulation with angular frequency m_ϕ .

We define the quark-mass splittings

$$\Delta_{sd} \equiv m_s - m_d, \quad \Delta_{bd} \equiv m_b - m_d, \quad \Delta_{bs} \equiv m_b - m_s. \quad (98)$$

For numerical estimates, we use PDG central values for CKM magnitudes, quark masses, and neutral-meson mass splittings, rounded to the precision required here [46]: $|V_{cd}| = 0.22487$, $|V_{cs}| = 0.97349$, $|V_{cb}| = 0.04183$, $|V_{td}| = 8.6 \times 10^{-3}$, $|V_{ts}| = 41.5 \times 10^{-3}$, $|V_{tb}| \simeq 1$, $m_d = 4.7$ MeV, $m_s = 93$ MeV, $m_b = 4.18$ GeV, which imply $\Delta_{sd} \simeq 88$ MeV, $\Delta_{bd} \simeq 4.18$ GeV, and $\Delta_{bs} \simeq 4.09$ GeV. For the experimental mass splittings and their relative uncertainties, we take $\Delta m_K = 0.5293 \times 10^{-2} \text{ ps}^{-1}$ and $\delta_K = 1.70 \times 10^{-3}$, $\Delta m_d = 0.5069 \text{ ps}^{-1}$ and $\delta_{B_d} = 3.75 \times 10^{-3}$, $\Delta m_s = 17.765 \text{ ps}^{-1}$ and $\delta_{B_s} = 3.38 \times 10^{-4}$. We parametrize the local ULDM density by ρ_{DM} , and use $0.325 \text{ GeV cm}^{-3}$ as a reference value [29].

For an estimate of the modulation in Δm_K , we retain the dominant charm contribution to the real part of the short-distance amplitude [74]. The fractional modulation is

$$\frac{\delta(\Delta m_K)(t)}{\Delta m_K} \simeq 2\text{Re} \left(\frac{\delta \lambda_c^K(t)}{\lambda_c^{K,\text{SM}}} \right), \quad \lambda_c^{K,\text{SM}} = V_{cs}^* V_{cd}. \quad (99)$$

At linear order in the ULDM-induced CKM shifts from Eq. (17), one finds

$$\frac{\delta \lambda_c^K}{\lambda_c^{K,\text{SM}}} = \phi \left[\frac{\lambda_{12}}{\Delta_{sd}} \frac{|V_{cd}|^2 - |V_{cs}|^2}{V_{cs}^* V_{cd}} - \frac{\lambda_{23}}{\Delta_{bs}} \frac{V_{cb}^*}{V_{cs}} - \frac{\lambda_{13}}{\Delta_{bd}} \frac{V_{cb}}{V_{cd}} \right]. \quad (100)$$

We take the ULDM background as in Eq. (6). Let δ_K denote the allowed fractional modulation amplitude of Δm_K . A sufficient condition is

$$2|\bar{\phi}| \left[\frac{|\lambda_{12}|}{\Delta_{sd}} \frac{||V_{cd}|^2 - |V_{cs}|^2|}{|V_{cs}||V_{cd}|} + \frac{|\lambda_{23}|}{\Delta_{bs}} \frac{|V_{cb}|}{|V_{cs}|} + \frac{|\lambda_{13}|}{\Delta_{bd}} \frac{|V_{cb}|}{|V_{cd}|} \right] \leq \delta_K. \quad (101)$$

This yields the single-coupling limits

$$|\lambda_{12}| \leq \frac{\delta_K}{2|\bar{\phi}|} \Delta_{sd} \frac{|V_{cs}||V_{cd}|}{||V_{cd}|^2 - |V_{cs}|^2|}, \quad (102)$$

$$|\lambda_{13}| \leq \frac{\delta_K}{2|\bar{\phi}|} \Delta_{bd} \frac{|V_{cd}|}{|V_{cb}|}, \quad (103)$$

$$|\lambda_{23}| \leq \frac{\delta_K}{2|\bar{\phi}|} \Delta_{bs} \frac{|V_{cs}|}{|V_{cb}|}. \quad (104)$$

With the numerical inputs, we obtain

$$|\lambda_{12}| \leq 8.2 \times 10^{-16} \left[\frac{m_\phi}{10^{-22} \text{ eV}} \right] \left[\frac{0.325 \text{ GeV cm}^{-3}}{\rho_{\text{DM}}} \right]^{1/2}, \quad (105)$$

$$|\lambda_{13}| \leq 8.5 \times 10^{-13} \left[\frac{m_\phi}{10^{-22} \text{ eV}} \right] \left[\frac{0.325 \text{ GeV cm}^{-3}}{\rho_{\text{DM}}} \right]^{1/2}, \quad (106)$$

$$|\lambda_{23}| \leq 3.7 \times 10^{-12} \left[\frac{m_\phi}{10^{-22} \text{ eV}} \right] \left[\frac{0.325 \text{ GeV cm}^{-3}}{\rho_{\text{DM}}} \right]^{1/2}. \quad (107)$$

The same CKM modulation also induces constraints from $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixing. The short-distance amplitude is dominated by the top contribution, leading to the estimates

$$\frac{\delta(\Delta m_d)(t)}{\Delta m_d} \simeq 2\text{Re} \left(\frac{\delta\lambda_t^{(d)}(t)}{\lambda_{t,\text{SM}}^{(d)}} \right), \quad \lambda_{t,\text{SM}}^{(d)} = V_{td}^* V_{tb}, \quad (108)$$

$$\frac{\delta(\Delta m_s)(t)}{\Delta m_s} \simeq 2\text{Re} \left(\frac{\delta\lambda_t^{(s)}(t)}{\lambda_{t,\text{SM}}^{(s)}} \right), \quad \lambda_{t,\text{SM}}^{(s)} = V_{ts}^* V_{tb}. \quad (109)$$

Keeping the parametrically dominant terms in the single-coupling limits, one obtains

$$|\lambda_{13}| \leq \frac{\delta_{B_d}}{2|\bar{\phi}|} \Delta_{bd} \frac{|V_{td}|}{|V_{tb}|}, \quad |\lambda_{23}| \leq \frac{\delta_{B_s}}{2|\bar{\phi}|} \Delta_{bs} \frac{|V_{ts}|}{|V_{tb}|}. \quad (110)$$

Numerically,

$$|\lambda_{13}| \leq 3.0 \times 10^{-15} \left[\frac{m_\phi}{10^{-22} \text{ eV}} \right] \left[\frac{0.325 \text{ GeV cm}^{-3}}{\rho_{\text{DM}}} \right]^{1/2}, \quad (111)$$

$$|\lambda_{23}| \leq 1.3 \times 10^{-15} \left[\frac{m_\phi}{10^{-22} \text{ eV}} \right] \left[\frac{0.325 \text{ GeV cm}^{-3}}{\rho_{\text{DM}}} \right]^{1/2}. \quad (112)$$

These bounds are stronger than the kaon-mixing limits for λ_{13} and λ_{23} , while kaon mixing provides the leading constraint on λ_{12} .

IV. CONSTRAINTS ON THE ULDM FIELD AS A QUANTUM PARTICLE

As discussed in Sec. II, the scalar field ϕ can also be treated as a quantum field describing particle excitations. In this section, we use the interaction terms and the fifth-force expressions derived above to obtain constraints from quark flavor-violating decays and searches for exotic fifth forces.

A. Quark-flavor violating decays

The off-diagonal interaction induces quark flavor-violating decays, which at the hadronic level lead to flavor-changing processes such as

$$B \rightarrow K\phi \quad \text{and} \quad K \rightarrow \pi\phi. \quad (113)$$

Since ϕ is invisible to detectors, these processes contribute to the experimental signatures $B \rightarrow K + \text{Inv}$ and $K \rightarrow \pi + \text{Inv}$. Measurements of the corresponding branching ratios therefore constrain the coupling strengths λ_{ij} .

Following Ref. [75], the branching ratios for the two-body decays $B \rightarrow K\phi$ and $K \rightarrow \pi\phi$ can be written as

$$\begin{aligned} \mathcal{B}(B \rightarrow K\phi) &= \frac{\tau_B}{8\pi m_B^2} \lambda_{23}^2 \left(\frac{m_B^2 - m_K^2}{m_b - m_s} \right)^2 f_{0,BK}(m_\phi^2)^2 \frac{\lambda^{1/2}(m_B^2, m_K^2, m_\phi^2)}{2m_B}, \\ \mathcal{B}(K \rightarrow \pi\phi) &= \frac{\tau_K}{8\pi m_K^2} \lambda_{12}^2 \left(\frac{m_K^2 - m_\pi^2}{m_s - m_d} \right)^2 f_{0,K\pi}(m_\phi^2)^2 \frac{\lambda^{1/2}(m_K^2, m_\pi^2, m_\phi^2)}{2m_K}, \\ \mathcal{B}(B \rightarrow \pi\phi) &= \frac{\tau_B}{8\pi m_B^2} \lambda_{13}^2 \left(\frac{m_B^2 - m_\pi^2}{m_b - m_d} \right)^2 f_{0,B\pi}(m_\phi^2)^2 \frac{\lambda^{1/2}(m_B^2, m_\pi^2, m_\phi^2)}{2m_B}, \end{aligned} \quad (114)$$

where τ_B (τ_K) and m_B (m_K) denote the lifetime and mass of the B (K) meson, respectively, and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$ is the Källén function. The scalar form factors $f_{0,BK}(q^2)$, $f_{0,K\pi}(q^2)$, and $f_{0,B\pi}(q^2)$ are defined by

$$\begin{aligned}
\langle K|\bar{s}b|B\rangle &= \frac{m_B^2 - m_K^2}{m_b - m_s} f_{0,BK}(q^2), & q = p_B - p_K, \\
\langle \pi|\bar{s}d|K\rangle &= \frac{m_K^2 - m_\pi^2}{m_s - m_d} f_{0,K\pi}(q^2), & q = p_K - p_\pi, \\
\langle \pi|\bar{d}b|B\rangle &= \frac{m_B^2 - m_\pi^2}{m_b - m_d} f_{0,B\pi}(q^2), & q = p_B - p_\pi,
\end{aligned} \tag{115}$$

with lattice QCD values $f_{0,BK}(q^2) = 0.309$, $f_{0,K\pi}(q^2) \approx 1$, and $f_{0,B\pi}(q^2) = 0.26$ from Refs. [76–78].

For conservative estimates, we use the following representative 90% CL bounds on final states with missing energy, based on Refs. [54–56]

$$\begin{aligned}
\mathcal{B}(B^+ \rightarrow K^+ + \text{Inv}) &\lesssim 1.9 \times 10^{-5} \quad (90\% \text{ CL}), \\
\mathcal{B}(K^+ \rightarrow \pi^+ + \text{Inv}) &\lesssim 2 \times 10^{-10} \quad (90\% \text{ CL}), \\
\mathcal{B}(B^+ \rightarrow \pi^+ + \text{Inv}) &\lesssim 8 \times 10^{-6} \quad (90\% \text{ CL}).
\end{aligned} \tag{116}$$

Combining these limits with Eq. (114) yields bounds on the couplings

$$\begin{aligned}
\lambda_{23} &\lesssim \left[\frac{8\pi m_B^2}{\tau_{B^+}} \left(\frac{m_b - m_s}{m_B^2 - m_K^2} \right)^2 \frac{2m_B}{\lambda^{1/2}(m_B^2, m_K^2, m_\phi^2)} \frac{\mathcal{B}(B^+ \rightarrow K^+ + \text{Inv})}{f_{0,BK}(m_\phi^2)^2} \right]^{1/2}, \\
\lambda_{12} &\lesssim \left[\frac{8\pi m_K^2}{\tau_{K^+}} \left(\frac{m_s - m_d}{m_K^2 - m_\pi^2} \right)^2 \frac{2m_K}{\lambda^{1/2}(m_K^2, m_\pi^2, m_\phi^2)} \frac{\mathcal{B}(K^+ \rightarrow \pi^+ + \text{Inv})}{f_{0,K\pi}(m_\phi^2)^2} \right]^{1/2}, \\
\lambda_{13} &\lesssim \left[\frac{8\pi m_B^2}{\tau_{B^+}} \left(\frac{m_b - m_d}{m_B^2 - m_\pi^2} \right)^2 \frac{2m_B}{\lambda^{1/2}(m_B^2, m_\pi^2, m_\phi^2)} \frac{\mathcal{B}(B^+ \rightarrow \pi^+ + \text{Inv})}{f_{0,B\pi}(m_\phi^2)^2} \right]^{1/2}.
\end{aligned} \tag{117}$$

In the ultralight regime $m_\phi \ll m_K, m_B$, the phase-space dependence on m_ϕ can be neglected in deriving the constraint, which yields

$$\lambda_{23} < 2.1 \times 10^{-8}, \quad \lambda_{12} < 2.1 \times 10^{-13}, \quad \lambda_{13} < 1.8 \times 10^{-8}. \tag{118}$$

The corresponding results are shown in Fig. 4.

B. Constraints from meson oscillations

Consider the interaction $\mathcal{L} \supset -\lambda_{12}\phi(\bar{s}d + \bar{d}s)$ in Eq. (1). Tree-level ϕ exchange generates a bilocal $\Delta S = 2$ interaction. Adopting the equal-time approximation for the bilocal quark densities and performing the time-difference integral of the ϕ propagator yields a static Yukawa kernel. We parametrize the resulting momentum weighting by a single effective scale $q_{\text{eff}}(K)$, which defines

$$H_{\text{eff}}^{\Delta S=2} \simeq \frac{1}{2} \lambda_{12}^2 \bar{\Delta}_K(m_\phi) [O_2 + O'_2 + 2O_4], \quad \bar{\Delta}_K(m_\phi) \equiv \frac{1}{q_{\text{eff}}(K)^2 + m_\phi^2}, \tag{119}$$

where O_i are the local $\Delta S = 2$ operators in the SUSY basis [79]. For the kaon we take $q_{\text{eff}}(K) = 0.3759$ GeV from the light-front quark model (LFQM) variational fit of Ref. [80].

We evaluate the required matrix elements at $\mu = 3$ GeV in $\overline{\text{MS}}$ using lattice bag parameters [81] and FLAG [82],

$$\langle \bar{K}^0 | O_2 | K^0 \rangle \simeq -0.153 \text{ GeV}^4, \quad \langle \bar{K}^0 | O_4 | K^0 \rangle \simeq 0.344 \text{ GeV}^4. \tag{120}$$

Parity invariance implies $\langle O'_2 \rangle = \langle O_2 \rangle$ for pseudoscalar external states. The dispersive contribution is

$$\Delta m_K^\phi \simeq 2\text{Re}M_{12}^\phi, \quad M_{12}^\phi = \frac{\lambda_{12}^2 \bar{\Delta}_K(m_\phi)}{4m_K} [2\langle O_2 \rangle + 2\langle O_4 \rangle] \simeq \lambda_{12}^2 \bar{\Delta}_K(m_\phi) \times 0.192 \text{ GeV}^3, \tag{121}$$

with m_K and $(\Delta m_K)_{\text{exp}}$ taken from the PDG [46]. In the ultralight regime $m_\phi \ll q_{\text{eff}}(K)$,

$$\Delta m_K^\phi \simeq (2.72 \text{ GeV}) \lambda_{12}^2, \quad |\Delta m_K^\phi| \leq (\Delta m_K)_{\text{exp}} \Rightarrow \lambda_{12} \lesssim 3.5 \times 10^{-8}. \quad (122)$$

The interactions

$$\mathcal{L} \supset -\lambda_{13} \phi (\bar{b}d + \bar{d}b) - \lambda_{23} \phi (\bar{b}s + \bar{s}b) \quad (123)$$

induce $\Delta B = 2$ transitions via the same mechanism. The static-kernel result can be expressed in terms of the HPQCD reduced matrix elements $R_{2,4}^{(q)}$ at $\mu = m_b$ [83],

$$\Delta m_{B_q}^\phi \simeq 2 |M_{12}^\phi(B_q)| = \lambda_{q3}^2 \bar{\Delta}_{B_q}(m_\phi) \left[\frac{|R_2^{(q)} + R_4^{(q)}|}{4} f_{B_q}^2 m_{B_q} \right], \quad \bar{\Delta}_{B_q}(m_\phi) \equiv \frac{1}{q_{\text{eff}}(B_q)^2 + m_\phi^2}, \quad (124)$$

with $q = d, s$.

We use $q_{\text{eff}}(B_d) = 0.4226 \text{ GeV}$ and $q_{\text{eff}}(B_s) = 0.4412 \text{ GeV}$ from Ref. [80]. We take $R_2^{(d)} = -2.059$, $R_4^{(d)} = 3.822$, $R_2^{(s)} = -2.180$, $R_4^{(s)} = 3.653$ from HPQCD [83], $f_{B_d} = 189.5 \text{ MeV}$ and $f_{B_s} = 230.3 \text{ MeV}$ from FLAG [82], and $m_{B_d} = 5.2796 \text{ GeV}$, $m_{B_s} = 5.3669 \text{ GeV}$, $\Delta m_d = 0.5069 \text{ ps}^{-1}$, and $\Delta m_s = 17.765 \text{ ps}^{-1}$ from the PDG [46]. In the ultralight regime $m_\phi \ll q_{\text{eff}}(B_q)$, imposing $|\Delta m_{B_q}^\phi| \leq (\Delta m_{B_q})_{\text{exp}}$ yields

$$\lambda_{13} \lesssim 8.4 \times 10^{-7}, \quad \lambda_{23} \lesssim 4.7 \times 10^{-6}. \quad (125)$$

C. Exotic force measurements

As described in Eq. (28), our model mediates a composition-dependent “fifth” force: the exotic acceleration it induces on a test body depends on the atomic species. Precision tests of the weak equivalence principle (WEP) that compare the free-fall accelerations of different materials therefore provide direct constraints on this interaction.

A particularly stringent bound comes from the MICROSCOPE satellite mission, which operates on a nearly circular Earth orbit with radius $R_{\text{MS}} \approx 7000 \text{ km}$ [84]. MICROSCOPE compares the accelerations of platinum and titanium test masses and reports a limit on the Eötvös parameter

$$\eta \equiv \frac{2|a_1 - a_2|}{a_1 + a_2} \leq 2.7 \times 10^{-15}, \quad (126)$$

where a_i is the orbital free-fall acceleration of the i th material. The common (composition-independent) part of the acceleration is gravitational,

$$\frac{a_1 + a_2}{2} = \frac{GM_E}{R_{\text{MS}}^2}, \quad (127)$$

with M_E the Earth mass and G the gravitational constant [46]. Any nonzero η would then be sourced by the composition dependence of the exotic force in Eq. (28).

In our setup, however, the effective coupling to nucleons is loop-generated, and the resulting differential acceleration is therefore parametrically suppressed. Translating the MICROSCOPE limit in Eq. (126) into a bound on the nucleon effective scale, one finds that the constraint on the reciprocal coupling is weak,

$$\frac{1}{\Lambda_{p/n}} \lesssim \mathcal{O}(1) \text{ GeV}^{-1}, \quad (128)$$

which implies a very mild restriction on λ_{ij} , much weaker than the bounds derived in the previous sections. We therefore do not display this constraint in the final plots.

V. ASTROPHYSICAL CONSTRAINTS ON ULD M

One potential concern for our model is whether it is subject to astrophysical or stellar-cooling constraints, which commonly arise in searches for ultralight particles [85–89]. However, in the present setup—where the scalar field ϕ couples to quarks exclusively through flavor-off-diagonal interactions—such limits are either inapplicable or strongly suppressed.

Cooling processes in stars, red giants [90], white dwarfs [91], or supernovae [92] take place in low-energy thermal plasmas ($T \sim \text{keV} - \mathcal{O}(10) \text{ MeV}$) whose relevant degrees of freedom are electrons, nucleons, and nuclei, i.e., matter that is effectively flavor-diagonal at hadronic scales. In this environment, the interaction $\phi \bar{d}_i d_j$ with $i \neq j$ corresponds, at the hadronic level, to transitions that change strangeness or bottomness. The emission of a single ϕ therefore requires either (i) external strange/bottom degrees of freedom in the medium, or (ii) the production of strange/bottom hadrons (or hypernuclear excitations) in the final state. Both possibilities are strongly disfavored in ordinary stellar interiors: there is no thermal population of strange or

bottom hadrons, and the relevant hadronic thresholds (set by the masses of strange/bottom hadrons or the corresponding excitation energies) are generically $\mathcal{O}(100 \text{ MeV})$ or larger, far above the available thermal energies. As a result, flavor-changing emission channels such as $d \rightarrow s + \phi$ or $s \rightarrow b + \phi$ are either kinematically inaccessible or exponentially Boltzmann suppressed, rendering ϕ production through these off-diagonal couplings negligible for standard stellar-cooling considerations.

In dense neutron-star cores, ϕ emission could in principle occur if strange degrees of freedom—such as hyperons or kaon condensation—are present, in which case the medium is no longer purely flavor-diagonal. However, the existence and abundance of such states depend sensitively on the (still poorly constrained) equation of state of dense matter. Moreover, any coupling of ϕ to strange quarks relevant for these environments is already stringently constrained by terrestrial flavor experiments. In particular, rare kaon decays such as $K \rightarrow \pi + \phi$ directly probe the same flavor-changing interaction and provide robust, model-independent bounds that are typically stronger and far less uncertain than prospective neutron-star cooling limits.

Consequently, the usual stellar-cooling mechanisms are absent unless the model also contains flavor-diagonal components of the scalar coupling. Hence, for a scalar that couples exclusively through off-diagonal down-type quark interactions, astrophysical cooling constraints are subdominant, model-dependent, and less robust than laboratory flavor-physics bounds.

VI. CONCLUSIONS

This work presents a systematic study of a flavor-violating ULDM portal in which a real scalar field ϕ couples off-diagonally to down-type quarks. We analyzed the same interaction in both regimes relevant to ultralight dark matter: a coherent classical background and a propagating quantum particle. In the coherent regime, the dominant effects are oscillatory shifts of down-type quark masses and CKM elements; in the particle regime, the same couplings contribute to flavor-violating decays and meson oscillations.

Using indicative direct CKM estimates, time-resolved ^{37}K and tritium data, quark-mass constraints, atomic-clock data, compiled NANOGrav-derived limits, and meson mixing, we derived complementary bounds on λ_{ij} across the mass range considered. In the coherent-field regime, the NANOGrav-derived effective mass-ratio curve compiled in Ref. [21] and translated through the m_ϕ response gives the strongest ultralight bounds, reaching approximately $\lambda_{12} < 1.4 \times 10^{-20}$, $\lambda_{13} < 9.5 \times 10^{-20}$, and $\lambda_{23} < 3.7 \times 10^{-19}$ near $m_\phi \simeq 4 \times 10^{-24} \text{ eV}$. At higher masses, the Rb/Quartz clock comparison yields the strongest clock bounds, reaching approximately $\lambda_{12} < 1.7 \times 10^{-13}$, $\lambda_{13} < 1.1 \times 10^{-12}$, and $\lambda_{23} < 7.0 \times 10^{-12}$ near $m_\phi \simeq 6 \times 10^{-18} \text{ eV}$. The tritium

decay analysis provides an independent nuclear-decay constraint, with its strongest quadratic bounds $\lambda_{12} \lesssim 3.4 \times 10^{-16}$, $\lambda_{13} \lesssim 2.3 \times 10^{-15}$, and $\lambda_{23} \lesssim 5.0 \times 10^{-14}$ near $m_\phi \simeq 1.7 \times 10^{-23} \text{ eV}$. Meson mixing provides additional complementary coverage, while the ^{37}K time-series analysis offers dedicated midfrequency reach over $5.4 \times 10^{-15} \text{ eV} \lesssim m_\phi \lesssim 4.1 \times 10^{-14} \text{ eV}$, with the strongest displayed points giving $\lambda_{12} \lesssim 6.3 \times 10^{-5}$ and $\lambda_{13} \lesssim 3.0 \times 10^{-3}$.

In the particle interpretation, invisible meson decays and meson-mixing observables provide additional constraints, including $\lambda_{12} < 2.1 \times 10^{-13}$ from $K \rightarrow \pi + \text{Inv}$, $\lambda_{23} < 2.1 \times 10^{-8}$ from $B \rightarrow K + \text{Inv}$, and $\lambda_{13} < 1.8 \times 10^{-8}$ from $B \rightarrow \pi + \text{Inv}$, while meson oscillations provide further independent coverage of the parameter space. Loop-induced fifth-force effects remain subdominant. Overall, combining coherent-field and particle probes significantly narrows flavor-violating ULDM parameter space over $m_\phi \sim 10^{-24} - 10^{-12} \text{ eV}$ and motivates future time-domain CKM analyses and dedicated flavor-violating decay searches.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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