

Negative- $\Sigma\bar{\Sigma}$ angular-parameter puzzle in J/ψ and $\psi(2S)$ decays

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Experimental measurements of J/ψ and $\psi(2S)$ decays to octet baryon pairs reveal a negative- $\Sigma\bar{\Sigma}$ puzzle in the angular distributions, $\alpha_{\Sigma}^{J/\psi} < 0$, whereas the other measured J/ψ octet channels and the corresponding $\psi(2S)$ channels have positive angular parameters. We show that an $SU(3)_F$ -exact singlet description is strongly disfavored in a χ^2 fit, giving $\chi^2 \simeq 10^5$. By contrast, a reduced scenario in which a common final-state interaction mixes the S- and D-wave amplitudes gives $\chi^2 = 4.21$ for the 22 isospin-reduced inputs and two nominal degrees of freedom. We find that the negative- $\Sigma\bar{\Sigma}$ puzzle can be reproduced with direct-production $SU(3)_F$ breaking at the 40% level. This is larger than the conventional quark-mass expectation of about 30%, but numerically smaller than the order-one channel variation suggested by the experimental data. The sizable observable effect is amplified by $SU(3)_F$ -breaking rescattering paths involving S–D mixing. These paths feed the much larger D-wave source into the final $SU(3)_F$ -breaking S-wave amplitude. The largest remaining pull comes from the $\psi(2S) \rightarrow N\bar{N}$ angular input, motivating a renewed experimental check of $\psi(2S) \rightarrow n\bar{n}$.

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I. INTRODUCTION

The experimental angular distributions for baryonic J/ψ and $\psi(2S)$ decays show an anomalous sign pattern in the $\Sigma\bar{\Sigma}$ channel. Here and below, $N = p, n$, $\Sigma = \Sigma^{+,0,-}$, and $\Xi = \Xi^{0,-}$. In J/ψ decays, the data find $\alpha_{\Sigma}^{J/\psi} < 0$, whereas the measured $N\bar{N}$, $\Lambda\bar{\Lambda}$, and $\Xi\bar{\Xi}$ channels, as well as the baryonic decays of $\psi(2S)$, have positive angular parameters [1–12]. The relevant data are collected in Table I. The phase measurements sharpen this observation. In J/ψ decay, $\Delta\Phi_{\Sigma}$ is negative, whereas the measured $\Lambda\bar{\Lambda}$ and $\Xi\bar{\Xi}$ phases are positive. Thus, the $\Sigma\bar{\Sigma}$ channel is singled out both by the sign of $\alpha_{\Sigma}^{J/\psi}$ and by a distinct strong-phase pattern. This makes final-state interaction (FSI) a natural place to look: it can modify phases after the short-distance production step and thereby turn an $SU(3)_F$ -breaking source into a larger channel-dependent effect. A useful precedent is provided by two-body hadronic D -meson decays, where long-distance rescattering and exchange topologies are known to be important and can generate sizable effective $SU(3)_F$

breaking [13–15]. We therefore ask whether a common baryon-antibaryon FSI, combined with the $SU(3)_F$ -breaking production, can account for the data. The left panel of Fig. 1 shows the corresponding short-distance three-gluon production topology, while the right panel illustrates, for instance, the rescattering of $N\bar{N}$ to $\Sigma\bar{\Sigma}$.

TABLE I. Experimental inputs for $\psi \rightarrow \mathbf{B}\bar{\mathbf{B}}$ with $\psi = J/\psi$, $\psi(2S)$. The J/ψ branching fractions are in units of 10^{-3} , while the $\psi(2S)$ branching fractions are in units of 10^{-4} [7]. The phase $\Delta\Phi$ is quoted in degrees.

Channel	State	$\mathcal{B}$	α	$\Delta\Phi$
$\Sigma^0\bar{\Sigma}^0$	J/ψ [1,5]	1.172(32)	−0.413(9)	−4.74(43)
	$\psi(2S)$ [1,5]	2.35(9)	0.814(40)	29.3(5.2)
$\Sigma^+\bar{\Sigma}^-$	J/ψ [16,17]	1.07(4)	−0.505(2)	−15.72(20)
	$\psi(2S)$ [16,17]	2.43(10)	0.713(11)	24.47(1.27)
$p\bar{p}$	J/ψ [18]	2.121(29)	0.595(19)	...
	$\psi(2S)$ [19]	2.94(8)	1.03(7)	...
$n\bar{n}$	J/ψ [18]	2.09(16)	0.50(21)	...
	$\psi(2S)$ [19]	3.06(15)	0.68(16)	...
$\Lambda\bar{\Lambda}$	J/ψ [1,6]	1.89(9)	0.475(4)	43.09(45)
	$\psi(2S)$ [1,20]	3.81(13)	0.830(22)	21.0(3.8)
$\Xi^0\bar{\Xi}^0$	J/ψ [8,21]	1.17(4)	0.514(16)	66.9(1.5)
	$\psi(2S)$ [9,21]	2.3(4)	0.768(38)	14.7(3.5)
$\Xi^-\bar{\Xi}^+$	J/ψ [11,22]	0.98(4)	0.585(6)	69.9(1.0)
	$\psi(2S)$ [22,23]	2.87(11)	0.693(69)	38.2(7.2)

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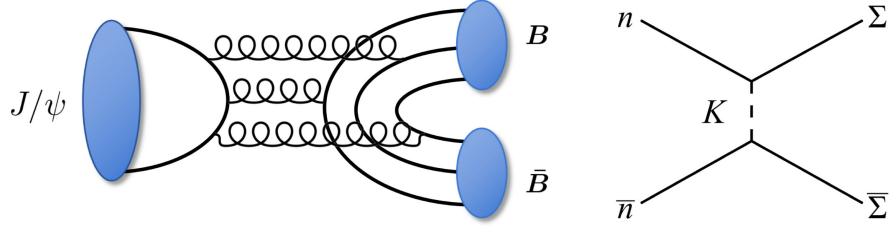


FIG. 1. Direct production and subsequent rescattering. Left: three-gluon topology for $J/\psi, \psi(2S) \rightarrow B\bar{B}$, where B denotes an octet baryon. Right: $n\bar{n} \rightarrow \Sigma\bar{\Sigma}$ through K -exchange, representing the FSI transition following direct production.

Branching fractions and angular distributions for J/ψ and $\psi(2S)$ decays to octet baryons are now measured with good precision, and BESIII polarization analyses have made several electric-magnetic relative phases accessible. The helicity-amplitude and timelike form-factor descriptions are standard [24–28]. Existing theoretical treatments include perturbative hard scattering, mass corrections, $SU(3)_F$ and electromagnetic amplitudes, factorization in quantum chromodynamics, and explicit rescattering [29–39]. The measured relative phases provide information beyond rates and make it possible to separate flavor structure from final-state phases.

The negative value of $\alpha_{\Sigma}^{J/\psi}$ indicates a sizable channel-dependent change in the relative S- and D-wave content. We test this possibility in a minimal framework in which the two charmonia have independent real production amplitudes but share a common baryon-antibaryon FSI operator. The setup follows the Watson-type separation of a short-distance production kernel from final-state phases [40,41]. The remainder of this paper is organized as follows. In Sec. II, we define the helicity amplitudes, the flavor basis, and the common S–D FSI parametrization. In Sec. III, we present the isospin-reduced inputs and the reduced fits. We conclude in Sec. IV with implications for the negative $\Sigma\bar{\Sigma}$ angular parameter and future measurements.

II. FORMALISM

Consider the decays $\psi \rightarrow B\bar{B}$. The field B denotes the standard matrix representation of the light-baryon $SU(3)_F$ octet

$$B_j^i = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}. \quad (2.1)$$

The indices $i, j = 1, 2, 3$ label the u, d, s flavor components. For each partial wave $L = S, D$, we use one $SU(3)_F$ -singlet contraction and two structures generated by the octet spurion $\bar{m} = \text{diag}(1, 1, -2)$:

$$|\mathbf{1}\rangle = \frac{1}{\sqrt{8}} |\bar{\mathbf{B}}_j^i \mathbf{B}_i^j\rangle, \quad (2.2)$$

$$|\mathbf{8}_a\rangle = \frac{\bar{m}_k^j}{6} (|\bar{\mathbf{B}}_j^i \mathbf{B}_i^k\rangle - |\bar{\mathbf{B}}_i^k \mathbf{B}_j^i\rangle), \quad (2.3)$$

$$|\mathbf{8}_s\rangle = \frac{\bar{m}_k^j}{\sqrt{20}} (|\bar{\mathbf{B}}_j^i \mathbf{B}_i^k\rangle + |\bar{\mathbf{B}}_i^k \mathbf{B}_j^i\rangle). \quad (2.4)$$

The factors 6 and $\sqrt{20}$ normalize the octet states. In the $SU(3)_F$ limit, a flavor-singlet ψ produces only the $\mathbf{1}$ component. The $\mathbf{8}_a$ and $\mathbf{8}_s$ components therefore represent explicit $SU(3)_F$ -breaking productions induced by the spurion $\bar{m}$. For a given charmonium state ψ and partial wave L , the direct-transition amplitudes are collected as

$$\vec{\mathcal{A}}_{L,\text{dir}}^\psi = \begin{pmatrix} \mathcal{A}_{L,\text{dir}}^{\psi,\mathbf{1}} \\ \mathcal{A}_{L,\text{dir}}^{\psi,\mathbf{8}_a} \\ \mathcal{A}_{L,\text{dir}}^{\psi,\mathbf{8}_s} \end{pmatrix} = \mathcal{A}_{L,\text{dir}}^{\psi,\mathbf{1}} \begin{pmatrix} 1 \\ r_L^{\psi,\mathbf{8}_a} \\ r_L^{\psi,\mathbf{8}_s} \end{pmatrix}. \quad (2.5)$$

These reduced production amplitudes are taken to be real because they correspond to short-distance transitions. Observable complex phases are generated by the common FSI. For each charmonium state, we collect the S- and D-wave amplitudes as

$$\vec{\mathcal{V}}_{\text{dir}}^\psi = \begin{pmatrix} \vec{\mathcal{A}}_{S,\text{dir}}^\psi \\ \vec{\mathcal{A}}_{D,\text{dir}}^\psi \end{pmatrix} \quad (2.6)$$

and use the all-order transformation

$$\vec{\mathcal{V}}_{\text{phys}}^\psi = \exp(i\Phi_{\text{FSI}}) \vec{\mathcal{V}}_{\text{dir}}^\psi, \quad \Phi_{\text{FSI}} = \begin{pmatrix} \Phi_{S,\text{FSI}} & \mathcal{K}_{SD} \\ \mathcal{K}_{SD} & \Phi_{D,\text{FSI}} \end{pmatrix}. \quad (2.7)$$

For each partial wave L , the flavor block of the dimensionless FSI phase matrix is written in the same $(\mathbf{1}, \mathbf{8}_a, \mathbf{8}_s)$ basis as

$$\Phi_{L,\text{FSI}} = \begin{pmatrix} \delta_{L0} & c_{La} & c_{Ls} \\ c_{La} & \delta_{La} & 0 \\ c_{Ls} & 0 & \delta_{Ls} \end{pmatrix}_{(\mathbf{1}, \mathbf{8}_a, \mathbf{8}_s)}. \quad (2.8)$$

The diagonal entries $\delta_{L0}, \delta_{La}, \delta_{Ls}$ are elastic phases, while c_{La} and c_{Ls} describe $SU(3)_F$ -breaking singlet-octet mixings. We take these mixing entries to be real in the leading closed-channel approximation motivated by one-meson exchange. Complex off-diagonal entries would parametrize higher-order corrections. The direct FSI between $\mathbf{8}_a$ and $\mathbf{8}_s$ is ignored because it requires two $SU(3)_F$ -breaking insertions and is therefore second order in the breaking expansion. We count the direct-transition ratios $r_L^{\psi, \mathbf{8}_i}$ and the phase-generator mixings $c_{La}/\pi, c_{Ls}/\pi$ as first-order $SU(3)_F$ -breaking quantities. The diagonal phases $\delta_{L0}, \delta_{La}, \delta_{Ls}$ are $SU(3)_F$ -conserving entries and may be larger than the singlet-octet mixings entries. If the FSI is closed under $\mathbf{B}\bar{\mathbf{B}} \rightarrow \mathbf{B}'\bar{\mathbf{B}}'$ rescattering [42], Φ_{FSI} is Hermitian, and $(\vec{\mathcal{V}}_{\text{phys}}^\psi)^\dagger \vec{\mathcal{V}}_{\text{phys}}^\psi = (\vec{\mathcal{V}}_{\text{dir}}^\psi)^\dagger \vec{\mathcal{V}}_{\text{dir}}^\psi$. This norm conservation is not identical to conservation of the phase-space-weighted sum of the measured branching fractions. In the reduced numerical benchmark, we take the S–D mixing block to be

$$\mathcal{K}_{\text{SD}} = \text{diag}(k_{\text{SD}}^1, k_{\text{SD}}^{\mathbf{8}_a}, k_{\text{SD}}^{\mathbf{8}_s}). \quad (2.9)$$

This keeps the diagonal S–D rescattering entries needed in the numerical benchmark below, while still neglecting the off-diagonal $\mathbf{8}_a$ – $\mathbf{8}_s$ FSI entries, which are second order in the breaking expansion.

For a physical $\mathbf{B}\bar{\mathbf{B}}$ channel, we define the flavor projection coefficients

$$g_{\mathbf{B}}^1 = \langle \mathbf{B}\bar{\mathbf{B}} | \mathbf{1} \rangle, \quad g_{\mathbf{B}}^a = \langle \mathbf{B}\bar{\mathbf{B}} | \mathbf{8}_a \rangle, \quad g_{\mathbf{B}}^s = \langle \mathbf{B}\bar{\mathbf{B}} | \mathbf{8}_s \rangle. \quad (2.10)$$

These Clebsch-type factors project reduced amplitudes onto physical charge or isospin-reduced channels and are derived from Eq. (2.1). Explicitly,

$$\begin{pmatrix} g_N^1 & g_\Lambda^1 & g_\Sigma^1 & g_\Xi^1 \\ g_N^a & g_\Lambda^a & g_\Sigma^a & g_\Xi^a \\ g_N^s & g_\Lambda^s & g_\Sigma^s & g_\Xi^s \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} \\ \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ -\frac{1}{\sqrt{20}} & -\frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{20}} \end{pmatrix}. \quad (2.11)$$

The corresponding projected S-wave and reduced D-wave amplitudes used in the fit are

$$S_{\mathbf{B}}^\psi = g_{\mathbf{B}}^1 \mathcal{A}_{\text{S,phys}}^{\psi, \mathbf{1}} + g_{\mathbf{B}}^a \mathcal{A}_{\text{S,phys}}^{\psi, \mathbf{8}_a} + g_{\mathbf{B}}^s \mathcal{A}_{\text{S,phys}}^{\psi, \mathbf{8}_s}, \quad (2.12)$$

$$D_{\mathbf{B}}^\psi = g_{\mathbf{B}}^1 \mathcal{A}_{\text{D,phys}}^{\psi, \mathbf{1}} + g_{\mathbf{B}}^a \mathcal{A}_{\text{D,phys}}^{\psi, \mathbf{8}_a} + g_{\mathbf{B}}^s \mathcal{A}_{\text{D,phys}}^{\psi, \mathbf{8}_s}. \quad (2.13)$$

With the standard form-factor normalization, the partial width is

$$\Gamma_{\mathbf{B}}^\psi = \frac{|\vec{p}_{\mathbf{B}}|}{6\pi} \left(|G_{M,\mathbf{B}}^\psi|^2 + \frac{2m_{\mathbf{B}}^2}{m_\psi^2} |G_{E,\mathbf{B}}^\psi|^2 \right), \quad (2.14)$$

where

$$|\vec{p}_{\mathbf{B}}| = \frac{1}{2} \sqrt{m_\psi^2 - 4m_{\mathbf{B}}^2}. \quad (2.15)$$

To convert widths to branching fractions, we use $\Gamma_{J/\psi} = 92.6$ keV and $\Gamma_{\psi(2S)} = 293$ keV [7]. Writing the total and relative four-momenta as $P = p_{\mathbf{B}} + p_{\bar{\mathbf{B}}}$, $Q = p_{\mathbf{B}} - p_{\bar{\mathbf{B}}}$, we use [28]

$$\mathcal{M} = \bar{u}(p_{\mathbf{B}}) \left[G_{M,\mathbf{B}}^\psi \gamma_\mu - \frac{2m_{\mathbf{B}}}{Q^2} (G_{M,\mathbf{B}}^\psi - G_{E,\mathbf{B}}^\psi) Q_\mu \right] v(p_{\bar{\mathbf{B}}}) \epsilon_\mu^\mu, \quad (2.16)$$

where the form factors are evaluated at $P^2 = m_\psi^2$. The labels “E” and “M” are inherited from the Sachs decomposition and refer to the leading elastic Breit-frame structures. In the spacelike region, at leading order in the momentum transfer $\mathbf{k}$, the current is given by

$$J_{\text{NR}}^0 \propto G_E \chi_f^\dagger \chi_i, \quad \mathbf{J}_{\text{NR}} \propto G_M \chi_f^\dagger i(\boldsymbol{\sigma} \times \mathbf{k}) \chi_i, \quad (2.17)$$

where χ_i and χ_f are Pauli spinors. In the present timelike decay, however, $\mathbf{k}$ is not small, and $G_{E,\mathbf{B}}^\psi$ and $G_{M,\mathbf{B}}^\psi$ are complex production-plus-FSI transition form factors (effective couplings) at the physical charmonium point.

The partial wave amplitudes are related to the form factors by [24,42]

$$S_{\mathbf{B}}^\psi = \left(G_{M,\mathbf{B}}^\psi + \frac{m_{\mathbf{B}}}{m_\psi} G_{E,\mathbf{B}}^\psi \right), \quad D_{\mathbf{B}}^\psi = \frac{m_\psi^2}{\sqrt{2}|\vec{p}_{\mathbf{B}}|^2} \left(G_{M,\mathbf{B}}^\psi - \frac{2m_{\mathbf{B}}}{m_\psi} G_{E,\mathbf{B}}^\psi \right). \quad (2.18)$$

The angular parameter $\alpha_{\mathbf{B}}^\psi$ is defined by [24,28,43]

$$\alpha_{\mathbf{B}}^\psi = \frac{m_\psi^2 |G_{M,\mathbf{B}}^\psi|^2 - 4m_{\mathbf{B}}^2 |G_{E,\mathbf{B}}^\psi|^2}{m_\psi^2 |G_{M,\mathbf{B}}^\psi|^2 + 4m_{\mathbf{B}}^2 |G_{E,\mathbf{B}}^\psi|^2}. \quad (2.19)$$

The relative phase measured through polarization observables is

$$\Delta\Phi_{\mathbf{B}}^\psi = \arg\left(\frac{G_{E,\mathbf{B}}^\psi}{G_{M,\mathbf{B}}^\psi}\right). \quad (2.20)$$

Thus, $\alpha_{\mathbf{B}}^\psi$ determines the magnitude ratio G_E/G_M , while $\Delta\Phi_{\mathbf{B}}^\psi$ specifies their relative phase.

TABLE II. Isospin-reduced data and predictions for the $\mathbf{8}_a$ -active fit with full diagonal k_{SD} . Branching-fraction units follow Table I, and $\Delta\Phi$ is in degrees.

B	ψ	$\mathcal{B}$		α		$\Delta\Phi$	
		Data	Fit	Data	Fit	Data	Fit
N	J/ψ [18]	2.12(3)	2.12(4)	0.59(2)	0.59(3)	...	-15(6)
	$\psi(2S)$ [19]	2.97(7)	3.0(1)	0.97(6)	0.87(6)	...	16(13)
Λ	J/ψ [1,6]	1.89(9)	1.9(1)	0.475(4)	0.475(6)	43.1(5)	43.1(6)
	$\psi(2S)$ [1,20]	3.81(13)	3.8(1)	0.83(2)	0.85(3)	21(4)	22(4)
Σ	J/ψ [1,5,16,17]	1.13(3)	1.13(4)	-0.500(2)	-0.500(3)	-13.8(2)	-13.8(3)
	$\psi(2S)$ [1,5,16,17]	2.39(7)	2.4(1)	0.72(1)	0.72(2)	24.7(1.2)	25(2)
Ξ	J/ψ [8,11,21,22]	1.08(3)	1.07(4)	0.578(5)	0.578(8)	69.1(8)	69(1)
	$\psi(2S)$ [9,21-23]	2.83(11)	2.8(1)	0.75(3)	0.72(4)	19(3)	19(5)

III. NUMERICAL RESULTS AND DISCUSSIONS

After charge partners are combined, we work with the $SU(2)_F$ -exact isospin channels N , Λ , Σ , Ξ . The input then contains 22 isospin-reduced data points: 16 values of $\mathcal{B}$ and α plus 6 isospin-reduced hyperon phase inputs formed from 10 measured charge-resolved phases. These inputs are collected in Table II, together with the fit values and predictions used below. The $p\bar{p}$ and $n\bar{n}$ phase entries marked by “...” in Table I are unmeasured and are excluded from the χ^2 function; they are neither imputed nor represented by additional floating observables. Consequently, only six isospin-reduced hyperon phase inputs, formed by combining the 10 measured charge-resolved phases into two inputs each for Λ , Σ , and Ξ , enter the quoted total of 22 inputs. The $N\bar{N}$ phases shown in the Fit column of Table II are derived predictions, one for each charmonium state, and they do not contribute to the minimized χ^2 . Because charge

TABLE III. Best-fit parameters for the $\mathbf{8}_a$ -active fit with full diagonal k_{SD} . Phases are in radians, and parentheses give local one-standard-deviation errors.

FSI parameters			
δ_{Sa}	0.93(21)	δ_{Ss}	2.82(6)
δ_{D0}	-0.21(18)	δ_{Da}	-3.05(11)
δ_{Ds}	-0.12(5)		
c_{Sa}	-0.18(5)	c_{Ss}	-0.57(7)
c_{Da}	-0.82(27)	c_{Ds}	-0.41(2)
k_{SD}^1	0.10(1)	$k_{SD}^{\mathbf{8}_a}$	-0.30(2)
$k_{SD}^{\mathbf{8}_s}$	-1.3(1)		
Direct transition			
$\mathcal{A}_{S,\text{dir}}^{J/\psi,1}$	$-1.3(2) \times 10^{-3}$	$r_S^{J/\psi, \mathbf{8}_a}$	0.34(35)
$\mathcal{A}_{D,\text{dir}}^{J/\psi,1}$	$2.25(14) \times 10^{-2}$	$r_D^{J/\psi, \mathbf{8}_a}$	0.32(21)
$\mathcal{A}_{S,\text{dir}}^{\psi(2S),1}$	$-3.48(8) \times 10^{-3}$	$r_S^{\psi(2S), \mathbf{8}_a}$	-0.12(4)
$\mathcal{A}_{D,\text{dir}}^{\psi(2S),1}$	$-8.7(7) \times 10^{-3}$	$r_D^{\psi(2S), \mathbf{8}_a}$	-0.41(21)

partners are combined before minimization, the quoted χ^2 tests the model against the isospin-reduced averages.

The parameter count is organized around the $SU(3)_F$ -exact singlet limit. This baseline has the four real singlet production amplitudes $\mathcal{A}_{S,\text{dir}}^{\psi,1}$ and $\mathcal{A}_{D,\text{dir}}^{\psi,1}$, with $\psi = J/\psi$, $\psi(2S)$, together with three diagonal S-D rescattering parameters k_{SD}^1 , $k_{SD}^{\mathbf{8}_s}$, and $k_{SD}^{\mathbf{8}_a}$. First-order $SU(3)_F$ breaking in production adds the two octet structures $\mathbf{8}_a$ and $\mathbf{8}_s$ for each ψ and each partial wave, giving eight additional real direct-transition amplitudes. The common FSI block contains six diagonal phases, while first-order $SU(3)_F$ -breaking FSI adds four singlet-octet mixing entries. After fixing the unphysical overall phase by $\delta_{S0} = 0$, a fully unrestricted common-FSI fit therefore contains $4 + 8 + 6 + 4 + 3 - 1 = 24$ real parameters. This parameter count motivates reduced common-FSI scenarios. In addition to the $SU(3)_F$ -exact case, we consider an $\mathbf{8}_a$ -active case with $\mathcal{A}_{L,\text{dir}}^{\psi, \mathbf{8}_s} = 0$. This assumption removes four direct-production parameters. With the diagonal rescattering block used above, the reduced fit has 20 parameters and two nominal degrees of freedom.

Numerically, the $SU(3)_F$ -exact scenario gives $\chi^2 \simeq 10^5$, showing that $SU(3)_F$ breaking is needed to explain the data. To keep the reduced benchmark within a QCD-motivated naturalness range, we impose $|r| \leq 0.49$ and $|c_{Li}/\pi| \leq 0.4$. These phenomenological restrictions are motivated by the conventional QCD breaking scale, but are not strict bounds derived from first-principles QCD. The $\mathbf{8}_a$ -active scenario with full diagonal k_{SD} gives $\chi^2 = 4.21$.¹ The parameter values in this scenario are collected in Table III. The fitted first-order flavor-breaking quantities reach $\max |r| = 0.41$, exceeding the

¹The corresponding $\mathbf{8}_s$ -active full-diagonal fit gives $\chi^2 = 58.79$.

conventional quark-mass benchmark $(m_s - m_{u,d})/\Lambda_{\text{QCD}} \sim 0.3$; the independently normalized FSI-mixing measure reaches $\max |c|/\pi = 0.26$. In their respective normalizations, these fitted first-order quantities are at the level of the conventional benchmark while remaining much smaller than the order-one channel-level departure from the $\text{SU}(3)_F$ -symmetric pattern indicated by the J/ψ data, most visibly the sign reversal of $\alpha_{\Sigma}^{J/\psi}$ relative to the other measured J/ψ octet channels. The sizable fitted k_{SD}^8 shows that the symmetric octet still participates through the common rescattering block rather than through direct production.

$$e^{i\Phi_{\text{FSI}}} \simeq \begin{pmatrix} 0.91 - 0.09i & 0.07 - 0.15i & 0.26 - 0.04i \\ 0.07 - 0.15i & 0.56 + 0.80i & -0.01 - 0.03i \\ 0.26 - 0.04i & -0.01 - 0.03i & -0.72 - 0.30i \\ -0.01 + 0.12i & -0.05 + 0.05i & -0.10 - 0.10i \\ 0.01 - 0.02i & -0.10 - 0.06i & 0.01 + 0.04i \\ -0.14 - 0.18i & -0.01 + 0.02i & 0.54 - 0.11i \end{pmatrix} \begin{pmatrix} -0.01 + 0.12i & 0.01 - 0.02i & -0.14 - 0.18i \\ -0.05 + 0.05i & -0.10 - 0.06i & -0.01 + 0.02i \\ -0.10 - 0.10i & 0.01 + 0.04i & 0.54 - 0.11i \\ 0.79 + 0.03i & -0.48 + 0.04i & -0.08 - 0.30i \\ -0.48 + 0.04i & -0.85 + 0.14i & -0.04 + 0.11i \\ -0.08 - 0.30i & -0.04 + 0.11i & 0.54 - 0.49i \end{pmatrix}. \quad (3.1)$$

The numerical entries of Eq. (3.1) connecting the two direct singlet sources to the final S-wave components give, for $\psi = J/\psi$,

$$\begin{aligned} \mathcal{A}_{\text{S,phys}}^{\psi,1} &\simeq (0.91 - 0.09i)\mathcal{A}_{\text{S,dir}}^{\psi,1} + (-0.01 + 0.12i)\mathcal{A}_{\text{D,dir}}^{\psi,1}, \\ \mathcal{A}_{\text{S,phys}}^{\psi,8} &\simeq (0.26 - 0.04i)\mathcal{A}_{\text{S,dir}}^{\psi,1} + (-0.10 - 0.10i)\mathcal{A}_{\text{D,dir}}^{\psi,1}. \end{aligned} \quad (3.2)$$

Here, the entry $(0.91 - 0.09i)$ connects the singlet S-wave source to the singlet S-wave final state and survives in the $\text{SU}(3)_F$ limit, whereas $(0.26 - 0.04i)$ and $(-0.10 - 0.10i)$ feed the symmetric-octet final state and vanish in that limit. Relative to the leading singlet entry, these flavor-changing entries are of the expected $\text{SU}(3)_F$ -breaking size, $\mathcal{O}(30\%)$. However, the direct D-wave source dominates, with $|\mathcal{A}_{\text{D,dir}}^{\psi,1}/\mathcal{A}_{\text{S,dir}}^{\psi,1}| \simeq 17$. Consequently, the physical singlet and symmetric-octet S-wave amplitudes are comparable in magnitude. Thus, the enhancement comes from the source hierarchy acting through flavor-changing S–D rescattering, not from a large transition element.

For $\psi(2S)$, the measurements $\alpha_p = 1.03(7)$ and $\alpha_n = 0.68(16)$ give $\delta_\alpha^{\psi(2S)} \equiv (\alpha_p - \alpha_n)/(\alpha_p + \alpha_n) = 0.20(12)$ when the quoted errors are treated as uncorrelated. The two measurements differ by $0.35(17)$, or about 2.0σ . Electromagnetic production and $m_d - m_u$ effects can generate isovector amplitudes. For scale, the $\psi(2S)$ mixed-hyperon data [44] give the phase-space-corrected amplitude benchmark

The fit results in Table II show that $\alpha_{\Sigma}^{J/\psi}$ is reproduced within this framework. When the full rescattered S_{Σ} and D_{Σ} amplitudes are converted back to G_E and G_M , they also reproduce the phase clue in Table II, $\Delta\Phi_{\Sigma}^{J/\psi} \simeq -13.8^\circ$, while the corresponding J/ψ phases for $\Lambda\bar{\Lambda}$ and $\Xi\bar{\Xi}$ are positive. Thus, the same rescattered S- and D-wave pattern that drives the negative $\alpha_{\Sigma}^{J/\psi}$ also makes $\Sigma\bar{\Sigma}$ carry a different strong phase from the other measured J/ψ channels.

To see the origin of the apparently large $\text{SU}(3)_F$ -breaking effect, we display the full FSI operator. Using the rounded central values in Table III, its matrix in the ordered basis of Eq. (2.7) is

$$r_{\Sigma\Lambda}^{\psi(2S)} \equiv \left[\frac{|\vec{p}_{\Lambda\bar{\Lambda}}^{\psi(2S)}| |\mathcal{B}(\psi(2S) \rightarrow \Lambda\bar{\Sigma}^0 + \text{c.c.})|}{|\vec{p}_{\Lambda\bar{\Sigma}^0}^{\psi(2S)}| 2\mathcal{B}(\psi(2S) \rightarrow \Lambda\bar{\Lambda})} \right]^{1/2} \simeq 0.046. \quad (3.3)$$

In isospin-breaking power counting, both $\delta_\alpha^{\psi(2S)}$ and $r_{\Sigma\Lambda}^{\psi(2S)}$ enter at first order. We therefore expect $\delta_\alpha^{\psi(2S)} = \mathcal{O}(r_{\Sigma\Lambda}^{\psi(2S)})$, which is parametrically consistent with the electromagnetic counting $\mathcal{O}(\alpha_{\text{em}})$. The central value $\delta_\alpha^{\psi(2S)} = 0.20$ is larger than the above first-order isospin-breaking benchmark. Since the estimated isospin-breaking effect is too small to account for the observed charge-channel difference, we suspect that one or both experimental central values may be shifted by an unaccounted systematic effect. The reduced fit does not use the two charge-specific measurements separately but uses their inverse-variance average $\alpha_N^{\psi(2S)} = 0.97(6)$ as a single isospin-reduced input. This central-value inconsistency is therefore inherited by the reduced input and produces its nominal $\Delta\chi^2 \simeq 2.48$. The reduced fit makes a sharp prediction, $\alpha_p^{\psi(2S)} \simeq \alpha_n^{\psi(2S)} \simeq 0.87$. This common value is 2.3σ below the measured $\alpha_p^{\psi(2S)} = 1.03(7)$ and 1.2σ above the measured $\alpha_n^{\psi(2S)} = 0.68(16)$, using each measurement's quoted uncertainty. Renewed charge-resolved measurements will directly test this prediction and establish whether a genuine proton-neutron difference remains. Such measurements could also be pursued at the proposed Super Tau-Charm Facility [45].

IV. CONCLUSION

We have studied the negative- $\Sigma\bar{\Sigma}$ puzzle in $J/\psi, \psi(2S) \rightarrow B\bar{B}$ angular distributions. The data show $\alpha_{\Sigma}^{J/\psi} < 0$, whereas the other measured J/ψ octet channels and the corresponding $\psi(2S)$ channels have positive angular parameters. In a singlet plus octet basis with a common FSI, the $SU(3)_F$ -exact singlet description is strongly disfavored, giving $\chi^2 \simeq 10^5$.

Allowing the common FSI to mix the S- and D-wave amplitudes, with an antisymmetric direct octet and a full diagonal k_{SD} block, gives $\chi^2 = 4.21$ for two nominal degrees of freedom using the 22 isospin-reduced inputs. This goodness-of-fit statement does not assess the mutual consistency of the underlying charge modes. The fit values and predictions are shown in Tables II and III. The negative $\Sigma\bar{\Sigma}$ angular parameter is traced to the different rescattered S- and D-wave pattern in $J/\psi \rightarrow \Sigma\bar{\Sigma}$. The direct 8_a source affects $N\bar{N}$ already before rescattering, while the $\Sigma\bar{\Sigma}$ effect is generated through the common FSI. The opposite sign of the measured $J/\psi \rightarrow \Sigma\bar{\Sigma}$ phase relative to the measured Λ and Ξ phases gives the same message: the puzzle is tied to strong phases. Seen this way, the fit does not require an independent Σ -specific $SU(3)_F$ -breaking source. The quantities assigned as first order in the breaking counting reach $\max |r| = 0.41$ and $\max |c|/\pi = 0.26$. The former exceeds the conventional quark-mass benchmark of about 0.3, so the fitted breaking is not small on that scale; both values

nevertheless remain below the order-one channel dependence indicated by the data. Their impact on the physical symmetric-octet S-wave contribution is amplified because breaking-dependent rescattering paths combine the much larger D-wave source with the common k_{SD} block. This produces $\alpha_{\Sigma}^{J/\psi} < 0$, while the $\psi(2S)$ production amplitudes keep $\alpha_{\Sigma}^{\psi(2S)}$ positive. The charge-averaged $\alpha_N^{\psi(2S)}$ gives the largest contribution to the χ^2 of the reduced fit. We therefore recommend revisiting the proton and neutron measurements entering this average; the fit predicts $\alpha_p^{\psi(2S)} = \alpha_n^{\psi(2S)} = 0.87 \pm 0.06$.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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