

Selection toolkit for standard and non-standard GUTs

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Under a reasonable set of *ab initio* assumptions, we define and chart the atlas of simple gauge theories with families of fermions whose masses are forbidden by gauge invariance. We propose a compass to navigate the atlas based on counting degrees of freedom. When searching for Grand Unification Theories with three matter generations, the free energy singles out the SU(5) Georgi-Glashow model as the minimal one, closely followed by SO(10) with spinorial matter. The atlas also defines the dryland of grand-unifiable gauge extensions of the standard model. We further provide examples relevant for gauge dual completions of the standard model as well as extensions by an additional SU(N) gauge symmetry.

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I. INTRODUCTION

The Standard Model (SM) of particle physics [1–3] is a remarkable construction, both in its theoretical content and in its success to provide an accurate description of the fundamental forces, excluding gravity. It is largely based on the gauge principle, introducing interactions via local symmetries [4]: SU(3) for the strong chromomagnetic ones and $SU(2) \times U(1)$ for the electroweak ones. This structure has been experimentally tested up to energies of few TeV and below the percent level in most sectors, with the energy frontier currently met at the LHC. Nevertheless, the SM must be considered as a low-energy effective theory, due to its theoretical shortcomings and the observed phenomena

beyond its realms (such as the baryon asymmetry in the Universe, neutrino masses, and inflation). However, when considering higher-energy models, it is not at all granted that their low-energy limit uniquely yields the SM. Usually, a selection is made in terms of matter content, gauge groups, and symmetry-breaking patterns following the primer of low-energy physics. A tantalizing example is given by Grand Unification Theories (GUTs), where the semisimple gauge symmetry of the SM is replaced by a more minimal structure, with the final goal of describing interactions via a single gauge coupling at higher energies. The first attempt to unify quarks and leptons was made by means of a fourth color [5], whereas it soon became clear that the complete SM structure could be derived from SU(5) [6] or SO(10) [7]. The exceptional group E_6 [8] offers a popular alternative, mainly motivated by heterotic string theory [9].

Inspired by GUTs, in this article, we propose to start, instead, from a general atlas of gauge theories characterized by a single gauge coupling. These *grand unified theories* differ from GUTs as they are not required to contain the SM (see Ref. [10] for an early attempt to the latter). We propose using the atlas to chart the dryland of grand-unifiable models that extend the SM gauge symmetry. We put

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forward three concrete application. First, we map the GUT landscape to the atlas by identifying the minimal theoretical requirements that may lead to a selection of the SM at low energies. Second, we apply the atlas to SM duals, i.e., theories that do not contain the SM but rather flow to it at low energies. Finally, we consider theories that contain additional gauge factors to the SM ones at low energies.

Remaining within the realm of quantum field theory in four-dimensional space-time, one can reasonably expect that, below the Planck scale, any theory of Nature features a number of Weyl fermions and, at least, one gauge group. This is in line with the observed particle spectrum of the SM, which is constituted predominantly by fermions and gauge fields. Furthermore, we follow the principle that any mass that is not forbidden by a local symmetry exists and is as large as possible. Hence, we will only consider fermions whose mass is forbidden by gauge invariance, because they are naturally light compared to any higher scale such as the Planck one. Note that we do not consider (light) scalar fields, as their masses cannot be forbidden. This reasoning can be extended to a supersymmetric (SUSY) framework, with Weyl fermions and gauge bosons replaced by chiral and vector superfields in $\mathcal{N} = 1$ [11,12] or to nonsupersymmetric gauge-Yukawa theories featuring ultraviolet fixed points of either completely safe [13–19] or free nature [20–24].

Henceforth, we define the *Grand Unified Theory Atlas* (GTA) as being populated by gauge theories that meet the following requirements:

- (1) One non-Abelian simple Lie group as gauge symmetry. In this article, we consider $SU(N)$, $SO(N)$, $Sp(2N)$, and exceptional gauge groups.
- (2) Anomaly-free fermionic matter families. By a *family*, we mean an irreducible set of massless fermions so that a mass term is forbidden by gauge invariance [25].
- (3) Asymptotic freedom in the gauge coupling, which is the only renormalizable interaction, in the absence of scalars.

The requirement 1 stems from the simplest realization of a single gauge coupling. Alternatively, one could consider products of identical groups related by cyclical symmetries, in the spirit of trinification [27,28]. An atlas of these models will be presented in a forthcoming publication. Asymptotic freedom in 3, instead, is crucial in order to obtain well-defined theories in the ultraviolet, which allows us to characterize their properties. We will see a concrete use of this property in the first application of the atlas below.

We will restrict our considerations to $d = 4$ space-time and renormalizable theories, where asymptotic freedom allows the theory to be in principle viable at arbitrarily high energies. Each nonequivalent theory is characterized by a number N_g of families (also known as generations), either identical or not, limited by asymptotic freedom. In our

GTA, irreducible families are defined in two complementary ways:

- (i) *Chiral families* are made of gauge anomaly-free combinations of complex representations [29]. They emerge from several representations of $SU(N)$ with $N \geq 3$ [30], the spinorial of $SO(10 + 4k)$ with $k \in \mathbb{N}$, and the fundamental of the exceptional group E_6 . Here, multiple identical families are allowed.
- (ii) *Pseudoreal families* are made of a single representation whose (Majorana) mass is forbidden. They correspond to the spinorial of $SO(11 + 8k)$, $SO(12 + 8k)$, and $SO(13 + 8k)$ with $k \in \mathbb{N}$, some representations of $SU(N)$ and $Sp(2N)$, and the fundamental of E_7 . For pseudoreal representations of $Sp(2N)$ groups, one needs to ensure the absence of the π_4 Witten anomalies [33,34]; hence, combinations of two representations may be necessary.

Pseudoreal families only appear as a single copy, since, for an even number, mass terms can be written by pairing them. The low-energy properties of theories with pseudoreal families remain uncertain [33], and they will be studied in a forthcoming publication [35]. A complete classification of $SU(N)$ and $Sp(2N)$ families can be found in the Appendix. We remind the reader that we do not impose on the theories in the GTA to lead to the SM at low energies, contrary to GUTs. Detailed phenomenological considerations, therefore, are beyond the scope of the GTA definition, and they should be considered as part of a further step toward a proper use of the atlas.

Scalar fields may play an important role in phenomenology, as they may be required in order to break the gauge group and generate fermion masses via Yukawa couplings. Details depend on the usage of the GTA. Scalars are less constrained by the atlas criteria, as they do not contribute to anomalies and their mass is unconstrained. Furthermore, the presence of additional Yukawa and quartic couplings change the high-energy behavior of the theory. The inclusion of scalars should be considered as adding a third dimension to the atlas, which should be studied case by case depending on how the atlas is used.

In the following, we will present three complementary applications of the GTA.

II. FINDING THE STANDARD MODEL

Starting from the GTA at very high energies, one can hope to find a selection rule that gives a strong preference for a GUT model that may lead to the SM at low energies. Outside the atlas, one could also envision gravity-free, asymptotically safe GUTs, but these typically require a more involved high energy dynamics [36,37]. GUTs within phenomenological string models have also been considered [9,38,39], where a unified dynamics emerges at the string scale.

To our purpose, we need a criterion to rank the theories in the GTA. It may be tempting to count the degrees of

freedom in each family [10], defined via the number of Weyl spinors

$$n_f = \sum_{r \in f} d_r, \quad (1)$$

where d_r is the dimension of the representation r and the sum spans over the representations that define a family. In Table I, we list the first few families, ranked by increasing n_f . However, to minimize the choice of the gauge group and matter representation simultaneously, the question of the relative weight between bosons and fermions becomes essential. A simple ansatz would consist in the naive counting of degrees of freedom without relative weight. Instead, we prefer to rely on meaningful physical quantities, which are well defined in quantum field theory. One such example is based on using the free energy of the system as a way to count the degrees of freedom of the theory [40,41]. As the theories are asymptotically free at high energy, the associated free energy can be computed as the one of a relativistic gas of free particles at high temperature. In this regime, the free energy $\mathcal{F}$, normalized to the temperature, reads

$$f_{FE} = -\frac{45\mathcal{F}}{\pi^2 T^4} = d_G + \frac{7}{8} \sum_f n_g^f n_f, \quad (2)$$

where d_G is the dimension of the adjoint representation and n_g^f counts the copies of the irreducible family f

TABLE I. Models with massless families with less than 40 fermions. The multiplicities are computed for one family. For the indicated type, C means chiral (complex), while PR means pseudoreal. Values in parentheses correspond to the supersymmetric case, where allowed by asymptotic freedom. The n_g^f column is set at maximum to 1 for PR representations as only one massless generation is possible in that case.

Group	Family	Type	n_f	ξ_f	$n_g^f \leq$	f_{FE}	f_a
SU(5)	$\mathbf{10}_A + \bar{\mathbf{5}}_F$	C	15	4/55	13 (7)	37.1 (73.1)	4.36
Sp(4)	16	PR	16	12/33	1 (1)	24 (48.7)	1.97
SO(10)	$\mathbf{16}_{\text{Spin}}$	C	16	4/88	21 (11)	59 (114)	7.99
SU(6)	$\mathbf{20}_{A3}$	PR	20	6/66	1 (1)	52.5 (103)	6.33
Sp(6)	$\mathbf{6}_F + \mathbf{14}_{A3}$	PR	20	6/44	1 (1)	38.5 (76.9)	3.92
SU(3)	$\mathbf{6}_S + 7 \times \bar{\mathbf{3}}_F$	C	27	12/33	2 (1)	31.6 (65.6)	1.79
SU(3)	$\mathbf{15} + 2 \times \bar{\mathbf{6}}_S$	C	27	30/33	1 (0)	31.6(−)	1.79
SU(6)	$\mathbf{15}_A + 2 \times \bar{\mathbf{6}}_F$	C	27	6/66	10 (5)	58.6 (116)	6.44
E_6	27	C	27	1/22	21 (11)	102 (197)	13.8
SO(11)	$\mathbf{32}_{\text{Spin}}$	PR	32	8/99	1 (1)	83 (163)	9.96
SO(12)	$\mathbf{32}_{\text{Spin}}$	PR	32	8/110	1 (1)	94 (184)	11.9
SU(4)	$\mathbf{20} + \bar{\mathbf{6}}_S + \mathbf{4}_F$	C	34	38/44	1 (0)	44.8(−)	3.10

(so that $N_g \equiv \sum_f n_g^f$). For SUSY theories, the analogous quantity reads

$$f_{FE}^S = \frac{15}{8} \left(d_G + \sum_f n_g^f n_f \right). \quad (3)$$

The values of the free energy counters are listed in the left-hand columns of Table I for models of one family.

One can also envision using another scalar quantity to minimize with respect to gauge and matter content. To compare with free energy, we will also consider the degree-of-freedom count f_a arising from the a -function [42]. In the asymptotically free regime of the theories, it can be perturbatively computed and reads [43]

$$f_a = (4\pi)^2 a = \frac{124d_G + 11 \sum_f n_g^f n_f}{720}. \quad (4)$$

The above expression indicates that the function f_a increases the contribution of the gauge bosons compared to the free-energy counterpart. Note that both f_a and f_{FE} have been used in the literature to study the flow of theories from the ultraviolet to the infrared [40,44,45].

In Table I, we recognize renown examples of chiral theories: the Georgi-Glashow ones based on fundamental and two-index antisymmetric starting with SU(5) [6,40] and the Bars-Yankielowicz ones based on fundamental and two-index symmetric starting with SU(3) [40,46]. Both counters favor theories with the smallest possible gauge group, hence preferring the two Sp(4) and SU(3) theories over the SU(5) one. The same ranking occurs in the SUSY cases, with some theories already excluded by the loss of asymptotic freedom (indicated by a zero in the max n_g^f column). However, note that the same ranking may change for models with more than one generation. Asymptotic freedom plays a crucial role in defining the theory degrees of freedom. Furthermore, if lost, it would deem the theory effective, as some couplings would diverge at a high scale. To search for the asymptotically free theories, one requires the first coefficient of the one-loop beta function to be negative. This implies

$$b_{1\text{-loop}} = -\frac{11}{3} C_G \left(1 - \sum_f n_g^f \xi_f \right) < 0, \quad (5)$$

with

$$\xi_f = \frac{2}{11 C_G} \sum_{r \in f} T_r, \quad (6)$$

where T_r is the trace normalization of the representation r and C_G is the quadratic Casimir of the adjoint. This allows us to define the maximal multiplicity for complex families

$$n_g^f < 1/\xi_f. \quad (7)$$

For SUSY theories,

$$b_{1\text{-loop}}^S = -3C_G \left(1 - \frac{11}{6} \sum_f n_g^f \xi_f \right), \quad (8)$$

hence reducing the number of allowed families, $n_g^f < 6/(11\xi_f)$.

It is evident that minimizing either f_{FE} or f_a can only lead to the trivial minimal number of families, i.e., $N_g = 0$ (a pure Yang-Mills theory). So, for now, we will consider N_g as an input and compare theories with the same number of families. In Table II, we list all the theories with $N_g = 3$ identical chiral families listed for increasing f_{FE} (up to $f_{FE} < 200$). As explained above, they all correspond to complex representations. The model with minimal free energy is the SU(5) one of Georgi-Glashow [6] with $f_{FE} = 63.4$, closely followed by SO(10) [7] with $f_{FE} = 87$. Note also that the only model in the list that cannot contain the SM gauge symmetry is based on SU(4). The same ranking is valid in the SUSY case, where the SU(4) model is excluded for loss of asymptotic freedom. As expected, f_a gives larger weights to the gauge part, so the SU(5) model is only surpassed by SU(4), which has a very similar score.

For completeness, we also considered the case in which the three families consist of different representations. There are many such examples, with smaller free energy achieved when only one generation is different from the minimal case. We list the most minimal cases in Table III; all these models, however, have f_{FE} scores higher than the minimal SU(5) and SO(10) theories.

Our first finding, therefore, is that the free energy allows us to select the canonical SU(5) and SO(10) GUTs as the preferred models *with three generations*. So far, we have

TABLE II. Models with $N_g = 3$ identical generations and $f_{FE} < 200$. The models are listed with increasing free energy f_{FE} . The SU(N) representations are the fundamental (F), the two-index symmetric (S), and the two-index antisymmetric (A) and three-index (A_3) antisymmetric.

Group	Family	n_f	ξ_f	f_{FE}	f_a
SU(5)	$\mathbf{10}_A + \bar{\mathbf{5}}_F$	15	4/55	63.4 (129)	4.82
SO(10)	$\mathbf{16}_{\text{Spin}}$	16	4/88	87 (174)	8.48
SU(6)	$\mathbf{15}_A + 2 \times \bar{\mathbf{6}}_F$	27	6/66	106 (218)	7.27
SU(4)	$\mathbf{10}_S + 8 \times \bar{\mathbf{4}}_F$	42	14/44	125 (–)	4.51
E_6	27	27	6/132	149 (298)	14.7
SU(7)	$\mathbf{21}_A + 3 \times \bar{\mathbf{7}}_F$	42	8/77	158 (326)	10.2
SU(7)	$\mathbf{35}_{A_3} + 2 \times \bar{\mathbf{7}}_F$	49	12/77	177 (366)	10.5
SU(5)	$\mathbf{15}_S + 9 \times \bar{\mathbf{5}}_F$	60	16/55	181 (–)	6.88

TABLE III. SU(5) models with two generations of $\mathbf{10}_A + \bar{\mathbf{5}}_F$ and a third family of different type. We list all cases with $f_{FE} < 120$ for illustration.

Group	3rd family	$n_{f,3}$	$\xi_{f,3}$	f_{FE}	f_a
SU(5)	$\mathbf{10}_A + \bar{\mathbf{5}}_F$	15	4/55	63.4 (129)	4.82
SU(5)	$\mathbf{15}_S + 9 \times \bar{\mathbf{5}}_F$	60	16/55	103 (214)	5.51
SU(5)	$\mathbf{45} + 6 \times \bar{\mathbf{5}}_F$	75	30/55	116 (–)	5.74

not discussed the scalar sector, which is crucial for breaking the GUT symmetry and generating appropriate Yukawa couplings. In our framework, it would be ideal to require that the additional couplings are also asymptotically free, hence imposing nontrivial constraints on the model, which we leave for future investigation.

III. SWAMPLAND OF GAUGE DUALITIES

The traditional approach to GUTs starts from the SM at low energies and aims at finding a minimal GUT that leads to the breaking of the unified gauge symmetry. However, the gauge structure of the SM may be modified before the GUT scale is reached. For example, partial unification of leptons and quarks [5] could occur at energies as low as 10^6 GeV. A complete classification of possible gauge extensions of the SM has been presented in Ref. [47].

Another possibility is offered by gauge-gauge dualities, where one of the SM gauge groups emerges as the effective infrared limit of a more fundamental ultraviolet theory. A concrete example was first provided for $\mathcal{N} = 1$ SUSY SU(N) theories [48,49] and extended to non-SUSY theories in Refs. [50–52] due to the marginal role played by scalars. Such dualities can be applied straightforwardly to the SU(3) gauge of the SM [53] (for the SUSY case, see Refs. [54,55]). An interesting feature of this ultraviolet completion is that the electroweak symmetry breaking is due to composite scalars at low energies and a multi-Higgs model naturally emerges, with features similar to scalar democracy [56]. Furthermore, the number of colors in the dual theory is linked to the number of generations [51] in the SM: at high energies, the dual gauge symmetry is SU($2N_g - 3$), where the duality is only possible for $N_g = 3$ [SU(3)], $N_g = 4$ [SU(5)], and $N_g = 5$ [SU(7)]. Does the GTA allow us to select one high-energy theory out of these three, hence fixing the number of generations in the SM?

The fermionic matter content of the dual SM is listed in Table IV, where $X = 2N_g - 3$. For $N_g = 3$, this is exactly equivalent to a scalarless SM. The free energy considerations from Table II already suggest that the $N_g = 3$ case is to be preferred, as it can be embedded into the minimal- f_{FE} SU(5) model with three generations. The straightforward extension to $N_g = 4, 5$ generations would require to consider GUTs based on SU(7) and SU(9), respectively; however, no model in the GTA contains the chiral matter

TABLE IV. Fermion content of the high-energy dual SM, where $X = 2N_g - 3$ and $N_g = 3, 4, 5$ is the number of allowed fermion generations. Hypercharge is normalized to give electric charge $Q = T_L^3 + Y$.

	SU(X)	SU(2) _L	U(1) _Y
Q_L	F	2	$1/(2X)$
U_R^c	$\bar{F}$	1	$-1/2 - 1/(2X)$
D_R^c	$\bar{F}$	1	$1/2 - 1/(2X)$
l_L	1	2	$-1/2$
e_R^c	1	1	1

content in Table IV. This is due to a special feature of $X = 3$ [57], namely, the fact that a two-index antisymmetric representation is equal to the antifundamental of SU(3), $A \equiv \bar{F}$. This feature allows embedding of Q_L , T_R^c , and e_R^c in the same 10 of SU(5). This embedding is not extendable to SU(7) nor SU(9).

Similar considerations apply to the SO(10) case, which could be extended to SO(14) and SO(18), respectively, for $N_g = 4, 5$. The embedding of the $X = 3$, $N_g = 3$, case in SO(10) is based on the fact that $\text{SO}(10) \supset \text{SO}(6) \times \text{SO}(4)$, where $\text{SO}(6) \sim \text{SU}(4)$ is equivalent to the fourth color gauge structure [5]. Also, $\text{SO}(10) \supset \text{SU}(5)$, hence containing the Georgi-Glashow model. Such considerations and embeddings cannot be extended to SO(14) and SO(18). Finally, $E_6 \supset \text{SU}(5) \times \text{SU}(2) \times \text{U}(1)$ does not contain the fermion spectrum in Table IV. Hence, we find that the dual SM with 4 and 5 generations is not in the GTA dryland. Note that the dual theory, which emerges from the GTA, does not contain any Yukawa couplings: they are generated by the duality itself at low energies [53]. Hence, a realistic dual GUT would only require a single scalar field at high scale for the breaking of the gauge symmetry down to the dual SM one.

IV. WHICH GAUGE EXTENSION?

The GTA dryland can also be used to constrain possible extensions of the SM gauge group at low energies. This is common practice in model building: for instance, in replacing the Higgs sector with a strongly coupled gauge interaction [58] or in including a dark sector to explain dark matter [59,60]. In both cases, the extended gauge group has a factorized form $\mathcal{G}_{\text{SM}} \times \mathcal{G}_{\text{BSM}}$. What extensions of this type are in the dryland of the GTA?

As an example, we analyzed models consisting of $\text{SU}(5) \times \text{SU}(N) \subset \text{SU}(N+5)$, where the SM is contained in the SU(5) gauge part. The minimal requirement is $N \geq 2$ in order to obtain a non-Abelian BSM (beyond the standard model) sector that confines at low energies. The $\text{SU}(N+5)$ chiral families should contain three generations of Georgi-Glashow multiplets, i.e., $\mathbf{10} + \bar{\mathbf{5}}$, plus states charged under SU(N). After analyzing all chiral families for $N \geq 2$, we

found a unique viable model. It is based on SU(8), with one family of

$$\bar{\mathbf{56}}_{A3} + 2 \times \mathbf{28}_A + 3 \times \bar{\mathbf{8}}_F, \quad (9)$$

which has $n_f = 136$ and $\xi_f = 30/88$. Under $\text{SU}(8) \rightarrow \text{SU}(5) \times \text{SU}(3) \times \text{U}(1)$, each representation decomposes as

$$\bar{\mathbf{56}} \rightarrow (10, 1)_{-9} \oplus (\bar{10}, \bar{3})_{-1} \oplus (\bar{5}, 3)_7 \oplus (1, 1)_{15}, \quad (10)$$

$$\mathbf{28} \rightarrow (10, 1)_6 \oplus (5, 3)_{-2} \oplus (1, \bar{3})_{-10}, \quad (11)$$

$$\bar{\mathbf{8}} \rightarrow (\bar{5}, 1)_{-3} \oplus (1, \bar{3})_5. \quad (12)$$

Taking into account the multiplicities in the SU(8) family, we note the presence of three SU(5) families plus one singlet, and various fields charged under SU(3):

$$F \Rightarrow (\bar{5}, 3)_7 + 2 \times (5, 3)_{-2}, \quad (13)$$

$$\bar{F} \Rightarrow (\bar{10}, \bar{3})_1 + 2 \times (1, \bar{3})_{-10} + 3 \times (1, \bar{3})_5. \quad (14)$$

If this SU(3) gauge interaction generated confinement and chiral symmetry breaking at low energies, the mesons will contain scalars that transform as 5 and 45 of SU(5), where both contain a Higgs component. If a vacuum expectation value is generated for such mesons, they could be identified with the source of electroweak symmetry breaking, and hence play the role of the SM Higgs. Naturally, the breaking of $\text{SU}(8) \rightarrow \text{SU}(5) \times \text{SU}(3) \rightarrow \mathcal{G}_{\text{SM}} \times \text{SU}(3)$ must be obtained via (heavy) elementary scalars, while the confinement of SU(3) must occur not far from the electroweak scale. Baryons that transform as one family of SM fermions are also generated, and hence they could play the role of partial compositeness partners for the third generation of quarks (and leptons), as in Ref. [61].

V. CONCLUSIONS

In this article, we proposed an atlas of asymptotically free gauge theories, based on a simple unified gauge factor, with massless families of fermions. The free energy at high temperatures can be used as a counter for the degrees of freedom of each theory, thus allowing us to chart the atlas. By requiring three families of fermions, the free energy naturally selects the SU(5) Georgi-Glashow GUT as the most minimal theory, closely followed by the canonical SO(10) GUT. Hence, the SM may have a natural position within the GTA. The atlas can also be used to check if gauge extensions of the SM allow for a grand unified extension at high energies. We illustrated this in two examples, one based on a gauge-gauge duality on the chromomagnetic interactions and the other based on an SU(N) extension of the standard gauge symmetry. In the former case, the GTA yields a strong preference for three families.

We have not considered yet scalar fields, as their mass cannot be prevented by any symmetry. Including spontaneous symmetry breaking of the gauge symmetry and generation of Yukawa couplings could imprint further constraints on the atlas, providing a phenomenological compass to navigate us toward the optimal high-energy theory. In our analysis, asymptotic freedom plays a crucial role in counting the degrees of freedom of each theory.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX

1. $\text{Sp}(2N)$ pseudoreal families and the Witten anomalies

The well-known $\text{SU}(2)$ Witten anomaly [33] admits a natural generalization to the symplectic groups $\text{Sp}(2n)$, as Witten's original argument relies on the nontriviality of the fourth homotopy group π_4 . The family of Lie groups $\text{Sp}(2n)$ is the only one for which $\pi_4 = \mathbb{Z}_2 \neq 0$. Furthermore, symplectic groups contain pseudoreal irreducible representations, which can be used to define theories within the GTA.

In this context, the candidate carriers of the anomaly are Weyl fermions in pseudoreal representations. The group $\text{Sp}(2n)$ admits only real or pseudoreal representations, due to the existence of an invariant symplectic form. Weyl fermions in real representations can be given gauge-invariant mass terms, allowing for a Pauli-Villars regularization scheme and thus avoiding any anomaly. In contrast, pseudoreal representations do not admit such mass terms.

The pseudoreal representations of $\text{Sp}(2n)$ are precisely those that appear in the odd tensor powers of the defining representation $\mathbf{F} \equiv 2n$ [62]. In Table V, we list all the pseudo-real representations that leave the gauge coupling asymptotically free (AF). We use the Dynkin labels, in the second column, to identify the irreducible representations.

Among these, only those with a *half-integer Dynkin index* contribute nontrivially to the Witten anomaly, assuming the normalization $T(2n) = 1/2$ [63]. Besides the fundamental, which always carries the Witten anomaly [33], the list contains three-index representations. For every $\text{Sp}(2n)$ group, either the symmetric or the antisymmetric carries the anomaly depending on n being even (symmetric) or odd (antisymmetric). Instead, the mixed representation $(1, 1, \dots)$ always has integer Dynkin index. Finally, the five-index antisymmetric is only relevant for $\text{Sp}(10)$, the first group for which it is defined, as it loses AF for larger groups.

In the presence of multiple fermion species, the anomaly is given by the sum of their Dynkin indices [34]. Accordingly, the anomaly can be canceled by including an even number of representations that are individually anomalous. If those representations are different, mass terms remain forbidden, hence allowing us to define anomaly-free pseudoreal families. Taking these considerations into account (along with asymptotic freedom descending from the GTA conditions), we construct Table VI.

Some of these models can be excluded by a second $\text{SU}(2)$ anomaly, discovered in Ref. [34]. This anomaly typically applies to three-index symmetric representations. However, it is based on the assumption that all fermion fields are in odd tensor powers of the defining representation (pseudoreal), while all bosons are in even tensors (real). This hypothesis could be invalidated by the presence of massive states, such as fermions in real representations. Nevertheless, in the main text, we have excluded theories that carry this anomaly.

2. Chiral $\text{SU}(N)$ families

Complex (chiral) families are constructed by complex representations, of which $\text{SU}(n)$ groups are rich. The only two constraints are given by the cancellation of the gauge anomalies and by asymptotic freedom. A list of all complex and AF representations is presented in Table VII, adapted from Ref. [64]. The AF limit on n in the next-to-last column is derived by counting the contribution of the representation alone. In practice, when constructing a chiral family, additional representations must be added to cancel the

TABLE V. List of $\text{Sp}(2n)$ irreducible pseudoreal representations that respect asymptotic freedom. They are labeled by their Dynkin label, dimension, and Dynkin index T_r . Conditions on their contribution to the Witten anomaly and asymptotic freedom are provided in the last two columns.

Name	Dynkin label	Dimension	$2T_r$	Anomaly?	AF
F	$(1, \dots)$	$2n$	1	Yes	Always
A3	$(0, 0, 1, \dots)$	$\frac{2n}{3}(2n+1)(n-2)$	$(n-1)(2n-3)-1$	n odd	$3 \leq n \leq 8$
A5	$(0, 0, 0, 0, 1, \dots)$	$\frac{n}{15}(n-4)(n-1)(4n^2-1)$	$\frac{1}{6}(n-1)(n-4)(2n-3)(2n-1)$	No	$n = 5$
S3	$(3, \dots)$	$\frac{2n}{3}(2n+1)(n+1)$	$(n+1)(2n+3)$	n even	$1 \leq n \leq 3$
R11	$(1, 1, \dots)$	$\frac{8}{3}n(n^2-1)$	$4(n^2-1)$	Never	$2 \leq n \leq 3$

TABLE VI. Complete list of GTA models based on $\text{Sp}(2N)$ gauge theories with pseudoreal families free of the Witten anomaly. The last column indicates if the model suffers from the Wang-Wen-Witten (WWW) anomaly [34], associated with the three-index symmetric representation.

Group	Family	n_f	ξ_f	$n_g^f \leq$	f_{FE}	f_a	WWW anomaly?
$\text{Sp}(2)$	$\mathbf{4}_{S3}$	4	14/22	1 (0)	6.5(−)	0.58	Yes
$\text{Sp}(4)$	$\mathbf{16}_{R11}$	16	12/33	1 (1)	24 (48.7)	1.97	No
$\text{Sp}(4)$	$\mathbf{4}_F + \mathbf{20}_{S3}$	24	22/33	1 (0)	31(−)	2.09	Yes
$\text{Sp}(6)$	$\mathbf{6}_F + \mathbf{14}_{A3}$	20	6/44	1 (1)	38.5 (76.9)	3.92	No
$\text{Sp}(6)$	$\mathbf{56}_{S3}$	56	36/44	1 (0)	70(−)	4.47	Yes
$\text{Sp}(6)$	$\mathbf{64}_{R11}$	64	32/44	1 (0)	77(−)	4.59	No
$\text{Sp}(8)$	$\mathbf{48}_{A3}$	48	14/55	1 (1)	78 (158)	6.93	No
$\text{Sp}(10)$	$\mathbf{10}_F + \mathbf{110}_{A3}$	120	28/66	1 (1)	160 (328)	11.3	No
$\text{Sp}(10)$	$\mathbf{132}_{A5}$	132	42/66	1 (0)	170 (328)	11.5	No
$\text{Sp}(12)$	$\mathbf{208}_{A3}$	208	44/88	1 (1)	260 (536)	16.6	No
$\text{Sp}(14)$	$\mathbf{14}_F + \mathbf{350}_{A3}$	364	66/99	1(−)	432 (−)	23.6	No
$\text{Sp}(16)$	$\mathbf{544}_{A3}$	544	90/110	1(−)	612(−)	31.7	No

gauge anomaly. This does not affect the representations up to two tensor indices, $\mathbf{F}$, $\mathbf{A2}$, and $\mathbf{S2}$. For higher rank representation, however, this effects is crucial. To estimate a conservative range, we use the fact that $\mathbf{S2}$ is the most efficient representation, in the sense that it provides the smallest Dynkin index per unit of anomaly, and hence we used the inequality

$$2T_r + \left| \frac{\mathcal{A}_r}{\mathcal{A}_{S2}} \right| 2T_{S2} < 11n \quad (\text{A1})$$

to obtain the largest n allowing for a AF chiral family containing the representation r . The result is listed in the last column of Table VII.

The explicit construction of the AF chiral families crucially depends on the rank of the $\text{SU}(n)$ group and, consequently, on the number of involved representations. As the representations of rank less than 2, $\mathbf{F}$, $\mathbf{A2}$, and $\mathbf{S2}$, are always present for every n , it is first convenient to build all irreducible families made of these three. The result crucially depends on n . Nevertheless, one can identify three families that are always present for every n :

$$\mathbf{A2} + (n-4)\bar{\mathbf{F}} \quad \text{for } n \geq 5, \quad (\text{A2})$$

$$\mathbf{S2} + (n+4)\bar{\mathbf{F}} \quad \text{for } n \geq 3, \quad (\text{A3})$$

$$\mathbf{S2} + \overline{\mathbf{A2}} + 8\bar{\mathbf{F}} \quad \text{for } n \geq 5. \quad (\text{A4})$$

TABLE VII. List of $\text{SU}(n)$ irreducible complex representations that respect asymptotic freedom. They are labeled by their Dynkin label, dimension, and Dynkin index T_r . In the last three columns, we indicate the contribution to the gauge anomaly and the conditions for asymptotic freedom. Note that $\mathbf{Ra} \equiv \mathbf{R13}$ for $\text{SU}(5)$.

Name	Dynkin label	Dimension	$2T_r$	Anomaly $\mathcal{A}_r$	AF	AF in family
$\mathbf{F}$	(1, ...)	n	1	1	$n \geq 3$	All
$\mathbf{A2}$	(0, 1, ...)	$\frac{1}{2}n(n-1)$	$(n-2)$	$n-4$	$n \geq 5$	All
$\mathbf{S2}$	(2, ...)	$\frac{1}{2}n(n+1)$	$(n+2)$	$n+4$	$n \geq 3$	All
$\mathbf{A3}$	(0, 0, 1, ...)	$\frac{1}{6}n(n-1)(n-2)$	$\frac{1}{2}(n-2)(n-3)$	$\frac{1}{2}(n-3)(n-6)$	$7 \leq n \leq 26$	$7 \leq n \leq 17$
$\mathbf{A4}$	(0, 0, 0, 1, ...)	$\frac{1}{24}n(n-1)(n-2)(n-3)$	$\frac{1}{6}(n-2)(n-3)(n-4)$	$\frac{1}{6}(n-3)(n-4)(n-8)$	$9 \leq n \leq 12$	$9 \leq n \leq 11$
$\mathbf{S3}$	(3, ...)	$\frac{1}{6}n(n+1)(n+2)$	$\frac{1}{2}(n+2)(n+3)$	$\frac{1}{2}(n+3)(n+6)$	$3 \leq n \leq 16$	Never
$\mathbf{R11}$	(1, 1, ...)	$\frac{1}{3}n(n^2-1)$	(n^2-3)	(n^2-9)	$4 \leq n \leq 11$	$4 \leq n \leq 6$
$\mathbf{R22}$	(0, 2, ...)	$\frac{1}{12}n^2(n^2-1)$	$\frac{1}{3}n(n^2-4)$	$\frac{1}{3}n(n^2-16)$	$5 \leq n \leq 6$	$n=5$
$\mathbf{Ra}$	(1, 0, ..., 1, 0)	$\frac{1}{2}n(n+1)(n-2)$	$\frac{1}{2}(n-2)(3n+1)$	$\frac{1}{2}(-n^2+7n+2)$	$5 \leq n \leq 9$	$5 \leq n \leq 8$
$\mathbf{R13}$	(1, 0, 1, ...)	$\frac{1}{8}n(n^2-1)(n-2)$	$\frac{1}{2}(n-2)(n^2-n-4)$	$\frac{1}{2}(n-4)(n^2-n-8)$	$n=6$	Never
$\mathbf{Rb}$	(2, ..., 1)	$\frac{1}{2}n(n-1)(n+2)$	$\frac{1}{2}(n+2)(3n-1)$	$\frac{1}{2}(n^2+7n-2)$	$3 \leq n \leq 5$	$n=3$

The first corresponds to a generalization of the Georgi-Glashow model [6], while the second class was first identified by Bars and Yankielowicz [46]. For every n , other irreducible combinations are also allowed, which crucially depend on n . On the other hand, for $n \geq 18$, only these three representations are allowed by AF, leading to classifiable families.

In the following, we will present the complete list of families for all $SU(n)$ group, starting from the smallest.

3. $SU(3)$ and $SU(4)$

The $SU(3)$ and $SU(4)$ are the smallest groups with complex representations, and they are characterized by the absence of the $\mathbf{A2}$ representation [which is equivalent to the $\bar{\mathbf{F}}$ for $SU(3)$ and real for $SU(4)$]. For both, only three representations allow to build chiral families; cf. Tables VIII and IX, top.

There are only three irreducible anomaly-free combinations that preserve asymptotic freedom in $SU(3)$, and they

TABLE VIII. Complete list of relevant representations (top table) and chiral families (bottom table) for $SU(3)$. In the top table, the representations are labeled by their dimension. The maximum number of generations is shown in the last column, with the supersymmetric case in parentheses.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
$\mathbf{F}$	3	1	1
$\mathbf{S2}$	6	5	7
$\mathbf{Rb}$	15	20	14
$SU(3)$ family	n_f	$33\xi_f$	$n_g^f \leq$
$6_{S2} + 7\bar{3}_F$	27	12	2 (1)
$15_{Rb} + \bar{6}_{S2} + 7\bar{3}_F$	42	32	1 (0)
$15_{Rb} + 2\bar{6}_{S2}$	27	30	1 (0)

TABLE IX. Complete list of relevant representations (top table) and chiral families (bottom table) for $SU(4)$. In the top table, the representations are labeled by their dimension. The maximum number of generations is shown in the last column, with the supersymmetric case in parentheses.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
$\mathbf{F}$	4	1	1
$\mathbf{S2}$	10	6	8
$\mathbf{R11}$	20	13	7
$SU(4)$ family	n_f	$44\xi_f$	$n_g^f \leq$
$10_{S2} + 8\bar{4}_F$	42	14	3 (1)
$20_{R11} + 7\bar{4}_F$	48	20	2 (1)
$20_{R11} + \bar{10}_{S2} + 4_F$	34	20	2 (1)
$2\ 20_{R11} + \bar{10}_{S2} + 6\bar{4}_F$	74	38	1 (0)

are listed in the Table VIII bottom. For $SU(4)$, we found four combinations, as listed in the Table IX bottom.

4. $SU(5)$

The group $SU(5)$ is the smallest one that features a complex nontrivial $\mathbf{A2}$ representation. In total, there are six representation that can be used to build chiral families; cf. Table X top. The general strategy for $SU(n)$ groups with $n \geq 5$ is to first construct all irreducible families made of the three lowest-rank representations: $\mathbf{F}$, $\mathbf{A2}$, and $\mathbf{S2}$. The number and structure of the independent families depend crucially on the contribution to the anomaly of the two rank-2 representations. For $SU(5)$, as $\mathcal{A}_{A2} = \mathcal{A}_F = 1$, the fundamental and antisymmetric are interchangeable. This implies that, besides the Georgi-Glashow and Bars-Yankielowicz families, one can build a class of families made of $\mathbf{S2} + x\bar{\mathbf{A2}} + (9-x)\bar{\mathbf{F}}$, as listed in the top block of Table X bottom.

The second step consists in ranking the remaining higher-order representations by increasing $\mathcal{A}_r$; cf. the lower block of Table X top. The families are then constructed iteratively, by first using the $\bar{\mathbf{F}}$ to cancel the anomaly and then substituting the needed number of fundamentals with the other representations. For instance, taking the $\mathbf{Ra}$ representation, with $\mathcal{A}_{Ra} = 6$, we first build $\mathbf{Ra} + 6\bar{\mathbf{F}}$; then, one can substitute $\mathbf{F} \rightarrow \mathbf{A2}$ to construct the class $\mathbf{Ra} + x\bar{\mathbf{A2}} + (6-x)\bar{\mathbf{F}}$. Then, one substitutes $9\mathbf{F} \rightarrow \mathbf{S2}$: as $\mathcal{A}_{Ra} < \mathcal{A}_{S2}$, the anomaly mismatch can be compensated as $\mathbf{Ra} + \bar{\mathbf{S2}} + 3\mathbf{F}$. Finally, the substitution $\mathbf{F} \rightarrow \mathbf{A2}$ is performed again. This leads to the families in the second block of Table X bottom. This step can be repeated for the other two representations, $\mathbf{R22}$ and $\mathbf{R11}$. Finally, one can consider combinations a starting point, where the only usable one turns out to be $\mathbf{R11} + \bar{\mathbf{R22}}$. The complete list of 52 independent families is reported in Table X bottom.

5. Large n families

For large n , namely, $n \geq 18$, only three representations can be used to define chiral families: $\mathbf{F}$, $\mathbf{A2}$, and $\mathbf{S2}$. This allows us to classify all possible irreducible and anomaly-free combinations, as listed in Table XI. In fact, this pattern also continues for $n = 16, 17$, with the only difference that additional families can be added, containing one $\mathbf{A3}$ representation (see Tables XXII and XXIII).

For $n \leq 15$, the patterns change, and more representations must be considered; hence, the construction of chiral families must proceed case by case.

6. $SU(6)$ to $SU(17)$

We complete the classification by presenting the complete list of chiral families for $SU(6)$ in Table XII, $SU(7)$ in Table XIII, $SU(8)$ in Table XIV, $SU(9)$ in Table XV, $SU(10)$ in Table XVI, $SU(11)$ in Table XVII, $SU(12)$ in

TABLE X. Complete list of chiral families (top table) for SU(5), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	5	1	1
A2	10	3	1
S2	15	7	9
Ra	45	24	6
R22	50	35	15
R11	40	22	16

SU(5) family	n_f	$55\xi_f$	$n_g^f \leq$	Condition
$\mathbf{10}_{A_2} + \bar{\mathbf{5}}_F$	15	4	13 (7)	
$\mathbf{15}_{S_2} + 9\bar{\mathbf{5}}_F$	60	16	3 (1)	
$\mathbf{15}_{S_2} + \mathbf{10}_{A_2} + 8\bar{\mathbf{5}}_F$	65	18	3 (1)	
$\mathbf{15}_{S_2} + x\mathbf{10}_{A_2} + (9-x)\bar{\mathbf{5}}_F$	$60 + 5x$	$(16 + 2x)$	2 (1)	$2 \leq x \leq 5$
$\mathbf{15}_{S_2} + x\mathbf{10}_{A_2} + (9-x)\bar{\mathbf{5}}_F$	$60 + 5x$	$(16 + 2x)$	1 (1)	$x = 6$
$\mathbf{15}_{S_2} + x\mathbf{10}_{A_2} + (9-x)\bar{\mathbf{5}}_F$	$60 + 5x$	$(16 + 2x)$	1 (0)	$7 \leq x \leq 9$
$\mathbf{45}_{Ra} + x\mathbf{10}_{A_2} + (6-x)\bar{\mathbf{5}}_F$	$75 + 5x$	$(30 + 2x)$	1 (0)	$0 \leq x \leq 6$
$\mathbf{45}_{Ra} + \mathbf{15}_{S_2} + x\mathbf{10}_{A_2} + (3-x)\mathbf{5}_F$	$75 + 5x$	$(34 + 2x)$	1 (0)	$0 \leq x \leq 3$
$\mathbf{50}_{R22} + x\mathbf{10}_{A_2} + (15-x)\bar{\mathbf{5}}_F$	$125 + 5x$	$(50 + 2x)$	1 (0)	$0 \leq x \leq 2$
$\mathbf{50}_{R22} + \mathbf{15}_{S_2} + x\mathbf{10}_{A_2} + (6-x)\bar{\mathbf{5}}_F$	$95 + 5x$	$(48 + 2x)$	1 (0)	$0 \leq x \leq 3$
$\mathbf{50}_{R22} + 2\mathbf{15}_{S_2} + 3\mathbf{5}_F$	95	52	1 (0)	
$\mathbf{50}_{R22} + 2\mathbf{15}_{S_2} + \mathbf{10}_{A_2} + 2\mathbf{5}_F$	100	54	1 (0)	
$\mathbf{40}_{R11} + x\mathbf{10}_{A_2} + (16-x)\bar{\mathbf{5}}_F$	$120 + 5x$	$(38 + 2x)$	1 (0)	$0 \leq x \leq 8$
$\mathbf{40}_{R11} + \mathbf{15}_{S_2} + x\mathbf{10}_{A_2} + (7-x)\bar{\mathbf{5}}_F$	$90 + 5x$	$(36 + 2x)$	1 (0)	$0 \leq x \leq 7$
$\mathbf{40}_{R11} + 2\mathbf{15}_{S_2} + 2\mathbf{5}_F$	80	38	1 (0)	
$\mathbf{40}_{R11} + 2\mathbf{15}_{S_2} + \mathbf{10}_{A_2} + \mathbf{5}_F$	85	40	1 (0)	
$\mathbf{40}_{R11} + 2\mathbf{15}_{S_2} + 2\mathbf{10}_{A_2}$	90	42	1 (0)	
$\mathbf{40}_{R11} + \mathbf{45}_{Ra} + \mathbf{15}_{S_2} + \bar{\mathbf{5}}_F$	105	54	1 (0)	

TABLE XI. Complete list of chiral SU(n) families for $n \geq 16$ (for $n = 16, 17$, additional family exists with one A_3); see Tables XXII and XXIII.

Family	n_f	$11n\xi_f$	$n_g^f <$	
$\mathbf{A2} + (n-4)\bar{\mathbf{F}}$	$\frac{3}{2}n(n-3)$	$2(n-3)$	$\frac{11n}{2(n-3)}$	$n \geq 5$
$\mathbf{S2} + (n+4)\bar{\mathbf{F}}$	$\frac{3}{2}n(n+3)$	$2(n+3)$	$\frac{11n}{2(n+3)}$	$n \geq 3$
$\mathbf{S2} + \mathbf{A2} + 8\bar{\mathbf{F}}$	$n(n+8)$	$2(n+4)$	$\frac{11n}{2(n+4)}$	$n \geq 5$
$\mathbf{S2} + 2\mathbf{A2} + (n-12)\mathbf{F}$	$\frac{5}{2}n(n-5)$	$2(2n+7)$	$\frac{11n}{2(2n+7)}$	$n \geq 16$
$2\mathbf{S2} + 3\mathbf{A2} + (20-n)\bar{\mathbf{F}}$	$\frac{3}{2}n(n+13)$	$2(2n+9)$	$\frac{11n}{2(2n+9)}$	$16 \leq n \leq 20$
$2\mathbf{S2} + 3\mathbf{A2} + (n-20)\mathbf{F}$	$\frac{1}{2}n(7n-41)$	$2(3n-11)$	$\frac{11n}{2(3n-11)}$	$n \geq 20$
$3\mathbf{S2} + 4\mathbf{A2} + (28-n)\bar{\mathbf{F}}$	$\frac{5}{2}n(n+11)$	$2(3n+13)$	$\frac{11n}{2(3n+12)}$	$21 \leq n \leq 28$
$3\mathbf{S2} + 4\mathbf{A2} + (n-28)\mathbf{F}$	$\frac{3}{2}n(3n-19)$	$2(4n-15)$	$\frac{11n}{2(4n-15)}$	$n \geq 28$
$4\mathbf{S2} + 5\mathbf{A2} + (36-n)\bar{\mathbf{F}}$	$\frac{1}{2}n(7n+71)$	$2(4n+17)$	$\frac{11n}{2(4n+17)}$	$29 \leq n \leq 36$
$4\mathbf{S2} + 5\mathbf{A2} + (n-36)\mathbf{F}$	$\frac{1}{2}n(11n-73)$	$2(5n-19)$	$\frac{11n}{2(5n-19)}$	$n \geq 36$
$3\mathbf{S2} + 5\mathbf{A2} + (2n-32)\mathbf{F}$	$3n(2n-11)$	$2(5n-18)$	$\frac{11n}{2(5n-18)}$	$16 \leq n \leq 19$

TABLE XII. Complete list of chiral families (top table) for SU(6) with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	6	1	1
A2	15	4	2
S2	21	8	10
Ra	84	38	4
R11	70	33	27

SU(6) family	n_f	$66\xi_f$	$n_g^f \leq$	Condition
15 _{A2} + 2 $\bar{6}_F$	27	6	10 (5)	
21 _{S2} + $x\bar{15}$ _{A2} + (10 - 2x) $\bar{6}_F$	81 + 3x	(18 + 2x)	3 (1)	0 ≤ x ≤ 1
21 _{S2} + $x\bar{15}$ _{A2} + (10 - 2x) $\bar{6}_F$	81 + 3x	(18 + 2x)	2 (1)	2 ≤ x ≤ 5
84 _{Ra} + $x\bar{15}$ _{A2} + (4 - 2x) $\bar{6}_F$	108 + 3x	(42 + 2x)	1 (0)	0 ≤ x ≤ 2
84 _{Ra} + 21 _{S2} + $x15$ _{A2} + (6 - 2x) $\bar{6}_F$	141 + 3x	(52 + 2x)	1 (0)	0 ≤ x ≤ 3
70 _{R11} + $x\bar{15}$ _{A2} + (27 - 2x) $\bar{6}_F$	232 + 3x	(60 + 2x)	1 (0)	0 ≤ x ≤ 2
70 _{R11} + 21 _{S2} + $x\bar{15}$ _{A2} + (17 - 2x) $\bar{6}_F$	193 + 3x	(58 + 2x)	1 (0)	0 ≤ x ≤ 3
70 _{R11} + 221 _{S2} + $x\bar{15}$ _{A2} + (7 - 2x) $\bar{6}_F$	154 + 3x	(56 + 2x)	1 (0)	0 ≤ x ≤ 3
70 _{R11} + 321 _{S2} + 36 _F	151	60	1 (0)	
70 _{R11} + 321 _{S2} + 15 _{A2} + 6 _F	154	62	1 (0)	

TABLE XIII. Complete list of chiral families (bottom table) for SU(7), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	7	1	1
A2	21	5	3
S2	28	9	11
Ra	140	55	1
A3	35	10	2

SU(7) family	n_f	$77\xi_f$	$n_g^f \leq$	Condition
21 _{A2} + 3 $\bar{7}_F$	42	8	9 (5)	
28 _{S2} + 11 $\bar{7}_F$	105	20	3 (2)	
28 _{S2} + $x\bar{21}$ _{A2} + (11 - 3x) $\bar{7}_F$	105	(20 + 2x)	3 (1)	1 ≤ x ≤ 2
28 _{S2} + $x\bar{21}$ _{A2} + (11 - 3x) $\bar{7}_F$	105	(20 + 2x)	2 (1)	x = 3
28 _{S2} + 421 _{A2} + 7 _F	119	30	2 (1)	
228 _{S2} + 721 _{A2} + $\bar{7}_F$	210	54	1 (0)	
140 _{Ra} + $\bar{7}_F$	147	56	1 (0)	
140 _{Ra} + 21 _{A2} + 27 _F	175	62	1 (0)	
140 _{Ra} + 28 _{S2} + $x21$ _{A2} + (10 - 3x) $\bar{7}_F$	238	(74 + 2x)	1 (0)	0 ≤ x ≤ 1
35 _{A3} + 2 $\bar{7}_F$	49	12	6 (3)	
35 _{A3} + 21 _{A2} + 7 _F	63	16	4 (2)	
35 _{A3} + 28 _{S2} + $x21$ _{A2} + (9 - 3x) $\bar{7}_F$	126	(30 + 2x)	2 (1)	0 ≤ x ≤ 3

(Table continued)

TABLE XIII. (*Continued*)

SU(7) family	n_f	$77\xi_f$	$n_g^f \leq$	Condition
$235_{A3} + \overline{21}_{A2} + \overline{7}_F$	98	26	2 (1)	
$235_{A3} + \overline{28}_{S2} + x\overline{21}_{A2} + (7 - 3x)\overline{7}_F$	147	$(36 + 2x)$	2 (1)	$0 \leq x \leq 1$
$235_{A3} + \overline{28}_{S2} + x\overline{21}_{A2} + (7 - 3x)\overline{7}_F$	147	$(36 + 2x)$	1 (1)	$x = 2$
$335_{A3} + 2\overline{21}_{A2}$	147	40	1 (1)	
$335_{A3} + \overline{28}_{S2} + x\overline{21}_{A2} + (5 - 3x)\overline{7}_F$	168	$(44 + 2x)$	1 (0)	$0 \leq x \leq 1$
$335_{A3} + \overline{28}_{S2} + 2\overline{21}_{A2} + \overline{7}_F$	182	50	1 (0)	
$435_{A3} + \overline{28}_{S2} + 3\overline{7}_F$	189	52	1 (0)	
$435_{A3} + \overline{28}_{S2} + \overline{21}_{A2}$	189	54	1 (0)	
$535_{A3} + \overline{28}_{S2} + \overline{7}_F$	210	60	1 (0)	
$140_{Ra} + 35_{A3} + \overline{21}_{A2}$	196	70	1 (0)	
$140_{Ra} + \overline{35}_{A3} + \overline{7}_F$	182	66	1 (0)	

TABLE XIV. Complete list of chiral families (bottom table) for SU(8), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	8	1	1
A2	28	6	4
S2	36	10	12
Ra	216	75	-3
A3	56	15	5

SU(8) family	n_f	$88\xi_f$	$n_g^f \leq$	Condition
$28_{A2} + 4\overline{8}_F$	60	10	8 (4)	
$36_{S2} + 12\overline{8}_F$	132	22	3 (2)	
$36_{S2} + x\overline{28}_{A2} + (12 - 4x)\overline{8}_F$	$132 - 4x$	$(22 + 2x)$	3 (1)	$1 \leq x \leq 3$
$216_{Ra} + 3\overline{8}_F$	240	78	1 (0)	
$216_{Ra} + \overline{28}_{A2} + \overline{8}_F$	252	82	1 (0)	
$56_{A3} + 5\overline{8}_F$	96	20	4 (2)	
$56_{A3} + \overline{28}_{A2} + \overline{8}_F$	92	22	3 (2)	
$56_{A3} + 2\overline{28}_{A2} + 3\overline{8}_F$	136	30	2 (1)	
$56_{A3} + \overline{36}_{S2} + x\overline{28}_{A2} + (7 - 4x)\overline{8}_F$	$148 - 4x$	$(32 + 2x)$	2 (1)	$0 \leq x \leq 1$
$56_{A3} + \overline{36}_{S2} + 2\overline{28}_{A2} + \overline{8}_F$	156	38	2 (1)	
$256_{A3} + \overline{36}_{S2} + \overline{28}_F$	164	42	2 (1)	
$256_{A3} + \overline{36}_{S2} + \overline{28}_{A2} + \overline{28}_F$	192	48	1 (0)	
$356_{A3} + \overline{36}_{S2} + 3\overline{8}_F$	228	58	1 (0)	
$356_{A3} + \overline{36}_{S2} + \overline{28}_{A2} + \overline{8}_F$	240	62	1 (0)	

TABLE XV. Complete list of chiral families (bottom table) for SU(9), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	9	1	1
A2	36	7	5
S2	45	11	13
A4	126	35	5
A3	84	21	9

SU(9) family	n_f	$99\xi_f$	$n_g^f \leq$	Condition
$36_{A2} + 5\bar{9}_F$	81	12	8 (4)	
$45_{S2} + 13\bar{9}_F$	162	24	4 (2)	
$45_{S2} + \bar{36}_{A2} + 8\bar{9}_F$	153	26	3 (2)	
$45_{S2} + 2\bar{36}_{A2} + 3\bar{9}_F$	144	28	3 (1)	
$45_{S2} + 3\bar{36}_{A2} + 2\bar{9}_F$	171	34	2 (1)	
$245_{S2} + 5\bar{36}_{A2} + \bar{9}_F$	279	58	1 (0)	
$345_{S2} + 8\bar{36}_{A2} + 9_F$	432	90	1 (0)	
$126_{A4} + 5\bar{9}_F$	171	40	2 (1)	
$126_{A4} + \bar{36}_{A2}$	162	42	2 (1)	
$126_{A4} + \bar{45}_{S2} + x36_{A2} + (8 - 5x)9_F$	$243 - 9x$	$(54 + 2x)$	1 (0)	$0 \leq x \leq 1$
$126_{A4} + \bar{45}_{S2} + 236_{A2} + 2\bar{9}_F$	261	62	1 (0)	
$2126_{A4} + \bar{45}_{S2} + 39_F$	324	84	1 (0)	
$2126_{A4} + \bar{45}_{S2} + 36_{A2} + 2\bar{9}_F$	351	90	1 (0)	
$84_{A3} + x\bar{36}_{A2} + (9 - 5x)\bar{9}_F$	$165 - 9x$	$(30 + 2x)$	3 (1)	$0 \leq x \leq 1$
$84_{A3} + 2\bar{36}_{A2} + 9_F$	165	36	2 (1)	
$84_{A3} + \bar{45}_{S2} + 49_F$	165	36	2 (1)	
$84_{A3} + \bar{45}_{S2} + 36_{A2} + \bar{9}_F$	174	40	2 (1)	
$284_{A3} + \bar{45}_{S2} + 5\bar{9}_F$	258	58	1 (0)	
$284_{A3} + \bar{45}_{S2} + \bar{36}_{A2}$	249	60	1 (0)	
$84_{A3} + 126_{A4} + \bar{45}_{S2} + \bar{9}_F$	264	68	1 (0)	
$84_{A3} + \bar{126}_{A4} + 4\bar{9}_F$	246	60	1 (0)	
$84_{A3} + \bar{126}_{A4} + \bar{36}_{A2} + 9_F$	255	64	1 (0)	
$84_{A3} + 2\bar{126}_{A4} + 9_F$	345	92	1 (0)	

TABLE XVI. Complete list of chiral families (bottom table) for SU(10), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	10	1	1
A2	45	8	6
S2	55	12	14
A3	120	28	14
A4	210	56	14

SU(10) family	n_f	$110\xi_f$	$n_g^f \leq$	Condition
45 _{A2} + 6 $\overline{\mathbf{10}}$ _F	105	14	7 (4)	
55 _{S2} + 14 $\overline{\mathbf{10}}$ _F	195	26	4 (2)	
55 _{S2} + 45 _{A2} + 8 $\overline{\mathbf{10}}$ _F	180	28	3 (2)	
55 _{S2} + 245 _{A2} + 2 $\overline{\mathbf{10}}$ _F	165	30	3 (1)	
55 _{S2} + 345 _{A2} + 410 _F	230	40	2 (1)	
255 _{S2} + 545 _{A2} + 210 _F	355	66	1 (0)	
355 _{S2} + 745 _{A2}	480	92	1 (0)	
120 _{A3} + $x\overline{\mathbf{45}}$ _{A2} + $(14 - 6x)\overline{\mathbf{10}}$ _F	$260 - 15x$	$(42 + 2x)$	2 (1)	$0 \leq x \leq 2$
120 _{A3} + 345 _{A2} + 410 _F	295	56	1 (1)	
120 _{A3} + 55 _{S2}	175	40	2 (1)	
120 _{A3} + 55 _{S2} + 545 _{A2} + 210 _F	420	82	1 (0)	
120 _{A3} + 255 _{S2} + 745 _{A2}	545	108	1 (0)	
2120 _{A3} + 545 _{A2} + 210 _F	485	98	1 (0)	
210 _{A4} + $x\overline{\mathbf{45}}$ _{A2} + $(14 - 6x)\overline{\mathbf{10}}$ _F	$350 - 15x$	$(70 + 2x)$	1 (0)	$0 \leq x \leq 2$
210 _{A4} + 345 _{A2} + 410 _F	385	84	1 (0)	
210 _{A4} + 55 _{S2}	265	68	1 (0)	
210 _{A4} + 120 _{A3}	330	84	1 (0)	

TABLE XVII. Complete list of chiral families (bottom table) for SU(11), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	11	1	1
A2	55	9	7
S2	66	13	15
A3	165	36	20
A4	330	84	28

SU(11) family	n_f	$121\xi_f$	$n_g^f \leq$	Condition
55 _{A2} + 7 $\overline{\mathbf{11}}$ _F	132	16	7 (4)	
66 _{S2} + $x\overline{\mathbf{55}}$ _{A2} + $(15 - 7x)\overline{\mathbf{11}}$ _F	$231 - 22x$	$(28 + 2x)$	4 (2)	$0 \leq x \leq 1$
66 _{S2} + $x\overline{\mathbf{55}}$ _{A2} + $(15 - 7x)\overline{\mathbf{11}}$ _F	$231 - 22x$	$(28 + 2x)$	3 (2)	$x = 2$
66 _{S2} + 355 _{A2} + 611 _F	297	46	2 (1)	
266 _{S2} + 355 _{A2} + 911 _F	396	62	1 (1)	

(Table continued)

TABLE XVII. (*Continued*)

SU(11) family	n_f	$121\xi_f$	$n_g^f \leq$	Condition
$266_{S2} + 5\overline{55}_{A2} + 511_F$	462	76	1 (0)	
$366_{S2} + 7\overline{55}_{A2} + 411_F$	627	106	1 (0)	
$165_{A3} + x\overline{55}_{A2} + (20 - 7x)\overline{11}_F$	$385 - 22x$	$(56 + 2x)$	2 (1)	$0 \leq x \leq 2$
$165_{A3} + 3\overline{55}_{A2} + 11_F$	341	64	1 (1)	
$165_{A3} + \overline{66}_{S2} + 5\overline{11}_F$	286	54	2 (1)	
$165_{A3} + \overline{66}_{S2} + \overline{55}_{A2} + 211_F$	308	60	2 (1)	
$165_{A3} + 2\overline{66}_{S2} + x\overline{55}_{A2} + (10 - 7x)11_F$	$407 - 22x$	$(72 + 2x)$	1 (0)	$0 \leq x \leq 1$
$165_{A3} + 2\overline{66}_{S2} + 255_{A2} + 4\overline{11}_F$	451	84	1 (0)	
$165_{A3} + 3\overline{66}_{S2} + 455_{A2} + 3\overline{11}_F$	616	114	1 (0)	
$330_{A4} + x\overline{55}_{A2} + (28 - 7x)\overline{11}_F$	$638 - 22x$	$(112 + 2x)$	1 (0)	$0 \leq x \leq 4$
$330_{A4} + \overline{66}_{S2} + x\overline{55}_{A2} + (13 - 7x)\overline{11}_F$	$539 - 22x$	$(110 + 2x)$	1 (0)	$0 \leq x \leq 1$
$330_{A4} + \overline{66}_{S2} + 255_{A2} + 11_F$	517	116	1 (0)	
$330_{A4} + 2\overline{66}_{S2} + 211_F$	484	112	1 (0)	

TABLE XVIII. Complete list of chiral families (bottom table) for SU(12), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	12	1	1
A2	66	10	8
S2	78	14	16
A3	220	45	27

SU(12) family	n_f	$132\xi_f$	$n_g^f \leq$	Condition
$66_{A2} + 8\overline{12}_F$	162	18	7 (3)	
$78_{S2} + x\overline{66}_{A2} + (16 - 8x)\overline{12}_F$	$270 - 30x$	$(30 + 2x)$	4 (2)	$0 \leq x \leq 1$
$78_{S2} + 2\overline{66}_{A2}$	210	34	3 (2)	
$220_{A3} + x\overline{66}_{A2} + (27 - 8x)\overline{12}_F$	$544 - 30x$	$(72 + 2x)$	1 (0)	$0 \leq x \leq 3$
$220_{A3} + 4\overline{66}_{A2} + 512_F$	544	90	1 (0)	
$220_{A3} + \overline{78}_{S2} + 11\overline{12}_F$	430	70	1 (1)	
$220_{A3} + \overline{78}_{S2} + \overline{66}_{A2} + 3\overline{12}_F$	400	72	1 (0)	
$220_{A3} + \overline{78}_{S2} + 2\overline{66}_{A2} + 512_F$	490	84	1 (0)	
$220_{A3} + 2\overline{78}_{S2} + 512_F$	436	78	1 (0)	
$220_{A3} + 2\overline{78}_{S2} + 66_{A2} + 3\overline{12}_F$	478	86	1 (0)	

TABLE XIX. Complete list of chiral families (bottom table) for SU(13), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	13	1	1
A2	78	11	9
S2	91	15	17
A3	286	55	35

SU(13) family	n_f	$143\xi_f$	$n_g^f \leq$	Condition
$78_{A2} + 9\overline{13}_F$	195	20	7 (3)	
$91_{S2} + x\overline{78}_{A2} + (17 - 9x)\overline{13}_F$	$312 - 39x$	$(32 + 2x)$	4 (2)	$0 \leq x \leq 1$
$91_{S2} + 2\overline{78}_{A2} + 13_F$	260	38	3 (2)	
$291_{S2} + 3\overline{78}_{A2} + 7\overline{13}_F$	507	70	2 (1)	
$391_{S2} + 5\overline{78}_{A2} + 6\overline{13}_F$	741	106	1 (0)	
$491_{S2} + 7\overline{78}_{A2} + 5\overline{13}_F$	975	142	1 (0)	
$286_{A3} + x\overline{78}_{A2} + (35 - 9x)\overline{13}_F$	$741 - 39x$	$(90 + 2x)$	1 (0)	$0 \leq x \leq 3$
$286_{A3} + 4\overline{78}_{A2} + 13_F$	611	100	1 (0)	
$286_{A3} + 91_{S2} + x\overline{78}_{A2} + (18 - 9x)\overline{13}_F$	$611 - 39x$	$(88 + 2x)$	1 (0)	$0 \leq x \leq 2$
$286_{A3} + 291_{S2} + \overline{13}_F$	481	86	1 (0)	
$286_{A3} + 291_{S2} + \overline{78}_{A2} + 813_F$	650	104	1 (0)	
$286_{A3} + 391_{S2} + x\overline{78}_{A2} + (16 - 9x)13_F$	$767 - 39x$	$(116 + 2x)$	1 (0)	$0 \leq x \leq 1$
$286_{A3} + 391_{S2} + 278_{A2} + 2\overline{13}_F$	741	124	1 (0)	

TABLE XX. Complete list of chiral families (bottom table) for SU(14), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	14	1	1
A2	91	12	10
S2	105	16	18
A3	364	66	44

SU(14) family	n_f	$154\xi_f$	$n_g^f \leq$	Condition
$91_{A2} + 10\overline{14}_F$	231	22	6 (3)	
$105_{S2} + x\overline{91}_{A2} + (18 - 10x)\overline{14}_F$	$357 - 49x$	$(34 + 2x)$	4 (2)	$0 \leq x \leq 1$
$105_{S2} + 29\overline{1}_{A2} + 214_F$	315	42	3 (1)	
$2105_{S2} + 39\overline{1}_{A2} + 6\overline{14}_F$	567	74	2 (1)	
$3105_{S2} + 59\overline{1}_{A2} + 4\overline{14}_F$	826	112	1 (0)	
$4105_{S2} + 79\overline{1}_{A2} + 2\overline{14}_F$	1085	150	1 (0)	
$364_{A3} + x\overline{91}_{A2} + (44 - 10x)\overline{14}_F$	$980 - 49x$	$(110 + 2x)$	1 (0)	$0 \leq x \leq 4$
$364_{A3} + 59\overline{1}_{A2} + 614_F$	903	132	1 (0)	
$364_{A3} + 105_{S2} + x\overline{91}_{A2} + (26 - 10x)\overline{14}_F$	$833 - 49x$	$(108 + 2x)$	1 (0)	$0 \leq x \leq 2$
$364_{A3} + 105_{S2} + 39\overline{1}_{A2} + 414_F$	798	122	1 (0)	
$364_{A3} + 2105_{S2} + 8\overline{14}_F$	686	106	1 (0)	
$364_{A3} + 2105_{S2} + 9\overline{1}_{A2} + 214_F$	693	112	1 (0)	
$364_{A3} + 3105_{S2} + 1014_F$	819	124	1 (0)	
$364_{A3} + 3105_{S2} + 91_{A2}$	770	126	1 (0)	

TABLE XXI. Complete list of chiral families (bottom table) for SU(15), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	15	1	1
A2	105	13	11
S2	120	17	19
A3	455	78	54

SU(15) family	n_f	$165\xi_f$	$n_g^f \leq$	Condition
$105_{A2} + 11\overline{15}_F$	270	24	6 (3)	
$120_{S2} + x\overline{105}_{A2} + (19 - 11x)\overline{15}_F$	$405 - 60x$	$(36 + 2x)$	4 (2)	$0 \leq x \leq 1$
$120_{S2} + 2\overline{105}_{A2} + 315_F$	375	46	3 (1)	
$2120_{S2} + 3\overline{105}_{A2} + 5\overline{15}_F$	630	78	2 (1)	
$3120_{S2} + 5\overline{105}_{A2} + 2\overline{12}_F$	915	118	1 (0)	
$4120_{S2} + 7\overline{105}_{A2} + 15_F$	1230	160	1 (0)	
$455_{A3} + x\overline{105}_{A2} + (54 - 11x)\overline{15}_F$	$1265 - 60x$	$(132 + 2x)$	1 (0)	$0 \leq x \leq 4$
$455_{A3} + 5\overline{105}_{A2} + 15_F$	995	144	1 (0)	
$455_{A3} + \overline{120}_{S2} + x\overline{105}_{A2} + (35 - 11x)\overline{15}_F$	$1100 - 60x$	$(130 + 2x)$	1 (0)	$0 \leq x \leq 3$
$455_{A3} + \overline{120}_{S2} + 4\overline{105}_{A2} + 915_F$	1130	156	1 (0)	
$455_{A3} + 2\overline{120}_{S2} + x\overline{105}_{A2} + (16 - 11x)\overline{15}_F$	$935 - 60x$	$(128 + 2x)$	1 (0)	$0 \leq x \leq 1$
$455_{A3} + 2\overline{120}_{S2} + 2\overline{105}_{A2} + 615_F$	995	144	1 (0)	
$455_{A3} + 3\overline{120}_{S2} + 315_F$	860	132	1 (0)	
$455_{A3} + 3\overline{120}_{S2} + 105_{A2} + 8\overline{15}_F$	1040	150	1 (0)	

TABLE XXII. Complete list of chiral families (bottom table) for SU(16), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	16	1	1
A2	120	14	12
S2	136	18	20
A3	560	91	65

SU(16) family	n_f	$176\xi_f$	$n_g^f \leq$	Condition
$120_{A2} + 12\overline{16}_F$	312	26	6 (3)	
$136_{S2} + x\overline{120}_{A2} + (20 - 12x)\overline{16}_F$	$456 - 72x$	$(38 + 2x)$	4 (2)	$0 \leq x \leq 1$
$136_{S2} + 2\overline{120}_{A2} + 416_F$	440	50	3 (1)	
$2136_{S2} + 3\overline{120}_{A2} + 4\overline{16}_F$	696	82	2 (1)	
$3136_{S2} + 5\overline{120}_{A2}$	1008	124	1 (0)	
$560_{A3} + x\overline{120}_{A2} + (65 - 12x)\overline{16}_F$	$1600 - 72x$	$(156 + 2x)$	1 (0)	$0 \leq x \leq 5$
$560_{A3} + \overline{136}_{S2} + x\overline{120}_{A2} + (45 - 12x)\overline{16}_F$	$1416 - 72x$	$(154 + 2x)$	1 (0)	$0 \leq x \leq 3$
$560_{A3} + \overline{136}_{S2} + 4\overline{120}_{A2} + 316_F$	1224	168	1 (0)	
$560_{A3} + 2\overline{136}_{S2} + x\overline{120}_{A2} + (25 - 12x)\overline{16}_F$	$1232 - 72x$	$(152 + 2x)$	1 (0)	$0 \leq x \leq 2$
$560_{A3} + 3\overline{136}_{S2} + 5\overline{16}_F$	1048	150	1 (0)	
$560_{A3} + 3\overline{136}_{S2} + \overline{120}_{A2} + 716_F$	1200	166	1 (0)	

TABLE XXIII. Complete list of chiral families (bottom table) for SU(17), with representations labeled by their dimension.

Representation	d_r	$2T_r$	$\mathcal{A}_r$
F	17	1	1
A2	136	15	13
S2	153	19	21
A3	680	105	77

SU(17) family	n_f	$187\xi_f$	$n_g^f \leq$	Condition
136 _{A2} + 1317 _F	357	28	6 (3)	
153 _{S2} + $x\overline{\mathbf{136}}$ _{A2} + $(21 - 13x)\overline{\mathbf{17}}$ _F	$510 - 85x$	$(40 + 2x)$	4 (2)	$0 \leq x \leq 1$
153 _{S2} + $2\overline{\mathbf{136}}$ _{A2} + 517 _F	510	54	3 (1)	
2153 _{S2} + $3\overline{\mathbf{136}}$ _{A2} + $3\overline{\mathbf{17}}$ _F	765	86	2 (1)	
3153 _{S2} + $5\overline{\mathbf{136}}$ _{A2} + 217 _F	1173	134	1 (0)	
680 _{A3} + $x\overline{\mathbf{136}}$ _{A2} + $(77 - 13x)\overline{\mathbf{17}}$ _F	$1989 - 85x$	$(182 + 2x)$	1 (0)	$0 \leq x \leq 2$
680 _{A3} + 153 _{S2} + $x\overline{\mathbf{136}}$ _{A2} + $(56 - 13x)\overline{\mathbf{17}}$ _F	$1785 - 85x$	$(180 + 2x)$	1 (0)	$0 \leq x \leq 3$
680 _{A3} + $2\overline{\mathbf{153}}$ _{S2} + $x\overline{\mathbf{136}}$ _{A2} + $(35 - 13x)\overline{\mathbf{17}}$ _F	$1581 - 85x$	$(178 + 2x)$	1 (0)	$0 \leq x \leq 2$
680 _{A3} + $3\overline{\mathbf{153}}$ _{S2} + $x\overline{\mathbf{136}}$ _{A2} + $(14 - 13x)\overline{\mathbf{17}}$ _F	$1377 - 85x$	$(176 + 2x)$	1 (0)	$0 \leq x \leq 1$

Table XVIII, SU(13) in Table XIX, SU(14) in Table XX, SU(15) in Table XXI, SU(16) in Table XXII, and SU(17) in Table XXIII.

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