

Dynamical up-quark mass generation in QCD-like theories

Csaba Csáki^{1,*}, Tuhin S. Roy^{2,†}, Maximilian Ruhdorfer^{3,‡} and Taewook Youn^{1,4,§}

¹Laboratory for Elementary Particle Physics, *Cornell University*, Ithaca, New York 14853, USA

²Department of Theoretical Physics, *Tata Institute of Fundamental Research*,
Homi Bhabha Road, Mumbai 400005, India

³Stanford Institute for Theoretical Physics, *Stanford University*, Stanford, California 94305, USA

⁴School of Physics, *Korea Institute for Advanced Study*, Seoul 02455, Republic of Korea



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We calculate the dynamically generated up-quark mass in some QCD-like theories with $F = 3$ light flavors, obtained from supersymmetric QCD perturbed via anomaly mediated supersymmetry breaking. We match the low-energy effective theory to the traditional chiral Lagrangian of QCD and determine the coefficients to next-to-leading order in chiral perturbation theory, while also varying the number of colors N . We find that the dynamically generated up-quark mass vanishes in the large N limit and is small for $F < N$; however, for $F = N$ there is a sizable $\mathcal{O}(1)$ contribution. While our results are reliable only for small supersymmetry breaking, we observe that extrapolating the $F = N$ result to large supersymmetry breaking would lead to a dynamical up-quark mass that is large enough to account for its entire physical mass.

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I. INTRODUCTION

The strong CP problem of QCD is one of the most puzzling open questions in the standard model (SM) of particle physics. The θ -parameter of QCD explicitly violates CP and feeds into the expression for the electric dipole moment of the neutron, which can be reconciled with experimental bounds only if $\bar{\theta} < 10^{-10}$, even though *a priori* any value of $\bar{\theta}$ seems equally possible. The simplest resolution to this puzzle would arise if one of the quark masses (in practice the up-quark mass) were to vanish. This would provide an anomalous, but classically unbroken, $U(1)$ axial symmetry that could be used to rotate away the $\bar{\theta}$ angle and render it unphysical. While this solution seems theoretically very appealing, the physical up-quark mass appears to be nonvanishing. However, it was pointed out already by Kaplan and Manohar in 1986 [1] that dynamical effects contributing to the pion mass can effectively shift the up-quark mass, possibly allowing a successful fit to the pseudoscalar meson masses even if the underlying

up-quark Yukawa coupling was vanishing. One simple interpretation of this is that nonperturbative effects generate a dynamical quark mass contribution, which corresponds to an additive renormalization of the quark mass. One often cited example would be instanton contributions to the up-quark mass in a three flavor theory, as depicted in Fig. 1. Such an instanton would provide corrections $\Delta m_u \propto m_d^* m_s^*$, and could be sizable. Of course, the instantons are strongly coupled and incalculable using standard instanton calculus (see e.g. [2] for simple power counting rules to estimate instanton effects). The important point is that the strong CP problem would still be solved if all of the up-quark mass was dynamically generated, as long as the underlying up-Yukawa coupling was vanishing.

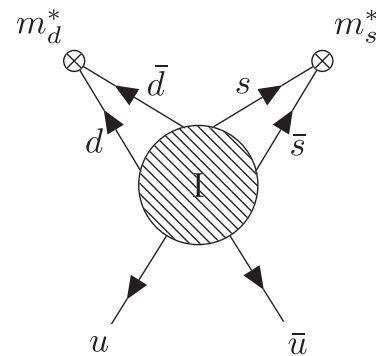


FIG. 1. Feynman diagram for the 't Hooft operator corresponding to the instanton induced up-quark mass.

*Contact author: csaki@cornell.edu

†Contact author: tuhin@theory.tifr.res.in

‡Contact author: m.ruhdorfer@stanford.edu

§Contact author: taewook.youn@cornell.edu

Generally it is very difficult to calculate the dynamical quark masses from first principles due to the strong dynamics. The best route is via lattice simulations (see e.g. [3,4] for an overview), though there is some ambiguity to disentangle the underlying up-quark mass from the dynamical corrections [5]. Another possibility is to use phenomenological fits of the low-energy constants in the chiral Lagrangian to experimental data [6–9]. However, the current precision is not sufficient to significantly narrow down the dynamical contribution to the up-quark mass. Finally large- N QCD allows one to obtain estimates for the low-energy constants in the chiral Lagrangian [10–12]. In particular, Ref. [12] recently determined the low-energy constants to leading order in the large- N expansion, finding that the dynamically generated contribution to the up-quark mass is small in the large- N limit.

In this paper we pursue a different approach. Rather than trying to calculate directly in QCD, we will be considering a different theory, which has many similarities to QCD itself; hence we call it a QCD-like theory. It is obtained from a supersymmetric (SUSY) version of QCD with three flavors ($F = 3$), where SUSY is broken via anomaly mediated supersymmetry breaking (AMSB). As shown in [13,14], this theory has the right massless degrees of freedom in the UV (i.e. the squarks and gluinos get masses proportional to the SUSY breaking scale m), while there is a QCD-like vacuum (which in many cases is the global minimum of the potential, depending on the number of flavors and colors) in which the $SU(3)_L \times SU(3)_R$ flavor symmetry is broken to $SU(3)_V$. The corresponding Goldstone bosons are massless in the limit of vanishing quark masses, and a systematic evaluation of the chiral Lagrangian is possible and mostly calculable, up to a few incalculable constants for the case when $F = N$. Note that the calculation of the chiral Lagrangian is reliable only in the limit of small supersymmetry breaking, while true QCD would be obtained in the limit of large SUSY breaking, which is outside the region of validity of our results.

In this work, we determine the chiral Lagrangian for these QCD-like models to second order in the quark masses (which for $F = 3$ should capture the leading effects of dynamical up-quark mass generation) and to fourth order in derivatives. This will entail the matching of a linear sigma model obtained from the SUSY theory to the nonlinear

theory of χ PT (chiral perturbation theory). Using our second-order chiral Lagrangian we will determine the dynamically generated up-quark mass. We find that the coefficient L_4 and L_6 (which as we will explain contains the information on the dynamical up-quark mass) is unsuppressed for finite N, F compared to the other coefficients in the chiral Lagrangian. If one were to extrapolate to large SUSY breaking $m/\Lambda \sim \mathcal{O}(1)$ (beyond the validity of our approximations), one would numerically obtain large dynamical up-quark mass corrections for the $F = N = 3$ case that could be accounting for the entire observed up-quark mass, while for $F < N$ we find smaller corrections.

II. DYNAMICAL UP-QUARK MASS FROM χ PT

First, we review how the dynamical quark mass generation is incorporated in traditional χ PT. The leading-order χ PT Lagrangian is simply given by

$$\mathcal{L}_0 = \frac{1}{4} F_0^2 \{ \text{tr}[\partial^\mu U^\dagger \partial_\mu U] + \text{tr}[\chi^\dagger U + \chi U^\dagger] \}, \quad (1)$$

where U is the unitary three-by-three matrix containing the eight pseudoscalar Goldstone bosons, F_0 is the leading-order pion decay constant and $\chi = 2B_0 M_Q$ with $B_0 = -\langle \bar{q}q \rangle / (3F_0^2)$ related to the quark condensate [15]. Here χ does not depend on the renormalization scale and scheme, while B_0 and M_Q do. This Lagrangian will determine the leading four-pion interactions as well as the usual Gell-Mann Okubo mass relations. However, as we saw in the Introduction, there are dynamical contributions to the effective mass which scale like $\Delta m_u \propto m_d^* m_s^*$, implying that one needs to include terms which are higher order in the quark masses or derivatives in the chiral Lagrangian. In the absence of external gauge fields the next-to-leading order chiral Lagrangian is composed of terms with two powers of the quark mass, one power of the quark mass and two derivatives, or four derivatives. χ PT traditionally uses the following basis of eight operators [6]:

$$\mathcal{L}_1 = \sum_{i=1}^8 L_i \mathcal{O}_i, \quad (2)$$

where

$$\begin{aligned} \mathcal{O}_1 &= \text{tr}[\partial_\mu U \partial^\mu U^\dagger]^2, & \mathcal{O}_2 &= \text{tr}[\partial_\mu U \partial_\nu U^\dagger] \text{tr}[\partial^\mu U \partial^\nu U^\dagger], \\ \mathcal{O}_3 &= \text{tr}[(\partial_\mu U \partial^\mu U^\dagger)^2], & \mathcal{O}_4 &= \text{tr}[\partial_\mu U \partial^\mu U^\dagger] \text{tr}[\chi^\dagger U + \chi U^\dagger], \\ \mathcal{O}_5 &= \text{tr}[\partial_\mu U \partial_\nu U^\dagger (\chi^\dagger U + \chi U^\dagger)], & \mathcal{O}_6 &= \text{tr}[\chi^\dagger U + U^\dagger \chi]^2, \\ \mathcal{O}_7 &= \text{tr}[\chi^\dagger U - U^\dagger \chi]^2, & \mathcal{O}_8 &= \text{tr}[\chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger]. \end{aligned} \quad (3)$$

These $\mathcal{O}_{1,\dots,8}$ operators form a complete set and capture all nonperturbative effects including for example instanton contributions. To see this, consider the effective operator generated by the instanton diagram in Fig. 1:

$$\mathcal{O}_{-1} = \text{tr}[\text{cof} \chi^\dagger U^\dagger] + \text{H.c.}, \quad (4)$$

where the cofactor is given by $\text{cof}\chi^\dagger = \det(\chi^\dagger)(\chi^\dagger)^{-1}$. As expected, this operator is not independent of the $\mathcal{O}_{1,\dots,8}$ set. The easiest way to see this is by using the Kaplan-Manohar identity for $SU(3)$ matrices [1]

$$\text{tr}[\text{cof}\chi^\dagger U^\dagger] = \frac{1}{2} \{ \text{tr}[\chi^\dagger U]^2 - \text{tr}[\chi^\dagger U \chi^\dagger U] \}, \quad (5)$$

which implies that

$$\mathcal{O}_{-1} = \frac{1}{4}(\mathcal{O}_6 + \mathcal{O}_7) - \frac{1}{2}\mathcal{O}_8. \quad (6)$$

The upshot is that all of the QCD dynamics (to the given order in expansion in the quark masses and derivatives) is encoded in the coefficients $L_{1,\dots,8}$.

In fact, the Kaplan-Manohar identity (5) allows one to further simplify the Lagrangian: it can be utilized as a gauge transformation on the χ PT Lagrangian at $\mathcal{O}(p^4)$. The physics remains invariant under the shift of the mass matrix [1] as long as some of the coefficients $L_{1,\dots,8}$ are also shifted according to be following transformation:

$$\begin{aligned} \chi &\rightarrow \chi + a \text{cof}(\chi^\dagger), & L_6 &\rightarrow L_6 - a \frac{F_0^2}{16} \\ L_7 &\rightarrow L_7 - a \frac{F_0^2}{16}, & \text{and } L_8 &\rightarrow L_8 + a \frac{F_0^2}{8}, \end{aligned} \quad (7)$$

for arbitrary a . One way of utilizing this redundancy is to eliminate the instanton generated operator in Eq. (4) by choosing a appropriately. Another way to utilize this redundancy is by choosing a such that the meson spectrum [at $\mathcal{O}(p^4)$] arises entirely from the leading terms in Eq. (1). For example, denoting the up and down type masses in this gauge by μ_u and μ_d , respectively, and taking the quark masses to be real which we will assume from now on, one gets

$$m_\pi^2 = B_0(\mu_u + \mu_d) \quad (8)$$

in this gauge even from the $\mathcal{O}(p^4)$ Lagrangian.

An essential consequence of the above ambiguity is that fundamental/ultraviolet (electroweak generated) quark mass parameters of the SM Lagrangian can be extracted from the measured pion spectrum *only* up to an arbitrary shift [1,16]. Therefore, one needs to find an indirect route in order to check whether the observed meson spectrum is consistent with a zero up-quark mass ansatz or not. The key is to first work in a gauge where the $\mathcal{O}(p^2)$ Lagrangian in Eq. (1) is matched to the quark Lagrangian of the SM (with mass parameters $m_{u,d,s}$). Then at $\mathcal{O}(p^4)$ one finds

$$m_\pi^2 = \beta_1(m_u + m_d) + \beta_2 m_s(m_u + m_d) + \mathcal{O}(m_{u,d}^2), \quad (9)$$

where

$$\beta_1 = B_0, \quad \beta_2 = 16 \frac{B_0^2}{F_0^2} (2L_6 - L_4). \quad (10)$$

This allows one to directly compare the $m_{u,d,s}$ parameters with the effective $\mu_{u,d,s}$ parameters since the pion masses are gauge invariant. In fact, it rather simplifies if the SM Lagrangian is characterized by a massless up quark, i.e. $m_u = 0$, and if one further argues that $\mu_d \approx m_d$ and $\mu_s \approx m_s$ [9],

$$\frac{\mu_u}{\mu_d} \simeq \frac{\beta_2}{\beta_1} \mu_s = 16 \frac{B_0 \mu_s}{F_0^2} (2L_6 - L_4). \quad (11)$$

Note that the value $\mu_u/\mu_d \simeq 0.5$ corresponds to $\beta_2/\beta_1 \simeq 5 \text{ GeV}^{-1}$, a value that is commonly cited in the literature. While lattice simulation results [17,18] report a highly constrained value of $\mu_u/\mu_d \lesssim 0.1$, unconstrained χ PT fits to experimental data [6–8] prefer larger values of L_4 and L_6 , allowing $\mu_u/\mu_d \simeq 0.5$ within 1σ uncertainty.

III. THE SCALAR POTENTIAL IN AMSB QCD

The low-energy constants L_i in the chiral Lagrangian, as well as B_0 and F_0 , cannot be computed from first principles within QCD, since QCD gets strongly coupled at low energies. Determinations of the L_i typically rely on phenomenological fits [6–8] or lattice simulations [3,4]. Their leading-order scaling can also be estimated as an expansion in the number of colors, in the large- N limit [6,10–12]. While it is generally believed that an extrapolation of large- N results to $N = 3$ gives a good qualitative description of QCD, conclusions derived from large- N QCD should always be taken with a grain of salt.

Here we consider an alternative setup based on supersymmetric QCD (SQCD) [19–21] (see [22] for a review), which allows us to derive the chiral Lagrangian from first principles for any number of colors N , including $N = 3$. We additionally assume that supersymmetry is softly broken via AMSB [23,24]. AMSB ensures that SQCD has a QCD-like, i.e. chiral symmetry-breaking vacuum for $F < 3N$ [14]. Supersymmetry breaking lifts the masses of the superpartners and ensures that the pseudo-Nambu-Goldstone bosons (pNGBs) from chiral symmetry breaking, i.e. the mesons, are the lightest states in the theory, just as in ordinary QCD. By explicitly integrating out heavier states in the scalar sector we can derive the chiral Lagrangian to any order in quark masses. However, note that, while this theory has many similarities with QCD, it is only QCD-like. The superpartners, and the gluinos in particular, still play an important role in the confinement dynamics, and only in the limit in which the supersymmetry breaking scale m gets larger than the confinement scale Λ , one would recover ordinary QCD. However, when $m \sim \Lambda$, perturbativity is lost and this limit is not straightforward to take.

Our setup is a supersymmetric $SU(N)$ gauge theory with F flavors of chiral superfield pairs in the fundamental and antifundamental representation of the gauge group. The effects of AMSB are incorporated using the conformal

compensator formalism [25], where the supersymmetry breaking scale m is the F -term of a chiral superfield that appears wherever scale invariance is broken. At tree level it generates a contribution to the scalar potential which is given by [14]

$$V_{\text{tree}} = \partial_i W g^{ij*} \partial_{j^*} W + m^* m (\partial_i K g^{ij*} \partial_{j^*} K - K) + m(\partial_i W g^{ij*} \partial_{j^*} K - 3W) + \text{c.c.}, \quad (12)$$

where g^{ij*} is the inverse of the Kähler metric $g_{ij^*} = \partial_i \partial_{j^*} K$. Given our specific interest in $F = 3$, this paper focuses on the cases where $F < N$ and $F = N$. The Kähler potential and superpotential of interest are given by

$$W^{F < N} = (N - F) \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{1/(N-F)} + \text{tr}[M_Q M],$$

$$K^{F < N} = 2\text{tr}[\sqrt{MM^\dagger}] \quad (13)$$

for $F < N$ [19,26], and

$$W^{F=N} = X \left(\frac{\det M - B\bar{B}}{\Lambda^{2N}} - 1 \right) + \text{tr}[M_Q M],$$

$$K^{F=N} = \frac{\text{tr}[MM^\dagger]}{\alpha|\Lambda|^2} + \frac{XX^\dagger}{\beta|\Lambda|^4} + \frac{B\bar{B}^\dagger}{\gamma|\Lambda|^{2N-2}} + \frac{\bar{B}\bar{B}^\dagger}{\delta|\Lambda|^{2N-2}} \quad (14)$$

for $F = N$ [20]. Here $M_{ff'} = \bar{Q}_f Q_{f'}$, $f, f' = 1, \dots, F$ is the meson superfield, constructed from quark chiral superfields $Q_f, \bar{Q}_f$. M_Q is the supersymmetric mass matrix for the quark superfields. $B, \bar{B}$ are completely antisymmetric color singlet combinations of the quark superfields Q and $\bar{Q}$, i.e. $B = \epsilon^{f_1 \dots f_F} \epsilon_{a_1 \dots a_N} Q_{f_1}^{a_1} \dots Q_{f_F}^{a_N}$, where a_i are color indices. X corresponds to a Lagrange multiplier that implements the quantum modified constraint $\det M - B\bar{B} = \Lambda^{2N}$ in the superpotential, and $\alpha, \beta, \gamma, \delta$ are incalculable numbers.

Note that truncating the Kähler potential to only quadratic terms for $F = N$ is valid as long as $M/\Lambda^2, X/\Lambda^3, B/\Lambda^N, \bar{B}/\Lambda^N \ll 1$, which is not the case. This implies that there are higher-order terms in the Kähler potential which are as important as the leading-order term. Our assumption is therefore that we still capture the qualitative behavior of the theory even if we neglect the higher-order terms. A more rigorous approach would be to start with $F = N + 1$, which is under perturbative control, and introduce a heavy supersymmetric mass μ for one flavor, satisfying $\Lambda > \mu > m > M_Q$, and then integrate it out. In this framework, the Lagrange multiplier field X naturally corresponds to the $M_{N+1, N+1}$ component of the meson field, providing a justification for the assumed form of its Kähler potential. It has been verified that this approach yields results consistent with our simplified method.

Using the explicit expressions for K and W , we obtain the scalar potential from Eq. (12) for each theory. Since we are interested in a QCD-like minimum and the chiral

perturbation theory, we have $B = \bar{B} = 0$, and integrate out X in the $F = N$ case. For simplicity, we also set $\theta = 0$, i.e. $\arg \Lambda = 0$ for the rest of the paper. For a derivation of the leading-order chiral Lagrangian with $\theta \neq 0$, see for example [27–29].

IV. THE CHIRAL LAGRANGIAN IN AMSB QCD

As a next step we determine the chiral Lagrangian in AMSB QCD by integrating out the heavy degrees of freedom in the scalar sector at tree level. Note that the fermionic superpartners of the mesons contribute to the scalar potential only at the loop level. We will assume that $\Lambda \gg m \gg M_Q$ in order to maintain perturbative control. Following [30] we parametrize the meson matrix as

$$M = SU, \quad S^\dagger = S, \quad U^\dagger U = \mathbf{I}_3. \quad (15)$$

Hereafter, M refers to the complex scalar component of the meson superfield. Here U contains the nine pNGBs in a nonlinear representation

$$U = \exp \left\{ -\frac{i}{3} \eta' \right\} \tilde{U}, \quad \det \tilde{U} = 1, \quad (16)$$

where the η' is the GB of the spontaneously broken $U(1)_A$, where we have absorbed the decay constant into the definition of the field. Note that U transforms as a bi-fundamental under $U(3)_L \times U(3)_R$, while S is a singlet under $SU(3)_L$ and transforms in the adjoint representation of $SU(3)_R$. Therefore, S contains 9 real degrees of freedom [8 that transform in the adjoint representation of $SU(3)_R$ and one singlet].

To zeroth order in the quark masses S obtains a vacuum expectation value (VEV) proportional to the identity, what spontaneously breaks $U(3)_L \times U(3)_R \rightarrow U(3)_V$, i.e.

$$\langle S \rangle = f_0^2 \mathbf{I}_3 + \mathcal{O}(M_Q), \quad (17)$$

with

$$f_0^{F < N} = \left(\frac{N+3}{3N-3} \frac{\Lambda}{m} \right)^{\frac{N-3}{2N}} \Lambda,$$

$$f_0^{F=N} = \Lambda - \frac{m^2}{6\alpha\beta\Lambda} + \mathcal{O}(m^4). \quad (18)$$

Plugging this into the scalar potential, one finds the leading-order chiral Lagrangian in Eq. (1) and identifies F_0 and $\chi = 2B_0 M_Q^\dagger$, where the explicit expression for F_0 and B_0 can be found in Table I.

To find higher-order terms in the chiral Lagrangian, we have to integrate out the radial modes in S ; i.e. we have to perturbatively solve the equation of motion for S as a function of U to a given order in M_Q , and we have to find $S = S[U]$, which minimizes the potential. To do so we parametrize S as

TABLE I. The parameters of χ PT in the lowest and next-to-leading order quark mass expansion for $F < N$ and $F = N$. F is specifically taken as 3. Large N counting is added in the $F < N$ column.

Theories	$F = 3 < N$	$F = N = 3$
F_0	$2f_0^{F < N} \sim \mathcal{O}(N^{1/2})$	$\frac{2\Lambda}{\sqrt{a}}$
B_0	$\frac{7N-3}{N+3} \frac{m}{2}$	$\frac{3am}{2}$
L_1	$-\frac{\Lambda^3}{36Nm^3} \sim \mathcal{O}(N^0)$	$-\frac{\Lambda^2}{6am^2} + \frac{4}{9a^2\beta}$
L_2	0	0
L_3	$\frac{\Lambda^3}{108m^3} \sim \mathcal{O}(N^1)$	$\frac{\Lambda^2}{2am^2} - \frac{1}{a^2\beta}$
L_4	$-\frac{\Lambda^3}{42Nm^3} \sim \mathcal{O}(N^0)$	$-\frac{\Lambda^2}{18am^2} + \frac{16}{81a^2\beta}$
L_5	$\frac{\Lambda^3}{378m^3} \sim \mathcal{O}(N^1)$	$\frac{\Lambda^2}{6am^2} - \frac{13}{27a^2\beta}$
L_6	$-\frac{5\Lambda^3}{1764Nm^3} \sim \mathcal{O}(N^0)$	$\frac{\Lambda^2}{216am^2} + \frac{5}{486a^2\beta}$
L_7	$-\frac{N\Lambda^3}{972m^3} \sim \mathcal{O}(N^2)$	$\frac{\Lambda^2}{36am^2} - \frac{19}{324a^2\beta}$
L_8	$\frac{\Lambda^3}{5292m^3} \sim \mathcal{O}(N^1)$	$\frac{\Lambda^2}{72am^2} - \frac{17}{324a^2\beta}$

$$S = f_0^2 \mathbf{I}_3 + S_1 + \dots \equiv f_0^2 \mathbf{I}_3 + H + T \mathbf{I}_3 + \dots, \quad (19)$$

where $H = S_1 - \frac{1}{3} \text{tr}[S_1] \mathbf{I}_3$ and $T = \frac{1}{3} \text{tr}[S_1]$. Note that H and T are of order $\mathcal{O}(M_Q)$ and $\mathcal{O}(\partial^2)$ by construction.¹ The relevant terms in the Lagrangian which include H and T take the form

$$\mathcal{L}^{(H,T)} = \frac{M_Q^2}{2} \text{tr}[H^2] - \text{tr}[A^{(H)}[U]H] + \frac{M_T^2}{2} T^2 - A^{(T)}[U]T, \quad (20)$$

where $M_{(H,T)}$ is the effective mass of H and T and $A^{(H,T)}[U]$ is a U -dependent source term, which is different for each potential. $A^{(H,T)}$ can be decomposed into three parts: $A_{M_Q}^{(H,T)}, A_{\text{kin}}^{(H,T)}, A_{\eta'}^{(H,T)}$, each coming from the mass term, the kinetic term, and η' -dependent terms, respectively:

$$\begin{aligned} A_{M_Q}^{(H)} &\propto \chi^\dagger U + U^\dagger \chi - \frac{1}{3} \text{tr}[\chi^\dagger U + U^\dagger \chi], \\ A_{M_Q}^{(T)} &\propto \text{tr}[\chi^\dagger U + U^\dagger \chi], \\ A_{\text{kin}}^{(H)} &\propto \partial_\mu U \partial^\mu U^\dagger - \frac{1}{3} \text{tr}[\partial_\mu U \partial^\mu U^\dagger], \\ A_{\text{kin}}^{(T)} &\propto \text{tr}[\partial_\mu U \partial^\mu U^\dagger], \\ A_{\eta'}^{(H)} &= 0, \\ A_{\eta'}^{(T)} &\propto 1 - \cos \eta'. \end{aligned} \quad (21)$$

¹Note that minimizing the potential with respect to S_1 will also correct the VEV in a form that does not respect the chiral symmetry, i.e. $\langle S \rangle = f_0 \mathbf{I}_3 + a \text{tr}[M_Q] \mathbf{I}_3 + b M_Q + \mathcal{O}(M_Q^2)$. This also happens in ordinary QCD [6].

Note that at this order in M_Q there is no mixing between H and T . Solving the equation of motion for H and T yields

$$H = \frac{A^{(H)}[U]}{M_H^2}, \quad T = \frac{A^{(T)}[U]}{M_T^2}. \quad (22)$$

Substituting the solution back into $\mathcal{L}^{(H,T)}$ results in the effective Lagrangian of the nonlinear sigma model

$$\begin{aligned} \mathcal{L}_1^{(H,T)} &= -\frac{1}{2M_H^2} \text{tr}[(A_{M_Q}^{(H)} + A_{\text{kin}}^{(H)} + A_{\eta'}^{(H)})^2] \\ &\quad - \frac{1}{2M_T^2} (A_{M_Q}^{(T)} + A_{\text{kin}}^{(T)} + A_{\eta'}^{(T)})^2. \end{aligned} \quad (23)$$

By the same token, we integrate out η' , which contains the η' meson:

$$\mathcal{L}_1^{(\eta')} = -\frac{1}{2M_{\eta'}^2} (A_{M_Q}^{(\eta')} + A_{\text{kin}}^{(\eta')})^2, \quad (24)$$

where

$$\begin{aligned} A_{M_Q}^{(\eta')} &\propto \text{tr}[\chi^\dagger U + U^\dagger \chi], \\ A_{\text{kin}}^{(\eta')} &= 0. \end{aligned} \quad (25)$$

This term generates $\mathcal{O}_7$ which gives an additional contribution to the mass of the η meson, due to $\eta - \eta'$ mixing.

The low-energy constants obtained from our matching procedure for $F < N$ and $F = N$ are listed in Table I. For $F < N$, they are expanded in powers of $1/N$, while for $F = N$, the expansion is in powers of $1/\beta$. Note that the limit $\beta \rightarrow \infty$ corresponds to X becoming nondynamical, i.e. an infinitely heavy genuine Lagrange multiplier. In this regime, the vacuum structure can still be determined even when $m \rightarrow \Lambda$, provided that $\beta \gg 1$.

The large- N counting is added for the $F < N$ case, using the N -dependence in the translation from the holomorphic to the canonical scale $|\Lambda| \propto N^{1/3} \Lambda_c$ [27,28], where Λ_c is the physical QCD scale

$$|\Lambda| = \left(\frac{b_0}{8\pi^2} \right)^{\frac{b_1}{2b_0^2}} \Lambda_c, \quad (26)$$

with $b_0 = 3N - F$ and $b_1 = 6N^2 - 2NF - \frac{4F(N^2-1)}{2N}$. Note that the large N behavior for the χ PT parameters, except for L_1 and L_2 , matches estimates in the literature [6,10–12]. L_1 and L_2 are known to scale as N^1 , and are typically dominated by the exchange of vector mesons [31]. However, since the potentials considered in this paper include only pseudoscalar fields and omit vector and axial-vector resonances, the contributions to L_1 and L_2 are suppressed.

V. DYNAMICAL UP-QUARK MASS IN AMSB QCD

We are now ready to present the dynamical up-quark mass in QCD-like theories. Using Eq. (11) along with the values of L_4 , L_6 , and B_0 from Table I, we find

$$\frac{\mu_u}{\mu_d} \simeq \begin{cases} \frac{16}{147} \frac{(N-1)(7N-3)}{N(N+3)^2} \left(\frac{N+3}{3N-3} \frac{\Lambda}{m} \right)^{3/N} \frac{\mu_s}{m} & (F = 3 < N), \\ \frac{7\alpha}{18} \frac{\mu_s}{m} & (F = N = 3). \end{cases} \quad (27)$$

Note that due to our assumption $m > M_Q$, μ_u/μ_d does not diverge when $m \rightarrow 0$ since m_s must also vanish in this limit. We can see that these expressions are sizable, and in particular for the $F = N$ case $\mu_u/\mu_d = \mathcal{O}(1)$ as long as $\alpha = \mathcal{O}(1)$ and μ_s/m is chosen to be not too small.

It remains unclear whether or how these AMSB QCD results can be reliably extrapolated to real QCD in the limit where the SUSY breaking scale becomes large ($m \gg \Lambda$). One naive guess would be to set $m \sim \mathcal{O}(\Lambda)$, assuming that all states obtaining masses from SUSY breaking decouple from the dynamics once they are heavier than Λ . Note that for $F < N$, the theory allows for a self-consistent extrapolation to $m \sim \mathcal{O}(\Lambda)$, as the effective theory is fully solvable in this limit. This also holds for $F = N$ as long as $\beta \gg 1$, which is expected since X acts as a Lagrange multiplier that enforces the quantum modified constraint on the moduli space. Using such a naive extrapolation one would obtain the following numerical values:

$$\frac{\mu_u}{\mu_d} \simeq 0.01 \quad \left(\begin{array}{l} F = 3, N = 4 \\ \Lambda_c = m, \mu_s/m = 0.5 \end{array} \right). \quad (28)$$

For $F = N$ the result depends on the incalculable parameter α , usually assumed to be $\alpha \sim \mathcal{O}(1)$. As an illustrative example, choosing $\alpha = 2$ leads to

$$\frac{\mu_u}{\mu_d} \simeq 0.4 \quad \left(\begin{array}{l} F = N = 3 \\ \alpha = 2, \mu_s/m = 0.5 \end{array} \right). \quad (29)$$

We should again emphasize that these numerical values are obtained by extrapolating outside the regime of validity of our theory.

The large- N behavior of our result matches the expectation that μ_u/μ_d is suppressed in this limit, vanishing altogether at leading order in $1/N$. However, in the case of $F = N = 3$ a sizable μ_u is generated. It is probably not too surprising that the result for this case is quite different (and much bigger) than for the $F < N$ case: the $F = N$ case is

truly strongly coupled, and the origin of chiral symmetry breaking lies in the confining dynamics (while for $F < N$ the theory is actually weakly coupled). In fact, the resulting dynamical contribution for the $F = N$ case could potentially account for the entire mass of the up quark without requiring an explicit up-quark Yukawa coupling, thereby rendering the QCD θ angle unphysical.

The fundamental origin of the dynamical up-quark mass is strongly tied to the generation of the effective low-energy superpotential from which it arises, in combination with the supersymmetric mass term and the supersymmetry breaking effects. It is well-known that for $F = N - 1$ the Affleck-Dine-Seiberg superpotential is generated by weakly coupled instantons [19], while for $F < N - 1$ it is due to gaugino condensation in the unbroken $SU(N - F)$. For $F = N$ a quantum modified constraint on the moduli space arises from the dynamics, which is implemented by the Lagrange multiplier field X . The exact origin of the quantum modified constraint remains unknown; however, it is consistent with being a strongly coupled instanton effect.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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