

Nonlocal positivity bounds: Islands in terra incognita

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The requirements of unitarity and causality lead to significant constraints on the Wilson coefficients of an effective field theory (EFT) expansion, known as positivity bounds. Their standard derivation relies on the crucial assumption of polynomial boundedness on the growth of scattering amplitudes in the complex energy plane, which is a property satisfied by local quantum field theories (QFTs), and by weakly coupled string theory in the Regge regime. The scope of this work is to clarify the role of locality by deriving generalized positivity bounds under the assumption of exponential boundedness, typical of nonlocal QFTs where the Froissart-Martin bound is usually not satisfied. Using appropriately modified dispersion relations, we derive new constraints and find regions in the EFT parameter space that do not admit a local UV completion. Furthermore, we show that there exist EFTs that satisfy IR causality and at the same time can admit a nonlocal UV completion, provided that the energy scale of nonlocality is of the same order or larger than the EFT cutoff. Finally, we provide an explicit example of an exponentially bounded amplitude that satisfies partial-wave unitarity and asymptotic causality.

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Introduction. To constrain the parameter space of viable effective field theories (EFTs) it is often not necessary to know the precise form of the ultraviolet (UV) completion. In many cases it is sufficient to assume certain physical properties in the UV regime, which can be translated into mathematical requirements on the S -matrix, from which restrictions on low-energy physics can then be derived [1–53] (see also [54] for a recent review). For example, standard requirements that are usually assumed are those of Lorentz invariance, unitarity, and causality. These properties are encoded in a set of mathematical conditions, such as positivity of the imaginary part of the partial waves and analyticity of the $2 \rightarrow 2$ scattering amplitude in the complex energy plane [55–58].

A further assumption that is typically made on the UV completion is that of *locality*, which states that the growth

of the scattering amplitude is *polynomially bounded* in the complex energy plane. For local gapped theories it can be proven that the amplitude for fixed physical momentum transfer is polynomially bounded by $|s|^2$ in the whole complex plane; this is known as Froissart-Martin bound¹ [59–61]. Although for theories with massless states no rigorous general proof of this bound exists, it is common to assume that the growth of the amplitude for fixed physical t is bounded by $|s|^2$ in the whole complex plane²

$$|A(s, t \leq 0)| < B|s|^2, \quad |s| \rightarrow \infty, \quad (1)$$

where B is a subdominant factor that may depend on s .

To clarify the role of locality, it is instructive to investigate what happens when we allow for a nonpolynomial growth of the scattering amplitude, i.e., for violations of the Froissart-Martin bound (1). In particular, we are

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¹It is worth to mention that Froissart [59] and Martin [60] initially showed that the amplitude was bounded by $|s|$ in the forward limit $t = 0$. Subsequently, Jin and Martin [61] showed that the amplitude for generic physical momentum transfer $t \leq 0$ is bounded by $|s|^2$. In any case, in this work we refer to the latter as Froissart-Martin bound.

²This can be verified at least for graviton-mediated amplitudes in $d > 4$, using the method of eikonal resummation [62,63].

going to consider amplitudes that are exponentially bounded in the complex s plane for fixed physical t

$$|A(s, t \leq 0)| < C|s|^{2\alpha}e^{\alpha|s|^{2g}}, \quad |s| \rightarrow \infty, \quad (2)$$

where α and g are two positive constant parameters, and C a subdominant factor that may depend on s .

QFTs in which scattering amplitudes are characterized by this type of exponential behavior could be classified according to Jaffe's criterion [64–67]: if $g < 1/4$ the corresponding QFT is said to be *localizable*, whereas if $g \geq 1/4$ the QFT is said to be *nonlocalizable*. When $g = 0$ we recover the standard local QFT description with polynomially bounded amplitudes. The name nonlocalizability is related to the fact that the fields must be delocalized through smeared test functions in order to make the position-space correlators (such as the Wightman function) finite and well defined.

It is worth mentioning that nonlocalizable amplitudes can characterize black-hole formation via high-energy scattering, as predicted by the classicalization mechanism [67,68]. Moreover, they naturally appear in perturbative string theory [69] and in some quantum field theory (QFT)-based UV completion of Einstein's gravity, such as infinite-derivative gravity [70] and asymptotically safe gravity [71]. Therefore, a deeper understanding of exponentially bounded amplitudes could shed new light on quantum gravity and field theoretic aspects of black holes.

The aim of this paper is to understand what type of constraints can be derived on the Wilson coefficients of the low-energy EFT expansion if the UV completion is assumed to be nonlocalizable according to (2), and study their physical implications. We work in four spacetime dimensions, and adopt the mostly plus metric signature $(-+++)$ and the Natural units system ($\hbar = 1 = c$). In what follows we use the words local (nonlocal) and localizable (nonlocalizable) interchangeably.

Dispersion relations. The central object of our investigation is the elastic $2 \rightarrow 2$ scattering amplitude $A(s, t, u)$ in some gapless scalar theory, where $s = -(p_1 + p_2)^2$, $t = -(p_1 - p_3)^2$, $u = -(p_1 - p_4)^2$ are the Mandelstam variables satisfying the relation $s + t + u = 0$; p_1, p_2 are the ingoing momenta and p_3, p_4 are the outgoing momenta. In particular, s is the center-of-mass energy squared, while t is the momentum transfer squared. The scattering amplitude can be chosen to be a function of two Mandelstam variables only, e.g., $A(s, t, u) = A(s, t)$.

We assume that the low-energy behavior of the amplitude can be captured by the following EFT expansion

$$\begin{aligned} A_{\text{EFT}}(s, t) = & g_2(s^2 + t^2 + u^2) + g_3stu + g_4(s^2 + t^2 + u^2)^2 \\ & + g_5(stu)(s^2 + t^2 + u^2) \\ & + g_6(s^2 + t^2 + u^2)^3 + \dots, \end{aligned} \quad (3)$$

which is manifestly crossing symmetric; the couplings g_n are called Wilson coefficients.

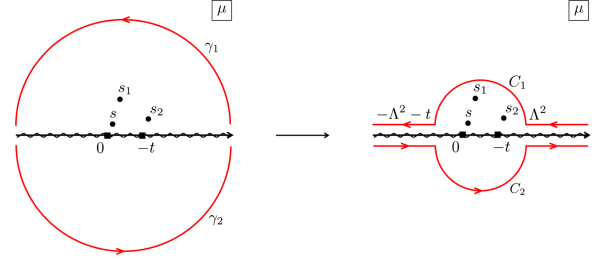


FIG. 1. Integration contour in the complex μ plane used to derive the twice-subtracted dispersion relations in Eqs. (6) and (8) (with $s_1 = s_2 = 0$). The initial contour $\gamma_1 \cup \gamma_2$ on the left is taken of infinite radius. It is then deformed into the contour on the right, where the circle $C_1 \cup C_2$ is centered at $\mu = -t/2$. The boxes denote the two branch points of the s -channel (from $\mu = 0$ to $+\infty$) and u -channel (from $\mu = -t$ to $-\infty$) branch cuts, respectively. The point s, s_1 , and s_2 are the poles in Eq. (4).

In the past literature [3,54], the Wilson coefficients have been constrained assuming that the UV completion is local, i.e., the amplitude is polynomially bounded in all complex directions in the s and t planes. In addition to these ingredients, the assumption of analyticity of the amplitude allows use to derive various dispersion relations at fixed $t \leq 0$, connecting the Wilson coefficients in A_{EFT} with positive contributions coming from the discontinuities along the branch cuts of the amplitude. Indeed, by choosing the contour $\gamma_1 \cup \gamma_2$ in Fig. 1, we can write [6,24]

$$\int_{\gamma_1 \cup \gamma_2} \frac{d\mu}{2\pi i} \frac{A(\mu, t)}{(\mu - s_1)(\mu - s_2)(\mu - s)} = 0, \quad (4)$$

where s_1 and s_2 are two subtraction points. Subsequently, by deforming the integration contour as shown in Fig. 1, Eq. (4) can be recast as

$$\begin{aligned} & \int_{C_1 \cup C_2} \frac{d\mu}{2\pi i} \frac{A(\mu, t)}{(\mu - s_1)(\mu - s_2)(\mu - s)} \\ & = \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi} \frac{\text{Disc}_s A(\mu, t)}{(\mu - s_1)(\mu - s_2)(\mu - s)} \\ & + \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi} \frac{\text{Disc}_s A(\mu, t)}{(\mu - u_1)(\mu - u_2)(\mu - u)}, \end{aligned} \quad (5)$$

where $\text{Disc}_s A(\mu, t) = \lim_{\epsilon \rightarrow 0^+} \frac{1}{2i} [A(\mu + i\epsilon, t) - A(\mu - i\epsilon, t)]$; we have used crossing symmetry to write $\text{Disc}_s A(-\mu - t, t) = -\text{Disc}_s A(\mu, t)$ and have defined analog subtraction points in terms of the Mandelstam variable u , i.e., $u_1 = -s_1 - t$ and $u_2 = -s_2 - t$. The scale Λ represents the EFT cutoff.

By computing the integral on the left-hand side of Eq. (5) through the residue theorem, we can obtain the twice-subtracted dispersion relation. For simplicity we choose $s_1 = s_2 = 0$, thus we can write

$$\begin{aligned} A_{\text{EFT}}(s, t) = & s^2 \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi} \frac{\text{Disc}_s A(\mu, t)}{\mu^2(\mu - s)} \\ & + s^2 \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi} \frac{\text{Disc}_s A(\mu, t)}{(\mu + t)^2(\mu - u)} + a(t) + sb(t), \end{aligned} \quad (6)$$

where

$$a(t) = A_{\text{EFT}}(0, t), \quad b(t) = \left. \frac{\partial A_{\text{EFT}}(s, t)}{\partial s} \right|_{s=0}, \quad (7)$$

are the two subtraction terms. Note that, we wrote $A_{\text{EFT}}(s, t)$ because the left-hand side of Eq. (6) satisfies the equation only for $s < \Lambda^2$. This aspect is what allows us to make the connection with the low-energy EFT expansion (3). In fact, more precisely, the left-hand side of Eq. (6) should read $A(s, t)|_{s < \Lambda^2} \simeq A_{\text{EFT}}(s, t)$.

If the assumption of polynomial boundedness is given up, the integral over $\gamma_1 \cup \gamma_2$ in Eq. (4) could blow up.³ This means that we need to implement a modified dispersion relation. Let us start considering the forward limit $t = 0$, and later on we will address the case of nonvanishing physical momentum transfer that will require a more careful treatment.

When $t = 0$, we can assume that the amplitude is bounded as $A(s, t=0) < s^2 \exp(-\alpha s^{2g})$ for any $s \in \mathbb{C}$, up to subdominant factors. This means that along the real axis the amplitude is exponentially suppressed in the forward limit, and can grow exponentially along some direction in the complex s plane. To write down a dispersion relation for this type of amplitude, we can adopt the following strategy. We multiply the amplitude by $\exp(\alpha s^{2g})$ and define $A'(s, t) \equiv \exp(\alpha s^{2g})A(s, t)$, which will be bounded by $|s|^2$ in the whole complex s plane. In other words, we compensate for the essential singularity at infinity, so that we can write a dispersion relation for $A'(s, t=0)$ that is analogous to Eq. (6)

$$\begin{aligned} \exp(\alpha s^{2g})A(s, 0) &= s^2 \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi} \frac{\text{Disc}_s A'(\mu, 0)}{\mu^2(\mu - s)} \\ &+ s^2 \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi} \frac{\text{Disc}_s A'(\mu, 0)}{\mu^2(\mu + s)} \\ &+ a'(0) + sb'(0), \end{aligned} \quad (8)$$

where $a'(0)$ and $b'(0)$ have the same form as those in Eq. (7), but with A replaced by A' . However, since our amplitude satisfies $A_{\text{EFT}}(0, 0) = 0$ and $\partial_s A_{\text{EFT}}(s, 0)|_{s=0} = 0$, the two subtraction terms actually vanish in the forward limit, i.e., $a'(0) = 0 = b'(0)$.

Positivity bounds for $t = 0$. Let us now derive the positivity bounds in the forward limit. Hereafter we study the nonlocalizable case with $g = 1$.

³It is worth to mention that for localizable QFTs, in which the amplitude is bounded as (2) with $g < 1/4$, it can actually be shown that the amplitude cannot grow faster than a polynomial in the complex s plane [66]. In this paper, instead, we are interested in nonlocalizable QFTs with $g > 1/4$ (in particular $g = 1$), for which a similar result does not necessarily apply. Therefore, modified dispersion relations that take into account a nonlocalizable exponential growth are needed in our case.

If we define the positive quantity

$$\left\langle \frac{1}{\mu^m} \right\rangle \equiv \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi} \text{Disc}_s A'(\mu, 0) \frac{1}{\mu^m}, \quad (9)$$

we can write

$$\begin{aligned} \langle 1/\mu^3 \rangle &= g_2, & \langle 1/\mu^5 \rangle &= 2g_4 + \alpha g_2, \\ \langle 1/\mu^7 \rangle &= 4g_6 + 2\alpha g_4 + \alpha^2 g_2/2, \\ \langle 1/\mu^9 \rangle &= g_8 + 4\alpha g_6 + \alpha^2 g_4 + \alpha^3 g_2/6, \end{aligned} \quad (10)$$

and analogous relations can be obtained for higher-order Wilson coefficients. In addition to the positivity relations above, we also have the “relaxing” inequality $\langle 1/\mu^n \rangle < \frac{1}{\Lambda^2} \langle 1/\mu^{n-1} \rangle$, and the two Cauchy-Schwarz inequalities $\langle 1/\mu^5 \rangle^2 < \langle 1/\mu^3 \rangle \langle 1/\mu^7 \rangle$ and $\langle 1/\mu^7 \rangle^2 < \langle 1/\mu^5 \rangle \langle 1/\mu^9 \rangle$. This set of inequalities provides,

$$\begin{aligned} 0 < 2\tilde{g}_4 + \tilde{\alpha} < 1, & \quad 0 < 4\tilde{g}_6 + 2\tilde{\alpha}\tilde{g}_4 + \tilde{\alpha}^2/2 < 2\tilde{g}_4 + \tilde{\alpha}, \\ 4\tilde{g}_6 + 2\tilde{\alpha}\tilde{g}_4 + \tilde{\alpha}^2/2 &\geq (2\tilde{g}_4 + \tilde{\alpha})^2, \end{aligned} \quad (11)$$

where we have defined the dimensionless quantities $\tilde{\alpha} \equiv \alpha \Lambda^4$, $\tilde{g}_4 \equiv g_4 \Lambda^4/g_2$, $\tilde{g}_6 \equiv g_6 \Lambda^8/g_2$, and so on. In the $\alpha \rightarrow 0$ limit, we recover the well-known local case [7].

The regions in the $\tilde{g}_4 - \tilde{g}_6$ plane that are allowed by the constraints in (11) are shown in Fig. 2(a) for different values of $\tilde{\alpha}$. We can track how the allowed region changes as a function of $\tilde{\alpha}$, going from $\tilde{\alpha} = 0$ to $\tilde{\alpha} = 1.0$. In particular, we found that the regions corresponding to $\tilde{\alpha} \geq \sqrt{2} - 1 \approx 0.41$ have no overlap with the local case $\tilde{\alpha} = 0$: this can happen because the dependence on α in (11) shifts the intervals for the interaction couplings and creates regions of no overlap with the case $\alpha = 0$. We also verified that using two Cauchy-Schwarz inequalities to derive the bounds on $\tilde{g}_4$ and $\tilde{g}_6$ is already sufficient to obtain the most optimal constraints, in agreement with the analyses performed in local cases [7,72]. In Fig. 2(b) we plotted the allowed regions in the three-dimensional parameter space spanned by $\tilde{g}_4, \tilde{g}_6, \tilde{g}_8$.

Positivity bounds for $t < 0$. The above derivation of positivity bounds does not give us any information about the couplings in the EFT amplitude (3) that disappear when $t = 0$, such as g_3 . Constructing the dispersion relation for $t < 0$ requires more details about the structure of the UV amplitude. We assume the following crossing-symmetric form, which is consistent with partial-wave unitarity (see the Supplemental Material [73]):

$$A(s, t, u) = F(t, u)e^{-\alpha s^2} + F(s, u)e^{-\alpha t^2} + F(s, t)e^{-\alpha u^2}, \quad (12)$$

where $F(s, t)$ is bounded by s^2 in the entire complex plane. We can then write the $s \leftrightarrow u$ crossing-symmetric representation as

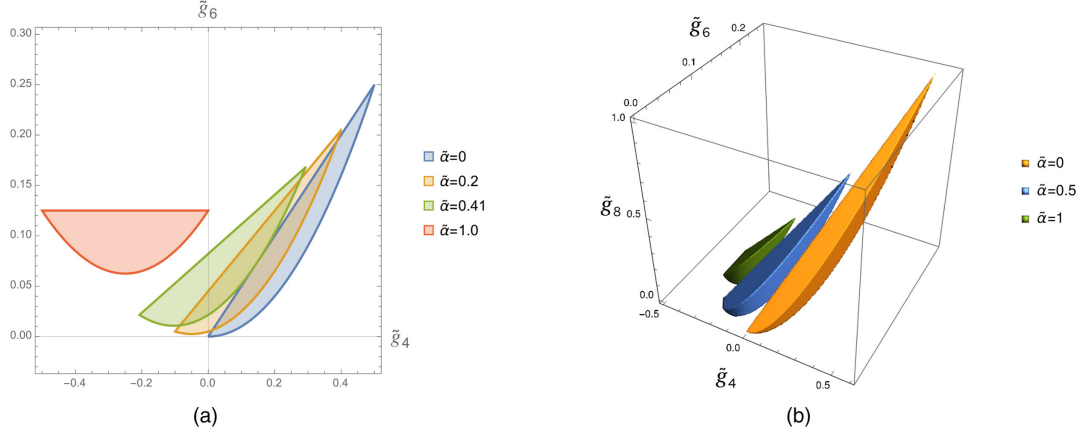


FIG. 2. (a) Allowed regions in the $\tilde{g}_4 - \tilde{g}_6$ plane when $\tilde{\alpha} = 0, 0.2, 0.41, 1$. (b) Allowed regions in the three-dimensional parameter space spanned by $\tilde{g}_4, \tilde{g}_6$ and $\tilde{g}_8$ when $\tilde{\alpha} = 0, 0.5, 1$.

$$A_{\text{EFT}}(s, t) = e^{-\alpha s^2} s^2 \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi} \frac{\text{Disc}_s A'(\mu, t)}{\mu^2(\mu - s)} + e^{-\alpha u^2} u^2 \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi} \frac{\text{Disc}_s A'(\mu, t)}{\mu^2(\mu - u)} + a(t), \quad (13)$$

where $A'(\mu, t) = A(\mu, t)e^{\alpha\mu^2}$. The expression for $a(t)$ can be derived by imposing full crossing symmetry and we have $a(0) = 0$. Note that, there is no subtraction term $sb(t)$, otherwise the $s \leftrightarrow u$ crossing symmetry would be broken.

The state-of-the-art technique to derive the bound on g_3 is based on the theory of moments and is described in [72,74]. We briefly review the formalism in the Supplemental Material [73] and also show that it does not depend on the particular definition of the brackets. In other words, up to additional α -dependent terms in some of the relevant relations, the constraints in the $g_3 - g_4$ plane in our nonlocal model can be derived by following the same steps as in the local case [72,74].

The result is given by the following two sets of inequalities:

$$\text{I:} \begin{cases} -\sqrt{\frac{427}{3}}(2\tilde{g}_4 + \tilde{\alpha}) < \tilde{g}_3 < 3\sqrt{2\tilde{g}_4 + \tilde{\alpha}}, \\ -\frac{\tilde{\alpha}}{2} \leq \tilde{g}_4 \leq \frac{961}{2562} - \frac{\tilde{\alpha}}{2}, \end{cases} \quad (14)$$

$$\text{II:} \begin{cases} -\frac{62}{6}(2\tilde{g}_4 + \tilde{\alpha}) - \sqrt{\frac{320}{9}[(2\tilde{g}_4 + \tilde{\alpha}) - (2\tilde{g}_4 + \tilde{\alpha})^2]} < \tilde{g}_3 < 3\sqrt{2\tilde{g}_4 + \tilde{\alpha}}, \\ \frac{961}{2562} - \frac{\tilde{\alpha}}{2} \leq \tilde{g}_4 \leq \frac{1-\tilde{\alpha}}{2}. \end{cases} \quad (15)$$

In Fig. 3(a) we have shown the allowed regions in the $\tilde{g}_3 - \tilde{g}_4$ plane given by the bounds (14) and (15), for different values of $\tilde{\alpha}$. The local constraint is just shifted down allowing for negative values of g_4 , which are forbidden in the standard local case.

IR causality bounds. The requirement of causality for the low-energy theory imposes a set of bounds on the Wilson coefficients of the EFT expansion (3), that must hold independently of the type of UV completion. Causality can be probed by considering a process in which a plane wave propagates on a classical background created by some local source [75–88], and its validity demands the absence of superluminal propagation. However, the uncertainty

principle prevents us from detecting very small time advances in the signal propagation, i.e., $|\Delta T| \lesssim 1/\omega$, where ω is the energy of the propagating wave. Therefore, unobservable causality violations could still be physically allowed because, thanks to their quantum stochastic nature, they do not sum up into macroscopic time advances in the signal propagation [75,84,89,90]. This means that we can physically admit sufficiently short time advances that are within the uncertainty principle, i.e., time delays ΔT can also get very small negative values

$$\Delta T > -1/\omega. \quad (16)$$

This condition is weaker than asymptotic causality (also known as Gao-Wald condition [91]) which demands a

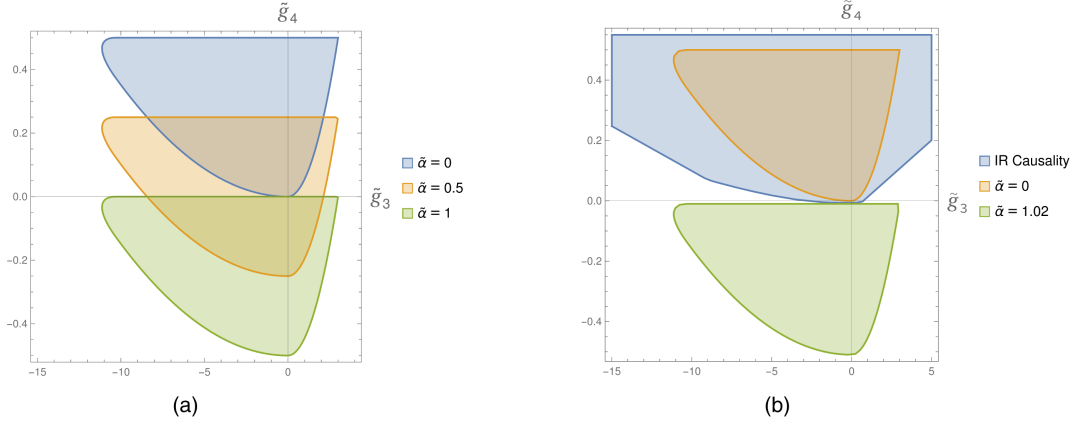


FIG. 3. (a) Allowed region in the $\tilde{g}_3 - \tilde{g}_4$ plane for $\tilde{\alpha} = 0, 0.5, 1$ from (14) and (15). (b) Allowed regions by IR causality (blue region) [75] and positivity bounds on $\tilde{g}_3$ and $\tilde{g}_4$ in both local ($\tilde{\alpha} = 0$) and nonlocal ($\tilde{\alpha} = 1.02$) cases. For $\tilde{\alpha} < 1.02$ there is an overlap between constraints from positivity bounds and IR causality, while larger values of $\tilde{\alpha}$ are in contrast with IR causality.

positive time delay, i.e. $\Delta T > 0$.⁴ From a quantum mechanical point of view, the latter might be a too strong requirement, for example it is known to exclude higher derivative interactions in the EFT expansion. Moreover, the weaker condition in Eq. (16) is enough to guarantee the absence of closed timelike curves [3,92].

A comprehensive study of causality bounds in the case of shift-symmetric scalar theories based on the condition in Eq. (16) was performed in [75]. We plotted the causality constraints on $\tilde{g}_3$ and $\tilde{g}_4$ derived in [75] together with our bounds in Fig. 3(b). We find the following results. (i) IR causality bounds are noncompact and significantly weaker than local positivity constraints on the Wilson coefficients, thus leaving a place in the space of EFT parameters for causal EFTs whose UV completions can *only* be nonlocal. (ii) Exponentially bounded amplitudes allow for negative values of g_4 that are compatible with IR causality and a nonlocal UV completion. (iii) A causal EFT with cutoff Λ can admit a nonlocal UV completion only if the scale of nonlocality is of the same order or larger than the EFT cutoff, i.e.,

$$\tilde{\alpha} = \alpha\Lambda^4 \leq 1.02 \Leftrightarrow M \geq (1.02)^{1/4} \Lambda \simeq \Lambda, \quad (17)$$

where the second inequality is written in terms of the nonlocal energy scale $M \equiv 1/\alpha^{1/4}$.

It is worth to mention that UV completions whose amplitudes are compatible with exponential boundedness, unitarity, analyticity, and IR causality, may in principle still show some causality violation at high energies. Our study cannot say anything about UV causality, but this is an aspect that will be investigated in future work.

However, we have managed to construct exponentially bounded, unitary amplitudes that satisfy UV causality, in

particular the condition of asymptotic causality. An example is given by $A(s, t, u) = C + [F(s) + F(t) + F(u)]$, where

$$F(z) = -\sqrt{\alpha} z_2 F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\alpha z^2\right) + \frac{\sqrt{\pi}}{2} \text{erf}(\sqrt{\alpha} z) [\log(\sqrt{\alpha} z) + i\pi], \quad (18)$$

and C is some constant. In the Supplemental Material [73] we explicitly show that partial-wave unitarity and asymptotic causality are satisfied by this amplitude.

Conclusions. In this paper, we derived generalized positivity bounds on the Wilson coefficients of the EFT expansion, assuming that the low-energy theory admits a nonlocal UV completion according to which the amplitudes are exponentially bounded in the complex s and t planes.

We showed that the allowed regions in the space of EFT parameters can either overlap or not with those in the local case, depending on the value of the ratio of the nonlocality energy scale to EFT cutoff. In particular, we found regions where the corresponding EFTs do not admit a local UV completion, but only a nonlocal one. We also compared the new positivity bounds with low-energy causality constraints [75] and found that causal EFTs can admit nonlocal UV completions only if the scale of nonlocality—defined as $M = \alpha^{-1/4}$ —is of the same order or larger than the EFT cutoff Λ .

To support our main findings, we provided an explicit example that serves as a proof-of-concept for the existence of amplitudes that satisfy all the standard desired properties (i.e., partial-wave unitarity, causality, crossing symmetry) except for the polynomial boundedness.

Understanding nonlocalizable amplitudes can be relevant for describing field-theoretic aspects of black-hole formation via high-energy scattering [67,68], and for formulating QFT-based UV completions of general relativity, such as infinite-derivative gravity [70] or asymptotically safe gravity [71].

⁴This condition is known to imply analyticity of the amplitudes in local theories [56]. However, to the best of our knowledge, it is still an open question whether the polynomial boundedness follows from it, i.e., whether locality and asymptotic causality imply each other.

Indeed, it would also be interesting to compare our positivity bounds with constraints derived in the context of asymptotically safe theories [93–96]. Exploring and classifying different types of viable exponentially bounded amplitudes could be an interesting direction for future studies. This has the potential to shed new light on the form of the quantum effective action in UV-complete gravitational theories, as well as on the question of the uniqueness of string theory as the only consistent UV completion of general relativity.

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Data availability. No data were created or analyzed in this study.

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