

# Pole trajectories from $S$ - and $P$ -wave $D\bar{D}^*$ interactions

Xiao-Xiao Chen<sup>✉</sup>, Zuo-Ming Ding<sup>✉</sup>, and Jun He<sup>✉\*</sup>

*School of Physics and Technology, Nanjing Normal University, Nanjing 210097, China*



(Received 21 April 2025; accepted 28 May 2025; published 10 June 2025)

In this work, we investigate the  $S$ - and  $P$ -wave interactions of the  $D\bar{D}^*$  system within the framework of the quasipotential Bethe-Salpeter equation, with the aim of exploring possible molecular states and their corresponding pole trajectories. The interaction potentials are constructed using the one-boson-exchange model, incorporating the exchanges of  $\pi$ ,  $\eta$ ,  $\rho$ ,  $\omega$ ,  $\sigma$ , and  $J/\psi$  mesons, based on heavy quark effective Lagrangians. The poles of the scattering amplitude are analyzed and their evolution on two Riemann sheets is systematically traced as the cutoff parameter increases up to 5 GeV. We identify four molecular states arising from the  $S$ -wave  $D\bar{D}^*$  interaction. Among them, the bound state with quantum numbers  $I^G(J^{PC}) = 0^+(1^{++})$  corresponds well to the experimentally observed  $X(3872)$ , while its isovector partner with  $I^G(J^{PC}) = 1^-(1^{++})$  is found to exist only as a virtual state. Additionally, a  $0^-(1^{+-})$  state appears as a bound state. The isovector  $1^{+-}$  state, which may be associated with the  $Z_c(3900)$ , is observed to evolve from a bound state to a virtual state as the interaction strength decreases. For the  $P$ -wave  $D\bar{D}^*$  interaction, the structure  $G(3900)$  recently observed at BESIII is likely connected to a  $0^-(1^{--})$  state. A  $0^+(0^{++})$  state is also predicted in this channel. Both can appear as either resonance or bound/virtual state depending on the interaction strength.

DOI: [10.1103/36vy-r2z8](https://doi.org/10.1103/36vy-r2z8)

## I. INTRODUCTION

Since 2003, an increasing number of XYZ particles have been observed in experiments [1–3]. The discovery of these exotic states has challenged the traditional quark model's classification and interpretation of hadrons. Consequently, various theoretical frameworks have been proposed to explore the origin and internal structure of the XYZ particles, including molecular states, and compact multi-quark configurations, hadro-quarkonium [4–13]. Among these, the molecular state interpretation has gained particular attention, especially given that many observed XYZ states lie close to hadronic thresholds [1].

In the original molecular picture, a molecular state is regarded as a loosely bound system of two hadrons, analogous to the deuteron [14]. The  $X(3872)$  was the first XYZ particle to be experimentally discovered, and its proximity to the  $D\bar{D}^*$  threshold has led to widespread interpretation as a weakly bound  $D\bar{D}^*$  molecular state [15–20]. In Ref. [21], the one-boson-exchange model was employed to investigate the possibility of  $X(3872)$  being a  $D\bar{D}^*$  molecular state with quantum numbers  $J^{PC} = 1^{++}$ .

The observation of hidden-charm pentaquarks at LHCb further supports this picture, as they form a rich spectrum of  $S$ -wave molecular states with masses slightly below the corresponding thresholds. However, the discovery of states, such as  $Z_c(3900)$ ,  $Z_b(10610)$ , and  $Z_b(10650)$ , poses a challenge to this simple molecular scenario [22–29]. Although their masses lie close to the  $D\bar{D}^*$  and  $B^{(*)}\bar{B}^*$  thresholds, respectively, they are slightly above the thresholds, which makes it difficult to interpret them as loosely bound molecular states in the traditional sense.

According to standard scattering theory, a bound state generated from a single-channel interaction appears as a pole on the real axis below the threshold in the complex energy plane. In addition to bound states, virtual states located on the real axis below threshold but on the second Riemann sheet and resonances positioned above the threshold in the complex plane can also arise [30]. All three types of states, bound, virtual, and resonance, stem from attractive interactions; however, forming a bound state generally requires a stronger attraction. A representative example is the  $\Lambda(1405)$ , which is widely interpreted as a dynamically generated state with a two-pole structure. Notably, the higher-energy  $\bar{K}N$  channel exhibits stronger attraction, sufficient to support a bound state, whereas the lower-energy  $\pi\Sigma$  channel has relatively weaker attraction, though still strong enough to generate a resonance [31,32]. In Refs. [33,34], detailed fits to experimental data suggest that the  $Z_c(3900)$  may originate from a virtual state. Although a

\*Contact author: [junhe@njnu.edu.cn](mailto:junhe@njnu.edu.cn)

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP<sup>3</sup>.

virtual state lies below the threshold, its location on the second Riemann sheet allows it to produce observable enhancements above the threshold, rather than below. In Ref. [35], a survey of heavy-antiheavy hadronic molecules was also performed, considering both bound and virtual states. Through a simultaneous analysis of BESIII data on the processes  $e^+e^- \rightarrow \gamma(D^0\bar{D}^{*0}\pi^0/J/\psi\pi^+\pi^-)$  and LHCb data on  $B^+ \rightarrow K^+(J/\psi\pi^+\pi^-)$ , an isovector partner of the  $X(3872)$  has also been identified as a virtual state in Ref. [36]. These findings further support the idea that many near-threshold structures, rather than being bound states, may instead be virtual states even resonance arising from strong but insufficiently attractive interactions.

In contrast to the well-established  $S$ -wave molecular states, the existence of  $P$ -wave molecular states remains a topic of ongoing debate. A possible new structure, referred to as  $G(3900)$ , was recently observed by the BESIII Collaboration through high-precision measurements of the Born cross section in the process  $e^+e^- \rightarrow D\bar{D}$  [37], which was also observed at the  $B$  factories [38,39]. This structure lies near the  $D\bar{D}^*$  threshold but possesses spin parity  $J^P = 1^-$ , making it a strong candidate for a  $P$ -wave molecular state. Recent theoretical studies and model calculations suggest that  $G(3900)$  can be interpreted as the first  $P$ -wave  $D\bar{D}^*$  resonance [22,40]. In addition, the possible existence of other  $D\bar{D}^*$  molecular states with spin parities  $0^-$  and  $1^-$  has also been investigated in the literature [22].

In this work, we carry out a systematic investigation of possible molecular states arising from both  $S$ - and  $P$ -wave interactions of the  $D\bar{D}^*$  system, including those potentially related to the newly observed  $G(3900)$  and the isovector partner of the  $X(3872)$ . Our previous studies have primarily focused on analyzing  $D\bar{D}^*$  interactions in the  $S$ -wave channel to identify molecular candidates and explore their correspondence with experimentally observed states

[29,34,41–43]. However, the trajectories of scattering amplitude poles generated by such interactions have not been thoroughly examined and remain scarcely discussed in the existing literature.

The study of pole trajectories, i.e., how poles in the complex energy plane evolve with variations in interaction parameters, can provide deeper insight into the internal structures of near-threshold states. As demonstrated in Ref. [44], analyzing the movement of poles as a function of interaction strength is particularly informative for distinguishing  $S$ -wave and  $P$ -wave dynamics and for identifying virtual, bound, or resonant states. Motivated by this, we systematically explore the pole trajectories associated with  $D\bar{D}^*$  interactions using the quasipotential Bethe-Salpeter equation (qBSE) approach in combination with a one-boson-exchange (OBE) model. To reduce complexity, the analysis is restricted to a single-channel framework. By varying the cutoff parameter, we search for poles corresponding to different isospins,  $G$  parities, and spin parities (corresponding to  $S$  and  $P$  waves) in the  $D\bar{D}^*$  system.

This article is organized as follows. Following the introduction, Sec. II presents the theoretical framework, including the flavor wave functions, effective Lagrangians, construction of the interaction potentials, and a brief overview of the qBSE approach. The numerical results, with particular focus on the pole trajectories arising from the  $S$ - and  $P$ -wave  $D\bar{D}^*$  interactions, are provided in Sec. III. Finally, Sec. IV summarizes our findings and discusses their implications for the understanding of near-threshold exotic states.

## II. THEORETICAL FRAME

First, we construct the flavor wave function of  $D\bar{D}^*$  state under  $SU(3)$  symmetry as [29,45],

$$\begin{aligned} |X_{D\bar{D}^*}^0\rangle_{I=0} &= \frac{1}{2}[(|D^{*+}D^- \rangle + |D^{*0}\bar{D}^0\rangle) + c(|D^+D^{*-}\rangle + |D^0\bar{D}^{*0}\rangle)], \\ |X_{D\bar{D}^*}^0\rangle_{I=1} &= \frac{1}{2}[(|D^{*+}D^- \rangle - |D^{*0}\bar{D}^0\rangle) + c(|D^+D^{*-}\rangle - |D^0\bar{D}^{*0}\rangle)], \\ |X_{D\bar{D}^*}^+\rangle_{I=1} &= \frac{1}{\sqrt{2}}(|D^{*+}\bar{D}^0\rangle + c|D^+\bar{D}^{*0}\rangle), \\ |X_{D\bar{D}^*}^-\rangle_{I=1} &= \frac{1}{\sqrt{2}}(|D^{*-}\bar{D}^0\rangle + c|D^-\bar{D}^{*0}\rangle), \end{aligned} \quad (1)$$

where  $c = \pm$  corresponds to  $C$  parity  $C = \mp$ , respectively.  $C$  parity cannot be defined in the isovector state, and  $G$  parity is used instead.

We use the one-boson-exchange model to construct the interaction potential, which includes the  $\pi$ ,  $\eta$ ,  $\rho$ ,  $\omega$ , and  $\sigma$  exchanges. Considering the chiral symmetry, we first give the Lagrangian for the coupling vertices of heavy meson-heavy meson and exchange light meson as follows [46–50],

$$\begin{aligned}
\mathcal{L}_{\mathcal{P}^* \mathcal{P}^* \mathbb{P}} &= -i \frac{2g}{f_\pi} \varepsilon_{\alpha\mu\nu\lambda} v^\alpha \mathcal{P}_b^{*\mu} \mathcal{P}_a^{*\lambda\dagger} \partial^\nu \mathbb{P}_{ba} + i \frac{2g}{f_\pi} \varepsilon_{\alpha\mu\nu\lambda} v^\alpha \tilde{\mathcal{P}}_a^{*\mu\dagger} \tilde{\mathcal{P}}_b^{*\lambda} \partial^\nu \mathbb{P}_{ab}, \\
\mathcal{L}_{\mathcal{P}^* \mathcal{P} \mathbb{P}} &= -\frac{2g}{f_\pi} (\mathcal{P}_b \mathcal{P}_{a\lambda}^\dagger + \mathcal{P}_{b\lambda}^* \mathcal{P}_a^\dagger) \partial^\lambda \mathbb{P}_{ba} + \frac{2g}{f_\pi} (\tilde{\mathcal{P}}_{a\lambda}^* \tilde{\mathcal{P}}_b + \tilde{\mathcal{P}}_a^\dagger \tilde{\mathcal{P}}_{b\lambda}^*) \partial^\lambda \mathbb{P}_{ab}, \\
\mathcal{L}_{\mathcal{P} \mathcal{P} \mathbb{V}} &= -\sqrt{2} \beta g_V \mathcal{P}_b \mathcal{P}_a^\dagger v \cdot \mathbb{V}_{ba} + \sqrt{2} \beta g_V \tilde{\mathcal{P}}_a^\dagger \tilde{\mathcal{P}}_b v \cdot \mathbb{V}_{ab}, \\
\mathcal{L}_{\mathcal{P}^* \mathcal{P} \mathbb{V}} &= -2\sqrt{2} \lambda g_V v^\lambda \varepsilon_{\lambda\mu\alpha\beta} (\mathcal{P}_b \mathcal{P}_a^{*\mu\dagger} + \mathcal{P}_b^{*\mu} \mathcal{P}_a^\dagger) (\partial^\alpha \mathbb{V}^\beta)_{ba} - 2\sqrt{2} \lambda g_V v^\lambda \varepsilon_{\lambda\mu\alpha\beta} (\tilde{\mathcal{P}}_a^{*\mu\dagger} \tilde{\mathcal{P}}_b + \tilde{\mathcal{P}}_a^\dagger \tilde{\mathcal{P}}_b^{*\mu}) (\partial^\alpha \mathbb{V}^\beta)_{ab}, \\
\mathcal{L}_{\mathcal{P}^* \mathcal{P}^* \mathbb{V}} &= \sqrt{2} \beta g_V \mathcal{P}_b^* \cdot \mathcal{P}_a^{\dagger\dagger} v \cdot \mathbb{V}_{ba} - i 2\sqrt{2} \lambda g_V \mathcal{P}_b^{*\mu} \mathcal{P}_a^{*\nu\dagger} (\partial_\mu \mathbb{V}_\nu - \partial_\nu \mathbb{V}_\mu)_{ba} - \sqrt{2} \beta g_V \tilde{\mathcal{P}}_a^{*\mu\dagger} \tilde{\mathcal{P}}_b^{*\nu} v \cdot \mathbb{V}_{ab} \\
&\quad - i 2\sqrt{2} \lambda g_V \tilde{\mathcal{P}}_a^{*\mu\dagger} \tilde{\mathcal{P}}_b^{*\nu} (\partial_\mu \mathbb{V}_\nu - \partial_\nu \mathbb{V}_\mu)_{ab}, \\
\mathcal{L}_{\mathcal{P} \mathcal{P} \sigma} &= -2g_s \mathcal{P}_b \mathcal{P}_b^\dagger \sigma - 2g_s \tilde{\mathcal{P}}_b \tilde{\mathcal{P}}_b^\dagger \sigma, \\
\mathcal{L}_{\mathcal{P}^* \mathcal{P}^* \sigma} &= 2g_s \mathcal{P}_b^* \cdot \mathcal{P}_b^{\dagger\dagger} \sigma + 2g_s \tilde{\mathcal{P}}_b^* \cdot \tilde{\mathcal{P}}_b^{\dagger\dagger} \sigma,
\end{aligned} \tag{2}$$

where the velocity  $v$  should be replaced by  $i \partial / 2 \sqrt{m_i m_f}$  with the  $m_{i,f}$  being the mass of the initial or final heavy meson.  $\mathcal{P}^{(*)T} = (D^{(*)0}, D^{(*)+}, D_s^{(*)+})$  and  $\tilde{\mathcal{P}}^{(*)T} = (\bar{D}^{(*)0}, \bar{D}^{(*)-}, \bar{D}_s^{(*)-})$ , and satisfy the normalization relations  $\langle 0 | \mathcal{P} | Q \bar{q} (0^-) \rangle = \sqrt{M_{\mathcal{P}}}$  and  $\langle 0 | \mathcal{P}_\mu^* | Q \bar{q} (1^-) \rangle = \epsilon_\mu \sqrt{M_{\mathcal{P}^*}}$ . The pseudoscalar and the  $\mathbb{P}$  and  $\mathbb{V}$  are the pseudoscalar and vector matrices

$$\mathbb{P} = \begin{pmatrix} \frac{\sqrt{3}\pi^0 + \eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\sqrt{3}\pi^0 + \eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \quad \mathbb{V} = \begin{pmatrix} \frac{\rho^0 + \omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^0 + \omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \tag{3}$$

In this Lagrangians, coupling constants— $g_s$ ,  $g$ ,  $\beta$ , and  $\lambda$ —are involved, along with the pion decay constant  $f_\pi = 132$  MeV and the parameter  $g_V = m_\rho / f_\pi = 5.9$ . Specifically, due to spontaneously broken chiral symmetry [51],  $g_s$  is related to the coupling  $\tilde{g}$  for the process  $D(0^+) \rightarrow D(0^-) \pi$  via  $g_s = \tilde{g} / (2\sqrt{6})$ , with  $\tilde{g} = 3.73$  taken from Ref. [52]. The  $D^* D^* \pi$  coupling constant is assumed to be equal to that of the  $D^* D \pi$  vertex,  $g = 0.59$ , which is extracted from the decay width of  $D^{*+}$  [53]. The value of  $\beta = 0.9$  is fixed based on vector meson dominance [53], and  $\lambda = 0.56 \text{ GeV}^{-1}$  is obtained by comparing form factors from light-cone sum rules and lattice QCD [53]. These values are widely accepted in the literature and have been adopted in many phenomenological studies [54,55]. For clarity, we have listed them in Table I.

This work also considers the couplings of heavy-light charmed mesons to  $J/\psi$ , and writes the correlation Lagrangians as [50,56],

$$\begin{aligned}
\mathcal{L}_{D_{(s)}^* \bar{D}_{(s)}^* J/\psi} &= -i g_{D_{(s)}^* D_{(s)}^* \psi} (\psi \cdot \bar{D}^* \overleftrightarrow{\partial} \cdot D^* \\
&\quad - \psi^\mu \bar{D}^* \cdot \overleftrightarrow{\partial}^\mu D^* + \psi^\mu \bar{D}^* \cdot \overleftrightarrow{\partial}^\tau D^{*\mu}), \\
\mathcal{L}_{D_{(s)}^* \bar{D}_{(s)} J/\psi} &= g_{D_{(s)}^* D_{(s)} \psi} \epsilon_{\beta\mu\alpha\tau} \partial^\beta \psi^\mu (\bar{D} \overleftrightarrow{\partial}^\tau D^{*\alpha} + \bar{D}^{*\alpha} \overleftrightarrow{\partial}^\tau D), \\
\mathcal{L}_{D_{(s)} \bar{D}_{(s)} J/\psi} &= i g_{D_{(s)} D_{(s)} \psi} \psi \cdot \bar{D} \overleftrightarrow{\partial} D,
\end{aligned} \tag{4}$$

where the couplings are related to a single parameter  $g_2$  as

$$\frac{g_{D^* D^* \psi}}{m_{D^*}} = \frac{g_{D_{(s)} D_{(s)} \psi}}{m_D} = g_{D_{(s)}^* D_{(s)} \psi} = 2g_2 \sqrt{m_\psi}, \tag{5}$$

with  $g_2 = \sqrt{m_\psi} / (2m_D f_\psi)$ . The coupling constants used in the calculation are shown in Table I below.

With the above Lagrangians, the OBE interaction potentials can be constructed using standard Feynman rules as follows [29,34,43,57]:

$$\mathcal{V}_{\mathbb{P},\sigma} = I_i^{(d,c)} \Gamma_1 \Gamma_2 P_{\mathbb{P},\sigma} f(q^2), \quad \mathcal{V}_{\mathbb{V}} = I_i^{(d,c)} \Gamma_{1\mu} \Gamma_{2\nu} P_{\mathbb{V}}^{\mu\nu} f(q^2), \tag{6}$$

where  $\Gamma_1$  and  $\Gamma_2$  denote the vertices at the upper and lower ends of the exchanged boson, respectively. The explicit expressions for these vertices can be found in our previous works [41]. To incorporate the off-shell effects of the

TABLE I. The coupling constants adopted in the calculation, which are cited from the literature [52–55]. The  $\lambda$  is in the units of  $\text{GeV}^{-1}$ . The  $f_{\pi,\psi}$  are in the units of MeV. Others are in the units of 1.

| $\beta$ | $g$  | $g_V$ | $\lambda$ | $g_s$ | $f_\pi$ | $f_\psi$ |
|---------|------|-------|-----------|-------|---------|----------|
| 0.9     | 0.59 | 5.9   | 0.56      | 0.76  | 132     | 405      |

TABLE II. The isospin factors  $I_i^d$  and  $I_i^c$  for direct and cross diagrams and different exchange mesons.

|                  | $I_i^d$  |          |                |               |          |          | $I_i^c$         |                |                 |                |          |          |
|------------------|----------|----------|----------------|---------------|----------|----------|-----------------|----------------|-----------------|----------------|----------|----------|
|                  | $\pi$    | $\eta$   | $\rho$         | $\omega$      | $\sigma$ | $J/\psi$ | $\pi$           | $\eta$         | $\rho$          | $\omega$       | $\sigma$ | $J/\psi$ |
| $[D\bar{D}^*]^T$ | $\cdots$ | $\cdots$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | 1        | 1        | $-\frac{1}{2}c$ | $\frac{1}{6}c$ | $-\frac{1}{2}c$ | $\frac{1}{2}c$ | $\cdots$ | $c$      |
| $[D\bar{D}^*]^S$ | $\cdots$ | $\cdots$ | $\frac{3}{2}$  | $\frac{1}{2}$ | 1        | 1        | $\frac{3}{2}c$  | $\frac{1}{6}c$ | $\frac{3}{2}c$  | $\frac{1}{2}c$ | $\cdots$ | $c$      |

exchanged mesons and the extended structure of the interaction vertices, a monopole form factor is introduced:  $f(q^2) = \Lambda_e^2/(q^2 - \Lambda_e^2)$ , as adopted in Refs. [43,58]. Physically, the form factor introduced at each vertex encodes the compositeness of heavy mesons when probed by exchange mesons, reflecting the finite size effects of the hadrons involved. Its precise functional form and the associated cutoff parameter  $\Lambda$  are not well constrained by either experimental data or first-principles calculations. As is commonly done in OBE models, we treat  $\Lambda$  as a free parameter that can absorb certain model-dependent uncertainties. All couplings in the OBE potential are taken from effective Lagrangians with phenomenologically fixed constants and are independent of  $\Lambda$ . In our framework, the short-range part of the interaction is modeled by including exchanges of heavier mesons such as  $\rho$ ,  $\omega$ , and  $J/\psi$ , rather than by introducing explicit local four-point contact terms, as typically done in effective field theory. The omission of such contact interactions—whose couplings are in general cutoff-dependent—implies that our cutoff variation only partially probes the full parameter space of the model. Nevertheless, it is expected that this approach provides a reasonable first approximation to the short-range dynamics within the limitations of the OBE framework.

When discussing the interaction of double heavy flavor state  $D\bar{D}^*$ , the coefficients are collected as flavor factors. In Table II, flavor factors  $I_i^d$  and  $I_i^c$  of certain meson exchange  $i$  of certain interaction are listed for direct and cross diagrams, respectively.

In the Bethe-Salpeter equation, the potential kernel is the central part that describes the interaction between two particles. The scattering amplitude can be obtained by inserting potential kernel into the Bethe-Salpeter equation. Through quasipotential approximation and partial wave decomposition, the four-dimensional integral equation of Minkowski space is reduced to a one-dimensional equation with spin parity  $J^P$ , as follows [29,34,43,59,60],

$$i\mathcal{M}_{\lambda'\lambda}^{J^P}(\mathbf{p}', \mathbf{p}) = i\mathcal{V}_{\lambda'\lambda}^{J^P}(\mathbf{p}', \mathbf{p}) + \sum_{\lambda''} \int \frac{\mathbf{p}''^2 d\mathbf{p}''}{(2\pi)^3} \cdot i\mathcal{V}_{\lambda'\lambda''}^{J^P}(\mathbf{p}', \mathbf{p}'') G_0(\mathbf{p}'') i\mathcal{M}_{\lambda''\lambda}^{J^P}(\mathbf{p}'', \mathbf{p}), \quad (7)$$

where the summation is limited to nonnegative helicities  $\lambda''$ . The propagator  $G_0(\mathbf{p}'')$  is reduced from its original

four-dimensional form using the quasipotential approximation and takes the form

$$G_0 = \frac{\delta^+(p_h''^2 - m_h^2)}{p_l''^2 - m_l^2} = \frac{\delta^+(p_h''^0 - E_h(\mathbf{p}''))}{2E_h(\mathbf{p}'')[(W - E_h(\mathbf{p}''))^2 - E_l^2(\mathbf{p}'')]} \quad (8)$$

Following the spectator approximation adopted in this work, the heavier particle (denoted as  $h$ ) in a given channel is placed on shell [61], satisfying  $p_h''^0 = E_h(\mathbf{p}'') = \sqrt{m_h^2 + \mathbf{p}''^2}$ . The energy of the lighter particle (denoted as  $l$ ) is then given by  $p_l''^0 = W - E_h(\mathbf{p}'')$ , where  $W$  is the total energy in the center-of-mass frame. Here and throughout the paper, the three-momentum in the center-of-mass frame is defined as  $\mathbf{p} = |\mathbf{p}|$ . The partial wave potential can be expressed by the potential in Eq. (6) as follows

$$\mathcal{V}_{\lambda'\lambda}^{J^P}(\mathbf{p}', \mathbf{p}) = 2\pi \int d\cos\theta [d_{\lambda\lambda'}^J(\theta) \mathcal{V}_{\lambda'\lambda}(\mathbf{p}', \mathbf{p}) + \eta d_{-\lambda\lambda'}^J(\theta) \mathcal{V}_{\lambda'-\lambda}(\mathbf{p}', \mathbf{p})], \quad (9)$$

where  $\eta = PP_1P_2(-1)^{J-J_1-J_2}$ .  $P$  and  $P_1, P_2$  represent the parity of the system and the parity of the two constituent heavy mesons, respectively.  $J$  and  $J_1, J_2$  represent the total spin of the system and the spins of the two constituent heavy mesons, respectively. The initial and final relative momenta are chosen as  $\mathbf{p} = (0, 0, \mathbf{p})$  and  $\mathbf{p}' = (\mathbf{p}' \sin\theta, 0, \mathbf{p}' \cos\theta)$ .  $d_{\lambda\lambda'}^J(\theta)$  is the Wigner  $d$  matrix, which describes the rotation relationship between the initial and final relative momenta. An exponential regularization was also introduced as a form factor in the reduced propagator as  $G_0(\mathbf{p}'') \rightarrow G_0(\mathbf{p}'') e^{-2(p_l''^2 - m_l^2)^2/\Lambda_e^4}$  [43].

### III. POLE TRAJECTORY OF $D\bar{D}^*$ INTERACTION

Based on the above theoretical framework, the scattering amplitude for the  $D\bar{D}^*$  interaction is reduced to a one-dimensional integral equation. Molecular states are identified as poles in the complex energy plane by solving the condition  $|1 - V(z)G(z)| = 0$ , where  $z = E_R - i\Gamma/2$  denotes the complex energy [43,58]. For simplicity, we set the two form factor cutoff parameters,  $\Lambda_e$  and  $\Lambda_s$ , equal and denote them collectively as a single cutoff parameter  $\Lambda$ . Since the coupling constants are fixed, we treat  $\Lambda$  as the only free parameter and investigate how the pole trajectories evolve with its variation. This effectively modulates the interaction strength, as a larger  $\Lambda$  generally leads to a stronger attractive potential. Although the physical value of  $\Lambda$  is unknown, scanning over a range allows us to assess the qualitative behavior and robustness of the predictions. We vary  $\Lambda$  up to 5 GeV on two Riemann sheets and focus on poles within 50 MeV of the  $D\bar{D}^*$  threshold. All relevant isospin, spin, parity, and  $G$ -parity quantum numbers arising

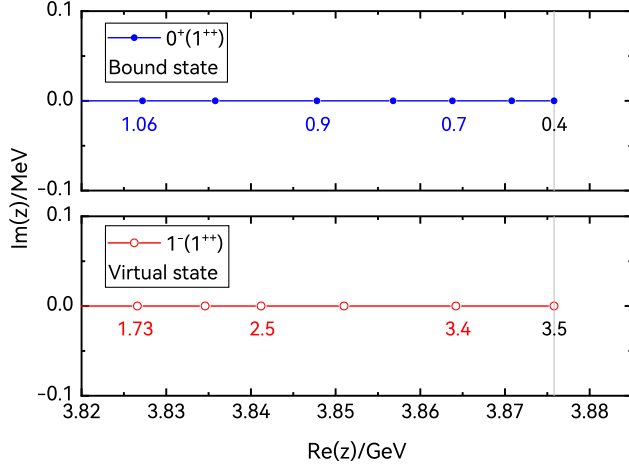


FIG. 1. Pole trajectories for the isoscalar (upper panel) and isovector (lower panel) states with quantum numbers  $1^{++}$ . For the  $0^+(1^{++})$  channel, the corresponding cutoff values from left to right are 1.06, 1.0, 0.9, 0.8, 0.7, 0.6, and 0.4 GeV. For the  $1^-(1^{++})$  channel, the cutoff values are 1.73, 2.0, 2.5, 3.2, 3.4, and 3.5 GeV. The gray solid line indicates the  $D\bar{D}^*$  threshold.

from  $S$ - and  $P$ -wave interactions are considered. Higher partial waves such as  $D$  and  $F$  waves may also contribute to the same spin-parity channels due to the structure of the partial wave decomposition. Only cases that produce bound states, virtual states, or resonances are included in the results.

### A. $S$ -wave pole trajectory

For the  $S$ -wave interaction, poles are found in the channels with quantum numbers  $I^G(J^{PC}) = 0^+(1^{++})$ ,  $1^-(1^{++})$ ,  $0^-(1^{+-})$ , and  $1^+(1^{+-})$ . The upper panel of Fig. 1 illustrates the evolution of the pole positions for the isoscalar state with  $1^{++}$ . When the cutoff parameter  $\Lambda$  is around 0.4 GeV, the pole is located at the threshold. As the interaction becomes stronger with increasing  $\Lambda$ , the pole moves below the threshold along the real axis on the first Riemann sheet, reaching approximately 50 MeV below threshold at  $\Lambda = 1.06$  GeV. With further increase of the cutoff, the pole continues to move deeper, but we limit our analysis to bound states within 50 MeV below the threshold. Although the cutoff is already quite small and further reduction may lack direct physical meaning, we nevertheless examine its behavior for completeness. Under such conditions, no virtual state appears on the second Riemann sheet. Therefore, for the  $0^+(1^{++})$  channel, only bound states exist, with no corresponding virtual state or resonance observed. This isoscalar bound state with  $I^G(J^{PC}) = 0^+(1^{++})$  can be naturally associated with the experimentally observed  $X(3872)$ , in agreement with previous studies [62,63].

As shown in the lower panel of Fig. 1, unlike the isoscalar case, only a virtual state is found for the isovector

channel with quantum numbers  $1^-(1^{++})$ . When the cutoff  $\Lambda$  is around 3.5 GeV, the pole appears near the threshold. As  $\Lambda$  decreases, the pole moves along the real axis on the second Riemann sheet, reaching approximately 50 MeV below the threshold when the cutoff is reduced to around 1.8 GeV, and continues to shift further as the cutoff decreases. As the interaction weakens, the pole is unable to enter the first Riemann sheet to form a bound state. Several recent studies [64–66] have predicted the existence of an isovector charmonium-like  $D\bar{D}^*$  hadronic molecule with  $1^-(1^{++})$ . In particular, the analysis in Ref. [36] suggests that such a state should manifest as a virtual state. Our results support this prediction, indicating the presence of a  $1^-(1^{++})$  molecular state as a virtual state.

In Fig. 2, the pole trajectories for the  $J^{PC} = 1^{+-}$  channel are shown. The trajectory for the isoscalar state is presented in the upper panel, where only bound states are observed. The pole first appears at the threshold when the cutoff  $\Lambda$  is around 0.5 GeV. As the cutoff increases, the attraction becomes stronger, causing the pole to move along the real axis on the first Riemann sheet, reaching approximately 50 MeV below the threshold when the cutoff is reduced to around 1.08 GeV, and continues to shift further as the cutoff decreases.

The middle and lower panels of Fig. 2 show the pole trajectories of the isovector state with quantum numbers  $1^{+-}$ . When the cutoff is very large, corresponding to a strong attractive interaction, the pole appears deep in the first Riemann sheet, far below the  $D\bar{D}^*$  threshold. As the cutoff decreases, the pole moves upward along the real axis toward the threshold. As shown in the middle panel, the pole reaches a position approximately 50 MeV below the threshold when the cutoff is reduced to 2.30 GeV. Continuing to lower the cutoff, the pole gradually approaches the threshold and reaches it at around 1.70 GeV. Further reduction of the cutoff below 1.70 GeV weakens the interaction, causing the pole to transition into the second Riemann sheet, where it becomes a virtual state. This virtual state then moves away from the threshold as the cutoff continues to decrease, eventually disappearing at a pole position of approximately 3.847 GeV when the cutoff is reduced to 1.39 GeV. Thus, depending on the value of the cutoff, the isovector state with quantum numbers  $1^{+-}$  can appear either as a bound state or as a virtual state. The presence of a virtual state provides a plausible interpretation for the  $Z_c(3900)$ , which manifests as a peak above the  $D\bar{D}^*$  threshold, as discussed in Refs. [34,67].

### B. $P$ -wave pole trajectory

In this subsection, we present the results for the  $P$ -wave  $D\bar{D}^*$  interaction, where only two isoscalar states are found, with quantum numbers  $0^{-+}$  and  $1^{-+}$ . Figure 3 displays the evolution of the pole positions for the  $0^+(0^{-+})$  state. The results show that in the single-channel  $D\bar{D}^*$  interaction,

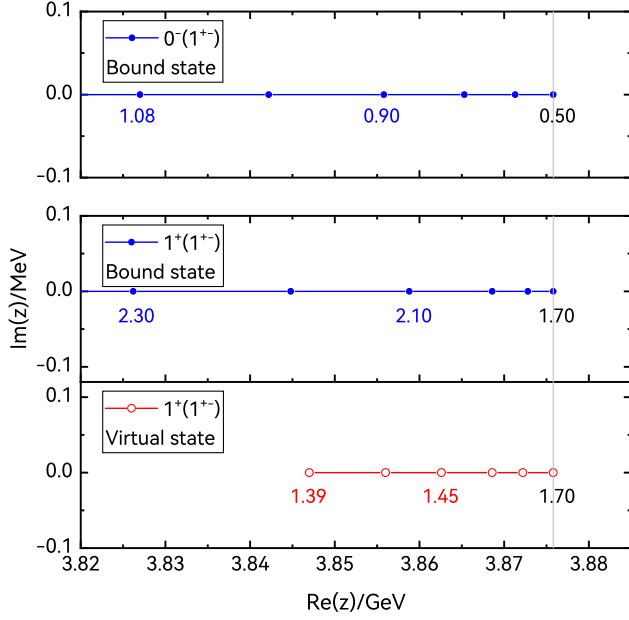


FIG. 2. Pole trajectories for the isoscalar bound (upper panel), isovector bound (middle panel), and isovector virtual (lower panel) states with quantum numbers  $1^{+-}$ . For the isoscalar state  $0^-(1^{+-})$ , the corresponding cutoff parameters from left to right are 1.08, 1.00, 0.90, 0.80, 0.70, and 0.50 GeV. For the isovector state  $1^+(1^{+-})$ , the bound state pole positions correspond to cutoff values (from left to right) of 2.3, 2.2, 2.1, 1.9, 1.8, and 1.7 GeV. The virtual state pole positions correspond to cutoff values (from right to left) of 1.70, 1.56, 1.50, 1.45, 1.41, and 1.39 GeV. The gray solid line indicates the  $D\bar{D}^*$  threshold.

both bound and virtual states can appear on the first and second Riemann sheets, respectively, when the cutoff parameter is below 0.93 GeV. These poles originate far from the threshold and gradually move toward it as the interaction becomes weaker. As shown in the figure, the bound and virtual states approach the threshold along the real axis and enter the range of interest, within 50 MeV below the threshold, at cutoff values of 1.15 and 1.50 GeV, respectively. Although the two poles move at different rates, they both reach the threshold simultaneously at a cutoff of 0.93 GeV, which corresponds to the branch point connecting the two Riemann sheets. If the cutoff is further reduced, the poles cross the threshold and enter the complex energy plane on the second Riemann sheet. In this region, they form a pair of complex conjugate poles, departing from the real axis, and eventually disappear at a cutoff of approximately 0.76 GeV, with a complex energy of  $3.882 \pm 0.004$  GeV. This trajectory is consistent with the behavior discussed in Ref. [44].

Figure 4 shows the evolution of the pole positions for the state with quantum numbers  $I^G(J^{PC}) = 0^-(1^{--})$  arising from the  $D\bar{D}^*$  interaction. Similar to the  $0^+(0^{++})$  case, it is evident that by appropriately tuning the cutoff parameter  $\Lambda$ , both a bound state and a virtual state can be generated below the threshold on the first and second Riemann sheets,

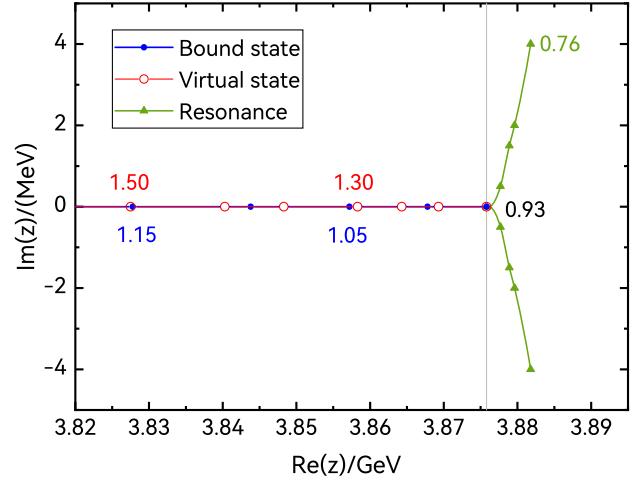


FIG. 3. Pole trajectory of the state with quantum numbers  $0^+(0^{++})$ . The blue line represents the bound state on the first Riemann sheet, with the corresponding cutoff parameters (from left to right) being 1.15, 1.10, 1.05, 1.00, and 0.93 GeV. The red line denotes the virtual state on the second Riemann sheet, with cutoff values (from left to right) of 1.50, 1.45, 1.40, 1.30, 1.20, 1.10, and 0.93 GeV. The green line corresponds to the pair of conjugate poles in the complex plane, with cutoffs (from left to right) of 0.93, 0.89, 0.85, 0.81, and 0.76 GeV. The gray solid line indicates the  $D\bar{D}^*$  threshold.

respectively. As the cutoff decreases, the two poles move along the real axis on their respective Riemann sheets at different rates but reach the branch point simultaneously at a cutoff of 1.07 GeV. Upon further reduction of the cutoff, weakening the interaction, the poles enter the complex energy plane in the second Riemann sheet as a pair of complex conjugate poles, forming a resonance. These poles eventually disappear at a cutoff of approximately 0.90 GeV, with a complex energy of  $3.884 \pm 0.006$  GeV. The newly observed structure  $G(3900)$ , located near the  $D\bar{D}^*$  threshold and carrying the quantum numbers  $I^G(J^{PC}) = 0^-(1^{--})$ , can thus be interpreted as a molecular state dynamically generated from the  $P$ -wave  $D\bar{D}$  interaction.

#### IV. SUMMARY

In the present work, we conduct a systematic investigation of the molecular states arising from the  $D\bar{D}^*$  interaction by analyzing their pole trajectories. Within the framework of the qBSE, the interaction potentials are constructed based on heavy quark and chiral symmetries. The poles of the scattering amplitude are examined to identify possible bound states, virtual states, and resonances, as summarized in Fig. 5.

For the states generated from the  $S$ -wave interaction, the isoscalar states with quantum numbers  $1^{++}$  and  $1^{+-}$  can only appear as bound states by adjusting the strength of the attraction through variations in the cutoff. The  $1^{++}$  state, in particular, can be associated with the well-known  $X(3872)$ .

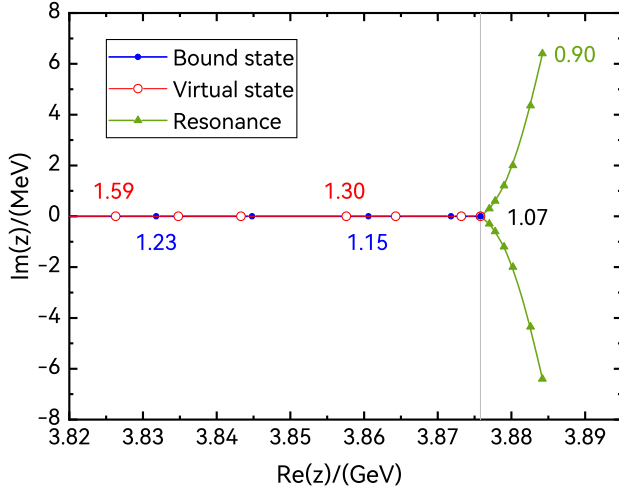


FIG. 4. Pole trajectory of the state with quantum numbers  $0^-(1^{--})$ . The blue line represents the bound state, with the corresponding cutoff parameters (from left to right) being 1.23, 1.20, 1.15, 1.10, and 1.07 GeV. The red line represents the virtual state, with cutoff parameters of 1.59, 1.50, 1.40, 1.30, 1.20, 1.10, and 1.07 GeV (from left to right). The green line represents the resonance trajectory, with cutoff parameters of 1.07, 1.05, 1.03, 1.01, 1.00, 0.95, and 0.90 GeV (from left to right). The gray solid line indicates the  $D\bar{D}^*$  threshold.

Moreover, an isovector partner of the  $X(3872)$  is also predicted to exist, but only as a virtual state. This result aligns with the analysis in Ref. [36].

Since virtual states can exist independently without being accompanied by bound states as  $1^-(1^{++})$  state, the existence of virtual states is often overlooked, as bound states tend to attract more attention. However, our findings suggest that bound states, virtual states, and resonances should all be considered when studying hadronic molecules. Notably, virtual states, which can also account for observed structures above thresholds, as discussed in Refs. [33,34], can be produced solely. In particular, our results support the interpretation of the isovector  $1^{+-}$  state, which may appear as either a bound or virtual state, as a possible explanation for the  $Z_c(3900)$ , observed above the  $D\bar{D}^*$  threshold.

The spin parity of a  $D\bar{D}^*$  system with negative parity must correspond to a  $P$ -wave state. If the recently observed isoscalar  $G(3900)$  state with quantum numbers  $1^{--}$  is interpreted as a molecular state arising from the  $D\bar{D}^*$  interaction, it should indeed be assigned as a  $P$ -wave state. Our results support the existence of two isoscalar  $P$ -wave states with quantum numbers  $0^{+-}$  and  $1^{--}$ , with the latter being associated with the  $G(3900)$ . For both states, the variation of the cutoff results in different possible configurations: bound states, virtual states, or resonances.

In the current work, we also study the behavior of pole trajectories. For the  $S$  wave, only bound states and virtual states are produced for the  $D\bar{D}^*$  interaction, with no

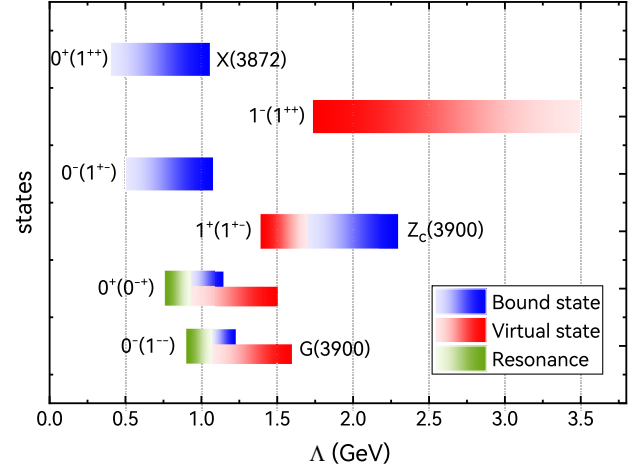


FIG. 5. Summary of the pole trajectories as a function of the cutoff parameters. Lighter colors represent poles located closer to the  $D\bar{D}^*$  threshold.

resonances observed. In contrast, for the  $P$ -wave, all three types of poles, bound states, virtual states, and resonances, are produced as the cutoff varies. The trajectories for the two states are in good agreement with the theoretical discussions presented in Refs. [30,44]. Thus, the current results provide a concrete example of the pole trajectory for  $P$ -wave interactions.

As shown in Fig. 5, the cutoff ranges for which solutions exist in different cases do not always overlap. In the present work, we focus on single-channel calculations to avoid the complexities associated with coupled-channel dynamics and to provide a clearer understanding of the pole trajectories. In principle, the inclusion of coupled-channel effects may shift the pole positions through interchannel interactions. However, such effects do not necessarily resolve the observed mismatch in cutoff values across different channels. For example, if a single cutoff reproduces the  $X(3872)$  but fails to generate the  $Z_c(3900)$  or  $G(3900)$ , this would suggest that these states cannot all be interpreted as molecular states within the same parameter setup in a coupled-channel framework. A definitive conclusion requires a full coupled-channel analysis.

In addition, we emphasize that the current OBE model does not include explicit local four-point contact interactions, which are essential ingredients in effective field theory descriptions of short-range dynamics. Although heavy meson exchanges such as  $\rho$ ,  $\omega$ , and  $J/\psi$  introduce short-range interactions that partially mimic contactlike contributions, the absence of genuine contact terms restricts the interaction to a particular slice of the full parameter space. As a result, the cutoff dependence in our analysis provides only a lower bound on the true systematic uncertainty, and a more complete description would require incorporating contact terms and performing a consistent renormalization procedure.

We present a systematic analysis of the  $S$ - and  $P$ -wave  $D\bar{D}^*$  interactions using a single-channel OBE framework, extracting the corresponding pole trajectories and identifying the conditions under which molecular states such as the  $X(3872)$ ,  $Z_c(3900)$ , and  $G(3900)$  may emerge. While the present model does not include explicit short-range contact interactions and is restricted to single-channel dynamics, it effectively captures the essential features of long-range forces and offers valuable insight into the formation mechanisms of near-threshold states. These results provide a solid baseline for future investigations incorporating coupled-channel effects and a more complete treatment of short-range dynamics, with the goal of achieving a more definitive understanding of the  $S$ - and  $P$ -wave  $D\bar{D}^*$

interactions and their implications for hadronic molecular states.

## ACKNOWLEDGMENTS

This project is supported by the National Science Foundation of China (Grant No. 12475080).

## DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

- 
- [1] P. A. Zyla *et al.* (Particle Data Group), Review of particle physics, *Prog. Theor. Exp. Phys.* **2020**, 083C01 (2020).
  - [2] S. L. Olsen, T. Skwarnicki, and D. Zieminska, Nonstandard heavy mesons and baryons: Experimental evidence, *Rev. Mod. Phys.* **90**, 015003 (2018).
  - [3] C. Z. Yuan, The XYZ states revisited, *Int. J. Mod. Phys. A* **33**, 1830018 (2018).
  - [4] M. B. Voloshin, Charmonium, *Prog. Part. Nucl. Phys.* **61**, 455 (2008).
  - [5] M. Alberti, G. S. Bali, S. Collins, F. Knechtli, G. Moir, and W. Söldner, Hadroquarkonium from lattice QCD, *Phys. Rev. D* **95**, 074501 (2017).
  - [6] H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, The hidden-charm pentaquark and tetraquark states, *Phys. Rep.* **639**, 1 (2016).
  - [7] Q. Wu, D. Y. Chen, and T. Matsuki, A phenomenological analysis on isospin-violating decay of  $X(3872)$ , *Eur. Phys. J. C* **81**, 193 (2021).
  - [8] F. K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao, and B. S. Zou, Hadronic molecules, *Rev. Mod. Phys.* **90**, 015004 (2018).
  - [9] A. Ali, J. S. Lange, and S. Stone, Exotics: Heavy Pentaquarks and Tetraquarks, *Prog. Part. Nucl. Phys.* **97**, 123 (2017).
  - [10] Y. Dong, A. Faessler, and V. E. Lyubovitskij, Description of heavy exotic resonances as molecular states using phenomenological Lagrangians, *Prog. Part. Nucl. Phys.* **94**, 282 (2017).
  - [11] F. K. Guo, X. H. Liu, and S. Sakai, Threshold cusps and triangle singularities in hadronic reactions, *Prog. Part. Nucl. Phys.* **112**, 103757 (2020).
  - [12] M. Cleven, F. K. Guo, C. Hanhart, Q. Wang, and Q. Zhao, Employing spin symmetry to disentangle different models for the XYZ states, *Phys. Rev. D* **92**, 014005 (2015).
  - [13] Z. G. Wang, Analysis of the Hidden-charm Tetraquark molecule mass spectrum with the QCD sum rules, *Int. J. Mod. Phys. A* **36**, 2150107 (2021).
  - [14] N. A. Tornqvist, From the deuteron to deusons, an analysis of deuteron—like meson meson bound states, *Z. Phys. C* **61**, 525 (1994).
  - [15] S. K. Choi *et al.* (Belle Collaboration), Observation of a narrow charmonium—like state in exclusive  $B^+ \rightarrow K^+ \pi^+ \pi^- J/\psi$  decays, *Phys. Rev. Lett.* **91**, 262001 (2003).
  - [16] I. W. Lee, A. Faessler, T. Gutsche, and V. E. Lyubovitskij,  $X(3872)$  as a molecular  $D\bar{D}^*$  state in a potential model, *Phys. Rev. D* **80**, 094005 (2009).
  - [17] N. A. Tornqvist, Isospin breaking of the narrow charmonium state of Belle at 3872-MeV as a deuson, *Phys. Lett. B* **590**, 209 (2004).
  - [18] F. E. Close and P. R. Page, The  $D^{*0}\bar{D}^0$  threshold resonance, *Phys. Lett. B* **578**, 119 (2004).
  - [19] T. Barnes and S. Godfrey, Charmonium options for the  $X(3872)$ , *Phys. Rev. D* **69**, 054008 (2004).
  - [20] Y. S. Kalashnikova, Coupled-channel model for charmonium levels and an option for  $X(3872)$ , *Phys. Rev. D* **72**, 034010 (2005).
  - [21] N. Li and S. L. Zhu, Isospin breaking, Coupled-channel effects and Diagnosis of  $X(3872)$ , *Phys. Rev. D* **86**, 074022 (2012).
  - [22] Z. Y. Lin, J. Z. Wang, J. B. Cheng, L. Meng, and S. L. Zhu, Identification of the  $G(3900)$  as the  $P$ -wave  $D\bar{D}^*/\bar{D}D^*$  resonance, *Phys. Rev. Lett.* **133**, 241903 (2024).
  - [23] Z. Q. Liu *et al.* (Belle Collaboration), Study of  $e^+e^- \rightarrow \pi^+\pi^- J/\psi$  and observation of a charged charmoniumlike state at Belle, *Phys. Rev. Lett.* **110**, 252002 (2013); *Phys. Rev. Lett.* **111**, 019901(E) (2013).
  - [24] T. Xiao, S. Dobbs, A. Tomaradze, and K. K. Seth, Observation of the charged hadron  $Z_c^\pm(3900)$  and evidence for the

- neutral  $Z_c^0(3900)$  in  $e^+e^- \rightarrow \pi\pi J/\psi$  at  $\sqrt{s} = 4170$  MeV, *Phys. Lett. B* **727**, 366 (2013).
- [25] Q. Wang, C. Hanhart, and Q. Zhao, Decoding the riddle of  $Y(4260)$  and  $Z_c(3900)$ , *Phys. Rev. Lett.* **111**, 132003 (2013).
- [26] Z. G. Wang and T. Huang, Possible assignments of the  $X(3872)$ ,  $Z_c(3900)$  and  $Z_b(10610)$  as axial-vector molecular states, *Eur. Phys. J. C* **74**, 2891 (2014).
- [27] J. R. Zhang, Improved QCD sum rule study of  $Z_c(3900)$  as a  $\bar{D}D^*$  molecular state, *Phys. Rev. D* **87**, 116004 (2013).
- [28] F. Aceti, M. Bayar, E. Oset, A. Martinez Torres, K. P. Khemchandani, J. M. Dias, F. S. Navarra, and M. Nielsen, Prediction of an  $I = 1$   $\bar{D}D^*$  state and relationship to the claimed  $Z_c(3900)$ ,  $Z_c(3885)$ , *Phys. Rev. D* **90**, 016003 (2014).
- [29] J. He, Study of the  $B\bar{B}^*/D\bar{D}^*$  bound states in a Bethe-Salpeter approach, *Phys. Rev. D* **90**, 076008 (2014).
- [30] J. R. Taylor, *Scattering Theory: The Quantum Theory on Non-relativistic Collisions* (John Wiley & Sons, Inc., New York, 1972).
- [31] T. Hyodo and D. Jido, The nature of the  $\Lambda(1405)$  resonance in chiral dynamics, *Prog. Part. Nucl. Phys.* **67**, 55 (2012).
- [32] J. He and P. L. Lu, The octet meson and octet baryon interaction with strangeness and the  $\Lambda(1405)$ , *Int. J. Mod. Phys. E* **24**, 1550088 (2015).
- [33] M. L. Du, M. Albaladejo, F. K. Guo, and J. Nieves, Combined analysis of the  $Z_c(3900)$  and the  $Z_{cs}(3985)$  exotic states, *Phys. Rev. D* **105**, 074018 (2022).
- [34] J. He and D. Y. Chen,  $Z_c(3900)/Z_c(3885)$  as a virtual state from  $\pi J/\psi - \bar{D}^*D$  interaction, *Eur. Phys. J. C* **78**, 94 (2018).
- [35] X. K. Dong, F. K. Guo, and B. S. Zou, A survey of heavy-antiheavy hadronic molecules, *Progr. Phys.* **41**, 65 (2021).
- [36] T. Ji, X. K. Dong, F. K. Guo, C. Hanhart, and U. G. Meißner, Precise determination of the properties of  $X(3872)$  and of its isovector partner  $W_{c1}$ , [arXiv:2502.04458](https://arxiv.org/abs/2502.04458).
- [37] M. Ablikim *et al.* (BESIII Collaboration), Precise measurement of born cross sections for  $e^+e^- \rightarrow D\bar{D}$  at  $s = 3.80$ – $4.95$  GeV, *Phys. Rev. Lett.* **133**, 081901 (2024).
- [38] B. Aubert *et al.* (BABAR Collaboration), Study of the exclusive initial-state radiation production of the  $D\bar{D}$  system, *Phys. Rev. D* **76**, 111105 (2007).
- [39] G. Pakhlova *et al.* (Belle Collaboration), Measurement of the near-threshold  $e^+e^- \rightarrow D\bar{D}$  cross section using initial-state radiation, *Phys. Rev. D* **77**, 011103 (2008).
- [40] Y. Huang and X. Chen, Direct evidence for the  $\bar{D}D^*/D\bar{D}^*$  molecular nature of  $G(3900)$  through triangular singularity mechanisms, [arXiv:2501.10992](https://arxiv.org/abs/2501.10992).
- [41] Z. M. Ding, H. Y. Jiang, and J. He, Molecular states from  $D^{(*)}\bar{D}^{(*)}/B^{(*)}\bar{B}^{(*)}$  and  $D^{(*)}D^{(*)}/\bar{B}^{(*)}\bar{B}^{(*)}$  interactions, *Eur. Phys. J. C* **80**, 1179 (2020).
- [42] Z. F. Sun, Z. G. Luo, J. He, X. Liu, and S. L. Zhu, A note on the  $B^*\bar{B}$ ,  $B^*\bar{B}^*$ ,  $D^*\bar{D}$ ,  $D^*\bar{D}^*$  molecular states, *Chin. Phys. C* **36**, 194 (2012).
- [43] J. He, The  $Z_c(3900)$  as a resonance from the  $D\bar{D}^*$  interaction, *Phys. Rev. D* **92**, 034004 (2015).
- [44] C. Hanhart, J. R. Pelaez, and G. Rios, Remarks on pole trajectories for resonances, *Phys. Lett. B* **739**, 375 (2014).
- [45] Z. F. Sun, J. He, X. Liu, Z. G. Luo, and S. L. Zhu,  $Z_b(10610)^\pm$  and  $Z_b(10650)^\pm$  as the  $B^*\bar{B}$  and  $B^*\bar{B}^*$  molecular states, *Phys. Rev. D* **84**, 054002 (2011).
- [46] H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, T. M. Yan, and H. L. Yu, Chiral Lagrangians for radiative decays of heavy hadrons, *Phys. Rev. D* **47**, 1030 (1993).
- [47] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, and H. L. Yu, Heavy quark symmetry and chiral dynamics, *Phys. Rev. D* **46**, 1148 (1992); *Phys. Rev. D* **55**, 5851(E) (1997).
- [48] M. B. Wise, Chiral perturbation theory for hadrons containing a heavy quark, *Phys. Rev. D* **45**, R2188 (1992).
- [49] G. Burdman and J. F. Donoghue, Union of chiral and heavy quark symmetries, *Phys. Lett. B* **280**, 287 (1992).
- [50] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phenomenology of heavy meson chiral Lagrangians, *Phys. Rep.* **281**, 145 (1997).
- [51] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Chiral multiplets of heavy—light mesons, *Phys. Rev. D* **68**, 054024 (2003).
- [52] A. F. Falk and M. E. Luke, Strong decays of excited heavy mesons in chiral perturbation theory, *Phys. Lett. B* **292**, 119 (1992).
- [53] C. Isola, M. Ladisa, G. Nardulli, and P. Santorelli, Charming penguins in  $B \rightarrow K^*\pi$ ,  $K(\rho, \omega, \phi)$  decays, *Phys. Rev. D* **68**, 114001 (2003).
- [54] X. Liu, Z. G. Luo, Y. R. Liu, and S. L. Zhu,  $X(3872)$  and other possible heavy molecular states, *Eur. Phys. J. C* **61**, 411 (2009).
- [55] R. Chen, Z. F. Sun, X. Liu, and S. L. Zhu, Strong LHCb evidence supporting the existence of the hidden-charm molecular pentaquarks, *Phys. Rev. D* **100**, 011502 (2019).
- [56] Y. s. Oh, T. Song, and S. H. Lee,  $J/\psi$  absorption by  $\pi$  and  $\rho$  mesons in meson exchange model with anomalous parity interactions, *Phys. Rev. C* **63**, 034901 (2001).
- [57] J. He, Study of  $P_c(4457)$ ,  $P_c(4440)$ , and  $P_c(4312)$  in a quasipotential Bethe-Salpeter equation approach, *Eur. Phys. J. C* **79**, 393 (2019).
- [58] F. Gross, J. W. Van Orden, and K. Holinde, Relativistic one boson exchange model for the nucleon-nucleon interaction, *Phys. Rev. C* **45**, 2094 (1992).
- [59] J. He, Internal structures of the nucleon resonances  $N(1875)$  and  $N(2120)$ , *Phys. Rev. C* **91**, 018201 (2015).
- [60] J. He, Nucleon resonances  $N(1875)$  and  $N(2100)$  as strange partners of LHCb pentaquarks, *Phys. Rev. D* **95**, 074031 (2017).
- [61] F. Gross, Charge conjugation invariance of the spectator equations, *Few Body Syst.* **30**, 21 (2001).
- [62] R. Aaij *et al.* (LHCb Collaboration), Determination of the  $X(3872)$  meson quantum numbers, *Phys. Rev. Lett.* **110**, 222001 (2013).
- [63] E. S. Swanson, Short range structure in the  $X(3872)$ , *Phys. Lett. B* **588**, 189 (2004).
- [64] Z. H. Zhang, T. Ji, X. K. Dong, F. K. Guo, C. Hanhart, U. G. Meißner, and A. Rusetsky, Predicting isovector charmonium-like states from  $X(3872)$  properties, *J. High Energy Phys.* **08** (2024) 130.
- [65] J. M. Dias, T. Ji, X. K. Dong, F. K. Guo, C. Hanhart, U. G. Meißner, Y. Zhang, and Z. H. Zhang, Dispersive analysis of

- the isospin breaking in the  $X(3872) \rightarrow J/\psi \pi^+ \pi^-$  and  $X(3872) \rightarrow J/\psi \pi^+ \pi^0 \pi^-$  decays, [Phys. Rev. D \*\*111\*\*, 014031 \(2025\)](#).
- [66] M. Sadl, S. Collins, Z. H. Guo, M. Padmanath, S. Prelovsek, and L. W. Yan, Charmoniumlike channels  $1^+$  with isospin 1 from lattice and effective field theory, [Phys. Rev. D \*\*111\*\*, 054513 \(2025\)](#).
- [67] S. X. Nakamura, X. H. Li, H. P. Peng, Z. T. Sun, and X. R. Zhou, Global coupled-channel analysis of  $e^+e^- \rightarrow c\bar{c}$  processes in  $\sqrt{s} = 3.75\text{--}4.7$  GeV, [arXiv:2312.17658](#).