

# **Erratum: Effect of interactions on the topological expression for the chiral separation effect [Phys. Rev. D **107**, 094021 (2023)]**

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In our paper, Eq. (3) as well as Eqs. (47), (83) are valid in the presented form only when  $\Sigma_3$  is the hypersurface consisting of the two hyperplanes  $p_4 = \pm\epsilon \rightarrow 0$ . The same refers also to Eq. (147) of [1], where the noninteracting case was considered. Here, at each  $x$  matrix  $\gamma^5$  should commute or anticommute with  $Q_W$  and  $G_W$  ( $\mathbf{Q}_W^{(0)}$  and  $\mathbf{G}_W^{(0)}$ ) in a vicinity of the space depending Fermi surface. By the space depending Fermi surface, we understand the position in momentum space of the singularities of expression standing in the integrals in Eqs. (3), (47), and (83). This vicinity is of the size that is much larger than the inverse correlation length of the considered system. In practice, this means that the space depending Fermi surface should be situated sufficiently far from the ultraviolet region of momentum space, where the chiral symmetry is violated.

These expressions may be formulated also for the more general form of  $\Sigma_3$ , but then certain clarifications should be added. More specifically, in Eq. (3), we should understand function  $Q_W$  as depending on spatial momenta  $\vec{p}$  and spatial coordinates  $\vec{x}$ , i.e. it should be substituted by

$$Q_W \rightarrow \tilde{Q}(\vec{x}, \vec{p}) \equiv Q_W((\vec{x}, 0), (\vec{p}, \pm\omega_{\pm}(\vec{p}, \vec{x}))). \quad (\text{A})$$

Here the three-dimensional vector  $\vec{p}$  parametrizes surface  $\Sigma_3(x)$ . Function  $\omega$  represents the dependence of Matsubara frequency on  $\vec{p}$ :  $p_4 = \pm\omega_{\pm}(\vec{p}, \vec{x})$ , the upper sign is to be chosen for the upper piece of  $\Sigma_3$ , the lower sign—for the lower piece. Both pieces are closed through the boundary of the Brillouin zone. The piece of  $\Sigma_3$  with nonzero  $\omega_{\pm}(\vec{p}, \vec{x})$  should also be situated sufficiently far from the region of momentum space, where there is no chiral symmetry. We took into account that in the considered equilibrium systems  $Q$  as well as  $G$  does not depend on  $x^4$ .

In turn,  $G_W$  in Eq. (3) should be substituted by the  $\star$ —inverse with respect to  $\tilde{Q}$ , i.e.  $G_W \rightarrow \tilde{G}$  that obeys

$$\tilde{Q}(\vec{x}, \vec{p}) e^{\frac{i}{2}(\overleftarrow{\partial}_{\vec{x}} \overrightarrow{\partial}_{\vec{p}} - \overleftarrow{\partial}_{\vec{p}} \overrightarrow{\partial}_{\vec{x}})} \tilde{G}(\vec{x}, \vec{p}) = 1. \quad (\text{B})$$

With these clarifications Eq. (3) reads (for a general form of  $\Sigma_3$ ),

$$\mathcal{N} = \frac{\epsilon_{ijk}}{48\pi^2 \mathbf{V}} \int d^3x \int_{\Sigma_3(x)} d^3p \operatorname{tr}[\gamma^5 \tilde{G} \star \partial_{p_i} \tilde{Q} \star \tilde{G} \star \partial_{p_j} \tilde{Q} \star \tilde{G} \star \partial_{p_k} \tilde{Q}], \quad (\text{C})$$

with the three-dimensional volume  $\mathbf{V} = \int d^3x$ , which is assumed to be large.

The above clarifications refer also to the interacting version of Eq. (3) derived in our paper, i.e., to Eqs. (47), (80). These equations are valid in their original form only for  $\Sigma_3$  given by  $p_4 = \pm\epsilon \rightarrow 0$ . For the case of the hypersurface of more general form, they are to be understood as Eq. (C), when

$$\tilde{Q}(\vec{x}, \vec{p}) \equiv \mathbf{Q}_W^{(0)}((\vec{x}, 0), (\vec{p}, \pm\omega_{\pm}(\vec{p}, \vec{x}))), \quad (\text{D})$$

while  $\tilde{G}$  is the  $\star$ —inverse to  $\tilde{Q}$ .

With this clarification, Eqs. (47), (80) represent topological invariants robust to both smooth modification of the system and smooth modification of the form of hypersurface  $\Sigma_3$ .

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In both cases, the singularities should be avoided; i.e., modifying the system, one should not pass over a phase transition, while modifying  $\Sigma_3$  one should not cross the position of singularities that extends the notion of Fermi surface to the interacting nonhomogeneous systems.

- [1] M. Suleymanov and M. A. Zubkov, Chiral separation effect in nonhomogeneous systems, [Phys. Rev. D \*\*102\*\*, 076019 \(2020\)](#).