

Twist-3 generalized parton distributions of sea quarks at zero skewness in the light-cone quark model

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We present a systematic study of twist-3 generalized parton distributions (GPDs) for $\bar{u}$ and $\bar{d}$ sea quarks in the proton at zero skewness ($\xi = 0$) using the light-cone formalism with overlap representation. The proton wave functions are derived from a meson-baryon fluctuation model that incorporates $|q\bar{q}B\rangle$ Fock states, providing a natural framework for investigating sea quark contributions. Within this approach, we compute the complete set of twist-3 GPDs and present the numerical results, including both chiral-odd (H_2 , E_2 , $\tilde{E}_2$, and $\tilde{H}'_2$) and chiral-even ($\tilde{E}_{2T}$, H'_{2T} , E'_{2T} , and $\tilde{H}_{2T}$) distributions for $\bar{u}$ and $\bar{d}$ quarks at zero skewness. By taking the forward limit, we also calculate the corresponding twist-3 parton distribution functions $e(x)$ of $\bar{u}$ and $\bar{d}$ quarks. The kinetic orbital angular momenta of sea quarks deduced from the twist-3 GPDs are studied and compared to those from the twist-2 GPDs.

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I. INTRODUCTION

Understanding the internal structure of hadrons composed of constituent quarks, gluon, and sea quarks remains a fundamental challenge in quantum chromodynamics (QCD) and hadronic physics. Generalized parton distributions (GPDs) [1–4] have emerged as crucial observables for probing the three-dimensional structure of nucleons, complementing the information provided by transverse momentum dependent parton distributions (TMDs). These distributions generalize conventional parton distribution functions (PDFs) to the off-forward scattering regime, naturally appearing in exclusive processes, such as deeply virtual Compton scattering (DVCS), $\gamma^* h(p) \rightarrow \gamma h(p')$, and hard exclusive meson production, $\gamma^* h_1(p) \rightarrow M h_2(p')$, where the target nucleon receives recoil momentum Δ . Significant experimental progress has been made through measurements by the H1 [5–7], ZEUS [8,9], HERMES [10–12], COMPASS [13], and JLab [14] collaborations.

GPDs encode richer structural information than one-dimensional PDFs through their dependence on three variables: the longitudinal momentum fraction x , the momentum transfer squared t , and the skewness parameter ξ . In the forward limit ($\xi \rightarrow 0$, $t \rightarrow 0$), GPDs reduce to standard PDFs, while their integration over x yields nucleon form factors. Notably, GPDs provide access to the quark

orbital angular momentum (OAM) through the Ji sum rule [2] and enable tomographic imaging of nucleon structure via impact-parameter dependent distributions obtained through Fourier transformation [15–17].

The twist decomposition of GPDs, analogous to that of PDFs and TMDs, organizes contributions in inverse powers of the hard scale Q . While leading-twist GPDs dominate high-energy processes [18–24], higher-twist contributions become significant at moderate Q^2 values accessible in current experiments. These subleading terms play essential roles in preserving fundamental symmetries: leading-twist DVCS amplitudes alone violate electromagnetic Ward identities and Lorentz invariance, requiring higher-twist corrections for consistency [25–29]. Moreover, precise extraction of twist-2 GPDs from experimental data necessitates accounting for power corrections.

In addition to correcting the leading-twist amplitudes and enabling more reliable extraction of twist-2 GPDs, there are further motivations for measuring higher-twist GPDs, particularly twist-3 ones. First, eight independent twist-3 GPDs contribute to nucleon DVCS at this level [25–28,30], with measurable effects at relatively low Q^2 values. Second, twist-3 distributions provide unique access to quark OAM in longitudinally polarized nucleons [26,31], enabling decomposition of spin and OAM contributions that remain entangled in twist-2 observables [32–35]. Third, these distributions offer insights into transverse color forces [36,37]. In addition, certain spin-orbit correlations of the nucleon and the twist-3 DVCS amplitude can be expressed in terms of twist-3 GPDs [38–40]. Finally, higher-twist GPDs can be related to generalized TMDs (GTMDs) [41–44], which are referred to as “mother

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distributions” and contain the most comprehensive structural information about hadrons.

In recent years, the kinematical twist-3 effects in the DVCS process have been studied extensively. Moreover, the twist-3 GPDs have also been studied through various approaches, including quark target [45–47] and diquark models [47], lattice QCD [48], spectator models [49], light-front quark-diquark model [50], and basis light-front quantization [51]. However, most calculations focus on valence quark contributions, leaving sea quark twist-3 GPDs largely unexplored.

In this paper, we apply the light-cone quark model (LCQM) to calculate the twist-3 GPDs of $\bar{u}$ and $\bar{d}$ quarks at zero skewness using the overlap representation. As proposed in Refs. [52,53], the sea quark degree of freedom can be generated by the meson-baryon fluctuation model, where the proton can fluctuate into a composite state with a meson M and a baryon B , and consequently, the meson M contains the $q\bar{q}$ Fock states. The light-cone wave functions (LCWFs) of the proton thus can be derived in terms of the $|q\bar{q}B\rangle$ Fock states, as calculated in Ref. [54]. The expressions of the twist-3 GPDs of $\bar{u}$ and $\bar{d}$ quarks, including the twist-3 chiral-odd GPDs $H_2^{\bar{q}/P}$, $E_2^{\bar{q}/P}$, $\tilde{E}_2^{\bar{q}/P}$, and $\tilde{H}_2^{\bar{q}/P}$ and

the twist-3 chiral-even GPDs $\tilde{E}_{2T}^{\bar{q}/P}$, $H_{2T}'^{\bar{q}/P}$, $E_{2T}'^{\bar{q}/P}$, and $\tilde{H}_{2T}'^{\bar{q}/P}$ at zero skewness, can be obtained within the overlap representation in terms of these LCWFs. The numerical results for these twist-3 GPDs are presented by properly choosing the values of the parameters. By taking the forward limit, the numerical results for the corresponding twist-3 PDFs $e^{\bar{u}/P}(x)$ and $e^{\bar{d}/P}(x)$ are also presented. Furthermore, the kinetic OAM of sea quarks defined by the twist-2 and twist-3 GPDs are compared.

The remainder of the paper is organized as follows. In Sec. II, we introduce the definition of the twist-3 GPDs. We derive the overlap representation for the twist-3 GPDs in Sec. III, where we also obtain the analytic expressions of these GPDs of the sea quarks using the LCWFs. In Sec. IV, we present the numerical results of twist-3 GPDs and PDFs. We summarize the paper in Sec. V.

II. TWIST-3 GENERALIZED PARTON DISTRIBUTIONS

The quark GPDs are defined through the off-forward matrix elements of the light-cone quark-quark correlation function,

$$F_{\Lambda'\Lambda}^{[\Gamma]}(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik \cdot z} \langle p', \Lambda' | \bar{\psi} \left(-\frac{1}{2}z \right) \Gamma \mathcal{W} \left(-\frac{1}{2}z, \frac{1}{2}z | n \right) \psi \left(\frac{1}{2}z \right) | p, \Lambda \rangle \Big|_{z^+ = \bar{z}_T = 0}. \quad (1)$$

Here, $\mathcal{W}(-z/2, z/2 | n)$ denotes the Wilson line ensuring gauge invariance, p^μ (p'^μ) and Λ (Λ') represent the four-momentum and helicity of the initial (final) nucleon state, and

$$x = k^+/P^+, \quad \Delta^\mu = p'^\mu - p^\mu, \quad \xi = -\Delta^+/(2P^+),$$

are the average light-cone momentum fraction carried by the active quark, the momentum transfer with $t = \Delta^2$ and the skewness parameter, respectively.

Following the systematic classification of Ref. [41], the complete set of twist-3 GPDs comprises sixteen independent functions, including eight chiral-odd GPDs,

$$\begin{aligned} F_{\Lambda'\Lambda}^{[1]} &= \frac{M}{2(P^+)^2} \bar{u}(p', \Lambda') \left[\gamma^+ H_2(x, \xi, t) + \frac{i\sigma^{+\Delta}}{2M} E_2(x, \xi, t) \right] u(p, \Lambda) \\ &= \frac{M}{P^+} \left[H_2 \delta_{\Lambda, \Lambda'} + \frac{(\Lambda \Delta_1 + i\Delta_2)}{2M} E_2 \delta_{-\Lambda, \Lambda'} \right], \end{aligned} \quad (2)$$

$$\begin{aligned} F_{\Lambda'\Lambda}^{[\gamma_5]} &= \frac{M}{2(P^+)^2} \bar{u}(p', \Lambda') \left[\gamma^+ \gamma_5 \tilde{H}_2(x, \xi, t) + \frac{P^+ \gamma_5}{M} \tilde{E}_2(x, \xi, t) \right] u(p, \Lambda) \\ &= \frac{M}{P^+} \left[\Lambda \tilde{H}_2 \delta_{\Lambda, \Lambda'} - \frac{(\Delta_1 + i\Lambda \Delta_2)}{2M} \tilde{E}_2 \delta_{-\Lambda, \Lambda'} \right], \end{aligned} \quad (3)$$

$$\begin{aligned} F_{\Lambda'\Lambda}^{[i\sigma^{ij}\gamma_5]} &= -\frac{i\varepsilon_T^{ij}M}{2(P^+)^2} \bar{u}(p', \Lambda') \left[\gamma^+ H_2'(x, \xi, t) + \frac{i\sigma^{+\Delta}}{2M} E_2'(x, \xi, t) \right] u(p, \Lambda), \\ &= -\frac{i\varepsilon_T^{ij}M}{P^+} \left[H_2' \delta_{\Lambda, \Lambda'} + \frac{(\Lambda \Delta_1 + i\Delta_2)}{2M} E_2' \delta_{-\Lambda, \Lambda'} \right], \end{aligned} \quad (4)$$

$$\begin{aligned}
F_{\Lambda'\Lambda}^{[i\sigma^{+-}\gamma_5]} &= \frac{M}{2(P^+)^2} \bar{u}(p', \Lambda') \left[\gamma^+ \gamma_5 \tilde{H}'_2(x, \xi, t) + \frac{P^+ \gamma_5}{M} \tilde{E}'_2(x, \xi, t) \right] u(p, \Lambda) \\
&= \frac{M}{P^+} \left[\Lambda \tilde{H}'_2 \delta_{\Lambda\Lambda'} - \frac{(\Delta_1 + i\Lambda\Delta_2)}{2M} \tilde{E}'_2 \delta_{-\Lambda\Lambda'} \right], \tag{5}
\end{aligned}$$

and eight chiral-even GPDs

$$\begin{aligned}
F_{\Lambda'\Lambda}^{[i\gamma^i]} &= \frac{M}{2(P^+)^2} \bar{u}(p', \Lambda') \left[i\sigma^{+i} H_{2T}(x, \xi, t) + \frac{\gamma^+ \Delta_i - \Delta^+ \gamma^i}{2M} E_{2T}(x, \xi, t) \right. \\
&\quad \left. + \frac{P^+ \Delta_i - \Delta^+ P_i}{M^2} \tilde{H}_{2T}(x, \xi, t) + \frac{\gamma^+ P_i - P^+ \gamma^i}{M} \tilde{E}_{2T}(x, \xi, t) \right] u(p, \Lambda) \\
&= \left[\frac{\Delta_i}{2P^+} E_{2T} + \frac{\Delta_i}{P^+} \tilde{H}_{2T} + \frac{i\Lambda\epsilon_T^{ij} \Delta_j}{2P^+} (\tilde{E}_{2T} - \xi E_{2T}) \right] \delta_{\Lambda\Lambda'} \\
&\quad + \left[\frac{-M(\Lambda\delta_{i1} + i\delta_{i2})}{P^+} H_{2T} - \frac{(\Lambda\Delta_1 + i\Delta_2)\Delta_i}{2MP^+} \tilde{H}_{2T} \right] \delta_{\Lambda-\Lambda'}, \tag{6}
\end{aligned}$$

$$\begin{aligned}
F_{\Lambda'\Lambda}^{[i\gamma^j\gamma_5]} &= \frac{i\epsilon_T^{ij} M}{2(P^+)^2} \bar{u}(p', \Lambda') \left[i\sigma^{+j} H'_{2T}(x, \xi, t) + \frac{\gamma^+ \Delta_j - \Delta^+ \gamma^j}{2M} E'_{2T}(x, \xi, t) \right. \\
&\quad \left. + \frac{P^+ \Delta_j - \Delta^+ P_j}{M^2} \tilde{H}'_{2T}(x, \xi, t) + \frac{\gamma^+ P_j - P^+ \gamma^j}{M} \tilde{E}'_{2T}(x, \xi, t) \right] u(p, \Lambda) \\
&= \left[\frac{i\epsilon_T^{ij} \Delta_j}{2P^+} E'_{2T} + \frac{i\epsilon_T^{ij} \Delta_j}{P^+} \tilde{H}'_{2T} - \frac{\Lambda\Delta_i}{2P^+} (\tilde{E}'_{2T} - \xi E'_{2T}) \right] \delta_{\Lambda\Lambda'} \\
&\quad + \left[\frac{M(\delta_{i1} + i\Lambda\delta_{i2})}{P^+} H'_{2T} - \frac{i\epsilon_T^{ij} (\Lambda\Delta_1 + i\Delta_2) \Delta_j}{2MP^+} \tilde{H}'_{2T} \right] \delta_{\Lambda-\Lambda'}, \tag{7}
\end{aligned}$$

where $\sigma^{ij} = i[\gamma^i, \gamma^j]/2$, and $\epsilon_T^{ij} = \epsilon^{-+ij}$. Among them, $\tilde{H}_2, H'_2, E'_2, \tilde{E}'_2, H_{2T}, E_{2T}, \tilde{H}_{2T}, \tilde{E}'_{2T}$ are odd functions of ξ , while $H_2, E_2, \tilde{E}_2, \tilde{H}'_2, \tilde{E}_{2T}, H'_{2T}, E'_{2T}, \tilde{H}'_{2T}$ are even functions of ξ .

A different parametrization with vector and axial-vector components is introduced in Ref. [55], where F^μ -type and $\tilde{F}^\mu$ -type GPDs are parameterized as

$$F^\mu = \bar{u}(p') \left[P^\mu \frac{\gamma^+}{P^+} H + P^\mu \frac{i\sigma^{+\nu} \Delta_\nu}{2MP^+} E + \Delta_T^\mu \frac{1}{2M} G_1 + \gamma_T^\mu (H + E + G_2) + \Delta_T^\mu \frac{\gamma^+}{P^+} G_3 + i\epsilon_T^{\mu\nu} \Delta_\nu \frac{\gamma^+ \gamma_5}{P^+} G_4 \right] u(p), \tag{8}$$

$$\tilde{F}^\mu = \bar{u}(p') \left[P^\mu \frac{\gamma^+ \gamma_5}{P^+} \tilde{H} + P^\mu \frac{\Delta^+ \gamma_5}{2MP^+} \tilde{E} + \Delta_T^\mu \frac{\gamma_5}{2M} (\tilde{E} + \tilde{G}_1) + \gamma_T^\mu \gamma_5 (\tilde{H} + \tilde{G}_2) + \Delta_T^\mu \frac{\gamma^+ \gamma_5}{P^+} \tilde{G}_3 + i\epsilon_T^{\mu\nu} \Delta_\nu \frac{\gamma^+}{P^+} \tilde{G}_4 \right] u(p). \tag{9}$$

The GPDs from these two different parameterizations can be connected through the following relations [55]

$$G_1 = 2\tilde{H}_{2T}, \quad G_2 = -(H + E) - \frac{1}{\xi} (1 - \xi^2) H_{2T} + \xi E_{2T} - \tilde{E}_{2T}, \tag{10}$$

$$G_3 = \frac{1}{2} (H_{2T} + E_{2T}), \quad G_4 = \frac{1}{2\xi} H_{2T}, \tag{11}$$

$$\tilde{G}_1 = -\tilde{E} + 2\tilde{H}'_{2T}, \quad \tilde{G}_2 = -\tilde{H} + (1 - \xi^2) H'_{2T} - \xi^2 E'_{2T} - \frac{\Delta_T^2}{2M^2} \tilde{H}'_{2T} + \xi \tilde{E}'_{2T}, \tag{12}$$

$$\tilde{G}_3 = -\frac{\xi}{2} (H'_{2T} + E'_{2T}) - \frac{\xi(M^2 + t/4)}{M^2} \tilde{H}'_{2T} + \frac{1}{2} \tilde{E}'_{2T}, \quad \tilde{G}_4 = -\frac{1}{2} (H'_{2T} + E'_{2T}) - \frac{M^2 + t/4}{M^2} \tilde{H}'_{2T}. \tag{13}$$

Using $+$ ($-$) to denote the positive (negative) helicity of the proton, the twist-3 chiral-odd GPDs ($i = 1, j = 2$), can be expressed as the combinations of the helicity amplitudes of the correlator (1),

$$\frac{M}{P^+} H_2 = \frac{F_{++}^{[1]} + F_{--}^{[1]}}{2}, \quad \frac{i\Delta_2}{2P^+} E_2 = \frac{F_{+-}^{[1]} + F_{-+}^{[1]}}{2}, \quad (14)$$

$$\frac{M}{P^+} \tilde{H}_2 = \frac{F_{++}^{[\gamma_s]} - F_{--}^{[\gamma_s]}}{2}, \quad -\frac{\Delta_1}{2P^+} \tilde{E}_2 = \frac{F_{+-}^{[\gamma_s]} + F_{-+}^{[\gamma_s]}}{2}, \quad (15)$$

$$-\frac{iM}{P^+} H'_2 = \frac{F_{++}^{[i\sigma^{12}\gamma_s]} + F_{--}^{[i\sigma^{12}\gamma_s]}}{2}, \quad \frac{\Delta_2}{2P^+} E'_2 = \frac{F_{+-}^{[i\sigma^{12}\gamma_s]} + F_{-+}^{[i\sigma^{12}\gamma_s]}}{2}, \quad (16)$$

$$\frac{M}{P^+} \tilde{H}'_2 = \frac{F_{++}^{[i\sigma^{+-}\gamma_s]} - F_{--}^{[i\sigma^{+-}\gamma_s]}}{2}, \quad -\frac{\Delta_1}{2P^+} \tilde{E}'_2 = \frac{F_{+-}^{[i\sigma^{+-}\gamma_s]} + F_{-+}^{[i\sigma^{+-}\gamma_s]}}{2}. \quad (17)$$

Similarly, the twist-3 chiral-even GPDs can be expressed as

$$\frac{\Delta_T^2}{2P^+} (E_{2T} + 2\tilde{H}_{2T}) = \frac{\Delta_1(F_{++}^{[\gamma^1]} + F_{--}^{[\gamma^1]})}{2} + \frac{\Delta_2(F_{++}^{[\gamma^2]} + F_{--}^{[\gamma^2]})}{2}, \quad (18)$$

$$\frac{i\Delta_T^2}{2P^+} (\tilde{E}_{2T} - \xi E_{2T}) = \frac{\Delta_2(F_{++}^{[\gamma^1]} - F_{--}^{[\gamma^1]})}{2} - \frac{\Delta_1(F_{++}^{[\gamma^2]} - F_{--}^{[\gamma^2]})}{2}, \quad (19)$$

$$\frac{2M}{P^+} \left(H_{2T} + \frac{\Delta_T^2}{4M^2} \tilde{H}_{2T} \right) = \frac{F_{+-}^{[\gamma^1]} - F_{-+}^{[\gamma^1]}}{2} + \frac{i(F_{+-}^{[\gamma^2]} + F_{-+}^{[\gamma^2]})}{2}, \quad (20)$$

$$\frac{i\Delta_1\Delta_2}{MP^+} \tilde{H}_{2T} = \frac{-(F_{+-}^{[\gamma^1]} + F_{-+}^{[\gamma^1]})}{2} + \frac{i(F_{+-}^{[\gamma^2]} - F_{-+}^{[\gamma^2]})}{2}, \quad (21)$$

$$\frac{i\Delta_T^2}{2P^+} (E'_{2T} + 2\tilde{H}'_{2T}) = \frac{\Delta_2(F_{++}^{[\gamma^1\gamma_s]} + F_{--}^{[\gamma^1\gamma_s]})}{2} - \frac{\Delta_1(F_{++}^{[\gamma^2\gamma_s]} + F_{--}^{[\gamma^2\gamma_s]})}{2}, \quad (22)$$

$$\frac{-\Delta_T^2}{2P^+} (\tilde{E}'_{2T} - \xi E'_{2T}) = \frac{\Delta_1(F_{++}^{[\gamma^1\gamma_s]} - F_{--}^{[\gamma^1\gamma_s]})}{2} + \frac{\Delta_2(F_{++}^{[\gamma^2\gamma_s]} - F_{--}^{[\gamma^2\gamma_s]})}{2}, \quad (23)$$

$$\frac{2M}{P^+} \left(H'_{2T} + \frac{\Delta_T^2}{4M^2} \tilde{H}'_{2T} \right) = \frac{F_{+-}^{[\gamma^1\gamma_s]} + F_{-+}^{[\gamma^1\gamma_s]}}{2} + \frac{i(F_{+-}^{[\gamma^2\gamma_s]} - F_{-+}^{[\gamma^2\gamma_s]})}{2}, \quad (24)$$

$$\frac{i\Delta_1\Delta_2}{MP^+} \tilde{H}'_{2T} = \frac{F_{+-}^{[\gamma^1\gamma_s]} - F_{-+}^{[\gamma^1\gamma_s]}}{2} - \frac{i(F_{+-}^{[\gamma^2\gamma_s]} + F_{-+}^{[\gamma^2\gamma_s]})}{2}. \quad (25)$$

III. TWIST-3 GPDS OF SEA QUARKS IN THE OVERLAP REPRESENTATION

In this section, we present the calculation of twist-3 GPDs for $\bar{u}$ and $\bar{d}$ sea quarks in the proton at zero skewness ($\xi = 0$) using the LCQM with overlap representation. The light-cone formalism has proved particularly powerful for studying hadron structure, having been successfully applied to calculations of nucleon and meson parton distribution functions [56], form factors [57,58], anomalous magnetic moments [57], TMDs [54,59,60], GPDs [61–64], as well as

quark Wigner distributions [65–67]. We now extend this framework to investigate the twist-3 GPDs of sea quarks.

In the light-cone approach, the hadronic state is expanded in terms of light-cone wave functions (LCWFs) on a Fock-state basis. To incorporate sea quark degrees of freedom, we employ the baryon-meson fluctuation model [52,53], where the proton can fluctuate into a meson-baryon two-body system,

$$|p\rangle \rightarrow |MB\rangle \rightarrow |q\bar{q}B\rangle, \quad (26)$$

where the meson is composed of the $q\bar{q}$ pair.

In our analysis, we consider the specific fluctuations $|p\rangle \rightarrow |\pi^+ n\rangle$ and $|p\rangle \rightarrow |\pi^- \Delta^{++}\rangle$. The corresponding LCWFs, derived in Ref. [54], factorize into baryonic and mesonic components:

$$\psi_{\lambda_B \lambda_q \lambda_{\bar{q}}}^{\lambda_N}(x, y, \mathbf{k}_T, \mathbf{r}_T) = \psi_{\lambda_B}^{\lambda_N}(y, \mathbf{r}_T) \psi_{\lambda_q \lambda_{\bar{q}}}(x, y, \mathbf{k}_T, \mathbf{r}_T), \quad (27)$$

where $\psi_{\lambda_B}^{\lambda_N}(y, \mathbf{r}_T)$ can be viewed as the wave function of the nucleon in terms πB components, and $\psi_{\lambda_q \lambda_{\bar{q}}}(x, y, \mathbf{k}_T, \mathbf{r}_T)$ is the pion wave function in terms of $q\bar{q}$ components. The indices λ_N , λ_B , λ_q , and $\lambda_{\bar{q}}$ denote the helicities of the proton, the baryon, the quark, and the sea quark, respectively. x and y represent their light-cone momentum fractions, and $\mathbf{k}_T$ and $\mathbf{r}_T$ denote the transverse momenta of the antiquark and the meson.

The baryonic wave functions for different helicity combinations are given by

$$\begin{aligned} \psi_+^+(y, \mathbf{r}_T) &= \frac{M_B - (1-y)M}{\sqrt{1-y}} \phi_1, \\ \psi_-^+(y, \mathbf{r}_T) &= \frac{r_1 + ir_2}{\sqrt{1-y}} \phi_1, \\ \psi_+^-(y, \mathbf{r}_T) &= \frac{r_1 - ir_2}{\sqrt{1-y}} \phi_1, \\ \psi_-^-(y, \mathbf{r}_T) &= \frac{(1-y)M - M_B}{\sqrt{1-y}} \phi_1, \end{aligned} \quad (28)$$

where M and M_B are proton and baryon masses, respectively, and ϕ_1 is the momentum-space wave function

$$\phi_1(y, \mathbf{r}_T) = -\frac{g(r^2) \sqrt{y(1-y)}}{r_T^2 + L_1^2(m_\pi^2)}, \quad (29)$$

with m_π the pion mass, $g(r^2)$ is the form factor for the coupling of the nucleon-pion meson-baryon vertex, and

$$L_1^2(m_\pi^2) = yM_B^2 + (1-y)m_\pi^2 - y(1-y)M^2. \quad (30)$$

The mesonic wave functions for the $q\bar{q}$ Fock state are:

$$\begin{aligned} \psi_{++}(x, y, \mathbf{k}_T, \mathbf{r}_T) &= \frac{my}{\sqrt{x(y-x)}} \phi_2, \\ \psi_{+-}(x, y, \mathbf{k}_T, \mathbf{r}_T) &= \frac{y(k_1 - ik_2) - x(r_1 - ir_2)}{\sqrt{x(y-x)}} \phi_2, \\ \psi_{-+}(x, y, \mathbf{k}_T, \mathbf{r}_T) &= \frac{y(k_1 + ik_2) - x(r_1 + ir_2)}{\sqrt{x(y-x)}} \phi_2, \\ \psi_{--}(x, y, \mathbf{k}_T, \mathbf{r}_T) &= \frac{-my}{\sqrt{x(y-x)}} \phi_2. \end{aligned} \quad (31)$$

Here, m is the mass of the quark and the sea quark. Again, $\phi_2(x, y, \mathbf{k}_T, \mathbf{r}_T)$ is the momentum space wave function of

the $|q\bar{q}\rangle$ Fock state,

$$\phi_2(x, y, \mathbf{k}_T, \mathbf{r}_T) = -\frac{g(k^2) \sqrt{\frac{x}{y} \left(1 - \frac{x}{y}\right)}}{\left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T\right)^2 + L_2^2(m^2)}, \quad (32)$$

$g(k^2)$ is the form factor for the coupling of the pion meson-quark-sea quark vertex, and

$$L_2^2(m^2) = \frac{x}{y} m^2 + \left(1 - \frac{x}{y}\right) m^2 - \frac{x}{y} \left(1 - \frac{x}{y}\right) m_\pi^2. \quad (33)$$

For the form factors $g(r^2)$ and $g(k^2)$, we adopt the dipolar form

$$g(r^2) = -g_1(1-y) \frac{r_T^2 + L_1^2(m_\pi^2)}{[r_T^2 + L_1^2(\Lambda_\pi^2)]^2}, \quad (34)$$

$$g(k^2) = -g_2 \left(1 - \frac{x}{y}\right) \frac{\left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T\right)^2 + L_2^2(m^2)}{\left[\left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T\right)^2 + L_2^2(\Lambda_q^2)\right]^2}. \quad (35)$$

When considering the quark as the active parton (with the antiquark as spectator), the quark carries momentum $-\mathbf{k}_T$ and longitudinal momentum fraction $-x$, while the spectator antiquark has momentum $(y+x, \mathbf{r}_T + \mathbf{k}_T)$. The corresponding LCWFs maintain consistent relations under this transformation,

$$\begin{aligned} \psi_{++}(-x, y, -\mathbf{k}_T, \mathbf{r}_T) &= \psi_{++}(x, y, \mathbf{k}_T, \mathbf{r}_T), \\ \psi_{+-}(-x, y, -\mathbf{k}_T, \mathbf{r}_T) &= -\psi_{+-}(x, y, \mathbf{k}_T, \mathbf{r}_T), \\ \psi_{-+}(-x, y, -\mathbf{k}_T, \mathbf{r}_T) &= -\psi_{-+}(x, y, \mathbf{k}_T, \mathbf{r}_T), \\ \psi_{--}(-x, y, -\mathbf{k}_T, \mathbf{r}_T) &= \psi_{--}(x, y, \mathbf{k}_T, \mathbf{r}_T). \end{aligned} \quad (36)$$

The antiquark distributions are defined by using this conjugate correlation function, which can be related to the quark-quark correlation function by the following expression [68]:

$$\begin{aligned} F_{\Lambda'\Lambda}^{C[\Gamma]}(x, \xi, t; m_q, m_{\bar{q}}) &= \begin{cases} -F_{\Lambda'\Lambda}^{[\Gamma]}(-x, \xi, t; m_{\bar{q}}, m_q) & \text{for } \Gamma = \gamma^j, \gamma^-, i\sigma^{ij}\gamma_5, i\sigma^{j-}\gamma_5 \\ +F_{\Lambda'\Lambda}^{[\Gamma]}(-x, \xi, t; m_{\bar{q}}, m_q) & \text{for } \Gamma = \mathbf{1}, \gamma^j\gamma_5. \end{cases} \end{aligned} \quad (37)$$

In the overlap representation, the quark-quark correlation functioning at $\xi = 0$ can be expressed as [69]

$$F_{\Lambda\Lambda}^{[\Gamma]}(x, 0, t) = \sum_{\lambda_i} \int [dx] [d^2 k_T] \psi_{\lambda'_1 \lambda'_i}^{\Lambda'^*}(x_i, \mathbf{k}_T^i) \psi_{\lambda_1 \lambda_i}^{\Lambda}(x_i, \mathbf{k}_T^i) [\delta_{\lambda'_i \lambda_i} (i = 2 \dots n)] \\ \times \frac{u^\dagger \left(x_1 P^+, \mathbf{k}_T^1 - \frac{\Delta_T}{2}, \lambda'_1 \right) \gamma^0 \Gamma u \left(x_1 P^+, \mathbf{k}_T^1 + \frac{\Delta_T}{2}, \lambda_1 \right)}{2x P^+}, \quad (38)$$

where $\lambda_1 (\lambda'_1)$ represents the helicity of the initial (final) struck quark, and $\lambda_i (\lambda'_i) (i = 2 \dots n)$ denotes the helicity of the initial (final) spectators. The spinor product corresponds to the higher-twist Dirac matrices and encodes struck quark helicity combinations,

$$[dx] [d^2 k_T] = \prod \frac{dx_i k_T^i}{16\pi^3} 16\pi^3 \delta \left(1 - \sum x_i \right) \delta_T^2 \left(\sum \mathbf{k}_T^i \right) \delta(x - x_1). \quad (39)$$

The overlap representation of the twist-3 chiral-odd GPDs of sea quarks can be found in Appendix. Using the LCWFs in Eqs. (28) and (31) as well as the overlap representation in Eqs. (A1)–(A7), we obtain the analytic results for the twist-3 chiral-odd GPDs of sea quarks,

$$H_2^{\bar{q}/P}(x, 0, t) = \frac{m}{xM} \int_x^1 \frac{dy}{y} \int d^2 \mathbf{k}_T \int d^2 \mathbf{r}_T \frac{g_1^2 g_2^2 y (1-y)^2 \left(1 - \frac{x}{y} \right)^2 \left[(M_B - (1-y)M)^2 + \mathbf{r}_T^2 - \frac{1}{4} (1-y) \Delta_T^2 \right]}{2(2\pi)^6 D_1(y, \mathbf{r}_T, \Delta_T) D_2\left(\frac{x}{y}, \mathbf{k}_T - \frac{x}{y} \mathbf{r}_T, \Delta_T\right)} \\ \times \left[m^2 + \left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T \right)^2 - \frac{1}{4} \left(1 - \frac{x}{y} \right)^2 \Delta_T^2 + \left(1 - \frac{x}{y} \right) \frac{\Delta_T^2}{2} \right], \quad (40)$$

$$E_2^{\bar{q}/P}(x, 0, t) = -\frac{m}{x} \int_x^1 \frac{dy}{y} \int d^2 \mathbf{k}_T \int d^2 \mathbf{r}_T \frac{g_1^2 g_2^2 y (1-y)^3 \left(1 - \frac{x}{y} \right)^2 (M_B - (1-y)M)}{(2\pi)^6 D_1(y, \mathbf{r}_T, \Delta_T) D_2\left(\frac{x}{y}, \mathbf{k}_T - \frac{x}{y} \mathbf{r}_T, \Delta_T\right)} \\ \times \left[m^2 + \left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T \right)^2 - \frac{1}{4} \left(1 - \frac{x}{y} \right)^2 \Delta_T^2 + \frac{1}{2} \left(1 - \frac{x}{y} \right) \Delta_T^2 \right], \quad (41)$$

$$\tilde{E}_2^{\bar{q}/P}(x, 0, t) = 0, \quad (42)$$

$$\tilde{H}_2^{\prime \bar{q}/P}(x, 0, t) = \frac{m}{Mx} \int_x^1 \frac{dy}{y} \int d^2 \mathbf{k}_T \int d^2 \mathbf{r}_T \frac{g_1^2 g_2^2 x (1-y)^3 \left(1 - \frac{x}{y} \right)^3 (\mathbf{r}_T \times \Delta_T)^2}{2(2\pi)^6 D_1(y, \mathbf{r}_T, \Delta_T) D_2\left(\frac{x}{y}, \mathbf{k}_T - \frac{x}{y} \mathbf{r}_T, \Delta_T\right)}, \quad (43)$$

and the twist-3 chiral-even GPDs of sea quarks

$$\tilde{E}_{2T}^{\bar{q}/P}(x, 0, t) = -\frac{1}{x} \int_x^1 \frac{dy}{y} \int d^2 \mathbf{k}_T \int d^2 \mathbf{r}_T \frac{g_1^2 g_2^2 y (1-y)^3 \left(1 - \frac{x}{y} \right)^2 (\mathbf{r}_T \times \Delta_T)}{2(2\pi)^6 D_1(y, \mathbf{r}_T, \Delta_T) D_2\left(\frac{x}{y}, \mathbf{k}_T - \frac{x}{y} \mathbf{r}_T, \Delta_T\right)} \\ \times \left[2 \left(m^2 + \left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T \right)^2 - \frac{1}{4} \left(1 - \frac{x}{y} \right)^2 \Delta_T^2 \right) \frac{\mathbf{k}_T \times \Delta_T}{\Delta_T^2} + \left(1 - \frac{x}{y} \right) \left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T \right) \times \Delta_T \right], \quad (44)$$

$$E_{2T}^{\prime \bar{q}/P}(x, 0, t) = \frac{1}{x} \int_x^1 \frac{dy}{y} \int d^2 \mathbf{k}_T \int d^2 \mathbf{r}_T \frac{g_1^2 g_2^2 y (1-y)^2 \left(1 - \frac{x}{y} \right)^2}{2(2\pi)^6 D_1(y, \mathbf{r}_T, \Delta_T) D_2\left(\frac{x}{y}, \mathbf{k}_T - \frac{x}{y} \mathbf{r}_T, \Delta_T\right)} \\ \times \left[(M_B - (1-y)M)^2 + \mathbf{r}_T^2 - \frac{1}{4} (1-y) \Delta_T^2 - 2(1-y)M(M_B - (1-y)M) \right] \\ \times \left[\left(m^2 + \left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T \right)^2 - \frac{1}{4} \left(1 - \frac{x}{y} \right)^2 \Delta_T^2 - 2 \left(1 - \frac{x}{y} \right) m^2 \right) - 2 \left(1 - \frac{x}{y} \right) \left(\left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T \right) \times \Delta_T \right) \frac{\mathbf{k}_T \times \Delta_T}{\Delta_T^2} \right], \quad (45)$$

$$\tilde{H}_{2T}^{\bar{q}/P}(x, 0, t) = \frac{1}{x} \int_x^1 \frac{dy}{y} \int d^2 \mathbf{k}_T \int d^2 \mathbf{r}_T \frac{g_1^2 g_2^2 y (1-y)^3 \left(1 - \frac{x}{y}\right)^2 M(M_B - (1-y)M)}{2(2\pi)^6 D_1(y, \mathbf{r}_T, \Delta_T) D_2\left(\frac{x}{y}, \mathbf{k}_T - \frac{x}{y} \mathbf{r}_T, \Delta_T\right)} \\ \times \left[\left(m^2 + \left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T \right)^2 - \frac{1}{4} \left(1 - \frac{x}{y} \right)^2 \Delta_T^2 - 2 \left(1 - \frac{x}{y} \right) m^2 \right) - 2 \left(1 - \frac{x}{y} \right) \left(\left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T \right) \times \Delta_T \right) \frac{\mathbf{k}_T \times \Delta_T}{\Delta_T^2} \right], \quad (46)$$

$$H_{2T}^{\bar{q}/P}(x, 0, t) = 0, \quad (47)$$

where

$$D_1(y, \mathbf{r}_T, \Delta_T) = \left[\left(\mathbf{r}_T - \frac{1}{2} (1-y) \Delta_T \right)^2 + L_1^2 \right]^2 \left[\left(\mathbf{r}_T + \frac{1}{2} (1-y) \Delta_T \right)^2 + L_1^2 \right]^2, \quad (48)$$

$$D_2\left(\frac{x}{y}, \mathbf{k}_T - \frac{x}{y} \mathbf{r}_T, \Delta_T\right) = \left[\left[\left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T \right) - \frac{1}{2} \left(1 - \frac{x}{y} \right) \Delta_T \right]^2 + L_2^2 \right]^2 \left[\left[\left(\mathbf{k}_T - \frac{x}{y} \mathbf{r}_T \right) + \frac{1}{2} \left(1 - \frac{x}{y} \right) \Delta_T \right]^2 + L_2^2 \right]^2. \quad (49)$$

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we present the numerical results for twist-3 GPDs of the $\bar{u}$ and $\bar{d}$ quarks. For the parameters g_1 , g_2 , $\Lambda_{\bar{q}}$, Λ_π in our model, we adopt the values from Ref. [54], where g_2 and Λ_π are fixed by adopting the GRV leading-order (LO) parametrization [70] to perform the fit for $f_1^{\bar{u}/\pi^-}$ (or $f_1^{\bar{d}/\pi^+}(x)$). The MSTW2008 LO parametrization [71] is adopted for $f_1^{\bar{u}/P}$ and $f_1^{\bar{d}/P}$ to obtain the values of the parameters g_1 and $\Lambda_{\bar{q}}$. The best values from the fits of the parameters are shown in Table I.

A. Results for twist-3 GPDs of sea quarks

To illustrate the x -dependence and Δ_T -dependence of the twist-3 GPDs for the $\bar{u}$ and $\bar{d}$ quarks simultaneously, we present their three-dimensional shapes in Figs. 1–4. In the upper and lower panels of Fig. 1, we plot the chiral-even GPDs associated with $\Gamma = 1$, namely, $H_2^{\bar{u}/P}$, $H_2^{\bar{d}/P}$, $E_2^{\bar{u}/P}$, and $E_2^{\bar{d}/P}$ at $\xi = 0$. We find that $xH_2^{\bar{u}/P}$ and $xH_2^{\bar{d}/P}$ are positive throughout the entire x and Δ_T region. For fixed values of x , both $xH_2^{\bar{u}/P}$ and $xH_2^{\bar{d}/P}$ show a monotonically decreasing trend with increasing Δ_T . Moreover, these distributions decrease rapidly as x increases. In contrast, we observe that the signs of $xE_2^{\bar{u}/P}$ and $xE_2^{\bar{d}/P}$ are negative

for both $\bar{u}$ and $\bar{d}$ quarks. The magnitudes of these distributions also decrease with increasing x and Δ_T , similar to the behavior of $xH_2^{\bar{u}/P}(x, 0, t)$ and $xH_2^{\bar{d}/P}$.

In Fig. 2, we present the twist-3 chiral-even GPDs associated with $\Gamma = i\sigma^{+-}\gamma_5$, particularly $x\tilde{H}_2^{\bar{u}/P}$ and $x\tilde{H}_2^{\bar{d}/P}$ at $\xi = 0$. It is evident that both $x\tilde{H}_2^{\bar{u}/P}$ and $x\tilde{H}_2^{\bar{d}/P}$ are positive. Their magnitudes initially increase with Δ_T before subsequently decreasing. Additionally, $x\tilde{H}_2^{\bar{q}/P}$ of the $\bar{u}$ and $\bar{d}$ quarks peak at lower values of x , and the peak of these distributions shift slightly towards a higher x with the increase of Δ_T .

In Fig. 3, we present the twist-3 chiral-odd GPDs associated with $\Gamma = \gamma^i$, specifically $x\tilde{E}_{2T}^{\bar{u}/P}$ and $x\tilde{E}_{2T}^{\bar{d}/P}$, as functions of x and Δ_T . Our findings indicate that $x\tilde{E}_{2T}^{\bar{q}/P}$ for both $\bar{u}$ and $\bar{d}$ quarks peaks at $\Delta_T = 0$. Notably, for any given Δ_T , the peak values of these curves increase first and then decrease with increasing x , and the position of the peak is found in the small x region. Furthermore, the signs of $x\tilde{E}_{2T}^{\bar{u}/P}$ and $x\tilde{E}_{2T}^{\bar{d}/P}$ remain negative in the entire x and Δ_T regimes.

In the upper and lower panels of Fig. 4, we plot the twist-3 chiral-odd GPDs corresponding to $\Gamma = \gamma^i\gamma_5$, namely, $x\tilde{E}_{2T}^{\bar{u}/P}$, $x\tilde{E}_{2T}^{\bar{d}/P}$ and $x\tilde{H}_{2T}^{\bar{u}/P}$, $x\tilde{H}_{2T}^{\bar{d}/P}$, as functions of x and Δ_T , respectively. It is shown that $x\tilde{E}_{2T}^{\bar{u}/P}$ and $x\tilde{E}_{2T}^{\bar{d}/P}$ exhibit both positive and negative values depending on the variation of x and Δ_T . Both $x\tilde{E}_{2T}^{\bar{u}/P}$ and $x\tilde{E}_{2T}^{\bar{d}/P}$ decrease as Δ_T increases. As x increases, $x\tilde{E}_{2T}^{\bar{u}/P}$ and $x\tilde{E}_{2T}^{\bar{d}/P}$ transition from positive to negative values, indicating a sign flip in the distribution at a specific value of x and Δ_T . Similar behavior is observed for $x\tilde{H}_{2T}^{\bar{u}/P}$ and $x\tilde{H}_{2T}^{\bar{d}/P}$, which exhibit analogous three-dimensional plots but with opposite signs compared to $x\tilde{E}_{2T}^{\bar{u}/P}$ and $x\tilde{E}_{2T}^{\bar{d}/P}$.

TABLE I. Values of the parameters obtained in Ref. [54].

Parameters	$\bar{u}$	$\bar{d}$
g_1	9.33	5.79
g_2	4.46	4.46
Λ_π (GeV)	0.223	0.223
$\Lambda_{\bar{q}}$ (GeV)	0.510	0.510

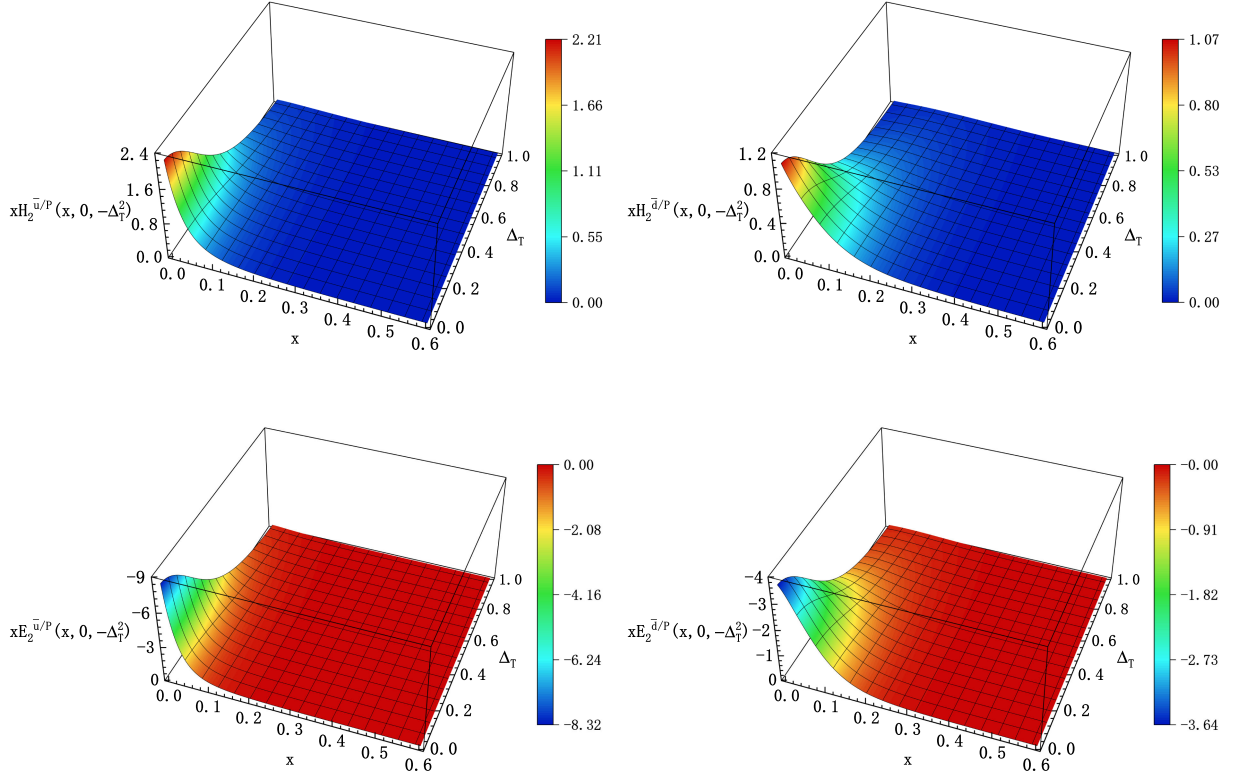


FIG. 1. Twist-3 chiral-even GPDs (multiplied with x) $H_2^{\bar{u}/P}$, $H_2^{\bar{d}/P}$, $E_2^{\bar{u}/P}$, and $E_2^{\bar{d}/P}$ in the light-cone quark model as functions of x and Δ_T .

B. Forward limit

Sum rules for the vector and axial-vector twist-3 GPDs G_i and $\tilde{G}_i$ have been derived in Ref. [72]. For x -moments of the vector GPD G_2 ,

$$\int_{-1}^1 dx x G_2(x, \xi, t) = \frac{1}{2} \left\{ G_A(t) - \int_{-1}^1 dx x [H(x, \xi, t) + E(x, \xi, t)] \right\}, \quad (50)$$

where $G_A(t)$ is the axial-vector form factor obtained from x^0 -moments of $\tilde{H}$. As shown in Ref. [31], the quark angular

momentum can be separated into the usual quark helicity and a gauge-invariant orbital contribution L^q . The kinetic OAM is related to the GPDs by Ji's sum rule,

$$L^q = \int dx \frac{1}{2} \{ x [H^{\bar{q}/P}(x, 0, 0) + E^{\bar{q}/P}(x, 0, 0)] - \tilde{H}^{\bar{q}/P}(x, 0, 0) \}. \quad (51)$$

Alternatively, the kinetic OAM can also be represented as [73]

$$L_z^q = - \int_{-1}^1 dx x G_2^q(x, 0, 0), \quad (52)$$

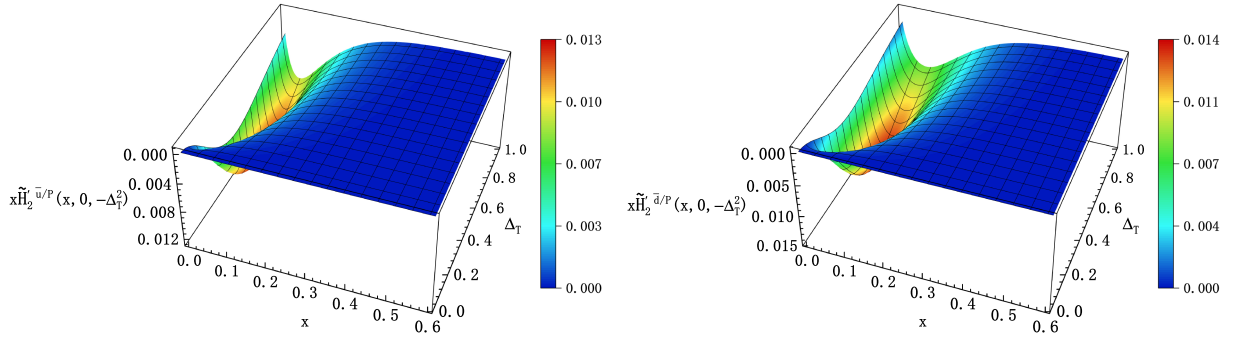


FIG. 2. Twist-3 chiral-even GPDs (multiplied with x) $\tilde{H}_2^{\bar{u}/P}$ and $\tilde{H}_2^{\bar{d}/P}$ in the light-cone quark model as functions of x and Δ_T .

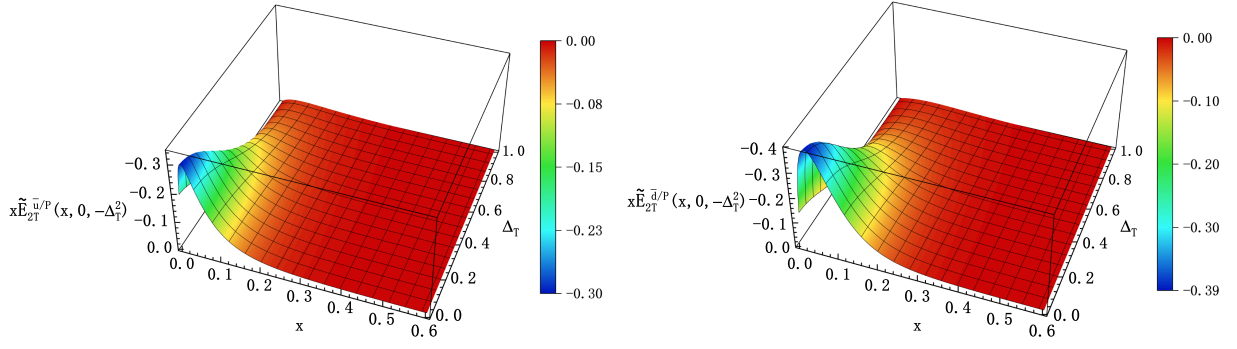


FIG. 3. Twist-3 chiral-odd GPDs (multiplied with x) $\tilde{E}_{2T}^{\bar{u}/P}$ and $\tilde{E}_{2T}^{\bar{d}/P}$ in the light-cone quark model as functions of x and Δ_T .

which is referred to as the Penttinen-Polyakov-Shuvaev-Strikman relation [26,72]. This implies that the relevance between twist-3 GPDs and physical observables cannot be neglected.

In the left and right panels of Fig. 5, we compare the x -dependence of the kinetic OAM of $\bar{u}$ and $\bar{d}$ quarks defined by the twist-2 and twist-3 GPDs, respectively. Our result reveals that the kinetic OAM defined by the twist-2 GPDs for both $\bar{u}$ and $\bar{d}$ quarks within the proton remains positive in the entire x range. While the kinetic OAM defined by the twist-3 GPDs exhibits negative values in the small x region. Similar trends are observed for both cases of kinetic OAM: the peaks of these curves initially increase

and then decrease as x increases. Furthermore, it is noteworthy that the kinetic OAM defined by the twist-2 GPDs peaks at a smaller x compared to that defined by the twist-3 GPDs. Utilizing the GPDs from our model, we calculate the kinetic OAM defined by the twist-3 GPDs and obtain $L^{\bar{u}} = 0.024$ and $L^{\bar{d}} = 0.046$. These results are consistent with the kinetic OAM values defined by the twist-2 GPDs, as calculated in Ref. [64], where $L^{\bar{u}} = 0.025$ and $L^{\bar{d}} = 0.046$. In Ref. [74], the light-front spectator model was employed to calculate the kinetic OAM of the sea quarks. The authors reported the sea quark OAMs derived from twist-2 GPDs as $L^{\bar{u}} = 0.040 \pm 0.003$ and $L^{\bar{d}} = 0.048 \pm 0.002$ (Model-I). Our result for the kinetic OAM

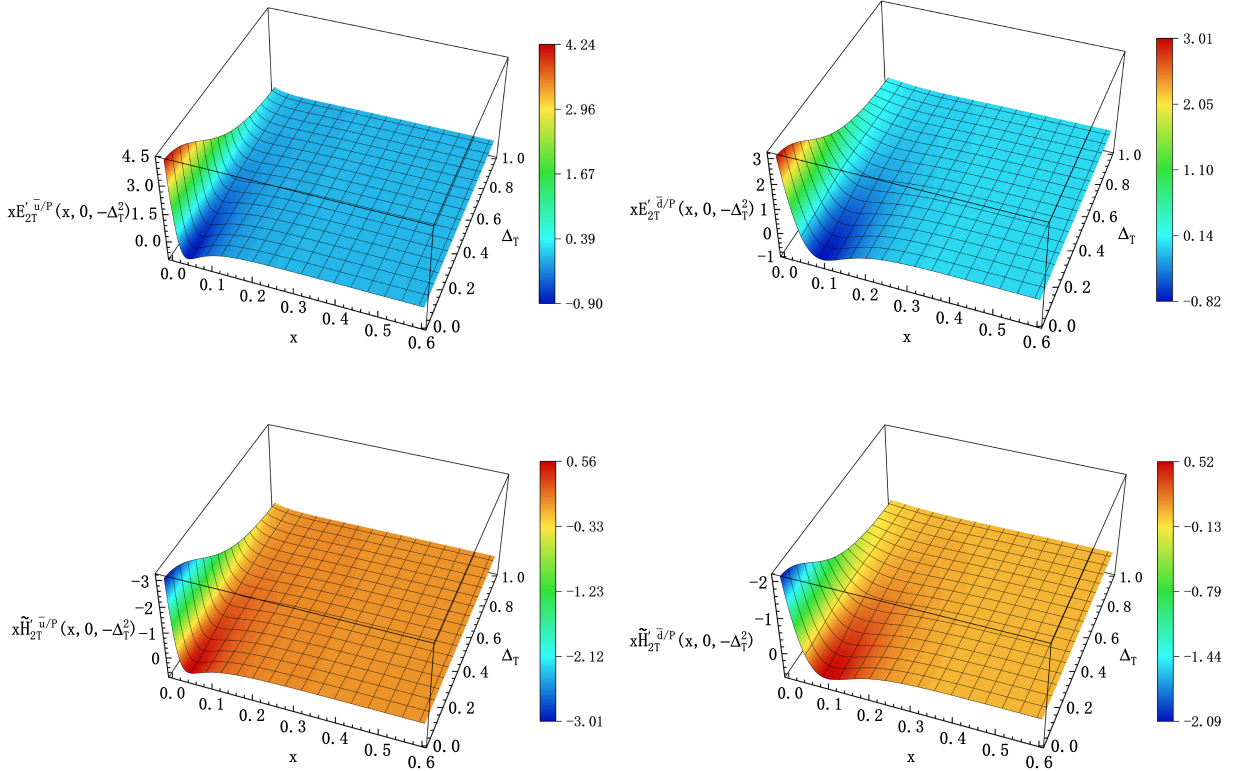


FIG. 4. Twist-3 chiral-odd GPDs (multiplied with x) $E_{2T}'^{\bar{u}/P}$, $E_{2T}'^{\bar{d}/P}$, $\tilde{H}_{2T}'^{\bar{u}/P}$, and $\tilde{H}_{2T}'^{\bar{d}/P}$ in the light-cone quark model as functions of x and Δ_T .

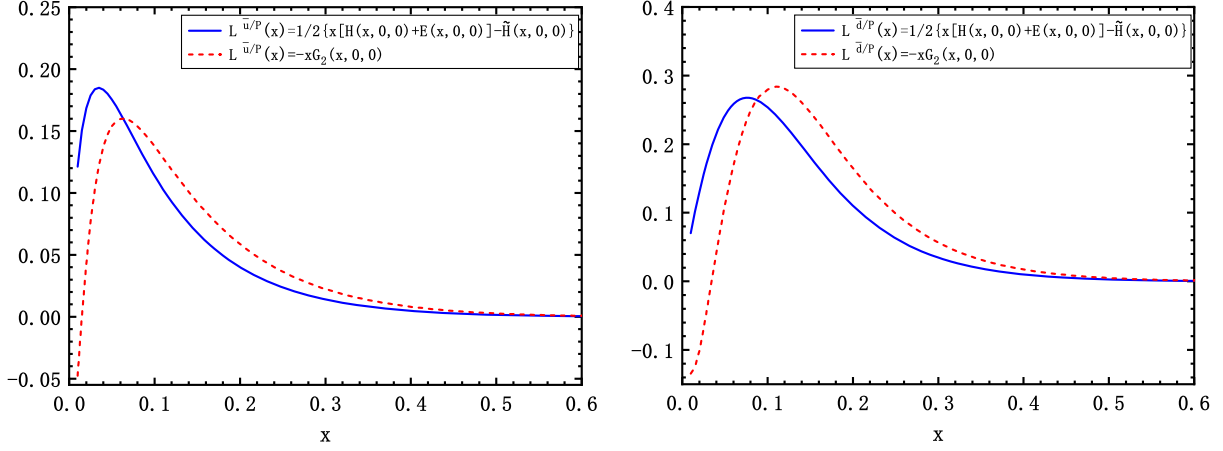


FIG. 5. Kinetic OAM of $\bar{u}$ (left panel) and $\bar{d}$ (right panel) quarks defined by the twist-2 and twist-3 GPDs as a function of x .

of the $\bar{d}$ quark is comparable to that in Ref. [74], while the kinetic OAM of the $\bar{u}$ quark in our model is smaller than that in their model. For further comparison, Ref. [74] also presented results using a different form of the light-front wave function (Model-II): $L^{\bar{u}} = 0.048 \pm 0.003$ and $L^{\bar{d}} = 0.059 \pm 0.002$. Although qualitatively similar to Model-I, both sea quark OAMs in Model-II are systematically larger than our model predictions. The agreement between OAM results obtained from twist-2 and twist-3 GPD formulations provides an important cross-check of our approach and suggests useful constraints on sea quark angular momentum contributions. The observed discrepancies with other theoretical approaches underscore the need for further theoretical refinements and experimental investigations. Such efforts will help elucidate the origins of these differences and advance our understanding of proton spin structure.

In the forward limit, the twist-3 GPDs H_2 , $\tilde{H}'_2$, and H_{2T} can reduce to their corresponding PDFs,

$$e(x) = \lim_{\Delta \rightarrow 0} H_2(x, 0, t), \quad (53)$$

$$h_L(x) = \lim_{\Delta \rightarrow 0} \tilde{H}'_2(x, 0, t), \quad (54)$$

$$g_T(x) = \lim_{\Delta \rightarrow 0} H'_{2T}(x, 0, t). \quad (55)$$

Among them, only H_2 has a nonzero twist-3 PDF $e(x)$. We can easily verify that our result satisfies the following relation:

$$e^{\bar{q}/P}(x) = \frac{m}{xM} f_1^{\bar{q}/P}(x), \quad (56)$$

where $f_1^{\bar{q}/P}(x)$ has been obtained in Ref. [54]. In Fig. 6, we depict the twist-3 PDF $xe^{\bar{q}/P}(x)$ of $\bar{u}$ and $\bar{d}$ quarks as a function of x . We find that the magnitudes of $xe^{\bar{u}/P}(x)$ and $xe^{\bar{d}/P}(x)$ decrease as x increases. The signs of $xe^{\bar{u}/P}(x)$ and

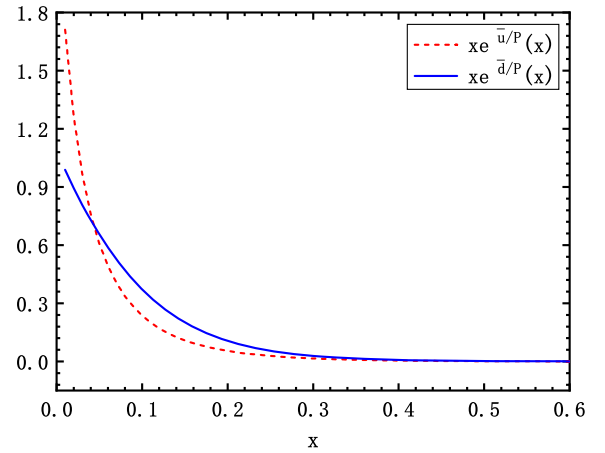


FIG. 6. Twist-3 PDFs $e^{\bar{q}/P}(x)$ (timed with x) of $\bar{u}$ and $\bar{d}$ quarks as a function of x .

$xe^{\bar{d}/P}(x)$ are positive throughout the entire x region. Particularly, the magnitude of $xe^{\bar{u}/P}(x)$ is much larger than that of $xe^{\bar{d}/P}(x)$.

V. CONCLUSION

In this work, we have investigated the twist-3 GPDs of sea quarks in the proton at zero skewness using the light-cone quark model. Our approach expresses these distributions through the overlap representation of LCWFs, providing a transparent framework for understanding sea quark contributions to nucleon structure. To model the sea quark degrees of freedom, we considered the proton Fock state expansion, which includes meson-baryon fluctuations, specifically treating the proton as a composite system of a pion and a baryon, where the pion itself consists of a quark-antiquark ($q\bar{q}$) pair. Within this framework, we derived analytic expressions for both chiral-odd and chiral-even twist-3 GPDs of $\bar{u}$ and $\bar{d}$ sea quarks.

Our analysis reveals two notable vanishing results within our model: the distributions $\tilde{E}_2^{\bar{q}/P}(x, 0, t)$ and $H'_{2T}{}^{\bar{q}/P}(x, 0, t)$ are identically zero. Through careful parameter selection, we obtained numerical results for these twist-3 GPDs as functions of both the longitudinal momentum fraction x and the transverse momentum transfer Δ_T . Furthermore, we presented the numerical results for the corresponding twist-3 PDFs by taking the forward limit of the GPDs. Particularly, we verify the relations between the twist-3 PDF $e^{\bar{q}/P}(x)$ and unpolarized PDF $f_1^{\bar{q}/P}(x)$ of the $\bar{u}$ and $\bar{d}$ quarks. In addition, the kinetic OAMs of sea quarks defined by the twist-2 GPDs and those defined by the twist-3 GPDs are compared in our work. The results demonstrate consistent OAM values between twist-2 and twist-3 definitions, providing important validation of our approach. This work offers new theoretical insights into sea quark dynamics, with implications for understanding nucleon structure and guiding future experimental measurements of higher-twist effects.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: OVERLAP REPRESENTATION OF TWIST-3 GPDS OF SEA QUARKS

Using Eq. (38) as well as the relations in Eqs. (36) and (37), the overlap representation of the twist-3 chiral-odd GPDs of sea quarks in terms of the LCWFs can be expressed as

$$H_2(x, 0, t) = \int_x^1 \frac{dy}{y} \int \frac{d^2 \mathbf{k}_T}{16\pi^3} \int \frac{d^2 \mathbf{r}_T}{16\pi^3} \sum_{\lambda_B \lambda_q} \times \left[\frac{m}{2Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) \right. \\ \left. + \frac{-\Delta_1 + i\Delta_2}{4Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) + \frac{\Delta_1 + i\Delta_2}{4Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) \right], \quad (A1)$$

$$\frac{i\Delta_2}{2} E_2(x, 0, t) = \int_x^1 \frac{dy}{y} \int \frac{d^2 \mathbf{k}_T}{16\pi^3} \int \frac{d^2 \mathbf{r}_T}{16\pi^3} \sum_{\lambda_B \lambda_q} \times \left[\frac{m}{2x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) \right. \\ \left. + \frac{-\Delta_1 + i\Delta_2}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) + \frac{\Delta_1 + i\Delta_2}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) \right],$$

$$\frac{-\Delta_1}{2} \tilde{E}_2(x, 0, t) = \int_x^1 \frac{dy}{y} \int \frac{d^2 \mathbf{k}_T}{16\pi^3} \int \frac{d^2 \mathbf{r}_T}{16\pi^3} \sum_{\lambda_B \lambda_q} \times \left[\frac{\Delta_1 - i\Delta_2}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) + \frac{\Delta_1 + i\Delta_2}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) \right], \quad (A2)$$

$$\tilde{H}'_2(x, 0, t) = - \int_x^1 \frac{dy}{y} \int \frac{d^2 \mathbf{k}_T}{16\pi^3} \int \frac{d^2 \mathbf{r}_T}{16\pi^3} \sum_{\lambda_B \lambda_q} \times \left[\frac{m}{2Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) \right. \\ \left. + \frac{-k_1 + ik_2}{2Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) + \frac{-(k_1 + ik_2)}{2Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+) \right]. \quad (A3)$$

Similarly, the chiral-even twist-3 GPDs in the overlap representation can be expressed as

$$\begin{aligned} \frac{i\Delta_T^2}{2} \tilde{E}_{2T}(x, 0, t) = & - \int_x^1 \frac{dy}{y} \int \frac{d^2\mathbf{k}_T}{16\pi^3} \int \frac{d^2\mathbf{r}_T}{16\pi^3} \sum_{\lambda_B \lambda_q} \\ & \times \left\{ \Delta_2 \left[\frac{2k_1 + i\Delta_2}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) + \frac{2k_1 - i\Delta_2}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right] \right. \\ & \left. - \Delta_1 \left[\frac{2k_2 - i\Delta_1}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) + \frac{2k_2 + i\Delta_1}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right] \right\}, \quad (\text{A4}) \end{aligned}$$

$$\begin{aligned} \frac{i\Delta_T^2}{2} (E'_{2T} + 2\tilde{H}'_{2T}) = & \int_x^1 \frac{dy}{y} \int \frac{d^2\mathbf{k}_T}{16\pi^3} \int \frac{d^2\mathbf{r}_T}{16\pi^3} \sum_{\lambda_B \lambda_q} \\ & \times \left\{ \Delta_2 \left[\frac{m}{2x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^- + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right. \right. \\ & + \frac{-2k_1 + i\Delta_2}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) + \frac{2k_1 + i\Delta_2}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \left. \right] \\ & - \Delta_1 \left[\frac{im}{2x} (-\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^- + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right. \\ & \left. \left. + \frac{-2k_2 - i\Delta_1}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) + \frac{2k_2 - i\Delta_1}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right] \right\}, \quad (\text{A5}) \end{aligned}$$

$$\begin{aligned} H'_{2T} + \frac{\Delta_T^2}{4M^2} \tilde{H}'_{2T} = & \int_x^1 \frac{dy}{y} \int \frac{d^2\mathbf{k}_T}{16\pi^3} \int \frac{d^2\mathbf{r}_T}{16\pi^3} \sum_{\lambda_B \lambda_q} \\ & \times \left\{ \left[\frac{m}{4Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^- + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right. \right. \\ & + \frac{-2k_1 + i\Delta_2}{8Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) + \frac{2k_1 + i\Delta_2}{8Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \left. \right] \\ & + i \left[\frac{im}{4Mx} (-\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^- + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right. \\ & \left. \left. + \frac{-2k_2 - i\Delta_1}{8Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) + \frac{2k_2 - i\Delta_1}{8Mx} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right] \right\}, \quad (\text{A6}) \end{aligned}$$

$$\begin{aligned} \frac{i\Delta_1 \Delta_2}{M} \tilde{H}'_{2T}(x, 0, t) = & \int_x^1 \frac{dy}{y} \int \frac{d^2\mathbf{k}_T}{16\pi^3} \int \frac{d^2\mathbf{r}_T}{16\pi^3} \sum_{\lambda_B \lambda_q} \\ & \times \left\{ \left[\frac{m}{2x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^- + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right. \right. \\ & + \frac{-2k_1 + i\Delta_2}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) + \frac{2k_1 + i\Delta_2}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \left. \right] \\ & - i \left[\frac{im}{2x} (-\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ - \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^- + \psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right. \\ & \left. \left. + \frac{-2k_2 - i\Delta_1}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) + \frac{2k_2 - i\Delta_1}{4x} (\psi_{\lambda_B \lambda_q}^{+*} \psi_{\lambda_B \lambda_q}^+ + \psi_{\lambda_B \lambda_q}^{-*} \psi_{\lambda_B \lambda_q}^-) \right] \right\}. \quad (\text{A7}) \end{aligned}$$

For simplicity, here we use $\psi_{\lambda_B \lambda_q \lambda'_q}^{\Lambda^*}$ and $\psi_{\lambda_B \lambda_q \lambda_q}^{\Lambda}$ to represent $\psi_{\lambda_B \lambda_q \lambda'_q}^{\Lambda^*}(x, y, \mathbf{k}_T'', \mathbf{r}_T'')$ and $\psi_{\lambda_B \lambda_q \lambda_q}^{\Lambda}(x, y, \mathbf{k}_T', \mathbf{r}_T')$, respectively. And

$$\begin{aligned} \mathbf{k}_T'' &= \mathbf{k}_T - \frac{1}{2}(1-x)\Delta_T, \\ \mathbf{k}_T' &= \mathbf{k}_T + \frac{1}{2}(1-x)\Delta_T, \end{aligned} \quad (\text{A8})$$

are the transverse momenta for the final-state and initial-state struck antiquarks, and

$$\begin{aligned} -\mathbf{r}_T'' &= -\mathbf{r}_T + \frac{1}{2}(1-y)\Delta_T, \\ -\mathbf{r}_T' &= -\mathbf{r}_T - \frac{1}{2}(1-y)\Delta_T, \\ (\mathbf{r}_T - \mathbf{k}_T)'' &= (\mathbf{r}_T - \mathbf{k}_T) + \frac{1}{2}(y-x)\Delta_T, \\ (\mathbf{r}_T - \mathbf{k}_T)' &= (\mathbf{r}_T - \mathbf{k}_T) - \frac{1}{2}(y-x)\Delta_T, \end{aligned} \quad (\text{A9})$$

are the transverse momenta for the final and initial spectators B and q , respectively.

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