

Axial charges and magnetic moments of the decuplet pentaquark family

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(Received 18 November 2025; accepted 4 March 2026; published 19 March 2026)

We present a systematic calculation of the axial charges and magnetic moments for the decuplet of hidden-charm molecular pentaquarks within the framework of the constituent quark model. Our findings reveal that the axial charges of these states are comparable in magnitude to that of the nucleon. Furthermore, we find that their magnetic moments obey a set of sum rules which hold for states sharing the same spin-flavor configuration, even in the presence of SU(3) flavor symmetry breaking. These results provide crucial insights into the internal structure and dynamics of multi-quark hadrons, offering valuable guidance for future experimental and theoretical investigations, $\mu_{P_{\psi}^{\Delta+}} - \mu_{P_{\psi}^{\Delta 0}} + \mu_{P_{\psi s}^{\Sigma-}} - \mu_{P_{\psi s}^{\Sigma+}} + \mu_{P_{\psi ss}^{N0}} - \mu_{P_{\psi ss}^{N-}} = 0$, $\mu_{P_{\psi}^{\Delta++}} + \mu_{P_{\psi}^{\Delta-}} + \mu_{P_{\psi sss}^{\Lambda}} = 3\mu_{P_{\psi s}^{\Sigma 0}}$.

DOI: 10.1103/32n9-j3pp

I. INTRODUCTION

The quark model, established by Gell-Mann in 1964 [1], provides a foundational framework for classifying hadrons into singlets, octets, and decuplets under SU(3) flavor symmetry. Beyond successfully describing conventional mesons and baryons, the model naturally accommodates the existence of exotic multi-quark states. Decades of experimental progress have now confirmed this prediction, with numerous tetraquark and pentaquark candidates observed in various experiments [2–10]. In this paper, we adopt the updated nomenclature for these pentaquark states as detailed in Ref. [11].

A landmark achievement in this field was the 2015 discovery by the LHCb Collaboration of the first hidden-charm pentaquark candidates, $P_{\psi}^N(4380)^+$ and $P_{\psi}^N(4450)^+$, in the decay $\Lambda_b^0 \rightarrow J/\psi K^- p$ [5]. In 2019, an updated analysis by LHCb revealed three additional states: $P_{\psi}^N(4312)^+$, $P_{\psi}^N(4440)^+$, and $P_{\psi}^N(4457)^+$ [6]. Notably, the previously observed $P_{\psi}^N(4450)^+$ was resolved into two distinct peaks corresponding to $P_{\psi}^N(4440)^+$ and the $P_{\psi}^N(4457)^+$. Further expanding the family, LHCb observed the hidden-charm strange pentaquark $P_{\psi s}^{\Lambda}(4459)^0$ in 2020 via amplitude analysis of $\Xi_b^- \rightarrow J/\psi \Lambda K^-$ [7]. In 2021, a new pentaquark structure, $P_{\psi}^N(4337)^+$, was identified in

$B_s^0 \rightarrow J/\psi p \bar{p}$ decays [8]. Most recently, LHCb and Belle reported the observation of $P_{\psi s}^{\Lambda}(4338)^0$ in $B^- \rightarrow J/\psi \Lambda \bar{p}$ decays and $\Upsilon(1S, 2S)$ inclusive decays to $J/\psi \Lambda$ [9,10].

The experimental discoveries of hidden-charm pentaquarks have stimulated extensive theoretical work to determine their spin-parity quantum numbers [12–26], although definitive experimental assignments remain unavailable. A striking commonality among these states is their mass proximity to thresholds for conventional baryon-meson pair production. This near-threshold behavior strongly suggests a molecular structure, where the pentaquark is a bound state of a charmed baryon and an anticharmed meson, potentially fitting into the octet or decuplet representations of the quark model. While this molecular picture provides a compelling theoretical basis, elucidating the exact nature of these states requires a deeper investigation of their fundamental properties.

Among the most revealing hadronic properties are the axial charge and the magnetic moment. The axial charge, g_A , is a fundamental parameter of the Standard Model, bridging electroweak and strong interactions [27,28]. It governs weak interaction processes such as beta decay, while also revealing deep connections between weak and strong interactions. This interplay is clearly embodied in the Goldberger-Treiman relation [29], $g_A = f_{\pi} g_{\pi NN} / M_N$, which directly links the axial charge to the pion decay constant f_{π} , the pion-nucleon coupling $g_{\pi NN}$, and the nucleon mass M_N . As a result, the relationship between pion dynamics and axial charges lies at the heart of hadronic structure studies. Moreover, g_A serves as a clear indicator of chiral symmetry breaking in nonperturbative QCD, making it an essential parameter in low-energy effective theories. Theoretical and phenomenological

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studies of axial charges have been actively pursued over the years [30–34]. Therefore, investigating the axial charges of the hidden-charm pentaquark offers a complementary and independent approach to understanding these exotic states.

Beyond what can be learned from axial charge alone, exploring the properties of the hidden-charm pentaquark family within molecular models offers valuable insights into the internal structure of these exotic states. Electromagnetic properties of hadrons play a crucial role in deciphering strong interaction mechanisms and structural features, thereby shedding light on nonperturbative aspects of QCD in the low-energy regime. Among these properties, the magnetic moment provides key information on internal charge and magnetization distributions. The magnetic moments of hidden-charm pentaquark states have been investigated using quark-based approaches in Refs. [35–37]. With the framework of QCD light-cone sum rules, the authors extracted the magnetic dipole moment of the $P_\psi^N(4312)^+$, $P_\psi^N(4440)^+$, $P_\psi^N(4457)^+$, $P_\psi^N(4380)^+$, and $P_{\psi s}^\Lambda(4459)^0$ pentaquark states in the molecular and diquark-diquark-antiquark models in Ref. [38]. In Ref. [39], the authors determined the ground state configuration of hidden-charm pentaquark states with quantum numbers $J^P = \frac{3}{2}^-$, along with the corresponding magnetic dipole moments and electromagnetic coupling constants. These results provide valuable input for quark-model-based studies of pentaquark photoproduction processes.

The investigation of axial charges and magnetic moments of pentaquark states offers crucial insight into their internal structure and the interquark dynamics governed by the strong interaction. These observables shed light on how quarks are bound into composite hadrons through nonperturbative QCD mechanisms. With ongoing and future experimental advancements, the discovery of the complete octet and decuplet of hidden-charm molecular pentaquarks appears increasingly attainable. Such progress will intensify interest in deciphering the internal organization of these states, where studies of axial charges and magnetic moments can provide essential quantitative guidance.

In this work, we extend our previous analysis of the octet hidden-charm molecular pentaquarks [40,41] to the decuplet representation. We report a comprehensive quark-model calculation of the axial charges and magnetic moments for the complete decuplet family. Our results demonstrate that the axial charges are of the same order as the nucleon's, and that the magnetic moments follow characteristic patterns, culminating in the sum rules that persist even under $SU(3)$ symmetry breaking.

This paper is structured as follows. Section II introduces the molecular model and the corresponding wave functions. The axial charges of the decuplet hidden-charm molecular pentaquark are then calculated in Sec. III, followed by the

computation of their magnetic moments in Sec. IV. Finally, Sec. V provides a summary of our work.

II. WAVE FUNCTIONS

The wave function Ψ of a hadronic state is central to the computation of its properties. In the quark model, the total wave function is constructed as a direct product of its constituent parts; the flavor wave function ψ_{flavor} , spin wave function χ_{spin} , color wave function ξ_{color} and space wave function η_{space} , each contributing to the complete quantum mechanical description. This is represented as

$$\Psi = \psi_{\text{flavor}} \otimes \chi_{\text{spin}} \otimes \xi_{\text{color}} \otimes \eta_{\text{space}}. \quad (1)$$

The overall wave function must be antisymmetric under fermion exchange, as required by Pauli exclusion principle. In the ground state, where the color wave function ξ_{color} is antisymmetric and the spatial wave function η_{space} is symmetric, the flavor-spin component must be constructed to satisfy the overall antisymmetry condition in calculations of axial charges and magnetic moments.

In the molecular picture, the hidden-charm pentaquark family is composed of a singly charmed baryon and an anticharmed meson. The singly charmed baryons can be derived from light baryons under $SU(3)$ flavor symmetry by replacing one light quark with a charm quark. Depending on the symmetry of the two remaining light quarks, two configurations arise:

- (i) When the light quark pair is symmetric, the charmed baryon belongs to the $\mathbf{6}_f$ representation. Combining this with the anticharmed meson ($\bar{\mathbf{3}}_f$) yields pentaquark states in the $\mathbf{10}_f$ and $\mathbf{8}_{1f}$ representations; and
- (ii) When the light quark pair is antisymmetric, the charmed baryon lies in the $\bar{\mathbf{3}}_f$ representation, leading to pentaquarks in the $\mathbf{8}_{2f}$ and $\mathbf{1}_f$ representations when coupled with the anticharmed meson.

This structure follows from the group decomposition $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8}_1 \oplus \mathbf{8}_2 \oplus \mathbf{10}$. We extend our study of hidden-charm molecular pentaquarks from the octet [40] to the decuplet by performing a systematic quark-model calculation of their axial charges and magnetic moments. The decuplet hidden-charm molecular pentaquark states have three spin configurations $J^P(J_b^{P_b} \otimes J_m^{P_m})$: $\frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$, $\frac{1}{2}^-(\frac{1}{2}^+ \otimes 1^-)$, and $\frac{3}{2}^-(\frac{1}{2}^+ \otimes 1^-)$, where J^P is the total spin of pentaquark states, and $J_b^{P_b} \otimes J_m^{P_m}$ correspond to the angular momentum and parity of baryon and meson, respectively. In Table I, we present the flavor wave functions ψ_{flavor} and the spin wave functions χ_{spin} of the S -wave anticharmed mesons. In Table II, we present the flavor wave functions ψ_{flavor} and the spin wave functions χ_{spin} of the S -wave singly charmed baryons. The flavor wave functions of the hidden-charm molecular pentaquark under $SU(3)$ symmetry are listed in Table III.

TABLE I. The flavor wave functions ψ_{flavor} and the spin wave functions χ_{spin} of the S -wave anticharmed mesons. Here, S and S_3 are the spin and its third component, while the arrow denotes the third component of the quark spin.

Mesons	$ S, S_3\rangle$	$\psi_{\text{flavor}} \otimes \chi_{\text{spin}}$
$\bar{D}^0$	$ 0, 0\rangle$	$\bar{c}u \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$
D^-	$ 0, 0\rangle$	$\bar{c}d \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$
D_s^-	$ 0, 0\rangle$	$\bar{c}s \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$
$\bar{D}^{*0}$	$ 1, 1\rangle$	$\bar{c}u \otimes \uparrow\uparrow$
	$ 1, 0\rangle$	$\bar{c}u \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$
D^{*-}	$ 1, -1\rangle$	$\bar{c}u \otimes \downarrow\downarrow$
	$ 1, 1\rangle$	$\bar{c}d \otimes \uparrow\uparrow$
	$ 1, 0\rangle$	$\bar{c}d \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$
	$ 1, -1\rangle$	$\bar{c}d \otimes \downarrow\downarrow$
D_s^{*-}	$ 1, 1\rangle$	$\bar{c}s \otimes \uparrow\uparrow$
	$ 1, 0\rangle$	$\bar{c}s \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$
	$ 1, -1\rangle$	$\bar{c}s \otimes \downarrow\downarrow$

TABLE II. The flavor wave functions ψ_{flavor} and the spin wave functions χ_{spin} of the S -wave singly charmed baryon. Here, S and S_3 are the spin and its third component, while the arrow denotes the third component of the quark spin.

Baryons	$ S, S_3\rangle$	$\psi_{\text{flavor}} \otimes \chi_{\text{spin}}$
Σ_c^{++}	$ \frac{1}{2}, \frac{1}{2}\rangle$	$uuc \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$uuc \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \uparrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
Σ_c^+	$ \frac{1}{2}, \frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(udc + duc) \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(udc + duc) \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \uparrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
Σ_c^0	$ \frac{1}{2}, \frac{1}{2}\rangle$	$ddc \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$ddc \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \uparrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
$\Xi_c^{'+}$	$ \frac{1}{2}, \frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(usc + suc) \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(usc + suc) \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \uparrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
Ξ_c^0	$ \frac{1}{2}, \frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(dsc + sdc) \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(dsc + sdc) \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \uparrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
Ω_c^0	$ \frac{1}{2}, \frac{1}{2}\rangle$	$ssc \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$ssc \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \uparrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
Ξ_c^+	$ \frac{1}{2}, \frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(usc - suc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$
	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(usc - suc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)$
Ξ_c^0	$ \frac{1}{2}, \frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(dsc - sdc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$
	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(dsc - sdc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)$
Λ_c^+	$ \frac{1}{2}, \frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(udc - duc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$
	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(udc - duc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)$

TABLE III. The expressions for different flavor wave function of the decuplet hidden-charm molecular pentaquark states under $SU(3)$ symmetry in the molecular model.

(Y, I, I_3)	10_f	Wave function
$(1, \frac{3}{2}, \frac{3}{2})$	$P_{\psi}^{\Delta++}$	$\bar{D}^{(*)0}\Sigma_c^{++}$
$(1, \frac{3}{2}, \frac{1}{2})$	$P_{\psi}^{\Delta+}$	$\sqrt{\frac{2}{3}}\bar{D}^{(*)0}\Sigma_c^+ + \sqrt{\frac{1}{3}}\bar{D}^{(*)-}\Sigma_c^{++}$
$(1, \frac{3}{2}, -\frac{1}{2})$	$P_{\psi}^{\Delta 0}$	$\sqrt{\frac{2}{3}}\bar{D}^{(*)-}\Sigma_c^+ + \sqrt{\frac{1}{3}}\bar{D}^{(*)0}\Sigma_c^0$
$(1, \frac{3}{2}, -\frac{3}{2})$	$P_{\psi}^{\Delta-}$	$D^{(*)-}\Sigma_c^0$
$(0, 1, 1)$	$P_{\psi s}^{\Sigma+}$	$\sqrt{\frac{2}{3}}\bar{D}^{(*)0}\Xi_c'^+ + \sqrt{\frac{1}{3}}\bar{D}_s^{(*)-}\Sigma_c^{++}$
$(0, 1, 0)$	$P_{\psi s}^{\Sigma 0}$	$\sqrt{\frac{1}{3}}(D^{(*)-}\Xi_c'^+ + \bar{D}^{(*)0}\Xi_c'^0) + \sqrt{\frac{1}{3}}\bar{D}_s^{(*)-}\Sigma_c^+$
$(0, 1, -1)$	$P_{\psi s}^{\Sigma-}$	$\sqrt{\frac{2}{3}}\bar{D}^{(*)-}\Xi_c'^0 + \sqrt{\frac{1}{3}}\bar{D}_s^{(*)-}\Sigma_c^0$
$(-1, \frac{1}{2}, \frac{1}{2})$	$P_{\psi ss}^{N0}$	$\sqrt{\frac{1}{3}}\bar{D}^{(*)0}\Omega_c^0 + \sqrt{\frac{2}{3}}\bar{D}_s^{(*)-}\Xi_c'^+$
$(-1, \frac{1}{2}, -\frac{1}{2})$	$P_{\psi ss}^{N-}$	$\sqrt{\frac{1}{3}}\bar{D}^{(*)-}\Omega_c^0 + \sqrt{\frac{2}{3}}\bar{D}_s^{(*)-}\Xi_c'^0$
$(-2, 0, 0)$	$P_{\psi sss}^{\Lambda-}$	$D_s^{(*)-}\Omega_c^0$

III. AXIAL CHARGE OF THE DECUPLET HIDDEN-CHARM MOLECULAR PENTAQUARK FAMILY

The axial charge represents a fundamental quantity in the standard model, bridging electroweak and strong interaction physics. It also provides a key probe of spontaneous chiral symmetry breaking in the nonperturbative regime of QCD. Investigating the axial charges of the newly observed hidden-charm pentaquark states thus offers a promising avenue to shed new light on chiral symmetry realization in exotic hadronic systems.

At the quark level, the pion-quark interaction reads

$$\mathcal{L}_{\text{quark}} = \frac{1}{2} g_q \bar{\psi}_q \gamma^\mu \gamma_5 \partial_\mu \phi \psi_q, \quad (2)$$

where g_q is the coupling constant at the quark level, σ_z is the z-component Pauli matrix. $f_\pi = 92$ MeV is the decay constant of the pseudoscalar meson. ϕ is the pseudoscalar meson field. Considering only the z-component, the π_0 quark interaction reads

$$\mathcal{L}_{\text{quark}} = \frac{1}{2} \frac{g_q}{f_\pi} (\bar{u} \sigma_z \partial_z \pi_0 u - \bar{d} \sigma_z \partial_z \pi_0 d), \quad (3)$$

The nucleon-pion Lagrangian at hadron level reads

$$\mathcal{L}_N = \frac{1}{2} g_A \bar{N} \gamma^\mu \gamma_5 \partial_\mu \phi N, \quad (4)$$

where g_A is the axial charge of the nucleon. Considering only the z-component, the nucleon-pion Lagrangian reads

$$\mathcal{L}_N = \frac{g_A}{f_\pi} \bar{N} \frac{\Sigma_z}{2} \partial_z \pi_0 N, \quad (5)$$

thus,

$$\left\langle N, j_3 = \frac{1}{2}; \pi_0 | \mathcal{L}_N | N, j_3 = \frac{1}{2} \right\rangle = \frac{1}{2} \frac{q_z}{f_\pi} g_A, \quad (6)$$

where q_z denotes the z-component of the external momentum carried by π_0 . At the quark level,

$$\left\langle N, j_3 = \frac{1}{2}; \pi_0 | \mathcal{L}_{\text{quark}} | N, j_3 = \frac{1}{2} \right\rangle = \frac{5}{6} \frac{q_z}{f_\pi} g_q. \quad (7)$$

Within the framework of quark-hadron duality, we obtain the relation $g_q = \frac{3}{5} g_A$ from Eqs. (6) and (7).

The decuplet pentaquark Lagrangian with $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$ in $\mathbf{10}_f$ flavor representation reads

$$\mathcal{L}_1^{\frac{1}{2}} = \text{Tr}(d_1 \bar{\mathcal{P}} \gamma_\mu \gamma^5 \partial^\mu \Phi \mathcal{P}), \quad (8)$$

where d_1 is the axial coupling constant of pentaquark state with $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$ in $\mathbf{10}_f$ flavor representation. Φ represents the pseudoscalar meson field in $SU(3)$ flavor symmetry,

$$\Phi \equiv \begin{pmatrix} \pi_0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi_0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}. \quad (9)$$

$\mathcal{P}$ represents the decuplet hidden-charm molecular pentaquark states. We adopt the tensor field $\mathcal{P} \equiv \mathcal{P}^{abc}$,

$$\begin{aligned} \mathcal{P}^{111} &= P_\psi^{\Delta^{++}}, & \mathcal{P}^{112} &= \frac{1}{\sqrt{3}} P_\psi^{\Delta^+}, & \mathcal{P}^{122} &= \frac{1}{\sqrt{3}} P_\psi^{\Delta^0}, \\ \mathcal{P}^{222} &= P_\psi^{\Delta^-}, & \mathcal{P}^{113} &= \frac{1}{\sqrt{3}} P_{\psi s}^{\Sigma^+}, & \mathcal{P}^{123} &= \frac{1}{\sqrt{6}} P_{\psi s}^{\Sigma^0}, \\ \mathcal{P}^{223} &= \frac{1}{\sqrt{3}} P_{\psi s}^{\Sigma^-}, & \mathcal{P}^{133} &= \frac{1}{\sqrt{3}} P_{\psi ss}^{N^0}, & \mathcal{P}^{233} &= \frac{1}{\sqrt{3}} P_{\psi ss}^{N^-}, \\ \mathcal{P}^{333} &= P_{\psi sss}^{\Lambda^-}. \end{aligned} \quad (10)$$

Substituting the matrix representations of Φ and $\mathcal{P}$ into Eq. (8) and retaining only the π_0 meson term, we obtain

$$\begin{aligned} \mathcal{L}_{10}^{\frac{1}{2}} &= \frac{1}{f_\pi} d_1 \bar{P}_\psi^{\Delta^{++}} \Sigma_z \partial_z \pi_0 P_\psi^{\Delta^{++}} + \frac{1}{3f_\pi} d_1 \bar{P}_\psi^{\Delta^+} \Sigma_z \partial_z \pi_0 P_\psi^{\Delta^+} \\ &\quad - \frac{1}{3f_\pi} d_1 \bar{P}_\psi^{\Delta^0} \Sigma_z \partial_z \pi_0 P_\psi^{\Delta^0} - \frac{1}{f_\pi} d_1 \bar{P}_\psi^{\Delta^-} \Sigma_z \partial_z \pi_0 P_\psi^{\Delta^-} \\ &\quad + \frac{2}{3f_\pi} d_1 \bar{P}_{\psi s}^{\Sigma^+} \Sigma_z \partial_z \pi_0 P_{\psi s}^{\Sigma^+} - \frac{2}{3f_\pi} d_1 \bar{P}_{\psi s}^{\Sigma^-} \Sigma_z \partial_z \pi_0 P_{\psi s}^{\Sigma^-} \\ &\quad + \frac{1}{3f_\pi} d_1 \bar{P}_{\psi ss}^{N^0} \Sigma_z \partial_z \pi_0 P_{\psi ss}^{N^0} - \frac{1}{3f_\pi} d_1 \bar{P}_{\psi ss}^{N^-} \Sigma_z \partial_z \pi_0 P_{\psi ss}^{N^-}. \end{aligned} \quad (11)$$

Having established the axial coupling constants for the decuplet hidden-charm molecular pentaquarks in terms of the parameter d_1 , we note that Eq. (11) specifically details the components for the π_0 meson decay. The same formalism can be extended to derive the corresponding couplings for decays involving other light mesons, all of which are likewise determined by the same universal parameter d_1 .

To determine the coupling constant d_1 , similar to the procedure employed for the nucleon, we consider the π_0 meson decay of $P_\psi^{\Delta^{++}}$ in Eq. (11). The Lagrangian for the π_0 decay of $P_\psi^{\Delta^{++}}$ with the spin configuration $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$ at the hadron level reads

$$\mathcal{L}_{P_\psi^{\Delta^{++}}}^{\frac{1}{2}} = \frac{d_1}{f_\pi} \bar{P}_\psi^{\Delta^{++}} \frac{\Sigma_z}{2} \partial_z \pi_0 P_\psi^{\Delta^{++}}. \quad (12)$$

$\frac{\Sigma_z}{2}$ is the spin operator of the hidden-charm pentaquark states. At the hadron level, the axial charges read

$$\left\langle P_\psi^{\Delta^{++}}; \pi_0 \left| \frac{d_1}{f_\pi} \bar{P}_\psi^{\Delta^{++}} \frac{\Sigma_z}{2} \partial_z \pi_0 P_\psi^{\Delta^{++}} \right| P_\psi^{\Delta^{++}} \right\rangle = \frac{d_1}{2} \frac{q_z}{f_\pi}. \quad (13)$$

At the quark level, the axial charge of $P_\psi^{\Delta^{++}}$ with the spin configuration $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$ read

$$\left\langle P_\psi^{\Delta^{++}}, +\frac{1}{2}; \pi_0 \left| \mathcal{L}_{\text{quark}} \right| P_\psi^{\Delta^{++}}, +\frac{1}{2} \right\rangle = \frac{2}{3} \frac{q_z}{f_\pi} g_q. \quad (14)$$

By comparing the hadron-level and quark-level matrix elements for the nucleon and the pentaquark, we establish the relation,

$$\frac{\frac{1}{2} g_A}{\frac{5}{6} g_q} = \frac{\frac{d_1}{2}}{\frac{2}{3} g_q}. \quad (15)$$

Thus, we obtain $d_1 = \frac{4}{5} g_A$. Similarly, we can obtain the axial charge of the pentaquark states with other flavor-spin configurations. The Lagrangian of pentaquark state for $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 1^-)$ in $\mathbf{10}_f$ flavor representation reads

$$\mathcal{L}_1^{\frac{1}{2}} = \text{Tr}(d_2 \bar{\mathcal{P}} \gamma_\mu \gamma^5 \partial^\mu \Phi \mathcal{P}), \quad (16)$$

The Lagrangian of pentaquark state for $J^P = \frac{3}{2}^-(\frac{1}{2}^+ \otimes 1^-)$ in $\mathbf{10}_f$ flavor representation reads

$$\mathcal{L}_3^{\frac{3}{2}} = \text{Tr}(d_3 \bar{\mathcal{P}}^\nu \gamma_\mu \gamma^5 \partial^\mu \Phi \mathcal{P}_\nu). \quad (17)$$

Similarly, we can also obtain the coupling constants $d_2 = \frac{7}{15}g_A$ and $d_3 = \frac{7}{5}g_A$. The axial charges of the decuplet hidden-charm molecular family in $\mathbf{10}_f$ flavor representation are listed in Table IV. As presented in Table IV, the axial charges of the decuplet hidden-charm pentaquark family are comparable to that of the nucleon, and generally larger compared to that of the octet hidden-charm pentaquark

family as shown in our previous work in Ref. [40]. We also notice that the axial charges of $P_{\psi s}^{\Sigma 0}$ and $P_{\psi s s}^{\Lambda -}$ in $\mathbf{10}_f$ flavor representation are zero.

At the hadronic level, the axial charge serves as a discriminating marker for the flavor-spin structure of pentaquark states. The resulting variation in its calculated value, within the quark-hadron dual picture, provides a robust basis for elucidating the strong decay patterns of structurally distinct pentaquarks in future research. The tabulated axial charges in Table IV are thus poised to play a crucial role in guiding subsequent calculations within the framework of chiral perturbation theory (details in our previous work in Ref. [42]).

TABLE IV. The axial charges of the decuplet hidden-charm pentaquark family in $\mathbf{10}_f$ flavor representation. The $J_b^{P_b} \otimes J_m^{P_m}$ correspond to the angular momentum and parity of baryon and meson, respectively.

Couplings	Coefficients	Wave functions	$J_b^{P_b} \otimes J_m^{P_m}$	$I(J^P)$	Results
$P_{\psi}^{\Delta++} P_{\psi}^{\Delta++} \pi_0$	d_1	$P_{\psi}^{\Delta++} : \bar{D}^{(*)0} \Sigma_c^{++}$	$\frac{1}{2}^+ \times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{4}{5}g_A$
	d_2		$\frac{1}{2}^+ \times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{7}{15}g_A$
	d_3			$\frac{1}{2}(\frac{3}{2})^-$	$\frac{7}{5}g_A$
$P_{\psi}^{\Delta+} P_{\psi}^{\Delta+} \pi_0$	$\frac{1}{3}d_1$	$P_{\psi}^{\Delta+} : \sqrt{\frac{2}{3}}\bar{D}^{(*)0}\Sigma_c^+ + \sqrt{\frac{1}{3}}\bar{D}^{(*)-}\Sigma_c^{++}$	$\frac{1}{2}^+ \times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{4}{15}g_A$
	$\frac{1}{3}d_2$		$\frac{1}{2}^+ \times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{7}{45}g_A$
	$\frac{1}{3}d_3$			$\frac{1}{2}(\frac{3}{2})^-$	$\frac{7}{15}g_A$
$P_{\psi}^{\Delta 0} P_{\psi}^{\Delta 0} \pi_0$	$-\frac{1}{3}d_1$	$P_{\psi}^{\Delta 0} : \sqrt{\frac{2}{3}}\bar{D}^{(*)-}\Sigma_c^+ + \sqrt{\frac{1}{3}}\bar{D}^{(*)0}\Sigma_c^0$	$\frac{1}{2}^+ \times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{4}{15}g_A$
	$-\frac{1}{3}d_2$		$\frac{1}{2}^+ \times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{7}{45}g_A$
	$-\frac{1}{3}d_3$			$\frac{1}{2}(\frac{3}{2})^-$	$-\frac{7}{15}g_A$
$P_{\psi}^{\Delta-} P_{\psi}^{\Delta-} \pi_0$	$-d_1$	$P_{\psi}^{\Delta-} : D^{(*)-}\Sigma_c^0$	$\frac{1}{2}^+ \times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{4}{5}g_A$
	$-d_2$		$\frac{1}{2}^+ \times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{7}{15}g_A$
	$-d_3$			$\frac{1}{2}(\frac{3}{2})^-$	$-\frac{7}{5}g_A$
$P_{\psi s}^{\Sigma+} P_{\psi s}^{\Sigma+} \pi_0$	$\frac{2}{3}d_1$	$P_{\psi s}^{\Sigma+} : \sqrt{\frac{2}{3}}\bar{D}^{(*)0}\Xi_c'^+ + \sqrt{\frac{1}{3}}\bar{D}_s^{(*)-}\Sigma_c^{++}$	$\frac{1}{2}^+ \times 0^-$	$1(\frac{1}{2})^-$	$\frac{8}{15}g_A$
	$\frac{2}{3}d_2$		$\frac{1}{2}^+ \times 1^-$	$1(\frac{1}{2})^-$	$\frac{14}{15}g_A$
	$\frac{2}{3}d_3$			$1(\frac{3}{2})^-$	$\frac{14}{15}g_A$
$P_{\psi s}^{\Sigma 0} P_{\psi s}^{\Sigma 0} \pi_0$	0	$P_{\psi s}^{\Sigma 0} : \sqrt{\frac{1}{3}}(D^{(*)-}\Xi_c'^+ + \bar{D}^{(*)0}\Xi_c'^0) + \sqrt{\frac{1}{3}}\bar{D}_s^{(*)-}\Sigma_c^+$	$\frac{1}{2}^+ \times 0^-$	$1(\frac{1}{2})^-$	0
	0		$\frac{1}{2}^+ \times 1^-$	$1(\frac{1}{2})^-$	0
	0			$1(\frac{3}{2})^-$	0
$P_{\psi s}^{\Sigma-} P_{\psi s}^{\Sigma-} \pi_0$	$-\frac{2}{3}d_1$	$P_{\psi s}^{\Sigma-} : \sqrt{\frac{2}{3}}\bar{D}^{(*)-}\Xi_c'^0 + \sqrt{\frac{1}{3}}\bar{D}_s^{(*)-}\Sigma_c^0$	$\frac{1}{2}^+ \times 0^-$	$1(\frac{1}{2})^-$	$-\frac{8}{15}g_A$
	$-\frac{2}{3}d_2$		$\frac{1}{2}^+ \times 1^-$	$1(\frac{1}{2})^-$	$-\frac{14}{45}g_A$
	$-\frac{2}{3}d_3$			$1(\frac{3}{2})^-$	$-\frac{14}{15}g_A$
$P_{\psi s s}^{N 0} P_{\psi s s}^{N 0} \pi_0$	$\frac{1}{3}d_1$	$P_{\psi s s}^{N 0} : \sqrt{\frac{1}{3}}\bar{D}^{(*)0}\Omega_c^0 + \sqrt{\frac{2}{3}}\bar{D}_s^{(*)-}\Xi_c'^+$	$\frac{1}{2}^+ \times 0^-$	$1(\frac{1}{2})^-$	$\frac{4}{15}g_A$
	$\frac{1}{3}d_2$		$\frac{1}{2}^+ \times 1^-$	$1(\frac{1}{2})^-$	$\frac{7}{45}g_A$
	$\frac{1}{3}d_3$			$1(\frac{3}{2})^-$	$\frac{7}{15}g_A$
$P_{\psi s s}^{N-} P_{\psi s s}^{N-} \pi_0$	$-\frac{1}{3}d_1$	$P_{\psi s s}^{N-} : \sqrt{\frac{1}{3}}\bar{D}^{(*)-}\Omega_c^0 + \sqrt{\frac{2}{3}}\bar{D}_s^{(*)-}\Xi_c'^0$	$\frac{1}{2}^+ \times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{4}{15}g_A$
	$-\frac{1}{3}d_2$		$\frac{1}{2}^+ \times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{7}{45}g_A$
	$-\frac{1}{3}d_3$			$\frac{1}{2}(\frac{3}{2})^-$	$-\frac{7}{15}g_A$
$P_{\psi s s s}^{\Lambda-} P_{\psi s s s}^{\Lambda-} \pi_0$	0	$P_{\psi s s s}^{\Lambda-} : D_s^{(*)-}\Omega_c^0$	$\frac{1}{2}^+ \times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	0
	0		$\frac{1}{2}^+ \times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	0
	0			$\frac{1}{2}(\frac{3}{2})^-$	0

IV. MAGNETIC MOMENTS OF THE DECUPLET HIDDEN-CHARM MOLECULAR PENTAQUARK FAMILY

Within the framework of the constituent quark model, we calculate the magnetic moments of hadrons. The quark model has been extensively applied over the past decades to investigate a wide range of hadronic properties, with the magnetic moments of hadronic molecular states being a recent focus of interest [43–45]. The total magnetic moment of a hadron can be decomposed into two components; the spin magnetic moment (μ_{spin}) and the orbital magnetic moment (μ_{orbital}),

$$\mu = \mu_{\text{spin}} + \mu_{\text{orbital}}. \quad (18)$$

In this work, we investigate the magnetic moments of S -wave pentaquark states within the molecular picture, where they are considered as bound states of a singly charmed baryon and an anticharmed meson. Accordingly, the orbital magnetic moment μ_{orbital} arising from the relative motion between the hadronic constituents is omitted from our subsequent calculations. For the spin magnetic moment (μ_{spin}), the operators of the magnetic moments at the quark level are

$$\hat{\mu}_{\text{spin}} = \sum_i \frac{Q_i}{2M_i} \hat{\sigma}_i, \quad (19)$$

where Q_i , M_i , and $\hat{\sigma}_i$ represent charge, mass and Pauli spin matrix of the i th quark, respectively. For hidden-charm molecular pentaquark states, the magnetic moment of an S -wave molecular pentaquark state is composed of the baryon spin magnetic moment and the meson spin magnetic moment,

$$\hat{\mu} = \hat{\mu}_B + \hat{\mu}_M. \quad (20)$$

Here, the subscripts B and M denote the baryon and meson components, respectively. Table V summarizes the expressions for the magnetic moments of the singly charmed baryons and anticharmed mesons, where each is given as a sum over the magnetic moments of their constituent quarks. For the numerical analysis, the constituent quark masses are used as the primary input parameters [46],

$$m_u = m_d = 0.336 \text{ GeV},$$

$$m_s = 0.540 \text{ GeV},$$

$$m_c = 1.660 \text{ GeV}.$$

Building on the magnetic moments of the singly charmed baryons and anticharmed mesons, we derive the corresponding moments for the decuplet hidden-charm molecular pentaquark states. The resulting analytical expressions and their corresponding numerical values for

TABLE V. The magnetic moments of the singly charmed baryons and anticharmed mesons, in unit of the nuclear magnetic moment μ_N .

Flavor	Hadrons	Expressions	Results
6_f	Σ_c^{++}	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_c$	2.36
	Σ_c^+	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_c$	0.49
	Σ_c^0	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_c$	-1.37
	$\Xi_c'^{+}$	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_s - \frac{1}{3}\mu_c$	0.73
	$\Xi_c'^0$	$\frac{2}{3}\mu_d + \frac{2}{3}\mu_s - \frac{1}{3}\mu_c$	-1.13
	Ω_c^0	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_c$	-0.90
$\bar{3}_f$	Ξ_c^+	μ_c	0.38
	Ξ_c^0	μ_c	0.38
	Λ_c^+	μ_c	0.38
3_f	$\bar{D}^{*0}$	$\mu_u + \mu_{\bar{c}}$	1.48
	D^{*-}	$\mu_d + \mu_{\bar{c}}$	-1.31
	D_s^{*-}	$\mu_s + \mu_{\bar{c}}$	-0.96

the S -wave $\frac{1}{2}^+ \otimes 0^- = \frac{1}{2}^-$, $\frac{1}{2}^+ \otimes 1^- = \frac{1}{2}^-$ and $\frac{1}{2}^+ \otimes 1^- = \frac{3}{2}^-$ pentaquark family in $\mathbf{10}_f$ representation are summarized in Tables VI–VIII. Thus, we can summarize several points:

- (i) The magnetic moments of the S -wave decuplet hidden-charm pentaquark states are strongly dependent on their specific flavor and spin configurations. Notably, the magnetic moments of the $\frac{1}{2}^+ \otimes 1^- = \frac{1}{2}^-$ states are significantly smaller than those of the $\frac{1}{2}^+ \otimes 0^- = \frac{1}{2}^-$ and $\frac{1}{2}^+ \otimes 1^- = \frac{3}{2}^-$ configurations. Furthermore, for the S -wave $\frac{1}{2}^+ \otimes 0^- = \frac{1}{2}^-$ and $\frac{1}{2}^+ \otimes 1^- = \frac{3}{2}^-$ decuplet hidden-charm pentaquark states, the magnetic moments are approximately proportional to their electric charges. This is a natural outcome of the additive magnetic moments in the molecular model, which can serve as a distinguishing feature from other compact models. In fact, we have discussed the magnetic moments of

TABLE VI. The magnetic moments of the S -wave $\frac{1}{2}^+ \otimes 0^- = \frac{1}{2}^-$ decuplet hidden-charm pentaquark states (in unit of μ_N).

$\frac{1}{2}^+ \otimes 0^- = \frac{1}{2}^-$ States	Expressions	Results
$P_{\psi}^{\Delta^{++}}$	$\mu_{\Sigma_c^{++}}$	2.36
$P_{\psi}^{\Delta^+}$	$\frac{2}{3}\mu_{\Sigma_c^+} + \frac{1}{3}\mu_{\Sigma_c^{++}}$	1.12
$P_{\psi}^{\Delta^0}$	$\frac{2}{3}\mu_{\Sigma_c^0} + \frac{1}{3}\mu_{\Sigma_c^+}$	-0.13
$P_{\psi}^{\Delta^-}$	$\mu_{\Sigma_c^0}$	-1.37
$P_{\psi s}^{\Sigma^+}$	$\frac{2}{3}\mu_{\Xi_c'^+} + \frac{1}{3}\mu_{\Sigma_c^{++}}$	1.27
$P_{\psi s}^{\Sigma^0}$	$\frac{1}{3}\mu_{\Xi_c'^+} + \frac{1}{3}\mu_{\Xi_c'^0} + \frac{1}{3}\mu_{\Sigma_c^+}$	0.03
$P_{\psi s}^{\Sigma^-}$	$\frac{2}{3}\mu_{\Xi_c'^0} + \frac{1}{3}\mu_{\Sigma_c^0}$	-1.21
$P_{\psi ss}^{N^0}$	$\frac{2}{3}\mu_{\Xi_c'^+} + \frac{1}{3}\mu_{\Omega_c^0}$	0.19
$P_{\psi ss}^{N^-}$	$\frac{2}{3}\mu_{\Xi_c'^0} + \frac{1}{3}\mu_{\Omega_c^0}$	-1.05
$P_{\psi sss}^{\Lambda^-}$	$\mu_{\Omega_c^0}$	-0.90

TABLE VII. The magnetic moments of the S -wave $\frac{1}{2}^+ \otimes 1^- = \frac{1}{2}^-$ decuplet hidden-charm pentaquark states (in unit of μ_N).

$\frac{1}{2}^+ \otimes 1^- = \frac{1}{2}^-$ States	Expressions	Results
$P_{\psi}^{\Delta^{++}}$	$\frac{2}{3}\mu_{\bar{D}^{*0}} - \frac{1}{3}\mu_{\Sigma_c^{*+}}$	0.20
$P_{\psi}^{\Delta^+}$	$\frac{4}{9}\mu_{\bar{D}^{*0}} + \frac{2}{9}\mu_{D^{*-}} - \frac{2}{9}\mu_{\Sigma_c^{*+}} - \frac{1}{9}\mu_{\Sigma_c^{*+}}$	-0.002
$P_{\psi}^{\Delta^0}$	$\frac{4}{9}\mu_{D^{*-}} + \frac{2}{9}\mu_{\bar{D}^{*0}} - \frac{2}{9}\mu_{\Sigma_c^{*+}} - \frac{1}{9}\mu_{\Sigma_c^0}$	-0.21
$P_{\psi}^{\Delta^-}$	$\frac{2}{3}\mu_{D^{*-}} - \frac{1}{3}\mu_{\Sigma_c^0}$	-0.42
$P_{\psi}^{\Sigma^+}$	$\frac{4}{9}\mu_{\bar{D}^{*0}} + \frac{2}{9}\mu_{D_s^{*-}} - \frac{2}{9}\mu_{\Xi_c^{*+}} - \frac{1}{9}\mu_{\Sigma_c^{*+}}$	0.02
$P_{\psi}^{\Sigma^0}$	$\frac{2}{9}\mu_{D^{*-}} - \frac{1}{9}\mu_{\Xi_c^{*+}} + \frac{2}{9}\mu_{\bar{D}^{*0}} - \frac{2}{9}\mu_{\Xi_c^0} + \frac{2}{9}\mu_{D_s^{*-}} - \frac{1}{9}\mu_{\Sigma_c^{*+}}$	-0.18
$P_{\psi}^{\Sigma^-}$	$\frac{4}{9}\mu_{D^{*-}} + \frac{2}{9}\mu_{D_s^{*-}} - \frac{2}{9}\mu_{\Xi_c^0} - \frac{1}{9}\mu_{\Sigma_c^0}$	-0.39
$P_{\psi}^{N^0}$	$\frac{4}{9}\mu_{D_s^{*-}} + \frac{2}{9}\mu_{\bar{D}^{*0}} - \frac{2}{9}\mu_{\Xi_c^{*+}} - \frac{1}{9}\mu_{\Omega_c^0}$	-0.16
$P_{\psi}^{N^-}$	$\frac{4}{9}\mu_{D_s^{*-}} + \frac{2}{9}\mu_{D^{*-}} - \frac{2}{9}\mu_{\Xi_c^0} - \frac{1}{9}\mu_{\Omega_c^0}$	-0.36
$P_{\psi}^{\Lambda^-}$	$\frac{2}{3}\mu_{D_s^{*-}} - \frac{1}{3}\mu_{\Omega_c^0}$	-0.34

TABLE VIII. The magnetic moments of the S -wave $1^- \otimes \frac{3}{2}^+ = \frac{3}{2}^-$ decuplet hidden-charm pentaquark states (in unit of μ_N).

$1^- \otimes \frac{3}{2}^+ = \frac{3}{2}^-$ States	Expressions	Results
$P_{\psi}^{\Delta^{++}}$	$\mu_{\bar{D}^{*0}} + \mu_{\Sigma_c^{*+}}$	3.84
$P_{\psi}^{\Delta^+}$	$\frac{2}{3}\mu_{\bar{D}^{*0}} + \frac{2}{3}\mu_{\Sigma_c^{*+}} + \frac{1}{3}\mu_{D^{*-}} + \frac{1}{3}\mu_{\Sigma_c^{*+}}$	1.67
$P_{\psi}^{\Delta^0}$	$\frac{2}{3}\mu_{D^{*-}} + \frac{2}{3}\mu_{\Sigma_c^{*+}} + \frac{1}{3}\mu_{\bar{D}^{*0}} + \frac{1}{3}\mu_{\Sigma_c^0}$	-0.50
$P_{\psi}^{\Delta^-}$	$\mu_{D^{*-}} + \mu_{\Sigma_c^0}$	-2.67
$P_{\psi}^{\Sigma^+}$	$\frac{2}{3}\mu_{\bar{D}^{*0}} + \frac{2}{3}\mu_{\Xi_c^{*+}} + \frac{1}{3}\mu_{D_s^{*-}} + \frac{1}{3}\mu_{\Sigma_c^{*+}}$	1.94
$P_{\psi}^{\Sigma^0}$	$\frac{1}{3}\mu_{D^{*-}} + \frac{1}{3}\mu_{\bar{D}^{*0}} + \frac{1}{3}\mu_{D_s^{*-}} + \frac{1}{3}\mu_{\Xi_c^{*+}} + \frac{1}{3}\mu_{\Xi_c^0} + \frac{1}{3}\mu_{\Sigma_c^{*+}}$	-0.23
$P_{\psi}^{\Sigma^-}$	$\frac{2}{3}\mu_{D^{*-}} + \frac{2}{3}\mu_{\Xi_c^0} + \frac{1}{3}\mu_{D_s^{*-}} + \frac{1}{3}\mu_{\Sigma_c^0}$	-2.40
$P_{\psi}^{N^0}$	$\frac{2}{3}\mu_{D_s^{*-}} + \frac{2}{3}\mu_{\Xi_c^{*+}} + \frac{1}{3}\mu_{\bar{D}^{*0}} + \frac{1}{3}\mu_{\Omega_c^0}$	0.04
$P_{\psi}^{N^-}$	$\frac{2}{3}\mu_{D_s^{*-}} + \frac{2}{3}\mu_{\Xi_c^0} + \frac{1}{3}\mu_{D^{*-}} + \frac{1}{3}\mu_{\Omega_c^0}$	-2.13
$P_{\psi}^{\Lambda^-}$	$\mu_{D_s^{*-}} + \mu_{\Omega_c^0}$	-1.85

$P_{\psi}^{\Delta^0}$ in diquark-diquark-antiquark model and diquark-triquark model in our recent article [36]. In Ref [36], the results in Tables 10 and 15 indicate that the magnetic moments of S -wave $\frac{1}{2}^-$ and $\frac{3}{2}^- P_{\psi}^{\Delta^0}$ in diquark-diquark-antiquark model and diquark-triquark model are not proportional to the electric charges. This specific conclusion can be inferred uniquely from the molecular interpretation; and

- (ii) The magnetic moments of decuplet pentaquark states sharing the same spin-flavor configuration satisfy the following sum rules:

$$\begin{aligned} \mu_{P_{\psi}^{\Delta^+}} - \mu_{P_{\psi}^{\Delta^0}} + \mu_{P_{\psi}^{\Sigma^-}} - \mu_{P_{\psi}^{\Sigma^+}} + \mu_{P_{\psi}^{N^0}} - \mu_{P_{\psi}^{N^-}} &= 0, \\ \mu_{P_{\psi}^{\Delta^{++}}} + \mu_{P_{\psi}^{\Delta^-}} + \mu_{P_{\psi}^{\Lambda^-}} &= 3\mu_{P_{\psi}^{\Sigma^0}}. \end{aligned} \quad (21)$$

Formulated similarly to the Coleman-Glashow sum rule for octet baryon magnetic moments in the SU(3) flavor symmetry limit [47–49], the first sum rule is shown to be valid for decuplet pentaquark states of corresponding spin-flavor configurations. The second sum rule further delineates the relationships governing the remaining decuplet pentaquark magnetic moments. These empirical rules furnish novel perspectives for exploring the substructure of hidden-charm pentaquarks. Our predictions, established in the absence of coupled-channel effects, will therefore be instrumental in guiding subsequent research into their internal composition.

The sum rules for magnetic moments and the observed approximate charge proportionality for certain configurations are not generic predictions for any pentaquark model; they are direct consequences of the specific molecular wave function constructed from color-singlet hadrons (a charmed baryon in the symmetric 6_f representation and an anti-charmed meson) combined into the overall 10_f flavor-spin representation. Their potential violation in compact or diquark-based models stems from fundamental differences in the internal color and spatial correlations.

In the molecular picture, the wave function is essentially a product of the constituent hadron wave functions. The magnetic moment operator, being a sum over individual quark magnetic moments, factors conveniently when acting on this loosely bound system. The resulting sum rules emerge from the exact Clebsch-Gordan coefficients governing the coupling of the $6_f \otimes \bar{3}_f$ flavor representations and their associated spins. This algebraic structure is rigid and leads to the specific linear relations among the magnetic moments of the decuplet members, which persist even when SU(3) mass symmetry is broken because the underlying coupling coefficients remain unchanged.

In contrast, the internal wave function of a compact pentaquark or a diquark-based state (e.g., diquark-diquark-antiquark or diquark-triquark) involves fundamentally different color-spatial symmetries:

1. Color structure: In compact models, all five quarks are in a overall color singlet, but this is achieved through different intermediate color couplings (e.g., $\bar{3}_c \otimes 3_c \otimes \bar{3}_c$ pathways in diquark models). The color wave function is typically more entangled and does not naturally separate into two color-singlet clusters. This different color confinement geometry can significantly alter the relative weight and phase of different flavor-spin components when the magnetic moment operator is applied.

2. Spatial wave function: For an S -wave molecular state, the dominant spatial configuration is weakly bound with a large relative separation between hadrons. For a compact state, all quarks are confined within a volume comparable to a conventional baryon. The spatial wave function is often assumed to be symmetric (or of mixed symmetry) in the relative coordinates, which couples differently to the flavor-

spin degrees of freedom via the Pauli principle. This different spatial overlap can modify the matrix elements of the spin-dependent magnetic moment operator.

3. Spin-flavor correlations: In diquark models, the internal spin is often carried by strongly correlated quark pairs (diquarks) with fixed spin (0 or 1). The total spin coupling scheme (e.g., coupling diquark spins to the antiquark spin) differs from the molecular scheme of coupling baryon spin and meson spin. This alternative spin coupling topology will generate different interference terms between the quark magnetic moments, making it highly unlikely to accidentally reproduce the same sum rules derived from the molecular spin-flavor Clebsch-Gordan coefficients.

Although the full decuplet of molecular pentaquarks has not yet been observed, our calculations provide a clear predictive framework for future experimental searches and identification. The axial charge is directly related to coupling constants in strong decays such as $P_{\psi}^{\Delta^{++}} \rightarrow J/\psi p \pi \pi$. Future measurements of the branching fractions of such decays could be compared with our predicted axial charge values to test the molecular coupling picture.

As the lifetime of pentaquark state is very short, experimenters can measure the magnetic moment of pentaquark state by using the Thomas precession of particles with magnetic moment in a uniform magnetic field. When the polarized pentaquark state whose spin axis is perpendicular to the external magnetic field and parallel to the direction of motion is hit into the annular orbit of the magnetic storage ring. Under the action of a highly uniform stable magnetic field in the orbit, the pentaquark state will make a uniform circular motion along the magnetic storage ring. The magnetic moment can be measured with the revolution angular velocity of the pentaquark state.

Thus, while the specific numerical patterns and the sum rules are necessary outcomes of the assumed $\mathbf{10}_f$ molecular wave function symmetry, they serve as decisive discriminators between structural hypotheses. Their experimental confirmation would provide strong evidence for the molecular, weakly-bound picture. Conversely, the observation of magnetic moments that significantly deviate from these relations would point towards a more compact or internally correlated structure, where the specific color-spatial symmetries inherent to the molecular model do not apply. This underlines the predictive power and structural sensitivity of these electromagnetic observables.

V. SUMMARY

The systematic study of exotic hadrons continues to provide critical tests for our understanding of

nonperturbative QCD. In this context, the axial charge and the magnetic moment stand out as fundamental observables that probe distinct aspects of hadronic structure: the former is intimately connected to chiral symmetry and spin content, while the latter reveals the spatial distribution of internal currents and charges.

In this work, we have performed a comprehensive analysis of the decuplet hidden-charm molecular pentaquarks within the quark model. Our calculation of the axial charges demonstrates that their magnitudes are comparable to that of the nucleon, suggesting significant quark spin contributions. These results provide essential input for constructing effective theories, such as chiral perturbation theory, for these exotic states.

Regarding the magnetic moments, our findings reveal a rich and structured pattern. These magnetic moments are highly sensitive to the specific flavor and spin configuration of the pentaquark. Notably, the magnetic moments for the $\frac{1}{2}^+ \otimes 0^- = \frac{1}{2}^-$ and $\frac{1}{2}^+ \otimes 1^- = \frac{3}{2}^-$ states are found to be approximately proportional to their electric charges. A key outcome of this work is the derivation of the sum rules, a set of relations obeyed by the magnetic moments of decuplet pentaquarks sharing the same spin-flavor structure, which persist even under SU(3) symmetry breaking.

As fundamental physical observables, axial charges and magnetic moments offer valuable pathways toward experimental characterization. Although pentaquark states are short-lived, their magnetic moments could, in principle, be accessed via Thomas precession in a uniform magnetic field. Axial couplings, on the other hand, can be more directly extracted from the decay widths of strong decays. We anticipate that the predictions presented here-for both axial charges and magnetic moments will serve as a valuable benchmark for future experimental analyses and theoretical studies, thereby advancing our understanding of the molecular nature and internal dynamics of multi-quark systems.

ACKNOWLEDGMENTS

This project is supported by the National Natural Science Foundation of China under Grant No. 11905171. This work is also supported by Natural Science Basic Research Program of Shaanxi (Program No. 2025JC-YBMS-039) and Young Talent Fund of Xi'an Association for Science and Technology (Grant No. 959202413087).

DATA AVAILABILITY

The data that support the findings of this article are openly available [40].

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