

Decay and production properties of a strange double-charm pentaquark

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In this work, we investigate the decay and production properties of the strange double-charm pentaquark P_{ccs}^{++} with strangeness $S = -1$. Building upon our previous work predicting its $J^P = 1/2^-$ molecular configuration, we employ three-point QCD sum rules to calculate its strong decay widths and estimate its production branching ratio via Ξ_{bc}^+ baryon decays. The total strong decay width to the $\Xi_{cc}\bar{K}$ and $\Omega_{cc}\pi$ final-state channels is determined as 85 ± 19 MeV. Furthermore, using a rescattering mechanism, we analyze the $\Xi_{bc}^+ \rightarrow D_s^{*-}\Xi_{cc}^{++} \rightarrow D^-P_{ccs}^{++}$ process and estimate the production branching ratio to be $\mathcal{B}r(\Xi_{bc}^+ \rightarrow D^-P_{ccs}^{++}) = (4.3_{-1.5}^{+2.0}) \times 10^{-6}$. The relatively narrow width and detectable branching ratio suggest the possibility searching for this pentaquark state in the future.

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I. INTRODUCTION

The study of exotic multiquark states, proposed early in 1964 [1,2], has become a pivotal frontier in hadronic physics, offering profound insights into the nonperturbative dynamics of quantum chromodynamics (QCD) [3–15]. Since the discovery of the first hidden-charm pentaquarks $P_c(4380)$ and $P_c(4450)$ by the LHCb Collaboration [16], significant theoretical and experimental efforts have been devoted to unraveling the nature of these states, which lie beyond the conventional quark model. The recent observation of the double-charm tetraquark $T_{cc}^+(3875)$ [17,18] and the strange-charm tetraquark $T_{c\bar{s}}(2900)$ [19] further highlights the rich spectrum of exotic hadrons and underscores the potential existence of double-heavy counterparts in the baryon sector, such as double-charm pentaquarks.

Theoretical attempts have been made to study the mass spectrum from both the hadronic molecular picture [20–29]

and the compact pentaquark picture [30–34], as well as their electromagnetic properties [28,29,35–38]. In our previous work [39], we systematically investigated the mass spectra of strange double-charm pentaquarks with quark content $ccus\bar{d}$ and strangeness $S = -1$, employing QCD sum rules for both molecular and compact configurations. Among the predicted configurations, the $J^P = 1/2^-$ molecular pentaquark with the $\Xi_{cc}\bar{K}$ structure stands out: its mass (4.20 GeV) lies slightly above the $\Xi_{cc}\bar{K}$ threshold, allowing strong decays only into $\Xi_{cc}\bar{K}$ and $\Omega_{cc}\pi$ channels. This suggests a relatively narrow resonance that could manifest as a discernible peak in experimental invariant mass spectra. The proximity to the threshold suppresses the phase space for strong decays, potentially enhancing its experimental detectability.

Meanwhile, the ongoing experimental quest for doubly heavy Ξ_{bc}^+ baryons offers a pivotal opportunity to unravel the double-charm pentaquark state. Cabibbo-favored decays of Ξ_{bc}^+ , such as $\Xi_{bc}^+ \rightarrow D_s^{*-}\Xi_{cc}^{++}$, could generate the $\Xi_{cc}\bar{K}$ pentaquark through rescattering processes. At the quark level, the diagram for the process $\Xi_{bc}^+ \rightarrow D^-P_{ccs}^{++}$ is shown in the left panel of Fig. 1. The weak decay arises from a Cabibbo-favored weak transition $b \rightarrow c(\bar{c}s)$ along with the creation of a $d\bar{d}$ pair from the strong interaction. This diagram, known as the external W emission diagram,

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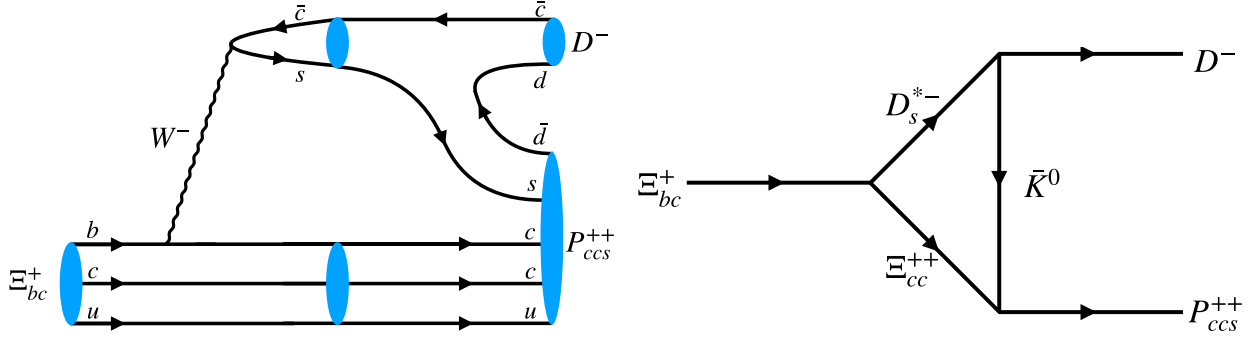


FIG. 1. The production of the strange double-charm pentaquark at the quark level (left) and hadronic level (right).

is nonfactorizable because the $s\bar{c}$ pair produced in the weak interaction ends up in different final-state hadrons. Thus, long-distance contributions play a significant role in the $\Xi_{bc}^+ \rightarrow D^- P_{ccs}^{++}$ process, where the weakly produced $c\bar{s}$ and ucc pairs hadronize via $\bar{K}^0$ meson exchange interaction, and the D_s^{*-} and Ξ_{cc}^{++} pair converts into a D^- and the pentaquark P_{ccs}^{++} . The corresponding interaction at the hadronic level is shown in the right panel of Fig. 1. Such a mechanism for final-state interaction effects has been successfully applied to D meson decays [40], B meson decays [41,42], and charm baryon decays [43,44]. Recently, this mechanism has also been applied to the production of tetraquark states in B meson decays [45,46] and of hidden-charm pentaquark states P_c in Ξ_b decays [47]. In this work, we apply this mechanism to the production of the double-charm pentaquark via the $\Xi_{bc}^+ \rightarrow D_s^{*-} \Xi_{cc}^{++} \rightarrow D^- P_{ccs}^{++}$ process with $\bar{K}^0$ exchange.

A precise calculation of the decay width and production branching ratios of the double-charm pentaquark is thus critical to quantify its production rates in such channels and to guide experimental searches at facilities like LHCb and Belle II. In Ref. [39], we also predicted the masses of possible compact pentaquark states with the same quark content. Notably, two compact configurations were found with masses of 4.25 ± 0.4 GeV, which are close to the mass of the molecular state studied here. It is therefore a natural question how their decay widths and production rates compare with those of the molecular configuration. However, our calculations for these particular compact interpolating currents show that the sum rules analyses for the coupling constants $g_{P_{ccs}\Xi_{cc}\bar{K}}$ and $g_{P_{ccs}\Omega_{cc}\pi}$ did not exhibit a sufficiently stable Borel platform within a reasonable M_B^2 window, leading to unreliable predictions of these couplings. Since the hadron decay widths and production rates are highly sensitive to the hadronic couplings, it is difficult to provide quantitative estimations for these compact pentaquark states within the framework of three-point QCD sum rules. In this work, we focus on the well-behaved molecular configuration to study the decay and production properties of strange double-charm pentaquark state. We continue our study by calculating the decay width of the double-charm

pentaquark using the QCD sum rule method and estimating its production branching ratios in Ξ_{bc} decays. This paper is organized as follows: In Sec. II, we outline the formalism for computing decay widths within the QCD sum rule approach. Section III presents the numerical results for the $J^P = 1/2^-$ pentaquark's decay properties. Section IV discusses its production mechanism via Ξ_{bc}^+ decays and estimates detectable branching ratios. A brief summary is presented in Sec. V.

II. THREE-POINT QCD SUM RULE

Over past several decades, the method of the QCD sum rule has been proven to be very powerful to study hadron properties [48–51]. In this section, we study the three-point correlation function of several two-body strong decay processes $M \rightarrow X + Y$. For the strong decay process $M \rightarrow X + Y$, the corresponding correlator is written as

$$\Pi(p, p', q) = \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \langle 0 | T \{ J_X(x) J_Y(y) J_M^\dagger(0) \} | 0 \rangle, \quad (1)$$

where $J_{M(X,Y)}$ is the interpolating current for the initial (final) state. In this section, we consider the $P_{ccs}\Xi_{cc}\bar{K}$ and $P_{ccs}\Omega_{cc}\pi$ strong decay vertices with $K(\pi)$ off shell. We use the following interpolating current for P_{ccs}^{++} by considering it as a $\Xi_{cc}\bar{K}$ molecule [39],

$$\xi_1 = [\epsilon_{abc} (c_a^T C \gamma_\mu c_b) \gamma_\mu \gamma_5 u_c] [\bar{d}_d \gamma_5 s_d], \quad (2)$$

where C denotes the charge conjugate operator, subscript $a \cdots d$ denotes the color index, and u, d, s, c denote the up, down, strange, and charm quark field, respectively. This current can couple to the P_{ccs}^{++} state with $J^P = 1/2^-$ via

$$\langle 0 | \xi_1 | P_{ccs}^{1/2^-} \rangle = \lambda_{P_{ccs}}^- u(p), \quad (3)$$

in which the values of the coupling constant $\lambda_{P_{ccs}}^-$ are determined from the two-point mass sum rules established in Ref. [39],

$$\lambda_{P_{ccs}}^- = (2.3 \pm 0.7) \times 10^{-3} \text{ GeV}^6. \quad (4)$$

The interpolating currents for $\bar{K}$ and π^+ mesons can be constructed as

$$J_{\bar{K}} = i\bar{d}_a\gamma_5 s_a, \quad J_{\pi^+} = i\bar{d}_a\gamma_5 u_a, \quad (5)$$

which can couple to the meson states via

$$\begin{aligned} \langle 0|J_{\bar{K}}|\bar{K}\rangle &= f_{\bar{K}} \frac{m_{\bar{K}}^2}{m_s} \equiv \lambda_{\bar{K}}, \\ \langle 0|J_{\pi^+}|\pi^+\rangle &= f_{\pi} \frac{m_{\pi}^2}{m_u + m_d} \equiv \lambda_{\pi}. \end{aligned} \quad (6)$$

The interpolating currents for double-charm baryons are taken as [52]

$$\begin{aligned} J_{\Xi_{cc}} &= \epsilon_{abc}(c_a^T C \gamma_{\mu} c_b) \gamma_{\mu} \gamma_5 u_c, \\ J_{\Omega_{cc}} &= \epsilon_{abc}(c_a^T C \gamma_{\mu} c_b) \gamma_{\mu} \gamma_5 s_c, \end{aligned} \quad (7)$$

which can couple to the baryon states via

$$\begin{aligned} \langle 0|J_{\Xi_{cc}}|\Xi_{cc}(p, s)\rangle &= f_{\Xi_{cc}} u(p, s), \\ \langle 0|J_{\Omega_{cc}}|\Omega_{cc}(p, s)\rangle &= f_{\Omega_{cc}} u(p, s). \end{aligned} \quad (8)$$

The coupling constants $g_{P_{ccs}\Xi_{cc}\bar{K}}$ and $g_{P_{ccs}\Omega_{cc}\pi}$ are defined via the effective Lagrangian [53],

$$\begin{aligned} \mathcal{L}_{P_{ccs}\Xi_{cc}\bar{K}} &= g_{P_{ccs}\Xi_{cc}\bar{K}} P_{ccs} \Xi_{cc} \bar{K}, \\ \mathcal{L}_{P_{ccs}\Omega_{cc}\pi} &= g_{P_{ccs}\Omega_{cc}\pi} P_{ccs} \Omega_{cc} \pi. \end{aligned} \quad (9)$$

Thus, the transition matrix element can be obtained as

$$\begin{aligned} \langle \Xi_{cc}(p', s') \bar{K}(q) | P_{ccs}(p) \rangle &= g_{P_{ccs}\Xi_{cc}\bar{K}} \bar{u}_{\Xi_{cc}}(p', s') u_{P_{ccs}}(p, s), \\ \langle \Omega_{cc}(p', s') \pi(q) | P_{ccs}(p) \rangle &= g_{P_{ccs}\Omega_{cc}\pi} \bar{u}_{\Omega_{cc}}(p', s') u_{P_{ccs}}(p, s). \end{aligned} \quad (10)$$

With the above coupling relations and transition matrix element, we can obtain the three-point correlation function Eq. (1) for $P_{ccs}^{++} \rightarrow \Xi_{cc} \bar{K}$ on the phenomenological side,

$$\begin{aligned} \Pi(p, p', q) &= \int d^4x d^4y e^{ip' \cdot x} e^{-iq \cdot y} \langle 0 | T \{ J_{P_{ccs}}(x) J_K^\dagger(y) J_{\Xi_{cc}}^\dagger(0) \} | 0 \rangle \\ &= \frac{\lambda_{P_{ccs}}^- \lambda_{\Xi_{cc}} \lambda_K g_{P_{ccs}\Xi_{cc}\bar{K}}}{(p^2 - m_{P_{ccs}}^2)(p'^2 - m_{\Xi_{cc}}^2)(q^2 - m_K^2)} (\not{p} + m_{P_{ccs}})(\not{p}' + m_{\Xi_{cc}}) + \dots, \end{aligned} \quad (11)$$

and for $P_{ccs}^{++} \rightarrow \Omega_{cc} \pi$,

$$\begin{aligned} \Pi(p, p', q) &= \int d^4x d^4y e^{ip' \cdot x} e^{-iq \cdot y} \langle 0 | T \{ J_{P_{ccs}}(x) J_{\pi}^\dagger(y) J_{\Omega_{cc}}^\dagger(0) \} | 0 \rangle \\ &= \frac{\lambda_{P_{ccs}}^- \lambda_{\Omega_{cc}} \lambda_{\pi} g_{P_{ccs}\Omega_{cc}\pi}}{(p^2 - m_{P_{ccs}}^2)(p'^2 - m_{\Omega_{cc}}^2)(q^2 - m_{\pi}^2)} (\not{p} + m_{P_{ccs}})(\not{p}' + m_{\Omega_{cc}}) + \dots. \end{aligned} \quad (12)$$

On the operator product expansion (OPE) side, we can evaluate the correlation function with the standard QCD sum rule approach. To establish a sum rule for the coupling constant, we pick out the $1/Q^2$ terms around the pole $Q^2 \sim 0$ (where $Q^2 = -q^2$) with the structure $\not{p}$ in the OPE series and then match both sides of the sum rule. The mass of the off-shell meson ($\bar{K}$ or π) is much smaller than the masses of the baryons (Ξ_{cc} , Ω_{cc} , P_{ccs}). This justifies the approximation of extracting the $1/q^2$ pole term directly in the QCD sum rule analysis, making an exponential extrapolation to the on-shell point unnecessary. After performing the Borel transform $P^2 \rightarrow M_B^2$ on both phenomenological and OPE sides, we obtain the strong coupling for the $P_{ccs}\Xi_{cc}\bar{K}$ vertex,

$$g_{P_{ccs}\Xi_{cc}\bar{K}}(s_0, M_B^2) = \frac{1}{\lambda_{P_{ccs}}^- \lambda_{\Xi_{cc}} \lambda_K (m_{P_{ccs}} + m_{\Xi_{cc}})} \frac{m_{P_{ccs}}^2 - m_{\Xi_{cc}}^2}{e^{-m_{\Xi_{cc}}^2/M_B^2} - e^{-m_{P_{ccs}}^2/M_B^2}} \left(\int_{s_0}^{s_0} ds \rho(s) e^{-s/M_B^2} + R(M_B^2) \right), \quad (13)$$

and strong coupling for the $P_{ccs}\Omega_{cc}\pi$ vertex,

$$g_{P_{ccs}\Omega_{cc}\pi}(s_0, M_B^2) = \frac{1}{\lambda_{P_{ccs}}^- \lambda_{\Omega_{cc}} \lambda_{\pi} (m_{P_{ccs}} + m_{\Omega_{cc}})} \frac{m_{P_{ccs}}^2 - m_{\Omega_{cc}}^2}{e^{-m_{\Omega_{cc}}^2/M_B^2} - e^{-m_{P_{ccs}}^2/M_B^2}} \left(\int_{s_0}^{s_0} ds \rho(s) e^{-s/M_B^2} + R(M_B^2) \right), \quad (14)$$

where the continuum threshold $s_0 = 22.3 \text{ GeV}^2$ is taken from the two-point mass sum rules in Ref. [39].

Using the OPE method, the three-point function can also be evaluated at the quark-gluonic level as a function of various QCD parameters. To evaluate the Wilson coefficients, we adopt the heavy quark propagator in momentum space and the light quark propagator in coordinate space,

$$iS_Q^{ab}(p) = \frac{i\delta^{ab}}{p - m_Q} + \frac{i}{4} g_s \frac{\lambda_{ab}^n}{2} G_{\mu\nu}^n \frac{\sigma^{\mu\nu}(p + m_Q) + (p + m_Q)\sigma^{\mu\nu}}{(p^2 - m_Q^2)^2} + \frac{i\delta^{ab}}{12} \langle g_s^2 GG \rangle m_Q \frac{p^2 + m_Q p}{(p^2 - m_Q^2)^4}, \quad (15)$$

$$\begin{aligned} iS_q^{ab}(x) = & \frac{i\delta^{ab}}{2\pi^2 x^4} \not{x} - \frac{\delta^{ab}}{12} \langle \bar{q}q \rangle + \frac{i}{32\pi^2} \frac{\lambda_{ab}^n}{2} g_s G_{\mu\nu}^n \frac{1}{x^2} (\sigma^{\mu\nu} \not{x} + \not{x} \sigma^{\mu\nu}) \\ & + \frac{\delta^{ab} x^2}{192} \langle \bar{q} g_s \sigma \cdot G q \rangle - \frac{m_q \delta^{ab}}{4\pi^2 x^2} + \frac{i\delta^{ab} m_q \langle \bar{q}q \rangle}{48} \not{x} - \frac{im_q \langle \bar{q} g_s \sigma \cdot G q \rangle \delta^{ab} x^2 \not{x}}{1152}, \end{aligned} \quad (16)$$

where Q represents the heavy quark c or b , q represents the light quark u, d, s , and the superscripts a, b denote the color indices. In this work, we evaluate Wilson coefficients of the correlation function up to dimension nine condensates at the leading order in α_s . The spectral function $\rho(s)$ in Eqs. (13) and (14) are given in the Appendix. We discuss the details to obtain suitable parameter working regions in QCD sum rule analysis in the next section.

III. NUMERICAL ANALYSIS

In this section, we perform the three-point QCD sum rule analysis for double heavy molecular pentaquark systems using the interpolating currents in Eq. (2). We use the standard values of various QCD condensates as $\langle \bar{q}q \rangle(1 \text{ GeV}) = -(0.24 \pm 0.03)^3 \text{ GeV}^3$, $\langle \bar{q} g_s \sigma \cdot G q \rangle(1 \text{ GeV}) = -M_0^2 \langle \bar{q}q \rangle$, $M_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$, $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.8 \pm 0.1$, and $\langle g_s^2 GG \rangle(1 \text{ GeV}) = (0.48 \pm 0.14) \text{ GeV}^4$ at the energy scale $\mu = 1 \text{ GeV}$ [51,54–60] and $m_s(2 \text{ GeV}) = 95_{-3}^{+9} \text{ MeV}$, $m_c(m_c) = 1.27_{-0.04}^{+0.03} \text{ GeV}$, and $m_b(m_b) = 4.18_{-0.03}^{+0.04} \text{ GeV}$ from the Particle Data Group [61]. We also take into account the energy-scale dependence of the above parameters from the renormalization group equation [62],

$$\begin{aligned} m_s(\mu) &= m_s(2 \text{ GeV}) \left[\frac{\alpha_s(\mu)}{\alpha_s(2 \text{ GeV})} \right]^{\frac{12}{33-2n_f}}, \\ m_c(\mu) &= m_c(m_c) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{33-2n_f}}, \\ m_b(m_b) &= m_b(m_b) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)} \right]^{\frac{12}{33-2n_f}}, \\ \langle \bar{q}q \rangle(\mu) &= \langle \bar{q}q \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}}, \\ \langle \bar{s}s \rangle(\mu) &= \langle \bar{s}s \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}}, \\ \langle \bar{q} g_s \sigma \cdot G q \rangle(\mu) &= \langle \bar{q} g_s \sigma \cdot G q \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2n_f}}, \\ \langle \bar{s} g_s \sigma \cdot G s \rangle(\mu) &= \langle \bar{s} g_s \sigma \cdot G s \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2n_f}}, \\ \alpha_s(\mu) &= \frac{1}{b_0 t} \left[1 - \frac{b_1 \log t}{b_0 t} + \frac{b_1^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right], \end{aligned} \quad (17)$$

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857-5033n_f+325n_f^2}{128\pi^3}$, and $\Lambda = 210 \text{ MeV}$, 292 MeV , and 332 MeV for the flavors $n_f = 5, 4$, and 3 , respectively. In this work, we evolve all the input parameters to the energy scale $\mu = m_c$ for our sum rule analysis. The parameters adopted for the K and π mesons and double-charm baryons are displayed in Table I.

TABLE I. The values of the hadronic parameters m_H and f_H in the works taken from Refs. [61,63–65].

Meson(M)	Mass m_M [GeV]	Decay constant f_M [GeV]	Baryon(B)	Mass m_B [GeV]	Decay constant f_B [GeV]
π	0.140	0.16 ± 0.04	Ξ_{cc}^{++}	3.621	0.109
K	0.494	0.16 ± 0.02	Ω_{cc}^+	3.738	0.138

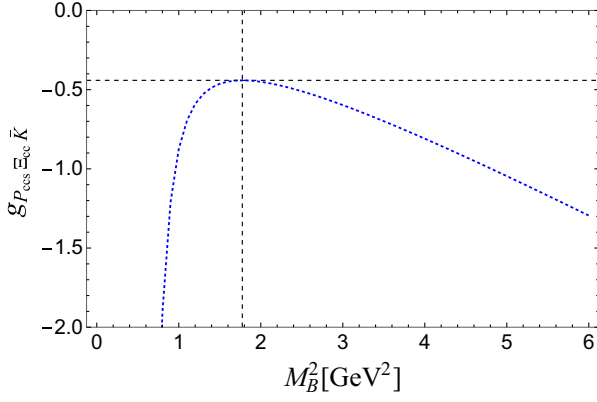


FIG. 2. The dependence of the strong coupling $g_{P_{ccs}\Xi_{cc}\bar{K}}$ on the Borel mass M_B^2 . The transfer momentum is set to be $Q^2 = m_{\Xi_{cc}}^2 \sim 13.1 \text{ GeV}^2$.

A. Strong coupling $g_{P_{ccs}\Xi_{cc}\bar{K}}$

In Fig. 2, we show the variation of the coupling constant $g_{P_{ccs}\Xi_{cc}\bar{K}}(Q^2)$ with the Borel mass M_B^2 at $Q^2 = m_{\Xi_{cc}}^2 \approx 13.1 \text{ GeV}^2$. This momentum point is chosen far away from the kaon pole m_K^2 so that the approximation of extracting the $1/Q^2$ term in the QCD sum rule is valid. We find that the coupling constant has a maximum value at $M_B^2 \sim 1.77 \text{ GeV}^2$, where it shows minimal dependence on the unphysical Borel parameter. Since the kaon mass is much smaller than the masses of the baryons Ξ_{cc} and P_{ccs} , the Q^2 dependence of the coupling constant is negligible in

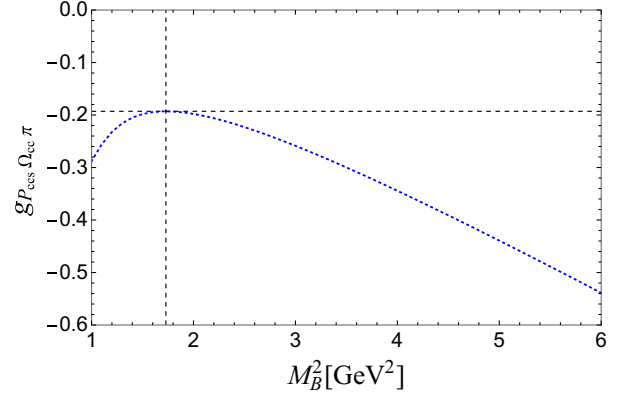


FIG. 3. The dependence of the strong coupling $g_{P_{ccs}\Omega_{cc}\pi}$ on the Borel mass M_B^2 . The transfer momentum is set to be $Q^2 = m_{\Omega_{cc}}^2 \sim 13.8 \text{ GeV}^2$.

the range from the sum rule working point $Q^2 = m_{\Xi_{cc}}^2$ to the physical on-shell point $Q^2 = -m_K^2$. Therefore, we can directly use the coupling constant evaluated at $Q^2 = m_{\Xi_{cc}}^2$ to compute the decay width without performing an extrapolation. The extracted coupling constant is

$$g_{P_{ccs}\Xi_{cc}\bar{K}}(m_{\Xi_{cc}}^2) = -(0.45 \pm 0.05) \text{ GeV}^{-3}. \quad (18)$$

This value is then used to compute the decay width. From the matrix element, we can obtain the decay width for $P_{ccs}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0$ process,

$$\Gamma(P_{ccs}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0) = \frac{\sqrt{\lambda(m_{P_{ccs}}^2, m_{\Xi_{cc}}^2, m_K^2)}}{8\pi m_{P_{ccs}}^2} g_{P_{ccs}\Xi_{cc}\bar{K}}(m_{\Xi_{cc}}^2) ((m_{P_{ccs}} + m_{\Xi_{cc}})^2 - m_K^2). \quad (19)$$

Substituting the above coupling, we can obtain the decay width as

$$\Gamma(P_{ccs}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0) = 65 \pm 16 \text{ MeV}. \quad (20)$$

B. Strong coupling $g_{P_{ccs}\Omega_{cc}\pi}$

For the $P_{ccs}\Omega_{cc}\pi$ vertex, we show the variation of the coupling constant $g_{P_{ccs}\Omega_{cc}\pi}(Q^2)$ with the Borel mass M_B^2 at $Q^2 = m_{\Omega_{cc}}^2 \approx 13.8 \text{ GeV}^2$ in Fig. 3. Similar to the discussion above, we extract the $1/Q^2$ term in the QCD sum rule since

the negligible pion mass. The dependence of the coupling constant on the unphysical Borel parameter achieves minimum around $M_B^2 \sim 1.74 \text{ GeV}^2$. Neglecting the small Q^2 dependence, we compute the coupling constant at $Q^2 = m_{\Omega_{cc}}^2$ as

$$g_{P_{ccs}\Omega_{cc}\pi}(m_{\Omega_{cc}}^2) = -(0.19_{-0.06}^{+0.05}) \text{ GeV}^{-3}. \quad (21)$$

From the matrix element, we obtain the decay width for the $P_{ccs}^{++} \rightarrow \Omega_{cc}^{+} \pi^{+}$ process,

$$\Gamma(P_{ccs}^{++} \rightarrow \Omega_{cc}^{+} \pi^{+}) = \frac{\sqrt{\lambda(m_{P_{ccs}}^2, m_{\Omega_{cc}}^2, m_{\pi}^2)}}{8\pi m_{P_{ccs}}^2} g_{P_{ccs}\Omega_{cc}\pi}(m_{\Omega_{cc}}^2) ((m_{P_{ccs}} + m_{\Omega_{cc}})^2 - m_{\pi}^2). \quad (22)$$

Substituting the above coupling, we can obtain the decay width as

$$\Gamma(P_{ccs}^{++} \rightarrow \Omega_{cc}^+ \pi^+) = 20_{-10}^{+11} \text{ MeV}. \quad (23)$$

Thus, we can obtain the total strong decay width as

$$\begin{aligned} \Gamma_{P_{ccs}^{++}} &= \Gamma(P_{ccs}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0) + \Gamma(P_{ccs}^{++} \rightarrow \Omega_{cc}^+ \pi^+) \\ &= (85 \pm 19) \text{ MeV}. \end{aligned} \quad (24)$$

IV. PRODUCTION VIA FINAL-STATE INTERACTION

In the framework of the rescattering mechanism, the decay $\Xi_{bc}^+ \rightarrow P_{ccs}^{++} D^-$ can most likely proceed as $\Xi_{bc}^+ \rightarrow D_s^{*-} \Xi_{cc}^{++} \rightarrow D^- P_{ccs}^{++}$ with K^0 exchange. Under the factorization approach [66–68], we can get the decay amplitude of $\Xi_{bc}^+ \rightarrow D_s^* \Xi_{cc}$,

$$\mathcal{A}(\Xi_{bc}^+ \rightarrow D_s^* \Xi_{cc}) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} a_1 \epsilon^{*\mu} \bar{u}_{\Xi_{cc}} \left(A_1 \gamma_\mu \gamma_5 + A_2 \frac{p_{\Xi_{cc}\mu}}{m_{\Xi_{bc}}} \gamma_5 + B_1 \gamma_\mu + B_2 \frac{p_{\Xi_{cc}\mu}}{m_{\Xi_{bc}}} \right). \quad (25)$$

The above decay amplitudes in the factorization approach are expressed as

$$A_1 = -\lambda_{D_s^*} \left[g_1(m_{D_s^*}^2) + g_2(m_{D_s^*}^2) \frac{m_{\Xi_{cc}} - m_{\Xi_{bc}}}{m_{\Xi_{bc}}} \right], \quad (26)$$

$$A_2 = -2\lambda_{D_s^*} g_2(m_{D_s^*}^2), \quad (27)$$

$$B_1 = \lambda_{D_s^*} \left[f_1(m_{D_s^*}^2) - f_2(m_{D_s^*}^2) \frac{m_{\Xi_{cc}} + m_{\Xi_{bc}}}{m_{\Xi_{bc}}} \right], \quad (28)$$

$$B_2 = 2\lambda_{D_s^*} f_2(m_{D_s^*}^2), \quad (29)$$

where G_F is the Fermi constant, V_{ik} is the Cabibbo-Kobayashi-Maskawa matrix elements, a_1 is the effective Wilson coefficients obtained by the factorization approach [69], and $f_{1,2}$ and $g_{1,2}$ are transition form factors of the $\Xi_{bc}^+ \rightarrow \Xi_{cc} D_s^*$ weak decay process. The above form factors can be parametrized as

$$F(Q^2) = \frac{F(0)}{1 - \frac{Q^2}{m_{\text{fit}}^2} + \delta \left(\frac{Q^2}{m_{\text{fit}}^2} \right)^2}, \quad (30)$$

where the parameters F_0 , m_{fit} , and δ are taken from Ref. [68] and are listed in Table II. The amplitude for the $\Xi_{bc}^+ \rightarrow D_s^{*-} \Xi_{cc}^{++} \rightarrow D^- P_{ccs}^{++}$ process can be written as

$$\begin{aligned} \mathcal{A}(\Xi_{bc}^+ \rightarrow D_s^{*-} \Xi_{cc}^{++} \rightarrow D^- P_{ccs}^{++}) \\ = i \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} a_1 \int_{-1}^1 d \cos \theta \int_0^{2\pi} d\phi \frac{|\mathbf{p}_{D_s^*}|}{32\pi^2 m_{\Xi_{bc}}^2 m_{D_s^*}^2} \frac{g_{D_s^* DK}(-t) g_{P_{ccs} \Xi_{cc} \bar{K}}(-t)}{t - m_K^2} \bar{u}_{P_{ccs}}(p_P, s_P) (\not{p}_{\Xi_{cc}} + m_{\Xi_{cc}}) H u_{\Xi_{bc}}(p_{\Xi_{bc}}, s_{\Xi_{bc}}), \end{aligned} \quad (31)$$

where

$$H = -(p_D \cdot p_{D_s^*})(m_{\Xi_{bc}} \not{p}_{D_s^*}(A_1 \gamma_5 + B_1) + p_{D_s^*} \cdot p_{\Xi_{cc}}(A_2 \gamma_5 + B_2)) + m_{D_s^*}^2(m_{\Xi_{bc}} \not{p}_D(A_1 \gamma_5 + B_1) + p_D \cdot p_{\Xi_{cc}}(A_2 \gamma_5 + B_2)). \quad (32)$$

The corresponding decay width can be written as

$$\Gamma(\Xi_{bc}^+ \rightarrow P_{ccs}^{++} D^-) = \frac{\sqrt{\lambda(m_{\Xi_{bc}}^2, m_D^2, m_{P_{ccs}}^2)}}{16\pi m_{\Xi_{bc}}^3} |\mathcal{A}(\Xi_{bc}^+ \rightarrow D_s^{*-} \Xi_{cc}^{++} \rightarrow D^- P_{ccs}^{++})|^2. \quad (33)$$

TABLE II. The values of the parameters $F(0)$, m_{fit} and δ for the form factors in Eqs. (26)–(29) for the $\Xi_{bc}^+ \rightarrow D_s^* \Xi_{cc}$ process taken from Ref. [68].

Form factor	$F(0)$	m_{fit}	δ	Form factor	$F(0)$	m_{fit}	δ
f_1	0.550	4.45	0.43	g_1	0.530	4.57	0.44
f_2	-0.230	4.07	0.47	g_2	-0.043	3.90	0.48

It should be noted that in some works of the final-state interaction formalism [45,47] the decay amplitude contains the form factor $F(t, m) = (\Lambda^2 - m_K^2)/(\Lambda^2 - t)$ for each strong vertex, which is introduced to compensate the off-shell effect of the exchanged particle at the vertices [70]. In this work, we can obtain the strong coupling, such $g_{P_{ccs}\Xi_{cc}\bar{K}}$. We take the result of strong coupling $g_{D_s^* DK}(Q^2)$ with the QCD sum rule formalism as follows [46]:

$$g_{D_s^* DK}(Q^2) = (2.8_{-0.8}^{+1.3} \text{ GeV}^{-2}) e^{(0.2 \pm 0.0 \text{ GeV}^{-2}) Q^2}. \quad (34)$$

With the values of the strong couplings given in Eqs. (18) and (34), the result of Eq. (33), and the mass of Ξ_{bc}^+ taken from the lattice result [63], the decay width of the $\Xi_{bc}^+ \rightarrow P_{ccs}^{++} D^-$ process can be calculated as

$$\Gamma(\Xi_{bc}^+ \rightarrow P_{ccs}^{++} D^-) = (1.2_{-0.4}^{+0.6}) \times 10^{-17} \text{ GeV}. \quad (35)$$

Taking the lattice result of lifetime of Ξ_{bc}^+ [63], the production branching fraction of the $\Xi_{bc}^+ \rightarrow P_{ccs}^{++} D^-$ process becomes

$$\text{Br}(\Xi_{bc}^+ \rightarrow P_{ccs}^{++} D^-) = (4.3_{-1.5}^{+2.0}) \times 10^{-6}. \quad (36)$$

V. SUMMARY

Based on our previous calculations of mass spectroscopy [39], we further study the decay and production properties of the exotic strange double-charm pentaquark state P_{ccs}^{++} with $J^P = 1/2^-$. We perform the three-point QCD sum rules to calculate the coupling constants of $P_{ccs}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0$ and $P_{ccs}^{++} \rightarrow \Omega_{cc}^+ \pi^+$ strong decay processes. The partial decay widths of these two processes are obtained as

$$\begin{aligned} \Gamma(P_{ccs}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0) &= 65 \pm 16 \text{ MeV}, \\ \Gamma(P_{ccs}^{++} \rightarrow \Omega_{cc}^+ \pi^+) &= 20_{-10}^{+11} \text{ MeV}, \end{aligned} \quad (37)$$

yielding a relative branching ratio $\Gamma(P_{ccs}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0) : \Gamma(P_{ccs}^{++} \rightarrow \Omega_{cc}^+ \pi^+) \approx 3.3:1$. The total decay width is predicted as

$$\Gamma_{P_{ccs}^{++}} = 85 \pm 19 \text{ MeV}. \quad (38)$$

Furthermore, we study the $\Xi_{bc}^+ \rightarrow D_s^* \Xi_{cc}^{++} \rightarrow D^- P_{ccs}^{++}$ process via the rescattering mechanism to estimate the branching ratio of $\Xi_{bc}^+ \rightarrow D^- P_{ccs}^{++}$ as $(4.3_{-1.5}^{+2.0}) \times 10^{-6}$. This value of the branching ratio aligns with the productions of hidden-charm pentaquark states in Ξ_b decays [71–73] (having the same $b \rightarrow c\bar{c}s$ weak transition), which typically lie around $\text{Br} \sim 10^{-6}$ – 10^{-5} . Combining with the decay branching ratio $\text{Br}(P_{ccs}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0)$, we obtain $\text{Br}(\Xi_{bc}^+ \rightarrow D^- P_{ccs}^{++} \rightarrow D^- \bar{K}^0 \Xi_{cc}^{++}) \approx \text{Br}(\Xi_{bc}^+ \rightarrow D^- P_{ccs}^{++}) \text{Br}(P_{ccs}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0) = 3.3 \times 10^{-6}$, providing a benchmark for future experiments.

To date, LHCb has pursued the search of the Ξ_{bc}^+ state exploring the invariant mass of $\Lambda_c \pi$, $\Xi_c \pi$ and DKp final states without significant signals [74–76]. Nevertheless, high Ξ_{bc}^+ yields have been expected at future facilities such as MuIC ($\sim 10^8$ events/year) [77], CEPC/FCC-ee ($\sim 10^7$ events/year) [78], LHeC ($\sim 10^5$ events/year) [79], and LHCb Run 3 ($\sim 10^4$ events/year) [80], which will produce a considerable amount of double-charm pentaquark states in the future. Our analysis is specifically applied to the molecular interpretation of the P_{ccs}^{++} state. While compact pentaquark configurations with similar masses are also possible [39], a quantitative analysis of their decay properties within the same QCD sum rule framework proved inconclusive due to the instability of the operator product expansion series for the relevant correlation functions. Future studies with alternative theoretical approaches or improved sum rule constructions may help to clarify the properties of such compact states and their potential distinguishability from molecular candidates in experiments.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: SPECTRAL FUNCTION FOR THREE-POINT CORRELATION OF STRONG VERTICES

The spectral function $\rho(s)$ and $R(M_B^2)$ in Eq. (13) are shown as follows:

$$\rho(s) = \int_{x_{\min}}^{x_{\max}} dx \int_{y_{\min}}^{y_{\max}} dy \frac{3}{512\pi^5} y \left(\pi^2 \langle GG \rangle m_s (2\langle \bar{q}q \rangle - \langle \bar{s}s \rangle) ((x-2)y+1) - \frac{x}{y-1} (\langle GG \rangle + 16\pi^2 m_s (\langle \bar{s}s \rangle - 2\langle \bar{q}q \rangle)) \right. \\ \left. \times \Delta(x, y, s) (3(x-1)\Delta(x, y, s) + m_c^2 + 2s(x-1)(y-1)y) \right), \quad (\text{A1})$$

$$R(M_B^2) = \int_0^1 dx \int_0^1 dy \frac{\langle GG \rangle m_s (\langle \bar{s}s \rangle - 2\langle \bar{q}q \rangle) ((x-2)y+1)}{512\pi^3 (x-1)^2 (y-1)^2} \\ \times (s_1 (x-1)^2 (y-1)^2 y - m_c^2 (x(y(4((x-1)x+1)y-2x-5)+3) + y-1)) e^{-s_1/M_B^2}, \quad (\text{A2})$$

$$\text{where } x_{\min} = 0, \quad x_{\max} = \frac{1-2\sqrt{m_c^2/s}}{(1-\sqrt{m_c^2/s})^2}, \quad y_{\min} = \frac{s(1-x)+m_c^2 x - \sqrt{(s(1-x)+m_c^2 x)^2 - 4m_c^2 s(1-x)}}{2s(1-x)}, \quad y_{\max} = \frac{s(1-x)+m_c^2 x + \sqrt{(s(1-x)+m_c^2 x)^2 - 4m_c^2 s(1-x)}}{2s(1-x)},$$

$$\text{and } \Delta(x, y, s) = -s(1-y)y + \frac{m_c^2(1-xy)}{1-x}, \quad s_1 = \frac{m_c^2(1-xy)}{(1-x)(1-y)y}.$$

The spectral function $\rho(s)$ and $R(M_B^2)$ in Eq. (14) are shown as follows:

$$\rho(s) = \int_{x_{\min}}^{x_{\max}} dx \int_{y_{\min}}^{y_{\max}} dy \frac{3}{512\pi^5 (y-1)} \langle GG \rangle xy \Delta(x, y, s) (-3(x-1)\Delta(x, y, s) - m_c^2 - 2s(x-1)(y-1)y) \\ + \int_{z_{\min}}^{z_{\max}} dz \frac{3\langle GG \rangle m_s \langle \bar{s}s \rangle (z-1)}{128\pi^3} \quad (\text{A3})$$

$$R(M_B^2) = \int_0^1 dz \frac{\langle GG \rangle m_s \langle \bar{s}s \rangle}{256\pi^3 z} (m_c^2 z - s_2(4z^2 - 6z + 2)) e^{-s_2/M_B^2} \quad (\text{A4})$$

$$\text{where } z_{\min} = \frac{1}{2}(1 - \sqrt{1 - 4m_c^2/s}), \quad z_{\max} = \frac{1}{2}(1 + \sqrt{1 - 4m_c^2/s}), \quad \text{and } s_2 = \frac{m_c^2}{(1-z)z}.$$

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