

Quantum-entangled pions

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We show that two- and three-pion states produced in the decay of neutral kaons are contextual, entangled, and Bell nonlocal in isospin space. By reinterpreting the experimental values of the different isospin amplitudes, we can determine the amount of entanglement enjoyed by these states and the extent to which they violate the noncontextuality and Bell locality inequalities. Notably, the two-pion case provides an example of a state that is more Bell nonlocal, as witnessed by the Collins-Gisin-Linden-Massar-Popescu inequality, than the maximally entangled one. The three-pion state offers a genuine multipartite test of Bell nonlocality with qutrits. This is the first instance where quantum properties like entanglement and contextuality are demonstrated to exist within the internal space of isospin.

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Introduction. Particles produced in scattering and decay processes can become entangled in their spin state. Similarly, internal space degrees of freedom, such as isospin in flavor space, can also become entangled.

Pions carry isospin $I = 1$ and the three pion states, π^+ , π^- and π^0 , are members of an isospin triplet. In analogy with the case of the spin of a massive spin-one particle, we can discuss the quantum properties of the pion isospin triplet. We expect the two- and three-pion final states from the CP conserving decays of neutral kaons K_S and K_L to show quantum contextuality [1,2], entanglement [3], and Bell nonlocality [4]—and the extent of these properties will be influenced by the relative weight assigned to the various isospin channels that govern the decays. These properties show that pions are proper quantum entities that cannot be visualized, let alone described, as classical point particles.

Once CP -violating effects are taken into account, the actual final states are incoherent mixtures containing a small component of a three-pion state in the predominantly two-pion state produced by the K_S decay, and vice versa for the K_L . However, the weight of these extra components is of the order of 10^{-3} in both cases, so they can be safely ignored for our purposes.

For quantum properties to manifest, it is essential to model the isospin using a non-Abelian group. Particles

whose properties are defined by internal spaces described by Abelian groups, such as electric charge, do not exhibit quantum correlations. This is because these properties are described by one-dimensional representations that do not allow for different choices of orientations within their internal space. Consequently, for instance, the state of two opposite charged particles (which are otherwise identical) is described by a factorizable state, and the correlations between their charge states remain purely classical.

The quantum properties we discuss characterize the states of the two- and three-pion systems. These states can be reconstructed by means of quantum state tomography—which is a particularly simple task to achieve in this case. We utilize the experimental values of the different isospin amplitudes that regulate the decays to compute the amount of entanglement, contextuality, and Bell nonlocality in isospin space. The two-pion state remains entangled when generated in the decay from the kaon—which is induced by the weak interactions and not isospin conserving. It is an example of a state that, though nonmaximally entangled, is more Bell nonlocal than the maximally entangled one. The three-pion state is of particular interest because it provides a multipartite test of Bell nonlocality with qutrits.

While these quantum properties of the states are indeed real and influence the physical processes, it is crucial to recognize that there is a difference with respect to when the same quantities are measured in spin space. For instance, the various average values entering a Bell inequality cannot be directly measured experimentally because we cannot rotate our apparatus in isospin space, contrary to what we do with polarimeters. Instead, as discussed below, a Bell

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nonlocality witness can be certified using quantum state tomography in isospin space [5].

To the best of our knowledge, this is the first experimental measure of entanglement in isospin space. Correlations due to Bose symmetry in pion scattering were analyzed in [6] (for a review with more recent results see [7,8]). Such correlations differ from those considered here because they are generated by an incoherent superposition of states. More recently, entanglement in flavor space has been discussed in scattering processes in [9–14] to explore possible connections between entanglement and symmetry enhancements.

Two-pion state. The two pions emerging from the decay of a neutral kaon K^0 in its rest reference frame are described by the state

$$|\Psi\rangle = \frac{1}{\sqrt{2 + |\gamma|^2}} [|\pi^+\pi^-\rangle + \gamma|\pi^0\pi^0\rangle + |\pi^-\pi^+\rangle], \quad (1)$$

in which the two pions are distinguished by their opposite momenta. The case $\gamma = -1$ corresponds to an unbroken isospin and the pions triplet falling into an irreducible representation of the group and into a state with maximal entanglement.

The configuration of the state is determined by the requirement $Q = I_z = 0$ of the final system as a whole. Neglecting CP violating effects, the two states $|\pi^+\pi^-\rangle$ and $|\pi^-\pi^+\rangle$ enter with the same weight, and therefore one constant, γ , is sufficient to fully determine the state. The state in Eq. (1) is formally the same as that describing the final spin state $J = J_z = 0$ in the decay of a scalar state into two massive spin-one states: the π^+ and π^- play respectively the role of the $+1$ and -1 components of spin as measured along an arbitrary direction, whereas the π^0 plays that of the component 0.

Why are the two pions in Eq. (1) in a coherent superposition rather than a mixture? In the unbroken isospin limit, these processes are the same, and the two possibilities are to be coherently summed with the $\pi^0\pi^0$ and $\pi^+\pi^-$ final state produced as a superposition. This is the case of the isospin-conserving strong interactions. Weak interactions, being small perturbations, cause only minor distortions in the strong eigenstates.

The density matrix $\rho = |\Psi\rangle\langle\Psi|$ obtained from Eq. (1) satisfies $\rho^2 = \rho$, it being a pure state. The reduced density matrix of one pion, obtained by tracing over the degrees of freedom of the other, is given, in both cases, by

$$\rho_1 = \frac{1}{|\gamma|^2 + 2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & |\gamma|^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2)$$

and describes an incoherent mixture of the three physical π^+ , π^0 , and π^- states with weights given by the elements on the diagonal.

The reduced matrix in Eq. (2) determines whether the single pions produced in the decay feature quantum contextuality or not by means of the noncontextuality inequality [15]

$$\overline{\text{CNTXT}}_9 = \sum_{k=1}^4 \lambda_k^{\rho_1} \lambda_k^{\Pi} \leq 3, \quad (3)$$

in which $\lambda_i^{\rho_1}$ are the eigenvalues of the matrix ρ_1 and λ_i^{Π} those of the matrix

$$\Pi = \begin{pmatrix} 10/3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 8/3 \end{pmatrix}, \quad (4)$$

combined as to obtain the largest value in the sum in Eq. (3).

We find that

$$\overline{\text{CNTXT}}_9 = \begin{cases} \frac{8|\gamma|^2 + 19}{3|\gamma|^2 + 6} & \text{if } |\gamma|^2 < 1, \\ \frac{10|\gamma|^2 + 17}{3|\gamma|^2 + 6} & \text{if } |\gamma|^2 \geq 1, \end{cases} \quad (5)$$

which can be used to determine whether the pions are in a noncontextual state or not.

As the pions are in a pure state, entanglement can be quantified via the entanglement entropy [3], equal to

$$\mathcal{E}[\rho] = \frac{1}{|\gamma|^2 + 2} \left[|\gamma|^2 \log \left(\frac{|\gamma|^2 + 2}{|\gamma|^2} \right) + 2 \log (|\gamma|^2 + 2) \right], \quad (6)$$

which can vary from 0 to $\log 3$. The quantity

$$\mathcal{I}_3 = \text{Tr}[\mathcal{B}\rho], \quad (7)$$

with the matrix $\mathcal{B}$ given by [16]

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (8)$$

can be used to test Bell locality for two qutrits by means of the inequality

$$\mathcal{I}_3 \leq 2. \quad (9)$$

The maximum attainable value within quantum mechanics (QM) is $\mathcal{I}_3 \simeq 2.915$ [16], whereas in completely nonlocal theories it is $\mathcal{I}_3 = 4$ [17].

To determine the value of the entanglement entropy and of the two inequalities in Eqs. (5) and (9), we must determine the value of the coefficient γ entering the density matrix ρ . Assuming CP conservation, the kaon decay process is regulated by the real isospin amplitudes A_0 and A_2 , which are related to the decay modes as

$$\begin{aligned} \langle \pi^+ \pi^- | \mathcal{H}_W | K^0 \rangle &= A_0 e^{i\delta_0} + \frac{A_2}{\sqrt{2}} e^{i\delta_2} \\ \langle \pi^0 \pi^0 | \mathcal{H}_W | K^0 \rangle &= A_0 e^{i\delta_0} - \sqrt{2} A_2 e^{i\delta_2}, \end{aligned} \quad (10)$$

and determine the value of γ as

$$\gamma = \frac{\langle \pi^0 \pi^0 | \mathcal{H}_W | K^0 \rangle}{\langle \pi^+ \pi^- | \mathcal{H}_W | K^0 \rangle} = \frac{\sqrt{2} A_0 e^{i(\delta_0 - \delta_2)} - 2 A_2}{\sqrt{2} A_0 e^{i(\delta_0 - \delta_2)} + A_2}, \quad (11)$$

where $\mathcal{H}_W$ is the weak Hamiltonian. If the $I = 2$ amplitude is neglected, $\gamma = 1$ and the two pions are in the $I = 0$, $I_z = 0$ eigenstate that maximizes the isospin entanglement. The introduction of an $I = 2$, $I_z = 0$ component, suppressed by the $\Delta I = 1/2$ rule, causes a small departure from the $\gamma = -1$ limit and enhances the nonlocal character of the state. Had the $I = 2$ component been of the same order as that with $I = 0$, the state would have been significantly less entangled. The phases in Eq. (10) come from the strong interactions of the final state pions which undergo $\pi - \pi$ scattering. It is important to stress that both electroweak (causing the kaon decay) and strong (in the final state scattering) interactions are responsible for the presence of entanglement in the pion final state. For this

reason, QM has here been tested in the presence of the full Standard Model.

By using the isospin amplitudes given in [18,19]

$$A_0 = (2.704 \pm 0.001) \times 10^{-7} \text{ GeV}$$

$$A_2 = (1.210 \pm 0.002) \times 10^{-8} \text{ GeV}$$

$$\delta_0(m_K) - \delta_2(m_K) = (47.5 \pm 0.9)^\circ, \quad (12)$$

we obtain

$$|\gamma|^2 = 0.880 \pm 0.002. \quad (13)$$

The quantum properties depend strongly on the coefficient γ , and for large values of the parameter the system becomes disentangled because the components containing charged pions are suppressed. In the opposite limit, $\gamma \rightarrow 0$, entanglement remains and involves the charged pions only. For the value in Eq. (13) we find

$$\mathcal{E}[\rho] = 1.097 \pm 0.001, \quad (14)$$

close to the largest value achievable, $\log 3 \simeq 1.099$ (obtained for $|\gamma|^2 = 1$).

Taking the same value for γ , we find that

$$\text{CNTXT}_9 = 3.014 \pm 0.001, \quad (15)$$

$$\mathcal{I}_3 = 2.890 \pm 0.001. \quad (16)$$

The noncontextuality and Bell locality inequalities in Eqs. (5) and (9) are both violated with a significance well in excess of the 5σ level. The state in Eq. (1) is thus certified to be nonlocally entangled and contextual. The large significances obtained in all these tests are driven by the precision achieved in the experimental determination of the CP conserving kaon decay amplitudes. In addition, as a value larger than $\mathcal{I}_3 = 2.915$ is not reached, QM is confirmed.

If isospin breaking corrections are included, the values of the isospin amplitudes change [19], but not sufficiently to have an appreciable impact on the quantum properties of the two-pion state.

Three-pion state. The three pions coming from the K_L decay are describe by the state

$$\begin{aligned} |\Phi\rangle &= \frac{1}{\sqrt{6 + |\Gamma|^2}} [|\pi^0 \pi^+ \pi^- \rangle + |\pi^+ \pi^0 \pi^- \rangle + |\pi^+ \pi^- \pi^0 \rangle \\ &\quad + \Gamma |\pi^0 \pi^0 \pi^0 \rangle + |\pi^0 \pi^- \pi^+ \rangle + |\pi^- \pi^0 \pi^+ \rangle + |\pi^- \pi^+ \pi^0 \rangle], \end{aligned} \quad (17)$$

in which the pions are distinguished by their momenta p_i , with $\sum_i p_i = p_K$ (with p_K being the kaon momentum).

The relative weight Γ of the $|\pi^0\pi^0\pi^0\rangle$ state is determined by the experimental values of the amplitudes $A(\pi^0\pi^0\pi^0)$ and $A(\pi^+\pi^-\pi^0)$; it depends on the two Dalitz plot variables describing the process. The corresponding density matrix $\eta = |\Phi\rangle\langle\Phi|$ satisfies $\eta^2 = \eta$, it being a pure state.

Contextuality can be tested for each pion produced in the decay by considering the corresponding reduced density matrices

$$\eta_0 = \eta_- = \eta_+ = \frac{1}{|\Gamma|^2 + 6} \begin{pmatrix} 2 & 0 & 0 \\ 0 & |\Gamma|^2 + 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad (18)$$

obtained from η by tracing over the degrees of freedom of the remaining two pions.

The noncontextuality inequality, as given by Eq. (3), is found to be

$$\text{CNTXT}_9 = \frac{10|\Gamma|^2 + 54}{3(6 + |\Gamma|^2)}, \quad (19)$$

and it is the same for each of the pions.

The classification of entanglement in multipartite systems is richer than in the bipartite case (see, for instance, [3,20,21]), and a concept of particular importance is that of genuine multipartite entanglement (GME): a state features GME when it cannot be decomposed as a convex combination of biseparable states (states which are separable on, at least, one bipartition of the system), and thus exhibits quantum correlations involving all its parts [22]. In the case of the three-pion state (17), GME can be quantified by

$$\mathcal{C}[\eta] = \min\{\mathcal{C}_{1(23)}, \mathcal{C}_{2(13)}, \mathcal{C}_{3(12)}\}, \quad (20)$$

where

$$\mathcal{C}_{i(jk)} = \sqrt{2[1 - \text{Tr}(\eta_{(jk)}^2)]}, \quad (21)$$

in which $\eta_{(jk)} = \text{Tr}_{(i)}[\eta]$ is the reduced density matrix obtained by tracing out two of the pions. We find

$$\mathcal{C}_{1(23)} = \mathcal{C}_{3(12)} = \mathcal{C}_{2(31)} = \frac{4\sqrt{|\Gamma|^2 + 3}}{|\Gamma|^2 + 6}, \quad (22)$$

which, in the present case, quantify the entanglement of one of the pions with the system composed by the other two.

Another quantitative measure of tripartite entanglement is provided by the concurrence triangle [23,24], given by

$$F_3 = 4\sqrt{\frac{\mathcal{Q}(\mathcal{Q} - \mathcal{C}_{1(23)})(\mathcal{Q} - \mathcal{C}_{2(13)})(\mathcal{Q} - \mathcal{C}_{3(12)})}{3}}, \quad (23)$$

in which

$$\mathcal{Q} = 6\frac{\sqrt{|\Gamma|^2 + 3}}{|\Gamma|^2 + 6}, \quad (24)$$

yielding

$$F_3 = 16\frac{|\Gamma|^2 + 3}{(|\Gamma|^2 + 6)^2} \quad (25)$$

in the case of the three pions. As in the case of two pions, entanglement among the three pions only vanishes for $|\Gamma| \rightarrow \infty$.

Genuine multipartite Bell nonlocality can be probed in a system composed by three qutrits by means of an operator $\mathcal{B}'$ defined as [22,25]

$$\begin{aligned} \mathcal{B}' = & A_0 \otimes B_0 \otimes C_0 + A_0 \otimes B_0 \otimes C_1 + A_0 \otimes B_1 \otimes C_0 \\ & + A_1 \otimes B_0 \otimes C_0 - A_1 \otimes B_1 \otimes C_1 - A_0 \otimes B_1 \otimes C_1 \\ & - A_1 \otimes B_0 \otimes C_1 - A_1 \otimes B_1 \otimes C_0, \end{aligned} \quad (26)$$

in which the six 3×3 Hermitian matrices A_k , B_k , and C_k , $k = 0, 1$, satisfy $A_k^2 = B_k^2 = C_k^2 = \mathbb{1}_3$. The Svetlichny inequality [22,25]

$$\mathcal{I}'_3 = \text{Tr}[\eta \mathcal{B}'] \leq 4 \quad (27)$$

tests Bell locality in a tripartite system. The largest possible value in QM is $4\sqrt{2}$.

We find the six, for simplicity, orthogonal matrices in Eq. (26) performing a maximization of $\mathcal{I}'_3$. These matrices are given in terms of rational numbers as

$$A_0 = -B_1 = -C_0 = \begin{pmatrix} \frac{8}{9} & \frac{4}{9} & \frac{1}{9} \\ \frac{4}{9} & -\frac{7}{9} & -\frac{4}{9} \\ \frac{1}{9} & -\frac{4}{9} & \frac{8}{9} \end{pmatrix} \quad (28)$$

$$A_1 = B_0 = -C_1 = \begin{pmatrix} \frac{8}{9} & -\frac{4}{9} & \frac{1}{9} \\ -\frac{4}{9} & -\frac{7}{9} & \frac{4}{9} \\ \frac{1}{9} & \frac{4}{9} & \frac{8}{9} \end{pmatrix}, \quad (29)$$

within a 1% approximation. We find that $\Gamma = 1.372$ yields the maximum value $\mathcal{I}'_3 = 4.569$, as computed by using the state in Eq. (17) in the test of Eq. (27).

Experimental determinations of the coefficient Γ depend on two Dalitz plot variables,

$$x = \frac{s_2 - s_1}{m_{\pi^+}^2} \quad \text{and} \quad y = \frac{s_3 - s_0}{m_{\pi^+}^2}, \quad (30)$$

in which $s_i = (p_j + p_k)^2$ and $s_0 = (m_{K_L}^2 + m_{\pi^+}^2 + m_{\pi^-}^2 + m_{\pi^0}^2)/3$. In terms of these, the three-pion decay amplitudes are then written as [18]

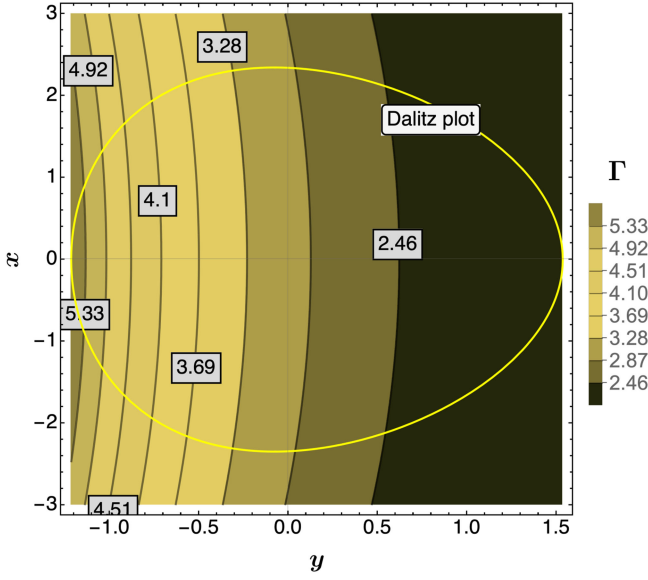


FIG. 1. Variation of the coefficient Γ of the three-pion state across the kinematic space of the two variables x and y of the Dalitz plot, which uses the central values [26] of the coefficients in Eqs. (31)–(32) and refers to the final state containing charged pions—which is the most restrictive. The area inside the closed curve shows the allowed values for the variables x and y .

$$A(K_L \rightarrow \pi^0 \pi^0 \pi^0) = 3(\alpha_1 + \alpha_3) + 3(\zeta_1 - 2\zeta_3) \left(y^2 + \frac{1}{3}x^2 \right) \quad (31)$$

and

$$\begin{aligned} A(K_L \rightarrow \pi^0 \pi^+ \pi^-) &= (\alpha_1 + \alpha_3) - (\beta_1 + \beta_2)y \\ &+ (\zeta_1 - 2\zeta_3) \left(y^2 + \frac{1}{3}x^2 \right) \\ &+ (\xi_1 - 2\xi_3) \left(y^2 - \frac{1}{3}x^2 \right), \end{aligned} \quad (32)$$

in which the coefficients α_i , ζ_i , β_i , and ξ_i are given by chiral perturbation theory. We take their values and related uncertainties from the best fit in [26]. The values of the kaon and pion masses are taken to be $m_{\pi^\pm} = 139.570$ MeV, $m_{\pi^0} = 134.977$ MeV, and $m_{K^0} = 497.611$ MeV [19].

Figure 1 depicts the variation of the coefficient Γ in Eq. (17) as the variables x and y vary. Only the values within the closed curve are permitted by the kinematics of the decay process. The value $\Gamma = 1.372$ that maximizes the expectation value of the Bell operator given in Eq. (26) lies outside the range of possible values. Figure 2 shows the values of $\mathcal{I}'_3$ as a function of the same Dalitz plot variables. For values of the Dalitz plot variable x and y within the region on the left of the white dashed line in Fig. 2, the three-pion isospin state does not violate (27) despite being entangled. This is possible because the system is tripartite,

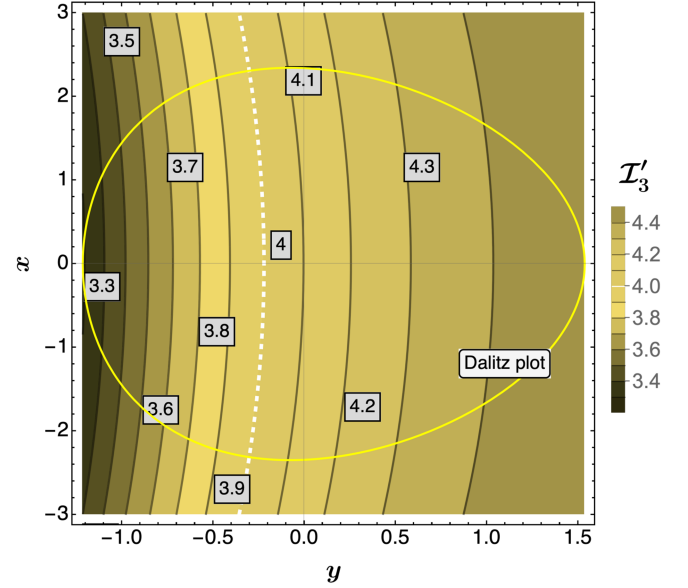


FIG. 2. Variation of the value of $\mathcal{I}'_3$ across the kinematic space of the two variables x and y of the same Dalitz plot as in Fig. 1. The area inside the closed curve shows the allowed values for the variables x and y . The critical value $\mathcal{I}'_3 = 4$ is marked by the white dashed contour.

as entanglement inherently implies Bell nonlocality for bipartite pure states.

The three-pion state in Eq. (17) is entangled and violates noncontextuality and Bell locality inequalities for several values that the Dalitz plot variables within the accessible kinematic region.

Taking as a benchmark the value $\Gamma = 2.600 \pm 0.008$ —which is among those allowed in the Dalitz plot and for which $\mathcal{I}'_3$ is larger than 4—and whose uncertainty is obtained by propagating those of the coefficients in Eqs. (31)–(32) as given by [26], we find that the non-contextuality inequality is violated with a very large significance as

$$\text{CNTXT}_9 = 3.177 \pm 0.001, \quad (33)$$

and entanglement is quantified in

$$\mathcal{C}[\eta] = 0.979 \pm 0.001 \quad \text{and} \quad F_3 = 0.959 \pm 0.002. \quad (34)$$

Moreover,

$$\mathcal{I}'_3 = 4.257 \pm 0.003, \quad (35)$$

and the Svetlichny's inequality in Eq. (27) is also violated with a significance well above the 5σ level.

Outlook. The isospin space, though more abstract than the ordinary spin space, provides an elegant and simple testing

ground for studying the quantum properties of particle states.

The state describing the two pions that emerge from the decay of the neutral kaon K_S exhibits quantum contextuality, entanglement, and Bell nonlocality. These quantum properties emerge from the interplay between the electro-weak and strong interactions. We use experimental data to determine how much of these properties remain after symmetry breaking. The two-qutrit state of the two pions exhibits greater Bell nonlocality, as witnessed by the Collins-Gisin-Linden-Massar-Popescu inequality [17], compared to the maximally entangled state.

The state describing the decay of the neutral kaon K_L into three pions exhibits genuine multipartite entanglement and genuine multipartite Bell nonlocality, making it a system of great value for exploring the quantum properties of multipartite states, which are still relatively unexplored.

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- [1] J. S. Bell, On the problem of hidden variables in quantum mechanics, *Rev. Mod. Phys.* **38**, 447 (1966).
 - [2] S. Kochen and E. Specker, The problem of hidden variables in quantum mechanics, *J. Math. Mech.* **17**, 59 (1968).
 - [3] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, *Rev. Mod. Phys.* **81**, 865 (2009).
 - [4] J. Bell, On the Einstein Podolsky Rosen paradox, *Physics (Long Island City, N.Y.)* **1**, 195 (1964).
 - [5] M. Fabbrichesi, R. Floreanini, and L. Marzola, About testing Bell locality at colliders, [arXiv:2503.18535](https://arxiv.org/abs/2503.18535).
 - [6] G. Goldhaber, S. Goldhaber, W.-Y. Lee, and A. Pais, Influence of Bose-Einstein statistics on the anti-proton proton annihilation process, *Phys. Rev.* **120**, 300 (1960).
 - [7] D. H. Boal, C. K. Gelbke, and B. K. Jennings, Intensity interferometry in subatomic physics, *Rev. Mod. Phys.* **62**, 553 (1990).
 - [8] G. Alexander, Bose-Einstein and Fermi-Dirac interferometry in particle physics, *Rep. Prog. Phys.* **66**, 481 (2003).
 - [9] S. R. Beane, D. B. Kaplan, N. Klco, and M. J. Savage, Entanglement suppression and emergent symmetries of strong interactions, *Phys. Rev. Lett.* **122**, 102001 (2019).
 - [10] S. R. Beane, R. C. Farrell, and M. Varma, Entanglement minimization in hadronic scattering with pions, *Int. J. Mod. Phys. A* **36**, 2150205 (2021).
 - [11] I. Low and T. Mehen, Symmetry from entanglement suppression, *Phys. Rev. D* **104**, 074014 (2021).
 - [12] Q. Liu, I. Low, and T. Mehen, Minimal entanglement and emergent symmetries in low-energy QCD, *Phys. Rev. C* **107**, 025204 (2023).
 - [13] M. Carena, I. Low, C. E. M. Wagner, and M.-L. Xiao, Entanglement suppression, enhanced symmetry, and a standard-model-like Higgs boson, *Phys. Rev. D* **109**, L051901 (2024).
 - [14] K. Kowalska and E. M. Sessolo, Entanglement in flavored scalar scattering, *J. High Energy Phys.* **07** (2024) 156.
 - [15] M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, Tests of quantum contextuality in particle physics, *Phys. Rev. D* **112**, 033005 (2025).
 - [16] A. Acin, T. Durt, N. Gisin, and J. I. Latorre, Quantum nonlocality in two three-level systems, *Phys. Rev. A* **65**, 052325 (2002).
 - [17] D. Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, Bell inequalities for arbitrarily high-dimensional systems, *Phys. Rev. Lett.* **88**, 040404 (2002).
 - [18] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, and J. Portoles, Kaon decays in the standard model, *Rev. Mod. Phys.* **84**, 399 (2012).
 - [19] M. Antonelli *et al.* (FlaviaNet Working Group on Kaon Decays Collaboration), An evaluation of $|V_{us}|$ and precise tests of the standard model from world data on leptonic and semileptonic kaon decays, *Eur. Phys. J. C* **69**, 399 (2010).
 - [20] V. Coffman, J. Kundu, and W. K. Wootters, Distributed entanglement, *Phys. Rev. A* **61**, 052306 (2000).
 - [21] Z.-H. Ma, Z.-H. Chen, J.-L. Chen, C. Spengler, A. Gabriel, and M. Huber, Measure of genuine multipartite entanglement with computable lower bounds, *Phys. Rev. A* **83**, 062325 (2011).
 - [22] G. Svetlichny, Distinguishing three-body from two-body nonseparability by a bell-type inequality, *Phys. Rev. D* **35**, 3066 (1987).
 - [23] S. Xie and J. H. Eberly, Triangle measure of tripartite entanglement, *Phys. Rev. Lett.* **127**, 040403 (2021).
 - [24] Z.-X. Jin, Y.-H. Tao, Y.-T. Gui, S.-M. Fei, X. Li-Jost, and C.-F. Qiao, Concurrence triangle induced genuine multipartite entanglement measure, *Results Phys.* **44**, 106155 (2023).
 - [25] M. Seevinck and G. Svetlichny, Bell-type inequalities for partial separability in n -particle systems and quantum mechanical violations, *Phys. Rev. Lett.* **89**, 060401 (2002).
 - [26] J. Bijnens, P. Dhonte, and F. Borg, $K \rightarrow 3\pi$ decays in chiral perturbation theory, *Nucl. Phys.* **B648**, 317 (2003).