

Equivalent electric dipole moment in SMEFT

Marco Ardu^{*} and Nicola Valori[†]

*Instituto de Física Corpuscular, Universidad de Valencia and CSIC,
Edificio Institutos Investigación, C/Catedrático José Beltrán 2, 46980 Paterna, Spain*



(Received 9 April 2025; accepted 10 December 2025; published 23 January 2026)

The electric dipole moment of the electron (eEDM) is typically investigated in experiments using paramagnetic molecules. However, the physical observable in these searches consists of a linear combination of CP -violating interactions, rather than the eEDM alone, which is commonly referred to as the equivalent EDM of the system. Assuming the presence of new CP -odd physics from heavy degrees of freedom, we parametrize its effects within the Standard Model effective field theory (SMEFT) framework. We systematically compute the contributions to the full low-energy direction probed by EDM searches, focusing on leading-order effects at dimension six and one-loop level, while also discussing selected two-loop and dimension-eight contributions. We find that eEDM experiments are sensitive to a broader class of SMEFT operators than previously recognized.

DOI: [10.1103/2yfc-jbx2](https://doi.org/10.1103/2yfc-jbx2)

I. INTRODUCTION

The Standard Model (SM) of particle physics provides a remarkably precise description of fundamental interactions, successfully accounting for most observed phenomena up to energies near the TeV scale. However, despite its successes, several key observations remain unexplained within the SM framework. These include the existence of nonzero neutrino masses, the presence of dark matter, and the observed baryon asymmetry of the Universe. Addressing these issues requires physics beyond the SM (BSM), motivating both theoretical developments and experimental searches for new interactions.

In particular, the baryon-antibaryon asymmetry of the Universe cannot be dynamically generated within the SM alone [1,2]. A key requirement for successful baryogenesis is additional sources of CP violation beyond those present in the Cabibbo-Kobayashi-Maskawa (CKM) and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices, which govern quark and lepton mixing, respectively. This motivates experimental searches for CP -odd observables that could serve as direct signatures of new CP -violating interactions. Among these, electric dipole moments (EDMs) of SM particles provide some of the most stringent probes of CP -violating BSM physics. Theoretically, EDMs are

particularly well suited for new physics (NP) searches because their SM contributions are extremely suppressed. In the quark sector, CP violation is parametrized by the Jarlskog invariant [3], $J = \text{Im}(V_{tb}V_{td}^*V_{cb}V_{cb}^*) \sim 3 \times 10^{-5}$ [4], which leads to EDM contributions only appearing at three-loop order or higher due to the required multiple CKM insertions and the antisymmetry of J [5].¹ In the lepton sector, an analogous Jarlskog invariant can be constructed from the PMNS matrix, but no definite sign of CP violation in neutrino oscillations has been measured so far.² Additionally, the PMNS-induced EDMs also appear at high loop orders, leading to suppressed contributions. Experiments looking for EDMs are sensitive to much larger values than those predicted by the SM and would thus be able to unambiguously detect or constraint CP -odd new physics interactions. The absence of EDM signals in present searches has led to stringent upper limits on the EDMs of leptons and the neutron, summarized in Table I, with future experiments expected to further improve the sensitivities. The projection for future electron EDM searches anticipates an improvement of up to 3 orders of magnitude [10–12]. Dedicated searches for a muon EDM at PSI are also expected to run in the upcoming years, with a target sensitivity of $d_\mu \lesssim 6 \times 10^{-23} e \cdot \text{cm}$ [13]. Improvement for the neutron EDM are also expected from the n2EDM experiment [14].

^{*}Contact author: marco.ardu@ific.uv.es

[†]Contact author: nicola.valori@uv.es

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

¹Long-distance contributions can lead to larger values [6,7], but are still far from the experimental sensitivities.

²Although there is a small preference for a nonzero Dirac phase when including Super-Kamiokande atmospheric neutrino data [8,9].

TABLE I. Current upper limit on the EDMs of charged leptons and the neutron. As explained in the text, the electron EDM bounds are found assuming that the coefficient of the semileptonic interactions that enter in the equivalent EDM of the paramagnetic HfF^+ molecule is zero.

EDM	Upper limit at 90% CL
d_e	$4.1 \times 10^{-30} \text{ e} \cdot \text{cm}$ [15]
d_μ	$1.8 \times 10^{-19} \text{ e} \cdot \text{cm}$ [16]
d_τ	$\sim 10^{-17}$ [17]
d_n	$1.8 \times 10^{-26} \text{ e} \cdot \text{cm}$ [18]

Assuming that new physics lies at a high-energy scale, its low-energy effects can be systematically described in a model-independent manner using the Standard Model effective field theory (SMEFT). In this approach, the contributions of the heavy degrees of freedom are encoded in the Wilson coefficients of higher-dimensional operators constructed from SM fields and observables can be calculated in terms of the operator coefficients. EDMs, in particular, are related to the imaginary coefficient of the dipole operator and are sensitive to a wide variety of CP -odd operators that can contribute to it. A number of SMEFT analyses of EDMs exist in the literature, including complete studies of one-loop effects at dimension six [19,20], as well as selected two-loop contributions [21].

Given the exceptional sensitivity of electron EDM experiments, constraints derived from these measurements often set the most stringent limits on BSM CP violation. In the EFT framework, the experimental reach of electron EDM searches corresponds to the largest UV scales among all EDM observables. The state of the art of electron EDM best limit come from searches of the EDMs of paramagnetic molecules, like HfF^+ and ThO . However, these experiments are sensitive not only to the permanent EDM of the electron, but also to other CP -odd interactions between the electron and the nuclei. They effectively constrain a combination of the electron EDM and semileptonic scalar interactions, collectively referred to as the “equivalent EDM” of the system (d^{equiv}). The reported upper limits on the electron EDM (eEDM) assume no contribution from the CP -odd scalar, an assumption that may not hold, in general. If the CP -violating physics affects electron-nuclei interactions, paramagnetic EDM searches can provide direct constraints on its magnitude. These semileptonic interactions have been studied in the context of leptoquark models [22] and can also be used to indirectly bound the QCD θ term [23]. Moreover, searches for CP -violating effects in paramagnetic molecules are also sensitive to long-distance nucleon-nucleon interactions [24]. Although it is possible to combine measurements on different paramagnetic systems to get model-independent upper limits on the eEDM and the semileptonic interactions independently, the resulting bounds are generally weaker than those obtained by assuming that only one source contributes. So when using

the upper-limit quoted in Table I, d^{equiv} rather than d_e should be considered whenever the semileptonic contribution is not negligible. Previous EFT studies focused on the eEDM contributions in SMEFT, while the goal of this work is to systematically study the full experimental direction probed by paramagnetic EDM searches within the SMEFT framework. This may also provide insight into potential flat directions in the EFT space, since cancellations between the two contributions are, in principle, possible. By including all relevant renormalization group mixing effects at the one-loop level and dimension six, we aim to provide a comprehensive analysis of how electron EDM experiments constrain the space of CP -violating operators. In all cases where $d_e \sim d^{\text{equiv}}$, our analysis reproduces the results in [19,20], while whenever the semileptonic interactions are relevant we assess the correct operator sensitivities. Selected two-loop and dimension-eight contributions are also discussed.

The paper is organized as follows. In Sec. II, we review the low-energy operators that enter in the definition of the equivalent EDM and discuss the combinations probed by the different paramagnetic molecules. In Sec. III, we introduce the SMEFT Lagrangian and calculate the contributions of high-energy operators onto the low-energy experimental direction to constrain the SMEFT operator coefficients. Finally, we summarize our results in Sec. IV.

II. THE EQUIVALENT ELECTRIC DIPOLE MOMENT

As discussed in the Introduction, the most sensitive searches of the electron EDM are conducted on paramagnetic systems, where the EDM of the unpaired electrons is inferred from the frequency shift of paramagnetic atoms or molecules in the presence of an electric field. However, these energy shifts depend not only on the eEDM, but also on other CP -odd semileptonic interactions. At the experimental scale, the relevant interactions to which experiments are most sensitive can be parametrized according to the Lagrangian [25],

$$\mathcal{L} = -i \frac{d_e}{2} \bar{e} \sigma_{\mu\nu} \gamma_5 e F^{\mu\nu} + \frac{G_F}{\sqrt{2}} C_S \bar{e} i \gamma_5 e \bar{N} N, \quad (2.1)$$

where $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$, $N = (p, n)$ is the nucleon isospin doublet, G_F is the Fermi constant, and C_S is a dimensionless Wilson coefficient.

Adopting this parametrization, the CP -odd energy shift is expressed in terms of the linear combination $hf = W_d d_e + W_S C_S$, where W_d is a coefficient proportional to the effective electric field, and W_S is a molecule-specific structure constant defined as [26]

$$W_S = \frac{G_F}{\sqrt{2}} Z \langle \Psi | \sum_{j=1}^n \gamma_j^0 \gamma_j^5 \rho_K(\vec{r}_j) | \Psi \rangle, \quad (2.2)$$

where Z is the proton number, j runs over the electron in the molecule, $\rho_K(\vec{r}_j)$ is the nuclear charge density at the position of electron j , and Ψ is the system wave function. By factoring out W_d , it is possible to define an equivalent eEDM (d_e^{equiv}) in terms of the operator coefficients in (2.1) that correspond to the observable measured in paramagnetic EDM searches. Currently, the best experimental sensitivity is obtained using HfF^+ molecular ions, where the equivalent EDM reads³ [27–30]

$$d_{\text{HfF}^+}^{\text{equiv}} = d_e + C_S \times 0.9 \times 10^{-20} e \cdot \text{cm}. \quad (2.3)$$

As of today, $d_{\text{HfF}^+}^{\text{equiv}}$ is constrained to be [15]

$$|d_{\text{HfF}^+}^{\text{equiv}}| < 4.1 \times 10^{-30} e \cdot \text{cm}. \quad (2.4)$$

Within the SM, d_e^{equiv} is predicted to be $\sim 10^{-35} e \cdot \text{cm}$ [31], which is still far from the reach of future eEDM experiments [10–12]. It is then possible to constrain CP -violating operators from the d_e^{equiv} upper limit.

A single experiment cannot distinguish the contributions of the two operators in Eq. (2.1), making it impossible to determine whether a potential signal arises from d_e or C_S or to impose a model-independent bound on each coefficient. In order to understand the typical contributions to d_e^{equiv} coming from the eEDM and the semileptonic interactions, we consider the following representative UV matching from CP -odd NP at the scale Λ ,⁴

$$d_e \sim \frac{v}{16\pi^2 \Lambda^2} \quad C_S \sim \frac{1}{\Lambda^2 G_F}, \quad (2.5)$$

where $v \sim 174$ GeV is the Higgs vacuum expectation value (VEV). This implies that the eEDM still remains the most sensitive observable to heavy new physics, given that it can probe energy scales $\Lambda \lesssim 10^7$ TeV which are $10 \sim 10^2$ times larger than those probed by the semileptonic contribution. However, the dipole coefficient could be suppressed by small couplings or additional loop factors, potentially allowing the scalar contribution to compete or dominate in d_e^{equiv} .

Since the low-energy EFT space spanned by the eEDM and the semileptonic scalar operator is two-dimensional, a

³We are neglecting the isospin breaking effects that are proportional to the coefficient of the semileptonic operator $\bar{e}i\gamma_5 e \bar{N}\tau^3 N$, with τ^3 being the diagonal Pauli matrix, because they are suppressed by the small $Z - N$ difference in the molecule, where Z is the number of protons and N is the number of neutrons.

⁴Here, we assume $\mathcal{O}(1)$ coefficients at the lowest loop level at which these operators can arise in the UV, in order to capture the typical size of their contributions to the experimental observable. Note that the factor of G_F in C_S arises from the standard normalization convention used in Eq. (2.1).

pair of experiments using different molecules would, in principle, be sufficient to resolve the degeneracy. This holds true provided that the experimental sensitivities define sufficiently orthogonal directions in the coefficient plane; otherwise, both experiments would probe nearly the same linear combination of operators, leaving the distinction ambiguous.

Searches for a permanent EDM in thorium monoxide ThO are sensitive to the equivalent EDM combination [32–34]

$$d_{\text{ThO}}^{\text{equiv}} = d_e + C_S \times 1.5 \times 10^{-20} e \cdot \text{cm}, \quad (2.6)$$

which is currently bounded to be $< 1.1 \times 10^{-29} e \cdot \text{cm}$ [35]. The HfF^+ direction can be decomposed into a projection onto $d_{\text{ThO}}^{\text{equiv}}$ and the perpendicular direction as

$$d_{\text{HfF}^+} \sim 0.75(\cos \phi d_{\text{ThO}} + \sin \phi d_{\text{ThO}\perp}), \quad (2.7)$$

where $\sin \phi \simeq 0.25$ and the orthogonal combination is

$$d_{\text{ThO}\perp} = 1.5d_e - C_S \times 10^{-20} e \cdot \text{cm}. \quad (2.8)$$

From Eq. (2.7), in case of successful detection of d_e^{equiv} , the two directions can be disentangled if the two measurements have a relative accuracy of order $\sim \tan \phi$, rendering it possible to measure d_e and C_S separately. A global fit of the current two independent measures leads to [15]

$$|d_e| < 2.1 \times 10^{-29} e \cdot \text{cm} \quad |C_S| < 1.9 \times 10^{-9} \quad (2.9)$$

at 90% CL, which is roughly 1 order of magnitude milder than the bounds inferred from the most sensitive measure assuming that one of the two coefficients is negligible. Note that the loss in sensitivity could be traced to the less precise measurement on ThO , but even with comparable precision, the loss of 1 order of magnitude in the combined bound is expected from Eq. (2.7), because the two experimental directions are aligned up to the small angle $\sin \phi \sim 0.25$.

A. Matching at the nucleon scale

In this section, we derive the matching conditions at $\mu \sim 2$ GeV, the scale where QCD becomes nonperturbative, relating nucleon- and quark-level operators relevant for the interactions contributing to the equivalent EDMs. We consider operators with chiral fields because it facilitates the matching with the SMEFT.

The scalar quark operators relevant for the matching onto the CP -odd semileptonic operator of Eq. (2.1) are

$$O_{XY}^{\text{eq}} = (\bar{e}P_X e)(\bar{q}P_Y q), \quad (2.10)$$

where $P_{X,Y}$ stands for the chirality projectors with $X, Y = L, R$, and q labels the SM quarks. These are added

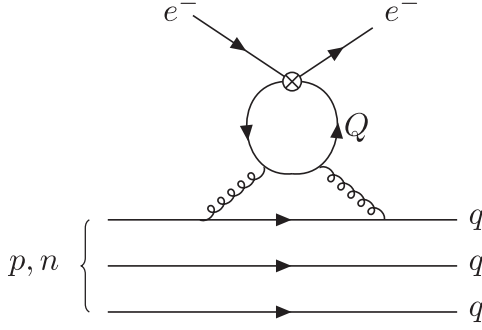


FIG. 1. Contribution to the semileptonic interaction C_S from closing a heavy-quark loop in a CP -odd scalar contact interaction, and gluons are exchanged with the nucleons.

to the Lagrangian as

$$\mathcal{L}_q = \sum_q \sum_{X,Y=L,R} \frac{C_{XY}^{\text{eq}}}{\Lambda^2} O_{XY}^{\text{eq}}, \quad (2.11)$$

where we have divided for the appropriate power of the energy scale Λ to have dimensionless coefficients. Starting from the chiral operators, the interaction featuring a leptonic pseudoscalar current and a quark scalar current corresponds to the combination

$$\frac{i}{2} \text{Im} [C_{RL}^{\text{eq}} + C_{RR}^{\text{eq}}] (\bar{e} \gamma_5 e) (\bar{q} q) \equiv C_s^q (\bar{e} i \gamma_5 e) (\bar{q} q), \quad (2.12)$$

since the Hermiticity of the Lagrangian imposes that $C_{XY}^* = C_{\bar{X}\bar{Y}}$, where $\bar{L} = R$ and $\bar{R} = L$.

Operators with valence quarks ($q = u, d, s$) match directly onto the nucleon-level operators because the scalar light-quark currents have nonzero matrix elements with the nucleon state, $\langle N | \bar{q} q | N \rangle \sim G_s^{N,q} \langle N | \bar{N} N | N \rangle$, where the QCD form factors $G_s^{N,q}$ are given in Table II. Scalar operators with heavy quarks ($Q = c, b, t$) instead match at the one-loop level with the dimension-seven gluon operator $O_{eG} = (\bar{e} i \gamma_5 e) (G_{\mu\nu}^a G_a^{\mu\nu})$ [36],

$$C_{eG} = -\frac{\alpha_s(m_Q)}{12\pi^2 m_Q} C_s^Q(m_Q), \quad (2.13)$$

and the gluon bilinear, in turn, matches onto the nucleon current at the confinement scale $\mu \sim 2 \text{ GeV}$ [36–38],⁵

$$\langle N | G_{\mu\nu}^a G_a^{\mu\nu} | N \rangle = -\frac{8\pi m_N}{9\alpha_s(\mu)} \langle N | \bar{N} N | N \rangle. \quad (2.14)$$

⁵These contributions are depicted in the diagram of Fig. 1. Note that the scalar contact interactions represented in the diagram could also result from CP -odd Higgs couplings stemming from modified Yukawa interactions.

TABLE II. Values of the scalar matching coefficients from [39]. The values for u, d result from chiral perturbation theory (χ PT) calculations [40,41], while for $q = s$ lattice results are also used [42].

	$q = u$	$q = d$	$q = s$
$G_s^{P,q}$	9	8.2	0.42
$G_s^{N,q}$	8.1	9	0.42

Taking into account the QCD running of the gluon operator coefficient C_{eG} (which has the same anomalous dimension as α_s), we can write down a general expression for the matching onto the CP -odd scalar with nucleons,

$$\frac{\Lambda^2 G_F}{\sqrt{2}} C_S(\mu) = \sum_{q=u,d,s} G_s^{N,q} C_s^q(\mu) + \sum_{Q=c,b,t} \frac{2m_N}{27m_Q} C_s^Q(m_Q). \quad (2.15)$$

III. EQUIVALENT EDMS IN SMEFT

Assuming the presence of heavy physics above the electroweak scale, we can parametrize the low-energy effects with the SMEFT Lagrangian,

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_d \sum_{i,\xi} \frac{C_i^\xi}{\Lambda^{d-4}} O_i^\xi, \quad (3.1)$$

where i runs over all the operators of a given dimension d and ξ represents the flavor indices. We define the SM fields and interactions in the notation of [43]. Here, we neglect the contributions of the operator with dimension higher than six, which we take in the on-shell Warsaw basis [43,44]. Dimension-eight contributions are discussed in Sec. III A. We define left-handed quark doublets in the up basis, where the up Yukawa matrix is diagonal and the quark doublets are defined as

$$q_L = \begin{pmatrix} u_L \\ d'_L \end{pmatrix}; \quad d'_{L,i} = V_{\text{CKM}}^{ij} d_{L,j}, \quad (3.2)$$

where primed fields are the down-type quarks in the weak basis, while the unprimed are the corresponding fields in the mass eigenstate basis. Flavor change involving the up sector is possible in matching at the electroweak scale because the operator O_{uH} contributes to the quark masses when the Higgs acquires a VEV. Note that quark flavor mixing already occurs at the renormalizable level, because CKM causes the matrix Y_d and Y_u to rotate with the renormalization scale and also contributes to off-diagonal wave function corrections. Since we work in the up basis, the flavor-changing effects are controlled by matrix elements of $Y_d^\dagger Y_d$, which due to the small down-type Yukawas leads to relatively small rotation angles. For this reason,

although we rotate back to the up basis at lower scales, we do not find interesting flavor-changing effects from the dimension-four renormalization group equations (RGEs).⁶

We also consider singlet and doublet lepton fields in the charged lepton mass eigenstate basis, so the lepton Yukawas can have off-diagonal entries in the presence of the operator $\mathcal{O}_{eH}$. However, we consider only lepton flavor-conserving operators in our analysis.

Contact interactions dressed with SM loops generally lead to divergences, which, after renormalization, cause the operator coefficients in Eq. (3.1) to evolve with the renormalization scale according to their RGEs. The solution to the dimension-six RGEs, which are fully known at the one-loop level [45–47], takes the following form:

$$\vec{C}(\mu_f) = \vec{C}(\mu_i)U(\mu_f, \mu_i), \quad (3.3)$$

having aligned the operator coefficients in a row vector $\vec{C}$ and where $U(\mu_f, \mu_i)$ is an evolution matrix that is related to the anomalous dimension of the coefficients. Hence, operators at the high scale will, in general, mix with each other at lower energy, allowing one to constrain operators that are otherwise difficult to probe directly.

Starting from a single SMEFT operator at a time at the scale $\mu \sim \Lambda$, we run down to the electroweak scale, where the heavy SM particles (t quark and H , W , and Z bosons) are integrated out. At this stage, the operators generated in the SMEFT RGE evolution are matched onto effective interactions that are invariant under $SU(3)_c \times U(1)_{\text{em}}$. Then, we continue running the low-energy operators down to the relevant energy scales, and we subsequently match onto the EDM experimental direction of Eq. (2.3). The matching onto C_S is given in Eq. (2.15), where semileptonic scalar operators involving light quarks are evolved down to the confinement scale $\mu \sim 2$ GeV, while the coefficients of operators involving heavy quarks cease running at their respective mass thresholds. Note that in using Eq. (2.15) we are not including the running of C_S from 2 GeV to the electron mass $\mu \sim m_e$, which is the scale of the EDM experiments. We comment on the impact of some sub-GeV effects later in the text.

Below the electroweak scale, we parametrize the dipole operator with the electrons as

$$\mathcal{L}_{\text{dip}} = C_{e\gamma} \bar{e} \sigma_{\mu\nu} P_R e F^{\mu\nu} + \text{H.c.}, \quad (3.4)$$

which directly matches onto the eEDM of Eq. (2.1) through the relation

$$d_e = -2 \text{Im}[C_{e\gamma}(m_e)], \quad (3.5)$$

⁶With the exception of operators mixing with the top tensor $C_{lequ(3)}^{eett}$, which we include in Fig. 2.

where the operator coefficient $C_{e\gamma}(m_e)$ is at the electron mass scale.

The full one-loop running, as well as the tree-level matching between the SMEFT and the low-energy EFT, is performed using the public package WILSON [48].

We take $\Lambda = 10$ TeV as the initial energy scale for the running and normalization of the operators. The one-loop RGE evolution is computed numerically, so it includes effects beyond the leading-log approximation.

Operators for which the contribution to d_e dominates over the semileptonic term C_S are shown in Fig. 2. For these operators, considering d^{equiv} or d_e as the experimental observable does not change the results. The operators $O_{\phi\bar{V}(V)}$ and $C_{lequ(3)}$ are the only ones that mix directly with the dipole at one loop. In contrast, operators such as $O_{lequ(1)}$, $O_{\bar{W}}$, and O_{fV} , with $f \neq e$, contribute to the dipole operators via second-order effects in the one-loop RGEs. We obtain results compatible with [20], although we relaxed the assumption of minimal flavor violation (MFV), so we do not define chirality-flipping operators with a built-in Yukawa suppression. Note that, from the EFT perspective, this is merely a normalization of the operators. The MFV scenario can be recovered by appropriately rescaling the sensitivity on the operator coefficients. While the MFV assumption is often invoked to ensure, for example, that the same operators do not generate excessively large contributions to the electron mass at one-loop level (or tree-level in case of C_{eq}^{11}), the numerical values shown in Fig. 2 confirm that such effects remain subdominant compared to the tree-level SM contribution once the EDM constraints are taken into account.

In general, we do not account for finite one-loop mixing effects,⁷ as they are known to depend on the renormalization scheme, the choice of evanescent operators [51], and the treatment of γ_5 [52]. This dependence introduces ambiguities in the UV matching and would require the inclusion of the two-loop RGEs to obtain scheme-independent observables. Selected two-loop effects have been computed in [21], including the mixing of the vector operator C_{le}^{ette} and the scalar operator C_{ledq}^{eebb} with the dipole via insertions of the τ and b Yukawa couplings. The operators C_{le}^{ette} and C_{ledq}^{eebb} also contribute to the dipole operator at higher orders in the one-loop RGEs through a chain of mixings. Specifically, they first mix with $C_{lequ(1)}^{eett}$ via a Yukawa insertion involving either the τ lepton or b quark, which then mixes into the tensor operator with tops before ultimately contributing to the dipole. While this process is formally suppressed at three

⁷With the only exception of $O_{\bar{W}}$, where the one-loop finite mixing has been computed in dimensional regularization (DR) and it is found to be comparable to the second-order contribution in the one-loop RGE. While the finite contribution depends on the regularization scheme, we stress that the only scheme consistent with an EFT approach is DR [49,50].

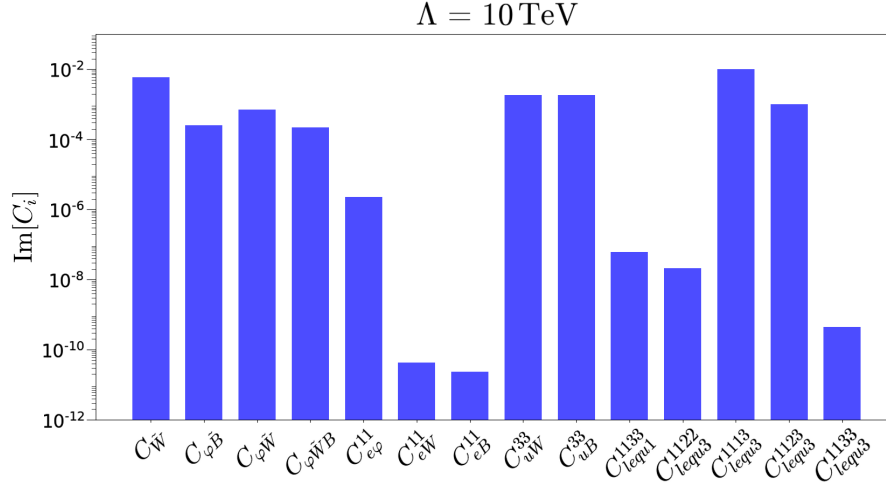


FIG. 2. Sensitivities on the imaginary part of the SMEFT Wilson coefficients for the operators that primarily contribute to d_e , from the EDM limit in Eq. (2.4), assuming $\Lambda = 10$ TeV as the initial scale for the running. The first four columns refer to the real part of the correspondent WILSON coefficients, due to the fact that the violation of CP stems from the operators themselves and not from an imaginary coupling.

loops, it is also enhanced by a cubic logarithm, $\log^3(\Lambda/m_t)$, whereas the direct two-loop mixing contains only a single power of $\log(\Lambda/m_t)$. Moreover, the anomalous dimensions in the one-loop mixing chain are sizable, particularly in the tensor-to-dipole diagrams, while the two-loop mixing coefficient is comparatively suppressed. We account for the leading-log two-loop running of the scalar operator C_{ledq}^{eebb} , resumming the one-loop QCD running, following the results of [21]. However, we find that the higher-order one-loop RGE-induced mixing is already ~ 20 times bigger than the two-loop contribution for $\Lambda = 10$ TeV.⁸ We also do not consider effects beyond the one-loop RGEs mixing for the vector $C_{le}^{e\tau\tau e}$ because the result would be scheme dependent⁹ and its contribution to C_S is suppressed, so previous eEDM SMEFT analyses would apply even when considering d^{equiv} . In contrast, C_{ledq}^{eebb} contributes significantly to the semileptonic operator featuring in d^{equiv} , so it can be constrained directly via the matching onto the scalar.

In addition to the contributions from operators that mix with C_{eV} in the RGEs, there are additional effects due to the modification of SM couplings when matching operators at the electroweak scale. Indeed, after spontaneous symmetry breaking, $C_{\psi\phi}$ modifies the SM Yukawas, which are not anymore proportional to the fermion mass matrix. As a result, if $C_{\psi\phi}$ is complex, a CP -odd interaction with the

Higgs and the SM fermions survives even after rotating the fields in their mass eigenstate basis. The dominant contributions to the eEDM from modified Yukawa interactions stem from the Barr-Zee diagrams [53]. This is a two-loop effect where the external photon is attached to a loop of charged particles, which is then connected to the lepton line via a photon and a Higgs boson. This contribution dominates over the one loop, which is suppressed by the small electron Yukawa, because in the Barr-Zee diagram the Higgs couples to a heavy particle in the loop. The largest contribution arises from a top or W loop, with the CP -violating phase coming from a $C_{e\phi}$ insertion in the electron line. A complex $C_{e\phi}$ also contributes to the CP -odd scalar operators discussed in Sec. II A, when the Higgs boson is removed at the electroweak scale.¹⁰ However, since d_e is more sensitive to heavy new physics than C_S , contributions to the semileptonic interaction, though sizable, are subdominant.

Figure 3 compares the sensitivities on Wilson coefficients from d_e and $d_{\text{Hff}^+}^{\text{equiv}}$, for the operators where the contribution to C_S is sizable. The semileptonic operators that are diagonal in the quark flavor match directly onto C_S and the dominant RGE effect is the QCD running that rescales the operator coefficients. We find that the relative scaling between the different generations is in agreement with Eq. (2.15). As a result, this class of operators gives the most significant contribution to C_S , with the only exception of $C_{lequ(1)}^{1133}$, where the corresponding matching coefficient is suppressed by m_N/m_t and its contribution to eEDM

⁸This ratio strongly depends on the initial scale of the running, given the $\log^3$ dependence in the higher-order one-loop mixing.

⁹The authors of [21] compute the two-loop mixing with the dipole of the four-lepton scalar $(\bar{l}_e e_e)(\bar{e}_\tau l_\tau)$, which is Fierz equivalent to $C_{le}^{e\tau\tau e}$ in $d = 4$ dimensions. However, in $d = 4 - 2\epsilon$ a set of evanescent operators should be specified in order to translate the results for the vector, which may also result in one-loop finite mixing as discussed in [20].

¹⁰Since the top is also integrated out at the same time, there is a $\propto C_{eH}$ matching contribution at one loop to the gluon operator $(\bar{e}i\gamma_5 e)(G_{\mu\nu}^a G_a^{\mu\nu})$, following from a diagram similar to Fig. 1 with a t loop and the crossed dot replaced by a Higgs exchange.

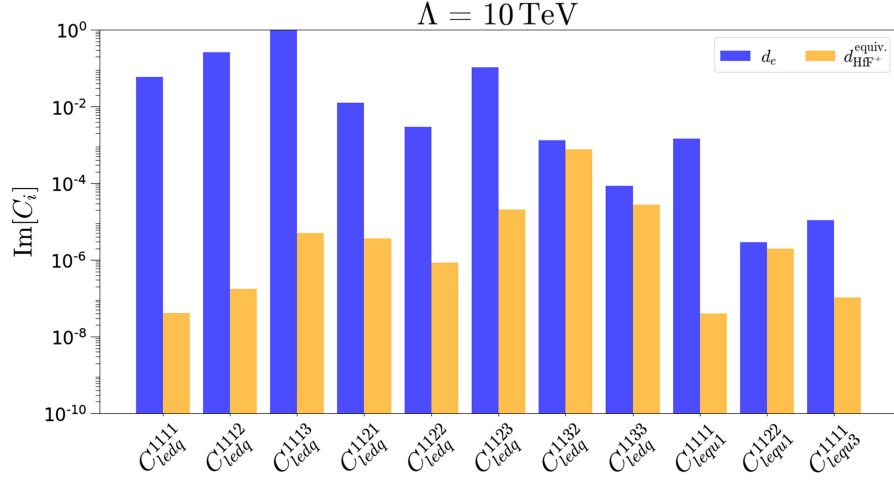


FIG. 3. Comparison between the sensitivities on the imaginary part of SMEFT WILSON coefficients from $d_{\text{HF}^+}^{\text{equiv}}$ and d_e , considering the experimental limit in Eq. (2.4) and $\Lambda = 10$ TeV as the initial scale for the running. The figure shows only the operators where the upper bound from $d_{\text{HF}^+}^{\text{equiv}}$ is stronger than from d_e .

dominates. This is due to the fact that the mixing with the dipole occurs at second order, but it is enhanced by a large anomalous dimension and features the Yukawa of the top.

Tensor operators $C_{lequ(3)}$ mix to both the eEDM and semileptonic scalar at one loop but with different anomalous dimensions. In the former case, the one-loop mixing is proportional to the Yukawa of the up-type quarks while, for the latter, the mixing occurs via electroweak gauge bosons exchange. Therefore, as a consequence of the Yukawa suppression, $d_{\text{HF}^+}^{\text{equiv}}$ is significantly more sensitive to operators with the up quark compared to d_e alone, while for operators with tops and tensor with charms C_S is negligible with respect to d_e . Note that for no operator the $d_{\text{HF}^+}^{\text{equiv}}$ sensitivity is worse than the one of d_e , implying that no flat directions arise when considering only one operator at a time. We remind the reader that we are only including perturbative contributions above 2 GeV. However, it has been shown that sub-GeV mixing with the dipole in χ PT can be significant for scalar and tensor operators involving first-generation quarks [54]. While these effects are estimated to be 1–2 orders of magnitude larger than the RGE-induced mixing above 2 GeV, they do not alter the conclusion that $d_{\text{HF}^+}^{\text{equiv}}$ remains the most sensitive observable, because the contributions to C_S are still several orders of magnitude larger.

To summarize, semileptonic scalars and tensors, with the exception of operators involving top quarks and tensors with charm quarks, are generally expected to contribute more significantly to C_S than to d_e . Scalars involving down-type quarks are primarily constrained through their contributions to C_S , since there is no dimension-six tensor operator involving down-type singlets, and mixing with the dipole operator is therefore a higher-order effect. As a result, we expect that models generating complex C_{ledq} at tree level, such as those with a singlet vector leptoquark

(see [55–59]), are constrained by paramagnetic EDM searches due to their contribution to C_S . In contrast, if new physics generates scalar or tensor operators involving heavy up-type quarks, then $d_{\text{HF}^+}^{\text{equiv}} \sim d_e$.

In general, when new physics couples to first-generation quarks, the contribution from C_S tends to dominate. Although such scenarios may be disfavored because first-generation couplings are the most constrained, it is important to emphasize that $d_{\text{HF}^+}^{\text{equiv}}$ is one of the most sensitive probes of new physics, and it can further constrain couplings that are small enough to remain compatible with other observations [which is testified by the fact that, for $\sim \mathcal{O}(1)$ couplings, contributions from scales $\Lambda \sim 10^5$ TeV can be within the experimental reach]. Finally, if the MFV assumption holds, most of the sensitivities shown in Fig. 3 would be significantly relaxed, since the built-in suppression from the electron and/or first-generation quark Yukawa couplings would allow for semileptonic interactions at the TeV scale.

A. Dimension-eight effects

Given the exceptional sensitivity of EDM experiments to NP interactions, the next to leading effects in the EFT expansion can fall within the experimental reach. These appear with dimension-eight operators, which are added to the SMEFT Lagrangian as specified in Eq. (3.1), so they are suppressed by four powers of the NP scale Λ . The complete on-shell basis for dimension-eight SMEFT operators can be found in [60,61], and we adopt the notation of Ref. [60]. In this section, we only provide estimates for the contribution of selected dimension-eight operators, because a complete analysis is beyond the scope of this work and would require the dimension-eight RGEs, which are largely unknown, although some partial results are available [62–70]. We also disregard the contribution of derivative operators, as they

are expected to be suppressed at low energies (at least at the tree level), as well as operators that differ from a dimension-six operator only by an additional $\varphi^\dagger \varphi$ factor, since it is unlikely that a UV model would generate the dimension-eight term without also generating its dimension-six counterpart.

Tensor operators involving down-type fermions—either down-type quarks or charged leptons—which are nonzero only if the chirality projector in the two-fermion bilinears is the same ($P_X \otimes P_X$, with $X = L, R$), as well as scalar operators with the same structure, appear at leading order at dimension eight. This is because gauge-invariant interactions of this type require the insertion of two Higgs fields. The relevant operators are¹¹

$$\mathcal{O}_{l^2 e^2 \varphi^2(4)}^{ijkl} = (l_i \varphi \sigma_{\alpha\beta} e_j)(l_k \varphi \sigma^{\alpha\beta} e_l), \quad (3.6)$$

$$\mathcal{O}_{leq d \varphi^2(3)}^{ijkl} = (l_i \varphi e_j)(q_k \varphi d_l) \quad (3.7)$$

$$\mathcal{O}_{leq d \varphi^2(4)}^{ijkl} = (l_i \varphi \sigma_{\alpha\beta} e_j)(q_k \varphi \sigma^{\alpha\beta} d_l), \quad (3.8)$$

and their respective Hermitian conjugates. The tensor operator will mix with the dimension-eight dipoles with a Yukawa suppressed mixing and contribute to the eEDM as

$$d_e \sim \frac{C_T}{16\pi^2} \frac{m_f}{\Lambda^2} \frac{v^2}{\Lambda^2} \log\left(\frac{\Lambda}{m_f}\right), \quad (3.9)$$

where C_T is the dimension-eight tensor coefficient, and m_f is the mass of the respective fermion running in the loop. As in the dimension-six case, the tensor operator can also mix with the associated scalar via gauge boson exchanges, which, in turn, will match onto four-fermion interactions with a v^2/Λ^2 suppression when the electroweak symmetry is broken¹² and contribute to the semileptonic interaction of the equivalent EDM as described in Eq. (2.15). The tensors with quarks lighter than the strange contribute predominantly to the semileptonic operator by mixing with the scalar via gauge interactions, while the tensors with bottom quarks and τ leptons mainly contribute to the dipole as estimated in Eq. (3.9). By analogy with dimension-six operators involving up-type quarks, dimension-eight scalar operators with bottom quarks are expected to contribute comparably to both the semileptonic operator and the dipole, with the latter arising via an intermediate mixing with the tensor. Another class of four-fermion operators that

are not present at dimension six are scalars with up quarks having the chirality structure $P_R \otimes P_L$,

$$C_{lequ\varphi^2(5)}^{ijkl} = (\bar{l}_i \varphi e_j)(\bar{u}_k \tilde{\varphi}^\dagger q_l), \quad (3.10)$$

and the respective Hermitian conjugates. These operators will match onto C_S because they give tree-level contributions to the scalars of Eq. (2.15) with up-type quarks. Note that contrary to the dimension-six case, the scalar $C_{lequ\varphi^2(5)}^{eett}$ should contribute more to C_S than d_e , because there is no tensor with the same chirality structure to mix with as an intermediate step.

Finally, the operator

$$\mathcal{O}_{leG^2\varphi}^{ee} = (\bar{l}_e \varphi e) G_{\mu\nu}^A G^{A\mu\nu} \quad (3.11)$$

matches onto the gluon operator $\mathcal{O}_{eG}$ of Sec. II A when the Higgs acquires a VEV and contributes to the semileptonic C_S at the QCD confinement scale as described by Eq. (2.14).

Additional dimension-eight effects include flavor-changing operators in the up-quark and lepton sectors that contribute to the EDM.¹³ This arises because the operators $\mathcal{O}_{e\varphi}$ and $\mathcal{O}_{u\varphi}$ modify fermion masses when electroweak symmetry is broken, leading to nondiagonal Yukawa matrices in the mass eigenstate basis. Consequently, if the SMEFT is initially defined in a basis where up-type quark or charged-lepton Yukawas are diagonal, matching onto low-energy interactions requires rotating the fields to their mass eigenstates, potentially introducing flavor violation. The rotation angles are determined by the size of the $\mathcal{O}_{\psi\varphi}$ coefficient, allowing flavor-changing operators to induce contributions to the flavor-conserving interactions relevant for the equivalent EDM. For instance, if at the electroweak scale both the operator $C_{lequ(1)}^{eeij}$ and $C_{u\varphi}^{ji}$ are present, then

$$C_{lequ(1)}^{eejj}(m_W) \sim C_{lequ(1)}^{eeij} C_{u\varphi}^{ji} \frac{v^3}{(\max(m_i, m_j) \Lambda^2)}. \quad (3.12)$$

Neglecting terms suppressed by small Yukawa couplings, the rotation angles of the quark doublets are proportional to the matrix elements $C_{u\varphi}^{ij}$ with $j > i$. In contrast, the mixing of right-handed up-type quarks, which is not suppressed by Yukawa couplings, is governed by the size of the lower-left block, i.e., the elements $C_{u\varphi}^{ji}$ with $j > i$. Note that this rotation is formally a double insertion of dimension-six operators, hence a consistent treatment of these contributions may also require calculating the

¹¹The tensor operators with leptons that we consider here are Fierz equivalent to a combination of scalars $\mathcal{O}_{l^2 e^2 \varphi^2(3)}^{ijkl} = (l_i \varphi e_j)(l_k \varphi e_l)$ present in the basis of Ref. [60].

¹²The full tree-level matching at $\mathcal{O}(1/\Lambda^4)$ between SMEFT and the low-energy EFT can be found in [71].

¹³We do not discuss down quarks because in the up basis the down Yukawa features CKM mixing which clearly dominates over the dimension-eight effects.

TABLE III. Estimated sensitivities of $d_{\text{HfF}^+}^{\text{equiv}}$ on the imaginary part of dimension-eight coefficients or double insertion of dimension-six operators, with operators normalized at $\Lambda = 10$ TeV. The index i can take values $i = 2, 3$ while $j = 1, 2$.

Operator	$d_{\text{HfF}^+}^{\text{equiv}}$ sensitivity on $\text{Im}(C)$
$C_{l^2 e^2 q^2}^{\text{eerr}}(4)$	$\lesssim 10^{-4}$
$C_{leq d q^2}^{\text{eedd}}(3)$	$\lesssim 10^{-5}$
$C_{leq d q^2}^{\text{eedd}}(4)$	$\lesssim 10^{-4}$
$C_{leq d q^2}^{\text{eess}}(3)$	$\lesssim 10^{-3}$
$C_{leq d q^2}^{\text{eess}}(4)$	$\lesssim 10^{-2}$
$C_{leq d q^2}^{\text{eebb}}(3)$	$\lesssim 10^{-2}$
$C_{leq d q^2}^{\text{eebb}}(4)$	$\lesssim 10^{-5}$
$C_{leq u q^2}^{\text{eeuu}}(5)$	$\lesssim 10^{-5}$
$C_{leq u q^2}^{\text{eecc}}(5)$	$\lesssim 10^{-3}$
$C_{leq u q^2}^{\text{eett}}(5)$	$\lesssim 10^{-1}$
$C_{le G^2 q}^{\text{ee}}$	$\lesssim 10^{-2}$
$C_{lequ(1)}^{\text{eei1}} \sim C_{lequ(1)}^{\text{eei1}} C_{u\varphi}^{li}, C_{lequ(3)}^{\text{eei1}} \sim C_{lequ(3)}^{\text{eei1}} C_{u\varphi}^{li}$	$\lesssim 10^{-5}/y_{u_i}$
$C_{lequ(1)}^{\text{eei1}} \sim C_{lequ(1)}^{\text{eei1}} C_{u\varphi}^{li}, C_{lequ(3)}^{\text{eei1}} \sim C_{lequ(3)}^{\text{eei1}} C_{u\varphi}^{li}$	$\lesssim 10^{-4}/y_{u_i}$
$C_{lequ(1)}^{\text{ee12}} \sim C_{lequ(1)}^{\text{ee12}} C_{u\varphi}^{21}, C_{lequ(3)}^{\text{ee12}} \sim C_{lequ(3)}^{\text{ee12}} C_{u\varphi}^{21}$	$\lesssim 10^{-3}/y_c$
$C_{lequ(1)}^{\text{ee22}} \sim C_{lequ(1)}^{\text{ee22}} C_{u\varphi}^{21}, C_{lequ(3)}^{\text{ee22}} \sim C_{lequ(3)}^{\text{ee22}} C_{u\varphi}^{21}$	$\lesssim 10^{-5}/y_c$
$C_{lequ(1)}^{\text{ee22}} \sim C_{lequ(1)}^{\text{ee22}} C_{u\varphi}^{32}, C_{lequ(3)}^{\text{ee22}} \sim C_{lequ(3)}^{\text{ee22}} C_{u\varphi}^{32}$	$\lesssim 10^{-3}/y_t$
$C_{lequ(1)}^{\text{ee22}} \sim C_{lequ(1)}^{\text{ee22}} C_{u\varphi}^{32}, C_{lequ(3)}^{\text{ee22}} \sim C_{lequ(3)}^{\text{ee22}} C_{u\varphi}^{32}$	$\lesssim 10^{-5}/y_t$
$C_{lequ(1)}^{\text{eej3}} \sim C_{lequ(1)}^{\text{eej3}} C_{u\varphi}^{3j}, C_{lequ(3)}^{\text{eej3}} \sim C_{lequ(3)}^{\text{eej3}} C_{u\varphi}^{3j}$	$\lesssim 10^{-4}/y_t$
$C_{lequ(1)}^{\text{eej3}} \sim C_{lequ(1)}^{\text{eej3}} C_{u\varphi}^{3j}, C_{lequ(3)}^{\text{eej3}} \sim C_{lequ(3)}^{\text{eej3}} C_{u\varphi}^{3j}$	$\lesssim 10^{-7}/y_t$

dimension-eight RGEs proportional to the product of two dimension-six operators. In the charged lepton sector, a similar rotation should be performed in the presence of $\mathcal{O}_{e\varphi}^{ij}$, with $i \neq j$. However, we do not consider lepton flavor-violating operators in this work, which we aim to explore in a future project. The estimated sensitivities on the dimension-eight coefficients and on dimension-six flavor-changing operators in the up quark sector are summarized in Table III.

IV. CONCLUSION

The search for sources of CP violation beyond the Standard Model is essential for shedding light on the mechanism behind baryogenesis. At the present time, one of the best places to look for such effects is in electron EDM experiments, which can probe NP scales up to $\sim 10^7$ TeV. Currently, the best upper limit on the eEDM is given by measurements on paramagnetic molecules.

However, these experiments are sensitive to a linear combination of CP -violating interactions known as the equivalent EDM d^{equiv} , which contains the eEDM and the Wilson coefficient C_S of a semileptonic operator with electrons and nuclei. We review the definition of d^{equiv} in Sec. II for different paramagnetic molecules.

In this work, we study the contributions of higher-dimensional SMEFT operators to the equivalent EDM of HfF^+ molecular ions, which currently has the most stringent upper limit $|d_{\text{HfF}^+}^{\text{equiv}}| < 4.1 \times 10^{-30} e \cdot \text{cm}$. To do so, we consider one SMEFT operator at a time for a fixed initial scale of $\Lambda = 10$ TeV, which we run down to the scale of the experiments to compute the low-energy contributions. The one-loop RGE running, as well as all the matching between the different effective theories when crossing the SM particle masses, has been performed with the public package WILSON. Selected two-loop running effects are also included. The matching at the QCD confinement scale between quark- and nucleon-level operators relevant for the semileptonic interactions entering in d^{equiv} is given in Eq. (2.15) of Sec. II A, where the effects of heavy quarks have also been included.

We divide the discussion of the results (Sec. III) in two parts. First, we show in Fig. 2 the sensitivity on classes of operators where the eEDM dominates over C_S . Our results are compatible with previous SMEFT analysis of the eEDM. The new results of this work are summarized in Fig. 3, which presents the single-operator limits on the coefficients of those operators that contribute more significantly to d_e^{equiv} than to d_e alone. Semileptonic scalars and tensors are found to give larger contribution to C_S than d_e , with the only exception of operators with top quarks and tensors with charms.

Finally, in Sec. III A, we discuss selected dimension-eight effects, which, given the impressive sensitivities of EDM experiments, are estimated to be within the reach of the experimental searches.

ACKNOWLEDGMENTS

We thank Sacha Davidson for her useful comments on the manuscript. We acknowledge financial support from the Spanish Grant No. PID2023-151418NB-I00 funded by MCIU/AEI/10.13039/501100011033/ FEDER, UE and from Generalitat Valenciana Projects No. CIPROM/2021/054 and No. CIPROM/2022/66.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

- [1] M. B. Gavela, P. Hernandez, J. Orloff, and O. Pene, Standard model CP violation and baryon asymmetry, *Mod. Phys. Lett. A* **9**, 795 (1994).
- [2] P. Huet and E. Sather, Electroweak baryogenesis and standard model CP violation, *Phys. Rev. D* **51**, 379 (1995).
- [3] C. Jarlskog, Commutator of the quark mass matrices in the standard electroweak model and a measure of maximal CP nonconservation, *Phys. Rev. Lett.* **55**, 1039 (1985).
- [4] Particle Data Group, Review of particle physics, *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
- [5] M. Pospelov and A. Ritz, CKM benchmarks for electron electric dipole moment experiments, *Phys. Rev. D* **89**, 056006 (2014).
- [6] Y. Yamaguchi and N. Yamanaka, Large long-distance contributions to the electric dipole moments of charged leptons in the standard model, *Phys. Rev. Lett.* **125**, 241802 (2020).
- [7] Y. Yamaguchi and N. Yamanaka, Quark level and hadronic contributions to the electric dipole moment of charged leptons in the standard model, *Phys. Rev. D* **103**, 013001 (2021).
- [8] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, The fate of hints: Updated global analysis of three-flavor neutrino oscillations, *J. High Energy Phys.* **09** (2020) 178.
- [9] F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri, and A. Palazzo, Unfinished fabric of the three neutrino paradigm, *Phys. Rev. D* **104**, 083031 (2021).
- [10] A. Hiramoto *et al.*, SiPM module for the ACME III electron EDM search, *Nucl. Instrum. Methods Phys. Res., Sect. A* **1045**, 167513 (2023).
- [11] N. J. Fitch, J. Lim, E. A. Hinds, B. E. Sauer, and M. R. Tarbutt, Methods for measuring the electron's electric dipole moment using ultracold ybf molecules, *Quantum Sci. Technol.* **6**, 014006 (2020).
- [12] A. C. Vutha, M. Horbatsch, and E. A. Hessels, Orientation-dependent hyperfine structure of polar molecules in a rare-gas matrix: A scheme for measuring the electron electric dipole moment, *Phys. Rev. A* **98**, 032513 (2018).
- [13] M. Sakurai *et al.*, muEDM: Towards a search for the muon electric dipole moment at PSI using the Frozen-spin technique, *J. Phys. Soc. Jpn. Conf. Proc.* **37**, 020604 (2022).
- [14] nEDM Collaboration, The design of the nEDM experiment: nEDM Collaboration, *Eur. Phys. J. C* **81**, 512 (2021).
- [15] T. S. Roussy, L. Caldwell, T. Wright, W. B. Cairncross, Y. Shagam, K. B. Ng *et al.*, An improved bound on the electron's electric dipole moment, *Science* **381**, 46 (2023).
- [16] Muon (g-2) Collaboration, An improved limit on the muon electric dipole moment, *Phys. Rev. D* **80**, 052008 (2009).
- [17] Belle Collaboration, An improved search for the electric dipole moment of the τ lepton, *J. High Energy Phys.* **04** (2022) 110.
- [18] C. Abel *et al.*, Measurement of the permanent electric dipole moment of the neutron, *Phys. Rev. Lett.* **124**, 081803 (2020).
- [19] J. Aebischer, W. Dekens, E. E. Jenkins, A. V. Manohar, D. Sengupta, and P. Stoffer, Effective field theory interpretation of lepton magnetic and electric dipole moments, *J. High Energy Phys.* **07** (2021) 107.
- [20] J. Kley, T. Theil, E. Venturini, and A. Weiler, Electric dipole moments at one-loop in the dimension-6 SMEFT, *Eur. Phys. J. C* **82**, 926 (2022).
- [21] G. Panico, A. Pomarol, and M. Riembau, EFT approach to the electron electric dipole moment at the two-loop level, *J. High Energy Phys.* **04** (2019) 090.
- [22] W. Dekens, J. de Vries, M. Jung, and K. K. Vos, The phenomenology of electric dipole moments in models of scalar leptoquarks, *J. High Energy Phys.* **01** (2019) 069.
- [23] H. Mulder, R. Timmermans, and J. de Vries, Probing the QCD $\bar{\theta}$ term with paramagnetic molecules, *J. High Energy Phys.* **07** (2025) 232.
- [24] C. Baruch, P. B. Changala, Y. Shagam, and Y. Soreq, Constraining CP violating nucleon-nucleon long-range interactions in diatomic eEDM searches, *Phys. Rev. Lett.* **133**, 113202 (2024).
- [25] M. Pospelov and A. Ritz, Electric dipole moments as probes of new physics, *Ann. Phys. (Amsterdam)* **318**, 119 (2005).
- [26] T. Fleig, $\mathcal{P}$, $\mathcal{T}$ -odd and magnetic hyperfine-interaction constants and excited-state lifetime for hff^+ , *Phys. Rev. A* **96**, 040502 (2017).
- [27] T. Chupp, P. Fierlinger, M. Ramsey-Musolf, and J. Singh, Electric dipole moments of atoms, molecules, nuclei, and particles, *Rev. Mod. Phys.* **91**, 015001 (2019).
- [28] A. N. Petrov, N. S. Mosyagin, T. A. Isaev, and A. V. Titov, Theoretical study of Hff^+ in search of the electron electric dipole moment, *Phys. Rev. A* **76**, 030501 (2007).
- [29] T. Fleig and M. K. Nayak, Electron electric-dipole-moment interaction constant for hff^+ from relativistic correlated all-electron theory, *Phys. Rev. A* **88**, 032514 (2013).
- [30] L. V. Skripnikov, Communication: Theoretical study of HfF^+ cation to search for the T,P-odd interactions, *J. Chem. Phys.* **147**, 021101 (2017).
- [31] Y. Ema, T. Gao, and M. Pospelov, Standard model prediction for paramagnetic electric dipole moments, *Phys. Rev. Lett.* **129**, 231801 (2022).
- [32] E. R. Meyer and J. L. Bohn, Prospects for an electron electric-dipole moment search in metastable ThO and ThF^+ , *Phys. Rev. A* **78**, 010502 (2008).
- [33] V. A. Dzuba, V. V. Flambaum, and C. Harabati, Relations between matrix elements of different weak interactions and interpretation of the parity-nonconserving and electron electric-dipole-moment measurements in atoms and molecules, *Phys. Rev. A* **84**, 052108 (2011).
- [34] L. V. Skripnikov, A. N. Petrov, and A. V. Titov, Communication: Theoretical study of ThO for the electron electric dipole moment search, *J. Chem. Phys.* **139**, 221103 (2013).
- [35] ACME Collaboration, Improved limit on the electric dipole moment of the electron, *Nature (London)* **562**, 355 (2018).
- [36] S. Davidson, Y. Kuno, Y. Uesaka, and M. Yamanaka, Probing $\mu e \gamma \gamma$ contact interactions with $\mu \rightarrow e$ conversion, *Phys. Rev. D* **102**, 115043 (2020).
- [37] V. Cirigliano, R. Kitano, Y. Okada, and P. Tuzon, On the model discriminating power of $\mu \rightarrow e$ conversion in nuclei, *Phys. Rev. D* **80**, 013002 (2009).
- [38] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Remarks on Higgs boson interactions with nucleons, *Phys. Lett.* **78B**, 443 (1978).

- [39] S. Davidson, Completeness and complementarity for $\mu \rightarrow e\gamma\mu \rightarrow e\bar{e}e$ and $\mu A \rightarrow eA$, *J. High Energy Phys.* **02** (2021) 172.
- [40] M. Hoferichter, J. Ruiz de Elvira, B. Kubis, and U.-G. Meißner, High-precision determination of the pion-nucleon σ term from Roy-Steiner equations, *Phys. Rev. Lett.* **115**, 092301 (2015).
- [41] J. M. Alarcon, J. Martin Camalich, and J. A. Oller, The chiral representation of the πN scattering amplitude and the pion-nucleon sigma term, *Phys. Rev. D* **85**, 051503 (2012).
- [42] P. Junnarkar and A. Walker-Loud, Scalar strange content of the nucleon from lattice QCD, *Phys. Rev. D* **87**, 114510 (2013).
- [43] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, Dimension-six terms in the standard model Lagrangian, *J. High Energy Phys.* **10** (2010) 085.
- [44] W. Buchmuller and D. Wyler, Effective Lagrangian analysis of new interactions and flavor conservation, *Nucl. Phys.* **B268**, 621 (1986).
- [45] R. Alonso, E. E. Jenkins, A. V. Manohar, and M. Trott, Renormalization group evolution of the standard model dimension six operators III: Gauge coupling dependence and phenomenology, *J. High Energy Phys.* **04** (2014) 159.
- [46] E. E. Jenkins, A. V. Manohar, and M. Trott, Renormalization group evolution of the standard model dimension six operators I: Formalism and lambda dependence, *J. High Energy Phys.* **10** (2013) 087.
- [47] E. E. Jenkins, A. V. Manohar, and M. Trott, Renormalization group evolution of the standard model dimension six operators II: Yukawa dependence, *J. High Energy Phys.* **01** (2014) 035.
- [48] J. Aebischer, J. Kumar, and D. M. Straub, WILSON: A Python package for the running and matching of WILSON coefficients above and below the electroweak scale, *Eur. Phys. J. C* **78**, 1026 (2018).
- [49] F. Boudjema, K. Hagiwara, C. Hamzaoui, and K. Numata, Anomalous moments of quarks and leptons from nonstandard $WW\gamma$ couplings, *Phys. Rev. D* **43**, 2223 (1991).
- [50] B. Gripaios and D. Sutherland, Searches for cp -violating dimension-6 electroweak gauge boson operators, *Phys. Rev. D* **89**, 076004 (2014).
- [51] S. Herrlich and U. Nierste, Evanescent operators, scheme dependences and double insertions, *Nucl. Phys.* **B455**, 39 (1995).
- [52] M. Ciuchini, E. Franco, L. Reina, and L. Silvestrini, Leading order QCD corrections to $b \rightarrow s\gamma$ and $b \rightarrow sg$ decays in three regularization schemes, *Nucl. Phys.* **B421**, 41 (1994).
- [53] S. M. Barr and A. Zee, Electric dipole moment of the electron and of the neutron, *Phys. Rev. Lett.* **65**, 21 (1990).
- [54] W. Dekens, E. E. Jenkins, A. V. Manohar, and P. Stoffer, Non-perturbative effects in $\mu \rightarrow e\gamma$, *J. High Energy Phys.* **01** (2019) 088.
- [55] J. Aebischer, G. Isidori, M. Pesut, B. A. Stefanek, and F. Wilsch, Confronting the vector leptoquark hypothesis with new low- and high-energy data, *Eur. Phys. J. C* **83**, 153 (2023).
- [56] J. Fuentes-Martín, G. Isidori, M. König, and N. Selimović, Vector leptoquarks beyond tree level, *Phys. Rev. D* **101**, 035024 (2020).
- [57] J. Fuentes-Martín, G. Isidori, M. König, and N. Selimović, Vector leptoquarks beyond tree level. II. $\mathcal{O}(\alpha_s)$ corrections and radial modes, *Phys. Rev. D* **102**, 035021 (2020).
- [58] C. Cornella, J. Fuentes-Martín, and G. Isidori, Revisiting the vector leptoquark explanation of the B-physics anomalies, *J. High Energy Phys.* **07** (2019) 168.
- [59] I. Doršner, S. Fajfer, A. Greljo, J. F. Kamenik, and N. Košnik, Physics of leptoquarks in precision experiments and at particle colliders, *Phys. Rep.* **641**, 1 (2016).
- [60] C. W. Murphy, Dimension-8 operators in the standard model effective field theory, *J. High Energy Phys.* **10** (2020) 174.
- [61] H.-L. Li, Z. Ren, J. Shu, M.-L. Xiao, J.-H. Yu, and Y.-H. Zheng, Complete set of dimension-eight operators in the standard model effective field theory, *Phys. Rev. D* **104**, 015026 (2021).
- [62] L. Vale Silva, Effects of squared four-fermion operators of the standard model effective field theory on meson mixing, *Phys. Rev. D* **110**, 016006 (2024).
- [63] M. Chala, Constraints on anomalous dimensions from the positivity of the S matrix, *Phys. Rev. D* **108**, 015031 (2023).
- [64] M. Chala and X. Li, Positivity restrictions on the mixing of dimension-eight SMEFT operators, *Phys. Rev. D* **109**, 065015 (2024).
- [65] S. D. Bakshi, M. Chala, A. Díaz-Carmona, Z. Ren, and F. Vilches, Renormalization of the SMEFT to dimension eight: Fermionic interactions I, *J. High Energy Phys.* **12** (2025) 214.
- [66] S. Das Bakshi, M. Chala, A. Díaz-Carmona, and G. Guedes, Towards the renormalisation of the Standard Model effective field theory to dimension eight: Bosonic interactions II, *Eur. Phys. J. Plus* **137**, 973 (2022).
- [67] M. Chala, G. Guedes, M. Ramos, and J. Santiago, Towards the renormalisation of the standard model effective field theory to dimension eight: Bosonic interactions I, *SciPost Phys.* **11**, 065 (2021).
- [68] S. Davidson and M. Gorbahn, Charged lepton flavor change and nonstandard neutrino interactions, *Phys. Rev. D* **101**, 015010 (2020).
- [69] M. Ardu, S. Davidson, and M. Gorbahn, Sensitivity of $\mu \rightarrow e$ processes to τ flavor change, *Phys. Rev. D* **105**, 096040 (2022).
- [70] R. Boughezal, Y. Huang, and F. Petriello, Renormalization-group running of dimension-8 four-fermion operators in the SMEFT, *Phys. Rev. D* **110**, 116015 (2024).
- [71] S. Hamoudou, J. Kumar, and D. London, Dimension-8 SMEFT matching conditions for the low-energy effective field theory, *J. High Energy Phys.* **03** (2023) 157.