

Scalar, vector, and tensor form factors of pion and kaon

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(Received 6 May 2025; accepted 16 June 2025; published 30 June 2025)

We calculate the possible scalar, vector, and tensor form factors (FFs) for pion and kaon in the light-cone quark model (LCQM). These FFs are calculated from the leading and subleading twist generalized parton distribution functions (GPDs) of the pion and kaon. The vector and tensor FFs correspond to twist-2 GPDs, whereas the scalar FFs correspond to twist-3 scalar GPDs. We calculate the FFs from the GPDs quark-quark correlator and express them in the form of light-front wave functions. The behavior of these FFs was found to be in agreement with experimental data, other model predictions, and lattice simulation results. We have also calculated the charge radii corresponding to different FFs and compared them with other models, lattice simulation results, and experimental data. The scalar radii of the pion and kaon of our model are found to be 0.528 and 0.409 fm, respectively.

DOI: [10.1103/2wpb-jgkc](https://doi.org/10.1103/2wpb-jgkc)

I. INTRODUCTION

Being the lightest bound states in the hadron spectrum, the pion and kaon mesons are essential to comprehend the strong interaction in the nonperturbative domain of QCD [1,2]. They are pseudo-Goldstone bosons that originate from the spontaneous breaking of chiral symmetry and provide a novel perspective on the dynamical mass creation and confinement mechanisms in QCD [3,4]. Moreover, pions and kaons play a crucial role in explaining the long-range dynamics of the strong interactions. Understanding the internal structure of these hadrons is essential to provide a deeper insight into them and can be studied using multidimensional distribution functions like generalized transverse momentum parton distribution functions (GTMDs) [5–7], generalized parton distributions (GPDs) [7–12], transverse momentum parton distribution functions (TMDs) [13–18], parton distribution functions (PDFs) [19–21], electromagnetic form factors (EMFFs) [22–24], etc. GTMDs are called the mother distributions as they carry all the information about each degree of freedom of the hadron. The three-dimensional GPDs and TMDs carry information about the spatial and transverse structure along with the longitudinal momentum fraction of the valence quark inside a hadron. The quark GPDs are functions of

longitudinal momentum fraction (x), skewness parameter (ξ), and transverse momentum transferred $\Delta_{\perp}^2$, while the TMDs are the functions of x and transverse momentum of quark $\mathbf{k}_{\perp}$. The one-dimensional PDFs carry information about the longitudinal momentum fraction (x) carried by the valence quark from the hadron momenta [19–21]. Therefore, to understand a complete picture of the internal structure of a hadron, one needs to study all these distribution functions nonperturbatively using low-energy models. These distributions can further be evolved to a high-energy scale to compare with experimental observables with a proper evolution toolkit like Dokshitzer-Gribov-Lipatov-Altarelli-Parisi, Collins-Soper-Sterman, Balitsky-Fadin-Kuraev-Lipatov, etc. formalisms [25–27].

In this work, we have mainly focused on the form factors (FFs) of pion and kaon extracted from GPDs. This work is an extension of our previous study of vector FFs of pion and kaon [28]. There are a total of eight GPDs for pseudoscalar mesons up to twist 4 [29]. The vector and tensor FFs are the result of leading twist unpolarized $F_1(x, \xi, -\Delta_{\perp}^2)$ and transversely polarized $H_1(x, \xi, -\Delta_{\perp}^2)$ GPDs, respectively, while the scalar FF is derived from the scalar $E_2(x, \xi, -\Delta_{\perp}^2)$ GPD, which is twist 3 in nature. The coupling of a photon to the meson is described by the widely studied EMFFs, which are the pion and kaon vector FFs and can be accessed directly through experiments [30–42]. However, there is still a lack of information about this FF at a high- Q^2 region. The pion vector FF has been investigated through lattice QCD simulations [43–45], light-front holographic model [46], anti-de Sitter/QCD [11], light-cone quark model (LCQM) [28], Nambu–Jona-Lasinio model [47], contact interaction [48], etc. The kaon vector FF has been explored through experiments [40,41] and lattice simulations [49,50] and in different

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theoretical models [28,51,52]. The upcoming electron-ion collider (EIC) and upgraded 22 GeV Jefferson Lab (JLAB) E12-09-001 experiments aim to provide more insight into the vector FFs of pions and kaons [53,54]. The EIC is going to study the vector FFs of pion and kaon through the Sullivan process [55]. In contrast, scalar and tensor FFs have remained relatively unexplored due to their lack of direct experimental accessibility and have not received sufficient attention. Similarly, very limited work has been reported for tensor and scalar FFs in theoretical models [56]. Notwithstanding, these two FFs still deserve scrutiny. The scalar FF of the pion plays a significant role in hadron and nuclear physics, particularly due to its connection with low-energy $\pi\pi$ scattering processes. It is especially relevant in the isoscalar S-wave channel, where it provides insight into the dynamics of chiral symmetry breaking [56,57]. The scalar FF can be used to extract the pion's scalar radius and to constrain low-energy constants within chiral perturbation theory. The study of tensor FFs not only provides deeper insights into the pion and kaon internal dynamics, but it also provides information about the dipolelike distortion of the quark density in the transverse plane and can be used to study the gravitational FFs [58].

In this work, we have solved the quark-quark correlator for unpolarized and polarized valence quark GPDs to calculate the respective FFs using the LCQM. As we are mainly focusing on the comparative analysis of valence quarks, we have not considered the gluon in the present work. We have presented the FFs in the overlap form of light-front wave functions (LFWFs) and also in the explicit form by using the total wave function (spin and momentum space wave function). For a comprehensive understanding, we have plotted the FFs of pion, kaon, and their constituents as a function of Q^2 along with a comparison with available experimental data, lattice simulation data, and theoretical predictions. We have also evolved the FFs to high Q^2 using next-to-leading-order (NLO) evolutions [59]. The vector, tensor, and scalar radii have also been calculated from the respective FFs and compared with available data for both the pion and kaon. For this work, we have calculated the FFs using the minimal Fock-state of LCQM [60,61]. Being gauge invariant and relativistic by nature, LCQM is a nonperturbative method. Its primary focus is on valence quarks, which are the essential building blocks that determine the general structure and inherent characteristics of hadrons.

The paper is arranged as follows. In Sec. II, we discuss the LCQM along with the spin and momentum wave function in LF formalism. The relations among the FFs and GPDs along with the GPDs quark-quark correlator are discussed in Sec. II B. The overlap form with explicit expressions for all FFs are presented in this section, while in Sec. III, we present the plots for each FFs. The radii corresponding to each FFs are also calculated in this section. We finally summarize our work in Sec. IV.

II. METHODOLOGY

A. Light-cone quark model

The light-cone (LC) formalism provides a helpful framework for relativistically characterizing the hadrons in terms of quark and gluon degrees of freedom. The mesonic wave function can be expressed using the LC Fock-state expansion as [1,62,63]

$$|M\rangle = \sum |q\bar{q}\rangle\psi_{q\bar{q}} + \sum |q\bar{q}g\rangle\psi_{q\bar{q}g} + \sum |q\bar{q}gg\rangle\psi_{q\bar{q}gg} + \dots, \quad (1)$$

where $|M\rangle$ denotes the meson eigenstate. For this work, we have considered only the leading-order eigenstates of mesons having a quark-antiquark pair. However, the higher Fock-state contribution will provide a complete understanding of the gluons and sea quarks inside a meson [63]. The mesonic wave function based on the LC quantization of QCD using two particle Fock-state expansion can be expressed as [64–66]

$$|\pi^+(K^+)(P, S_z = 0)\rangle = \sum_{\lambda_q, \lambda_{\bar{q}}} \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3 \sqrt{x(1-x)}} \Psi_{\pi(K)} \times (x, \mathbf{k}_\perp, \lambda_q, \lambda_{\bar{q}}) |xP^+, \mathbf{k}_\perp, \lambda_q, \lambda_{\bar{q}}\rangle. \quad (2)$$

Here, $|\pi^+(K^+)(P, S_z = 0)\rangle$ is the eigenstate of pion (kaon) with four-vector momenta $P = (P^+, P^-, P_\perp)$ with S_z being the longitudinal spin projection. $\mathbf{k} = (\mathbf{k}^+, \mathbf{k}^-, \mathbf{k}_\perp)$ is the four-vector momenta of the constituent quark with helicity λ_q . $x = \mathbf{k}^+/P^+$ and $1-x$ are the longitudinal momentum fraction of the constituent quark and antiquark. $\Psi_{\pi(K)}(x, \mathbf{k}_\perp, \lambda_q, \lambda_{\bar{q}})$ is the LC total wave function, which can be expanded in terms of momentum space wave function and spin wave function with different spin and helicity projections can be expressed as

$$\Psi_{\pi(K)}(x, \mathbf{k}_\perp, \lambda_q, \lambda_{\bar{q}}) = J_{S_z}(x, \mathbf{k}_\perp, \lambda_q, \lambda_{\bar{q}}) \psi_{\pi(K)}(x, \mathbf{k}_\perp). \quad (3)$$

Here, $J_{S_z}(x, \mathbf{k}_\perp, \lambda_q, \lambda_{\bar{q}})$ and $\psi_{\pi(K)}(x, \mathbf{k}_\perp)$ are the spin and momentum space wave functions of the mesons, respectively. The momentum space wave function in Eq. (3) can be expressed using quark masses and quark momentum through Brodsky-Huang-Lepage [60,65] as

$$\psi_{\pi(K)}(x, \mathbf{k}_\perp) = A_{\pi(K)} \exp \left[-\frac{\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}{8\beta_{\pi(K)}^2} - \frac{(m_q^2 - m_{\bar{q}}^2)^2}{8\beta_{\pi(K)}^2 \left(\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x} \right)} + \frac{(m_q^2 + m_{\bar{q}}^2)}{4\beta_{\pi(K)}^2} \right], \quad (4)$$

$m_{q(\bar{q})}$ are the masses of quark and antiquark of the meson, where $A_{\pi(K)}$ and $\beta_{\pi(K)}$ are the normalization constant and harmonic scale parameter of the pion (kaon), respectively. The normalization constant can be calculated through the normalizing condition as

$$\int \frac{dx d^2 \mathbf{k}_\perp}{2(2\pi)^3} |\psi_{\pi(K)}(x, \mathbf{k}_\perp)|^2 = 1. \quad (5)$$

$J_{S_z}(x, \mathbf{k}_\perp, \lambda_q, \lambda_{\bar{q}})$ in Eq. (3) is the front-form spin wave function of pseudoscalar mesons. This spin wave function can be calculated by deriving from the instant form by Melosh-Wigner rotation [60,65,67] or can be calculated by using the proper vertex chosen for the meson [65,68]. Both the above methods give rise to the same form of spin wave function. The spin wave function for pseudoscalar mesons ($S_z = 0$) with different helicities is expressed as

$$\begin{cases} J_{(S_z=0)}(x, \mathbf{k}_\perp, \uparrow, \uparrow) = \frac{1}{\sqrt{2}} \omega_{\pi(K)}^{-1} (-\mathbf{k}_{1\perp} + i\mathbf{k}_{2\perp}) (\mathcal{M}_{\pi(K)} + m_q + m_{\bar{q}}), \\ J_{(S_z=0)}(x, \mathbf{k}_\perp, \uparrow, \downarrow) = \frac{1}{\sqrt{2}} \omega_{\pi(K)}^{-1} ((1-x)m_q + xm_{\bar{q}}) (\mathcal{M}_{\pi(K)} + m_q + m_{\bar{q}}), \\ J_{(S_z=0)}(x, \mathbf{k}_\perp, \downarrow, \uparrow) = \frac{1}{\sqrt{2}} \omega_{\pi(K)}^{-1} (-(1-x)m_q - xm_{\bar{q}}) (\mathcal{M}_{\pi(K)} + m_q + m_{\bar{q}}), \\ J_{(S_z=0)}(x, \mathbf{k}_\perp, \downarrow, \downarrow) = \frac{1}{\sqrt{2}} \omega_{\pi(K)}^{-1} (\mathbf{k}_{1\perp} + i\mathbf{k}_{2\perp}) (\mathcal{M}_{\pi(K)} + m_q + m_{\bar{q}}), \end{cases} \quad (6)$$

with $\omega_{\pi(K)} = (\mathcal{M}_{\pi(K)} + m_q + m_{\bar{q}}) \sqrt{x(1-x)[\mathcal{M}_{\pi(K)}^2 - (m_q - m_{\bar{q}})^2]}$. Here, $\mathcal{M}_{\pi(K)}$ is the invariant mass of the pion (kaon) defined in terms of its quark mass m_q and antiquark mass $m_{\bar{q}}$ as

$$\mathcal{M}_{\pi(K)}^2 = \frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x}. \quad (7)$$

The four-vector average momenta of the meson (P), constituent quark (k_q), and antiquark ($k_{\bar{q}}$) in the LC frame are, respectively, defined as

$$P \equiv \left(P^+, \frac{\mathcal{M}_{\pi(K)}^2}{P^+}, \mathbf{0}_\perp \right), \quad (8)$$

$$k_q \equiv \left(xP^+, \frac{\mathbf{k}_\perp^2 + m_q^2}{xP^+}, \mathbf{k}_\perp \right), \quad (9)$$

$$k_{\bar{q}} \equiv \left((1-x)P^+, \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{(1-x)P^+}, -\mathbf{k}_\perp \right). \quad (10)$$

B. Form factors

The vector, tensor, and scalar quark FFs of pseudoscalar mesons can be calculated from the leading twist and subleading twist quark GPDs by integrating over longitudinal momentum fraction at zero skewness as [69]

$$F_v^q(Q^2) = \int dx F_1(x, \xi = 0, -\Delta_\perp^2), \quad (11)$$

$$F_T^q(Q^2) = \int dx H_1(x, \xi = 0, -\Delta_\perp^2), \quad (12)$$

$$F_S^q(Q^2) = \int dx E_2(x, \xi = 0, -\Delta_\perp^2). \quad (13)$$

Here, v , T , and S are noted for vector, tensor, and scalar, respectively. F_1 , H_1 , and E_2 correspond to vector, tensor, and scalar pseudoscalar meson quark GPDs, respectively, with F_1 and H_1 being the twist-2 quark GPDs, whereas E_2 is the twist-3 quark GPD. Q^2 is the squared four-momentum transfer between the incoming and outgoing pion and kaon. Δ is the transverse momentum transfer between the initial- and final-state meson. The meson FFs can be calculated by taking into account each flavor of the meson, and we have

$$F_{v,T,S}^{\pi(K)} = e_q F_{v,T,S}^q + e_{\bar{q}} F_{v,T,S}^{\bar{q}}, \quad (14)$$

where $e_{q(\bar{q})}$ is the charge of the quark (antiquark). The antiquark FFs can be calculated from antiquark GPDs, which are discussed further in the manuscript. The matrix elements of the quark operators at a lightlike separation are defined as GPDs [8]. The valence quark GPDs for pseudoscalar mesons can be expressed through quark-quark correlators as [28,29]

$$\begin{aligned} \Phi_q^{[\Gamma]}(x, \xi, -\Delta_\perp^2) &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik \cdot z} \langle \pi^+(K^+) (P^*, S_z = 0) | \\ &\quad \times \bar{\psi}(-z/2) \Gamma \mathcal{W} \psi(z/2) | \pi^+(K^+) \\ &\quad \times (P, S_z = 0) \rangle |_{z^+ = z_\perp = 0}, \end{aligned} \quad (15)$$

where P^* and P are the four-vector momenta of the final and initial meson. $z = (z^+, z^-, z_\perp)$ and ψ are the position four vector and the quark field operator. $\mathcal{W}$ is the Wilson line which connects the quark field operator from $-z/2$ to $z/2$ and ensures the color gauge invariance. The variables $\Delta = (\Delta^+, \Delta^-, \Delta_\perp) = (P^* - P)$ and $\xi = -\Delta^+/(P^* + P)$ are the momentum transfer between the initial- and final-state meson

and the skewness parameter. Here, $Q^2 = \Delta^2 = -\Delta_\perp^2$ as $\Delta^+ = 0$ for our case. In this work, we mainly focus on the transverse momentum transferred $\Delta_\perp^2$, which gives rise to FFs at $\xi = 0$. Γ is the gamma matrix specific for each GPD. $\Gamma = \gamma^+, \iota\sigma^{j+}\gamma_5$, and 1 correspond to the vector, tensor, and scalar quark GPDs, respectively. These GPDs can be expressed in the form of $\Phi_q^{[\Gamma]}(x, \xi, -\Delta_\perp^2)$ as

$$F_1(x, \xi = 0, -\Delta_\perp^2) = \Phi_q^{[\gamma^+]}(x, \xi, -\Delta_\perp^2), \quad (16)$$

$$H_1(x, \xi = 0, -\Delta_\perp^2) = -\frac{\iota\mathcal{M}_{\pi(K)}}{\epsilon^{ij}\Delta_\perp^i}\Phi_q^{[\iota\sigma^{j+}\gamma_5]}(x, \xi, -\Delta_\perp^2), \quad (17)$$

$$E_2(x, \xi = 0, -\Delta_\perp^2) = \frac{P^+}{\mathcal{M}_{\pi(K)}}\Phi_q^{[1]}(x, \xi, -\Delta_\perp^2). \quad (18)$$

Now, using the pion and kaon meson Fock-state of Eq. (2) in the quark-quark correlator of Eq. (15), the overlap form of different FFs with all possibilities of helicities are found to be

$$F_v^q(Q^2) = \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} [\Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \uparrow, \uparrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \uparrow, \uparrow) + \Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \uparrow, \downarrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \uparrow, \downarrow) \\ + \Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \downarrow, \uparrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \downarrow, \uparrow) + \Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \downarrow, \downarrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \downarrow, \downarrow)], \quad (19)$$

$$F_T^q(Q^2) = \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \mathcal{M}_{\pi(K)} [\iota(\Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \downarrow, \uparrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \uparrow, \uparrow) + \Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \downarrow, \downarrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \uparrow, \downarrow)) \\ - \iota(\Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \uparrow, \uparrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \downarrow, \uparrow) + \Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \uparrow, \downarrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \downarrow, \downarrow))], \quad (20)$$

$$F_S^q(Q^2) = \int \frac{dx d^2\mathbf{k}_\perp}{\mathcal{M}_{\pi(K)} 16\pi^3} \left[\frac{m_q}{x} (\Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \uparrow, \uparrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \uparrow, \uparrow) + \Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \uparrow, \downarrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \uparrow, \downarrow) \\ + \Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \downarrow, \uparrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \downarrow, \uparrow) + \Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \downarrow, \downarrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \downarrow, \downarrow)) \right. \\ \left. + \frac{(\Delta_1 + \iota\Delta_2)}{2x} (\Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \downarrow, \uparrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \uparrow, \uparrow) + \Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \downarrow, \downarrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \uparrow, \downarrow)) \right. \\ \left. - \frac{(\Delta_1 - \iota\Delta_2)}{2x} (\Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \uparrow, \uparrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \downarrow, \uparrow) + \Psi_{\pi(K)}^*(x, \mathbf{k}_\perp'', \uparrow, \downarrow)\Psi_{\pi(K)}(x, \mathbf{k}'_\perp, \downarrow, \downarrow)) \right]. \quad (21)$$

Here, $\mathbf{k}_\perp'' = \mathbf{k}_\perp - \Delta_\perp/2$ and $\mathbf{k}'_\perp = \mathbf{k}_\perp + \Delta_\perp/2$ are the transverse momenta carried by the quark in the final- and initial-state meson. For this work, we have chosen the symmetric frame or Breit frame for our calculations. For tensor FF, we have taken $j = 2$. Taking $j = 1$ does not affect the final results. The explicit forms of quark FFs for pion by using the spin and momentum space wave function are expressed as

$$F_v^{q(\pi)}(Q^2) = \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \left[\mathbf{k}_\perp^2 - (1-x)^2 \frac{\Delta_\perp^2}{4} + (m_u(1-x) + xm_d)^2 \right] \frac{\psi_\pi^*(x, \mathbf{k}_\perp'')\psi_\pi(x, \mathbf{k}'_\perp)}{\omega_\pi''\omega_\pi'}, \quad (22)$$

$$F_T^{q(\pi)}(Q^2) = \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} [\mathcal{M}_\pi(1-x)(m_u(1-x) + m_d x)] \frac{\psi_\pi^*(x, \mathbf{k}_\perp'')\psi_\pi(x, \mathbf{k}'_\perp)}{\omega_\pi''\omega_\pi'}, \quad (23)$$

$$F_S^{q(\pi)}(Q^2) = \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \frac{2m_u}{x\mathcal{M}_\pi} \left[\mathbf{k}_\perp^2 - (1-x)^2 \frac{\Delta_\perp^2}{4} + (m_u(1-x) + xm_d)^2 + m_u(1-x)\Delta_\perp^2 \right] \frac{\psi_\pi^*(x, \mathbf{k}_\perp'')\psi_\pi(x, \mathbf{k}'_\perp)}{\omega_\pi''\omega_\pi'}, \quad (24)$$

where

$$\omega_\pi'' = (\mathcal{M}_\pi'' + m_u + m_d)\sqrt{x(1-x)[\mathcal{M}_\pi''^2 - (m_u - m_d)^2]}, \quad (25)$$

$$\omega_\pi' = (\mathcal{M}_\pi' + m_u + m_d)\sqrt{x(1-x)[\mathcal{M}_\pi'^2 - (m_u - m_d)^2]}. \quad (26)$$

With $\mathcal{M}_\pi$ being the bound state pion mass, represented as

$$\mathcal{M}_\pi'' = \sqrt{\frac{(\mathbf{k}_\perp - \frac{\Delta_\perp}{2})^2 + m_u^2}{x} + \frac{(\mathbf{k}_\perp - \frac{\Delta_\perp}{2})^2 + m_d^2}{1-x}},$$

$$\mathcal{M}_\pi' = \sqrt{\frac{(\mathbf{k}_\perp + \frac{\Delta_\perp}{2})^2 + m_u^2}{x} + \frac{(\mathbf{k}_\perp + \frac{\Delta_\perp}{2})^2 + m_d^2}{1-x}}.$$

Similarly, the explicit forms of the quark FFs of the kaon are found to be

$$F_v^{q(K)}(Q^2) = \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \left[\mathbf{k}_\perp^2 - (1-x)^2 \frac{\Delta_\perp^2}{4} + (m_u(1-x) + xm_s)^2 \right] \frac{\psi_K^*(x, \mathbf{k}_\perp'') \psi_K(x, \mathbf{k}_\perp')}{\omega_K'' \omega_K'}, \quad (27)$$

$$F_T^{q(K)}(Q^2) = \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} [\mathcal{M}_K(1-x)(m_u(1-x) + m_s x)] \frac{\psi_K^*(x, \mathbf{k}_\perp'') \psi_K(x, \mathbf{k}_\perp')}{\omega_K'' \omega_K'}, \quad (28)$$

$$F_S^{q(K)}(Q^2) = \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \frac{1}{x \mathcal{M}_K} \left[2m_u(\mathbf{k}_\perp^2 - (1-x)^2 \frac{\Delta_\perp^2}{4} + (m_u(1-x) + xm_s)^2) + 2((1-x)m_u + xm_s)^2(1-x)\Delta_\perp^2 \right] \times \frac{\psi_K^*(x, \mathbf{k}_\perp'') \psi_K(x, \mathbf{k}_\perp')}{\omega_K'' \omega_K'}. \quad (29)$$

The antiquark FFs can be calculated from antiquark GPDs of the meson. The antiquark GPDs can be calculated from the quark GPDs as

$$\text{quark GPD}(x, \xi, -\Delta_\perp^2, m_q, m_{\bar{q}}) = \text{antiquark GPD}(1-x, \xi, -\Delta_\perp^2, m_{\bar{q}}, m_q). \quad (30)$$

Both the quark and antiquark FFs will give rise to meson FFs as already mentioned in Eq. (14).

III. RESULTS AND DISCUSSIONS

For the numerical predictions of FFs of pion and kaon, we have considered the LCQM. In this model, we need only two input parameters: quark masses (m_q) and harmonic scale parameter ($\beta_{\pi(K)}$) for the calculations. $m_{u(\bar{d})}$ and β_π are taken to be 0.2 and 0.410, respectively, for the pion [67], whereas $m_{u(\bar{s})}$ and β_K are taken to be 0.2(0.556) and 0.405 for the case of the kaon [28,67]. These input parameters have been calculated by fitting the mass spectra, and they successfully produce the decay constant, PDFs, and charge radii quite well compared to other parameter values [28,67]. The calculated FFs of pion and kaon are

functions of only $Q^2 = -\Delta_\perp^2$ (GeV²). This work is the extension of our previous work on EMFF or vector FF of the pion and kaon [28]. Here, we are mainly trying to understand the scalar and tensor FFs, which have been less studied theoretically as well as experimentally. We observed that the vector FFs of the pion and kaon also obey the sum rule $F_v(Q^2 = 0) = 1$. We have already compared our LCQM result of pion and kaon EMFFs with experimental as well as lattice simulation results in our previous work [28]. In Fig. 1(a), we plot the second moment of vector FF $A_2(Q^2)$ at model scale $\mu^2 = 0.19$ as well as at scale $\mu^2 = 2$ for the case of pion and found to be smoothly decreasing behavior with increasing Q^2 .

The second moment of the pion and kaon is calculated from the second moment of GPDs as

$$A_2(Q^2) = \int dx x F_1(x, 0, -\Delta_\perp^2), \quad B_2(Q^2) = \int dx x H_1(x, 0, -\Delta_\perp^2). \quad (31)$$

Here, A and B denote the second moment of the vector and tensor FFs, respectively. The evolution of FFs has been done by using the NLO kernel as [59,73,74]

$$FF(Q^2, \mu^2) = \left(\frac{\alpha(\mu^2)}{\alpha(\mu_0^2)} \right)^{4/27} \left[1 - \frac{337}{486\pi} (\alpha(\mu_0^2) - \alpha(\mu^2)) \right] FF(Q^2, \mu_0^2), \quad (32)$$

with the NLO strong coupling constant

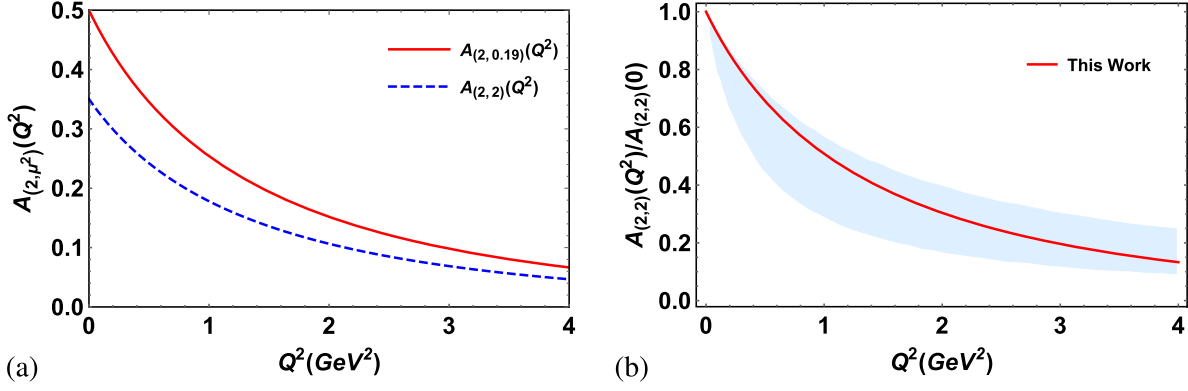


FIG. 1. (a) The second moment of pion vector FF $A_{(2,\mu^2)}(Q^2)$ is plotted at model scale $\mu^2 = 0.19 \text{ GeV}^2$ and at $\mu^2 = 2 \text{ GeV}^2$. (b) The normalized second moment pion vector FF is compared with lattice simulation results (shaded region) [70]. For reference, in our previous work [28], we have plotted the vector FFs of pion with available experimental as well as lattice simulation data [30,36–39,71,72].

$$\alpha(\mu^2) = \frac{4\pi}{9 \ln(\mu^2/\Lambda_{\text{QCD}}^2)} \left[1 - \frac{64}{81} \frac{\ln(\mu^2/\Lambda_{\text{QCD}}^2)}{\ln(\mu^2/\Lambda_{\text{QCD}}^2)} \right]. \quad (33)$$

The model scale μ_0^2 of LCQM has been taken as 0.19 GeV^2 [75] for this work. Λ_{QCD}^2 is the QCD scale, which is taken to be 0.30 GeV^2 . In Fig. 1(c), we observed that the evolved $A_2(Q^2)$ is reduced by 30% at $Q^2 = 0$ compared to the unevolved FF at model scale. The normalized second moment FF $A_{(2,2)}$ at $\mu^2 = 2 \text{ GeV}^2$ has been compared with

only available lattice simulation data [70] and found to be in agreement with it. The tensor FF is the result of polarized GPDs and spin-flip wave function. As there is no experimental result available for the tensor FF of the pion, so we have tried to compare our results with available lattice simulation results and theoretical predictions. The first moment $F_T(Q^2)$ and second moment $B_2(Q^2)$ of tensor FFs are presented and compared with lattice simulation data [69] in Figs. 2(a) and 2(c), respectively. Our $F_T(Q^2)$ FF is

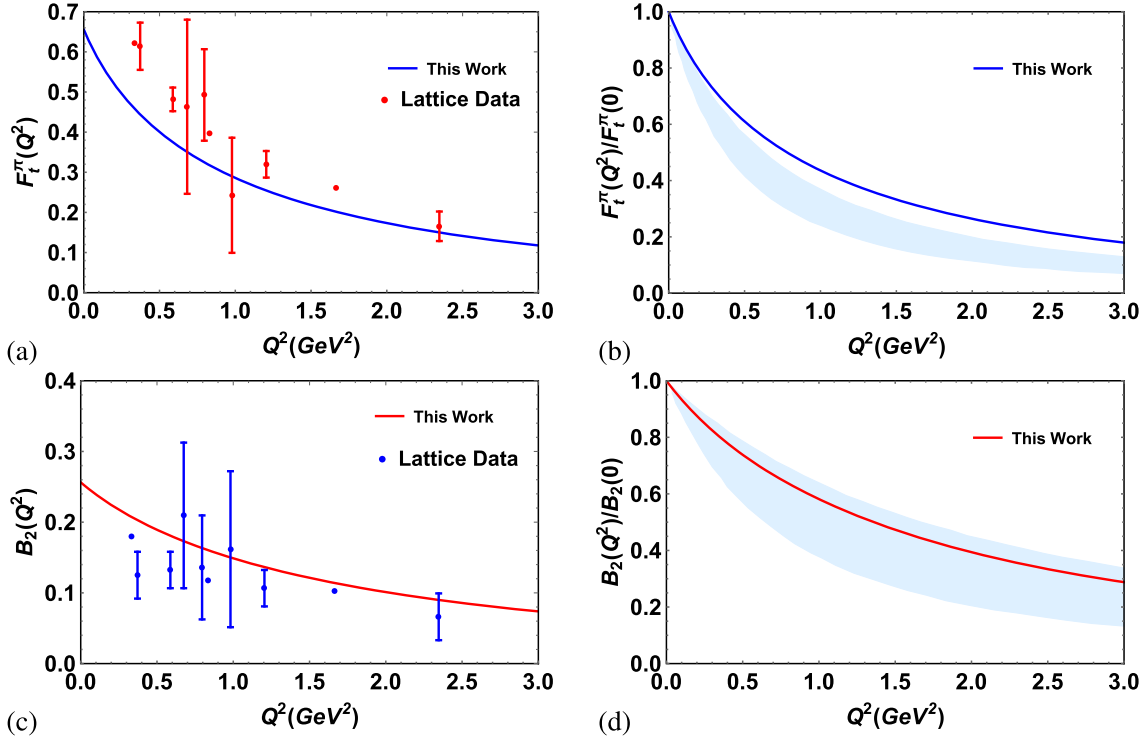


FIG. 2. The tensor FF of pion is plotted with respect to $Q^2 \text{ GeV}^2$. The first $[F_T^\pi(Q^2)]$ and second moment $[B_2(Q^2)]$ of the tensor FF of the pion are compared with lattice simulation data [69] in parts (a) and (c), respectively. The normalized first and second moments of the tensor FF of the pion are compared with lattice simulation data (shaded region) shown in Refs. [70,73] in parts (b) and (d), respectively.

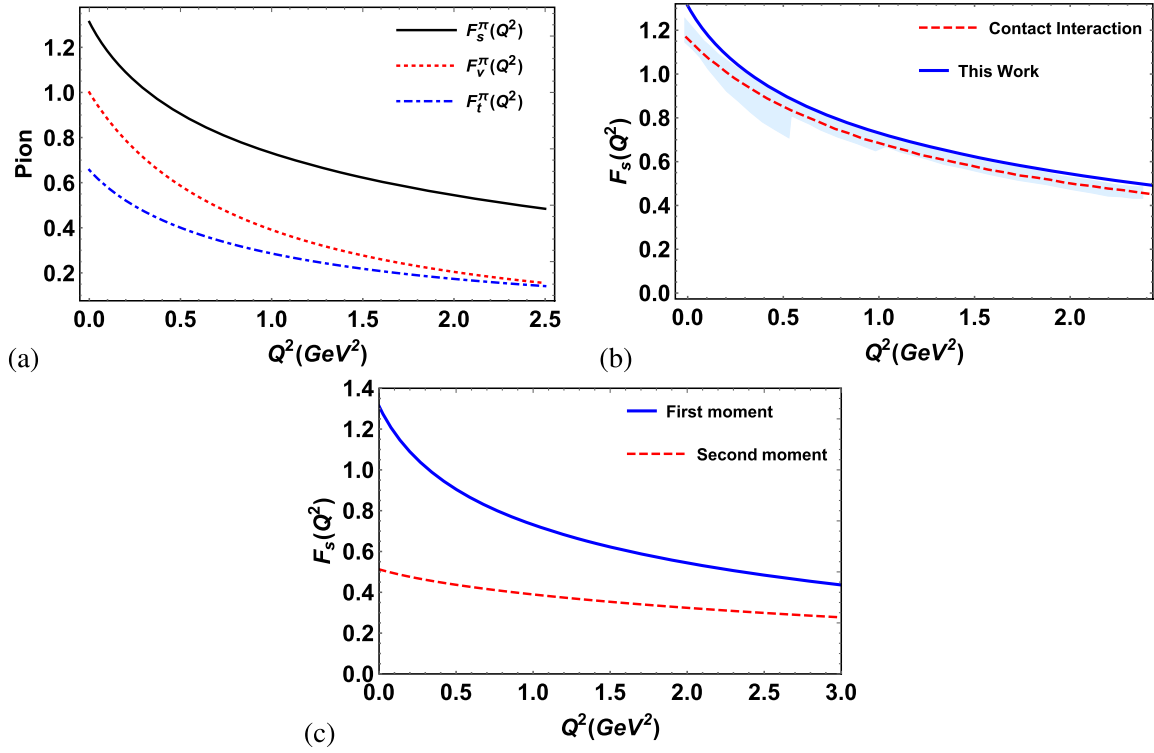


FIG. 3. (a) The scalar [$F_S^\pi(Q^2)$], vector [$F_V^\pi(Q^2)$], and tensor [$F_T^\pi(Q^2)$] FF of the pion are plotted with respect to Q^2 GeV². (b) The scalar FF [$F_S(Q^2)$] of our work is compared with only available lattice simulation data (shaded region) [50] and theoretical model (contact interaction) predictions [56]. (c) The first and second moments of the scalar FF of the pion are plotted with respect to Q^2 GeV².

found to have a similar kind of behavior to contact interaction results [56], instanton vacuum [76], chiral quark model [77], and lattice simulation results [50] but has a higher magnitude in distribution compared to all at low Q^2 . At $Q^2 = 0$, our $F_T^\pi(Q^2)$ comes out to be 0.655 compared to 0.856 in Ref. [69], 0.1612 in Ref. [73], 0.362 in Ref. [56], and 0.4 in Ref. [50]. While the evolved $F_T^\pi(Q^2 = 0)$ FF at $\mu^2 = 2$ GeV² is found to be 0.460 of our model compared to 0.216 in Ref. [76], 0.149 in Ref. [77], and 0.216 in Ref. [70]. In Figs. 2(b) and 2(d), we have compared our normalized first and second moment tensor FFs with only available lattice simulation data [70]. The normalized second tensor FF $B_2(Q^2)$ perfectly matches with the lattice simulation result, while the normalized $F_T^\pi(Q^2)$ has a higher distribution at $Q^2 \geq 0.3$ compared to the lattice simulation data [70]. The scalar FF $F_S^\pi(Q^2)$ has been derived from twist-3 scalar GPDs for the case of pion and kaon. This FF is plotted with respect to Q^2 for the case of pion in Fig. 3. Similar to tensor FF, there are no experimental data available for the case of scalar FFs, and even fewer studies have been reported for this FF [50,56]. The scalar FF of the pion is found to have a higher distribution compared to vector and tensor FFs, as shown in Fig. 3(a). This kind of behavior has also been seen in contact interaction [56] as well as lattice simulation result [50]. This may be due to the factor x in the denominator of

$F_S(Q^2)$. We have compared our scalar FF $F_S(Q^2)$ result with available lattice simulation [50] and contact interaction results [56] in Fig. 3(b), which is found to be in good agreement with them. Our result has a slightly higher distribution compared to both of them. At $Q^2 = 0$, the $F_S(Q^2)$ is found to be 1.323 compared to 1.16 for the case of contact interaction [56]. In Fig. 3(c), we have plotted the second moment of scalar FF along with the first moment. Overall, the scalar, vector, and tensor FFs calculated in our model are found to be in good agreement with available results.

Now, we move on to the results of FFs of kaon and its constituents. We have demonstrated the vector FFs of positively charged kaon, u quark, and $\bar{s}$ antiquark of the kaon in Fig. 4. The $\bar{s}$ antiquark is found to have a higher charge distribution compared to the u quark and the kaon. This is due to the heavier mass of the $\bar{s}$ antiquark compared to the u quark. Similar kinds of observations have also been seen in Refs. [78,79]. All the vector FFs show $F(Q^2 = 0) = 1$ for the case of the kaon. With the increase in Q^2 , all the FFs saturate. The vector FFs ratio of $F_v^u(Q^2)/F_v^K(Q^2)$, $F_v^K(Q^2)/F_v^{\bar{s}}(Q^2)$, and $F_v^{\bar{s}}(Q^2)/F_v^K(Q^2)$ are plotted in Fig. 4(d) and were found to decrease smoothly with Q^2 . All these ratios are found to be unity at $Q^2 = 0$. It is observed that the ratio $F_v^u(Q^2)/F_v^K(Q^2)$ decreases slowly compared to that of others. The ratio $F_v^u(Q^2)/F_v^{\bar{s}}(Q^2)$ of

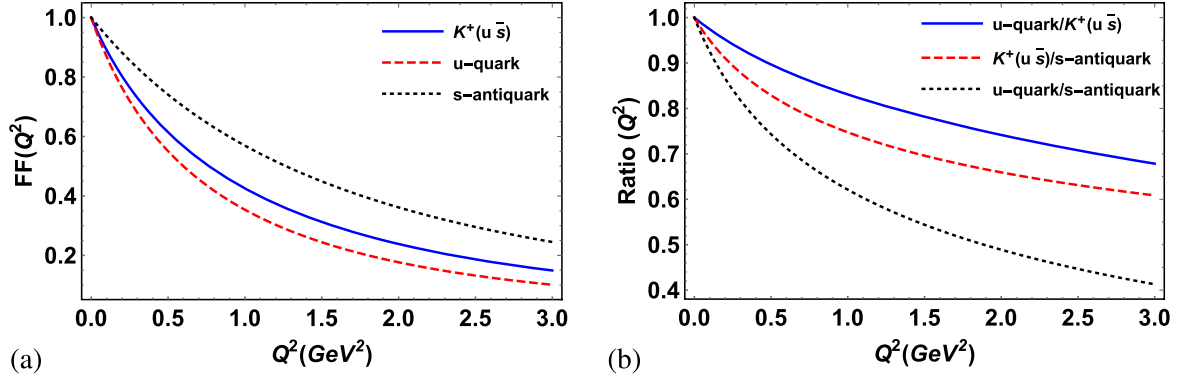


FIG. 4. The vector FF of kaon and its constituent (u quark and s antiquark) are plotted with respect to Q^2 GeV². (a) The vector FF of $K^+(u \bar{s})$, the u quark, and the s antiquark are plotted with respect to Q^2 up to 3 GeV². (b) The u -quark/ K^+ , $K^+/\bar{s}$ -antiquark, and u -quark/ $\bar{s}$ -antiquark FF ratios are plotted with respect to Q^2 GeV². However, in our previous work [28], we plotted the vector FF of the kaon with available experimental results [40,41].

our result is found to follow a similar kind of trend when compared with Refs. [50,79]. The ratio of $F_v^s(Q^2)/F_v^u(Q^2)$ is found at 3.45 at $Q^2 = 6$ GeV², which is quite high compared to Ref. [79]. This $F_v^u(Q^2)/F_v^s(Q^2)$ is found to decrease up to some positive value of Q^2 and then saturate at unity as $Q^2 \rightarrow \infty$, as mentioned in Refs. [78,79]. Now, looking into the tensor FF of the kaon, it is found that the u quark has higher distributions compared to the kaon and $\bar{s}$ antiquark in the region $0 \leq Q^2 \leq 0.73$ GeV² and vice versa

after, $Q^2 \geq 0.75$ as shown in Fig. 5. There are no experimental data, and theoretical models have been reported for the case of the kaon tensor and scalar FFs before now, but only lattice simulation data are available in Ref. [50]. In the case of Ref. [50], it has been predicted that the tensor FF of the u quark has a higher distribution than the $\bar{s}$ antiquark of the kaon, which can also be seen in our calculations. At $Q^2 = 0$, the tensor FFs of the u quark, $\bar{s}$ antiquark, and kaon are found to be 1.02, 0.71, and 0.93, respectively. In

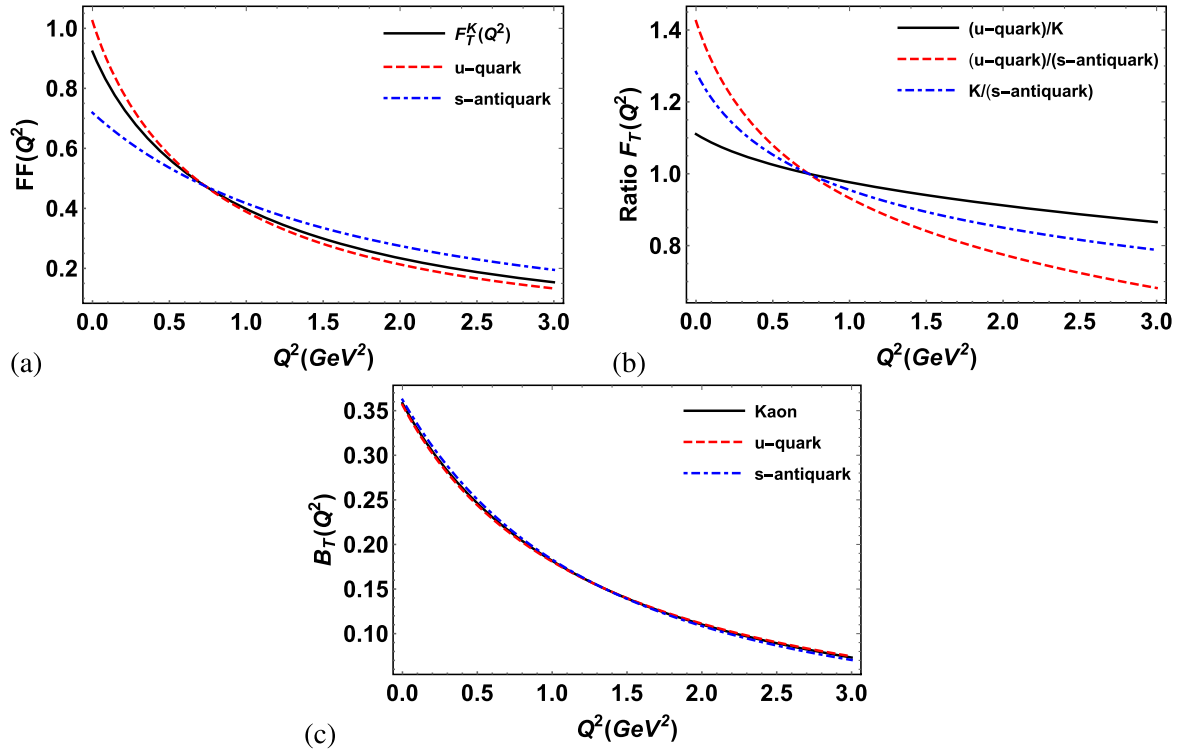


FIG. 5. (a) The tensor FF of the kaon [$F_T^K(Q^2)$] and its constituents [$F_T^q(Q^2)$] (u quark and s antiquark) are plotted with respect to Q^2 GeV². (b) The u -quark/ K^+ , u -quark/ $\bar{s}$ -antiquark, and $K^+/\bar{s}$ -antiquark tensor FFs ratios are plotted. (c) The second moment $B_T(Q^2)$ of the kaon and its constituents is plotted.

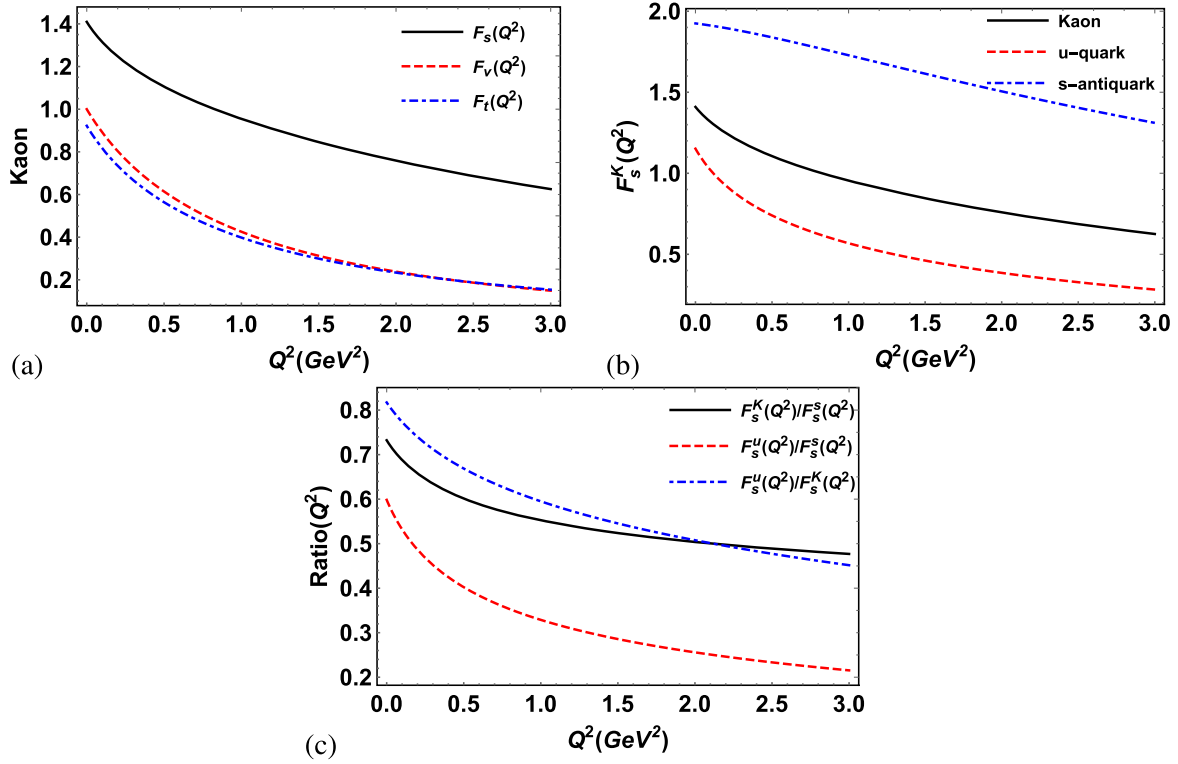


FIG. 6. (a) The scalar [$F_S(Q^2)$], vector [$F_V(Q^2)$], and tensor [$F_T(Q^2)$] FFs of kaon are plotted with respect to Q^2 GeV². (b) The scalar FF of the kaon [$F_S^K(Q^2)$] and its constituents [$F_S^q(Q^2)$] (the u quark and $\bar{s}$ -antiquark) are plotted. (c) The $K^+/\bar{s}$ -antiquark, u -quark/ $\bar{s}$ -antiquark, and u -quark/ K^+ scalar FF ratio are plotted.

Fig. 5(b), we have shown the ratios $F_T^u(Q^2)/F_T^K(Q^2)$, $F_T^u(Q^2)/F_T^s(Q^2)$, and $F_T^K(Q^2)/F_T^s(Q^2)$ of the kaon. The $F_T^u(Q^2)/F_T^s(Q^2)$ tensor FF ratio is found to follow the same trend as lattice simulations [50]. The second moment of the kaon tensor FF along with its constituent is plotted in Fig. 5(c). All are found to have exactly the same distributions. The kaon scalar $F_S^K(Q^2)$, vector $F_V^K(Q^2)$, and tensor $F_T^K(Q^2)$ are plotted in Fig. 6(a). The scalar $F_S^K(Q^2)$ FF is found to have a higher distribution as compared to other FFs as the pion has a value of 1.42 at $Q^2 = 0$. However, both vector $F_V^K(Q^2)$ and tensor $F_T^K(Q^2)$ have nearly similar behavior after $Q^2 \geq 1.75$. Like with vector FFs results, the scalar $\bar{s}$ -antiquark FF is found to have a higher distribution than the u quark and kaon, as shown in Fig. 6(b). The scalar FFs ratios among the kaon and its constituents are plotted in Fig. 6 and were found to have behavior similar to that in Ref. [50]. From both the pion and kaon, we can conclude that all the FFs show

$$F_S^{\pi(K)}(Q^2) \geq F_V^{\pi(K)}(Q^2) \geq F_T^{\pi(K)}(Q^2) \quad (34)$$

at the low $Q^2 \leq 1$ GeV² region. The pion to kaon, u -quark, and $\bar{s}$ -antiquark FFs ratios are shown in Fig. 7. The scalar FF $F_S^{\pi(K)}(Q^2)/F_S^{u(K)}(Q^2)$ ratio has contradictory behavior with

lattice simulation results [50], while others have a similar kind of trend with lattice data.

We have also calculated the radii corresponding to different FFs for both the pion and kaon. The scalar, vector, and tensor radii can be calculated as [56]

$$\langle r_S^2 \rangle = \frac{-6}{F_S(0)} \frac{\partial F_S(Q^2)}{\partial Q^2} \Big|_{Q^2 \rightarrow 0}, \quad (35)$$

$$\langle r_V^2 \rangle = \frac{-6}{F_V(0)} \frac{\partial F_V(Q^2)}{\partial Q^2} \Big|_{Q^2 \rightarrow 0}, \quad (36)$$

$$\langle r_T^2 \rangle = \frac{-6}{F_T(0)} \frac{\partial F_T(Q^2)}{\partial Q^2} \Big|_{Q^2 \rightarrow 0}. \quad (37)$$

In our case, $F_V(0)$ is found to be unity for both the pion and kaon. The radii for the u quark and $\bar{s}$ antiquark are calculated from quark FFs. The predicted values of the pion, u quark of the kaon, the $\bar{s}$ antiquark, and the charged kaon are presented in Table I. We have also compared our results with available experimental data [30,41,80,81], lattice simulations [50,82,83], and theoretical predictions [56,79,84]. For the case of the pion, we found that the radii follow the order

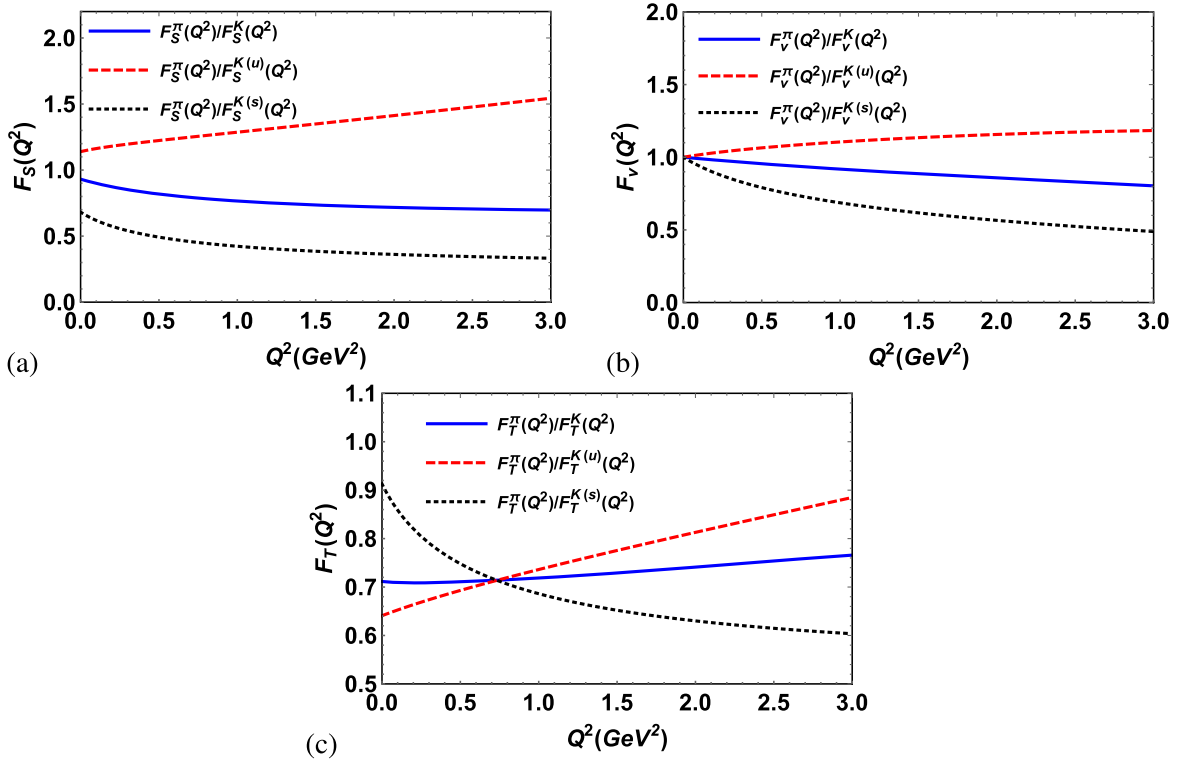


FIG. 7. (a) The scalar, vector, and tensor FFs ratio of the pion to kaon (along with the u quark and $\bar{s}$ antiquark) are plotted with respect to Q^2 in parts (a), (b), and (c), respectively.

$$\sqrt{\langle r_T^2 \rangle} \text{ (fm)} \geq \sqrt{\langle r_V^2 \rangle} \text{ (fm)} \geq \sqrt{\langle r_S^2 \rangle} \text{ (fm)}. \quad (38)$$

Similar behavioral orders are reported in Refs. [50,56], while for the case of the u quark of the kaon, the $\bar{s}$ antiquark of the kaon, and the charged kaon, the order is found to be

$$\sqrt{\langle r_V^2 \rangle} \text{ (fm)} \geq \sqrt{\langle r_T^2 \rangle} \text{ (fm)} \geq \sqrt{\langle r_S^2 \rangle} \text{ (fm)}. \quad (39)$$

However, in Ref. [50], the tensor radii have a higher value than the vector radii, which contradicts our case. The $\sqrt{\langle r_V^2 \rangle} / \sqrt{\langle r_T^2 \rangle}$ vector charge radii ratio of the kaon is found to be 0.61 compared to 0.69 of Ref. [79]. As there are no data available for the case of scalar and tensor radii of the kaon, so we cannot compare them. However, the overall radii are found to be in good agreement with other results. The upcoming EIC and JLab will provide more insight into the vector charge radii of the pion and kaon.

TABLE I. The scalar, vector, and tensor radii of the pion, kaon, and their constituents in unit of fermi (fm) in our results are compared with available experimental results [30,41,80,81], lattice QCD simulation results [50,82,83], and theoretical models [56,79,84].

	Pion			u quark (kaon)			s antiquark (kaon)			Kaon		
	$\sqrt{\langle r_S^2 \rangle}$	$\sqrt{\langle r_V^2 \rangle}$	$\sqrt{\langle r_T^2 \rangle}$	$\sqrt{\langle r_S^2 \rangle}$	$\sqrt{\langle r_V^2 \rangle}$	$\sqrt{\langle r_T^2 \rangle}$	$\sqrt{\langle r_S^2 \rangle}$	$\sqrt{\langle r_V^2 \rangle}$	$\sqrt{\langle r_T^2 \rangle}$	$\sqrt{\langle r_S^2 \rangle}$	$\sqrt{\langle r_V^2 \rangle}$	$\sqrt{\langle r_T^2 \rangle}$
This work	0.528	0.558	0.567	0.440	0.663	0.605	0.338	0.406	0.326	0.409	0.568	0.529
LQCD [50]	0.482	0.539	0.679	0.386	...	0.618	0.321	...	0.5	...	0.538	...
CI-MRL [56]	0.434	0.558	0.583	...	...	...	...	...	...	...	...	...
Ref. [84]	...	0.640	...	...	...	...	...	...	...	...	0.530	...
NA7 [30,80]	...	0.657, 0.662	...	...	...	...	...	...	...	...	...	...
JLQCD [82]	...	0.677	...	...	...	...	...	...	...	...	0.616	...
CERN-NA007 [41]	...	...	...	...	...	...	...	...	...	...	0.583	...
PDG [81]	...	0.659	...	...	...	...	...	...	...	...	0.560	...
LQCD [83]	0.635	...	...	...	...	...	...	...	...	...	...	...
Ref. [79]	...	0.65	...	...	0.62	...	...	0.43	...	...	...	...

IV. CONCLUSION

In this work, we have investigated the scalar, vector, and tensor form factors of the pion and kaon within the light-cone quark model framework. These FFs have been derived from the leading and subleading twist generalized parton distribution functions. By solving the quark-quark correlator, we have presented the FFs in the form of light-front wave functions and calculated their explicit forms by using the proper spin wave functions. We have demonstrated these FFs through two-dimensional plots. We have compared our results with available experimental data, lattice simulation results, and available theoretical predictions. We have also calculated the second moment of these FFs and evolved them to a higher scale to compare with the available lattice simulation results. All our results are found to be in good agreement with other data. The vector FFs of the pion and kaon obey the $F_v(Q^2 = 0) = 1$ property. We observed that the scalar FFs have higher distributions compared to the vector and tensor FFs for both the pion and kaon. We have also studied the FFs of the u quark and $\bar{s}$ antiquark of the kaon. The $\bar{s}$ antiquark shows a higher distribution compared to the u quark for both the vector and scalar FFs, while the opposite behavior is seen for the case of the tensor FF. There is no work reported in any theoretical models for tensor and scalar FFs yet. The kaon results were found to have the same trend as lattice

simulation results, but the magnitude differs from them. We have also predicted the scalar, vector, and tensor radii of the pion and kaon (along with its constituents), calculated from their respective FFs. The radii are found to be in good agreement with available experimental results, lattice simulation data, and theoretical model predictions.

There are still no experimental data available for scalar and tensor FFs; however, some data are available for the vector FFs case for pion and kaon. Even though there are limited studies going on for the case of kaon FFs in experiment and lattice simulation, this study will definitely provide deeper insights into the importance of tensor and scalar contributions of pseudoscalar mesons in understanding the internal structure of hadrons.

ACKNOWLEDGMENTS

H. D. would like to thank the Science and Engineering Research Board, Anusandhan-National Research Foundation, Government of India, under the scheme SERB-POWER Fellowship (Grant No. SPF/2023/000116) for financial support.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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