Chiral anomaly cancellation and neutral triple gauge boson vertices in the SM EFT

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We demonstrate the cancellation of chiral anomalies in the Standard Model (SM) effective field theory (EFT), achieved through a consistent choice of loop momentum routing in triangle diagrams with dimension-6 operator insertions. By enforcing gauge invariance and Bose symmetry, we show that Goldstone-boson contributions cancel anomalies arising from massive gauge-boson vertices, thereby preserving the consistency of the SM EFT. We compute neutral triple gauge-boson vertices at one loop, revealing dominant contributions from dimension-6 operators at all energies below the EFT cutoff. A UV-complete anomaly-free model with a heavy vectorlike electron validates our approach, illustrating how heavy fermion decoupling generates SM EFT operators while maintaining anomaly cancellation. Our results highlight the phenomenological relevance of these vertices for probing new physics at colliders, particularly through dimension-6 effects that scale as the inverse of the center of mass energy squared, $1/s$, offering a viable pathway for experimental detection.

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I. INTRODUCTION

Chiral anomalies represent a profound insight into the interplay between quantum mechanics and symmetries in particle physics. An anomaly occurs when a symmetry present at the classical level of a theory fails to be preserved after quantization. Chiral anomalies specifically pertain to the nonconservation of chiral currents in the presence of gauge fields, which has crucial implications for the consistency of quantum field theories [1–3]. In the Standard Model (SM) of particle physics, anomaly cancellation is essential to ensure gauge invariance, unitarity, and renormalizability. The SM achieves this through a delicate balance of the electromagnetic charges, the color, and the weak isospin representations of quarks and leptons within one generation [4–6].

When extending the SM to include the effects of new physics at higher-energy scales, we often use an effective field theory approach known as the Standard Model effective field theory (EFT) (for reviews see, Refs. [7,8]). This article discusses the relevance of chiral anomalies within the SM EFT, as well as their intimate connections to the triple gauge-boson vertices. The SM EFT extends the SM by adding higher-dimensional operators formed from SM fields that respect the gauge symmetries of the SM. At low energies, the coefficients in front of these operators are suppressed by powers of a high-energy scale Λ, reflecting the effects of new physics that resides above this scale. The SM EFT Lagrangian can be expressed as

$$\mathcal{L}_{\text{SM EFT}} = \mathcal{L}_{\text{SM}} + \sum_{d>4} \frac{C_d}{\Lambda^{d-4}} \mathcal{O}_d, \tag{1.1}$$

where $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian, $\mathcal{O}_d$ represents the d-mass dimensional operators, and C_d are the corresponding Wilson coefficients. When extending the SM to the SM EFT, the potential reemergence of anomalies becomes a critical concern. The added higher-dimensional operators could, in principle, introduce new anomalies or modify existing ones. Therefore, constructing a consistent SM EFT demands careful attention to anomaly cancellation at each operator level. If a beyond the Standard Model (BSM) theory exists and is gauge anomaly-free, then the SM EFT should also be gauge anomaly-free in the sense that it preserves all Ward identities necessary for gauge invariance. In other words, it is imperative to match a consistent theory in the ultraviolet with another consistent one in the infrared following the decoupling of heavy particles at the scale Λ [9].

A first idea that comes into one's mind is that cancellation of chiral anomalies happens *à la* SM and therefore leads to sum rules among Wilson coefficients that should be obeyed in any anomaly-free fundamental (UV) theory above scale Λ [10]. However, this procedure does not work: there are certain UV models invalidating these

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anomaly-cancellation sum rules [11]. The current state of the art is that chiral anomalies can be removed with an appropriate choice of local and finite counterterms [12–14].

Regarding chiral anomalies in the SM EFT, our study serves as a complement to these works, for it follows a somewhat different direction. By working exclusively in the broken electroweak phase of the SM EFT [15], we calculate triangle one-loop diagrams strictly in four dimensions with $\{\gamma^\mu, \gamma^5\} = 0$, and demand that Ward identities be satisfied by a certain routing of the momenta in the triangle loop. This is certainly a scheme choice which is consistent with gauge invariance and Bose symmetry, as we readily confirm. We prove that the procedure can always be performed with higher-dimensional operator insertions in the SM EFT, with the key points of its success being (a) the SM is an anomaly-free gauge theory and (b) the Goldstone-boson triangle diagrams canceling massive vector-boson triangle diagrams in the SM EFT.

We demonstrate the above within a bottom-up and a top-down approach and compare it to the full theory result for a decoupling of a heavy vectorlike fermion. Our outcome in both approaches follows the picture found by D'Hoker and Farhi [16,17]: the decoupling of a fermion whose mass is generated by a Yukawa coupling results in a gauge invariant action functional with a gauge and Higgs field term. In our UV-considered theory in Sec. VA, the vectorlike electron, which has a large gauge invariant mass M, receives an additional small part from a Yukawa coupling y_E, proportional to $|y_E|^2 v^2/M$ as a result of the mixing of heavy and light (SM) electrons after the Higgs field, acquiring a vacuum expectation value v. The induced terms are gauge invariant and are encoded in the SM EFT in agreement with the full theory.

Following the idea of Ref. [18],[1] we use the routing momenta parameters consistent with the Ward identities and Bose symmetry to calculate neutral triple gauge boson vertices (nTGVs) in the SM EFT with dimension-6 operator insertions for all combinations V^*VV, with $V = \gamma, Z$. In doing so, we compare the results for the form factor $h_3^\gamma(s)$ [24,25] with the full (UV) theory example mentioned above, merely as a demonstration of the validity of our procedure. We find, to our knowledge for the first time, that the effects of dimension-6 operators [26]

$$\begin{aligned}
&\mathcal{O}_{\varphi\ell}^{(1)} = (\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{\ell}_L \gamma^\mu \ell_L), \qquad \mathcal{O}_{\varphi\ell}^{(3)} = (\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar{\ell}_L \tau^I \gamma^\mu \ell_L), \qquad \mathcal{O}_{\varphi e} = (\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{e}_R \gamma^\mu e_R),\\
&\mathcal{O}_{\varphi q}^{(1)} = (\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{q}_L \gamma^\mu q_L), \qquad \mathcal{O}_{\varphi q}^{(3)} = (\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar{q}_L \tau^I \gamma^\mu q_L), \qquad \mathcal{O}_{\varphi d} = (\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{d}_R \gamma^\mu d_R),\\
&\mathcal{O}_{\varphi u} = (\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{u}_R \gamma^\mu u_R)
\end{aligned} \tag{1.2}$$

are far more important than dimension-8 operator insertions [27], and for all energies below the new physics (NP) resonances scale like $1/s$, where $\sqrt{s}$ is the center of mass energy. There is a promising region where their effects could, in principle, be measured at future e^+e^--colliders (subject to electroweak constraints and collider sensitivity) at energies above but near the electroweak scale. The LHC and the upgraded High-Luminosity LHC (HL-LHC) hold similar conclusions at high (TeV) energies, despite the small size of the form factors.

Our study, as introduced above, is an interesting probe for collider phenomenology. Since the Large Electron-Positron (LEP) collider [28–30], searches for neutral triple gauge-boson vertices have attracted sufficient attention, both experimentally at Tevatron [31,32] and LHC [33–35] and theoretically; a partial list of references includes [18,36–49]. Furthermore, applications of triple gauge-boson vertices to dark matter [50–53], neutron stars [54], and cosmology [55] complement an interesting topic in the current literature.

The outline of our study goes as follows: In Sec. II we define the relevant Ward identities (WIs) for addressing chiral anomalies through the gauge-fixing-parameter independence. In Sec. III we show how these WIs can be made satisfied in SM EFT with dimension-6 operator insertions. In Sec. IV we analytically discuss the impact of EFT operators in nTGVs, and in Sec. V we work through a particular UV model to verify our SM EFT results and go beyond them to discuss higher-dimensional operator effects. The reader who is solely interested in nTGVs can start directly in Sec. IV without losing any content and then continue to Sec. VI, where phenomenological implications are briefly stated. We conclude in Sec. VII. All the relevant vertices discussed in this article are included in the Appendix.

II. A WARD IDENTITY FROM GAUGE-FIXING-PARAMETER INDEPENDENCE

Let us first determine the relevant WI and then show why it is important to be anomaly-free. As an example, we consider the s-channel of a positron-electron scattering process with momenta p_1 and p_2, respectively, to the final state on-shell vector bosons V_j^ν and V_k^ρ, with momenta k_1

[1]The same procedure has been applied to the decay $Z \to Z'\gamma$ in Refs. [19,20] for extending the SM with a $U(1)'$ symmetry or a Stückelberg gauge boson. See also [21–23].

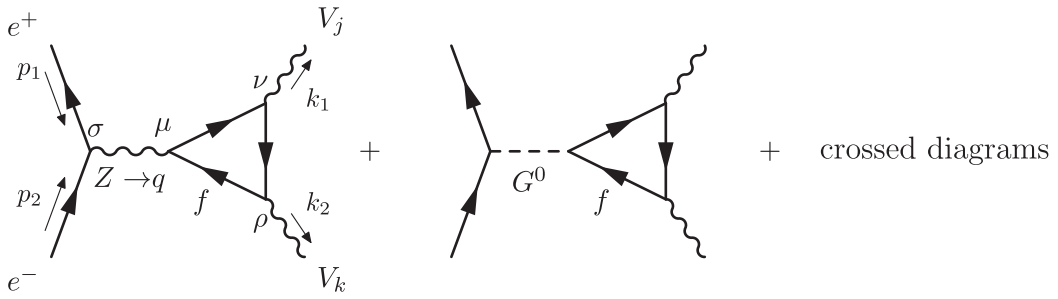


FIG. 1. Feynman diagrams for s-channel contributions to the $e^+e^- \to V_jV_k$ amplitude, where $V = (Z, W, \gamma)$ and mediates the Z-gauge-boson and Goldstone-boson (G^0) triangle loop circulated by a fermion of flavor f.

and k_2, respectively, $e^+(p_1) + e^-(p_2) \to V_i^*(q) \to V_j^\nu(k_1) + V_k^\rho(k_2)$, with V being either the massive neutral (Z) or charged ($W^\pm$) gauge bosons or the photon γ. Momentum conservation implies that $q_\mu = (k_1 + k_2)_\mu = (p_1 + p_2)_\mu$, and the Greek letters $\mu, \nu, \rho, \ldots$ indicate spacetime indices. Working in linear R_ξ gauges,[2] the relevant Feynman diagrams are those with γ or Z mediators, with the latter associated with the corresponding Goldstone boson G^0, which is depicted in Fig. 1. The scattering amplitude, in standard textbook notation (e.g., [56]), reads

$$-\bar{v}(p_1)[\gamma^\sigma(a_Z^{(e)} + b_Z^{(e)}\gamma^5)]u(p_2)\frac{1}{q^2 - M_Z^2}\left(g_{\sigma\mu} - \frac{q_\sigma q_\mu}{M_Z^2}\right)\sum_f \Delta^{\mu\nu\rho(f)}_{ZV_jV_k}(k_1,k_2)\epsilon^*_\nu(k_1)\epsilon^*_\rho(k_2)$$
$$+\frac{2b_Z^{(e)}m_e}{q^2 - \xi_Z M_Z^2}\frac{1}{M_Z^2}\bar{v}(p_1)\gamma^5 u(p_2)\sum_f [q_\mu \Delta^{\mu\nu\rho(f)}_{ZV_jV_k}(k_1,k_2) + iM_Z\Delta^{\nu\rho(f)}_{G^0V_jV_k}(k_1,k_2)]\epsilon^*_\nu(k_1)\epsilon^*_\rho(k_2). \quad (2.1)$$

The one-particle irreducible (1PI) vertices $\Delta^{\mu\nu\rho(f)}_{ZV_jV_k}(k_1,k_2)$ and $\Delta^{\nu\rho(f)}_{G^0V_jV_k}(k_1,k_2)$ denote the fermion f triangle vertices with the Z and Goldstone G^0 bosons in the ith vertex, respectively. Only the term in the second line of (2.1) depends on the gauge-fixing parameter ξ_Z through the pole in the Goldstone-boson propagator. One can convince themselves that in the entire amplitude there is no other contribution to such a term,[3] and as a consequence, the expression inside the parentheses must vanish. Keeping in mind current conservation for V_i, the Ward identity among on-shell gauge bosons V_j and V_k states[4] can be generalized as follows:

$$q_\mu\Delta^{\mu\nu\rho}_{V_iV_jV_k} + iM_{V_i}\Delta^{\nu\rho}_{G^0V_jV_k} = 0, \qquad V_i = Z,\gamma, \quad V_{j,k} = Z,\gamma,W^\pm, \quad (2.2)$$

where $\Delta^{\mu\nu\rho}_{V_iV_jV_k} = \sum_f \Delta^{\mu\nu\rho(f)}_{V_iV_jV_k}$ and, similarly, $\Delta^{\nu\rho}_{G^0V_jV_k} = \sum_f \Delta^{\nu\rho(f)}_{G^0V_jV_k}$, with the three-point 1PI triangle graphs shown in Fig. 1 summed over all relevant fermions f. Note that the WI (2.2) is an important piece for theory consistency; otherwise, the scattering amplitude in (2.1) would depend on the gauge-fixing parameter ξ_Z (or ξ_γ for a photon mediator). It is straightforward to observe that Eq. (2.2) is equivalent to the validity of partial conservation of axial-vector current at the classical level.

Since in the SM EFT there are two neutral weak bosons Z and γ, at least two out of three gauge bosons should be identical and Bose symmetry should apply. For example, in Fig. 1, if $V_j = Z$ and $V_k = \gamma$, the S-matrix element should be identical to the one with $q \leftrightarrow -k_1$. Considering this approach, there are WIs analogous to Eq. (2.2) for the fields V_j^ν and V_k^ρ, namely,

$$-k_{1\nu}\Delta^{\mu\nu\rho}_{V_iV_jV_k} + iM_{V_j}\Delta^{\mu\rho}_{V_iG^0V_k} = 0, \qquad V_j = Z,\gamma, \quad V_{i,k} = Z,\gamma,W^\pm, \quad (2.3)$$

[2]We are working in Warsaw basis [26], adopting the notation and the Feynman rules in the mass basis from Ref. [15]. The Feynman rule for the gauge-boson (V) vertex to fermions (f) is expressed as $[-i\gamma_\mu(a_V^{(f)} + b_V^{(f)}\gamma^5)]$ throughout. Their explicit form in SM EFT is provided in the Appendix.

[3]It is straightforward, albeit tedious, to demonstrate this result, but there is a quick way to do so by using the three gauge-fixing terms, one for each vector-boson propagator ξ_γ, ξ_Z, and $\xi_{W^\pm}$. Since there is no other triangle or box diagram with all involved fermions mediated by the Z boson (or the photon), we arrive at the only two options for diagrams in Fig. 1.

[4]The outside product of polarization vectors will be kept implicit in all the WIs below unless stated otherwise.

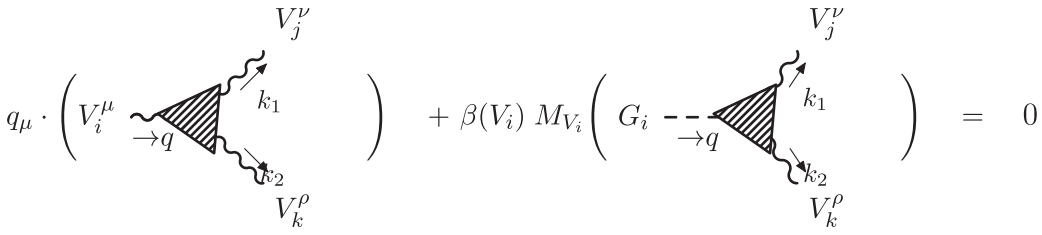


FIG. 2. Graphical representation of the WI in Eq. (2.6).

$$-k_{2\rho}\Delta^{\mu\nu\rho}_{V_iV_jV_k} + iM_{V_k}\Delta^{\mu\nu}_{V_iV_jG^0} = 0, \qquad V_k = Z,\gamma, \quad V_{i,j} = Z,\gamma,W^\pm. \tag{2.4}$$

One other, somewhat laborious way of deriving Eqs. (2.3) and (2.4) is the following: stick a two-fermion current in every outgoing vector boson of Fig. 1 and inspect gauge invariance in the sense of ξ independence. Not only are these Ward identities [together with Eq. (2.2)] needed for gauge invariance to hold, but reduced Ward identities of the form

$$q_\mu\Delta^{\mu,\ldots}_{V_i\ldots} + iM_{V_i}\Delta_{G_i\ldots} = 0 \tag{2.5}$$

are also in place, where the ellipses represent a combination of Goldstone- and/or vector-boson fields. Therefore, Eqs. (2.2)–(2.4) should always be valid at the same time. Their violation renders the SM, and therefore the SM EFT, meaningless.

One can derive similar identities when working with charged currents too, e.g., similar to Fig. 1 but with the process $\bar{\nu}_e(p_1) + e^-(p_2) \to V_i^*(q) \to V_j(k_1) + V_k(k_2)$, where two V's are W-gauge bosons. In fact, it is possible to group all the WIs in a single equation for every incoming gauge boson V_i (associated with the Goldstone boson G_i) with four-momentum transfer $q = k_1 + k_2$ and mass M_{V_i},

$$q_\mu\Delta^{\mu\nu\rho}_{V_iV_jV_k}(k_1,k_2) + \beta(V_i)M_{V_i}\Delta^{\nu\rho}_{G_iV_jV_k}(k_1,k_2) = 0, \quad \{\beta(Z) = i, \beta(W^+) = -1, \beta(W^-) = +1\}, \tag{2.6}$$

graphically pictured in Fig. 2, where the shaded triangles also include the crossed diagrams.

However, the WI in (2.6) is, in general, false; i.e., it may be anomalous. The reason being the fact that the triangle diagrams $\Delta^{\mu\nu\rho(f)}_{V_iV_jV_k}$ in the SM and the SM EFT, and $\Delta^{\nu\rho(f)}_{G^0V_jV_k}$ only in SM EFT, are linearly divergent. The calculation of these diagrams leads to anomalous terms that depend on the routing of the momentum circulating in both triangle loops, which reflects upon the ambiguity in the dimensional regularization scheme of treating γ^5 consistently in more than four dimensions.

Strictly speaking, the identity in (2.6) is valid for on-shell particles V_j and V_k. As such, it was proven to all orders by Chanowitz and Gaillard [57]. Naturally, as the reader may already be aware, Eq. (2.6) serves as the foundation for the renowned Goldstone-boson equivalence theorem [58–61].

III. CHIRAL ANOMALIES IN THE SM EFT

A. The SM EFT at tree level and WIs

Because of gauge invariance and Bose symmetry, in the SM EFT with up to dimension-6 operators, there are no triple neutral gauge-boson vertices at tree level. There are, however, triple vertices with charged vector bosons of the form γW^+W^- and ZW^+W^- and higher particle multiplicity vertices arising from one CP-even and two CP-odd operators. We have explicitly checked the validity of Eq. (2.6) for those vertices at tree level by using the Feynman rules in Ref. [15] in R_ξ gauges.

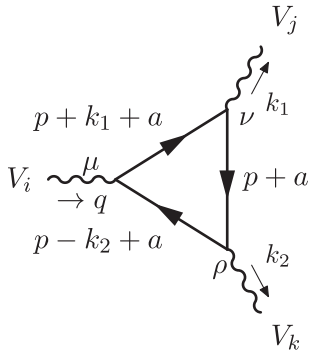
Nevertheless, for inspecting chiral anomalies in triangle diagrams in SM EFT, the application of (2.5) is very useful. It is trivial to show that, dotting a $V\bar{f}f$ vertex for every fermion flavor in the SM EFT with the momentum of a massive vector boson V, we obtain a vertex proportional to the $G\bar{f}f$ vertex, with the vector boson replaced by its associate Goldstone boson G.

B. Cancellations of the triangle anomalies in the SM EFT

The naive WIs in (2.6) contain two parts: the first triangle $\Delta^{\mu\nu\rho}_{V_iV_jV_k}$ attached to gauge bosons V and the second triangle $\Delta^{\nu\rho}_{G_iV_jV_k}$, where V_i has been replaced by a Goldstone boson. Both triangles and their crossed diagrams are linearly divergent. This means that each amplitude depends on the routing of the momenta circulating the triangles.

For the neutral[5] external vector-boson triangle and for each fermion f, we have

[5]For external W bosons, the triangle fermion mass m_f is not universal, but the diagrams share the same anomaly factor [cf. (3.9)] as in the case of neutral external gauge bosons. As far as the chiral anomaly factor is concerned, the result is independent of m_f. See Ref. [18] for the direct calculation in the scheme adopted here.



$+$ crossed diagram $[a \leftrightarrow b,\ k_1 \leftrightarrow k_2,\ (\nu,\rho) \leftrightarrow (\rho,\nu)]$

$$
\begin{aligned}
\Delta^{\mu\nu\rho(f)}_{V_iV_jV_k}(k_1,k_2;a,b) = (-1)\mathrm{Tr}\int \frac{d^4p}{(2\pi)^4}\left[-i\gamma^\mu\left(a^{(f)}_{V_i}+b^{(f)}_{V_i}\gamma^5\right)\right]\frac{i}{(\not{p}-\not{k}_2)+\not{a}-m_f}\left[-i\gamma^\rho\left(a^{(f)}_{V_k}+b^{(f)}_{V_k}\gamma^5\right)\right] \\
\times\frac{i}{\not{p}+\not{a}-m_f}\left[-i\gamma^\nu\left(a^{(f)}_{V_j}+b^{(f)}_{V_j}\gamma^5\right)\right]\frac{i}{(\not{p}+\not{k}_1)+\not{a}-m_f} \\
+(-1)\mathrm{Tr}\int \frac{d^4p}{(2\pi)^4}\left[-i\gamma^\mu\left(a^{(f)}_{V_i}+b^{(f)}_{V_i}\gamma^5\right)\right]\frac{i}{(\not{p}-\not{k}_1)+\not{b}-m_f}\left[-i\gamma^\nu\left(a^{(f)}_{V_j}+b^{(f)}_{V_j}\gamma^5\right)\right] \\
\times\frac{i}{\not{p}+\not{b}-m_f}\left[-i\gamma^\rho\left(a^{(f)}_{V_k}+b^{(f)}_{V_k}\gamma^5\right)\right]\frac{i}{(\not{p}+\not{k}_2)+\not{b}-m_f},
\end{aligned}
\tag{3.1}
$$

where (-1) is the fermionic loop factor and m_f is a common mass for the fermion f inside the triangle loop. In both diagrams, we shift the internal loop momenta with arbitrary four-vectors a^μ and b^μ. This shift with arbitrary vectors is not necessary for the triangle diagram $\Delta^{\mu\nu\rho}$ itself, which is convergent, after all; it is nevertheless necessary for its divergence $q_\mu\Delta^{\mu\nu\rho}$, which we are going to examine below [see Eq. (3.2)]. All vector ($a^{(f)}_{V_i}$) or axial-vector ($b^{(f)}_{V_i}$) couplings are general SM EFT couplings to all orders of EFT expansion.[6] Dotted in (3.1) with q_μ and exploiting standard identities,[7] we find that

$$
q_\mu\Delta^{\mu\nu\rho(f)}_{V_iV_jV_k}(k_1,k_2;a,b) = -2m_f b^{(f)}_{V_i}\Gamma^{\nu\rho(f)}_{V_jV_k}(k_1,k_2) + \Pi^{\nu\rho(f)}_{V_iV_jV_k}(k_1,k_2;a,b). \tag{3.2}
$$

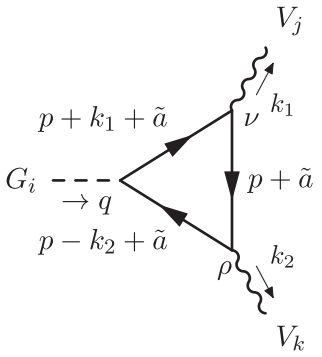
The integral $\Gamma^{\nu\rho(f)}_{V_jV_k}$ is finite and is given in Eqs. (B4) and (B5) of Ref. [18]. Because it cancels when the Goldstone diagram is included, as shown below, it is not a concern for the anomaly. The important piece is the integral $\Pi^{\nu\rho(f)}_{V_iV_jV_k}(k_1,k_2;a,b)$. It is divided into two parts: one chiral (i.e., γ^5 dependent) and one nonchiral (i.e., γ^5 independent). For the nonchiral part, the choice $b^\mu=-a^\mu$ [18,63,64] of the arbitrary vectors results in $\Pi^{\nu\rho(f)}_{V_iV_jV_k}(k_1,k_2;a,-a)|_{\text{nonchiral}}=0$, which is expected because there are no nonchiral anomalies. Following this choice, we are left with one arbitrary vector a^μ. We shall return to the explicit formula of $\Pi^{\nu\rho(f)}_{V_iV_jV_k}(k_1,k_2;a,-a)|_{\text{chiral}}$ after examining the Goldstone-boson diagram below.[8]

In the SM EFT, by replacing the vector boson V_i with its associate Goldstone boson G_i and utilizing a general form from Eqs. (A10)–(A13) in the Appendix, we have

[6]Note that possible tensor (dipole) SM EFT couplings do not contribute to the chiral anomalies [62]. They vanish when dotted with q_μ. Moreover, dipole Wilson coefficients arise at one-loop order in a perturbative decoupled UV theory and are therefore formally a two-loop effect in the closed fermion loop diagrams. Our analysis is strictly at one-loop order in the SM EFT.

[7]Appendix B of Ref. [18] contains all the details.

[8]Admittedly, the use of the arbitrary shift-vector technique, while well suited to chiral anomalies, may be cumbersome when applied to a general amplitude. See, however, Ref. [65].



$+$ crossed diagram $[\tilde{a} \leftrightarrow \tilde{b},\ k_1 \leftrightarrow k_2,\ (\nu, \rho) \leftrightarrow (\rho, \nu)]$

$$\Delta^{\nu\rho(f)}_{G_i V_j V_k}(k_1, k_2; \tilde{a}, \tilde{b}) = i\left[(-1)^f \frac{m_f}{v Z_{G^0}}\right]\Gamma^{\nu\rho(f)}_{V_j V_k}(k_1, k_2) + \frac{i}{M_Z} q_\mu \Delta^{\mu\nu\rho(f)}_{\tilde{V}_i V_j V_k}(k_1, k_2; \tilde{a}, \tilde{b}), \tag{3.3}$$

where $(-1)^f = +1$ for $f = u, \nu$ and $(-1)^f = -1$ for $f = d, e$. Note that the arbitrary momentum shift vectors $\tilde{a}^\mu$ and $\tilde{b}^\mu$ are now, in general, different from the vector-boson triangle. Following the same reasoning as before, we choose $\tilde{b}^\mu = -\tilde{a}^\mu$. The factor Z_{G^0} is a wave-function normalization of the Goldstone boson and a function of Wilson coefficients and the Higgs vacuum expectation value v (see the Appendix and Refs. [15,66] for definitions). Our calculation below shows that it cancels together with all other field redefinitions. Anomaly terms originate from the second term in the rhs of (3.3), with the difference with respect to (3.1) being that the coupling of $\tilde{V}_i = Z$ to the fermions is a pure EFT (dimension-6) coupling; i.e., it depends solely on Wilson coefficients, as shown in the Appendix.

As we saw in Sec. II, the sufficient and necessary chiral anomaly-free conditions to get gauge-fixing-parameter independence are Eq. (2.2) and its variants, Eqs. (2.3) and (2.4). We now have all the ingredients to check upon these WIs. For the WI in (2.2), we combine Eqs. (3.2) and (3.3) and, by making use of the $V_i = Z$ boson with mass $M_Z = \frac{1}{2} g_Z v Z_{G^0}$ and denoting that $b^{(f)}_Z = \hat{b}^{(f)}_Z + \tilde{b}^{(f)}_Z$, with $\hat{b}^{(f)}_Z = -(-1)^f \frac{g_Z}{4}$ being the SM part, and $\tilde{b}^{(f)}_Z$ the dimension-6 insertion of the axial $Zf\bar{f}$ vertex (see the Appendix), we find without loss of generality that[9]

$$\begin{aligned} &q_\mu \Delta^{\mu\nu\rho}_{V_i V_j V_k}(k_1, k_2; a) + i M_{V_i} \Delta^{\nu\rho}_{G^0 V_j V_k}(k_1, k_2; \tilde{a}) \\ &= \sum_f [\Pi^{\nu\rho(f)}_{V_i V_j V_k}(k_1, k_2; a) - \Pi^{\nu\rho(f)}_{\tilde{V}_i V_j V_k}(k_1, k_2; \tilde{a})], \end{aligned} \tag{3.4}$$

where, the tilde in $\Pi^{\nu\rho(f)}_{\tilde{V}_i V_j V_k}$ indicates a pure dimension-6 insertion of the vertex V_i to the fermions, whereas all other vertices without the tilde are completely general, of the SM + EFT type. Note that Eq. (3.4) is exact for every fermion mass m_f and that the rescaling factor Z_{G^0} has disappeared. Moreover, $\Pi^{\nu\rho(f)}_{V_i V_j V_k}$ contains only the chiral (i.e., dependent on γ^5) part since, as we pointed out previously, we can always choose the nonchiral part of $\Pi^{\nu\rho(f)}_{V_i V_j V_k}$ to vanish by taking the opposite routing of momentum shifts between the triangle diagram and its crossed cousin. This leaves just two unknown arbitrary vectors, a^μ and $\tilde{a}^\mu$ in (3.4), related to $\Pi^{\nu\rho(f)}_{V_i V_j V_k}$ and $\Pi^{\nu\rho(f)}_{\tilde{V}_i V_j V_k}$, respectively. These two vectors can only depend upon a linear combination of the external momenta k_1 and k_2, namely,

$$a^\mu = z k_1^\mu + w k_2^\mu, \qquad \tilde{a}^\mu = \tilde{z} k_1^\mu + \tilde{w} k_2^\mu, \tag{3.5}$$

with the set $\{w, z, \tilde{w}, \tilde{z}\}$ being real and arbitrary numbers. Therefore, the routing of the momenta needs two parameters w and z for the vector-boson diagrams and two parameters $\tilde{w}$ and $\tilde{z}$ for the Goldstone-boson diagrams. We will show that these parameters can be chosen such that chiral anomalies do cancel in the SM EFT for each SM fermion f contribution. There is an exception to this rule which is famous [4–6]: the SM couplings introduce chiral anomalies which cancel only if we add all leptons and quarks for each generation.

It is now time to discuss the explicit form of the chiral anomaly pieces, $\Pi^{\nu\rho(f)}_{V_i V_j V_k}$. Following the steps in Appendix B of Ref. [18] (see also Refs. [63,64,67]), using an anticommuting γ^5 and traces of γ matrices strictly in four dimensions, and, after a bit of algebra, the rhs of Eqs. (2.2)–(2.4), which is zero, now becomes

[9]Although we have taken $V_i = Z$, this equation is valid as well for $V_i = \gamma$ with $M_\gamma = 0$ and $\Pi^{\nu\rho(f)}_{\tilde{\gamma} V_j V_k} = 0$ since, apart from dipole operators, there are no other $d = 6$ EFT insertions to the photon-fermion-fermion vertex (they can be absorbed in the redefinition of the electric charge [66]).

$$\sum_f [\Pi^{\nu\rho(f)}_{V_iV_jV_k}(k_1,k_2;a) - \Pi^{\nu\rho(f)}_{\tilde{V}_iV_jV_k}(k_1,k_2;\tilde{a})] = \frac{1}{4\pi^2} L^{\nu\rho} \sum_f [\mathcal{A}^{(f)}_{V_iV_jV_k}(w-z) - \mathcal{A}^{(f)}_{\tilde{V}_iV_jV_k}(\tilde{w}-\tilde{z})], \tag{3.6}$$

$$\sum_f [\Pi^{\mu\rho(f)}_{V_iV_jV_k}(k_1,k_2;a) - \Pi^{\mu\rho(f)}_{V_i\tilde{V}_jV_k}(k_1,k_2;\tilde{a})] = \frac{1}{4\pi^2} L^{\mu\rho} \sum_f [\mathcal{A}^{(f)}_{V_iV_jV_k}(w-1) - \mathcal{A}^{(f)}_{V_i\tilde{V}_jV_k}(\tilde{w}-1)], \tag{3.7}$$

$$\sum_f [\Pi^{\mu\nu(f)}_{V_iV_jV_k}(k_1,k_2;a) - \Pi^{\mu\nu(f)}_{V_iV_j\tilde{V}_k}(k_1,k_2;\tilde{a})] = \frac{1}{4\pi^2} L^{\mu\nu} \sum_f [\mathcal{A}^{(f)}_{V_iV_jV_k}(z+1) - \mathcal{A}^{(f)}_{V_iV_j\tilde{V}_k}(\tilde{z}+1)], \tag{3.8}$$

where $L^{\mu\nu} = \epsilon^{\mu\nu\kappa\lambda} k_{1\kappa} k_{2\lambda}$ is a Lorentz kinematic factor for the anomaly and $\mathcal{A}^{(f)}_{V_iV_jV_k}$ is the anomaly factor

$$\mathcal{A}^{(f)}_{V_iV_jV_k} = a^{(f)}_{V_i} a^{(f)}_{V_j} b^{(f)}_{V_k} + a^{(f)}_{V_i} b^{(f)}_{V_j} a^{(f)}_{V_k} + b^{(f)}_{V_i} a^{(f)}_{V_j} a^{(f)}_{V_k} + b^{(f)}_{V_i} b^{(f)}_{V_j} b^{(f)}_{V_k} \tag{3.9}$$

for each fermion f, within one generation, circulating the triangles. The first three terms in (3.9) represent the vector-vector-axial chiral anomaly, whereas the last term is the axial-axial-axial (AAA) one. The coefficients $a^{(f)}$ and $b^{(f)}$ can be expanded as

$$a^{(f)}_{V_i} = \hat{a}^{(f)}_{V_i} + \tilde{a}^{(f)}_{V_i}, \qquad b^{(f)}_{V_i} = \hat{b}^{(f)}_{V_i} + \tilde{b}^{(f)}_{V_i}, \tag{3.10}$$

where $\hat{a}^{(f)}_{V_i}$ is the vector part of the SM-like[10] dimension-4 vertex with fermion f and vector boson V_i and $\tilde{a}^{(f)}_{V_i}$ is the pure dimension-6 corresponding part. All these couplings are explicitly produced in the Appendix. Similarly, for the axial-vector couplings $b^{(f)}_{V_i}$. We can further simplify Eqs. (3.6)–(3.8) by noting from Eqs. (3.10) and (3.9) that $\mathcal{A}^{(f)}_{\tilde{V}_iV_jV_k} = \mathcal{A}^{(f)}_{V_iV_jV_k} - \mathcal{A}^{(f)}_{\hat{V}_iV_jV_k}$. Consequently, for the chiral anomalies to cancel in the SM EFT, the rhs's of Eqs. (3.6)–(3.8) must vanish, that is,

$$\sum_f \mathcal{A}^{(f)}_{V_iV_jV_k}(w-z-\tilde{w}+\tilde{z}) + \sum_f \mathcal{A}^{(f)}_{\hat{V}_iV_jV_k}(\tilde{w}-\tilde{z}) = 0, \tag{3.11}$$

$$\sum_f \mathcal{A}^{(f)}_{V_iV_jV_k}(w-\tilde{w}) + \sum_f \mathcal{A}^{(f)}_{V_i\hat{V}_jV_k}(\tilde{w}-1) = 0, \tag{3.12}$$

$$\sum_f \mathcal{A}^{(f)}_{V_iV_jV_k}(z-\tilde{z}) + \sum_f \mathcal{A}^{(f)}_{V_iV_j\hat{V}_k}(\tilde{z}+1) = 0. \tag{3.13}$$

We now observe that the first terms on the lhs's of Eqs. (3.11)–(3.13) always vanish through the choices

$$w = \tilde{w}, \qquad z = \tilde{z}. \tag{3.14}$$

This selection indicates equal routing of momenta for the gauge and Goldstone-boson diagrams in Fig. 1. In other words, for such a choice, the Goldstone-boson triangle, the pure $1/\Lambda^2$ contribution, cancels the corresponding triangle vector-boson contribution. With (3.14), the rhs's of the Ward identities in Eqs. (2.2)–(2.4) become

$$\sum_f \mathcal{A}^{(f)}_{\hat{V}_iV_jV_k}(w-z) = 0, \tag{3.15}$$

$$\sum_f \mathcal{A}^{(f)}_{V_i\hat{V}_jV_k}(w-1) = 0, \tag{3.16}$$

$$\sum_f \mathcal{A}^{(f)}_{V_iV_j\hat{V}_k}(z+1) = 0. \tag{3.17}$$

Every WI contains one distinct SM-like vertex in a different vertex of the triangles, but the other two vertices, the uncharted ones, are generic SM EFT vertices. Note, however, that if we naively take $w = z = 0$ so that the first WI [Eq. (3.15)] is satisfied, as in the case implied in Ref. [11], the other two vertices never do so, in general.

First, let us assume that all vertices, except for one that we contract with the external momentum in the triangle, are SM-like. For these, there is no solution set for w and z such that anomalies cancel from all Ward identities—i.e., the system of (3.15)–(3.17) has no solution—unless it is $\mathcal{A}^{(f)}_{\hat{V}_i\hat{V}_j\hat{V}_k} = 0$. This is what happens in the SM, but only when we add contributions from all leptons (ℓ) and quarks (q) within a generation and for each term in (3.9),

$$\sum_{f=\ell,q} \mathcal{A}^{(f)}_{\hat{V}_i\hat{V}_j\hat{V}_k} = 0. \tag{3.18}$$

However, what about the cancellation of chiral anomalies when the vertex contracted is SM-like and the others are of

[10] As explicitly shown in the Appendix, the word *SM-like* here and below means the following: In the SM EFT, even at dimension-6 truncation, we need to rescale the gauge couplings by certain normalization constants (Z_g, $Z_{g'}$, etc.) that depend on flavor blind Wilson coefficients associated with operators such as $Q_{\varphi B}$ and $Q_{\varphi W}$ [15,66]. This rescaling cancels out in the anomaly-cancellation condition [cf. Eq. (3.18)], like in the SM.

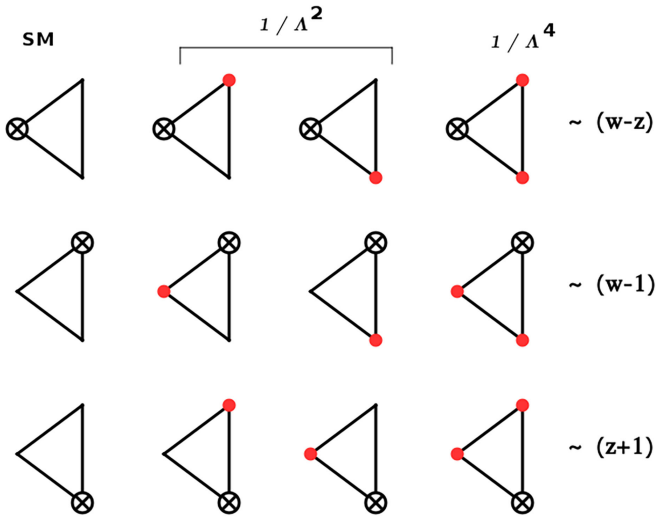


FIG. 3. The plan behind momentum routing choices for the pair of indices (w, z) in four-vector a^μ of Eq. (3.5) such that chiral anomalies cancel in the SM EFT. Every red blob indicates a pure EFT insertion, e.g., from $d = 6$ operators only. The circled multiplication symbol signifies momentum action on the vertex indicated, while the nondotted vertices are SM-like. From the top row to the bottom row, the triangles represent the three WIs in the SM EFT expansion, with anomaly terms in Eqs. (3.15)–(3.17).

the SM EFT type or, more broadly, when there are multiple higher-dimensional operators entering the triangle vertices?

It is enlightening to consider in detail one term from the rhs of $\mathcal{A}^{(f)}_{V_iV_jV_k}$ in Eq. (3.9), say, $a^{(f)}_{V_i} b^{(f)}_{V_j} a^{(f)}_{V_k} \equiv aba$ for clarity, but strictly in this order, and explain the logic behind anomaly cancellations at every order in EFT expansion. Then anomaly terms [apart from the pure SM one $\hat{a}\,\hat{b}\,\hat{a}$, which vanishes because of (3.18)] violating the three WIs found in Eqs. (3.15)–(3.17), can be summarized in the following table:

$\hat{a}\,\tilde{b}\,\hat{a}$	$\hat{a}\,\hat{b}\,\tilde{a}$	$\hat{a}\,\tilde{b}\,\tilde{a}$	$\propto (w-z)$,
$\tilde{a}\,\hat{b}\,\hat{a}$	$\hat{a}\,\hat{b}\,\tilde{a}$	$\tilde{a}\,\hat{b}\,\tilde{a}$	$\propto (w-1)$,
$\hat{a}\,\tilde{b}\,\hat{a}$	$\tilde{a}\,\hat{b}\,\hat{a}$	$\tilde{a}\,\tilde{b}\,\hat{a}$	$\propto (z+1)$.

The key point here is to notice that each term appears in at most two different rows of this matrix and, as such, the system for w and z always has a solution. For example, the $\hat{a}\,\tilde{b}\,\hat{a}$ term exists only in the first and third rows, so we can choose $w = z = -1$ to cancel it. Similarly, the $\hat{a}\,\hat{b}\,\tilde{a}$ term exists only in the first and second rows, so we can choose $w = z = 1$ to cancel it. Finally, the $\tilde{a}\,\hat{b}\,\hat{a}$ term exists only in the second and third rows, and we can choose $w = 1$ and $z = -1$ to cancel it. Therefore, one by one, all $1/\Lambda^2$ anomalous terms cancel. The third column contains, potentially, $1/\Lambda^4$ terms. However, they are all unique and they cancel by choosing w and z such that the terms in parentheses indicated next to them vanish. Therefore, $1/\Lambda^4$ anomaly terms cancel too. Furthermore, there are no $1/\Lambda^6$ terms in the anomaly expansion with dimension-6 operator insertions. This completes the proof for chiral anomaly cancellation with all insertions of $d = 6$ operators.

One could attempt to elucidate upon Eqs. (3.15)–(3.17) somewhat graphically. In Fig. 3 we have drawn the EFT expansion with $3 \times 4 = 12$ triangles for the three WIs. The circled multiplication symbol indicates that it is dotted with external momentum, which in turn means either cancellation of Wilson coefficients in that vertex between massive vector and Goldstone bosons leaving behind a pure SM vertex or simply a SM-like vertex. The red blob signifies an insertion of a dimension-6 operator. A vertex without a symbol is a SM-like vertex. It is also helpful to refer to triangles in a matrix notation; e.g., (1,1) is the top-row, left-column triangle. Again, it is more instructive to go through Fig. 3 with an example. We adopt the triangle γZZ (in order ijk) in the SM EFT with dimension-6 operators. A detailed expansion of the anomaly factors in (3.9) corresponding to each of the three WIs (3.15)–(3.17) and the use of Eq. (3.10) are

$$\begin{aligned}\mathcal{A}^{(f)}_{\gamma ZZ} &= \hat{a}^{(f)}_\gamma b^{(f)}_Z a^{(f)}_Z + \hat{a}^{(f)}_\gamma a^{(f)}_Z b^{(f)}_Z = \hat{a}^{(f)}_\gamma \hat{b}^{(f)}_Z \hat{a}^{(f)}_Z \\ &+ \hat{a}^{(f)}_\gamma \hat{a}^{(f)}_Z \hat{b}^{(f)}_Z + \hat{a}^{(f)}_\gamma \tilde{b}^{(f)}_Z \hat{a}^{(f)}_Z + \hat{a}^{(f)}_\gamma \hat{b}^{(f)}_Z \tilde{a}^{(f)}_Z \\ &+ \hat{a}^{(f)}_\gamma \tilde{a}^{(f)}_Z \hat{b}^{(f)}_Z + \hat{a}^{(f)}_\gamma \hat{a}^{(f)}_Z \tilde{b}^{(f)}_Z + \hat{a}^{(f)}_\gamma \tilde{b}^{(f)}_Z \tilde{a}^{(f)}_Z \\ &+ \hat{a}^{(f)}_\gamma \tilde{a}^{(f)}_Z \tilde{b}^{(f)}_Z, \end{aligned} \tag{3.19}$$

$$\begin{aligned}\mathcal{A}^{(f)}_{\gamma \hat{Z}\hat{Z}} &= a^{(f)}_\gamma \hat{b}^{(f)}_Z a^{(f)}_Z + a^{(f)}_\gamma \hat{a}^{(f)}_Z b^{(f)}_Z = \hat{a}^{(f)}_\gamma \hat{b}^{(f)}_Z \hat{a}^{(f)}_Z \\ &+ \hat{a}^{(f)}_\gamma \hat{a}^{(f)}_Z \hat{b}^{(f)}_Z + \hat{a}^{(f)}_\gamma \hat{b}^{(f)}_Z \tilde{a}^{(f)}_Z + \hat{a}^{(f)}_\gamma \hat{a}^{(f)}_Z \tilde{b}^{(f)}_Z, \end{aligned} \tag{3.20}$$

$$\begin{aligned}\mathcal{A}^{(f)}_{\gamma Z\hat{Z}} &= a^{(f)}_\gamma b^{(f)}_Z \hat{a}^{(f)}_Z + a^{(f)}_\gamma a^{(f)}_Z \hat{b}^{(f)}_Z = \hat{a}^{(f)}_\gamma \hat{b}^{(f)}_Z \hat{a}^{(f)}_Z \\ &+ \hat{a}^{(f)}_\gamma \hat{a}^{(f)}_Z \hat{b}^{(f)}_Z + \hat{a}^{(f)}_\gamma \tilde{b}^{(f)}_Z \hat{a}^{(f)}_Z + \hat{a}^{(f)}_\gamma \tilde{a}^{(f)}_Z \hat{b}^{(f)}_Z. \end{aligned} \tag{3.21}$$

The photon vertex to fermions is purely vectorial and there is no additional EFT insertion, apart from universal redefinitions of the gauge couplings. Therefore, triangles (2,2), (2,4), (3,3), and (3,4) do not exist for the γZZ triangle. The first line of these equations corresponds to the left column (1,1), (2,1), (3,1) of the triangles in Fig. 3. They all cancel (for every w and z) because of (3.18). The second line corresponds to the $1/\Lambda^2$ contributions. The first and third terms on the second line of (3.19) [triangle (1,2) in Fig. 3] exist only on the second line of (3.21) [triangle (3,2) in Fig. 3], and they cancel if we set the routing of momenta to $w = z = -1$ or, equivalently, the routing of the arbitrary vector $a^\mu = -k_1^\mu - k_2^\mu$. Similarly, the second and fourth terms on the second line of (3.19) [triangle (1,3) in Fig. 3]

exist only on the second line of (3.20) [triangle (2,3) in Fig. 3], and they cancel if we set the routing of momenta to $w = z = 1$ or, equivalently, the routing of the arbitrary vector $a^\mu = k_1^\mu + k_2^\mu$. Finally, the terms on the third line of (3.19) are of order $1/\Lambda^4$ and correspond to the triangle (1,4) in Fig. 3. They vanish automatically because we set $w = z$ in the previous cases. To persuade oneself, one may perform additional examples involving all other external vector-boson combinations.

The above examples, and other examples that the reader can come up with, have something in common. The rather strange picture of the choice of the routing parameters w and z applies only when we look at SM vertices. For instance, in the $\hat{a}\,\tilde{b}\,\hat{a}$, we only need to satisfy the WIs for the currents associated with the SM-vertex insertions $w = z$ and $z = -1$. For $\tilde{a}\,\tilde{b}\,\hat{a}$ only one WI has to be satisfied, the one associated with the k leg, and therefore we choose $z = -1$. For three EFT insertions, say, $\tilde{a}\,\tilde{b}\,\tilde{a}$, there is no WI to satisfy, and therefore no anomalies are associated with them. It is the same with all $V_iV_jV_k$ triangles, as the reader can personally verify. But what does this routing prescription imply at the end? It implies that the divergence of currents arising from EFT insertions does not really matter at low energies in regard to chiral anomalies. As we shall see below Eq. (5.2) within a UV model, these currents are associated with global symmetries that are not conserved anyway. In other words, chiral anomalies arising from EFT insertions are rather illusory in nature.

The freedom to choose arbitrary vectors a^μ and b^μ is thus twofold. First, the condition $b^\mu = -a^\mu$ eliminates the nonchiral anomaly and, second, our choice of (w, z) in $a^\mu = zk_1^\mu + wk_2^\mu$ as in Fig. 3 moves the anomaly with dimension-6 SM EFT couplings to those currents with SM-like couplings. The latter anomaly cancels out when all the SM fermion fields add up; i.e., this is so because the SM is an anomaly-free gauge theory.

Our analysis truncates the EFT expansion at dimension 6, and we do not include a complete basis of dimension-8 operators. Nevertheless, certain contributions that scale up to $1/\Lambda^4$ can arise even within a dimension-6 analysis. In this case, our choice for the pair (w, z) for anomaly cancellation of $1/\Lambda^2$ insertions displayed in Fig. 3 and highlighted by the examples above guarantees anomaly cancellation for the $1/\Lambda^4$ anomalous terms too. Though we lack proof, we are tempted to think that the same pattern will be repeated for higher-dimensional bases of operators.

In summary, the disappearance of triangle chiral anomalies in SM EFT takes place term by term in Eq. (3.9). The reason for cancellation is twofold: first, because the SM is an anomaly-free theory [e.g., Eq. (3.18)] and, second, because it is always possible to make a choice of the routing momenta for every SM fermion (f) circulating the loop triangle when there are one or more insertions of higher-dimensional operators.

C. Bose symmetry and chiral anomalies in the SM EFT

Is the routing of the momenta canceling the anomalies consistent with Bose symmetry? The answer is yes. This will be demonstrated here through several examples. Bose symmetry and anomaly cancellations for three identical gauge bosons imply the momentum routing parameters z and w which are displayed in Table I. First, consider the example aba given below Eq. (3.18), which corresponds to one of the various contributions in a ZZZ triangle vertex. Then Bose symmetry applies among identical bosons in the SM or among identical bosons in the SM EFT. For example, the term $\tilde{a}\,\hat{b}\,\hat{a}$ has a Bose symmetry among j and k external legs. Looking at Table I, Bose symmetry is satisfied with $w + z = 0$ and the cancellation of anomalies (with a single dimension-6 insertion) by $w = 1$ and $z = -1$. Another example is the term $\hat{a}\,\tilde{b}\,\hat{a}$, which exhibits a symmetry between the i and k legs. For anomalies to cancel, we need $w = z = -1$, which satisfies the requirement that $2z - w + 1 = 0$ for Bose symmetry. Another example, not considered thus far, is the bbb term in a ZZZ triangle. Take, for instance, the $\hat{b}\,\hat{b}\,\tilde{b}$ (AAA) anomaly term. This is Bose symmetric under the i and j legs, which from Table I means that $2w - z - 1 = 0$, and the anomaly cancels when $w = z = 1$. Hence, in the ZZZ triangle, Bose symmetry and the cancellation of anomalies in the SM EFT are inextricably linked.

Moving to a few more examples, we first consider the γZZ vertex, which was worked out in Eqs. (3.19)–(3.21), and we consider, for example, the $\hat{a}\,\tilde{b}\,\hat{a}$ term. This has a Bose symmetry after the j and k legs are interchanged in a full SM EFT vertex but has no Bose symmetry between the EFT insertion ($\tilde{b}$) and the SM ($\hat{a}$) vertices. Bose symmetry does not apply here. To the contrary, for the vertex with two EFT insertions, e.g., $\hat{a}\,\tilde{b}\,\tilde{a}$, which is of order $(1/\Lambda^4)$, we need $w + z = 0$ for Bose symmetry and $w - z = 0$ for anomaly cancellation, and are both satisfied when $w = z = 0$. Similarly, for the $Z\gamma Z$ triangle vertex, we have Bose symmetry among the i and k legs for identical Z bosons, which from Table I implies that $2z - w + 1 = 0$, and the anomaly cancellation in our prescription, e.g., $\tilde{b}_Z\hat{a}\,\tilde{a}$, requires $w = 1$. Both constraints are fulfilled when $w = 1$ (anomaly cancellation) and $z = 0$ (Bose symmetry). All the above results for Bose symmetries in triangle diagrams are in agreement with those found in Ref. [18].

TABLE I. Bose symmetry and chiral anomaly cancellations routing momentum rules with a single operator dimension-6 insertion, among external, identical gauge bosons.

External legs	Bose symmetry	Anomaly cancellation
$j \leftrightarrow k$	$w + z = 0$	$w = 1,\ z = -1$
$i \leftrightarrow j$	$2w - z - 1 = 0$	$w = z = 1$
$i \leftrightarrow k$	$2z - w + 1 = 0$	$w = z = -1$

We should note that our scheme here is possibly not unique, since there may be other choices of the arbitrary vectors' combinations (including those associated with the Goldstone boson) that preserve the WIs. We tried several scheme choices for arbitrary vectors based solely on Bose symmetry [even relaxing the constraint of Eq. (3.14)] but did not succeed in finding a simpler, meaningful picture when also trying to preserve the WIs.

Nevertheless, the important conclusion is that Bose symmetry is compatible with our prescription of choosing the routing of the momenta for anomaly cancellation (see Table I). Although Bose symmetry alone, i.e., without gauge symmetry, is not sufficient to resolve the chiral anomaly cancellation in SM EFT, determining the triple neutral gauge-boson vertices requires Bose symmetry, as we will see in the next section.

IV. NEUTRAL TRIPLE GAUGE-BOSON TRIANGLE VERTICES

Based on the previous analysis, we conclude that the ξ-parameter gauge dependence in our working amplitude, $e^+e^- \to V_jV_k$ [see Eq. (2.1) and Fig. 1], vanishes for all combinations of SM EFT dimension-6 vertex insertions in the triangle. This is a result of vanishing chiral anomalies in SM EFT with a certain routing of momenta in different SM EFT triangle diagrams. The master WI, Eq. (2.6), can always be rendered to hold.

It is certainly of phenomenological interest to check upon triple gauge-boson vertices (TGVs) in the SM EFT with dimension-6 insertions. nTGVs are of special interest since they do not exist at tree level in SM EFT in order $1/\Lambda^2$. We would like here to investigate whether finite threshold one-loop corrections appear from the triangle loop. To that end, we write the most general C-odd and P-odd, and hence CP-even, vertex for three external gauge bosons in the notation of the graph above Eq. (3.1) [or Fig. 1], in the form [18][11]

$$\begin{aligned}&\Delta^{\mu\nu\rho}_{V_iV_jV_k}(k_1,k_2;w,z)\\&= A_1(k_1,k_2;w)\epsilon^{\mu\nu\rho\alpha}k_{2\alpha} + A_2(k_1,k_2;z)\epsilon^{\mu\nu\rho\alpha}k_{1\alpha}\\&\quad + A_3(k_1,k_2)\epsilon^{\mu\rho\alpha\beta}k_2^\nu k_{1\alpha}k_{2\beta} + A_4(k_1,k_2)\epsilon^{\mu\rho\alpha\beta}k_1^\nu k_{1\alpha}k_{2\beta}\\&\quad + A_5(k_1,k_2)\epsilon^{\mu\nu\alpha\beta}k_2^\rho k_{1\alpha}k_{2\beta} + A_6(k_1,k_2)\epsilon^{\mu\nu\alpha\beta}k_1^\rho k_{1\alpha}k_{2\beta}.\end{aligned} \tag{4.1}$$

The Lorentz invariant vertex $\Delta^{\mu\nu\rho}$ has mass dimension $+1$ and parametrizes the result of the fermion triangle diagram. A_1 and A_2 are dimensionless, and a naive power-counting analysis suggests they are at most linearly divergent. Therefore, they depend on the regularization scheme or, in our case, the routing momentum parameters w, z. The form factors, $A_{3,\ldots,6}$, on the other hand, have the mass dimension -2 and are consequently finite; they can be calculated from the fermion triangle in any regularization scheme. The trick in [18] is to apply (4.1) in the already calculated Ward identities of Eqs. (2.3) and (2.4) together with the anomaly terms of Eqs. (3.7) and (3.8) on the rhs's and to find $A_1(k_1,k_2;w)$ and $A_2(k_1,k_2;z)$ in (4.1). The result is then expressed in terms of the finite integrals $A_{3\ldots6}$ plus anomalous terms proportional to w and z, which are defined from the anomaly-cancellation conditions and the Bose symmetry described in the previous section.

As an example, the result for the triple neutral vertex $V_i^*V_jV_k$, where V_i^* is an off-shell gauge boson and V_j and V_k are on-shell vector bosons $V=\gamma, Z$, in the momentum notation provided in Eq. (3.1) is[12]

$$\begin{aligned}&\Delta^{\mu\nu\rho}_{V_i^*V_jV_k}(k_1,k_2;w,z)\\&= \sum_f\Bigg\{\Bigg[k_1^2(A_4^{(f)}-A_3^{(f)}) - \frac{m_f^2 b_{V_j}^{(f)}}{\pi^2}I^{(f)}_{1V_iV_k}\\&\quad + \frac{\mathcal{A}^{(f)}_{V_iV_jV_k}}{4\pi^2}(w-1)\Bigg]\epsilon^{\mu\nu\rho\alpha}k_{2\alpha} + \Bigg[k_2^2(A_5^{(f)}+A_3^{(f)})\\&\quad - \frac{m_f^2 b_{V_k}^{(f)}}{\pi^2}I^{(f)}_{2V_iV_j} + \frac{\mathcal{A}^{(f)}_{V_iV_jV_k}}{4\pi^2}(z+1)\Bigg]\epsilon^{\mu\nu\rho\alpha}k_{1\alpha}\\&\quad + A_3^{(f)}q^\mu\epsilon^{\nu\rho\alpha\beta}k_{1\alpha}k_{2\beta}\Bigg\}.\end{aligned} \tag{4.2}$$

The integrals $A^{(f)}_{3,4,5}(k_1,k_2)$ and $I^{(f)}_{1,2}(k_1,k_2)$ are finite, i.e., independent of w and z, and their explicit expressions together with several asymptotic limits and identities can be found in Ref. [18].[13] The crucial terms in (4.2) are the anomaly terms proportional to the routing parameters $(w-1)$ and $(z+1)$: these must be taken precisely as described in Secs. III B and III C following the pattern indicated in Fig. 3 so that anomaly cancellation and Bose symmetry, and therefore gauge invariance, are preserved. In the following, we are going to apply Eq. (4.2) in deriving the neutral gauge boson form factors in the SM EFT with the dimension-6 operator insertions of (1.2).

A. The $Z^*\gamma\gamma$ vertex

We apply Bose symmetry between the two photons (j and k legs), and by setting $k_1^2=k_2^2=0$ and $b_{V_j}^{(f)}=b_{V_k}^{(f)}=0$ in Eq. (4.2), we find that

[11]Apart from the appearance of parameters w and z, Eq. (4.1) was utilized in an old work by Rosenberg [68]. Actually, there are two more form factors allowed in this expression, but they are reduced to (4.1) after using certain identities; see footnote 11 of [18].

[12]Both sides of Eq. (4.2) are multiplied by the product of polarization vectors $\epsilon_\nu^*(k_1)\epsilon_\rho^*(k_2)$, omitted here for clarity.

[13]The only notational difference is the anomaly factor. It is denoted here as $\mathcal{A}^{(f)}_{V_iV_jV_k}$, whereas it is denoted in Ref. [18] as c.

$$\Delta^{\mu\nu\rho}_{Z^*\gamma\gamma}(k_1,k_2;w,z)=\Delta^{\mu\rho\nu}_{Z^*\gamma\gamma}(k_2,k_1;w,z)\Rightarrow w+z=0 \quad \text{and} \quad A_3^{(f)}(k_1,k_2)=A_3^{(f)}(k_2,k_1). \tag{4.3}$$

By inspecting the anomaly factor,

$$\sum_f \mathcal{A}^{(f)}_{Z\gamma\gamma}=\sum_f b_Z^{(f)}a_\gamma^{(f)}a_\gamma^{(f)}=\sum_f \hat{b}_Z^{(f)}\hat{a}_\gamma^{(f)}\hat{a}_\gamma^{(f)}+\sum_f \tilde{b}_Z^{(f)}\hat{a}_\gamma^{(f)}\hat{a}_\gamma^{(f)}, \tag{4.4}$$

the first term cancels because SM is an anomaly-free theory and the second term cancels because of the choice of $w=-z=1$ for the anomaly cancellation in the SM EFT, following the pattern of Fig. 3 and therefore in accordance with the Bose symmetry of Eq. (4.3). The only term remaining from the rhs of Eq. (4.2) is the last one, which is proportional to q^μ. This is also compatible with the Landau [69] and Yang [70] theorem, which states that the vertex $Z\gamma\gamma$ must vanish when all particles are on shell due to invariance under rotation and inversion symmetries. Looking at the first line of Eq. (2.1), the $(s-M_Z^2)$ pole cancels, resulting in the amplitude

$$\sum_f \frac{2m_e b_Z^{(e)}}{M_Z^2}A^{(f)}_{3Z^*\gamma\gamma}(s)[\bar{v}(p_1)\gamma^5 u(p_2)]\epsilon^{\rho\nu\alpha\beta}k_{1\alpha}k_{2\beta}\epsilon_\nu^*(k_1)\epsilon_\rho^*(k_2), \tag{4.5}$$

where $s=(k_1+k_2)^2$ is the center of mass energy squared. Interestingly, in the limit of $s\gg m_t^2$, the integral A_3 is approximated with $\sum_f A_3^{(f)}(s\gg m_t^2)\approx\sum_f \mathcal{A}_{Z\gamma\gamma}/(2\pi^2 s)$. Because of (3.18), this term vanishes in the SM up to $1/s$, but not in the SM EFT. The conclusion is that there are threshold one-loop corrections, of the order $\frac{g^2}{4\pi^2}\frac{m_e^2}{M_Z^2}\frac{C^{(6)}v^2}{\Lambda^2}$, contributing to the s-channel at high energy from the triangle diagram in the SM EFT with dimension-6 operators from the list of (1.2), with Wilson coefficients collectively denoted here as $C^{(6)}$. This picture reveals the existence of heavy fermions, like the heavy electron in Sec. V, and/or vector bosons decoupled from UV physics at low energies. However, their effects in $Z^*\gamma\gamma$ are utterly invisible at current colliders' architecture due to the external SM fermion mass suppression. In the energy region below the top threshold, $M_Z<s<m_t$, the SM anomaly cancellation $\sum_{f\neq t}\mathcal{A}^{(f)}_{Z\gamma\gamma}\neq 0$ is incomplete and $A_3^{(f\neq t)}\approx-\frac{\mathcal{A}^{(f\neq t)}_{Z\gamma\gamma}}{24\pi^2 m_t^2}$. However, the contribution from dimension-6 operators is further suppressed relative to the SM by a factor $C^{(6)}v^2/\Lambda^2$.

B. The $V^*\gamma Z$ vertices

We now apply Eq. (4.2) to find the form factors h_3^V [24,25] in $V^*(q)\gamma(k_1)Z(k_2)$ vertices with $V=\gamma, Z$. For a $\gamma^*\gamma Z$ triangle vertex, we need $w=z=1$, and a short computation presents a gauge invariant form[14]

$$\Delta^{\mu\nu\rho}_{\gamma^*\gamma Z}(k_1,k_2)=\frac{s}{M_Z^2}\left(\epsilon^{\mu\nu\rho\alpha}k_{1\alpha}+\frac{\epsilon^{\nu\rho\alpha\beta}q^\mu k_{1\alpha}k_{2\beta}}{s}\right)\sum_f M_Z^2[A_3^{(f)}(s,m_f)]_{\gamma^*\gamma Z}, \tag{4.6}$$

with the associated form factor

$$eh_3^\gamma(s)=\sum_f M_Z^2[A_3^{(f)}(s,m_f)]_{\gamma^*\gamma Z}=\sum_f\frac{[\hat{a}_\gamma^{(f)}\hat{a}_\gamma^{(f)}\hat{b}_Z^{(f)}+\hat{a}_\gamma^{(f)}\hat{a}_\gamma^{(f)}\tilde{b}_Z^{(f)}]}{\pi^2}\int_0^1 dx\int_0^{1-x}dy\left[\frac{-M_Z^2xy}{M_Z^2x(x-1)-(s-M_Z^2)xy+m_f^2}\right]. \tag{4.7}$$

In this triangle there is only one dimension-6 insertion. The anomalous terms, i.e., those that depend on the routing momentum parameters w and z, played a crucial role in getting a gauge invariant result. The vertex vanishes when the external gauge bosons are on shell. The resulting $h_3^\gamma(s)$ from Eq. (4.7) can be written analytically in the limit where the center of mass energy is greater than the top-quark mass threshold,[15]

[14]Although the term proportional to q^μ is irrelevant for current collider phenomenology, we explicitly write it in all the following vertices to show explicit invariance under gauge symmetry (for $V=\gamma$) and Bose symmetry (for $V=Z$).

[15]The upper limit on s comes from the EFT validity.

$$h_3^\gamma(4m_t^2 \ll s \ll \Lambda^2) \simeq \frac{eg_Z}{8\pi^2}\left(\frac{M_Z^2}{s}\right)\left(\frac{v^2}{\Lambda^2}\right)\left[(C^{\varphi\ell(1)} + C^{\varphi\ell(3)} - C^{\varphi e}) + \frac{1}{3}(5C^{\varphi q(1)} - 3C^{\varphi q(3)} - 4C^{\varphi u} - C^{\varphi d})\right] - i\frac{1}{3\pi}(eg_Z)\left(\frac{M_Z^2}{s}\right)\left(\frac{m_t^2}{s}\right)\ln\left(\frac{s}{m_t^2}\right)\left[1 + \frac{v^2}{\Lambda^2}(C^{\varphi q(3)} - C^{\varphi q(1)} + C^{\varphi u})\right]. \tag{4.8}$$

In this limit, the leading SM effect vanishes due to the anomaly cancellation in Eq. (3.18), with the remaining pieces, not shown in (4.8), descending first as $\ln(s)/s^2$. The imaginary part of h_3^γ behaves like $M_Z^2 m_f^2 \ln(s/m_f^2)/s^2$ and therefore is non-negligible only for the top-quark mass. Therefore, in the SM EFT, Eq. (4.8) shows a kind of "delayed unitarity" at large s. On the other hand, for energies below the top threshold, there is an incomplete anomaly cancellation in the SM, and the SM EFT adds to this vertex, with terms proportional to Cv^2/Λ^2,

$$h_3^\gamma(M_Z^2 \ll s \ll 4m_t^2) \simeq \frac{eg_Z}{8\pi^2}\left\{\left(\frac{M_Z^2}{s}\right)\left[\frac{4}{3} + \frac{v^2}{\Lambda^2}(C^{\varphi\ell(1)} + C^{\varphi\ell(3)} - C^{\varphi e}) + \frac{v^2}{3\Lambda^2}(C^{\varphi q(1)} + C^{\varphi q(3)} - C^{\varphi d})\right] + \left(\frac{M_Z^2}{9m_t^2}\right)\left[1 + \frac{v^2}{\Lambda^2}(C^{\varphi q(3)} - C^{\varphi q(1)} + C^{\varphi u})\right]\right\}. \tag{4.9}$$

By examining Eqs. (4.8) and (4.9), we see that dimension-6 SM EFT effects are evident in both the low- and high-energy regions. Parametrically, however, the high-energy behavior of $h_3^\gamma(s)$ in Eq. (4.8) results in stronger deviations with respect to the SM because the latter scales at most like $\ln(s)/s^2$ due to the chiral anomaly-cancellation condition (3.18).

The $Z^*(q)\gamma(k_1)Z(k_2)$ triangle vertex behaves similarly. The absence of anomalies with dimension-6 insertions in $\hat{Z}\hat{\gamma}\tilde{Z}$ requires $w = 1$ and $z = 1$, while the triangle $\tilde{Z}\hat{\gamma}\hat{Z}$ needs $w = 1$ and $z = -1$ in Eq. (4.2). For two dimension-6 insertions $\tilde{Z}\hat{\gamma}\tilde{Z}$, we need $w = 1, \ \forall \ z$. Therefore, in this case of a double insertion, z is not defined by an anomaly-cancellation prescription. Nevertheless, Bose symmetry defines z precisely. From Table I, Bose symmetry in interchanging $i \leftrightarrow k$ legs with $w = 1$ results in $z = 0$. When all this is matched up with (3.18), the anomaly terms in Eq. (4.2) give $\sum_f \frac{1}{4\pi^2}\mathcal{A}_{Z\gamma Z}^{(f)}\epsilon^{\mu\nu\rho\alpha}k_{1\alpha}$ for all EFT insertions. Combining this result with the other terms in Eq. (4.2), we find that[16]

$$\Delta_{Z^*\gamma Z}^{\mu\nu\rho}(k_1,k_2) = \sum_f\left\{\frac{1}{2}[sA_3^{(f)} + M_Z^2(A_3^{(f)} + A_5^{(f)})]\epsilon^{\mu\nu\rho\alpha}k_{1\alpha} + A_3^{(f)}q^\mu\epsilon^{\nu\rho\alpha\beta}k_{1\alpha}k_{2\beta}\right\}_{Z^*\gamma Z}. \tag{4.10}$$

One can show that the expression in the squared parentheses is proportional to $(s - M_Z^2)$ for energies nearby the Z-pole mass, and therefore the entire vertex vanishes when all external particles are on shell. It is, therefore, customary to extract the factor $(s - M_V^2)$ and define

$$eh_3^Z(s) = \frac{M_Z^2}{2(s - M_Z^2)}\sum_f[sA_3^{(f)} + M_Z^2(A_3^{(f)} + A_5^{(f)})]_{Z^*\gamma Z}. \tag{4.11}$$

In the low- and high-s regimes, expressions similar to Eqs. (4.8) and (4.9) can be derived. In the high-energy region, for example, we find at leading order in s,

$$eh_3^Z(4m_t^2 \ll s \ll \Lambda^2) \simeq \frac{M_Z^2}{2\pi^2 s}\sum_f[a_Z^{(f)}a_\gamma^{(f)}b_Z^{(f)}] + i\frac{3M_Z^2}{\pi s^2}[a_Z^{(t)}a_\gamma^{(t)}b_Z^{(t)}]m_t^2\ln\left(\frac{s}{m_t^2}\right), \tag{4.12}$$

whereas in the low-s region,

$$eh_3^Z(M_Z^2 \ll s \ll 4m_t^2) \simeq \frac{M_Z^2}{2\pi^2 s}\sum_{f\neq t}[a_Z^{(f)}a_\gamma^{(f)}b_Z^{(f)}] - \frac{1}{8\pi^2}\left(\frac{M_Z^2}{m_t^2}\right)[a_Z^{(t)}a_\gamma^{(t)}b_Z^{(t)}] + \mathcal{O}(M_Z^4/s^2, M_Z^2 s/m_t^4). \tag{4.13}$$

[16]We suppress the arguments of the A_i and I_i functions in the following paragraphs, i.e., $A_3^{(f)} = A_3^{(f)}(k_1,k_2)$, etc.

By using the results from the Appendix for $a_\gamma^{(f)}$, $a_Z^{(f)}$, and $b_Z^{(f)}$ in Eqs. (4.12) and (4.13), we arrive at the dominant contributions to $h_3^Z(s)$, with the Wilson coefficients written explicitly in analogy to Eqs. (4.8) and (4.9) for $h_3^\gamma(s)$.

C. The V^*ZZ vertices

We start with $V=\gamma$. Bose symmetry and anomaly cancellation following the routing momenta described in Table I and Sec. III C results in the $\frac{A_{\gamma ZZ}^{(f)}}{4\pi^2}\epsilon^{\mu\nu\rho\alpha}(k_1-k_2)_\alpha$ anomaly term in (4.2) for both single and double dimension-6 insertions, which, after some algebra, finally yields

$$\Delta^{\mu\nu\rho}_{\gamma^*ZZ}(k_1,k_2)=\frac{s}{M_Z^2}\left[\epsilon^{\mu\nu\rho\alpha}(k_1-k_2)_\alpha+2\frac{\epsilon^{\nu\rho\alpha\beta}q^\mu k_{1\alpha}k_{2\beta}}{s}\right]\sum_f\frac{1}{2}M_Z^2[A_3^{(f)}(s,m_f)]_{\gamma^*ZZ}. \tag{4.14}$$

This is a gauge invariant result that vanishes for all particles on shell, as it should. According to the notation (up to a possible overall sign) of Refs. [24,25], we find the CP-even form factor

$$\begin{aligned} ef_5^\gamma(s)&=\sum_f\frac{1}{2}M_Z^2[A_3^{(f)}(s,m_f)]_{\gamma^*ZZ}\\ &=\sum_f\frac{[\hat{a}_\gamma^{(f)}a_Z^{(f)}b_Z^{(f)}]}{\pi^2}\int_0^1dx\int_0^{1-x}dy\left[\frac{-M_Z^2xy}{M_Z^2x(x-1)+M_Z^2y(y-1)-(s-2M_Z^2)xy+m_f^2}\right], \end{aligned} \tag{4.15}$$

where, as denoted in this study, uncharted $a_V^{(f)}$ and $b_V^{(f)}$ are the full vector and axial-vector couplings in the SM EFT (see the Appendix) or, in fact, in any theory extending the Standard Model with a single flavor fermion of mass m_f circulating the triangle. The double integral in Eq. (4.15) can be expanded for small or large s and written in a form analogous to Eqs. (4.8) and (4.9).

The Z^*ZZ vertex can be similarly derived using Eq. (4.2). For this vertex, the neutrino is also involved directly in the triangle graph. However, the analytical form is slightly more complicated here and takes the form

$$\Delta^{\mu\nu\rho}_{Z^*ZZ}(k_1,k_2)=\epsilon^{\mu\nu\rho\alpha}(k_1-k_2)_\alpha\sum_f\left[M_Z^2(A_3^{(f)}-A_4^{(f)})_{Z^*ZZ}+\frac{m_f^2}{\pi^2}b_Z^{(f)}I_{1ZZ}^{(f)}+\frac{\mathcal{A}_{ZZZ}^{(f)}}{6\pi^2}\right]+[A_3^{(f)}]_{Z^*ZZ}q^\mu\epsilon^{\nu\rho\alpha\beta}k_{1\alpha}k_{2\beta}. \tag{4.16}$$

This result vanishes for all Z bosons on shell, i.e., for $s=M_Z^2$, as one can show after a tedious calculation. It is again important to notice that the "anomaly" term proportional to $\mathcal{A}_{ZZZ}^{(f)}$ is a result of the specific routing of w and z depicted in Fig. 3 and Table I for chiral anomaly cancellation and Bose symmetry, respectively. If we had adopted a different routing of momenta, that is, a different prefactor in the anomaly term, Eq. (4.16) would not vanish on shell and would not be Bose symmetric. In fact, in the SM EFT and at large $s\ll\Lambda^2$, this is the dominant piece in the Z^*ZZ-vertex form factor $f_5^Z(s)$,

$$ef_5^Z(4m_t^2\ll s\ll\Lambda^2)\simeq\frac{M_Z^2}{6\pi^2 s}\sum_f\mathcal{A}_{ZZZ}^{(f)}=\frac{M_Z^2}{6\pi^2 s}\sum_f[(b_Z^{(f)})^3+3(a_Z^{(f)})^2(b_Z^{(f)})], \tag{4.17}$$

where $a_Z^{(f)}$ and $b_Z^{(f)}$ are the full SM EFT vertices read from the Appendix. Although the SM contributions [for both real and imaginary parts of $f_5^Z(s)$] vanish as $M_Z^2m_t^2/s^2$, the SM EFT higher-dimensional pieces do not, and they scale like $1/s$. As we will see in the next section, this reflects the nondecoupling[17] of the new physics fermions that reside at energy scales of order Λ. This situation, however, can be seen even at low energies far below the top-quark threshold, assuming the latter to be very heavy. Because of the identity $m_t^2\lim_{m_t^2\to\infty}I_1=-\frac{1}{6}\mathcal{A}_{ZZZ}^{(t)}$, the second term in the square brackets of Eq. (4.16) cancels against the anomaly term in Eq. (4.16), leaving behind an incomplete contribution from light quarks and leptons,

$$\begin{aligned} &ef_5^Z(M_Z^2\ll s\ll 4m_t^2)\\ &\simeq\frac{M_Z^2}{6\pi^2 s}\sum_{f\neq t}\mathcal{A}_{ZZZ}^{(f)}-\frac{\mathcal{A}_{ZZZ}^{(t)}}{3\pi^2}\left(\frac{M_Z^2}{s}\right)\left(\frac{M_Z^2}{4m_t^2}\right)+\mathcal{O}(M_Z^2/4m_t^2), \end{aligned} \tag{4.18}$$

[17]Here the terminology *nondecoupling* simply refers to the observable scaling behavior that survives in the low-energy EFT, not to a breakdown of the EFT expansion.

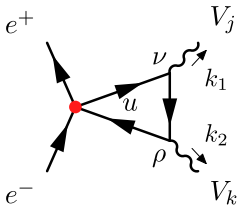
that scale like $1/s$ in the SM, affected trivially by SM EFT corrections to Zff vertices. The same happens for all vertices (or form factors); see, for instance, Eq. (4.9).

D. nTGVs through four-fermion vertices

Thus far, we have analyzed SM EFT effects from the set of operators in (1.2). These operators are induced, for example, after the decoupling of heavy fermions, in a way that we shall illustrate in the next section. There are, however, contributions to C-odd and P-odd form factors $h_3^V(s)$ and $f_5^V(s)$ induced by contact four-fermion vertices. Consider, for example, the operator (for simplicity, we assume one flavor generation)

$$O_{eu} = (\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R), \tag{4.19}$$

which arises from integrating out in an anomaly-free UV theory a heavy scalar leptoquark or a heavy vector boson (see, for instance, Refs. [71,72]) with appropriate gauge charges and couplings. We will again consider the amplitude, $e^+(p_1)e^-(p_2) \to V_j(k_1)V_k(k_2)$ as a working example. The relevant diagram is



$+$ crossed diagram ,

where the red dot is the operator O_{eu} insertion. For convenience, the rest of the notation can be adopted from the diagram above Eq. (3.1) and Fig. 1. This triangle diagram is linearly divergent, and techniques similar to those discussed previously apply. The WIs in (2.3) and (2.4) in legs V_j and V_k, respectively, must be satisfied. Consequently, there is an additional contribution to the amplitude $e^+(p_1)e^-(p_2) \to V_j^\nu(k_1)V_k^\rho(k_2)$ of Eq. (2.1) which reads

$$-\frac{1}{2}[\bar{v}(p_1)\gamma_\mu(1+\gamma^5)u(p_2)]\Delta^{\mu\nu\rho}_{\tilde{V}_i V_j V_k}(k_1,k_2;w,z). \tag{4.20}$$

The vertex $\Delta^{\mu\nu\rho}_{\tilde{V}_i V_j V_k}$ is given in Eq. (4.2). The arbitrary parameters w and z are defined by satisfying the WIs. These are $w=1$ and $z=-1$ for only one SM EFT insertion in vertex i. In this case, the fictitious gauge boson $\tilde{V}_i$ has only SM EFT couplings to fermions. For example, for the operator (4.19), we have [15]

$$a^{(eu)}_{\tilde{V}_i} = b^{(eu)}_{\tilde{V}_i} = \frac{C^{eu}}{2\Lambda^2}. \tag{4.21}$$

For example, the contribution (4.20) emerges at low energies after a heavy gauge boson (such as Z') is decoupled from a UV theory.

V. TOP-DOWN APPROACH FOR CHIRAL ANOMALIES AND NTGVS

An examination of a specific UV-model can enhance and clarify the bottom-up approach to chiral anomalies and TGVs discussed in the preceding sections. We will focus on the chiral anomalies and decoupling effects of an anomaly-free renormalizable gauge theory, which adds a heavy vectorlike electron E to the SM. We will next show how anomalies cancel in the full theory of this UV model after electroweak symmetry breaking. We then integrate out the heavy particles and derive the SM EFT of this model, verifying the bottom-up picture we found in Sec. III. As an illustration, we also calculate the nTGV form factor $h_3^\gamma(s)$ and compare it with our SM EFT results in Sec. IV. As far as the decoupling of heavy fermions is concerned, this simple model incorporates most of the necessary ingredients for studying the chiral anomalies and nTGVs in a top-down approach.

A. A simple UV-theory example

Consider a model with a heavy vectorlike electron $E(1,1,-1)$ which is a singlet under $SU(3)_c$ and $SU(2)_L$ gauge groups but has hypercharge identical to the right-handed electron, $Y_E=-1$. The field E has a Dirac mass M which is assumed to be much heavier than the electroweak scale, $M \gg v$, with v the vacuum expectation value of the Higgs field, φ. The full Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm BSM}, \tag{5.1}$$

where $\mathcal{L}_{\rm SM}$ is the SM Lagrangian[18] and

$$\begin{aligned}\mathcal{L}_{\rm BSM} = {}& \bar{E}_L i\not{D} E_L + \bar{E}_R i\not{D} E_R - M(\bar{E}_L E_R + \bar{E}_R E_L) \\ & - y_E \overline{\ell}_L \cdot \varphi E_R - y_E^* \bar{E}_R \varphi^\dagger \cdot \ell_L,\end{aligned} \tag{5.2}$$

where $D_\mu = \partial_\mu + ig' Y_E B_\mu$ is the covariant derivative and B_μ is the gauge field associated with the hypercharge symmetry.[19] The full Lagrangian $\mathcal{L}$ is gauge-anomaly-free since we have added to the SM Lagrangian a Dirac fermion E, which in turn means setting the vectorlike condition, $b_Z^{(E)}=0$, in the anomaly factor in Eq. (3.9). The axial current $J_A^\mu = \bar{E}\gamma^\mu\gamma^5 E$ associated with the chiral symmetry $E \to e^{i\theta\gamma^5}E$ is never conserved, i.e., $\partial_\mu J_A^\mu = 2iM\bar{E}\gamma^5 E$, because M is the mass of the "heavy" vectorlike fermion, and therefore the limit $M \to 0$ is unattainable by definition.

In the broken electroweak (EW) phase, $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$, the heavy ($E$) and light ($e$) electron

[18]We use the notation of Ref. [15] throughout.

[19]In $\mathcal{L}_{\rm BSM}$, terms like $m\bar{E}_L e_R + {\rm H.c.}$ that mix heavy and light electron fields are also allowed. It is easy to show that these terms can be eliminated by a redefinition of E_R and e_R fields, leaving (5.2) as a starting BSM Lagrangian.

fields' mass terms mix each other through the vacuum expectation value v of the Higgs field $\varphi^T = (G^+, \varphi_0)$ with $\varphi_0 = \frac{1}{\sqrt{2}}(v + h + iG^0)$ and the Yukawa coupling y_E. By assuming that the electron Yukawa coupling, $y_e \bar{\ell}_L \varphi e_R + \text{H.c}$, is zero for simplicity, we obtain

$$\mathcal{L}^{\text{mass}}_{\text{BSM}} = -(\,\bar{e}_L \quad \bar{E}_L\,)\mathcal{M}\begin{pmatrix} e_R \\ E_R \end{pmatrix} - (\,\bar{e}_R \quad \bar{E}_R\,)\mathcal{M}^\dagger\begin{pmatrix} e_L \\ E_L \end{pmatrix}, \tag{5.3}$$

with the mass matrix $\mathcal{M}$ being

$$\mathcal{M} = \begin{pmatrix} 0 & y_E \frac{v}{\sqrt{2}} \\ 0 & M \end{pmatrix}. \tag{5.4}$$

Applying the singular value decomposition theorem [73], we obtain $\mathcal{M} = U_e D_e W_e^\dagger$, where D_e is the diagonal mass matrix

$$D_e = \begin{pmatrix} 0 & 0 \\ 0 & M\sqrt{1+|\epsilon|^2} \end{pmatrix}, \quad \text{with} \quad \epsilon \equiv \frac{y_E v}{\sqrt{2}M}. \tag{5.5}$$

The matrices U_e and W_e are unitary, diagonalizing the Hermitian matrices $\mathcal{M}\mathcal{M}^\dagger$ and $\mathcal{M}^\dagger\mathcal{M}$, respectively. Explicitly, they are written as

$$U_e = \frac{1}{\sqrt{1+|\epsilon|^2}}\begin{pmatrix} 1 & \epsilon \\ -\epsilon^* & 1 \end{pmatrix}, \quad W_e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{5.6}$$

There are a few remarks to state here: First, the mass of the heavy fermion E is shifted upward from M to $M' = M\sqrt{1+|\epsilon|^2}$ in the new (mass) basis, while the light electron stays massless (since we have set $y_e = 0$); by increasing y_E, the scale M' will also increase for fixed M. Second, the transformation to the new basis affects only the left-handed heavy and light electrons

$$\begin{pmatrix} e'_L \\ E'_L \end{pmatrix} = U_e^\dagger \begin{pmatrix} e_L \\ E_L \end{pmatrix}, \tag{5.7}$$

not the right-handed ones [see Eq. (5.6)]; in other words, it is potentially anomalous. Although in the new "prime" basis we have decoupled propagators between heavy (E') and light fermions (e'),

$$\mathcal{L}^{\text{mass}}_{\text{BSM}} = (\,\bar{e}' \quad \bar{E}'\,)D_e\begin{pmatrix} e' \\ E' \end{pmatrix}, \tag{5.8}$$

contributions to the triangle anomalies will show up in the interaction of gauge-boson vertices with heavy-light electrons. It is straightforward to find the relevant interaction terms with the currents associated to the weak gauge bosons $W^\pm$ and Z and the photon A,

$$\mathcal{L} \supset -g(W^+_\mu J^{\mu+}_W + W^-_\mu J^{\mu-}_W) - g_Z Z_\mu J^\mu_Z - eA_\mu J^\mu_{\text{EM}}, \tag{5.9}$$

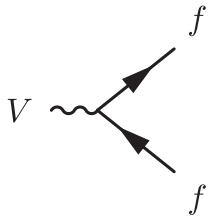
$$J^{\mu+}_W = \frac{1}{\sqrt{2}}[\bar{\nu}_L \gamma^\mu (U_e)_{11} e'_L + \bar{\nu}_L \gamma^\mu (U_e)_{12} E'_L], \qquad J^{\mu-}_W = (J^{\mu+}_W)^\dagger, \tag{5.10}$$

$$\begin{aligned} J^\mu_Z = {} & \bar{\nu}_L \gamma^\mu \left(\frac{1}{2}\right)\nu_L + \bar{e}'_L \gamma^\mu \left(-\frac{1}{2}|(U_e)_{11}|^2 + s_w^2\right)e'_L + \bar{E}'_L \gamma^\mu \left(-\frac{1}{2}|(U_e)_{12}|^2 + s_w^2\right)E'_L + \bar{e}'_L \gamma^\mu \left[-\frac{1}{2}(U_e)^*_{11}(U_e)_{12}\right]E'_L \\ & + \bar{E}'_L \gamma^\mu \left[-\frac{1}{2}(U_e)^*_{12}(U_e)_{11}\right]e'_L + \bar{e}'_R \gamma^\mu (s_w^2) e'_R + \bar{E}'_R \gamma^\mu (s_w^2) E'_R, \end{aligned} \tag{5.11}$$

$$J^\mu_{\text{EM}} = \bar{e}'_L \gamma^\mu (-1) e'_L + \bar{e}'_R \gamma^\mu (-1) e'_R + \bar{E}'_L \gamma^\mu (-1) E'_L + \bar{E}'_R \gamma^\mu (-1) E'_R, \tag{5.12}$$

with g the $SU(2)_L$-gauge coupling, $g_Z = \sqrt{g'^2 + g^2} = e/s_W c_W$,[20] and e the electron electric charge. Obviously, from Eqs. (5.10) and (5.11), there are heavy-light fermion mixed vertices in the weak charged and neutral currents but not in the electromagnetic current, Eq. (5.12). As we demonstrate below, chiral anomalies cancel out if and only if we consider these interactions. Turning all fermions into Dirac spinors and reading vector $(a_V^{(f)})$ and vector-axial couplings $(b_V^{(f)})$ from the vertex

[20] It is $s_W \equiv \sin(\theta_W)$ and $c_w \equiv \cos(\theta_w)$, with θ_w the weak mixing angle.



$$= -i\,\bar{f}\,\gamma^{\mu}(a_V^{(f)} + b_V^{(f)}\gamma^5)\,f\ , \tag{5.13}$$

with $V = Z, \gamma$, we find that

$$a_Z^{(\nu)} = \frac{g_Z}{4}, \qquad b_Z^{(\nu)} = -\frac{g_Z}{4}, \qquad a_\gamma^{(\nu)} = 0, \tag{5.14}$$

$$a_Z^{(e')} = \frac{g_Z}{2}\left(-\frac{1}{2}|(U_e)_{11}|^2 + 2s_w^2\right), \qquad b_Z^{(e')} = \frac{g_Z}{4}|(U_e)_{11}|^2, \qquad a_\gamma^{(e')} = -e, \tag{5.15}$$

$$a_Z^{(E')} = \frac{g_Z}{2}\left(-\frac{1}{2}|(U_e)_{12}|^2 + 2s_w^2\right), \qquad b_Z^{(E')} = \frac{g_Z}{4}|(U_e)_{12}|^2, \qquad a_\gamma^{(E')} = -e, \tag{5.16}$$

$$a_Z^{(e'-E')} = -\frac{g_Z}{4}(U_e)_{11}^*(U_e)_{12}, \qquad b_Z^{(e'-E')} = \frac{g_Z}{4}(U_e)_{11}^*(U_e)_{12}, \qquad a_\gamma^{(e'-E')} = 0, \tag{5.17}$$

plus $a_V^{(q)}$ and $b_V^{(q)}$ for quarks as in the SM.[21] In addition, from Eq. (5.11), we have $a_Z^{(E'-e')} = [a_Z^{(e'-E')}]^*$ and $b_Z^{(E'-e')} = [b_Z^{(e'-E')}]^*$. Of course, there are no axial-vector couplings to the photon, $b_\gamma^{(f)} = 0$.

To check the WIs in Eqs. (2.2)–(2.4), we also need the neutral Goldstone couplings to fermions. They can easily be found to be[22]

$$\mathcal{L}^{G^0} = -(iG^0\bar{E}'\gamma^5 E')\left(\frac{M'}{v}|(U_e)_{12}|^2\right) - (iG^0\bar{e}'P_R E')\left(\frac{M'}{v}(U_e)_{11}^*(U_e)_{12}\right) + (iG^0\bar{E}'P_L e')\left(\frac{M'}{v}(U_e)_{11}(U_e)_{12}^*\right). \tag{5.18}$$

In pure analogy with vector-boson vertices to fermions, here again there are contributions to Goldstone triangles from heavy and heavy/light fermion vertices. Moreover, and not without reason, these couplings look alike to the corresponding $V\bar{f}f$ couplings in Eq. (5.11). They are made as such to satisfy the WI in Eq. (2.5).

Even at this point, we can contact the EFT expansion of the theory. The true expansion parameter here is ϵ as defined in Eq. (5.5). Then from Eq. (5.6) we can expand the U_e-matrix elements and write

$$\begin{aligned} |(U_e)_{11}|^2 &= 1 - |\epsilon|^2 + |\epsilon|^4 + [\mathcal{O}(\epsilon)]^6, \\ |(U_e)_{12}|^2 &= |\epsilon|^2 - |\epsilon|^4 + [\mathcal{O}(\epsilon)]^6. \end{aligned} \tag{5.19}$$

It is interesting that we can directly read the Wilson coefficients in SM EFT arising from the decoupling of the heavy electron E' by matching the Feynman rule for the three-point Zee vertex of the model at hand [see Eq. (5.15)] with the corresponding SM EFT Feynman rule in the Appendix in the broken phase. By simultaneously noting that, for this particular model, the $Z\nu\nu$ vertex is the same as in the SM, we easily find the matching condition,

$$C^{\varphi e} = 0, \qquad C^{\varphi\ell(1)} = C^{\varphi\ell(3)} = -\frac{|y_E|^2}{4}, \qquad \Lambda = M, \tag{5.20}$$

which is in agreement with Ref. [71]. With this in mind, we now possess all the necessary components to ascertain whether the chiral anomaly cancellation exists, and we convey a few representative nTGVs for this particular model. We show these in the following examples.

1. Example: Anomaly cancellation in a $Z\gamma\gamma$ triangle and a nTGV

This is the simplest situation with only one possible AVV-type anomaly. The anomaly factor of Eq. (3.9) for light and heavy electrons contains

[21] In this model with heavy vectorlike electrons, the quark sector remains untouched with respect to the SM. Recall, however, that the pairs $a_V^{(q)}$ and $b_V^{(q)}$ for $q = u, d$ are necessary for canceling the chiral anomalies. In our notation, they can be read directly from Eq. (26) of Ref. [18].

[22] There are no $G^0\bar{e}'\gamma^5 e'$ terms, because we have set y_e to zero for simplicity.

$$\sum_{f=e',E'} \mathcal{A}^{(f)}_{Z\gamma\gamma} = b^{(e')}_Z a^{(e')}_\gamma a^{(e')}_\gamma + b^{(E')}_Z a^{(E')}_\gamma a^{(E')}_\gamma$$
$$= \frac{g_Z e^2}{4}[|(U_e)_{11}|^2 + |(U_e)_{12}|^2] = \frac{g_Z e^2}{4}, \quad (5.21)$$

where the last step follows from the fact that U_e is a unitary matrix. Needless to say, the anomaly piece of Eq. (5.21) cancels when we add the contribution from up and down quarks. In this UV-complete theory, the cancellation happens independent of the routing of the momenta. However, failure to include the heavy fermions E' in (5.21), even if those are vectorlike in the unbroken phase, will cause a chiral anomaly in the broken phase of order $|y_E|^2 v^2/M^2$. As we will show below, the effect of the heavy particle E' triangle loop is magically implemented in the SM EFT through the Goldstone-boson vertex to SM fermions.

One could also verify that pieces from finite anomalous terms arising from Goldstone- ($G^0\gamma\gamma$) and gauge-boson ($Z\gamma\gamma$) triangle diagrams cancel out explicitly by themselves satisfying the WIs in Eqs. (2.2)–(2.4). This is almost obvious from Eq. (3.2) with $m'_e = 0$ and $m'_E = M'$, and from Eq. (3.3) with the Goldstone- and Z-boson vertices to heavy electrons E' taken from Eqs. (5.18) and (5.16), respectively.

The anomaly factor in Eq. (5.21) is the same for all interchanges of the external legs $\mathcal{A}^{(f)}_{Z\gamma\gamma} = \mathcal{A}^{(f)}_{\gamma Z\gamma} = \mathcal{A}^{(f)}_{\gamma\gamma Z}$. An anomaly-free gauge extension of the SM does not mean that there are no effects in the triple gauge-boson vertices, although the truth is that the anomaly-cancellation condition $\sum_f \mathcal{A}^{(f)} = 0$ hugely suppresses these effects. It is interesting, phenomenologically at least, to find the $\gamma(q)^*\gamma(k_1)Z(k_2)$ vertex where the incoming photon is off shell and the outgoing photon and Z boson are on-shell particles.

Using Eq. (4.6) or, instead, directly using Eq. (4.7) for the model at hand, we find, in the "intermediate-energy" region between the top threshold and the high scale M,

$$\Re e[h^\gamma_3(s)] \simeq -\frac{g_Z e}{8\pi^2}\left(\frac{|y_E|^2 v^2}{M'^2}\right)\left[\frac{M_Z^2}{2s} + \frac{1}{24}\frac{M_Z^2}{M'^2} + \cdots\right],$$
$$4m_t^2 \ll s \ll M'^2. \quad (5.22)$$

The effect on $h^\gamma_3(s)$ from the decoupling of a heavy electron E is twofold: First, a dimension-6 contribution[23] which, however, drops off with the square of the center of mass energy like $1/s$ and, second, a constant with an energy dimension-8 contribution. The $1/2s$ term arises because of the reduced $Ze'e'$ vertex relative to the SM by a factor $|(U^e)_{11}|^2$ which does not cancel in the effective SM anomaly-cancellation condition. On the other hand, the $1/24M'^2$ term appears from the decoupling of the heavy E' with a suppression prefactor.

Hence, it appears from Eq. (5.22) that the dimension-6 contribution is always more important than the dimension-8 for center of mass energies $s < M'^2$. To our knowledge, this fact has been overlooked in the literature, where sometimes the opposite conclusions are stated. Although the dimension-8 contribution[24] of Eq. (5.22) agrees with the recent literature [47,48], the dimension-6 term of Eq. (5.22) has been entirely ignored there.

One may think that a conclusion from Eq. (5.22) is a result particularly attributed to the considered model with a heavy vectorlike electron. We believe, however, that it is generic to all models of decoupling anomaly-free clusters of heavy fermions with portal Yukawa couplings to the SM (like y_E here). If the dimension-8 term is larger than the $1/s$ term, then violation of unitarity will take place,[25] contrary to our consideration for a renormalizable and anomaly-free theory at high energies.

Furthermore, in the low-s region below the top threshold, we find that

$$h^\gamma_3(s) \simeq \frac{e g_Z}{8\pi^2}\left[\left(\frac{M_Z^2}{s}\right)\left(\frac{4}{3} - \frac{|y_E|^2 v^2}{2M'^2}\right) + \left(\frac{M_Z^2}{9m_t^2} - \frac{|y_E|^2}{24}\frac{v^2 M_Z^2}{M'^4}\right)\right],$$
$$M_Z^2 \ll s \ll 4m_t^2, \quad (5.23)$$

with a negligible imaginary part that is not shown. In this case, chiral anomalies do not cancel out due to the top-quark mass decoupling and the SM contribution is enhanced due to the light lepton and quark contribution. The dimension-6 contribution is the same as in Eq. (5.22), and in this model decreases the values of h^γ_3, in full agreement with the SM EFT outcome of Eq. (4.9) plugged into the matching condition (5.20). The dimension-8 term, i.e., the last term in (5.23), is completely negligible in this region.

Finally, in the energy region above the heavy fermion E mass $s \gg M'^2$, the form factor $h^\gamma_3(s)$ drops off much faster[26] than $1/s$ due to the exact chiral anomaly cancellation [see Eq. (5.21) and the discussion below]. We have numerically calculated $h^\gamma_3(s)$ without making approximations and discuss all the above results quantitatively in Sec. VI.

[23] We derived in Sec. IV B that this is precisely the dimension-6 SM EFT contribution we can see by applying the Wilson coefficients from Eq. (5.20) to Eq. (4.8).

[24] Following Ref. [47], the dimension-8 contribution arises from a linear combination of Wilson coefficients associated with four operators of the kind $\varphi^\dagger \tilde{V}^{\mu\nu}(D^\rho V_{\nu\rho})D_\mu\varphi$, with $(\tilde{V}_{\mu\nu})V_{\mu\nu}$ the (dual) field-strength tensor for either the $U(1)_Y$ or $SU(2)_L$ gauge group.

[25] By definition, $h^\gamma_3(s)$ is a monotonically decreasing function away from particle thresholds.

[26] In fact, as $h^\gamma_3(s \gg M'^2) \simeq \frac{M_Z^2 M'^2}{s^2}[\ln^2(\frac{s}{M'^2}) - \pi^2]$.

2. Example: Anomaly cancellation in a ZγZ triangle and a nTGV

In this case, there are also mixed $Z-E'-e'$ vertices in the triangle. The part of the anomaly factor for e' and E' is

$$\begin{aligned}\sum_{f=e',E'}\mathcal{A}^{(f)}_{Z\gamma Z} &= \sum_{f=e',E'}[b^{(f)}_Z a^{(f)}_\gamma a^{(f)}_Z + a^{(f)}_Z a^{(f)}_\gamma b^{(f)}_Z] \\ &= 2b^{(e')}_Z a^{(e')}_\gamma a^{(e')}_Z + 2b^{(E')}_Z a^{(E')}_\gamma a^{(E')}_Z + 2b^{(E'-e')}_Z a^{(e')}_\gamma a^{(e'-E')}_Z + 2b^{(e'-E')}_Z a^{(E')}_\gamma a^{(E'-e')}_Z \\ &= -\frac{g_Z^2 e}{4}\left\{-\frac{1}{2}[|(U_e)_{11}|^2+|(U_e)_{12}|^2]^2 + 2s_w^2[|(U_e)_{11}|^2+|(U_e)_{12}|^2]\right\} \\ &= -\frac{g_Z^2 e}{4}\left(-\frac{1}{2}+2s_w^2\right).\end{aligned} \tag{5.24}$$

This is precisely the opposite of the anomaly produced by the up and down quarks. The last step in (5.24) follows from the unitarity of the U_e matrix. Again, the heavy electrons do not completely decouple: apart from the heavy triangle loop, there are additional triangles in (5.24) with mixed $Z-e'-E'$ vertices of (5.11) which are essential for the anomaly cancellation. Obviously, the cancellation of chiral anomalies is an all orders result in the ϵ expansion defined in Eq. (5.19).

Additionally, we have carried out the calculation of the form factor $h_3^Z(s)$ defined in Eq. (4.11). At high energies, the real part is

$$\Re e[h_3^Z(s)] = -\frac{y_E^2}{4\pi^2}\frac{g_Z^2 v^2}{M'^2}\left[\frac{(1-2s_W^2)}{8}\frac{M_Z^2}{s}+\frac{(4-3s_W^2)}{144}\frac{M_Z^2}{M'^2}+\cdots\right], \qquad 4m_t^2 \ll s \ll M'^2. \tag{5.25}$$

The first term in the square brackets agrees with the dimension-6 result of Eq. (4.12) after replacing the matching outcome of Eq. (5.20). The second term inside the square brackets is the dimension-8 contribution to $h_3^Z(s)$. As in Eq. (5.22), this is always more than an order of magnitude smaller than the dimension-6 contribution. In the low-energy region below the top-threshold, we find

$$\begin{aligned}h_3^Z(s) = -\frac{g_Z^2}{4\pi^2}\Bigg\{&\frac{M_Z^2}{2s}\left[\left(-\frac{1}{2}+\frac{4}{3}s_W^2\right)+\frac{y_E^2 v^2}{M'^2}\frac{(1-2s_W^2)}{4}\right] \\ &+\left[\frac{M_Z^2}{24m_t^2}\left(-\frac{1}{2}+\frac{4}{3}s_W^2\right)+\frac{(y_E^2 v^2)M_Z^2}{M'^4}\frac{(4-3s_W^2)}{144}\right]+\cdots\Bigg\}, \\ &M_Z^2 \ll s \ll 4m_t^2.\end{aligned} \tag{5.26}$$

In this region, the SM contribution is the dominant one, although there is a tendency to cancel the dimension-6 contribution for a considerable Yukawa coupling $y_E \simeq 5$ where, however, the ϵ expansion is questionable (see the discussion in Sec. VI). For $y_E \sim O(1)$ and $M = 1$ TeV, the NP dimension-6 effect from the heavy electron is approximately $5\% \times y_E^2$ regarding the SM expectation. The dimension-8 contribution is more than an order of magnitude smaller than that.

3. Example: AAA anomaly cancellation in a ZZZ triangle

In our final example, we show the cancellation of the AAA anomaly in this model with heavy vectorlike electrons. The AAA-anomaly factor when adding e', E', and the neutrino ν in the triangle loop must vanish (as in the SM). From an analytical standpoint, it gives exactly

$$\begin{aligned}\sum_{f=\nu,e',E'}\mathcal{A}^{(f)}_{ZZZ(\text{axial-part})} &= \sum_{f=\nu,e',E'} b^{(f)}_Z b^{(f)}_Z b^{(f)}_Z \\ &= b^{(\nu)}_Z b^{(\nu)}_Z b^{(\nu)}_Z + b^{(e')}_Z b^{(e')}_Z b^{(e')}_Z + b^{(E')}_Z b^{(E')}_Z b^{(E')}_Z + 3b^{(e'-E')}_Z b^{(E'-e')}_Z b^{(e')}_Z + 3b^{(E'-e')}_Z b^{(e'-E')}_Z b^{(E')}_Z \\ &= \left(\frac{g_Z}{4}\right)^3\{-1+[|(U_e)_{11}|^2+|(U_e)_{12}|^2]^3\} \\ &= 0.\end{aligned} \tag{5.27}$$

The factor 3 on the rhs of the second line accounts for all possible diagram topologies with e' and E' in the triangle. The result is as expected from the SM: AAA-chiral anomalies cancel among leptons and quarks separately. Again, the cancellation is exact to all orders in the ϵ expansion of Eq. (5.19) thanks to the unitarity of the U_e matrix.

4. Remarks

The principal points highlighted in the aforementioned examples regarding the cancellation of chiral anomalies after electroweak symmetry breaking are

(i) Addition of leptons and quarks within a generation.
(ii) Inclusion of heavy and light fermions in the triangle loop.
(iii) Unitarity of the U_e matrix.
(iv) Anomaly cancellation can also be seen order by order in the $|\epsilon|^2 = |y_E|^2 v^2/2M^2$ expansion.

In this linear realization of symmetry breaking, the Goldstone-boson vertices to fermions result in finite contributions in triangle diagrams and have nothing to do with chiral anomalies, and neither does how the routing of the momenta in one-loop triangles is defined.

In all the above, we have set the electron Yukawa coupling to zero, $y_e = 0$. However, the result of the cancellation of the anomalies remains the same for $y_e \neq 0$: only the matrix U_e will be more detailed, but it will still be a unitary matrix.

B. SM EFT after the decoupling of a heavy vectorlike electron

The effective action at low energies consists of light SM fields, collectively called e in the following. It is obtained after integrating out from the path integral a heavy fermion field E of Eq. (5.2) [74–77],

$$\begin{aligned} e^{iS_{\rm eff}[e]} &= \int dE d\bar{E} e^{iS[e,E]} = \int dE d\bar{E} e^{iS_{\rm SM}[e]} e^{iS_{\rm BSM}[e,E]} \\ &= e^{iS_{\rm SM}[e]} \int dE d\bar{E} e^{i\int d^4x\{\bar{E}(i\not{D})E - M\bar{E}E - y_E\overline{\ell}\cdot\varphi P_R E - y_E^*\bar{E}\varphi^\dagger P_L\cdot\varphi\}} \\ &= e^{iS_{\rm SM}[e]} \int dE d\bar{E} e^{i\int d^4x\{\bar{E}KE + \bar{J}E + \bar{E}J\}}, \end{aligned} \tag{5.28}$$

where $K = i\not{D} - M$, $\bar{J} = -y_E\overline{\ell}\cdot\varphi P_R$, and $J = -y_E^*\varphi^\dagger P_L\cdot\varphi$. We can think of J and $\bar{J}$ as "sources" and "sinks" for the heavy fields, $\bar{E}$ and E, because without J's the fields E decouple completely. Solving the path integral in (5.28), we find that

$$e^{iS_{\rm eff}[e]} = e^{iS_{\rm SM}[e]} e^{-i\int d^4x \bar{J}(x)(i\not{D}-M)^{-1}J(x)} \det[i\not{D} - M], \tag{5.29}$$

with the covariant derivative now acting on light SM fields. The heavy fields E have been integrated out from the path integral. Expanding the operator $(i\not{D} - M)^{-1}$ in powers of small momenta ($p \ll M$), we find that

$$(i\not{D} - M)^{-1} = -\frac{1}{M}\left(1 + \frac{i\not{D}}{M} + \frac{(i\not{D})^2}{M^2} + \frac{(i\not{D})^3}{M^3} + \cdots\right). \tag{5.30}$$

Only odd powers of $(i\not{D})$ survive in the exponent of Eq. (5.29). At the zeroth order in the $\hbar$ expansion, Eq. (5.29) results in the local effective action $S_{\rm eff} = \int d^4x \mathcal{L}_{\rm eff}$, which up to dimension-8 operators reads

$$\begin{aligned} \mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} &+ \frac{|y_E|^2}{M^2}(\overline{\ell}\cdot\varphi)(i\not{D})(\varphi^\dagger\cdot\ell) \\ &+ \frac{|y_E|^2}{M^4}(\overline{\ell}\cdot\varphi)(i\not{D})^3(\varphi^\dagger\cdot\ell) + \mathcal{O}(1/M^6). \end{aligned} \tag{5.31}$$

The higher order operators are self-Hermitian. Note also that $(\varphi^\dagger\cdot\ell_L)$ is an $SU(2)_L$ singlet with hypercharge $Y = -1$. Hence, in the SM EFT, we have the appearance of a massless composite particle with the same quantum number as E_L. It is tempting to think that this particle potentially creates an extra chiral anomaly piece in SM EFT. This can be seen explicitly by expanding the dimension-6 operator in the broken $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$ phase of this model

$$\begin{aligned} &\frac{|y_E|^2}{M^2}(\overline{\ell}\cdot\varphi)(i\not{D})(\varphi^\dagger\cdot\ell) \\ &= \frac{|y_E|^2}{M^2}[|\varphi_0|^2\bar{e}_L i\partial\!\!\!/ e_L + g'|\varphi_0|^2(\bar{e}_L\gamma^\mu e_L)B_\mu \\ &\quad + (\bar{e}_L\gamma^\mu e_L)(\varphi_0 i\partial_\mu\varphi_0^*)] + \cdots, \end{aligned} \tag{5.32}$$

where the ellipsis indicates the neutrino and charged-Goldstone-boson contributions.[27] The first term on the rhs of Eq. (5.32) provides corrections to the kinetic term of the left-handed electron e_L, the second term couples the leptonic current of e_L to the gauge boson B_μ associated with the hypercharge $U(1)_Y$ gauge symmetry, and the last term couples the left-handed leptonic current to the Higgs current.[28] All these dimension-6 operators need to cooperate in the WIs of Eqs. (2.2)–(2.4). As an aside, it should be noted that in our analysis, the equations of motion for the SM fields are not utilized.

More specifically, in (5.32), we have for the left-handed electron kinetic term,

$$\mathcal{L}^{\rm Kin} = \bar{e}_L(1 + |\epsilon|^2)i\partial\!\!\!/ e_L \equiv \bar{e}'_L i\partial\!\!\!/ e'_L, \tag{5.33}$$

where ϵ is the same parameter as defined in Eq. (5.5). We therefore redefine the left-handed electrons as[29]

$$e'_L = (1 + |\epsilon|^2)^{1/2} e_L. \tag{5.34}$$

From the second term in (5.32) and in the notation of (5.9), the neutral currents read

[27] We are focusing mostly on neutral vector-boson vertices in this work. Charged-current effects can be demonstrated similarly.
[28] The neutral Higgs component φ_0 is defined above Eq. (5.3).
[29] An analogous field redefinition, but this time for left-handed SM neutrinos, can be found in Ref. [78] for a heavy neutral fermion decoupling, e.g., in the neutrino seesaw mechanism.

$$J_Z^\mu = \bar{e}'_L\gamma^\mu\left[-\frac{1}{2}(1+|\epsilon|^2)^{-1}+s_w^2\right]e'_L+\bar{e}_R\gamma^\mu(s_w^2)e_R+\cdots, \tag{5.35}$$

$$J_{\rm em} = \bar{e}'_L\gamma^\mu(-1)e'_L+\bar{e}_R\gamma^\mu(-1)e_R+\cdots, \tag{5.36}$$

where the ellipses refer to the rest of the SM-field unchanged currents with respect to the SM. In the more suggestive notation of the Appendix, we may separate SM couplings $\hat{a}_V^{(f)}$ and $\hat{b}_V^{(f)}$ from the $(d=6)$ SM EFT ones $\tilde{a}_V^{(f)}$ and $\tilde{b}_V^{(f)}$ as

$$\begin{aligned} a_Z^{(e')} &= \frac{g_Z}{2}\left[-\frac{1}{2}(1+|\epsilon|^2)^{-1}+2s_w^2\right] \equiv \hat{a}_Z^{(e')}+\tilde{a}_Z^{(e')}, \\ b_Z^{(e')} &= \frac{g_Z}{4}(1+|\epsilon|^2)^{-1} \equiv \hat{b}_Z^{(e')}+\tilde{b}_Z^{(e')}, \end{aligned} \tag{5.37}$$

where

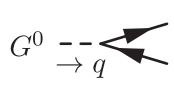
$$\begin{aligned} \hat{a}_Z^{(e')} &= \frac{g_Z}{2}\left[-\frac{1}{2}+2s_w^2\right], \qquad \hat{b}_Z^{(e')} = \frac{g_Z}{4}, \\ \tilde{a}_Z^{(e')} &= -\tilde{b}_Z^{(e')} = \frac{g_Z}{4}\left[\frac{|\epsilon|^2}{1+|\epsilon|^2}\right]. \end{aligned} \tag{5.38}$$

By comparing these to the corresponding currents in Eqs. (5.11) and (5.12) for e'_L in full theory, we observe that they are identical [recall Eq. (5.6): $|(U_e)_{11}|^2 = (1+|\epsilon|^2)^{-1}$]. Similarly, the Wilson coefficients of Eq. (5.20) are also confirmed by simply comparing the Zee and $Z\nu\nu$ generic dimension-6 vertices calculated in the Appendix to those in Eq. (5.38) expanded at $\mathcal{O}(|\epsilon|^2)$. However, to cancel chiral anomalies, we need the effect from the heavy E particles. How does the SM EFT reproduce their effects?

The Goldstone boson G^0 couples to the divergent current of the symmetry that is being broken. The last term of Eq. (5.32) results in

$$\frac{|y_E|^2}{M^2}(\bar{e}_L\gamma^\mu e_L)(\varphi_0 i\partial_\mu\varphi_0^*) \supset -\frac{i}{v}\left[\frac{|\epsilon|^2}{1+|\epsilon|^2}\right]\bar{e}'_L\gamma^\mu e'_L(i\partial_\mu G^0). \tag{5.39}$$

The term inside the square brackets is $|(U_e)_{12}|^2$, the same term that appears in the heavy electron E'_L contribution to the Z current in full theory [see Eq. (5.16)]. The corresponding Feynman rule (in the limit of zero electron mass, i.e., $y_e \to 0$) is



$$= \frac{1}{v}\left[\frac{|\epsilon|^2}{1+|\epsilon|^2}\right]\not{q}\,P_L \quad = \quad \frac{1}{M_Z}\not{q}\,(\tilde{a}_Z^{(e')}+\tilde{b}_Z^{(e')}\gamma^5)\,,$$

where Eq. (5.38) is utilized in the final equality.

It is important to notice that, when there is a SM EFT insertion in the Z-boson vertex, that is, $\tilde{a}_V^{(f)} \neq 0$ and $\tilde{b}_V^{(f)} \neq 0$, there is always a Goldstone boson coupled to the divergence of the spontaneously broken current with identical SM EFT insertions. In other words, Z bosons are longitudinally polarized when SM EFT insertions[30] exist in a Zff vertex. If we call $\Gamma^\mu_{\hat{Z}}$ (Γ_{G^0}) the dimension-6 SM EFT coupling of the Z boson (Goldstone boson) to e', then obviously the identity,

$$q_\mu\Gamma^\mu_{\hat{Z}e'e'}+iM_Z\Gamma_{G^0e'e'}=0, \tag{5.40}$$

holds. Hence, when we take the divergence of the Z-boson current, the dimension-6 SM EFT effects cancel out, leaving behind only the SM Z-boson coupling to e', that is, $\Gamma_{\hat{Z}}$,

$$q_\mu\Gamma^\mu_{Ze'e'}+iM_Z\Gamma_{G^0e'e'}=q_\mu\Gamma^\mu_{\hat{Z}e'e'}. \tag{5.41}$$

One can check this identity in general SM EFT in the broken phase by applying the Feynman rules of the Appendix (or those of Ref. [15]). Equation (5.41) holds for a generic dimension-6 truncation in the SM EFT, and we made great use of it in Sec. III B. The above results for the SM EFT in this particular model verifies all the ingredients used in the bottom-up approach. From now on, the cancellation of chiral anomalies repeats itself exactly as presented in Sec. III.

VI. SOME PHENOMENOLOGY

In this section, we present numerical results for the form factor $h_3^\gamma(s)$ in the SM, in the UV theory of Sec. V A 1, and in its SM EFT (Secs. IV B and V B), where a heavy vectorlike electron E of mass $M=1$ TeV is decoupled. As we have seen, corrections to $h_3^\gamma(s)$ relative to the SM are of the form $y_E^2v^2/M^2$ and may be significant for large y_E in the energy region between the Z-boson mass and the top mass. Furthermore, in the high-energy region, these effects are governed by the dominance of dimension-6 terms, $(\frac{M_Z^2}{s})$, with respect to dimension-8 terms of the form $\frac{M_Z^4}{M^4}$.

[30]That is, when one or more of the Wilson coefficients accompanying the operators in (1.2) are nonzero.

These features are depicted in Fig. 4, where the real part of $h_3^\gamma(s)$ versus the center of mass energy $\sqrt{s}$ is calculated numerically and plotted for various values of the Yukawa coupling y_E. The input value $y_E = 1$ is taken according to the result that is maximally allowed by the global SM EFT fit [79], particularly from electroweak precision observables. The bound on (assumed to be flavor universal) Wilson coefficients $C^{\varphi\ell^{(1,3)}}$, and hence on the Yukawa coupling through Eq. (5.20), is $y_E \lesssim 1.3$. The input value $y_E = 3$ is taken to be almost twice this bound just as a theoretical uncertainty and $y_E = 5$ for investigating the EFT validity versus the full theory results, although they are experimentally excluded. The mass of the heavy fermion is $M = 1$ TeV everywhere.

For two reference values of $\sqrt{s}$, the SM prediction is[31]

$$\text{SM: } h_3^\gamma(\sqrt{s} = 200\ \text{GeV}) = 7.2 \times 10^{-4},$$
$$h_3^\gamma(\sqrt{s} = 900\ \text{GeV}) = -2.4 \times 10^{-6} - 3.0 \times 10^{-5} i, \quad (6.1)$$

which is in agreement with the results of Ref. [38]. The heavy fermion E decoupling in full theory results in changing $h_3^\gamma(s)$ relative to the SM as

$$(y_E = 1): h_3^\gamma(\sqrt{s} = 200\ \text{GeV}) = 7.1 \times 10^{-4},$$
$$h_3^\gamma(\sqrt{s} = 900\ \text{GeV}) = -3.3 \times 10^{-6} - 3.0 \times 10^{-5} i,$$
$$(y_E = 3): h_3^\gamma(\sqrt{s} = 200\ \text{GeV}) = 6.2 \times 10^{-4},$$
$$h_3^\gamma(\sqrt{s} = 900\ \text{GeV}) = -8.9 \times 10^{-6} - 3.0 \times 10^{-5} i. \quad (6.2)$$

There is a reduction of about 2% for $y_E = 1$ (15% for $y_E = 3$) for $\sqrt{s} = 200$ GeV because of the negative Wilson coefficient [see (5.20)] inserted in Eq. (4.9). This is a particular effect of the fermion E being decoupled and is not generic to all gauge group representations of possible heavy fermions. Visible in Fig. 4 is the top-mass threshold peak, where the imaginary part of h_3^γ receives large values and tends to vanish asymptotically ($\sqrt{s} \gg 2m_t$), as expressed in the second line of Eq. (4.8).

As shown in the inset of Fig. 4, at energies near a high mass $M = 1$ TeV, the form factor $h_3^\gamma(s)$ takes on smaller values which peak at $\approx 10^{-6}$ (10^{-5}) for $y_E = 1$ ($y_E = 3$) and are about a factor of 1.4 (4) larger (in absolute value) than the SM expectation; e.g., compare (6.2) and (6.1) to $\sqrt{s} = 900$ GeV. As mentioned previously, in this region the SM drops like $1/s^2$, whereas the full theory (and also the SM EFT) drops like $1/s$.

The SM EFT of this model truncated at the dimension-6 level shows good agreement for Yukawa couplings with $y_E \lesssim 3$ compared to the full theory (the solid cyan line in Fig. 4). For larger values, i.e., $y_E = 5$, the dimension-8 and higher SM EFT contributions become important. However, in the latter case, the perturbation parameter of the theory

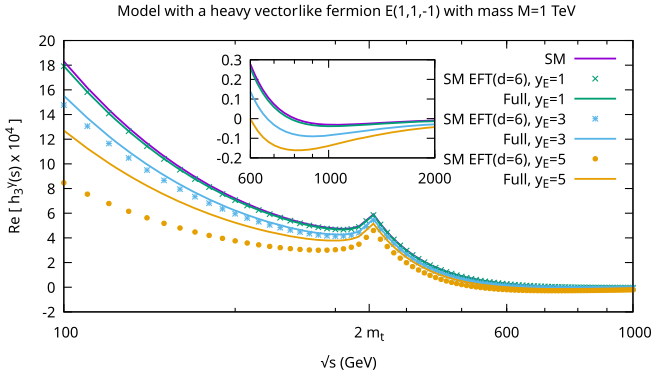


FIG. 4. The real part of the nTGV $\gamma^*\gamma Z$ form factor $\text{Re}[h_3^\gamma(s) \times 10^4]$ versus the center of mass energy $\sqrt{s}$. Solid purple lines refer to the SM and the full theory with a heavy vectorlike lepton $E(1, 1, -1)$ of mass $M = 1$ TeV, with Yukawa couplings $y_E = 1$ (green lines), $y_E = 3$ (cyan lines), and $y_E = 5$ (orange lines). The respective dimension-6 SM EFT predictions are presented with green crosses, cyan stars, and orange dots. Inset: graph focusing on the high-energy region and displaying only the full theory results (same axes and notation as in the main graph but with $\sqrt{s} \in [600, 2000]$ GeV).

in (5.19), $\epsilon \simeq 0.9$, signals the EFT-expansion breakdown,[32] if not the perturbative unitarity of the full model.

The first experimental search on neutral trilinear gauge-boson vertices[33] was performed at LEP through the precesses $e^+e^- \to Z\gamma$, with the Z boson decaying to quarks, charged leptons, or neutrinos [28–30]. The bound set at $|h_3^\gamma(s)| < 0.05$ in the region $\sqrt{s} \approx 200$ GeV was about 2 orders of magnitude above the SM prediction in (6.1). Tevatron searches [31,32] for nTGVs bypassed the LHC ones [33–35]. The process under experimental scrutiny, $pp \to Z(\nu\bar{\nu})\gamma$ with a high-energy photon $E^\gamma > 600$ GeV, sets a bound around $|h_3^\gamma(s)| \lesssim 4 \times 10^{-4}$. For reasons not clear to the authors, analyses at ATLAS and CMS treat $h_3^\gamma(s)$ as a static parameter. Hence, with some degree of uncertainty, we can assert that the LHC bound is approximately 2 orders of magnitude away from the SM prediction in (6.1) for $\sqrt{s} = 900$ GeV. Nonetheless, for $y_E = 3$, it narrows by an order of magnitude in the presence of a heavy vectorlike electron in proximity, as illustrated in (6.2).

If the sensitivity of future e^+e^- colliders for $\Re e[h_3^\gamma(s)]$ and $\sqrt{s} = 200$ GeV is going to be approximately $\sim 10^{-3}$ [80], then the SM prediction will be at the threshold of being measured. Almost certainly, the working model with a heavy electron is impossible to be searched for indirectly since, as Fig. 4 suggests, there is a reduction of about 15% on h_3^γ for $y_E = 3$. The question is if there are BSM effects

[31]We have set all light lepton and quark masses to zero. Their effects in $h_3^\gamma(s)$ for $\sqrt{s} \gtrsim M_Z$ are negligible.

[32]In this case the Z boson couples equally or even stronger to heavy (E) than to light (e) electrons, thus invalidating the EFT concept in general.

[33]Sometimes referred to as anomalous trilinear gauge-boson couplings.

from other than E single vectorlike fermion models that may push this value above 10^{-3}. The answer is given easily by the SM EFT prediction in Eq. (4.9) combined with the dictionary of Wilson coefficients at tree level in Ref. [71]. We find that enhanced predictions for h_3^γ relative to the SM appear in models with heavy vectorlike leptons $\Sigma(1,3)_0$ and $\Delta_3(1,2)_{-3/2}$ and heavy vectorlike quarks $T_2(3,3)_{2/3}$ and $Q_5(3,2)_{-5/6}$. A similar analysis can be performed in the high-energy region based on our analytic formula (4.8).

It is clear that in order to have appreciable nTGVs, the corresponding Yukawa coupling y_E has to be large. This raises questions about bounds from electroweak precision observables [81] since the Z-boson coupling to light fermions must be substantially modified. Needless to say, of course, direct LHC searches for vectorlike quarks and leptons [82,83]. A detailed phenomenological analysis, however, goes beyond the scope of our study.

A. Other UV models

The simple UV theory that we described with a single vectorlike fermion, although complete, is not the end of the story. There may be mass admixtures of multiple heavy fermions with the SM Higgs [47,48,71]. Such models avoid the heavy-light (SM) particle mixing, and therefore constraints from electroweak observables are relaxed. Related to the UV model with the single heavy vectorlike fermion $E(1,1,-1)$ of the previous section, one may envisage a model with an extra heavy vectorlike doublet $L(1,2,-1/2)$ with

$$\begin{aligned}\mathcal{L}_{\rm BSM} = \mathcal{L}_{\rm SM} + \bar{E} i \not{D} E + \bar{L} i \not{D} L - M_E \bar{E} E - M_L \bar{L} L \\ - y_R \bar{L}_L \cdot \varphi E_R - y_L \bar{L}_R \cdot \varphi E_L + \text{H.c.},\end{aligned} \tag{6.3}$$

where Yukawa couplings involving heavy-light fields, like $y_E \bar{\ell}_L \cdot \varphi E_R$ studied before, are now set to zero. After electroweak symmetry breaking, the two heavy fields L and E mix each other with a squared mass difference proportional to $Mv(y_R + y_L)$. In addition, vector-axial couplings to the Z boson, $b_Z^{(E)}$ and $b_Z^{(L)} \sim (y_R - y_L)$, are produced. All chiral anomalies in this model cancel as expected, and a calculation of $h_3^\gamma(s)$ in the full theory at high energies yields

$$\begin{aligned}h_3^\gamma(s) = \frac{e}{\pi^2}\left[-\frac{3}{4}\frac{M_Z^4 y_t^2}{s^2}\left(\ln^2\frac{s}{m_t^2} - \pi^2\right)\right. \\ \left. + \frac{M_Z^4}{96M^4}(y_R^2 - y_L^2) + \cdots\right], \qquad m_t^2 \ll s \ll M^2,\end{aligned} \tag{6.4}$$

where we have taken $M_E = M_L \equiv M$ and y_t is the top-Yukawa coupling. For the unnatural[34] choice $y_L \approx 0$ and $y_R \approx y_t$, the correction due to heavy leptons arises from dimension-8 operators in agreement with Refs. [47,48], but its effect is, in the best case, less than one per mile of the SM contribution for $M = 1$ TeV.

Furthermore, there may be additional heavy Higgs fields [84] or even anomaly-free sets of chiral fermions that receive masses from the SM Higgs boson, although the latter scenarios are highly disfavored by Higgs searches [85]. Other UV-complete models, not discussed in this article, may contain additional heavy vector bosons coupled to the SM (or even to extra) fermions. Then phenomenologically interesting phenomena can be handled with our analysis in Sec. IV D, for instance, Eq. (4.20). We leave quantitative results for all nTGV and related UV models to a future work.

VII. CONCLUSIONS

We have demonstrated the chiral anomalies in the SM EFT expansion with multiple dimension-6 insertions of operators are canceled, in the sense that the classical WIs [Eqs. (2.2)–(2.4)] can always be restored in one-loop triangle diagrams. This cancellation is achieved in the broken electroweak phase through a careful choice of loop momentum routing in triangle diagrams, depicted in Fig. 3 and Table I, involving higher-dimensional operators. The key to this result lies in the fact that the SM itself is anomaly-free [see Eq. (3.18)], and the Goldstone-boson contributions in the SM EFT ensure that gauge invariance and Bose symmetry are preserved. This cancellation holds for each SM fermion circulating in the one-loop triangle diagrams, regardless of the external gauge bosons involved.

We have calculated the C-odd and P-odd form factors for nTGVs, such as $Z^*\gamma\gamma$, $V^*\gamma Z$, and V^*ZZ, in the presence of dimension-6 operators. The master equation for those vertices is Eq. (4.2). These vertices, which do not exist at tree level in the SM EFT, receive finite one-loop corrections that are sensitive to the effects of higher-dimensional operators.

Form factors for nTGVs, such as $h_3^V(s)$ and $f_5^V(s)$, exhibit distinct behaviors at different energy scales. At low energies (below the top-quark threshold), the SM contributions dominate, but the effects of dimension-6 operators [Eq. (1.2)] modify the prediction, e.g., of $h_3^\gamma(s)$, by at most an amount of approximately 2% for perturbative and experimentally allowed Yukawa coupling $y_E = 1$ and $\sqrt{s} = 200$ GeV. Above the top-quark threshold, these form factors scale like $1/s$, rather than $1/s^2$ in the SM, reflecting the nondecoupling of heavy fermions from the UV theory and may have a significant impact on h_3^γ (up to a factor of 1.4 with respect to the SM). This behavior is consistent with the expectations from effective field theory, as shown in Fig. 4, and provides a clear signature for new physics beyond the SM.

[34]Rather than the custodial symmetry supported blind spot, $y_L = y_R$.

We have examined a specific UV-complete model involving a heavy vectorlike electron to validate our bottom-up approach. By integrating out the heavy fermion, we derived the corresponding SM EFT and confirmed that the chiral anomaly cancellation and the form factor $h_3^\gamma(s)$ are consistent with the full theory [see Eqs. (5.22) and (5.23)]. This top-down approach highlights the importance of including heavy fermion effects in the SM EFT and provides a concrete example of how UV physics manifests in the low-energy effective theory. Our analysis strongly suggests that dimension-6 operators have a more significant impact than dimension-8 (or higher) operators both at low energies near and above the Z-boson mass and at high energies near the heavy mass. This makes them potentially observable in future collider experiments.

In summary, this work establishes the consistency of chiral anomaly cancellation in the SM EFT, provides a detailed analysis of nTGVs, and highlights the potential for observing new physics effects in future collider experiments. The interplay between theoretical consistency and phenomenological observability underscores the importance of effective field theory in exploring the frontiers of particle physics.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: VECTOR AND GOLDSTONE-BOSON COUPLINGS TO FERMIONS IN SM EFT

Explicit results for the chiral anomalies and triple-neutral gauge-boson vertices appearing in Secs. III and IV, respectively, need as inputs the vector- and Goldstone-boson coupling to fermions in the SM EFT with the $d=6$ operators arranged in Eq. (1.2). These were found in Refs. [15,66] and are stated below for convenience. The Feynman rules for the $V\bar{f}f$ vertex are $-i\gamma^\mu(a_V^{(f)}+b_V^{(f)}\gamma^5)$, with $a_V^{(f)}\equiv\hat{a}_V^{(f)}+\tilde{a}_V^{(f)}$ and $b_V^{(f)}\equiv\hat{b}_V^{(f)}+\tilde{b}_V^{(f)}$ the "SM-like" and "EFT-like" parts. Then for $V=Z$ we obtain[35]

$$\hat{a}_Z^{(\nu)}=\frac{1}{4}g_Z,\quad \tilde{a}_Z^{(\nu)}=-\frac{1}{4}g_Z\frac{v^2}{\Lambda^2}(C^{\varphi\ell(1)}-C^{\varphi\ell(3)}), \tag{A1}$$

$$\hat{b}_Z^{(\nu)}=-\frac{1}{4}g_Z,\qquad \tilde{b}_Z^{(\nu)}=\frac{1}{4}g_Z\frac{v^2}{\Lambda^2}(C^{\varphi\ell(1)}-C^{\varphi\ell(3)}), \tag{A2}$$

$$\hat{a}_Z^{(e)}=-\frac{1}{4}(3g'Z_{g'}Z_{\gamma Z}^{21}+gZ_gZ_{\gamma Z}^{11}),$$
$$\tilde{a}_Z^{(e)}=-\frac{1}{4}g_Z\frac{v^2}{\Lambda^2}(C^{\varphi e}+C^{\varphi\ell(1)}+C^{\varphi\ell(3)}), \tag{A3}$$

$$\hat{b}_Z^{(e)}=\frac{1}{4}g_Z,\qquad \tilde{b}_Z^{(e)}=-\frac{1}{4}g_Z\frac{v^2}{\Lambda^2}(C^{\varphi e}-C^{\varphi\ell(1)}-C^{\varphi\ell(3)}), \tag{A4}$$

$$\hat{a}_Z^{(u)}=\frac{1}{12}(5g'Z_{g'}Z_{\gamma Z}^{21}+3gZ_gZ_{\gamma Z}^{11}),$$
$$\tilde{a}_Z^{(u)}=-\frac{1}{4}g_Z\frac{v^2}{\Lambda^2}(C^{\varphi u}+C^{\varphi q(1)}-C^{\varphi q(3)}), \tag{A5}$$

$$\hat{b}_Z^{(u)}=-\frac{1}{4}g_Z,\quad \tilde{b}_Z^{(u)}=-\frac{1}{4}g_Z\frac{v^2}{\Lambda^2}(C^{\varphi u}-C^{\varphi q(1)}+C^{\varphi q(3)}), \tag{A6}$$

$$\hat{a}_Z^{(d)}=-\frac{1}{12}(g'Z_{g'}Z_{\gamma Z}^{21}+3gZ_gZ_{\gamma Z}^{11}),$$
$$\tilde{a}_Z^{(d)}=-\frac{1}{4}g_Z\frac{v^2}{\Lambda^2}(C^{\varphi d}+C^{\varphi q(1)}+C^{\varphi q(3)}), \tag{A7}$$

$$\hat{b}_Z^{(d)}=\frac{1}{4}g_Z,\qquad \tilde{b}_Z^{(d)}=-\frac{1}{4}g_Z\frac{v^2}{\Lambda^2}(C^{\varphi d}-C^{\varphi q(1)}-C^{\varphi q(3)}), \tag{A8}$$

and for $V=\gamma$ we obtain

$$\hat{a}_\gamma^{(\nu)}=0,\qquad \hat{a}_\gamma^{(e)}=-g'Z_{g'}Z_{\gamma Z}^{22},\qquad \hat{a}_\gamma^{(u)}=\frac{2}{3}g'Z_{g'}Z_{\gamma Z}^{22},$$
$$\hat{a}_\gamma^{(d)}=-\frac{1}{3}g'Z_{g'}Z_{\gamma Z}^{22}. \tag{A9}$$

The coupling $g_Z=-g'Z_{g'}Z_{\gamma Z}^{21}+gZ_gZ_{\gamma Z}^{11}$, the normalization factors $Z_{g'}$ and Z_g, and the matrix $Z_{\gamma Z}$ can be stated as such to all orders in the EFT expansion. Their explicit expressions are given in Ref. [66] up to dimension-8 operators in the SM EFT. However, chiral anomalies with SM-like couplings $\hat{a}_V^{(f)}$ and $\hat{b}_V^{(f)}$ in Eq. (3.18) cancel without any reference to their specific mathematical expressions.

In this notation, the Feynman rules for the Goldstone-boson couplings to fermions, with incoming momentum q in the Goldstone-boson line, obtain a particularly simple form,

$$G^0(q)\bar{\nu}\nu\colon -\frac{m_e}{vZ_{G^0}}\gamma^5+\frac{\not{q}}{M_Z}(\tilde{a}_Z^{(\nu)}+\tilde{b}_Z^{(\nu)}\gamma^5), \tag{A10}$$

$$G^0(q)\bar{e}e\colon \frac{m_e}{vZ_{G^0}}\gamma^5+\frac{\not{q}}{M_Z}(\tilde{a}_Z^{(e)}+\tilde{b}_Z^{(e)}\gamma^5), \tag{A11}$$

[35]We do not include the dipole ($\sigma^{\mu\nu}$) parts in the following expressions.

$$G^0(q)\bar{u}u: -\frac{m_u}{vZ_{G^0}}\gamma^5 + \frac{\not{q}}{M_Z}(\tilde{a}_Z^{(u)} + \tilde{b}_Z^{(u)}\gamma^5), \tag{A12}$$

$$G^0(q)\bar{d}d: \frac{m_d}{vZ_{G^0}}\gamma^5 + \frac{\not{q}}{M_Z}(\tilde{a}_Z^{(d)} + \tilde{b}_Z^{(d)}\gamma^5). \tag{A13}$$

The normalization factor Z_{G^0} cancels in the calculation of anomalies, as noted in Eq. (3.3), and $M_Z = \frac{1}{2}g_Z v Z_{G^0}$. It is crucial to notice the similarity between the SM EFT parts of the $Z\bar{f}f$ and $G^0\bar{f}f$ vertices; exactly this feature was specifically identified in Sec. V B for the decoupling of a heavy electron.

Moreover, Eqs. (A10)–(A13) have been used in calculating the Goldstone-boson triangle diagram in Eq. (3.3), where $\not{q}$ is factored out from the anomaly terms $\Delta^{\mu\nu\rho(f)}_{\tilde{V}_i V_j V_k}$, with $\tilde{V}_i$ containing either $\tilde{a}_V^{(f)}$ or $\tilde{b}_V^{(f)}$ from Eqs. (A1)–(A9).

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