

Realistic method to access heavy meson light-cone distribution amplitudes from first-principle

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Light-cone distribution amplitudes (LCDAs) of heavy meson within heavy quark effective theory (HQET) are crucial for predicting physical observables in B decays, but unfortunately there is no first-principle result due to severe challenges. After analyzing these challenges, we propose a realistic method to determine heavy meson LCDA. We utilize equal-time correlations and incorporate a dynamic quark field for a fast moving heavy quark. To verify this method, we make use of lattice QCD simulation on a lattice ensemble with spacing $a = 0.05187$ fm. The preliminary findings for HQET LCDAs qualitatively align with phenomenological models, and the fitted result for the first inverse moment λ_B^{-1} is consistent with the experimentally constrained region from $B \rightarrow \gamma \ell \nu_\ell$. We explore how our findings can reduce model uncertainties in predictions of heavy-to-light form factors at large recoil. These results demonstrate the promise of our method in providing first-principle predictions for heavy meson LCDAs.

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Introduction. The investigation of heavy meson decays, exemplified by the B meson, offers profound insights into the nonperturbative realm of QCD and provides a stringent test for the boundaries of the standard model. At the heart of this analysis lies the concept of the light-cone distribution amplitude (LCDA) for heavy mesons [1–4]. These LCDAs are defined in heavy quark effective theory (HQET) and

encode the information about the probabilities of finding the quark and antiquark carrying certain momentum inside the heavy meson [5]. In addition, they are essential for understanding the dynamics of strong force at the interface between the long-range hadronic properties and the short-range quark-gluon degrees of freedom.

Though the ultraviolet behavior is understandable from QCD perturbation theory [6–11], establishing a reliable result on the full distribution of heavy meson LCDAs is not available yet. Constructing models or parametrizations [5,12,13] involves assumptions about the internal structure of heavy mesons, and the choice of model will unavoidably introduce uncertainties and biases. Recent studies have utilized these models in the framework of B meson light-cone sum rules (LCSRs) to calculate the form factors for $B \rightarrow K^*$ and $B \rightarrow \pi$ [14,15]. The obtained results at $q^2 = 0$ are as follows:

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$$\begin{aligned}
\mathcal{V}_{B \rightarrow K^*}(0) &= 0.359^{+0.141}_{-0.085} \big|_{\lambda_B - 0.019}^{+0.019} \big|_{\sigma_1 - 0.062}^{+0.001} \big|_{\mu} \\
&\quad^{+0.010}_{-0.004} \big|_{M^2}^{+0.016} \big|_{s_0 - 0.079}^{+0.153} \big|_{\varphi_{\pm}(\omega)}, \\
f_{B \rightarrow \pi}^0(0) &= 0.122 \times [1 \pm 0.07]_{s_0^{\pi}} \pm 0.11 \big|_{\Lambda_q} \\
&\quad \pm 0.02 \big|_{\lambda_E^2/\lambda_H^2}^{+0.05} \big|_{M^2}^{+0.05} \pm 0.05 \big|_{2\lambda_E^2 + \lambda_H^2} \\
&\quad^{+0.06}_{-0.10} \big|_{\mu_H} \pm 0.04 \big|_{\mu - 0.56}^{+1.36} \big|_{\lambda_B - 0.43}^{+0.25} \big|_{\sigma_1, \sigma_2}, \quad (1)
\end{aligned}$$

which uncertainties with the subscript λ_B and $\sigma_{1,2}$ in these form factors are from parametric errors in heavy meson LCDAs, while the one with the $\varphi_{\pm}(\omega)$ is induced by varying models for the LCDAs. It is evident that these uncertainties are dominant. Therefore, the lack of precise knowledge on LCDAs of heavy mesons from first-principles has been the long-standing primary obstacle in making reliable predictions for heavy meson decays.

In this Letter, we propose a novel and realistic method to determine heavy meson LCDAs from first-principles of QCD which can cure theoretical difficulties as we will explain in the following. Explicitly, we propose a sequential effective field theory and make use of equal-time correlations, also named as quasidistribution amplitudes (quasi-DAs) of heavy mesons with a large momentum component P^z . By designing a hierarchical ordering for the scales $P^z \gg m_H \gg \Lambda_{\text{QCD}}$, we point out that dynamics in these scales can be separated by integrating out the P^z and m_H in two steps, and obtain the required LCDAs in HQET at the final step. To verify this proposal, we perform a lattice QCD simulation of quasi-DAs on a lattice ensemble with $a = 0.05187$ fm. Our preliminary findings for HQET LCDAs qualitatively align with the existing phenomenological models. Using a model-independent parametrization, we also determine the first inverse moment and the obtained result $\lambda_B = 0.389(35)$ GeV lies in the experimentally constrained region $\lambda_B > 0.24$ GeV derived from the $B \rightarrow \gamma \ell \nu_{\ell}$ measurement [16]. The overall agreement demonstrates the promise of our method in providing first-principle predictions for heavy meson LCDAs.

Challenges in determination of heavy meson LCDA and the solution. The leading twist LCDA for a heavy meson is defined as [5]

$$\begin{aligned}
\varphi^+(\omega, \mu) &= \frac{1}{i\tilde{f}_H(\mu)m_H} \int_{-\infty}^{+\infty} \frac{dt}{2\pi} e^{i\omega_+ \cdot vt} \\
&\quad \times \langle 0 | \bar{q}(tn_+) \not{x}_+ \gamma_5 W_c(tn_+, 0) h_v(0) | H(v) \rangle, \quad (2)
\end{aligned}$$

where h_v denotes the heavy quark field in HQET, with the velocity v satisfying $v^2 = 1$. q denotes the light quark field with momentum ω . t denotes the spatial separation between the heavy and light quark fields. W_c is a Wilson line connecting the heavy and light quark fields to ensure the gauge invariance.

The first-principle calculation of heavy meson LCDA from lattice QCD would encounter significant difficulties.

First of all, simulating a light-cone distribution on the lattice is a challenging endeavor, made even more complex when incorporating an effective heavy quark field like h_v . More critically, the coexistence of a light-like Wilson line and the HQET field h_v leads to cusp divergences that will forbid all traditional lattice calculations. This can be elucidated by presenting a perturbative one-loop result for the involved operators [12]:

$$\begin{aligned}
O_v^{P,\text{ren}}(t, \mu) &= O_v^{P,\text{bare}}(t) + \frac{\alpha_s C_F}{4\pi} \left\{ \left(\frac{4}{\hat{\epsilon}^2} + \frac{4}{\hat{\epsilon}} \ln(it\mu) \right) O_v^{P,\text{bare}}(t) \right. \\
&\quad \left. - \frac{4}{\hat{\epsilon}} \int_0^1 du \frac{u}{1-u} [O_v^{P,\text{bare}}(ut) - O_v^{P,\text{bare}}(t)] \right\}, \quad (3)
\end{aligned}$$

with $d = 4 - \epsilon$ and the standard notation $2/\hat{\epsilon} = 2/\epsilon - \gamma_E + \ln(4\pi)$. In the local limit $t \rightarrow 0$, the $\ln(it\mu)$ term will diverge. Thus, the operator product expansion (OPE) for heavy meson LCDAs is ineffective, leading to ambiguities in defining the non-negative moments of heavy meson LCDAs. For these reasons, while there have been numerous lattice QCD calculations for light meson LCDAs—both moments [17–24] and full distributions [25–30]—no result for heavy meson LCDAs is available since the proposal in Ref. [5].

To circumvent the above challenges, two directions are in general possible. Using an off-light-cone Wilson line approach is one option [31–37], yet it encounters a significant signal-to-noise problem when dealing with the h_v field on the lattice [38–40]. The other option is to construct the LCDA using QCD heavy quark fields. The utilization of the heavy quark with finite masses serves as an analogy to critical phenomena. The effective heavy quark field is obtained with the $m_Q \rightarrow \infty$ limit, which represents a fixed point, usually rendering direct studies of correlations impractical. Accessing properties on fixed point can involve examining correlation functions in its vicinity and charting a trajectory for exploration. Shifting from this fixed point corresponds to the finite heavy quark mass and allows a direct simulation. It is important to emphasize that successfully implementing this approach is not trivial. Instead, it is crucial to ensure that a quantity is constructed with the same infrared behavior as the HQET LCDA we ultimately desire. This requires boosting the heavy meson and defining an LCDA that includes only QCD fields:

$$\begin{aligned}
\phi(y, \mu) &= \frac{1}{i\tilde{f}_H} \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{iy P_H \tau n_+} \\
&\quad \times \langle 0 | \bar{q}(\tau n_+) \not{x}_+ \gamma_5 W_c(\tau n_+, 0) Q(0) | H(P_H) \rangle. \quad (4)
\end{aligned}$$

The same infrared structure ensures that ϕ and φ^+ can be connected through a factorization

$$\varphi^+(\omega, \mu) = \frac{1}{m_H} \frac{\tilde{f}_H}{\tilde{f}_H} \frac{1}{\mathcal{J}_{\text{peak}}} \phi(y, \mu), \quad (5)$$

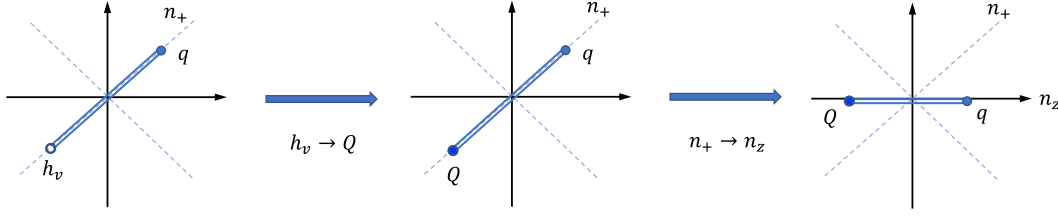


FIG. 1. Illustration of the two-step matching process to address issues in the first-principle calculation of heavy meson LCDA: implying the dynamic heavy quark field can eliminate the cusp divergence, and establishing that the equal-time correlation allows us to circumvent the necessity of light-cone quantities.

with $\omega = ym_H$. The heavy meson mass m_H is treated as a high scale in this factorization, which satisfies $m_H \gg \Lambda_{\text{QCD}}$. It will contribute to the hard-collinear modes of the QCD LCDA [41–43], and be collected into the perturbative jet function [43]

$$\mathcal{J}_{\text{peak}} = 1 + \frac{\alpha_s C_F}{4\pi} \left(\frac{1}{2} \ln^2 \frac{\mu^2}{m_H^2} + \frac{1}{2} \ln \frac{\mu^2}{m_H^2} + \frac{\pi^2}{12} + 2 \right) + \mathcal{O}(\alpha_s^2). \quad (6)$$

After surmounting the challenges posed by the HQET field h_v , the subsequent phase involves implementing the QCD LCDA on lattice QCD. Drawing inspiration from large momentum effective theory (LaMET) [44,45] (see Refs. [46,47] for recent reviews), the QCD LCDA is linked to an equal-time correlator with a substantial momentum P^z , referred to as the quasi-DA:

$$\tilde{\phi}(x, P^z) = \int \frac{dz}{2\pi} e^{-ixP^z} \langle 0 | \bar{q}(z) \Gamma W_c(z, 0) Q(0) | H(P^z) \rangle_R, \quad (7)$$

where the subscripted “ R ” denotes the renormalized matrix element. The matrix element contains an equal-time correlated nonlocal operator that can be directly simulated on lattice QCD [48]. As the P^z tends towards infinity, the quasi-DA will progressively converge towards the QCD LCDA. They also exhibit the same infrared structure, allowing them to be connected through a factorization:

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{(xP^z, \bar{x}P^z)^2}\right). \quad (8)$$

Therefore P^z serves as the ultraviolet scale, satisfying $P^z \gg m_H, \Lambda_{\text{QCD}}$.

Therefore, the above discussions can be summarized as a two-step factorization scheme, as illustrated by Fig. 1. To circumvent the challenges due to lattice simulations of HQET fields and the cusp divergence stemming from the nonlocal HQET operator matrix element, we first convert the HQET field into a QCD field, denoted by the first arrow in Fig. 1. This process is implemented within boosted HQET

factorization and establishes the hierarchy $m_H \gg \Lambda_{\text{QCD}}$. The QCD LCDA involves two distinct scales, m_H and Λ_{QCD} . However, as depicted by the second arrow in the figure, within the LaMET framework, one can derive the QCD LCDA via a lattice-computable equal-time correlated matrix element involving a highly boosted external state. To achieve this, P^z must satisfy $P^z \gg m_H, \Lambda_{\text{QCD}}$. Thus, in line with the essence of the two-step factorization, the hierarchical ordering of the three scales is $P^z \gg m_H \gg \Lambda_{\text{QCD}}$, imposing restrictions for the subsequent lattice validation.

Lattice QCD verification. It is crucial to emphasize that HQET LCDAs are not dependent on the heavy quark mass, which are due to heavy quark flavor symmetry. Hence, one has the flexibility to select m_Q as the charm quark mass m_c , the bottom quark mass m_b , or any other appropriate value, with the HQET LCDAs produced remaining unaffected. On the other hand, the heavy quark mass m_Q needs to be carefully selected to alleviate the impact of power corrections originating from $m_H^2/(P^z)^2$ and Λ_{QCD}/m_H in the factorization processes. Our optimal choice is to concentrate on the charmed meson in this study.

To validate our method, we conduct a numerical simulation using the gauge configurations produced by the CLQCD collaboration [49], with $N_f = 2 + 1$ flavor clover fermions and Symanzik gauge action. To guarantee the hierarchy among the three characteristic scales of quasi-DA and minimize the influence of power corrections in the two-step factorization process, we employ the finest ensemble at lattice spacing $a = 0.05187$ fm with sea quark masses corresponding to $m_\pi = 317.2$ MeV and $m_K = 536.1$ MeV. More details can be found in the supplemental paper [48].

We perform numerical simulations of the quasimatrix elements of the D meson, with a mass of $m_D \simeq 1.92$ GeV, on 549 gauge configurations using boosted momenta P^z up to $P^z \simeq 3.98$ GeV. By utilizing the state-of-the-art self renormalization scheme [50] and extrapolating the long-range correlations through a physics-inspired extrapolation form [51], we obtain the distributions of renormalized quasi-matrix elements of heavy D meson in coordinate space.

As mentioned in Eq. (7), the quasi-DA $\tilde{\phi}(x, P^z)$ of heavy D meson can be obtained from Fourier transforming its renormalized matrix elements into momentum space.

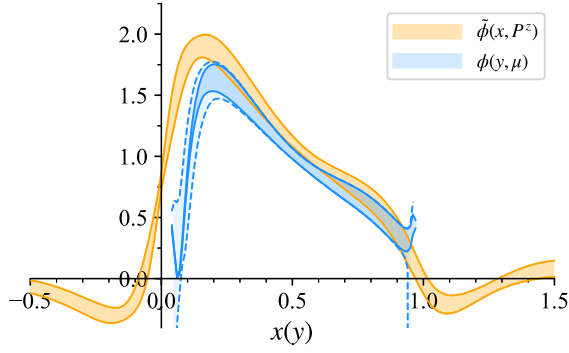


FIG. 2. Quasi-DA $\tilde{\phi}(x, P^z)$ with $P^z = 3.98$ GeV, and matched QCD LCDA $\phi(y, \mu)$ from next-to-leading-order (NLO) kernel. The scale μ is chosen as m_D . The result of $\tilde{\phi}(x, P^z)$ only contains a statistical error, while $\phi(y, \mu)$ contains both a statistical error (solid lines in the blue band) and a systematic one arising from the scale variation (dashed lines).

The numerical results of $\tilde{\phi}(x, P^z)$ at $P^z = 3.98$ GeV are illustrated as the orange band in Fig. 2.

Applying the matching relation given in Eq. (8), one can establish the connection between quasi-DA $\tilde{\phi}(x, P^z)$ and QCD LCDA $\phi(y, \mu)$ with the matching kernel at next-to-leading order in α_s given in Refs. [48,52]. In practice, we employ the renormalization group (RG) resummation starting from the lattice scale μ_0 to $\mu = m_D$, which satisfies both the factorization scale of $\tilde{\phi}(x, P^z)$, and the renormalization scale of $\phi(y, \mu)$. The numerical result of $\phi(y, \mu)$ is shown as the blue band in Fig. 2. The dashed lines represent the uncertainty associated with the initial resummation scale μ_0 , and we systematically vary the scale within a range from $0.8\mu_0$ to $1.2\mu_0$ to assess the robustness of the perturbative matching procedure and to estimate the pertinent systematic uncertainty.

The QCD LCDA can be divided into two distinct regions based on the order of the light quark momentum fraction y : when $y \sim \Lambda_{\text{QCD}}/m_H$, corresponding to the region where a pronounced peak appears in $\phi(y, \mu)$. As discussed above, it can be factorized into the HQET LCDA $\varphi^+(\omega, \mu)$ at the leading power of $\mathcal{O}(\Lambda_{\text{QCD}}/m_H)$. In the region $y \sim \mathcal{O}(1)$ which is known as the “tail region,” the QCD LCDA contributes starting from the one-loop order in perturbation theory, and its perturbative result was first provided in Eq. (16) of Ref. [7]. In Fig. 3 we exhibit the results of $\varphi_{\text{tail}}^+(\omega, \mu; \bar{\Lambda})$, where the parameter $\bar{\Lambda}$ reflects the power correction.

Based on the normalized result for QCD LCDA provided above, we match the multiplicative factorization form given in Eq. (5) to obtain the numerical result of HQET LCDA in the peak region. The result is shown by the pink band in Fig. 3. As mentioned above, we include both the statistical error (shown as the dashed lines) and the systematic ones arise from data analysis. The solid lines of φ_{peak}^+ represent the result incorporating both statistical and systematic errors.

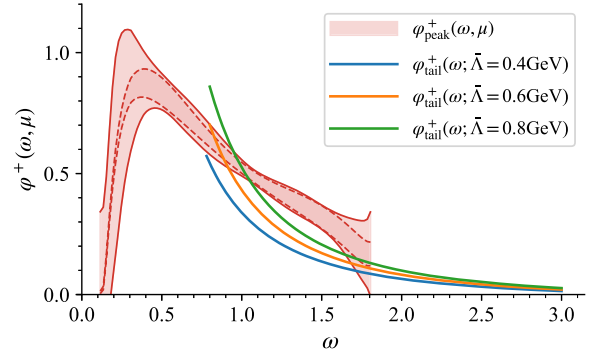


FIG. 3. The result of HQET LCDA in the peak region and tail region. The dashed lines in $\varphi_{\text{peak}}^+(\omega, \mu)$ indicate the upper and lower limits of the result containing only a statistical error, while the solid lines indicate the result including both statistical and systematic errors.

The power counting of parameters in HQET LCDA reveals that the momentum of the light quark in the peak region satisfies $\omega \sim \mathcal{O}(\Lambda_{\text{QCD}})$, while in the tail region it satisfies $\omega \sim \mathcal{O}(m_D)$. This suggests that the boundary ω_b between them lies in the range $\Lambda_{\text{QCD}} \ll \omega_b \ll m_D$. From Fig. 3, we observe that the intersection of the peak and tail regions occurs around 0.9 GeV, confirming the aforementioned statement. Consequently, we merge the peak and tail regions and smooth the vicinity around the intersection to finally obtain a continuous distribution, as illustrated in Fig. 4. The grey band highlights the range where our findings are notably affected by power corrections, specifically around $\omega \lesssim 0.19$ GeV.

In Fig. 4, we compare our results with the parametrizations for the HQET LCDA based on different models [5,12,13]. The default parameters of the first four models are collected in Eq. (93) of Ref. [53], and those of model V are given in Ref. [13]. Because of the challenges in estimating uncertainties in model-based analyses, we only display the central values as colored dashed lines. The spread of these lines serves as an estimate of systematic errors in the models.

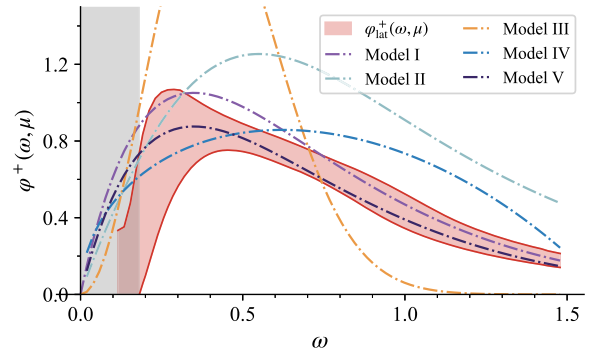


FIG. 4. Comparison of numerical results of $\varphi^+(\omega, \mu)$ from lattice QCD with commonly used phenomenological models. The renormalization scale μ of our result is chosen to be m_D . We only show the central values of the models as colored dashed lines, and interpret their spread as error estimate.

Our results do not favor models II, III, and IV with their default parameters given in Ref. [13]. From the figure one can see that the uncertainties are sizable, but future lattice simulations following similar methodologies can significantly improve precision, possibly eliminating the necessity for model-based analyses.

Phenomenological impacts. The first inverse moment λ_B^{-1} and inverse-logarithmic moments $\sigma_B^{(n)}$ play a pivotal role in QCD factorization theorems and LCSR studies in heavy flavor physics. Although the λ_B^{-1} and $\sigma_B^{(1)}$ have been studied in various approaches [5,7,12,16,54], there is considerable scope for enhancing their reliability and precision. In the case of $\sigma_B^{(2)}$ and subsequent orders, no existing results are currently available.

In the work, we employ model-independent parametrizations to extrapolate our results towards the region at small ω , in order to reconstruct the full distribution of HQET LCDA. One feasible approach to achieve it is to expand $\varphi^+(\omega, \mu)$ in the neighborhood where $\omega \rightarrow 0$:

$$\varphi^+(\omega, \mu) = \sum_{n=1}^N c_n \frac{\omega^n}{\omega_0^{n+1}} e^{-\omega/\omega_0}. \quad (9)$$

The advantage of this parametrization is that, for small values of $\omega \ll \omega_0$, the influence of higher-order terms in the expansion become less influential. Another form based on the expansion of generalized Laguerre polynomials, recently proposed by Ref. [55], is

$$\varphi^+(\omega, \mu) = \frac{\omega e^{-\omega/\omega_0}}{\omega_0^2} \sum_{k=0}^K \frac{a_k(\mu)}{1+k} L_k^{(1)}(2\omega/\omega_0), \quad (10)$$

where $L_k^{(1)}$ denote the associated Laguerre polynomials. The advantage of this parametrization lies in its expansion based on a complete set of orthogonal polynomials.

In the Supplemental Material [56], we detail the utilization of two distinct parametrizations of HQET LCDAs, along with expansions carried out up to the third order. Based on the extrapolated data, we determine the numerical values of λ_B and $\sigma_B^{(1)}$ of the HQET LCDA, as collected in the upper panel of Table I. The errors reported for λ_B and $\sigma_B^{(1)}$ in Table I originate from a combination of statistical uncertainties and systematic variations introduced by differing parametrizations and expansion orders.

A comparison of these results with the experimental data constraint [16] and phenomenological results [5,12,54] is given in Table I, which indicate our findings for λ_B are in agreement with them. While the results for $\sigma_B^{(1)}$ are only available from certain theoretical studies, our results are still in approximate agreement with them.

To investigate the impact of the obtained λ_B on the current studies of B meson weak decays, we use the results

TABLE I. Upper panel: Numerical results for λ_B and $\sigma_B^{(1)}$ at $\mu = 1$ GeV, which is consistent with the scale of theoretical results presented in the lower panel. Lower panel: λ_B and $\sigma_B^{(1)}$ from experiment and other theoretical approaches.

	λ_B (GeV)	$\sigma_B^{(1)}$
This work	0.376(63)	1.66(13)
Ref. [16]	>0.24	
Ref. [54]	0.383(153)	
Ref. [7]	0.48(11)	1.6(2)
Ref. [12]	0.46(11)	1.4(4)
Ref. [5]	0.35(15)	
Ref. [14]	$0.343_{-0.079}^{+0.064}$	
Ref. [58]	0.338(68)	

from LCSR [14] to investigate the $B \rightarrow K^*$ form factors. As an example, we present the results for $\mathcal{V}(q^2 = 0)$. Compared to replacing the λ_B used in Ref. [14] with our results, the relevant error estimates presented in Eq. (1) are refined as

$$\begin{aligned} \mathcal{V}_{B \rightarrow K^*}(0) &= 0.359_{-0.085}^{+0.141} \big|_{\lambda_B} \big|_{\varphi_{\pm}}^{+0.153}_{-0.079} \pm \dots \\ \Rightarrow \mathcal{V}_{B \rightarrow K^*}(0) &= 0.297 \pm 0.088 \big|_{\lambda_B} \pm \dots, \end{aligned} \quad (11)$$

in which we only focus on the terms originating from heavy meson LCDA. Central values of the form factors are smaller by 20%. Specifically, reduced form factors may offer a potential solution to the issue of phenomenological calculations surpassing experimental observations on differential branching fractions for $B \rightarrow K^* \ell^+ \ell^-$ [59,60].

Summary and prospect. The first-principles calculation of HQET LCDAs for a heavy meson has posed a persistent challenge ever since the concept was introduced in the 1990s [5], with no successful resolution achieved thus far. In this Letter, we have presented a first-principle method for calculating the leading-twist HQET LCDA of heavy mesons. To cure all the difficulties in the calculation, we have used a two-step matching method. In particular, we have constructed lattice QCD computable quasi-DAs characterized by three distinct dynamical scales arranged in a hierarchical order $P^z \gg m_H \gg \Lambda_{\text{QCD}}$. We have demonstrated that the separation of various dynamical scales can be achieved sequentially through a two-step factorization process, and the final outcome is the heavy meson LCDA.

To validate this approach, we have conducted a lattice QCD simulation of D meson quasi-DA and successfully matched it to the HQET LCDA. Our preliminary results exhibit qualitative agreement with several established phenomenological models. We have also extracted the first inverse moment, which lies in the experimentally constrained region derived from the $B \rightarrow \gamma \ell^+ \ell^-$ measurement and the phenomenological estimates. More results can be found in a detailed supplement [48]. Following the same spirit, the heavy quark spin symmetry has been used to

construct more useful matrix elements [61], and an estimate of target mass corrections and renormalon ambiguities are conducted in Ref. [62]. Dependence on heavy quark mass of QCD LCDAs is investigated in Ref. [63].

This study marks the first instance in the literature of predicting the HQET LCDAs from lattice QCD. After addressing various systematic effects, it can be anticipated that our methodology will deliver reliable and accurate predictions on LCDAs from first-principles of QCD, and facilitate precise calculations for decay rates of heavy mesons.

The computations in this paper were run on the Siyuan-1 cluster, and Advanced Computing East China Sub-center. The LQCD simulations were performed using the Chroma software suite [64] and QUDA [65–67] through HIP programming model [68].

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