

Threshold resummation for Z-boson pair production at NNLO + NNLL

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(Received 2 April 2025; accepted 15 December 2025; published 13 January 2026)

The production of a pair of on-shell Z-bosons is an important process at the LHC. Owing to its large production cross section at the LHC, this process is very useful for Standard Model precision studies, electroweak symmetry breaking sector as well as to unravel the possible new physics. In this work, we have performed the threshold resummation of the large logarithms that arise in the partonic threshold limit $z \rightarrow 1$, up to next-to-next-to-leading logarithmic (NNLL) accuracy. The presence of the two-loop contributions in the process-dependent resummation coefficient g_0 makes the numerical computation a nontrivial task. After matching the resummed predictions to the next-to-next-to-leading order (NNLO) fixed-order results, we present the invariant mass distribution to NNLO + NNLL accuracy in QCD for the current LHC energies. We find that in the high invariant mass region ($Q = 1$ TeV), while the NNLO corrections are as large as 83% with respect to the leading order, the NNLL contribution enhances the cross section by additional few percent, about 4% for 13.6 TeV LHC. In this invariant mass region, the conventional scale uncertainties in the fixed-order results get reduced from 3.4% at NNLO to about 2.6% at NNLO + NNLL, and this reduction is expected to be more for higher Q values.

DOI: [10.1103/2π7-5xv3](https://doi.org/10.1103/2π7-5xv3)

I. INTRODUCTION

The production of a pair of massive gauge bosons (Z) at the LHC is an important process which has been studied very well, both theoretically and experimentally. This process offers very clean signals that can be used to test the prediction of the Standard Model (SM) precisely, thanks to their moderately large production cross sections at the current LHC energies. The process can also be used for testing the SM electroweak symmetry breaking mechanism as well as in the study of fundamental weak interactions among elementary particles. Owing to the large coupling between the Higgs and massive gauge bosons, the process plays an important role in the Higgs sector. One of the important decay modes of Higgs is to a pair of massive gauge bosons, either ZZ or WW.

The experimental signature of this process typically involves either four charged leptons, two leptons plus missing energy, or two leptons plus two jets or four jet

events. Out of these, the decay to four charged leptons provides a very clean signal in the collider experiments, which led to the measurement of these final states both at the ATLAS and CMS experiments for different center of mass energies, e.g. 5.02 TeV [1], 7 TeV [2–5], 8 TeV [5–10], 13 TeV [10–15], and 13.6 TeV [16]. Such measurements can be used to probe the trilinear gauge couplings as well as to probe the possible hidden new physics. From the theoretical point of view, a similar study can easily be extended to the production of a pair of new massive gauge bosons.

Owing to the importance of this process, a precise knowledge of this process, specifically the production cross sections as well as various kinematic distributions at the current LHC and future high energetic hadron colliders, is very important. In the perturbative quantum chromodynamics (pQCD), the leading order (LO) predictions for this process have been available for a long time [17–20]. It is well known that the LO predictions are unreliable and are contaminated with large theoretical uncertainties. The next-to-leading order (NLO) calculations in pQCD were obtained for both on shell as well as off shell Z-bosons decaying to a pair of leptons. The NLO QCD corrections for this process can be found in Refs. [21–26]. The NLO QCD results, matched with parton shower (NLO + PS) has been studied in Monte Carlo programs such as POWHEG [27,28] and aMC@NLO [29]. The ZZ production at the LHC has been analyzed in the context of beyond Standard Model scenarios as well [30]. It is to be

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noted that at the lowest order in the perturbation theory, this is a quark-antiquark-initiated process similar to the Drell-Yan (DY) production of dileptons. However, the Z-boson pair production process has more similarities to the diphoton production process, as both involve identical particles in the final state, and both are t and u channel processes, whereas DY is an s -channel process (s, t , and u being the Mandelstam variables). For the diphoton production process even at LO, simple kinematic cuts are required to avoid divergences in the forward region, whereas for the case of ZZ production, the mass of the Z-boson avoids such divergences, and hence the total production cross section is finite even in absence of any kinematic cuts. However, in the high invariant mass region, or in the region where Z-boson carries much larger kinetic energy compared to the rest mass energy, both processes can have similar behavior in the cross sections, the special difference being the isolation algorithm to be used in the case of diphoton production process.

Precision studies entail going beyond NLO. However, for the Z-boson pair production process, even the NNLO results are challenging for both analytical as well numerical calculations. The full NNLO calculations have been carried out in Refs. [31–33] in QCD for the quark annihilation process. It is also worth noting that the NNLO corrections are computed for both the on shell Z-boson [34,35] case as well as for off shell Z-bosons [36,37] followed by their decay to lepton final states. With the availability of two loop helicity amplitudes [33], the differential distributions for the latter case also became possible. Fiducial cross sections and distributions are also available for the vector boson pair production processes [36,38]. Additionally, the LO process in the gluon fusion channel, $gg \rightarrow ZZ$, also contributes at $\mathcal{O}(\alpha_s^2)$. In the low invariant mass region, the gluon fluxes for LHC energies are very large and hence the contribution from this channel near the hadronic threshold region is very crucial and cannot be neglected. Going beyond NNLO for Z-boson pair production processes is a challenging task. On the other hand, for DY and Higgs production, this has been achieved. Recently the full N^3 LO results for DY have become available and can be found in Refs. [39–42], and soft-virtual (SV) [43–45] and next-to-soft-virtual (NSV) [46–48] threshold resummations of DY-type processes are available. The Higgs production through gluon fusion channel N^3 LO results are available in Refs. [49–53]; SV [54–57] and NSV [46,58,59] threshold resummation results are also available. For Higgs production via bottom quark annihilation, SV [60] and NSV [61] resummation results are also available. Rapidity resummations for DY processes are available for SV [62,63] and NSV [64–67]. Rapidity distribution for the Higgs production at the threshold to third order in the gluon fusion channel can be found in Ref. [68]. For the DY type processes, the threshold results up to third order can be found in Refs. [44,69,70]. This also enables the evaluation of the resummed parton distribution functions (PDFs),

which are important for high precision studies in the large and small x regions. To see the impact of resummed PDF's on DY and Higgs production; see Refs. [71,72]. Rapidity resummation for gluon fusion channel Higgs production is available for SV [73] and NSV [64,74] cases. Rapidity resummation for Higgs production via bottom quark annihilation is also available at NNLO + NNLL in Ref. [75]. Nevertheless, for the Z-boson pair in the final state, there has been a tremendous effort to go beyond NNLO. Transverse momentum resummation for vector boson pair production is available up to NNLO + N^3 LL [76,77]. The parton shower matched with NNLO (NNLO + PS) are recently studied using the multi-scale improved NNLO + PS (MINNLO_{PS}) [78] method for the ZZ production. The LO matching with parton shower results are available for the gluon fusion channel as well in Ref. [79]. The NLO corrections to this channel have also become available [80–83]. However, their contribution in the high invariant mass region (≥ 2000 GeV) becomes much smaller than those in the quark annihilation channel. The NLO results matching with parton shower (NLO + PS) results for the gluon fusion channel with massless quarks are also available [84]. Finally, at this precision level, the electroweak corrections cannot be ignored for precision studies and the NLO electroweak (EW) corrections to this process have been computed in Refs. [85–87]. Using soft-collinear effective theory formalism, threshold resummation for vector boson pair production is available up to NLO + NNLL [88].

While the threshold resummation for the final state on shell Z-bosons has been achieved to NLO + NNLL level, it is necessary to go beyond this accuracy. The Z-boson pair production cross section has been measured at the LHC experiments using 13 TeV data obtained from the 137 fb^{-1} luminosity. The measured total production cross section is 17.4 pb with an error of about 0.8 pb [15]. The invariant mass distribution also has been measured up to 1 TeV region. With the upcoming High-Luminosity LHC (HL-LHC), the integrated luminosity is expected to reach $3000\text{--}4000 \text{ fb}^{-1}$, which allows the measurement of ZZ events in this TeV region with more statistical data. Considering DY production of a lepton pair, the threshold resummation is known till N^3 LO + N^3 LL accuracy. In Ref. [44], invariant mass regions around 3 TeV were studied, which is also the same mass range for which experimental results exist in Ref. [89]. The experimental uncertainty reported for the DY process for the invariant mass in the Q range 1000–1500 GeV at 7 TeV LHC [90], is around 50%, which decreases by 25% for the 13 TeV in the same Q-range [89]. From the theory side, for the same $Q = 1.5$ TeV, the scale uncertainties decrease from 0.77% at NNLO level to about 0.28% at NNLO + NNLL level after incorporating threshold resummation [44].

For the planned future circular collider hadron [91], the increase in parton fluxes along with the integrated luminosity of $20\text{--}30 \text{ ab}^{-1}$ can enhance the invariant mass

distributions by a few orders of magnitude, thus allowing the measurement of the ZZ events beyond 1 TeV region even more precisely. At this level of experimental precision, adequate theoretical predictions are necessary. In this work, we attempt to increase the theoretical precision by performing the resummation at NNLL accuracy and give the results after matching them to the available fixed-order results at the NNLO level.

Our paper is organized as follows: we present the theoretical framework in Sec. II. Details of the phenomenological analysis and the numerical results are presented in Sec. III. Finally, in Sec. IV, we summarize our observations.

II. THEORETICAL FRAMEWORK

The hadronic cross section for Z-boson pair production can be written in terms of its partonic counterpart, as follows:

$$\frac{d\sigma}{dQ} = \sum_{a,b=\{q,\bar{q},g\}} \int_0^1 dx_1 \int_0^1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \int_0^1 dz \delta(\tau - zx_1x_2) \frac{d\hat{\sigma}_{ab}}{dQ}. \quad (2.1)$$

Here, Q is the invariant mass of the final states. The hadronic and partonic threshold variables τ and z are defined as

$$\tau = \frac{Q^2}{S}, \quad z = \frac{Q^2}{s}, \quad (2.2)$$

where S and s are the hadronic and partonic c.m. energies, respectively. τ and z are thus related by $\tau = x_1x_2z$.

The leading order DY type parton level process has the generic form

$$q(p_1) + \bar{q}(p_2) \rightarrow Z(p_3) + Z(p_4). \quad (2.3)$$

The LO cross section $\hat{\sigma}_{q\bar{q}}^{(0)}$ for Z-boson pair production can be written as

$$\frac{d\hat{\sigma}_{q\bar{q}}^{(0)}}{dQ} = \frac{1}{2s} \int dPS_2 \mathcal{M}_{(0,0)}, \quad (2.4)$$

where dPS_2 is the two-body phase space integration and $\mathcal{M}_{(0,0)}$ is the born amplitude, which in $d = 4 - 2\epsilon$ dimensions is given below,

$$\begin{aligned} \mathcal{M}_{(0,0)} = & B_f N (c_a^4 + 6c_a^2c_v^2 + c_v^4) \frac{4}{t^2u^2} \{ -m_z^4t^2 + 8m_z^4tu + t^3u - m_z^4u^2 + tu^3 - 4m_z^2tu(t+u) \\ & + e(-2m_z^4t^2 - 6m_z^4tu - 2t^3u + 2m_z^4u^2 + 2t^2u^2 - 2tu^3 + 4m_z^2tu(t+u)) \\ & + e^2(-m_z^4t^2 - 2m_z^4tu + t^3u - m_z^4u^2 + 2t^2u^2 + tu^3) \}. \end{aligned} \quad (2.5)$$

In Eq. (2.5), the kinematical variables s , t , and u are defined as

$$(p_1 + p_2)^2 = s, \quad (p_1 - p_3)^2 = t, \quad (p_2 - p_3)^2 = u \quad \text{and} \quad s + t + u = 2m_z^2, \quad (2.6)$$

$$B_f = \frac{(4\pi\alpha)^2}{4N^2}, \quad c_v = \frac{(T_3^f - 2\sin^2\theta_w Q_f)}{2\sin\theta_w \cos\theta_w}, \quad c_a = \frac{1}{4\sin\theta_w \cos\theta_w},$$

where the Q_f and T_3^f are the electric charge and the third component of weak isospin of the fermion f and θ_w is the weak mixing angle. Here, m_z is the mass of the Z-boson, N is the SU(N) color, and α is the fine structure constant. Here, we have dealt with γ_5 in d -dimension using Larin's prescription [92].

Beyond LO, the partonic cross section receives corrections originating from virtual and real contributions. It is interesting to study the cross section in the soft limit, which is defined by $z \rightarrow 1$. This means that the initial partonic c.m. energy is almost used to produce the final state pair of Z-bosons, and small energy is left to produce soft partons. In this limit, the partonic cross section can be organized as follows:

$$\frac{d\hat{\sigma}_{ab}}{dQ} = \frac{d\hat{\sigma}_{ab}^{(0)}}{dQ} (\Delta_{ab}^{\text{sv}}(z, \mu_F^2) + \Delta_{ab}^{\text{reg}}(z, \mu_F^2)). \quad (2.7)$$

The term Δ_{ab}^{sv} is known as the SV partonic coefficient and captures all the singular terms in the $z \rightarrow 1$ limit. Only quark-antiquark or gluon-gluon subprocesses contribute to this SV cross section. The Δ_{ab}^{reg} term contains regular (hard) contributions in the variable z . Both these contributions are expanded in a perturbative series of the strong coupling constant. In our work, we consider such an expansion up to NNLO in QCD. It is to be noted that the overall normalization factor $d\hat{\sigma}_{ab}^{(0)}/dQ$ depends on the process under study.

The singular part of the partonic coefficient has a universal structure which gets contributions from the underlying hard form factor [49,93–96], mass factorization kernels [97–99] and soft radiations [100–114]. According to the Kinoshita–Lee–Nauenberg theorem, these infrared divergences, when regularized and combined, yield finite contributions. After the infrared cancellation, the finite part of these has the universal structure in terms of $\delta(1-z)$ and plus distributions $\mathcal{D}_i = [\ln(1-z)^i/(1-z)]_+$. In the threshold limit, $z \rightarrow 1$, the plus distributions contribute dominantly to the SV cross section. These large distributions can be resummed to all orders in the threshold limit. Threshold resummation is conveniently performed in the Mellin (N) space where the convolution structures become simple product.

The partonic coefficient in the Mellin space is organized as follows:

$$\hat{\sigma}_N^{\text{N}^3\text{LL}} = \int_0^1 dz z^{N-1} \Delta_{ab}^{\text{sv}}(z) \equiv g_0 \exp(G_N). \quad (2.8)$$

The factor g_0 is independent of the Mellin variable, whereas the threshold enhanced large logarithms ($\ln^i N$ in Mellin space) are resummed through the exponent G_N . The resummed accuracy is determined through the successive terms from the exponent G_N which up to NNLL takes the form

$$G_N = \ln(\bar{N}) \bar{g}_1(\bar{N}) + \bar{g}_2(\bar{N}) + a_s \bar{g}_3(\bar{N}) + \cdots, \quad (2.9)$$

where $\bar{N} = N \exp(\gamma_E)$. These coefficients are universal and only depend on the partonic flavors being either quark or gluon. Their explicit form can be found in Refs. [111,115]. In order to achieve complete resummed accuracy one also needs to know the N -independent coefficient g_0 up to sufficient accuracies. In particular, up to NNLL, it takes the form

$$g_0 = 1 + a_s g_{01} + a_s^2 g_{02} + \cdots, \quad (2.10)$$

where $a_s = \frac{\alpha_s}{4\pi}$ and α_s is the strong coupling constant. Using the universal G_N and the process dependent g_0 , resummation for two Higgs boson production in the gluon fusion channel at $N^3\text{LO} + N^3\text{LL}$ has been achieved in Ref. [116].

It is also possible to resum part (or all) of the g_0 by including them in the exponent [54,55,117–119], which, however, have a subleading effect as these contributions are not dominated in the threshold region.

The N -independent coefficient g_0 is computed based on the formalism given in Ref. [120]; the expressions for g_{01} and g_{02} are given in the Appendix. The g_{01} requires one-loop virtual computations ($\mathcal{M}_{(0,1)}$); we have computed the amplitude $\mathcal{M}_{(0,1)}$ using our inhouse FORM [121] code and the expression is given in the Appendix. To obtain the results in z space, one needs to do the Mellin inversion as

$$\begin{aligned} \frac{d\sigma^{\text{N}^3\text{LL}}}{dQ} &= \frac{d\hat{\sigma}^{(0)}}{dQ} \sum_{a,b \in \{q,\bar{q}\}} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} \\ &\times \tau^{-N} f_{a,N}(\mu_F) f_{b,N}(\mu_F) \hat{\sigma}_N^{\text{N}^3\text{LL}}. \end{aligned} \quad (2.11)$$

This complex integral contains the Landau pole at $N = \exp(1/(2a_s\beta_0) - \gamma_E)$, which makes the choice of contour very important. The Mellin inversion is performed [122] along the contour $N = c + x \exp(i\phi)$, where x is real variable. Following the *minimal prescription* [123], we choose the value of c such that all the singularities except the Landau pole lie on the left side of the integration contour. For numerical results, we choose $c = 1.9$ and $\phi = 3\pi/4$. It may be noted that the results are dependent on the choice of the prescription. Several studies have been done in the past in this direction [124–128]. It is to be noted that there are other prescriptions to deal with the Landau pole, such as the Borel prescription [55]. Although these prescriptions differ by subleading terms, they give similar numerical results in the high Q region [55].

Finally, the matched results can be written as

$$\begin{aligned} \frac{d\sigma^{\text{N}^3\text{LO}+\text{N}^3\text{LL}}}{dQ} &= \frac{d\sigma^{\text{LO}}}{dQ} + \frac{d\hat{\sigma}^{(0)}}{dQ} \sum_{a,b \in \{q,\bar{q}\}} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} \\ &\times \tau^{-N} f_{a,N}(\mu_F) f_{b,N}(\mu_F) (\hat{\sigma}_N^{\text{N}^3\text{LL}} - \hat{\sigma}_N^{\text{N}^3\text{LL}}|_{\text{tr}}). \end{aligned} \quad (2.12)$$

In the above equation, $f_{a,N}$ is the Mellin transformed PDF, which one can obtain using publicly available code such as QCD-PEGASUS [122]. However, for numerical applications, it can also be approximated by employing the z -space PDF following Refs. [105,115]. The last term in the bracket of Eq. (2.12) indicates the truncation of the resummed partonic coefficient Eq. (2.8), which avoids double counting the regular terms already present in the fixed order.

Before we proceed to the numerical results, we add a few comments on the extraction of the resummation coefficient, g_{02} , which is a process-dependent quantity. To obtain the full g_{02} , we follow the procedure given in Ref. [120] and make use of the two-loop ($\mathcal{M}_{(0,2)}$) amplitudes as well as the one-loop squared amplitudes, ($\mathcal{M}_{(1,1)}$). The two-loop virtual amplitudes $\mathcal{M}_{(0,2)}$ are reconstructed using the VVamp package [33], while the $\mathcal{M}_{(1,1)}$ is obtained by squaring $\mathcal{M}_{(0,1)}$. After performing the UV renormalization, the IR pole structure of these amplitudes is then verified using Catani's I-operator formalism [129]. The finite parts of the two-loop amplitudes are quite large in size. These amplitudes are then simplified and optimized using our inhouse-developed FORM routines. We have used handyG [130] for numerical evaluations of the generalized polylogarithms in these one-loop and two-loop amplitudes. Finally, these two-loop amplitudes have been cross-checked numerically

for different phase space points with those implemented in the MATRIX [33,131–135]. The resummation formalism used here is developed in Refs. [62,73,105] and has been successfully used in the past in calculating NNLO + NNLL and N³LO + N³LL for Drell-Yan processes [43,44,56,136]. All the anomalous dimensions required for computing the resummed results are given in the Appendix.

III. NUMERICAL RESULTS

In this section, we present the numerical results for the Z-boson pair production process at the LHC. For the numerical computation, we take the fine structure constant to be $\alpha = 1/132.233193$. The mass of the weak gauge bosons $m_z = 91.1876$ GeV, $m_w = 80.385$ GeV. The Weinberg angle is $\sin^2 \theta_w = (1 - m_w^2/m_z^2) = 0.222897223$. This corresponds to the weak coupling $G_F = 1.166379 \times 10^{-5}$ GeV⁻². The default choice of center of mass energy of the incoming protons is 13.6 TeV. Unless specified otherwise, in our numerical analysis, we use MSHT20 [137] PDFs throughout, taken from the LHAPDF [138]. The LO, NLO, and NNLO cross sections are obtained by convoluting the respective coefficient functions with MSHT20lo_as130, MSHT20nlo_as120 and MSHT20nnlo_as118 PDF sets, using the central set (iset = 0) as the default choice. The strong coupling constant α_s is taken from LHAPDF [138], and it varies order by order in the perturbation theory. For our analysis, we consider the number of light quark flavors as $n_f = 5$. For the fixed-order calculations, we have used the package MATRIX [131]. The fixed-order results from MATRIX are checked against those obtained from the MadGraph package [139] up to NLO and are found to be in good agreement. The fixed-order NNLO results have been checked with the numbers quoted in the literature [34,36]. The resummation results are obtained using the inhouse developed numerical code. The unphysical renormalization and factorization scales are set to $\mu_R = \mu_F = Q$, where Q is the invariant mass of the Z-boson pair production in the final state. The scale uncertainties are estimated by varying the unphysical scales in the range so that $|\ln(\mu_R/\mu_F)| \leq \ln 2$. The symmetric scale uncertainty is calculated from the maximum of the absolute deviation of the cross section from that obtained with the central/default scale choice.

We first comment on how much the SV results are in comparison to the full fixed-order cross section. The SV results contain distributions, which contribute significantly in the threshold region. We find that at the SV corrections at the one-loop level are about 79% of the full first-order correction at $Q = 595$ GeV, and this contribution increases to 94% at $Q = 995$ GeV. Similarly, at NNLO level, the second-order SV contributions contribute about 57% of the full second-order result at $Q = 595$ GeV, and this contribution increased to 80% at $Q = 995$ GeV. A similar observation of large SV contributions has been reported in

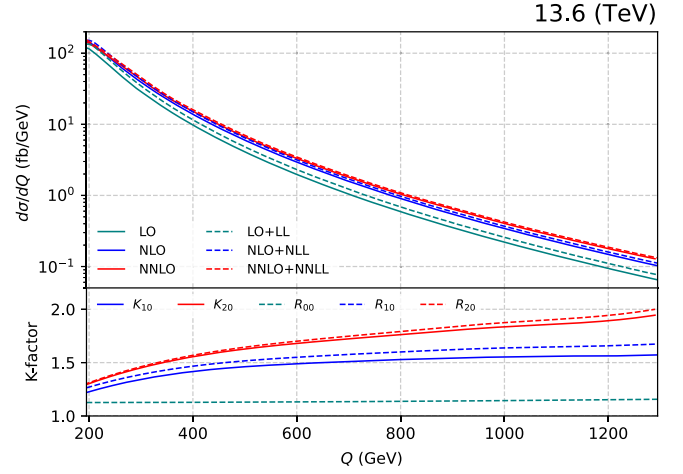


FIG. 1. Resummed predictions for the Z-boson pair invariant mass distribution (upper panel) and the corresponding K-factors (lower panel) up to NNLO + NNLL.

Ref. [68] in the context of Drell-Yan production of dileptons. With the available ingredients (i.e., without the three-loop virtual corrections), it is also possible to estimate the size of the soft-gluon contributions at three loop level. The SV corrections thus computed at the three-loop level are found to be around 0.2% of LO.

To estimate the impact of the higher-order corrections from FO and resummation, we define the following ratios of the cross sections, which are useful in the experimental analysis:

$$K_{ij} = \frac{\sigma_{N^i\text{LO}}}{\sigma_{N^j\text{LO}}}, \quad R_{ij} = \frac{\sigma_{N^i\text{LO}+N^j\text{LL}}}{\sigma_{N^j\text{LO}}} \quad \text{and} \quad L_{ij} = \frac{\sigma_{N^i\text{LO}+N^j\text{LL}}}{\sigma_{N^i\text{LO}+N^j\text{LL}}} \\ \text{with } i, j = 0, 1 \text{ and } 2. \quad (3.1)$$

In Fig. 1, we present the fixed-order results for the invariant mass distribution of the Z-boson pair production from $2m_z$ to 1300 GeV up to NNLO + NNLL in QCD. For the range of Q -variation considered here, the distribution varies over 3 orders of magnitude. In the lower panel, the corresponding fixed order as well as the resummed K-factors as defined in Eq. (3.1) are given. The NLO K-factor K_{10} here varies from about 1.22 at $Q = 200$ GeV to about 1.58 at $Q = 1300$ GeV. As can be seen from Fig. 1, the variation of K_{10} above $Q = 500$ GeV is mild. However, the NNLO K-factor K_{20} slowly but continuously increases from about 1.29 to about 1.94 for the Q range considered here. This clearly shows additional contributions coming from second-order corrections in the higher Q -region, which are due to the real correction subprocesses such as $qq' \rightarrow ZZqq'$. A more detailed discussion of this kind of contribution can be found in Ref. [86]. The resummation has been achieved in the Mellin space where large logarithms of kind $\ln(\bar{N})$ have been resummed to NNLL accuracy as outlined in the text. From R_{00} , we see that the LL resummation enhances the LO results by about 15% for

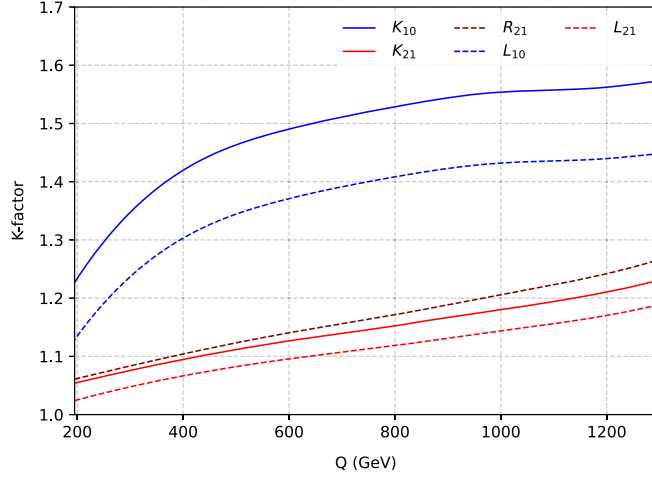


FIG. 2. K-factors for the fixed-order and resummed results for Z-boson pair production [see Eq. (3.1) for details].

$Q = 1300$ GeV. While the NLO corrections are as big as 58%, the corresponding NLO + NLL results are about 68% of LO for the same Q region. The corresponding contribution from NNLO is about 94%, where the NNLL resummation adds an additional few percent, making the NNLO + NNLL contributions sizable, about 99% of LO.

To better estimate the size of higher-order corrections beyond a certain fixed order, as well as that of resummed contributions at different logarithmic accuracy, we present various K-factors of the kind $K_{i,i-1}$, $R_{i,i-1}$ and $L_{i,i-1}$ in Fig. 2. We notice that, in general, L_{ij} are smaller compared to the respective K_{ij} , indicating that the threshold logarithms of a given accuracy (LL, NLL, ...) capture a substantial contribution of the higher-order corrections. Moreover, we notice that the gap between K_{21} and L_{21} is smaller than that between K_{10} and L_{10} , demonstrating a nice convergence of the higher-order QCD corrections.

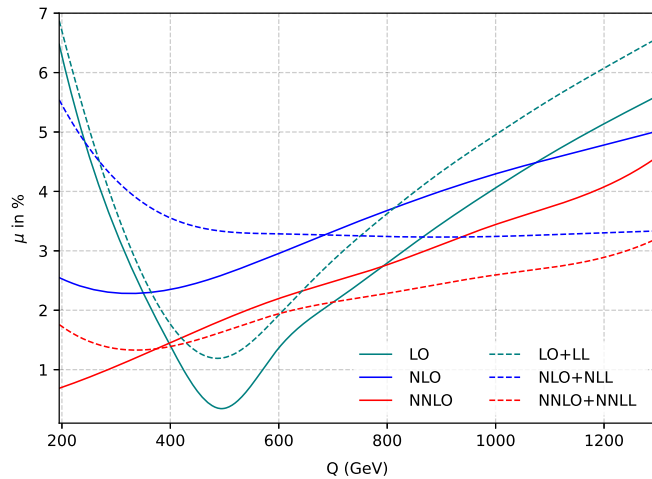


FIG. 3. Seven-point scale uncertainties for Z-boson pair production up to NNLO + NNLL.

Further, it is evident that the former gap is almost independent of Q , while the latter gap increases with Q . We also notice that while the L_{10} is about as large as 1.45, the L_{21} is smaller and is about 1.18. This indicates that the contribution of the second-order terms and the tower of further subleading logarithms (NNLL) that are not included in NLO + NLL are about 18% of NLO + NLL, and are still non-negligible for precision studies.

The underlying theory uncertainties in these distributions up to NNLO + NNLL due to the variation of arbitrary factorization and renormalization scales are presented in Fig. 3. Here, the complete seven-point scale variations, as discussed in the text, have been presented where the maximum uncertainty in the low Q -region at LO is about 6.5%. However, the inclusion of higher-order NLO and NNLO corrections reduce this uncertainty to as low as 2.6% and to less than 1.0%, respectively, for $Q = 200$ GeV. The general observation is that these scale uncertainties are found to increase with Q , which for the NNLO case are found to change from about 0.7% to about 4.5%.

For higher Q values, the threshold logarithms are dominant and as a result of resummation, the corresponding scale uncertainties are expected to be smaller than those in the fixed-order ones. We see that the scale uncertainties increase from LO to LO + LL; this is because for the process under study there is no μ_R scale dependence at LO. On the other hand, the seven-point scale uncertainties at NLO + NLL (NNLO + NNLL) become smaller than those at NLO(NNLO) beyond $Q = 700(400)$ GeV. While for NNLO, the scale uncertainties reach up to 4.5% in high Q regions, the corresponding ones at NNLO + NNLL level reach up to 3.2%.

In the low Q region, the resummation is found to enhance the scale uncertainties compared to the corresponding fixed-order results. This is due to the fact that in the low Q region, the regular terms coming from the hard noncollinear gluons as well as those from the other parton channels are also important. However being nonuniversal, they cannot be resummed to all orders in the perturbation theory. As a result, the resummation cannot improve the scale uncertainties in the fixed-order results in the low Q region.

In the left panel of Fig. 4, we present the uncertainties due to only the renormalization scale by keeping the $\mu_F = Q$ fixed. Similarly, in the right panel, the uncertainties due to only factorization scale variations for fixed $\mu_R = Q$ are given. It is also worth noting that while performing the resummation, the large partonic threshold logarithms are resummed to all orders in the perturbation series that is expanded in a_s , and hence the uncertainties due to μ_R are expected to be smaller for a given μ_F as shown in the left panel of Fig. 4. However, the scale μ_F enters both the PDFs as well as the parton coefficient functions. The uncertainty due to μ_F need not decrease as a result of resummation where the PDFs used are extracted at a particular fixed

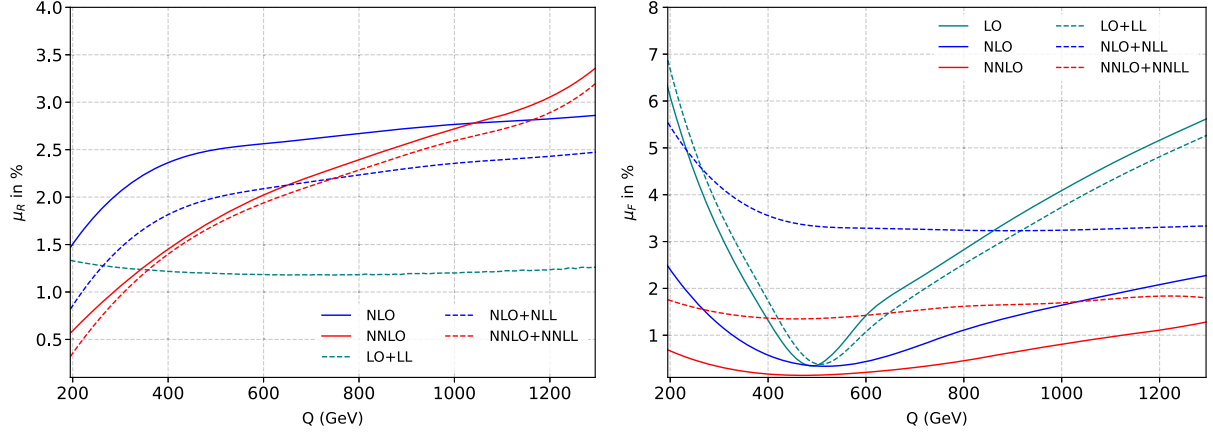


FIG. 4. Renormalization (left) and factorization (right) scale uncertainties for Z-boson pair production up to NNLO + NNLL.

order. Such a behavior of μ_F scale uncertainty can be seen in the right panel of Fig. 4 and has already been reported in the literature [43–47,58–60,62,73,74,140,141].

To study the convergence of the perturbation series, it is useful to keep the PDFs fixed and study how the cross sections vary at different orders. For this, we present the K-factors obtained from the invariant mass distribution computed with NNLO PDFs, to NNLO + NNLL accuracy. Thus, the same α_s is used both at NLO and NNLO. In the

top left panel of Fig. 5, we present the fixed-order K-factors, K_{10} , K_{20} and K_{21} . For a faster converging perturbation series, the factor $K_{i,i-1}$ is supposed to be as close to unity as possible. While the K_{10} has the usual NLO K-factor information and is as large as 55%, from the K_{21} we see that the NNLO corrections could contribute an additional 25% of NLO results. From the behavior of K_{21} , we see that the second-order corrections are small, but they increase with Q . Similar results are presented but for L_{ij} in

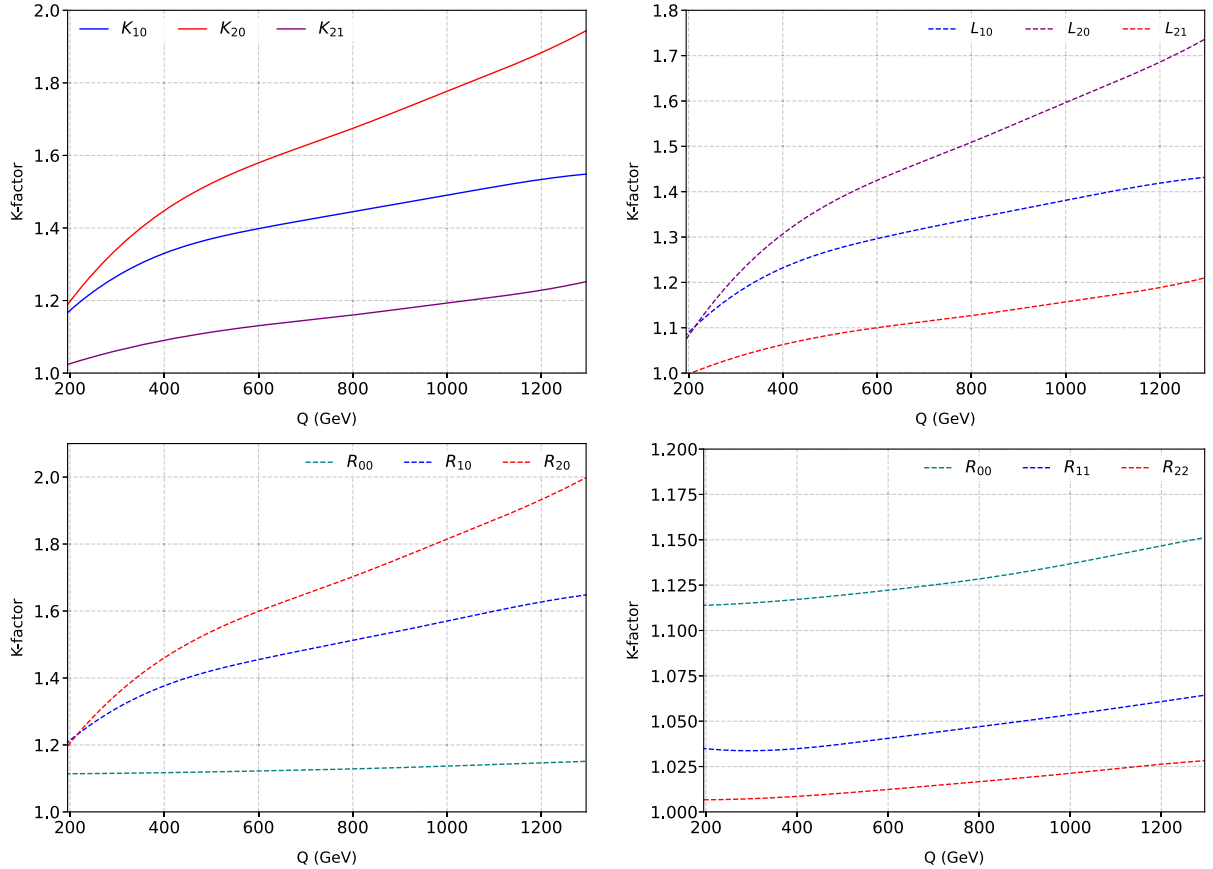


FIG. 5. K-factors as defined in Eq. (3.1) but using the same NNLO PDFs at various orders in the perturbation theory.

TABLE I. Inclusive cross section for Z-boson pair production for different c.m. energies of the incoming protons, along with the corresponding seven-point scale uncertainties. Also given are the ATLAS [12] and CMS [13] measurements with the total uncertainty from statistical, systematic, and luminosity uncertainties involved in the measurement.

$\sqrt{S}$	13.0 TeV	13.6 TeV	100.0 TeV
LO	$10.958 \pm 4.00\%$ pb	$11.664 \pm 4.24\%$ pb	$138.617 \pm 13.84\%$ pb
NLO	$14.380 \pm 1.92\%$ pb	$15.284 \pm 1.91\%$ pb	$169.090 \pm 5.13\%$ pb
NNLO _{$q\bar{q}$}	$15.427 \pm 1.01\%$ pb	$16.437 \pm 1.07\%$ pb	$184.438 \pm 1.69\%$ pb
NNLO	$16.418 \pm 2.22\%$ pb	$17.521 \pm 2.30\%$ pb	$212.344 \pm 3.81\%$ pb
LO + LL	$12.353 \pm 4.37\%$ pb	$13.143 \pm 4.62\%$ pb	$153.653 \pm 14.10\%$ pb
NLO + NLL	$14.890 \pm 4.49\%$ pb	$15.824 \pm 4.56\%$ pb	$174.162 \pm 7.76\%$ pb
NNLO _{$q\bar{q}$} + NNLL	$15.527 \pm 1.39\%$ pb	$16.543 \pm 1.42\%$ pb	$185.348 \pm 2.53\%$ pb
NNLO + NNLL	$16.518 \pm 1.84\%$ pb	$17.627 \pm 1.93\%$ pb	$213.254 \pm 3.64\%$ pb
ATLAS (13.0 TeV) [12]		17.3 ± 0.9 pb	
CMS (13.0 TeV) [13]		17.2 ± 1.0 pb	

the top right panel of Fig. 5. By definition, these ratios will estimate the contribution of higher-order corrections over and above at least LO + LL level. Hence, these are smaller than the corresponding fixed-order K-factors K_{ij} . In the bottom left panel of Fig. 5, we present the resummed K-factors R_{i0} to estimate the size of the resummed results above the LO predictions. We notice that the difference ($R_{20} - R_{10}$) is less than ($R_{10} - R_{00}$) for the whole invariant mass region considered here. In the bottom right panel of Fig. 5, we plot R_{ii} that gives the information about the resummed contributions over and above the corresponding fixed-order corrections. We notice that the contribution from the resummed corrections for any given i is smooth but slowly increasing as Q increases. From R_{22} , we can estimate the size of higher logarithmic terms beyond NNLO to be around 2.8% of NNLO for $Q = 1300$ GeV.

The total production cross sections, after integrating over the invariant mass region over the full kinematic region, for LHC energies are substantially large for the Z-boson pair production process. These production cross sections have been given in Table I up to NNLO + NNLL accuracy, along with the theory uncertainties due to the seven-point scale variations for different c.m. energies of the incoming protons. We notice that for any given c.m. energy, the cross sections increase while the uncertainties decrease as we go from LO to NLO in the fixed-order case. However, the uncertainties will increase from NLO to NNLO due to the gluon fusion channel opening up from the second order in perturbation theory, and hence new contributions will add up to the renormalization scale uncertainties. The gluon fluxes for the LHC energies near the $2m_z$ region are quite large, and this gluon fusion channel contributes about 9.3% of LO Z-boson pair production at 13.6 TeV LHC energy. To systematically quantify the uncertainties in the perturbation theory, we define the second-order cross section without this gluon fusion channel and call it NNLO _{$q\bar{q}$} . The scale uncertainties in the total production cross sections, LO, NLO, and NNLO _{$q\bar{q}$} are found to systematically decrease

from 4.24% to 1.07% for 13.6 TeV LHC energy. This behavior remains similar for other c.m. energies. We notice a similar behavior in the total production cross sections after the resummation has been performed, i.e., the scale uncertainties decrease from 4.62% to 1.42%, as we go from LO + LL to NNLO + NNLL. However, the NNLO _{$q\bar{q}$} + NNLL has a somewhat larger scale uncertainty compared to the one in NNLO _{$q\bar{q}$} . This is simply because of the definition of total production cross section where the invariant mass has been integrated out from $Q_{\min} = 2m_z$ to $\sqrt{S}$. In the lower Q -region the contribution from other channels like qg -subprocess cannot be ignored. However, with increasing $Q_{\min}$ in the total production cross section, the scale uncertainties in NNLO _{$q\bar{q}$} + NNLL are expected to be smaller than those in NNLO _{$q\bar{q}$} , as evident from Fig. 3. The NLO corrections for the gluon fusion channel, in the massless quark limit, are about 68% of its LO for the current LHC energies [80]. The inclusion of massive top quark loops is found to increase this correction to about 73% [83]. Finally, at this NNLO + NNLL accuracy in the perturbation theory, the NLO EW correction for Z-boson pair production also becomes important. The EW corrections are negative, and for total production, they are found to be around -6.2%, while the mixed NNLO QCD $\times$ EW corrections are about -5.7% for 13 TeV LHC. For the invariant mass distribution, the EW corrections amount to -15% at 1 TeV [86]. Finally, the NNLO + NNLL results match well within the theoretical and experimental uncertainties to the ATLAS [12] and CMS [13] measurements of the Z-boson pair production cross section at 13 TeV. The NNLO + NNLL enhances the NNLO cross section by 0.6% of NNLO at 13 TeV. We note that the uncertainties due to the nonperturbative PDFs are also important. These PDF uncertainties for the $q\bar{q}$ -initiated DY processes are found to be about 3% around the 1 TeV region [44]. The ZZ production process also takes place via $q\bar{q}$ initial states, and hence a similar PDF uncertainty is expected around $Q = 1$ TeV.

IV. SUMMARY

To summarize, we have performed the threshold resummation for the production of a pair of Z-bosons at the energies of LHC, by resumming the threshold logarithms to NNLL accuracy in QCD. The final state having two massive particles makes the process dependent one-loop and two-loop virtual corrections more difficult compared to the massless final state like dilepton or diphoton, or one massive final state like Higgs boson. The presence of such virtual amplitudes makes the resummation a challenging task to achieve numerically, in contrast to the case of $2 \rightarrow 1$ processes like DY and Higgs production cross section processes. It is also worth noting that compared to the previously available NLO + NNLL results, in the present NNLO + NNLL results, the presence of real corrections is also equally important and thanks to MATRIX, where such contributions have already been taken into account. In this work, we have performed this resummation by systematically matching to the known fixed-order NNLO results (from the package MATRIX) and present our phenomenological results to NNLO + NNLL accuracy for both the total production cross sections as well as for the invariant mass distribution of the Z-boson pair, for the current LHC energies, 13.0 and 13.6 TeV, as well as for the upcoming future 100 TeV collider. We notice that the NNLL resummed results in general enhance the cross sections and contribute an additional few percent to the known NNLO results. After performing the resummation at NNLO + NNLL accuracy, our results for the total production at 13 TeV c.m. energy are found to agree with experimental measurements (within the reported errors) from ATLAS and CMS. We have also presented the theory uncertainties by varying the unphysical renormalization and factorization scales from $Q/2$ to $2Q$. We find that, after

performing the resummation, the scale uncertainties of about 4.5% in NNLO cross sections get reduced to about 3.2% at NNLO + NNLL level for the invariant mass region $Q = 1.3$ TeV.

The availability of these resummed results is expected to provide a foundation for future theoretical and experimental studies of diboson processes at the LHC and beyond.

ACKNOWLEDGMENTS

The research work of M. C. K. is supported by SERB Core Research Grant (CRG) under Project No. CRG/2021/005270. P. B. wants to thank IIT Guwahati for using the computational resources at IIT Guwahati. The work of P. B. is supported by the INFN/QFT@COLLIDERS project (Italy). The authors would like to thank A. H. Ajith, M. Bonvini, L. Buonocore, G. Das and V. Ravindran for useful discussions. We acknowledge National Supercomputing Mission (NSM) for providing computing resources of PARAM Kamrupa at IIT Guwahati, which is implemented by C-DAC and supported by the Ministry of Electronics and Information Technology (MeitY) and Department of Science and Technology (DST), Government of India, where most of the computational work has been carried out.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: RESUMMATION COEFFICIENTS

The process-dependent g_0 coefficients defined in Eq. (2.10) are given as [defining $L_{qr} = \ln(Q^2/\mu_R^2)$, $L_{fr} = \ln(\mu_F^2/\mu_R^2)$],

$$g_{01} = 2\mathcal{K}_{(0,1)} + 2C_F\{-3L_{fr} + L_{qr}^2 + \zeta_2\}, \quad (\text{A1})$$

$$\begin{aligned} g_{02} = & \mathcal{K}_{(1,1)} + 2\mathcal{K}_{(0,2)} + C_F n_f \left\{ -\frac{328}{21} + \frac{2}{3}L_{fr} + \frac{112}{27}L_{qr} - \frac{20}{9}L_{qr}^2 - \frac{10}{9}\zeta_2 + \frac{16}{3}L_{fr}\zeta_2 - \frac{4}{3}L_{qr}\zeta_2 + \frac{32}{3}\zeta_3 \right\} \\ & + C_F^2 \{-3L_{fr} + 18L_{fr}^2 - 12L_{fr}L_{qr}^2 + 2L_{qr}^4 + 12L_{fr}\zeta_2 + 4L_{qr}^2\zeta_2 + 2\zeta_2^2 - 48L_{fr}\zeta_3\} \\ & + C_F \left\{ +3\beta_0 L_{fr}^2 - \frac{2}{3}\beta_0 L_{qr}^3 - 12L_{fr}\mathcal{K}_{(0,1)} + 4L_{qr}^2\mathcal{K}_{(0,1)} - 2\beta_0 L_{qr}\zeta_2 + 4\mathcal{K}_{(0,1)}\zeta_2 + \frac{46}{3}\beta_0\zeta_3 \right\} \\ & + C_F C_A \left\{ +\frac{2428}{81} - \frac{17}{3}L_{fr} - \frac{808}{27}L_{qr} + \frac{134}{9}L_{qr}^2 + \frac{67}{9}\zeta_2 - \frac{88}{3}L_{fr}\zeta_2 + \frac{22}{3}L_{qr}\zeta_2 - 4L_{qr}^2\zeta_2 - 12\zeta_2^2 \right. \\ & \left. - \frac{176}{3}\zeta_3 + 24L_{fr}\zeta_3 + 28L_{qr}\zeta_3 \right\}. \end{aligned} \quad (\text{A2})$$

Here, we define

$$\mathcal{K}_{(m,n)} = \frac{\mathcal{M}_{(m,n)}^{\text{fin}}}{\mathcal{M}_{(0,0)}}, \quad (\text{A3})$$

where

$$\mathcal{M}_{(m,n)}^{\text{fin}} \equiv \langle \mathcal{M}_m | \mathcal{M}_n \rangle, \quad (\text{A4})$$

and $|\mathcal{M}_n\rangle$ represents the UV-renormalized, IR-finite virtual amplitude at the n th order in a_s , as given in Eq. (C3) of Ref. [120]. The one-loop virtual contribution for Z -boson pair production is

$$\mathcal{M}_{(0,1)} = a_s(\mu_R^2) \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} (4\pi)^\epsilon \left(\frac{\mu_R^2}{s}\right)^\epsilon C_F \left[-4 \left\{ \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} \right\} \mathcal{M}_{(0,0)} + \mathcal{M}_{(0,1)}^f \right]. \quad (\text{A5})$$

Here,

$$\begin{aligned} \mathcal{M}_{(0,1)}^f = & B_f \mathcal{N} (c_v^4 + 6c_v^2 c_a^2 + c_a^4) \left(\frac{4}{t^2 u^2} \right) \left[2\zeta_2 \left(-20m_z^2 tu(t+u) - 4m_z^4(t^2 - 8tu + u^2) + tu(5t^2 + 4tu + 5u^2) \right. \right. \\ & + \frac{tu}{\kappa s(4m_z^2 - s)^2} (-32m_z^6 tu - 64m_z^8(t+u) + 8m_z^2 tu(t+u)^2 - (t+u)^3(3t^2 + 4tu + 3u^2) \\ & + m_z^4(22t^3 + 82t^2 u + 82tu^2 + 22u^3)) \Big) + \ln\left(\frac{s}{m_z^2}\right) \frac{1}{(4m_z^2 - s)^2} (8m_z^2 t^2 u^2(t+u) + 12m_z^8(t^2 - 8tu + u^2) \\ & - tu(t+u)^2(3t^2 + 4tu + 3u^2) + 4m_z^6(3t^3 - 5t^2 u - 5tu^2 + 3u^3) + m_z^4(3t^4 + 14t^3 u + 78t^2 u^2 + 14tu^3 + 3u^4)) \\ & + \frac{tu}{\kappa s(4m_z^2 - s)^2} (\ln(x) + 4\text{Li}_2(-x)) (-32m_z^6 tu - 64m_z^8(t+u) + 8m_z^2 tu(t+u)^2 - (t+u)^3(3t^2 + 4tu + 3u^2) \\ & + m_z^4(22t^3 + 82t^2 u + 82tu^2 + 22u^3)) + \frac{1}{(t - m_z^2)(u - m_z^2)(4m_z^2 - s)} (18m_z^{10}(t^2 - 8tu + u^2) \\ & + m_z^8(-9t^3 + 131t^2 u + 131tu^2 - 9u^3) - 7t^2 u^2(t^3 + t^2 u + tu^2 + u^3) - 4m_z^4 tu(4t^3 + 23t^2 u + 23tu^2 + 4u^3) \\ & + m_z^6(-9t^4 + 14t^3 u - 66t^2 u^2) + 14tu^3 - 9u^4) + m_z^2 tu(9t^4 + 32t^3 u + 82t^2 u^2 + 32tu^3 + 9u^4) \\ & + \ln\left(\frac{-t}{m_z^2}\right) \frac{u}{(t - m_z^2)^2} (3m_z^8(-4t + u) + 6m_z^6 t(2t + u) - 2m_z^2 t^2 u(4t + u) + 3t^3 u(2t + 3u) \\ & + m_z^4 t(-2t^2 + tu - 3u^2)) + \left(2\ln\left(\frac{-t}{m_z^2}\right) \ln\left(\frac{t - m_z^2}{m_z^2 s}\right) + 4\text{Li}_2\left(\frac{t}{m_z^2}\right) - \ln^2\left(\frac{-t}{m_z^2}\right) \right) u(m_z^4(8t - 2u) \\ & - 4m_z^2 t(2t + u) + t(2t^2 + 2tu + u^2)) + \ln\left(\frac{-u}{m_z^2}\right) \frac{t}{(u - m_z^2)^2} (3m_z^8(-4u + t) + 6m_z^6 u(2u + t) - 2m_z^2 u^2 t(4u + t) \\ & + 3u^3 t(2u + 3t) + m_z^4 u(-2u^2 + tu - 3t^2)) + \left(2\ln\left(\frac{-u}{m_z^2}\right) \ln\left(\frac{u - m_z^2}{m_z^2 s}\right) + 4\text{Li}_2\left(\frac{u}{m_z^2}\right) - \ln^2\left(\frac{-u}{m_z^2}\right) \right) \\ & \times t(m_z^4(8u - 2t) - 4m_z^2 u(2u + t) + u(2u^2 + 2tu + t^2)) \Big]. \\ x = & \frac{1 - \kappa}{1 + \kappa}; \quad \kappa = \sqrt{1 - 4 \frac{m_z^2}{s}}. \end{aligned} \quad (\text{A6})$$

The process-independent universal resum exponent defined in Eq. (2.9), which are used for DY-type processes, are given as

$$g_1 = \left[\frac{A_1}{\beta_0} \{ 2 - 2\ln(1 - \omega)1 + 2\ln(1 - \omega)1\omega^{-1} \} \right], \quad (\text{A7})$$

$$\begin{aligned} g_2 = & \left[\frac{D_1}{\beta_0} \left\{ \frac{1}{2} \ln(1 - \omega)1 \right\} + \frac{A_2}{\beta_0^2} \{ -\ln(1 - \omega)1 - \omega \} + \frac{A_1}{\beta_0} \left\{ \left(\ln(1 - \omega)1 + \frac{1}{2} \ln(1 - \omega)1^2 + \omega \right) \left(\frac{\beta_1}{\beta_0^2} \right) \right. \right. \\ & \left. \left. + (\omega)L_{fr} + (\ln(1 - \omega)1)L_{qr} \right\} \right], \end{aligned} \quad (\text{A8})$$

$$\begin{aligned}
g_3 = & \left[\frac{A_3}{\beta_0^2} \left\{ -\frac{\omega}{(1-\omega)} + \omega \right\} + \frac{A_2}{\beta_0} \left\{ \left(2\frac{\omega}{(1-\omega)} \right) L_{qr} + \left(3\frac{\omega}{(1-\omega)} + 2\frac{\ln(1-\omega)}{(1-\omega)} - \omega \right) \left(\frac{\beta_1}{\beta_0^2} \right) + (-2\omega) L_{fr} \right\} \right. \\
& + A_1 \left\{ -4\zeta_2 \frac{\omega}{(1-\omega)} + \left(-\frac{\ln(1-\omega)^2}{(1-\omega)} - \frac{\omega}{(1-\omega)} - 2\frac{\ln(1-\omega)}{(1-\omega)} + 2\ln(1-\omega)1 + \omega \right) \left(\frac{\beta_1}{\beta_0^2} \right)^2 \right. \\
& + \left(-\frac{\omega}{(1-\omega)} \right) L_{qr}^2 + \left(-\frac{\omega}{(1-\omega)} - 2\ln(1-\omega)1 - \omega \right) \left(\frac{\beta_2}{\beta_0^3} \right) + \left(\left(-2\frac{\omega}{(1-\omega)} - 2\frac{\ln(1-\omega)}{(1-\omega)} \right) \left(\frac{\beta_1}{\beta_0^2} \right) \right) L_{qr} + (\omega) L_{fr}^2 \left. \right\} \\
& \left. + \frac{D_2}{\beta_0} \left\{ \frac{\omega}{(1-\omega)} \right\} + D_1 \left\{ \left(-\frac{\omega}{(1-\omega)} \right) L_{qr} + \left(-\frac{\omega}{(1-\omega)} - \frac{\ln(1-\omega)}{(1-\omega)} \right) \left(\frac{\beta_1}{\beta_0^2} \right) \right\} \right]. \quad (A9)
\end{aligned}$$

Here, A_i are the universal cusp anomalous dimensions, D_i are the threshold noncusp anomalous dimensions, and $\omega = 2a_s\beta_0 \ln \bar{N}$. Note that all the perturbative quantities are expanded in powers of a_s . The cusp anomalous dimensions A_i are given as (four-loop results can be found in Refs. [142–144])

$$A_1 = C_F \{4\}, \quad (A10)$$

$$A_2 = C_F \left\{ n_f \left(-\frac{40}{9} \right) + C_A \left(\frac{268}{9} - 8\zeta_2 \right) \right\}, \quad (A11)$$

$$\begin{aligned}
A_3 = & C_F \left\{ n_f^2 \left(-\frac{16}{27} \right) + C_F n_f \left(-\frac{110}{3} + 32\zeta_3 \right) + C_A n_f \left(-\frac{836}{27} - \frac{112}{3}\zeta_3 + \frac{160}{9}\zeta_2 \right) \right. \\
& \left. + C_A^2 \left(\frac{490}{3} + \frac{88}{3}\zeta_3 - \frac{1072}{9}\zeta_2 + \frac{176}{5}\zeta_2^2 \right) \right\}. \quad (A12)
\end{aligned}$$

The coefficients D_i are given as

$$D_1 = C_F \{0\}, \quad (A13)$$

$$D_2 = C_F \left\{ n_f \left(\frac{224}{27} - \frac{32}{3}\zeta_2 \right) + C_A \left(-\frac{1616}{27} + 56\zeta_3 + \frac{176}{3}\zeta_2 \right) \right\}. \quad (A14)$$

Here,

$$C_A = N, \quad C_F = \frac{N^2 - 1}{2N}, \quad (A15)$$

$$\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f, \quad (A16)$$

$$\beta_1 = \frac{34}{3}C_A^2 - \frac{10}{3}n_f C_A - 2n_f C_F, \quad (A17)$$

$$\beta_2 = \frac{2857}{54}C_A^3 - \frac{1415}{54}n_f C_A^2 - \frac{205}{18}n_f C_F C_A + n_f C_F^2 + \frac{79}{54}n_f^2 C_A + \frac{11}{9}n_f^2 C_F. \quad (A18)$$

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