

Infinite-dimensional symmetries in plane wave spacetimes

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We study the asymptotic symmetries of the Nappi-Witten spacetime in four dimensions, a plane wave arising as the Penrose limit of $\text{AdS}_2 \times S^2$. Imposing suitable boundary conditions at large transverse distance, we uncover a new infinite-dimensional symmetry algebra allowing for nontrivial central extensions. The corresponding phase space encompasses the most general four-dimensional pp-wave metric, including in particular the Penrose limit of Kerr black holes.

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I. INTRODUCTION

Plane-fronted waves with parallel propagation, or *pp-waves*, form a class of geometries defined by the existence of covariantly constant null vector (see, e.g., [1] for a pedagogical introduction to pp-waves). Plane waves are a subclass of pp-waves defined, in Brinkmann coordinates, by

$$ds^2 = 2dudv + A_{ij}(u)x^i x^j du^2 + (dx^i)^2. \quad (1)$$

They are the generic result of a Penrose limit [2,3]: starting from a null geodesic γ in an arbitrary spacetime $\mathcal{M}$, the Penrose limit of $(\mathcal{M}, \gamma)$ is the spacetime $\mathcal{M}_\gamma$ consisting of the infinitesimal neighborhood of γ , now blown-up to infinity. This procedure can be generalized to include supergravity fields [4]. Plane waves have been put under the spotlight more than two decades ago. These backgrounds yield solvable string theories [5,6] and in particular, the maximally supersymmetric plane wave background of type IIB superstring theory was shown to arise as the Penrose-(Güven) limit of $\text{AdS}_5 \times S^5$ [7,8]. This led to the geometrical limit of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence known as the BMN limit [9,10], relating string theory on this plane wave background to a subsector of $\mathcal{N} = 4$ SYM theory. The causal boundary of this plane wave—and actually of all plane waves with nonpositive-definite wave profiles, in any

dimension—is a one-dimensional null line parametrized by u [2,3].

Holography in AdS spacetimes involves a bulk theory of gravity and a conformal field theory on its conformal boundary [11–14]. However, there are several examples where infinite-dimensional symmetries arise on different surfaces, such as in the near-horizon region of black holes [15–19]. More recently, it was shown that the eikonal regime of the quasinormal mode spectrum of black holes falls into highest-weight representations of a conformal $\text{SL}(2, \mathbb{R})$ symmetry algebra appearing in their near-photon ring region [20,21]. As the photon ring is essentially the locus of nearly bound unstable null geodesics, it was also shown that the eikonal regime quasinormal mode spectrum could be retrieved by solving the wave equation on the plane wave background resulting from the Penrose limit at the photon ring [22]. This was also recently generalized in [23] to the highly damped quasinormal mode spectrum of the Schwarzschild black hole by zooming onto the horizon.

In this paper, we investigate the asymptotic symmetries of plane wave spacetimes. Our focus will be the four-dimensional version of the BMN background, known as the *Nappi-Witten spacetime* [24], defined by the wave profile $A_{ij} = -\delta_{ij}/4$, $i, j = 1, 2$. This background is described by a Wess-Zumino-Witten (WZW) model built on the centrally extended two-dimensional Euclidean algebra, and can also be obtained as the Penrose limit of $\text{AdS}_2 \times S^2$, which is itself the near-horizon region of near-extremal Reissner-Nordström black holes. Here, the *asymptotic region* of interest is not the causal boundary of the plane wave, but rather the limit $r = (x^i x_i)^{1/2} \rightarrow \infty$, which drives the transverse coordinates of the plane wave (1) to infinity. This corresponds to the $x_1, x_2 \rightarrow +\infty$ limit associated with quasinormal mode boundary conditions posed in [22,23]. In this regime, the asymptotic symmetries capture a potential infinite-dimensional symmetry enhancement in the near-ring region of $\text{AdS}_2 \times S^2$, or equivalently, the

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neighborhood of null geodesics in the near-horizon region of extremal Reissner-Nordström black holes. We will present boundary conditions encompassing the Nappi-Witten spacetime, finite charge excitations thereof, and more general plane waves (1). We will then identify their asymptotic symmetries, which form an infinite-dimensional algebra that, to our knowledge, has not appeared elsewhere.

II. BROADER SIGNIFICANCE

While our construction is rooted in general relativity, the emergence of an infinite-dimensional, centrally extended algebra in the Nappi-Witten background has implications that reach far beyond gravitational physics. Similar algebraic structures govern symmetry enhancement in diverse contexts, from the Virasoro and BMS families familiar in holography [16,19,25,26] to nonequilibrium and hydrodynamic systems where effective Carrollian or Galilean limits appear [27–29]. The present results therefore point to a universal mechanism by which flux-carrying sectors generate state-dependent current algebras. In the holographic setting, this suggests new avenues for describing near-ring and Penrose-limit regimes of AdS spacetimes, while in quantum field theory it connects to recent discussions of anomalous symmetries and nonintegrable conserved quantities [30,31]. The algebra we uncover may thus serve as a bridge between gravitational asymptotics and broader symmetry phenomena in high-energy and condensed-matter theory alike.

III. THE NAPPI-WITTEN MODEL

The Nappi-Witten model is made a plane wave (1) with $A_{ij} = -\delta_{ij}/4$, $i, j = 1, 2$:

$$ds^2 = 2dudv - \frac{r^2}{4}du^2 + dr^2 + r^2d\theta^2, \quad (2)$$

with $x_1 = r \cos \theta$, $x_2 = r \sin \theta$, a nontrivial Neveu-Schwarz (NS) flux $H_{ur\theta} = r$ and a constant dilaton ϕ . As a WZW model, this model is an extremum of the NS-NS four-dimensional low-energy effective action

$$S = \int d^4x \sqrt{-g} e^{-2\phi} \left(R - \frac{1}{12} H^2 + 4\nabla_\mu \phi \nabla^\mu \phi \right). \quad (3)$$

A. Isometries

The four-dimensional nonsemisimple algebra $\mathfrak{nw}_4$ upon which this group manifold is constructed is defined by the generators $\{P_1, P_2, J, T\}$, whose commutation relations are given by

$$[J, P_i] = \varepsilon_{ij} P_j, \quad [P_i, P_j] = \varepsilon_{ij} T, \quad [T, P_i] = [T, J] = 0. \quad (4)$$

Notice that it is a central extension of the two-dimensional Euclidean algebra, with T acting as the central element, P_i the temporal and spatial translation generators and J the one of rotations. The isometries of (2) form an algebra can be seen as a double copy (left and right-moving) of $\mathfrak{nw}_4$, with the centers of each copy identified. The spacetime described by the metric (2) is actually the Penrose limit [32] of $\text{AdS}_2 \times S^2$, see, e.g., [9,33]. The isometries of (2) can then be seen as enhanced İnönü-Wigner contraction of $SO(1,2) \times SO(3)$ when the radii of AdS_2 and S^2 are equal [33].

B. Relation to Carroll symmetries

It was pointed out in [34] that the isometry group of a generic four-dimensional plane wave is identified as the Carroll group in one less dimension with broken rotations. In four-dimensional plane waves, the dimension of the isometry group is always equal to or larger than five. If the plane wave is conformally flat, then this number is brought to six and the Carroll rotations are restored. Depending on the wave profile, this number can be brought to seven. In the language of [28], we can show that the isometry algebra of the Nappi-Witten spacetime is isomorphic to the seven-dimensional $d = 3$, $z = 0$ type-D minimal extension of the Carroll algebra. What is more, on a fixed $u = k\pi/2$ surface (which are null hypersurfaces orthogonal to the causal boundary of the plane wave), one can check that the seven isometries are projected on the $d = 3$ type-K Carroll algebra. The infinite-dimensional algebra that we present in this letter is however *not* isomorphic to any infinite-dimensional conformal extension of the Carroll algebra. The bulk realization of conformal Carroll algebras in plane waves is discussed in [35].

IV. BOUNDARY CONDITIONS AND SYMMETRY ALGEBRA

We now introduce a conical defect (CD) in the geometry by forcing identifications for the u (acting as a time coordinate) and θ coordinates. Conical defects provide solutions of physical interest, as they model point-particle energy distributions [36]. Following the procedure implemented in [37], we find the following class of geometries¹:

$$ds_{\text{C.D.}}^2 = \frac{1}{\alpha^2 - A^2} dr^2 + r^2 d\theta^2 - \frac{1}{4} r^2 du^2 + 2adudv - 4Ad\theta dv, \quad (5)$$

This family of metrics is governed by both A and α , two constant parameters, for which we recover the Nappi-Witten metric when $A = 0$ and $\alpha = 1$. When $A = 0$, we can easily see that the geometry of the (r, θ) -plane is that of a cone. By defining $r \rightarrow \alpha r$, we have

¹This metric was derived with Gim Seng Ng.

$$ds_{r,\theta}^2 = dr^2 + \alpha^2 r^2 d\theta^2, \quad (6)$$

which corresponds to a cone whose conical defect is governed by α . The r -coordinate measures the distance from the geometric singularity. Performing the same coordinate transformations on the gauge field, one obtains

$$\mathbf{B}^{\text{C.D.}} = \frac{\alpha u - 2A\theta}{\alpha + A} r \left(dr \wedge d\theta - \frac{1}{2} du \wedge dr \right). \quad (7)$$

Acting on the CD with the Lie derivative along the exact Killing vectors, one generates BCs in the asymptotic region defined by $r \rightarrow \infty$. This yields

$$\begin{aligned} g_{uu} &= -\frac{r^2}{4} + \mathcal{O}(r), & g_{uv} &= \mathcal{O}(1), & g_{vv} &= 0, \\ g_{\theta\theta} &= r^2 + \mathcal{O}(r), & g_{rr} &= \mathcal{O}(1), & g_{vr} &= \mathcal{O}(1/r^2), \\ g_{ur} &= \mathcal{O}(1), & g_{v\theta} &= \mathcal{O}(1), & g_{u\theta} &= \mathcal{O}(r), \end{aligned} \quad (8)$$

where the u and θ dependencies of the arbitrary functions have been kept implicit, while they are independent in v . We also generate BCs for the $\mathbf{B}$ -field by acting on the conical defect gauge field $\mathbf{B}^{\text{C.D.}}$ with the generalized Killing couples $(\xi_i, \Lambda_i)^2$

$$B_{\mu\nu}^{\text{C.D.}} = (\mathcal{L}_\xi \mathbf{B}^{\text{C.D.}})_{\mu\nu} + (d\Lambda)_{\mu\nu}, \quad (9)$$

from which we find the following boundary conditions:

$$\begin{aligned} \mathbf{B} &= (ur + \mathcal{O}(r)) du \wedge dr + \left(\frac{1}{2} ur + \mathcal{O}(r) \right) dr \wedge d\theta \\ &+ \mathcal{O}(r) du \wedge d\theta. \end{aligned} \quad (10)$$

A. Asymptotic Killing vectors

Now that we have a set of BCs, we look for the isometries preserving this asymptotic behavior. We focus on the variation of those BCs under the effect of an arbitrary diffeomorphism ξ , whose components are expanded in power series of the radial coordinate r

$$\xi^\mu = \sum_{n=-\infty}^{+\infty} r^n \xi_n^\mu(u, v, \theta). \quad (11)$$

We manage to constrain orders of the power series by asking that each boundary condition is preserved, i.e.,

$$(\mathcal{L}_\xi g)_{\mu\nu} = \mathcal{O}(r^{p_{\mu\nu}}), \quad (12)$$

with $g_{\mu\nu} = g_{\mu\nu}^{\text{NW}} + \mathcal{O}(r^{p_{\mu\nu}})$, $g_{\mu\nu}^{\text{NW}}$ being the background metric. We obtain

$$\begin{cases} \xi^u = A + \mathcal{O}(1/r), \\ \xi^v = g_1(u, \theta)r + g_2(u, \theta) + \mathcal{O}(1/r), \\ \xi^r = h_1(u, \theta) + \mathcal{O}(1/r), \\ \xi^\theta = B + m_1(u, \theta)/r + \mathcal{O}(1/r^2). \end{cases} \quad (13)$$

We constrain the arbitrary functions appearing in this diffeomorphism by making use of our knowledge of the exact Killing vectors. In the form (13), the exact isometries satisfy

$$\begin{cases} 4\partial_u g_1(u, \theta) = -\partial_\theta m_1(u, \theta), \\ \partial_\theta g_1(u, \theta) = -\partial_u m_1(u, \theta), \end{cases} \quad (14)$$

which we will also impose in the asymptotic region. These relations encourage us to work in *light-cone coordinates* $x^\pm = \theta \pm u/2$. Expanding the arbitrary functions $g_1(u, \theta)$ and $m_1(u, \theta)$ into left- and right-moving modes and using Eq. (14), we end up with the asymptotic diffeomorphism

$$\begin{aligned} \xi &= \left[\frac{A}{2} + B + \frac{2}{r}(-g_+(x^+) + g_-(x^-)) \right] \partial_+ \\ &+ \left[-\frac{A}{2} + B + \frac{2}{r}(-g_+(x^+) + g_-(x^-)) \right] \partial_- \\ &+ [r(g_+(x^+) + g_-(x^-)) + g_2(x^+, x^-)] \partial_v \\ &+ [2(\partial_+ g_+(x^+) - \partial_- g_-(x^-))] \partial_r. \end{aligned} \quad (15)$$

We now Fourier expand these generators, with the coordinates $x^\pm$ inheriting the θ -periodicity. We extract three infinite towers of generators and two discrete ones

$$\begin{aligned} \xi_n^+ &= r e^{inx^+} \partial_v + 2i n e^{inx^+} \partial_r - \frac{2}{r} e^{inx^+} \partial_+ - \frac{2}{r} e^{inx^+} \partial_- \\ \xi_n^- &= r e^{inx^-} \partial_v - 2i n e^{inx^-} \partial_r + \frac{2}{r} e^{inx^-} \partial_+ + \frac{2}{r} e^{inx^-} \partial_-, \\ J^+ &= \partial_+, \quad J^- = \partial_-, \quad P_{n,m} = e^{inx^+} e^{imx^-} \partial_v. \end{aligned} \quad (16)$$

The exact Nappi-Witten symmetries (4) with generators T^+ and T^- identified $T^+ = T^- = T$ can be expressed in terms of these generators

$$\begin{aligned} P_1^+ &= \frac{1}{4}(\xi_{-1}^- + \xi_1^-), & P_1^- &= \frac{-i}{4}(\xi_1^+ - \xi_{-1}^+), \\ P_2^+ &= \frac{-i}{4}(\xi_{-1}^- - \xi_1^-), & P_2^- &= -\frac{1}{4}(\xi_1^+ + \xi_{-1}^+), \\ T &= -P_{0,0}, \end{aligned} \quad (17)$$

with $J^\pm \rightarrow \mp J^\mp$. We can also establish a correspondence with the Carroll algebra by introducing

$$A_n = i(\xi_{-n}^- + \xi_n^+) \quad \text{and} \quad B_n = (\xi_{-n}^- - \xi_n^+), \quad (18)$$

²They can be found in the appended *Mathematica* file.

such that we recover the three-dimensional Carroll generators by taking

$$\begin{aligned} C_1 &= -\frac{1}{2}A_1, & P_1 &= \frac{1}{2}A_{-1}, \\ C_2 &= \frac{1}{2}B_1, & P_2 &= -\frac{1}{2}B_{-1}, \\ J_{12} &= J^+ + J^-, & H &= P_{00}, \\ D &= \frac{1}{2}(J^+ - J^-), \end{aligned} \quad (19)$$

with C_1, C_2 the two Carroll boosts, P_1, P_2 the two spatial translations, H the time translation, J_{12} the rotation, as well as an additional dilation D .

Our asymptotic generators form a closed algebra with the following nonzero commutation relations

$$\begin{aligned} [\xi_m^+, \xi_n^+] &= 4i(m-n)P_{m+n,0}, \\ [\xi_m^-, \xi_n^-] &= -4i(m-n)P_{0,m+n}, \\ [J^\pm, \xi_n^\pm] &= in\xi_n^\pm, \\ [J^+, P_{m,n}] &= imP_{m,n}, \\ [J^-, P_{m,n}] &= inP_{m,n}, \end{aligned} \quad (20)$$

in the asymptotic $r \rightarrow \infty$ limit. One can verify that the commutation relations (20) satisfy the Jacobi identity and therefore define a Lie algebra. It admits the following central extensions

$$\begin{aligned} [\xi_m^+, \xi_n^+] &= 4i(m-n)P_{m+n,0} + k(n)\delta_{n+m,0}, \\ [\xi_m^-, \xi_n^-] &= -4i(m-n)P_{0,m+n} + \tilde{k}(n)\delta_{n+m,0}, \\ [J^\pm, \xi_n^\pm] &= in\xi_n^\pm, \\ [J^+, P_{m,n}] &= imP_{m,n}, \\ [J^-, P_{m,n}] &= inP_{m,n}, \\ [\xi_n^+, P_{m,0}] &= (c_3n^3 + c_1n)\delta_{n+m,0}, \\ [\xi_n^-, P_{0,m}] &= (\tilde{c}_3n^3 + \tilde{c}_1n)\delta_{n+m,0}, \\ [\xi_p^+, P_{m,n}] &= h(m,n)\delta_{p+m,0}, & n \neq 0, \\ [\xi_p^-, P_{n,m}] &= \tilde{h}(n,m)\delta_{p+m,0}, & n \neq 0, \end{aligned} \quad (21)$$

with $c_1, c_3, \tilde{c}_1$ and $\tilde{c}_3$ constant and $k(n), \tilde{k}(n), h(m,n)$ and $\tilde{h}(m,n)$ some odd functions in n , which cannot be reabsorbed by a redefinition of the generators.

B. Generalization of boundary conditions

Furthermore, we show that our BCs include the most general pp-wave metric. In Brinkmann coordinates, this family of metrics takes the following form [1]

$$\begin{aligned} ds^2 &= 2dudv + K(u, x^b)du^2 + dx_1^2 + dx_2^2 \\ &\quad + 2A_a(u, x^b)dx^a du, \end{aligned} \quad (22)$$

with $a, b = 1, 2$. Letting $x_1 = r \cos \theta$ and $x_2 = r \sin \theta$, one has

$$\begin{aligned} ds^2 &= 2dudv + K(u, r, \theta)du^2 + dr^2 + r^2d\theta^2 \\ &\quad + 2[A_1(u, r, \theta) \cos \theta + A_2(u, r, \theta) \sin \theta]dudr \\ &\quad + [-A_1(u, r, \theta)r \sin \theta + A_2(u, r, \theta)r \cos \theta]d\theta du. \end{aligned} \quad (23)$$

If $K(u, r, \theta)$ is at most quadratic in r , which is the case for plane waves (1), allowing $g_{uu} = \mathcal{O}(r^2)$ instead of $g_{uu} = -r^2/4 + \mathcal{O}(r)$ includes all plane waves in our boundary conditions. One can also verify that our asymptotic diffeomorphism (15) preserves these more relaxed BCs, giving rise to the same behavior of the charges³ as computed in the following section. Let us mention that this is an interesting result as one can already find in the literature some known examples satisfying these BCs, see, e.g., the Penrose limit of the Schwarzschild metric [1], the Penrose limit of the Kerr black hole, as well as the Penrose limit of the near-horizon near-extremal limit (near-NHEK) [22]. For instance, the Penrose limit of the Schwarzschild spacetime is given by the (Einstein) metric

$$ds^2 = 2dudv + \frac{1}{3M^2}(x^2 - y^2)du^2 + dx^2 + dy^2. \quad (24)$$

Going to polar coordinates, we find

$$K(u, r, \theta) = \frac{r^2 \cos 2\theta}{3M^2}, \quad A_i(u, r, \theta) = 0. \quad (25)$$

V. SURFACE CHARGES

We now compute the surface charges associated to the asymptotic isometries found in the previous section, using the machinery developed in [38,39] (see Refs. [40–42], along with the SurfaceCharges package⁴). Let us mention that the charges associated to the global isometries of the conical defect are finite and integrable.

A. An Abelian charge algebra

First, we consider the case where all BCs do not depend on v , at *any order*. The p -form contribution to the infinitesimal charges is identical for all the asymptotic generalized Killing symmetry couples and can be shown to vanish on-shell. The only contribution to the infinitesimal charges is gravitational. We set a phase space condition

$$g_{rr}^{(0)} g_v^{(0)} g_v^{(0)} = -1, \quad (26)$$

³See appended *Mathematica* file for the details of the computations.

⁴<https://ptm.ulb.be/gcompere/package.html>

where we use the notation

$$g_{\mu\nu} = r^n g_{\mu\nu}^{(0)} + r^{n-1} g_{\mu\nu}^{(-1)} + \dots \quad (27)$$

with n the highest power in r in a given metric component, and the metric corresponds to (8) expressed in the light-cone coordinates. The infinitesimal charges are integrable and given by

$$Q_{J^\pm} = \mp \frac{1}{16\pi G} \int dv \int_0^{2\pi} d\theta \sqrt{\frac{-g_{v\pm}^{(0)3}}{g_{rr}^{(0)} g_{v\mp}^{(0)}}}. \quad (28)$$

Moreover, the EOMs yield

$$\partial_\pm g_{v-}^{(0)} = 0 \quad \text{and} \quad \partial_\pm g_{v+}^{(0)} = 0, \quad (29)$$

resulting in an Abelian algebra

$$\{Q_{J^+}, Q_{J^-}\} = 0. \quad (30)$$

B. Toward an infinite-dimensional algebra

We have, so far, considered our BCs to be independent of the v coordinate. However, satisfying the asymptotic Killing equations allows us to keep a dependence on this coordinate at subleading orders. These slightly modified BCs still yield

$$\delta Q_{P_{m,n}} = 0 \quad (31)$$

when $r \rightarrow \infty$, but contribute nontrivially to the infinitesimal charges associated to $\xi_m^\pm$. On-shell, we find

$$g_{rr}^{(-1)} = 0, \quad (32)$$

and

$$\begin{aligned} \delta Q_{\xi_m^\pm} = & \frac{\pm \delta p}{32\pi G} g_{rr}^{(0)2} T'_m(x^\pm) \left[\frac{2}{g_{rr}^{(0)}} \partial_p \partial_v \left(g_{v-}^{(-1)} - g_{v+}^{(-1)} \right) \right. \\ & \left. + \left(g_{v-}^{(0)} \partial_v g_{v+}^{(-1)} + g_{v+}^{(0)} \partial_v g_{v-}^{(-1)} \right) \partial_p \left(g_{v+}^{(0)} - g_{v-}^{(0)} \right) \right]. \end{aligned} \quad (33)$$

The appearance of nonintegrable terms in the surface-charge variation is not a purely technical issue but signals the presence of genuine *fluxes through the asymptotic boundary*. Such fluxes encode exchanges of energy or information between the two chiral sectors, similar to the radiative fluxes associated with BMS symmetries at null infinity [26,30]. This observation places the asymptotic dynamics of plane-wave spacetimes squarely within the broader program that studies flux-dependent, state-dependent extensions of symmetry algebras in gravitational

and nongravitational settings. Modified brackets have been proposed in the literature, most notably the Barnich-Troessaert brackets [26,43], originally developed for the $\mathfrak{bm}\mathfrak{s}_4$ algebra, which require an arbitrary choice in splitting the charges into integrable and nonintegrable parts. In our case, this procedure yields partially nonantisymmetric results, suggesting that the modified Barnich-Troessaert brackets are not well suited to handle this form of non-integrability. Alternative approaches exist, such as the Wald-Zoupas prescription, which selects a preferred pre-symplectic potential, or a generalization of the Barnich-Troessaert bracket recently proposed to represent the extended corner algebra [44,45]. Finally, we note that while the charges associated with the $J^\pm$ generators are altered by these boundary conditions, they remain finite.

VI. CONCLUSION AND DISCUSSION

Starting from the four-dimensional Nappi-Witten space-time, we have constructed boundary conditions preserved by the infinite-dimensional symmetry algebra (20). The exact nature and relevance of this algebra in physical settings has yet to be fully explored, but we close this letter with some observations. This algebra describes an interplay between two chiral currentlike fields with modes $\xi_m^\pm$ and a bi-chiral field with modes $P_{m,n}^\pm$, with the global generators $J^\pm$ acting as momenta or scaling weights. These structures resemble currentlike algebras, but where the central extension (normally a constant) is replaced by a field/operator. Since the $\xi_m^\pm$ alone do not close, the $P_{m,n}^\pm$ generically represent fluxes responsible for charge non-conservation (but vanishing for our choice of boundary conditions). The study of gravitational phase spaces with nonvanishing fluxes, nonintegrable charges and state-dependent central extensions is a delicate and timely subject that would require further investigations (see, e.g., [26,45–50]). On the other hand, even though field theories with symmetries (20) have not been identified to our knowledge, one could attempt at engineering toy models capturing some of their features. A candidate for a simple example is given by the Lagrangian

$$\mathcal{L} = \partial_+ \chi \partial_- \chi + \chi \partial_+ \phi^+ \partial_- \phi^-, \quad (34)$$

where $\chi(x^+, x^-)$ and $\phi^\pm(x^\pm)$ are scalar fields (chiral for $\phi^\pm$). The model is invariant under independent chiral shifts of $\phi^\pm$: $\delta\phi^+ = \epsilon^+(x^+)$, $\delta\phi^- = \epsilon^-(x^-)$, $\delta\chi = 0$. The associated Noether charges are

$$Q_\pm[\epsilon^\pm] = \int_0^{2\pi} d\varphi \chi \partial_\mp \phi^\mp \epsilon^\pm(x^\pm), \quad (35)$$

while their Fourier modes can be identified with $\xi_m^\pm$. The system exhibits primary constraints which renders the canonical analysis nontrivial, requiring the use of Dirac brackets. This then suggests a charge algebra of

the form $\{\xi_+^m, \xi_+^n\}_D = 4i(m-n)P_{m+n,0}$ with $P_{m,n} = \int_0^{2\pi} d\varphi \chi(x^+, x^-) e^{i(mx^+ + nx^-)}$, and similarly for the “ $-$ ” sector. The field χ can be viewed as a flux mediator, coupling the left- and right-moving field terms $\phi^\pm$ through the interaction $\chi \partial_+ \phi^+ \partial_- \phi^-$, while its dynamics is governed by the equation of motion $2\partial_+ \partial_- \chi = \partial_+ \phi^+ \partial_- \phi^-$, where $\partial_+ \phi^+ \partial_- \phi^-$ plays the role of a flux density. Thus, χ can be interpreted physically as a potential field sourced by the flux associated to $\phi^\pm$. Additionally, the action is also invariant under global chiral dilations associated with global charges $J^\pm$. A detailed analysis of this model, including its canonical structure, quantization, the appearance of potential central extensions, partition function, is left for future work, along with the study of general properties of the symmetry algebra such as representation theory, coadjoint orbits, modular properties and geometric actions. Such a simple action is of course not believed to capture holographic properties of pp-wave spacetimes, no more than a massless scalar two dimensional CFT should describe AdS₃ gravity or a Carrollian boson being dual to gravity in flat space. However, it opens the way for studying new classes of potentially solvable field theories whose generic structure could be relevant in a gravity context. The toy field theory introduced above provides a minimal realization of this mechanism: its mixed chiral interactions reproduce the flux-induced nonclosure of the gravitational charge algebra. Beyond gravity, this construction resonates with field- theoretic descriptions of driven or open quantum systems, where symmetry currents are not strictly conserved but still organize dynamics through extended algebras. Analogous state-dependent symmetry structures appear in non-Lorentzian holography and hydrodynamic effective theories [29,47,49]. Investigating these parallels could reveal how infinite-dimensional symmetries arise universally in systems supporting flux exchange or information flow, from near-horizon holography to non-equilibrium condensed-matter dynamics.

On another hand, it has been shown [51] that black strings can be embedded in pp-wave spacetimes in five and more dimensions. Hence, a question of interest is how these solutions can be included in higher-dimensional generalizations of our boundary conditions and how this

would fit within the surface charges presented in [40]. We have also mentioned, in the Introduction, the intimate relation between Penrose limits and eikonal quasinormal modes, whose wave functions are generically obtained in terms of parabolic cylinder functions. It might be interesting to investigate whether the latter play, in the algebra (20), a role comparable to that of hypergeometric functions for the Virasoro algebra. Finally, let us mention that memory effects in plane waves have recently been discovered in [52] by exploiting the hidden symmetries of plane wave spacetimes. A natural question that this present letter puts forward is whether these memory effects can be related to our asymptotic symmetries and if an infrared triangle structure could be uncovered in asymptotically plane wave spacetimes, as plane waves can offer a simple generalization of results obtained in flat space.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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- [1] M. Blau, Plane waves and penrose limits, *Lecture Notes for the ICTP School on Mathematics in String and Field Theory* (2003) (2011).
 - [2] D. Marolf and S.F. Ross, Plane waves: To infinity and beyond!, *Classical Quantum Gravity* **19**, 6289 (2002).
 - [3] V.E. Hubeny and M. Rangamani, No horizons in pp-waves, *J. High Energy Phys.* **11** (2002) 021.

- [4] R. Güven, Plane wave limits and t-duality, *Phys. Lett. B* **482**, 255 (2000).
- [5] G.T. Horowitz and A.R. Steif, Strings in strong gravitational fields, *Phys. Rev. D* **42**, 1950 (1990).
- [6] G.T. Horowitz and A.A. Tseytlin, Exact solutions and singularities in string theory, *Phys. Rev. D* **50**, 5204 (1994).

- [7] R. Metsaev, Type IIB Green–Schwarz superstring in plane wave Ramond–Ramond background, *Nucl. Phys.* **B625**, 70 (2002).
- [8] R. Metsaev and A. A. Tseytlin, Exactly solvable model of superstring in plane wave ramond-ramond background, *Phys. Rev. D* **65**, 126004 (2002).
- [9] D. E. Berenstein, J. M. Maldacena, and H. S. Nastase, Strings in flat space and pp waves from $N = 4$ super-Yang-Mills, *J. High Energy Phys.* **04** (2002) 013.
- [10] D. Sadri and M. Sheikh-Jabbari, The plane-wave/super Yang-Mills duality, *Rev. Mod. Phys.* **76**, 853 (2004).
- [11] J. Maldacena, The large- N limit of superconformal field theories and supergravity, *Int. J. Theor. Phys.* **38**, 1113 (1999).
- [12] S. Gubser, I. Klebanov, and A. Polyakov, Gauge theory correlators from non-critical string theory, *Phys. Lett. B* **428**, 105 (1998).
- [13] E. Witten, Anti de Sitter space and holography, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
- [14] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Large N field theories, string theory and gravity, *Phys. Rep.* **323**, 183 (2000).
- [15] A. Strominger, Black hole entropy from near-horizon microstates, *J. High Energy Phys.* **02** (1998) 009.
- [16] S. Carlip, Black hole entropy from conformal field theory in any dimension, *Phys. Rev. Lett.* **82**, 2828 (1999).
- [17] M. Cvetič and F. Larsen, Near horizon geometry of rotating black holes in five dimensions, *Nucl. Phys.* **B531**, 239 (1998).
- [18] S. Carlip, Near-horizon conformal symmetry and black hole entropy, *Phys. Rev. Lett.* **88**, 241301 (2002).
- [19] M. Guica, T. Hartman, W. Song, and A. Strominger, The Kerr/CFT correspondence, *Phys. Rev. D* **80**, 124008 (2009).
- [20] S. E. Gralla, A. Lupsasca, and D. P. Marrone, The shape of the black hole photon ring: A precise test of strong-field general relativity, *Phys. Rev. D* **102**, 124004 (2020).
- [21] S. Hadar, D. Kapec, A. Lupsasca, and A. Strominger, Holography of the photon ring, *Classical Quantum Gravity* **39**, 215001 (2022).
- [22] K. Fransen, Quasinormal modes from penrose limits, *Classical Quantum Gravity* **40**, 205004 (2023).
- [23] D. Kapec and A. Sheta, pp-waves and the hidden symmetries of black hole quasinormal modes, *Classical Quantum Gravity* **42**, 155002 (2025).
- [24] C. R. Nappi and E. Witten, Wess-Zumino-Witten model based on a nonsemisimple group, *Phys. Rev. Lett.* **71**, 3751 (1993).
- [25] A. Strominger, AdS(2) quantum gravity and string theory, *J. High Energy Phys.* **01** (1999) 007.
- [26] G. Barnich and C. Troessaert, BMS charge algebra, *J. High Energy Phys.* **12** (2011) 105.
- [27] C. Duval, G. Gibbons, P. Horvathy, and P. Zhang, Carroll symmetry of plane gravitational waves, *Classical Quantum Gravity* **34**, 175003 (2017).
- [28] H. Afshar, X. Bekaert, and M. Najafizadeh, Classification of conformal carroll algebras, *J. High Energy Phys.* **12** (2024) 148.
- [29] L. Ciambelli and L. Lehner, Fluid-gravity correspondence and causal first-order relativistic viscous hydrodynamics, *Phys. Rev. D* **108**, 126019 (2023).
- [30] L. Freidel and D. Pranzetti, Gravity from symmetry: Duality and impulsive waves, *J. High Energy Phys.* **04** (2022) 125.
- [31] L. Ciambelli, From asymptotic symmetries to the corner proposal, *Proc. Sci., Modave2022* (2023) 002 [arXiv:2212.13644].
- [32] R. Penrose, Any space-time has a plane wave as a limit, in *Differential Geometry and Relativity: A Volume in Honour of André Lichnerowicz on His 60th Birthday* (Springer, New York, 1976), pp. 271–275.
- [33] M. Blau, J. Figueroa-O’Farrill, and G. Papadopoulos, Penrose limits, supergravity and brane dynamics, *Classical Quantum Gravity* **19**, 4753 (2002).
- [34] C. Duval, G. W. Gibbons, P. A. Horvathy, and P.-M. Zhang, Carroll symmetry of plane gravitational waves, *Classical Quantum Gravity* **34**, 175003 (2017).
- [35] E. Despontin, S. Detournay, S. Dutta, and D. Fontaine, Anisotropic conformal Carroll field theories and their gravity duals, *J. High Energy Phys.* **09** (2025) 056.
- [36] S. Deser and R. Jackiw, Three-dimensional cosmological gravity: Dynamics of constant curvature, *Ann. Phys. (N.Y.)* **153**, 405 (1984).
- [37] J. D. Brown and M. Henneaux, Central charges in the canonical realization of asymptotic symmetries: An example from three dimensional gravity, *Commun. Math. Phys.* **104**, 207 (1986).
- [38] V. Iyer and R. M. Wald, Some properties of the noether charge and a proposal for dynamical black hole entropy, *Phys. Rev. D* **50**, 846 (1994).
- [39] G. Barnich and F. Brandt, Covariant theory of asymptotic symmetries, conservation laws and central charges, *Nucl. Phys.* **B633**, 3 (2002).
- [40] G. Compère, Note on the first law with p-form potentials, *Phys. Rev. D* **75**, 124020 (2007).
- [41] S. Detournay and M. Guica, Stringy Schrödinger truncations, *J. High Energy Phys.* **08** (2013) 121.
- [42] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/2q94-5crl> for Mathematica file.
- [43] J. Bosma, M. Geiller, S. Majumdar, and B. Oblak, Radiative asymptotic symmetries of 3D Einstein-Maxwell theory, *SciPost Phys.* **16**, 092 (2024).
- [44] L. Freidel, A canonical bracket for open gravitational system, arXiv:2111.14747.
- [45] L. Freidel and D. Pranzetti, The extended corner symmetry algebra of 4d gravity, *J. High Energy Phys.* **04** (2021) 135.
- [46] L. Ciambelli, From asymptotic symmetries to the corner proposal, *Proc. Sci., Modave2022* (2023) 002 [arXiv:2212.13644].
- [47] H. Adami, M. M. Sheikh-Jabbari, V. Taghiloo, H. Yavartanoo, and C. Zwickel, Symmetries at null boundaries: Two and three dimensional gravity cases, *J. High Energy Phys.* **10** (2020) 107.
- [48] M. Geiller, C. Goeller, and C. Zwickel, 3d gravity in Bondi-Weyl gauge: Charges, corners, and integrability, *J. High Energy Phys.* **09** (2021) 029.
- [49] L. Ciambelli, A. Delfante, R. Ruzziconi, and C. Zwickel, Symmetries and charges in Weyl-Fefferman-Graham gauge, *Phys. Rev. D* **108**, 126003 (2023).

- [50] L. Ciambelli, C. Marteau, P.M. Petropoulos, and R. Ruzziconi, Fefferman-Graham and Bondi gauges in the fluid/gravity correspondence, [Proc. Sci., CORFU2019 \(2020\) 154](#) [[arXiv:2006.10083](#)].
- [51] E.G. Gimon, A. Hashimoto, O. Lunin, V.E. Hubeny, and M. Rangamani, Black strings in asymptotically plane wave geometries, [J. High Energy Phys. 08 \(2003\) 035](#).
- [52] J. Ben Achour and J.-P. Uzan, Displacement versus velocity memory effects from a gravitational plane wave, [J. Cosmol. Astropart. Phys. 08 \(2024\) 004](#).