

Weak bosons as partons below 10 TeV partonic center of momentum

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We investigate the modeling of weak boson number densities for leptons and hadrons in practical calculations in the Standard Model. In the framework of the Effective W Approximation (EWA) and in the R_ξ and axial gauges, we derive the unrenormalized, tree-level parton number densities for weak bosons from massless fermions at next-to-leading power in the collinear expansion. Corrections exhibit various pathologies and properties, including those conjectured but not proven, and parallel heavy quark factorization. We suppress pathologies through a new set of kinematical consistency conditions. When satisfied, shapes and normalizations of full matrix elements for many-leg processes can be well approximated by the EWA and fragmentation contributions at leading power, suggesting the onset of tree-level factorization. Findings also suggest that the EWA is testable at the LHC with $\mathcal{L} = 450 \text{ fb}^{-1}$ of same-sign WW scattering data at $\sqrt{s} = 13.6 \text{ TeV}$.

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I. INTRODUCTION

Despite confinement in quantum chromodynamics (QCD), the structure of hadrons can be reliably described with the QCD-improved parton model. A core tenet of this framework is the existence of a “parton sea” within a hadron, i.e., a near continuum of low-energy gluons and light quark-antiquark pairs. Importantly, sea partons originate from higher energy “valance” quarks that radiate gluons, which subsequently split and populate lower-energy modes.

In multi-TeV pp collisions, like those at the Large Hadron Collider (LHC), partonic scattering scales can be *much* larger than the masses of heavy-flavored quarks. Under such conditions, the parton sea can be extended to include perturbatively generated charm [1], bottom [2,3], and top quarks [4,5]. Since quarks also carry electric charges, and hence can radiate photons, the sea distributions in the proton [6–8] and neutron [6,9] also feature a photon component.

In the Standard Model of particle physics (SM), quarks (q) and leptons (ℓ) carry weak gauge charges, and the emergence of W and Z bosons as (perturbatively generated) sea partons in ultra-high-energy hadron collisions has long been predicted [10–12]. However, what practically constitutes “ultra-high energies” has never been firmly established.

In this work, we show that the answer is about 800 GeV. More specifically, when an incoming fermion radiates a (quasi)collinear W with at least $E_V = 800 \text{ GeV}$ of energy (900 GeV for Z) in the partonic center-of-momentum (pCM) frame, the partonic description of weak bosons becomes a reliable approximation of full matrix elements at tree level, suggesting the onset of factorization.

Presently, there is no all-orders factorization theorem that extends the parton model to the electroweak (EW) sector. It is unclear whether such a formulation is even possible [13–25]. Historically, predictions for EW boson parton number density functions (PDFs) have varied significantly [12,26–33]. But more recently, kinematical regimes have been proposed where predictions for W/Z PDFs might be reliable [32–37]. Here, we derive and firmly establish this regime.

Even at lowest order (LO), reliable predictions for weak boson PDFs can have broad impact. LHC data, for example, show that the same-sign WW scattering signal process, $W_1^\pm W_2^\pm \rightarrow W_3^\pm W_4^\pm \rightarrow \ell^\pm \ell'^\pm \nu_\ell \nu_{\ell'}$ is well

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$$\mu^-(p_i) \rightarrow \ell(p_f) V_\lambda(q) = \underbrace{\text{LLA} + \mathcal{O}\left(\frac{M_V^2}{\mu_f^2}\right)}_{\text{LP in } \theta_\ell} + \underbrace{\mathcal{O}\left(\frac{\mu_f^2}{E_V^2}, \frac{M_V^2}{E_V^2}\right)}_{\text{NLP in } \theta_\ell} + \dots$$

FIG. 1. Schematic relationship between the leading log (LLA), leading power (LP), and next-to-leading power (NLP) approximations in collinear $\mu^- \rightarrow \ell V_\lambda$ splitting at high energies.

described by LO matrix elements [38–44]. It would be appealing if LO predictions for W/Z PDFs of a proton could describe LHC data. Among applications, one could more easily conceptualize and simulate new phenomena [43,45,46], in analogy to heavy quark PDFs [4,5,47,48]. Measuring weak boson PDFs would also constitute laboratory-based probes of the universe’s EW epoch [49–51]. Above all, a complete understanding of EW symmetry breaking necessitates the determination of weak boson PDFs.

We report a breakthrough in this program. In the context of the Effective W Approximation (EWA) [10–12] we present the unrenormalized, tree-level PDFs for helicity-polarized weak bosons from massless, chiral leptons at next-to-leading power (NLP) in the collinear expansion. NLP corrections are process independent, exhibit phenomenologically interesting properties, and can reconcile reports of the EWA’s breakdown. Using these corrections, we derive a novel set of kinematical consistency conditions based on (i) collinear kinematics, (ii) gauge invariance, and (iii) positive definiteness. When enforced, shapes and normalizations of full matrix elements for many-leg processes at tree level can be well approximated by the EWA and fragmentation contributions.

Our work continues in the following order: In Sec. II we summarize our theoretical framework and report our NLP PDFs in Sec. III. In Sec. IV we discuss their phenomenological implications and in Sec. V illustrate our findings with lepton collisions. We finish with an outlook for testing the EWA at the LHC in Sec. VI. We discuss additional checks and technical implementation details of our work in Appendixes A, B, and C.

II. SETUP

We start by considering the inclusive, high-energy, muon-hadron deep-inelastic scattering process

$$\mu^-(p_i) + h(k) \xrightarrow{V_\lambda(q) h \rightarrow X} \ell(p_f) + X(k'). \quad (1)$$

Here $\ell \in \{\mu^-, \nu_\mu\}$ and $V \in \{W^\pm, Z\}$ is the exchange boson with helicity $\lambda \in \{\pm 1, 0, S\}$, momentum $q = p_i - p_f$, mass M_V , and squared virtuality $q^2 \equiv -Q^2 < 0$. The splitting $\mu^-(p_i) \rightarrow \ell(p_f) V_\lambda(q)$ is illustrated in the left side of Fig. 1. We assume $\sqrt{Q^2} \gtrsim \mathcal{O}(M_V)$ and adopt the following momentum assignments:

$$p_i^\mu = E_i(1, 0, 0, +1), \quad (2a)$$

$$\begin{aligned} p_f^\mu &= \left((1-z)E_i, \vec{p}_T, \sqrt{(1-z)^2 E_i^2 - p_T^2} \right) \\ &\equiv (1-z)E_i(1, \sin\theta_\ell \cos\phi_\ell, \sin\theta_\ell \sin\phi_\ell, \cos\theta_\ell), \end{aligned} \quad (2b)$$

$$\begin{aligned} q^\mu &= p_i^\mu - p_f^\mu \\ &\equiv (zE_i, |\vec{q}| \sin\theta_V \cos\phi_V, |\vec{q}| \sin\theta_V \sin\phi_V, |\vec{q}| \cos\theta_V). \end{aligned} \quad (2c)$$

Here, $z = E_V/E_i$ is the fraction of energy V carries from μ . From transverse-momentum conservation one can derive exact relationships between (θ_ℓ, ϕ_ℓ) and (θ_V, ϕ_V) .

To build our PDFs for V_λ , we use the following basis of polarization vectors in the R_ξ gauge:

$$\begin{aligned} \varepsilon^\mu(q, \lambda = \pm 1) &= \frac{1}{\sqrt{2}}(0, -\lambda \cos\theta_V \cos\phi_V + i \sin\phi_V \\ &\quad - \lambda \cos\theta_V \sin\phi_V - i \cos\phi_V, \lambda \sin\theta_V), \end{aligned} \quad (3a)$$

$$\varepsilon^\mu(q, \lambda = 0) = \frac{E_V}{\sqrt{q^2}|\vec{q}|} \left(\frac{|\vec{q}|^2}{E_V}, q_x, q_y, q_z \right) \quad (3b)$$

$$= \frac{q^\mu}{\sqrt{q^2}} + \tilde{\varepsilon}_0^\mu(q), \quad \text{where} \quad (3c)$$

$$\tilde{\varepsilon}_0^\mu(q) \equiv \frac{\sqrt{q^2}}{(E_V + |\vec{q}|)|\vec{q}|} (-|\vec{q}|, q_x, q_y, q_z), \quad (3d)$$

$$\varepsilon^\mu(q, \lambda = S) = \sqrt{\frac{1}{q^2} + \frac{(\xi - 1)}{q^2 - \xi M_V^2}} q^\mu. \quad (3e)$$

These obey the completeness relationship

$$\begin{aligned} \sum_{\lambda=\pm} \varepsilon_\mu(q, \lambda) \varepsilon_\nu^*(q, \lambda) + \varepsilon_\mu(q, 0) \varepsilon_\nu(q, 0) \\ - \varepsilon_\mu(q, S) \varepsilon_\nu(q, S) = -g_{\mu\nu} - \frac{(\xi - 1)q_\mu q_\nu}{(q^2 - \xi M_V^2)}. \end{aligned} \quad (4)$$

We split the $\lambda = 0$ polarization vector in Eq. (3b) into its Goldstone ($q^\mu/\sqrt{q^2}$) and gauge ($\tilde{\varepsilon}_0^\mu$) components. This

makes manifest the decoupling of Goldstone modes since q^μ is orthogonal to the $\mu \rightarrow \ell$ current [52]. The scalar polarization ($\lambda = S$) also decouples since $\varepsilon^\mu(q, S) \propto q^\mu$, implying results hold for all ξ .

We make the usual on-shell approximation [10], and use the polarization vectors for external states. These are obtained by making the replacement $\sqrt{q^2} \rightarrow M_V$ in Eqs. (3b)–(3d), but nowhere else. The implications of this have been discussed elsewhere [28,53,54].

III. BEYOND LEADING LOG

We proceed by computing the tree-level helicity amplitudes for $\mu \rightarrow V_\lambda \ell$ splitting. Next, we expand squared amplitudes to leading power (LP) and NLP in θ_ℓ . (In contrast, Refs. [55,56] evaluate amplitudes *after* Taylor expanding.) For $\lambda = 0$, only the leading $\mathcal{O}(M_V^2/E_V^2)$ term is kept. We integrate over the phase space of ℓ . Because the integral over $q^2 \sim \mathcal{O}(p_T^2)$ is ultraviolet divergent, we introduce a regulator $\mu_f > 0$ with mass dimension one, as in Ref. [10]. Alternatively, one can use dimensional regularization [12].

At LP in the θ_ℓ expansion, the tree-level, helicity-polarized W/Z PDFs for chiral leptons are

$$f_{V_+/\ell_L}^{\text{LP}}(z, \mu_f^2) = \frac{\tilde{g}^2 g_L^2 (1-z)^2}{4\pi^2 2z} \left[\log\left(\frac{\mu_f^2 + M_V^2}{M_V^2}\right) - \left(\frac{\mu_f^2}{\mu_f^2 + M_V^2}\right) \right], \quad (5a)$$

$$f_{V_-/\ell_L}^{\text{LP}}(z, \mu_f^2) = \frac{\tilde{g}^2 g_L^2}{4\pi^2 2z} \left[\log\left(\frac{\mu_f^2 + M_V^2}{M_V^2}\right) - \left(\frac{\mu_f^2}{\mu_f^2 + M_V^2}\right) \right], \quad (5b)$$

$$f_{V_0/\ell_L}^{\text{LP}}(z, \mu_f^2) = \frac{\tilde{g}^2 g_L^2 (1-z)}{4\pi^2 z} \left(\frac{\mu_f^2}{\mu_f^2 + M_V^2}\right), \quad (5c)$$

$$f_{V_\lambda/\ell_R}^{\text{LP}}(z, \mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_{-\lambda}/\ell_L}^{\text{LP}}(z, \mu_f^2), \quad (5d)$$

where the couplings are defined in Table I. Keeping the leading term when $\mu_f^2 \gg M_V^2$ recovers the usual PDFs in the leading log approximation (LLA) [10–12]. The relationship

TABLE I. Coupling definitions for fermions f, f' with weak isospin $(T_3)_L = \pm 1/2$ and electric charge Q^f .

Vertex	$\tilde{g}$	g_R^f	g_L^f
$W - f - f'$	$\frac{g}{\sqrt{2}}$	0	1
$Z - f - f$	$\frac{g}{\cos\theta_W}$	$-Q^f \sin^2\theta_W$	$(T_3^f)_L - Q^f \sin^2\theta_W$

between collinear expansions at LLA, LP, and NLP is illustrated in Fig. 1.

At NLP in the collinear expansion, we obtain

$$f_{V_+/\ell_L}^{\text{NLP}}(z, \mu_f^2) = f_{V_+/\ell_L}^{\text{LP}}(z, \mu_f^2) \left[1 + \frac{(2-z)}{(1-z)} \frac{M_V^2}{E_V^2} \right] - \frac{\mu_f^2}{4E_V^2} (2-z) f_{V_0/\ell_L}^{\text{LP}}(z, \mu_f^2), \quad (6a)$$

$$f_{V_-/\ell_L}^{\text{NLP}}(z, \mu_f^2) = f_{V_-/\ell_L}^{\text{LP}}(z, \mu_f^2) \left[1 + (2-z) \frac{M_V^2}{E_V^2} \right] - \frac{\mu_f^2}{4E_V^2} \frac{(2-z)}{(1-z)} f_{V_0/\ell_L}^{\text{LP}}(z, \mu_f^2), \quad (6b)$$

$$f_{V_0/\ell_L}^{\text{NLP}}(z, \mu_f^2) = f_{V_0/\ell_L}^{\text{LP}}(z, \mu_f^2) - \frac{M_V^2}{2E_V^2} [f_{V_+/\ell_L}^{\text{LP}}(z, \mu_f^2) + f_{V_-/\ell_L}^{\text{LP}}(z, \mu_f^2)], \quad (6c)$$

$$f_{V_\lambda/\ell_R}^{\text{NLP}}(z, \mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_{-\lambda}/\ell_L}^{\text{NLP}}(z, \mu_f^2). \quad (6d)$$

For quarks, weak boson PDFs at LLA, LP, and NLP are the same, up to gauge quantum numbers.

IV. A NOVEL SET OF CONSTRAINTS

From Eq. (6) several observations can be made:

- (i) NLP PDFs can be written as linear sums of LP PDFs for different helicities with mass-over-energy coefficients. This is suggestive of helicity inversion.
- (ii) NLP PDFs for $\lambda = \pm 1$ grow negative with increasing μ_f^2/E_V^2 , violating positive definiteness of PDFs at LO. Here, $\mu_f/E_V \sim p_T^V/p_z^V \sim \theta_V$, and the breakdown occurs because momenta are taken outside the collinear limit. We stress that taking $\mu_f \ll E_V$ is contrary to the standard practice of setting $\mu_f \sim \mathcal{O}(E_V)$. However, the practice is only justifiable when PDFs are resummed and μ_f is a renormalization scale as in Refs. [12,16,21,23].
- (iii) NLP PDFs for $\lambda = 0$ grow negative with decreasing E_V^2/M_V^2 , reflecting a violation of the $\mathcal{O}(M_V^2/E_V^2)$ expansion. Again, $z = E_V/E_\ell > M_V/E_\ell$ is a non-standard restriction when applying the EWA.
- (iv) In Eq. (6), $\mathcal{O}(M_V^2/E_V^2)$ and $\mathcal{O}(\mu_f^2/E_V^2)$ terms violate Bjorken scaling. In the deep-inelastic limit, E_V and $Q^2 \sim \mu_f^2 \gtrsim M_V^2$ are large, $z \propto Q^2/E_V$ is fixed, $\mathcal{O}(\mu_f^2/E_V^2)$ terms vanish, and scaling is recovered.

Corrections to weak boson PDFs beyond LP have been estimated for several processes [26,28,29,32,34,37,53,55,57], albeit with mixed findings regarding their size. After careful study of these works, we

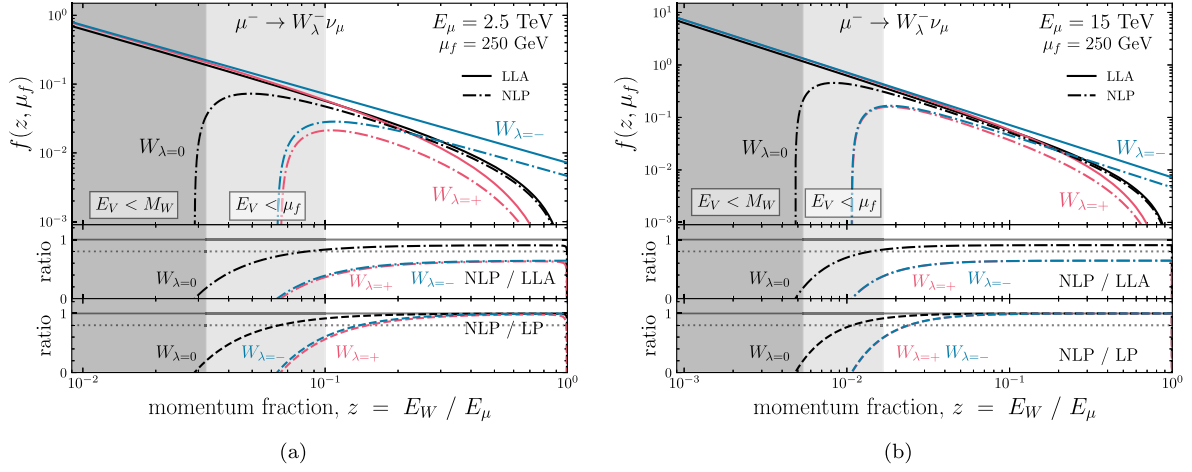


FIG. 2. Top: For (a) $E_\mu = 2.5$ TeV and (b) $E_\mu = 15$ TeV, the NLP (dash-dot) and LLA (solid) PDFs for W in $\mu^- \rightarrow W_\lambda^- \nu_\mu$, as a function $z = E_W/E_\mu$ with $\mu_f = 250$ GeV. Middle (bottom): ratio of NLP and LLA (LP) PDFs.

can attribute such disagreements to inappropriate choices of z and μ_f . For the photon, fermion-mass corrections beyond the LLA are well known [58]. The PDFs in Eq. (6) have not been previously reported.

Beyond the R_ξ gauge, we check gauge dependence using the EW axial gauge [59] with reference vectors

$$\text{lightcone (LC): } n^\mu = (1, -\hat{q}), \quad n^2 = 0, \quad (7a)$$

$$\text{temporal (TL): } n^\mu = (1, \vec{0}), \quad n^2 = +1, \quad (7b)$$

$$\text{spatial (SL): } n^\mu = (0, -\hat{q}), \quad n^2 = -1. \quad (7c)$$

The polarization vectors for $\lambda = \pm 1$ helicities are the same here as in the R_ξ gauge. For $\lambda = S$, we have $\varepsilon^\mu(q, \lambda = S) = 0$ while for $\lambda = 0$,

$$\varepsilon^\mu(q, \lambda = 0) = \frac{\sqrt{q^2} \left[\frac{n^2}{(q \cdot n)} q^\mu - n^\mu \right]}{\sqrt{(q \cdot n)^2 - n^2 q^2}}. \quad (8)$$

Together, these obey the completeness relationship

$$\begin{aligned} \sum_{\lambda=\pm} \varepsilon_\mu(q, \lambda) \varepsilon_\nu^*(q, \lambda) + \varepsilon_\mu(q, 0) \varepsilon_\nu(q, 0) - \varepsilon_\mu(q, S) \varepsilon_\nu(q, S) \\ = -g_{\mu\nu} + \frac{n_\mu q_\nu + n_\nu q_\mu}{(n \cdot q)} - \frac{n^2}{(q \cdot n)^2} q_\mu q_\nu. \end{aligned} \quad (9)$$

In this gauge the on-shell approximation [10] is obtained by making the replacement $q^2 \rightarrow M_V^2$ [54].

In the LC gauge, the q^μ term in Eq. (8) vanishes and the n^μ term reduces to $\tilde{\varepsilon}_0^\mu(q)$ in Eq. (3d). Hence, for all three helicities the PDFs at all powers of θ_ℓ are the same as those in the R_ξ gauge. In the TL and SL gauges, we also obtain the same PDFs at LP and LLA. The robustness of W/Z

PDFs at LP across multiple gauges has not been previously reported.

At NLP, only the transverse PDFs are the same in the TL and SL gauges. For the longitudinal PDF, we obtain $\mathcal{O}(M_V^2/E_V^2)$ terms that differ from Eq. (6c). This gauge dependence is a feature of axial gauges. Changing n^μ corresponds to shuffling terms between individual graphs. Indeed, interference terms in the R_ξ and LC gauges differ from those in the TL and SL gauges by $\mathcal{O}(M_V^2/E_V^2)$ and $\mathcal{O}(M_V \mu_f/E_V^2)$ terms.

Altogether, EWA predictions can be trusted but only when NLP corrections are negligible, i.e., when

$$(M_V^2/E_V^2) \ll 1 \text{ and } (\mu_f^2/E_V^2) \ll 1 \text{ (lab frame)}. \quad (10)$$

These kinematical criteria are the key finding of our work and can resolve historic discrepancies between EWA and full matrix element computations.

To illustrate their impact we show in the top panel of Fig. 2 the NLP (dash-dot) and LLA (solid) PDFs for W in $\mu^- \rightarrow W_\lambda^- \nu_\mu$ splitting as a function of energy fraction $z = E_W/E_\mu$, at (a) $E_\mu = 2.5$ TeV and (b) $E_\mu = 15$ TeV with $\mu_f = 250$ GeV. Configuration (a) corresponds to an LHC-like valance parton at $\sqrt{s} = 14$ TeV or a benchmark $\mu^+ \mu^-$ collider at $\sqrt{s} = 5$ TeV [60–62]. In the middle (bottom) panel, we show the NLP-to-LLA (NLP-to-LP) PDF ratios.

Pathologies emerge as $z \rightarrow 0$. When $E_W < M_W$, the NLP PDF for $\lambda = 0$ is negative, while for $\lambda = \pm 1$, this occurs when $E_W < \mu_f$. The bottom panels show that pure NLP terms are suppressed when $E_W > \mu_f$. This means that for moderate and large z the NLP-to-LLA ratio can be interpreted as the LP-to-LLA ratio. As $\mu_f = 250$, we are not in the large-log limit, and $\mathcal{O}(1)$ terms at LP are about $\delta f^{\text{LP}}/f^{\text{LLA}} \sim -10\%(-35\%)$ for $\lambda = 0(\pm 1)$ when $z \gtrsim 0.1$.

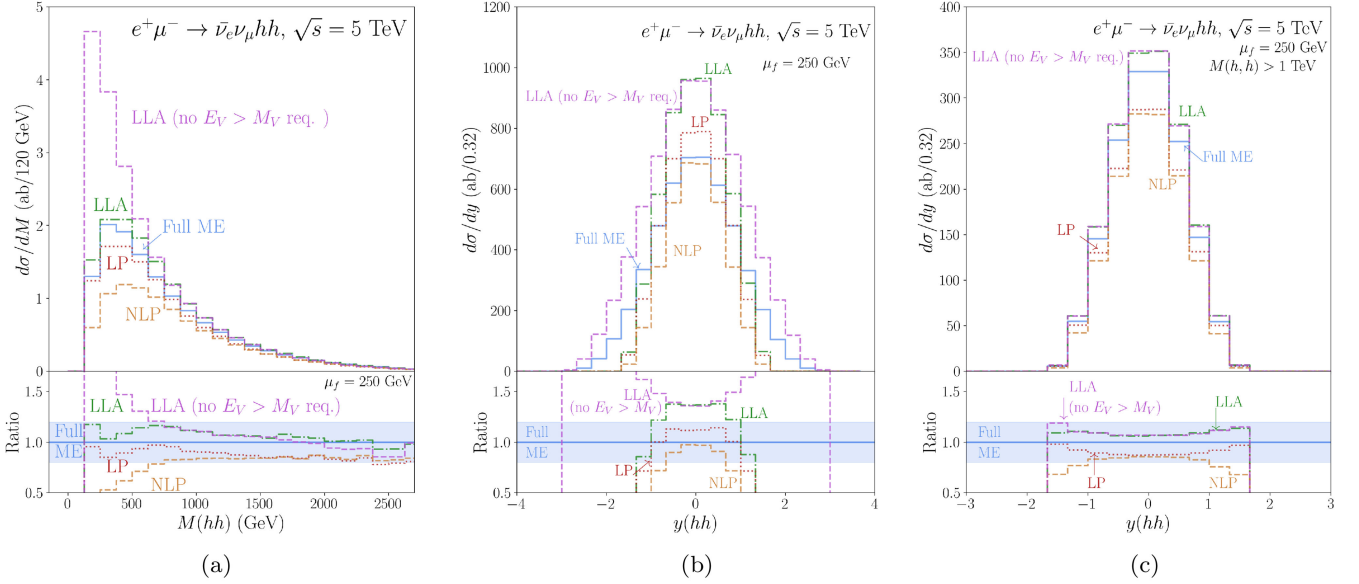


FIG. 3. (a) Top: The invariant mass distribution of the (hh) system in $e^+\mu^- \rightarrow \bar{\nu}_e \nu_\mu hh$ at LLA (without imposing $E_V > M_V$) (dashed), LLA (dot-dashed), LP (dotted), and NLP (dashed), along with the full $2 \rightarrow 4$ ME (solid) at $\sqrt{s} = 5$ TeV. Bottom: ratios with respect to the full ME. (b),(c) Same as (a) but for the rapidity of the (hh) system, and for $M(hh) > 1$ TeV.

V. EWA IN HIGH-ENERGY LEPTON COLLISIONS

To further illustrate the impact of Eq. (10), we consider in $e^+\mu^\mp$ collisions the following processes:

$$W_\lambda^+ W_{\lambda'}^- \rightarrow hh, \gamma\gamma\gamma, \text{ and } W_\lambda^+ W_{\lambda'}^+ \rightarrow W^+ W^+ \quad (11)$$

at LO and at $\sqrt{s} = 5$ TeV. We focus on these because (i) $W^+ W^- \rightarrow hh$ is mediated almost exclusively by $W_0 W_0$ scattering [37,63]. (ii) Similarly, $W^+ W^- \rightarrow \gamma\gamma\gamma$ is driven by $W_T^+ W_T^-$ ($T = \pm 1$) scattering [37,63]. The full $2 \rightarrow 5$ matrix element (ME) also contains non-trivial gauge cancellations and gauge-invariant, bremsstrahlung sub-contributions [64]. (iii) The full $2 \rightarrow 4$ ME for $W^+ W^+$ production contains large gauge cancellations and weak fragmentation contributions off initial- and final-state legs.

To simulate these processes, we implemented the LP and NLP PDFs given in Eqs. (5) and (6) for weak bosons from electrons and muons into the simulation framework MadGraph5_aMC@NLO [37,63,65,66]. (New features are available from v3.7.0 onward; see also Appendix A for usage support.) Total (σ) and differential ($d\sigma$) cross sections in the EWA are obtained assuming the factorization formula

$$\sigma_{e^+\mu^- \rightarrow \mathcal{F}+X} = f_{V_\lambda/e^+} \otimes f_{V_{\lambda'}/\mu^-} \otimes \hat{\sigma}_{V_\lambda V_{\lambda'} \rightarrow \mathcal{F}}. \quad (12)$$

Here, $\hat{\sigma}$ is the “partonic” cross section for the processes in Eq. (11). $\otimes$ denotes a convolution over z_i for incoming boson i . Unless noted, we require $z_{1(2)} > z_{\min} = (M_W/E_{e(\mu)})$. Initial-state W ’s are collinear to the beam ($|\vec{q}_{Ti}| = 0$) and on-shell, with $M_W^2 \neq 0$. Helicities and

energies are defined in the frame of the (WW) system. We set $\mu_f = 250(450)[340]$ GeV for $hh(\gamma\gamma\gamma)[W^+ W^+]$ production, which approximates the threshold scale for each process. For non-EWA MEs, all interfering diagrams are considered.

To regulate infrared poles in full and approximated MEs, we impose on $V \in \{\gamma, W\}$

$$p_T^V > 150 \text{ GeV}, |\eta^V| < 3, \text{ and } \Delta R_{\gamma\gamma} > 0.4. \quad (13)$$

Since $\sqrt{s} = 5$ TeV, Eq. (13) is sufficient to ensure MEs are well defined/perturbative in the sense of Collins, Soper, and Sterman (CSS) Ref. [67]. However, this is not guaranteed for larger $\sqrt{s}$ or for alternative multileg processes [23].

In the upper panel of Fig. 3(a) we show the invariant mass distribution of the (hh) system in $W^+ W^- \rightarrow hh$ as predicted by the EWA at LLA (dot-dashed), LP (dotted), and NLP (dashed), as well as by the full $2 \rightarrow 4$ ME for $e^+\mu^- \rightarrow \bar{\nu}_e \nu_\mu hh$ (solid). The lower panel shows the ratios with respect to the full ME with a $\pm 20\%$ tolerance band.

Over the entire $M(hh) = M(WW)$ mass range, disagreements between the full ME, LLA, and LP distributions fall within $\mathcal{O}(\pm 20\%)$. For low $M(WW)$, NLP predictions are far below the full ME. However, NLP and LP predictions converge for $M(WW) \gtrsim 1.6$ TeV, which corresponds to $E_W \sim 800$ GeV and $(M_W^2/E_W^2) \sim \mathcal{O}(1\%)$. For comparison, we also plot the LLA prediction allowing $z < M_V/E_\ell$ (dashed), which leads to the EWA overestimating the low $M(WW)$ region. We find similar agreement in the rapidity of the (hh) system when omitting [Fig. 3(b)] or applying [Fig. 3(c)] the criteria of Eq. (10).

TABLE II. Predicted cross sections (σ) (fb) for $e^+\mu^- \rightarrow \bar{\nu}_e \nu_\mu hh$ at $\sqrt{s} = 5$ TeV using the full ME (row 1) and the EWA at various accuracies (rows 2-4), along with standard 3-point scale uncertainties, for $M(hh) > 2m_h$ (1.6 TeV). Also shown (column 3) are the deviations [%] from the full ME.

σ [fb]	$M(hh) > 2m_h$ (1.6 TeV)	$(\sigma^{\text{EWA}} - \sigma^{\text{Full}})/\sigma^{\text{Full}}$
Full	1.66 (0.186)	—
LLA	$1.86^{+3\%}_{-1\%}$ ($0.194^{+1\%}_{<-0.5\%}$)	+12% (+4%)
LP	$1.51^{+18\%}_{-40\%}$ ($0.158^{+16\%}_{-39\%}$)	-9% (-14%)
NLP	$1.16^{+0\%}_{-29\%}$ ($0.157^{+15\%}_{-39\%}$)	-30% (-16%)

For $M(hh) > 2m_h$ (1.6 TeV), cross sections [fb] with 3-point scale uncertainties [%] are listed in Table II. Also shown are the deviations from the full ME [%]. The LLA best recovers the full ME at large $M(WW)$ but with an untrustworthy uncertainty of $\mathcal{O}(1\%)$ since $\mathcal{O}(M_W^2/\mu_f^2)$ terms are neglected. LP and NLP predictions feature slightly worse agreements but more reliable uncertainties. Interestingly, setting $\mu_f = 4m_h \sim 500$ GeV (not shown), which is roughly at the maximum of the $M(WW)$ distribution, reconciles (N)LP and full ME predictions.

To model the $\gamma\gamma\gamma$ process, we consider the incoherent sum of the EWA channel ($\sigma_{3\gamma}^{\text{EWA}}$) and a bremsstrahlung term ($\sigma_{3\gamma}^{\text{brem}}$). The $\sigma_{3\gamma}^{\text{brem}}$ contribution is estimated by scaling the rate for the $e^+\mu^- \rightarrow \bar{\nu}_e \nu_\mu \gamma\gamma$ process by a QED Sudakov factor:

$$\sigma_{3\gamma}^{\text{brem}} = S_{\text{QED}} \times \sigma^{\text{Full}}(e^+\mu^- \rightarrow \bar{\nu}_e \nu_\mu \gamma\gamma), \quad (14a)$$

$$S_{\text{QED}} = \frac{\alpha}{4\pi} \log^2 \left(\frac{\mu_S^2}{(p_{T,\min})^2} \right), \quad (14b)$$

with $p_{T,\min} = 150$ GeV and $\mu_S = \mu_f$. For each EWA contribution, we then rescale $\sigma_{3\gamma}^{\text{EWA}}$ to $\sigma_{3\gamma}^{\text{EWA+brem}} = \sigma_{3\gamma}^{\text{EWA}} + \sigma_{3\gamma}^{\text{brem}}$. Numerically, $\sigma_{3\gamma}^{\text{EWA}}$ and $\sigma_{3\gamma}^{\text{brem}}$ are comparable. Due to $\mathcal{O}(\mu_f^2/E_V^2)$ corrections, $\sigma_{3\gamma}^{\text{NLP}}$ goes negative for $M(\gamma\gamma\gamma) \lesssim 1250$ GeV.

In Fig. 4 we show the invariant mass of the $(\gamma\gamma\gamma)$ system for the full ME and EWA + brem combinations. Remarkably, we obtain good agreement despite the naïve modeling of $\sigma_{3\gamma}^{\text{brem}}$. Predictions for LLA + brem and LP + brem fall within $\mathcal{O}(20\%)$ of the full ME for $M(3\gamma) \gtrsim 800$ GeV. The NLP + brem curve achieves this for $M(WW) \gtrsim 1.8$ TeV and improves with larger $M(WW)$.

For the W^+W^+ process, we again combine the EWA channel (σ_{WW}^{EWA}) with a fragmentation term ($\sigma_{WW}^{\text{brem}}$). We estimate $\sigma_{WW}^{\text{brem}}$ by scaling the $e^+\mu^+ \rightarrow \bar{\nu}_e \nu_\mu \ell^+ W^+$ process by the Sudakov factor in Eq. (14b) but with the replacements $\alpha \mapsto \alpha_W$ and $p_{T,\min}^2 \mapsto p_{T,\min}^2 + M_W^2$ [68]. We approximate the momentum of W in final-state $\ell \rightarrow W\nu$ splitting by $p_W \approx p_\ell/2$ and adapt Eq. (13) for ℓ ($p_T^\ell > 300$ GeV, etc.).

In Fig. 5 we show the invariant mass of the (WW) system for the full ME and EWA + fragmentation. LP and NLP

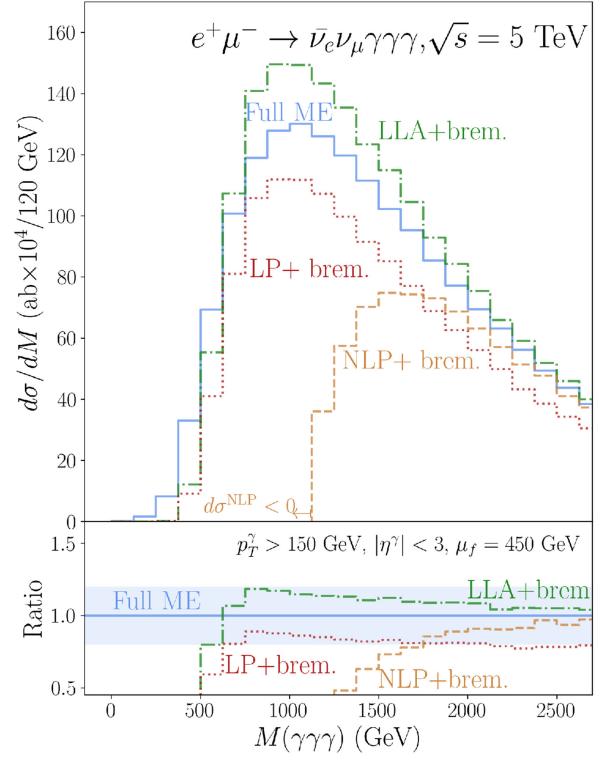


FIG. 4. Same as Fig. 3(a) but for the (3γ) system in the $e^+\mu^- \rightarrow \bar{\nu}_e \nu_\mu 3\gamma$ process, including EWA and bremsstrahlung contributions.

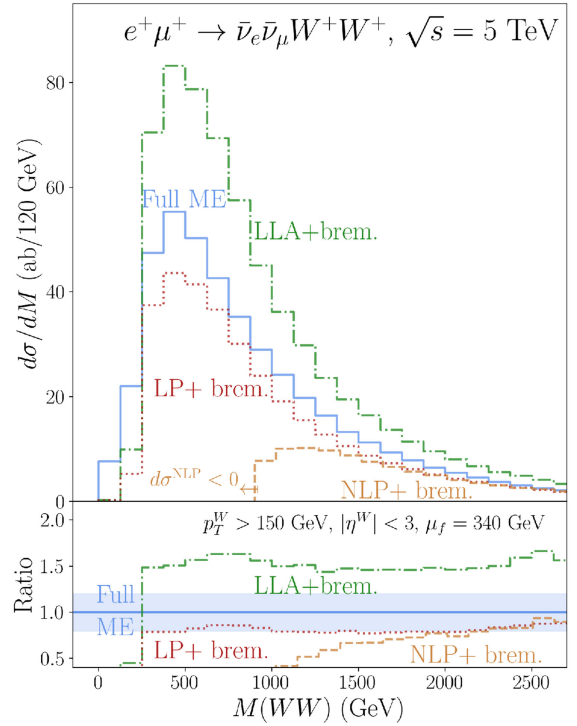


FIG. 5. Same as Fig. 3(a) but for the (WW) system in the process $e^+\mu^+ \rightarrow \bar{\nu}_e \nu_\mu W^+W^+$, including EWA and bremsstrahlung contributions.

curves again largely agree with the full ME. The LLA prediction, however, is $\mathcal{O}(50\%)$ too big due to missing $\mathcal{O}(M_W^2/\mu_f^2)$ terms.

When using renormalized collinear PDFs, the scale invariance of physical observables guarantees the existence of a (soft) Sudakov factor [69]. Assuming the formula of Eq. (12) holds, we anticipate better agreement between full MEs and predictions based on EWA and fragmentation when parton-shower, matching, and resummation methods are employed [37,70–73]. Even at LO, the impact can be significant [74,75], but demonstrating this requires investigation.

VI. CONCLUSION

We report the first NLP-accurate calculation of W/Z PDFs for high-energy fermions [Eq. (6)]. From these PDFs, we have derived kinematical criteria [Eq. (10)] that lead to good agreement between full and approximated MEs, including at the differential level, and suggest the onset of EW factorization at tree level. Partonic behavior emerges when weak bosons carry at least $E_V = 10 \times M_V$ of energy (pCM frame) and $p_T \sim \mu_f \ll E_V$ is near threshold. This corresponds to $\mathcal{O}(M_V^2/E_V^2) \sim 1\%$ power corrections.

Given this energy scale, it may be possible to observe the emergence of W bosons as partons of a proton at the LHC. At $\sqrt{s} = 13.6$ TeV, the same-sign WW scattering process $pp \rightarrow W^\pm W^\pm jj$ has a LO cross section of $\sigma_{\text{LHC}13.6} \sim 5.5(1.0)$ fb for $M(WW) > 1(1.6)$ TeV and $M(jj) > 500$ GeV. Taking the $W \rightarrow e/\mu$ branching rate $\text{BR}^2 \approx 4/81$, and assuming selection and acceptance rates of $\epsilon \times \mathcal{A} \approx 25\%$ [38–41], then about $N = \mathcal{L}\sigma\text{BR}^2\epsilon\mathcal{A} \approx 30(6)$ events are anticipated with $\mathcal{L} = 450 \text{ fb}^{-1}$, or $N = 300(60)$ with $\mathcal{L} = 4500 \text{ fb}^{-1}$. This is large enough to be interesting and motivates further study. Of particular importance is an EWA-compatible parton shower, which is now the last missing ingredient for simulating fully exclusive events at colliders.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [76].

APPENDIX A: USAGE IN MG5_aMC@NLO

Calling PDFs at LP and NLP for helicity-polarized W and Z bosons in electrons and muons is a new feature in the simulation framework MadGraph5_aMC@NLO (v3.7.0 and above) [65,66]. The development builds on releases for helicity-polarized parton scattering [63] and the implementation of the EVA at LLA [37]. Sample notebooks and run scripts may be found at Ref. [76].

To access LP and NLP PDFs, nearly identical syntax can be used as introduced in Ref. [37]. The additions is the `run_card` options are `evaorder` and `eva_xcut`. The available values for these two options and their impact are found in Tables III and IV.

NLP corrections are only available for weak bosons, not for the photon (since $m_\gamma = 0$). LP corrections to PDFs for photons from leptons are available as developed in Ref. [58]. The option `ievo_eva` for setting the EWA evolution variable, as developed in Ref. [37], is available only for LLA PDFs.

The syntax for simulating the $W_0^+ W_0^- \rightarrow hh$ process with the EWA at LP, as done in Fig. 3 is

```
set group_subprocesses false
generate w+{0} w- {0} > h h QED=2 QCD=0
output
launch
analysis=off
set ebeam 10000 # GeV
set nevents 10k
set lpp1 -3 # e+ beam
set lpp2 4 # mu- beam
set pdlabel eva
set evaorder 1 # 0=LLA (default), 1=LP,
2=NLP
set eva_xcut 1 # x > MV/Ebeam, 0 = no cut
set fixed_ren_scale true
set fixed_fac_scale1 true
set fixed_fac_scale2 true
set dsqrt_q2fact1 250
set dsqrt_q2fact2 250
set scalefact 1.0
set no_parton_cut
set use_syst true
done
```

TABLE III. `evaorder` settings for MadGraph5_aMC@NLO.

evaorder value [int]	PDF accuracy	Eqs.
0	LLA (default)	Ref. [37]
1	LP	Eq. (5)
2	NLP	Eq. (6)

TABLE IV. `eva_xcut` settings for `MadGraph5_aMC@NLO`.

<code>eva_xcut</code> value [int]	Impact on PDF
1	$f_{V/\ell}(z, \mu_f^2) = 0$ for $z < M_V/E_\ell$ (lab frame)
0	$f_{V/\ell}^{\text{LLA, LP, NLP}}(z, \mu_f^2) = \text{Ref. [37], Eq. (5), Eq. (6)}$ for $z < M_V/E_\ell$ (lab frame)

For descriptions of this syntax, see Refs. [37,63,66].

APPENDIX B: CHECKS AGAINST MG5_aMC@NLO

As a check of our implementation of LP [Eq. (5)] and NLP [Eq. (6)] PDFs into `mg5amc`, we independently computed the W^+W^- scattering process

$$W^+(p_A, \lambda_A)W^-(p_B, \lambda_B) \rightarrow h(p_1)h(p_2). \quad (\text{B1})$$

In the Unitary gauge, the matrix element is

$$-i\mathcal{M}(\lambda_A, \lambda_B) = \sum_{d \in \{s, t, u, 4\}} -i\mathcal{M}_d(\lambda_A, \lambda_B), \quad (\text{B2})$$

where the sum runs over four subamplitudes. For helicity $\{\lambda_k\}$ and momenta $\{p_k\}$, the amplitudes are

$$-i\mathcal{M}_s = +i \frac{3g^2 m_h^2}{2} \frac{[\varepsilon(p_A, \lambda_A) \cdot \varepsilon(p_B, \lambda_B)]}{Q^2 - m_h^2}, \quad (\text{B3a})$$

$$-i\mathcal{M}_4 = +i \frac{g^2}{2} [\varepsilon(p_A, \lambda_A) \cdot \varepsilon(p_B, \lambda_B)], \quad (\text{B3b})$$

$$-i\mathcal{M}_t = \frac{+ig^2}{t - M_W^2} \{M_W^2 [\varepsilon(p_A, \lambda_A) \cdot \varepsilon(p_B, \lambda_B)] + [\varepsilon(p_A, \lambda_A) \cdot p_1][\varepsilon(p_B, \lambda_B) \cdot p_2]\}, \quad (\text{B3c})$$

$$-i\mathcal{M}_u = \frac{+ig^2}{u - M_W^2} \{M_W^2 [\varepsilon(p_A, \lambda_A) \cdot \varepsilon(p_B, \lambda_B)] + [\varepsilon(p_A, \lambda_A) \cdot p_2][\varepsilon(p_B, \lambda_B) \cdot p_1]\}. \quad (\text{B3d})$$

In the partonic center-of-mass frame, momenta are

$$p_A = \frac{Q}{2}(1, 0, 0, +\beta_W), \quad |\vec{p}_A| = |\vec{p}_B| = \beta_W \frac{Q}{2}, \quad (\text{B4a})$$

$$p_B = \frac{Q}{2}(1, 0, 0, -\beta_W), \quad (\text{B4b})$$

$$p_1 = \frac{Q}{2}(1, \beta_h \sin \theta \sin \phi, \beta_h \sin \theta \cos \phi, \beta_h \cos \theta), \quad (\text{B4c})$$

$$p_2 = \frac{Q}{2}(1, -\vec{p}_1), \quad |\vec{p}_1| = |\vec{p}_2| = \beta_h \frac{Q}{2}. \quad (\text{B4d})$$

These assignments generate the following contractions among polarization vectors (all others vanish):

$$\varepsilon(p_A, \pm 1) \cdot \varepsilon(p_B, \pm 1) = -1, \quad (\text{B5a})$$

$$\varepsilon(p_A, 0) \cdot \varepsilon(p_B, 0) = -1 + \frac{Q^2}{2M_W^2}. \quad (\text{B5b})$$

Similarly, these assignments give the following contractions among polarization vectors and momenta:

$$\varepsilon(p_A, \pm 1) \cdot p_1 = \pm \frac{e^{\pm i\phi}}{2\sqrt{2}} Q \sqrt{1 - 4r_h} \sin \theta, \quad (\text{B6a})$$

$$\varepsilon(p_A, 0) \cdot p_1 = \frac{Q^2}{4M_W} [\sqrt{1 - 4r_W} - \sqrt{1 - 4r_h} \cos \theta], \quad (\text{B6b})$$

$$\varepsilon(p_A, \pm 1) \cdot p_2 = -[\varepsilon(p_A, \pm 1) \cdot p_1], \quad (\text{B6c})$$

$$\varepsilon(p_A, 0) \cdot p_2 = \frac{Q^2}{4M_W} [\sqrt{1 - 4r_W} + \sqrt{1 - 4r_h} \cos \theta], \quad (\text{B6d})$$

$$r_W = \frac{M_W^2}{Q^2}, \quad r_h = \frac{m_h^2}{Q^2}. \quad (\text{B6e})$$

The *helicity-polarized*, parton-level cross sections is determined (numerically) from the formula [63]

$$\hat{\sigma}(\lambda_A, \lambda_B) = \int dPS_2 \frac{d\hat{\sigma}(\lambda_A, \lambda_B)}{dPS_2}, \quad \text{where} \quad (\text{B7a})$$

$$\frac{d\hat{\sigma}(\lambda_A, \lambda_B)}{dPS_2} = \frac{1}{2Q^2 \sqrt{1 - 4r_W}} |\mathcal{M}(\lambda_A, \lambda_B)|^2, \quad (\text{B7b})$$

$$dPS_2 = \frac{d \cos \theta d\phi}{2(4\pi)^2} \sqrt{1 - 4r_h}. \quad (\text{B7c})$$

Polarized cross sections at the level of muons are obtained through the usual PDF convolutions [10]:

$$\sigma_{\lambda_A \lambda_B}(\mu^+ \mu^- \rightarrow hh + X) = f_{W^+/\mu^+} \otimes f_{W^-/\mu^-} \otimes \hat{\sigma}_{W^+W^- \rightarrow hh}(\lambda_A, \lambda_B) \quad (\text{B8})$$

$$= \int_{\tau_{\min}}^1 d\tau \int_{\tau}^1 \frac{dz_1}{z_1} f_{W^+/\mu^+}(z_1, \mu_f) \times f_{W^-/\mu^-}(z_2, \mu_f) \times \hat{\sigma}_{W^+W^- \rightarrow hh}(\lambda_A, \lambda_B), \quad (\text{B9})$$

TABLE V. Helicity-polarized partonic (column 2) and leptonic (columns 3–5) cross sections (pb) for the $W^+(\lambda_A)W^-(\lambda_B) \rightarrow hh$ subprocess in $\mu^+\mu^-$ collisions at $\sqrt{s} = 5$ TeV, assuming inputs of Eq. (B13), for various W^+W^- helicity configurations (column 1) and LLA/LP/NLP PDFs (column 3/4/5). Partonic cross sections (column 2) use a hard scattering scale of $Q = 1$ TeV.

Helicity configuration (λ_A, λ_B)	$\hat{\sigma}$ [pb] (no PDFs)	σ^{LLA} [pb]	σ^{LP} [pb]	σ^{NLP} [pb]
(+, +)	1.693×10^{-3}	119.6×10^{-9}	41.76×10^{-9}	13.56×10^{-9}
(+, -)	150.2×10^{-3}	2960×10^{-9}	1033×10^{-9}	581.2×10^{-9}
(+, 0)	433.3×10^{-6}	41.5×10^{-9}	17.34×10^{-9}	11.13×10^{-9}
(-, +)	150.2×10^{-3}	1351×10^{-9}	471.7×10^{-9}	236.8×10^{-9}
(-, -)	1.693×10^{-3}	119.6×10^{-9}	41.75×10^{-9}	13.56×10^{-9}
(-, 0)	433.3×10^{-6}	31.43×10^{-9}	13.14×10^{-9}	7.715×10^{-9}
(0, +)	433.3×10^{-6}	31.46×10^{-9}	13.15×10^{-9}	7.735×10^{-9}
(0, -)	433.3×10^{-6}	41.44×10^{-9}	17.32×10^{-9}	11.11×10^{-9}
(0, 0)	50.05	1825×10^{-6}	913.0×10^{-6}	817.8×10^{-6}

$$\text{with } z_2 = \frac{\tau}{z_1} \text{ and } \tau_{\min} = \frac{(2m_h)^2}{s}. \quad (\text{B10})$$

$$w_k^{\text{LLA}}(V_{\lambda=\pm}, \mu) = \log\left(\frac{\mu^2}{M_V^2}\right), \quad (\text{C2})$$

Importantly, we impose the additional threshold condition that $f_{W/\mu}(z_i) = 0$ for $E_{W_i} = z_i\sqrt{s}/2 < M_W$.

Unpolarized cross sections are recovered through averaging over initial-state spin configurations:

$$\hat{\sigma}_{\text{unpol}} = \frac{1}{3^2} \sum_{\lambda_A, \lambda_B \in \{\pm 1, 0\}} \hat{\sigma}(\lambda_A, \lambda_B), \quad (\text{B11})$$

$$\sigma = \frac{1}{3^2} \sum_{\lambda_A, \lambda_B \in \{\pm 1, 0\}} \sigma_{\lambda_A \lambda_B}. \quad (\text{B12})$$

We summarize our results in Table V for the following SM and collider inputs:

$$g \approx 0.6532, \quad M_W \approx 80.42 \text{ GeV}, \quad m_h = 125 \text{ GeV}, \\ M_Z \approx 91.19 \text{ GeV}, \quad \sqrt{s} = 5 \text{ TeV}, \quad \mu_f = m_h. \quad (\text{B13})$$

APPENDIX C: SCALE VARIATION AT LP AND NLP

Assuming a collection of events, each with cross section weight $\Delta\sigma_k$ at a baseline scale $\mu = \mu_0$, then the scale variation of the cross section weight ($\Delta\sigma'_k$) at a scale $\mu = \zeta\mu_0$ is given by the reweighting scheme

$$\Delta\sigma'_k(\zeta\mu_0) = \Delta\sigma_k(\mu_0) \times \frac{w_k(\zeta\mu_0)}{w_k(\mu_0)}. \quad (\text{C1})$$

Here, $w_k(\mu)$ is the scale weight. The scale weight varies according to PDF species and accuracy.

For transverse PDFs at LLA, LP [Eqs. (5a) and (5b)], and NLP [Eqs. (6a) and (6b)], the scale weights are

$$w_k^{\text{LP}}(V_{\lambda=\pm}, \mu) = \log\left(\frac{M_V^2 + \mu^2}{M_V^2}\right) - \left(\frac{\mu^2}{M_V^2 + \mu^2}\right), \quad (\text{C3})$$

$$w_k^{\text{NLP}}(V_{\lambda=\pm}, \mu) = (a_\lambda + b_T) \log\left(\frac{M_V^2 + \mu^2}{M_V^2}\right) \\ - \left(\frac{\mu^2}{\mu^2 + M_V^2}\right) (a_\lambda + c_T), \quad (\text{C4})$$

with coefficients

$$a_{\lambda=+} = (1-z), \quad a_{\lambda=-} = 1, \quad b_T = (2-z) \frac{M_V^2}{E_V^2}, \quad (\text{C5})$$

$$c_T = b_T + (2-z) \frac{\mu^2}{2E_V^2}. \quad (\text{C6})$$

For longitudinal PDFs at LLA, LP [Eq. (5c)], and NLP [Eq. (6c)], the scale weights are respectively,

$$w_k^{\text{LLA}}(V_{\lambda=0}, \mu) = 1, \quad (\text{C7})$$

$$w_k^{\text{LP}}(V_{\lambda=0}, \mu) = \left(\frac{\mu^2}{M_V^2 + \mu^2}\right), \quad (\text{C8})$$

$$w_k^{\text{NLP}}(V_{\lambda=0}, \mu) = \left(\frac{\mu^2}{M_V^2 + \mu^2}\right) - \left\{ \left[\frac{1 - (1-z)^2}{(1-z)} \right] \left(\frac{M_V^2}{4E_V^2}\right) \right. \\ \left. \times \left[\log\left(\frac{M_V^2 + \mu^2}{M_V^2}\right) - \left(\frac{\mu^2}{M_V^2 + \mu^2}\right) \right] \right\}. \quad (\text{C9})$$

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