

Stabilizing dark matter with quantum scale symmetry

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In the context of gauge-Yukawa theories with trans-Planckian asymptotic safety, quantum scale symmetry can prevent the appearance in the Lagrangian of couplings that would otherwise be allowed by the gauge symmetry. Such couplings correspond to irrelevant Gaussian fixed points of the renormalization group flow. Their absence in the theory implies that different sectors of the gauge-Yukawa theory are secluded from one another, in similar fashion to the effects of a global or a discrete symmetry. As an example, we impose the trans-Planckian scale symmetry on a model of grand unification based on the gauge group $SU(6)$, showing that it leads to the emergence of several fermionic weakly interacting massive particles dark matter candidates whose coupling strengths are entirely predicted by the UV completion.

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I. INTRODUCTION

Asymptotic safety (AS) is the property of a quantum field theory to develop fixed points of the renormalization group (RG) flow of the action [1]. Following the development of functional renormalization group (FRG) techniques [2,3], it was shown in several papers that AS may arise in quantum gravity and provide the key ingredient for the nonperturbative renormalizability of the theory [4–15]. Subsequent developments [16–29] further revealed that, given a particle theory in four space-time dimensions coupled to the gravitational action, the full system of gravity and matter may feature ultraviolet (UV) fixed points in the energy regime where gravitational interactions become strong, thus planting the seeds for the exploration of AS in models of particle physics.

In a theory with RG fixed points, the coefficients of the operators of the quantum effective action may be divided in two broad classes on the basis of their scaling behavior in the vicinity of the fixed point. *Irrelevant* couplings are infrared (IR)-attractive, they are properly scale-invariant at the quantum level so that in one interpretation [30–33] they may be thought of as the fundamental couplings of the microscopic theory. *Relevant* couplings, on the other hand,

are IR-repulsive and thus drive deviations from the scaling solution near the fixed point. In the RG flow relevant parameters can identify—at the point where they roughly become larger than 1—a characteristic scale of the low-energy theory. Typical examples include the dimensionless Newton constant in asymptotically safe quantum gravity, the dimensionless Higgs potential mass-squared parameter in the Standard Model (SM), and the strong gauge coupling in quantum chromodynamics (QCD), which generate, respectively, the Planck scale, the electroweak symmetry-breaking (EWSB) scale, and Λ_{QCD} . Since relevant couplings are free parameters of the macroscopic theory, they essentially play the role of *effective* quantities that cannot be predicted from first principles and thus must be determined experimentally.

Vice versa, the *a priori* free couplings of the matter Lagrangian that correspond to irrelevant directions become predictable in AS. The predictions of the scale-invariant theory can then be tested by phenomenological means. Early successes in this approach include a gravity-driven solution to the triviality problem in $U(1)$ gauge theories [34–36]; a ballpark prediction for the value of the Higgs mass (more precisely, of the quartic coupling of the Higgs potential) obtained a few years ahead of its discovery [37] (see also Refs. [38–40]); and the retroactive “postdiction” of the top-quark mass value [41].

When it comes to gauge-Yukawa theories of phenomenological interest, a gravity-driven prediction of the top/bottom mass ratio of the SM was extracted in Ref. [42]. Possible imprints of UV fixed points on the flavor structure of the SM and, in particular, the Cabibbo-Kobayashi-Maskawa matrix, were sought in Ref. [43], and an equivalent analysis for the Pontecorvo-Maki-Nakagawa-Sakata matrix elements can be found in Ref. [44]. The impact of asymptotically safe quantum-gravity calculations on the

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renormalization group equations (RGEs) of the Majorana mass term was investigated in detail in Refs. [45–47]. Predictions were also extracted for several models in relation to neutrino masses [48,49], flavor anomalies [50,51], the muon anomalous magnetic moment [52], baryon number [53,54], and the asymptotically safe SM [55,56].

In a couple of recent papers [44,57] (see also Ref. [58]) some of us showed that, in theories that include some Yukawa couplings constrained by low-scale observations to extremely small numbers (for example, the $\mathcal{O}(10^{-13})$ Yukawa coupling of a purely Dirac neutrino receiving its mass via the Higgs mechanism or, in more exotic cases, the tiny Yukawa coupling of a sterile-neutrino dark matter (DM) candidate whose relic abundance is determined via the freeze-in mechanism [59–63]), such minuscule values may arise dynamically if the RG flow of the Yukawa coupling develops a Gaussian irrelevant fixed point. In that case, in fact, the coupling in question receives a “natural” exponential suppression along the RG flow. A straightforward, yet unexplored consequence of the same idea is that the presence of an irrelevant Gaussian fixed point in the gauge-Yukawa sector can forbid the appearance of Lagrangian interactions that, while allowed by the gauge and global symmetries of the theory, may be in strong tension with observations. In other words, in certain situations of phenomenological interest the fundamental scale symmetry of the UV completion may act “transversely” to the gauge and other symmetries to switch off some of the unwanted couplings.

The presence of Gaussian fixed points can induce the emergence of DM in theories that have *a priori* no dark sector. We consider here a theory of grand unification (GUT) as a framework to accommodate the desired particle content beyond the SM (BSM). It has long been known that most nonsupersymmetric GUTs do not usually feature a fermionic DM candidate. The group (and its breaking chain) $\text{SO}(10) \rightarrow \text{SU}(5) \times \text{U}(1) \rightarrow \text{SM}$ does potentially contain fermion DM [64], because the particles of the dark and visible sectors are separated by their $\text{U}(1)$ charges. But this is typically not the case in GUTs based on $\text{SU}(N) \rightarrow \text{SU}(5) \times \text{SU}(N-5) \rightarrow \text{SM}$ [65]. In those cases, DM is either a pseudo-Nambu-Goldstone boson [66–69], or one is forced to introduce extra symmetries [70]. In this paper, we show that quantum scale symmetry, in the form of irrelevant Gaussian fixed points of the trans-Planckian RGEs, can act as *the* symmetry driving the emergence of DM in $\text{SU}(6)$ and, by extension, higher rank $\text{SU}(N)$ GUTs. Moreover, in this scenario the coupling strengths of the DM become entirely predicted by the asymptotically safe UV completion. Once the correct value of the relic abundance is factored in, the DM mass scale becomes a prediction too, thus providing a path for the complete testability of the theory.

It is worth pointing out that, while AS may be a viable alternative to grand unification, in the sense that one does

not really need a GUT to guarantee that all of the SM couplings remain finite up to arbitrary energies, a GUT provides nevertheless some advantages with respect to GUT-less scenarios with AS. On the one hand, it obviously creates an elegant framework for incorporating the BSM particle content. But, on a more subtle level, it also provides a theory that, unlike Refs. [44,57], features a completely relevant, asymptotically free gauge sector. As we shall see below, this quality allows one to derive phenomenological predictions that depend to great extent solely on the scaling behavior of the matter couplings and not on the fixed-point value of the coefficients of the gravitational action.

The paper is organized as follows. In Sec. II we recall basic notions of trans-Planckian AS. In Sec. III we describe the way quantum scale symmetry may act transversely to the gauge symmetry to prevent some of the Yukawa couplings from appearing in the Lagrangian at any scale. In Sec. IV we discuss a particular realization of this mechanism in the framework of an $\text{SU}(6)$ GUT. After defining the particle content of the model, we perform the UV fixed-point analysis of its gauge-Yukawa sector, derive predictions for the low-scale values of the gauge and Yukawa couplings, and comment on the qualitative UV features of the scalar potential. In Sec. V we discuss the DM properties of the model. We finally summarize our findings in Sec. VI. Appendices contain, respectively, discussion of the scalar sector of the $\text{SU}(6)$ model, trans-Planckian RGEs for the $\text{SU}(6)$ gauge and Yukawa couplings, a list of fixed-point solutions to the RGE system, some analytical formulas for the DM relic abundance, and the most generic form of the low-scale Yukawa Lagrangian.

II. GENERAL NOTIONS OF ASYMPTOTIC SAFETY

In AS, quantum gravity effects become important at approximately the Planck scale, where it is expected that graviton fluctuations start to contribute to the RG flow of the matter couplings. In a (renormalizable) matter theory with gauge and Yukawa interactions, the corresponding RGEs receive trans-Planckian contributions that at the leading order look like

$$\frac{dg_i}{dt} = \beta_i^{(\text{matter})} - f_g g_i \quad (1)$$

$$\frac{dy_j}{dt} = \beta_j^{(\text{matter})} - f_y y_j, \quad (2)$$

where $t = \ln \mu$ denotes the renormalization scale, g_i and y_j (with $i, j = 1, 2, 3, \dots$) are the set of gauge and Yukawa couplings, respectively, and $\beta_{i,j}^{(\text{matter})}$ stand for the matter beta functions without gravity.

The trans-Planckian objects f_g and f_y , which parametrize leading gravitational corrections to the running of the gauge and Yukawa couplings, are expected to be

universal (in the sense that they multiply linearly all matter couplings of the same kind), as gravity does not distinguish between the internal degrees of freedom in the matter sector. In the framework of the FRG, they are determined by the fixed points of the operators of the gravitational action, and can be computed from first principles. While the derivation of f_g and f_y is subject to extremely large uncertainties, including the choice of truncation in the gravity/matter action, the selected renormalization scheme, and the gauge-fixing parameters [6,7,12,16,17,22,71–75], it was nonetheless established that f_g is likely not negative, irrespective of the chosen RG scheme [21,26]. While $f_g = 0$ corresponds to respecting a particular classical symmetry of the gravitational action [21], $f_g \neq 0$ may arise not just in mass-dependent schemes like the FRG, but in dimensional regularization too [76,77]. Incidentally, as $f_g \geq 0$ supports asymptotic freedom in the non-Abelian gauge sector of the SM, these results lend credit to the consistency of AS.

Unlike f_g , the gravitational contribution to the Yukawa coupling, f_y , is subject to somewhat greater uncertainty. It was investigated in a set of simplified models [18,24,25,78], but no general results and definite conclusions regarding its size and sign are available (see, however, Ref. [55] for the most recent determination of f_y in the SM).

Once the flow of the gravitational action develops a fixed point dynamically, a trans-Planckian fixed point in the matter sector may emerge as well. Such a fixed point, which we henceforth indicate with an asterisk, $\{g_i^*, y_j^*\}$, corresponds to a zero of the beta functions of the system of Eqs. (1) and (2), given by the condition: $\beta_{i(j)}^{(\text{matter})}(g_i^*, y_j^*) - f_{g(y)} g_i^*(y_j^*) = 0$. One defines the stability matrix, M_{ij} , by linearizing the RG flow of the couplings $\{\alpha_k\} \equiv \{g_i, y_j\}$ around the fixed point,

$$M_{ij} = \partial \beta_i / \partial \alpha_j |_{\{\alpha_k^*\}}. \quad (3)$$

Critical exponents θ_i are then defined as opposite of eigenvalues of the stability matrix and they characterize the power-law evolution of the matter couplings in the vicinity of the fixed point. $\theta_i > 0$ corresponds to a relevant and UV-attractive eigendirection. All RG trajectories along this direction asymptotically reach the fixed point, so any deviation of a relevant coupling from the fixed point introduces a free parameter in the theory. It also means that the high-scale value of the coupling can always be adjusted to match its eventual measurement at the experimentally accessible energies.

If $\theta_i < 0$, the corresponding eigendirection is irrelevant and IR-attractive. In this case only one trajectory exists that the coupling's flow can follow toward the IR, thus either potentially providing a specific prediction for its value at the scale of phenomenological interest (if the fixed point is

nonzero), or preventing the coupling from appearing in the theory (if the fixed point is zero). Finally, $\theta_i = 0$ corresponds to a *marginal* eigendirection. The RG running is logarithmically slow along this direction and an analysis beyond the linear order is required in order to determine whether a fixed point is UV-attractive or IR-attractive.

In this study, we are going to work with $\beta_{i,j}^{(\text{matter})}$ at the one-loop level. When it comes to the predictions of AS for the matter theory, it was shown in Ref. [79] that uncertainties stemming from neglecting higher-order contributions are extremely small, especially when considering couplings that remain in the perturbative regime along the entire RG flow. Note also that, at the first order in perturbation theory, the parameters of the scalar potential do not affect the gauge-Yukawa system, as they only enter Eq. (1) from the third-loop level up, and Eq. (2) from the second loop. Therefore, the gauge-Yukawa sector can be treated independently.

As a matter of fact, the quartic couplings of the scalar potential should also be subject to gravitational contributions to the beta function. In the trans-Planckian regime, the scalar beta functions are modified similarly to Eqs. (1) and (2), with the gravitational corrections parametrized by f_λ . One should keep in mind, however, that gravitational corrections to the running couplings of the scalar potential do not need to be multiplicative, as some truncations of the gravitational action can generate contributions which are additive [80]. The impact of UV boundary conditions on the scalar potential of some realistic BSM theories was investigated, with respect to the relic abundance of DM in Refs. [80–82]; for axion models in Ref. [83]; in the context of dark energy and gravitational waves in Refs. [84,85]; and in GUTs in Refs. [86,87].

We conclude this section by mentioning that the issue of whether the FRG and other mass-dependent schemes may be the most appropriate tools for investigating/enforcing AS in quantum gravity remains a source of vital debate in the literature [88–94]. To this regard, it must be pointed out that significant theoretical progress was made by the AS community in recent years, with respect, particularly, to the Lorentzian FRG and UV unitarity [56,95–97].

III. VANISHING COUPLINGS FROM QUANTUM SCALE SYMMETRY

Let us consider a UV model that includes several copies of Weyl-fermion and scalar multiplets belonging to the same representation of a unified gauge group $\mathcal{G}$: $F_i = F_1, F_2, \dots \simeq N \in \mathcal{G}$ and $S_j = S_1, S_2, \dots \simeq M \in \mathcal{G}$. The model also includes at least one more fermion multiplet G belonging to the representation $\bar{N} \times \bar{M}$, and/or a scalar multiplet H included in $\bar{N} \times \bar{N}$. All Yukawa couplings of the form $GF_i S_j$ (or $F_i F_j H$) become allowed by the symmetry.

Let us focus on the $GF_i S_j$ case. One can schematically write down the Yukawa interaction part of the Lagrangian,

$$\mathcal{L} \supset \sum_{i,j=1,2,\dots} y_{ij} GF_i S_j + \text{other Yukawa terms} + \text{H.c.}, \quad (4)$$

where “other Yukawa terms” include all unspecified matter content that features a Yukawa coupling with one or the other F_i , S_j fields.

It is easy to show that differences in the fixed-point scaling behavior of the y_{ij} Yukawa couplings can emerge even though the Yukawa interactions in Eq. (4) feature identical gauge quantum numbers. Let us first write down the generic trans-Planckian RGE for Yukawa couplings y_{ij} ($i, j = 1, 2, \dots$) at one loop,

$$\begin{aligned} \frac{dy_{ij}}{dt} = \frac{1}{16\pi^2} & \left[\left(\sum_{k,l=1,2,\dots} a_{kl}^{(ij)} y_{kl}^2 + \sum_{y_r \in \text{other}} b_r^{(ij)} y_r^2 + c^{(ij)} g_G^2 \right) y_{ij} \right. \\ & \left. + \sum_{m,p,q \neq (ij)} d_{mpq}^{(ij)} y_m y_p y_q \right] - f_y y_{ij}, \end{aligned} \quad (5)$$

where f_y parametrizes the leading-order graviton contribution to the beta function, calculated at the fixed point of the gravitational action, $a_{kl}^{(ij)}$, $b_r^{(ij)}$, $c^{(ij)}$, and $d_{mpq}^{(ij)}$ are real coefficients specific to the matter beta function of coupling y_{ij} , the sum in y_r spans all the Yukawa couplings that are not indicated explicitly in Eq. (4), g_G is the gauge coupling of the theory, and the last addend inside square-brackets parametrizes the terms of the beta function that are not multiplicative in y_{ij} . As we shall see, system (5) in general admits several real fixed points. However, to make our case we can focus for the moment on the most minimal non-trivial solution, which is characterized by one single interactive Yukawa coupling, $y_{\hat{k}\hat{l}}^* \neq 0$, while all others are set at the Gaussian fixed point: $y_m^* = 0$, with $m \neq (\hat{k}\hat{l})$. It is straightforward to see that such a solution always exists,

$$y_{\hat{k}\hat{l}}^* = \sqrt{\frac{16\pi^2 f_y - c^{(\hat{k}\hat{l})} g_G^{*2}}{a_{\hat{k}\hat{l}}^{(\hat{k}\hat{l})}}}, \quad (6)$$

and that it can be real. We can further ground the discussion by assuming that the sole interactive Yukawa coupling of Eq. (6) is, e.g., $y_{22}^* \neq 0$, and consider the case with $g_G^* = 0$, which is typical of asymptotically free gauge theories. The critical exponents of two Gaussian Yukawa couplings, $y_{11}^* = y_{12}^* = 0$, may thus be approximated as

$$\theta_{11} \approx -\frac{\partial \beta_{11}}{\partial y_{11}} = -\frac{a_{22}^{(11)}}{16\pi^2} y_{22}^{*2} + f_y = \left(1 - \frac{a_{22}^{(11)}}{a_{22}^{(22)}} \right) f_y \quad (7)$$

$$\theta_{12} \approx -\frac{\partial \beta_{12}}{\partial y_{12}} = -\frac{a_{22}^{(12)}}{16\pi^2} y_{22}^{*2} + f_y = \left(1 - \frac{a_{22}^{(12)}}{a_{22}^{(22)}} \right) f_y \quad (8)$$

(the off-diagonal terms of the stability matrix can be neglected as they do not contribute to θ_{11} and θ_{12} at the fixed point $y_{11}^* = y_{12}^* = 0$). It follows straightforwardly that, depending on the relative size of $a_{22}^{(11)}$, $a_{22}^{(12)}$, and $a_{22}^{(22)}$, Yukawa couplings y_{11} and y_{12} may emerge from the Gaussian fixed point with different scaling behavior. If, for example, $f_y > 0$ and $a_{22}^{(11)} < a_{22}^{(22)}$, then $\theta_{11} > 0$ and y_{11} emerges from the fixed point along a relevant direction, effectively being a free parameter of the theory. At the same time, there is no guarantee that $a_{22}^{(12)} < a_{22}^{(22)}$ in Eq. (8), since we expect typically that $a_{22}^{(12)} > a_{22}^{(11)}$.

In order to clarify why this is the case, we present in the upper row of Fig. 1 some of the diagrams contributing to the one-loop beta function of Yukawa coupling y_{11} . The lower row shows the corresponding diagrams for y_{12} . At the Gaussian fixed point $y_{11}^* = y_{12}^* = y_{21}^* = 0$, Yukawa coupling $y_{22}^* \neq 0$ contributes equally to the two diagrams in Fig. 1(a), which give $a_{22}^{(11)}$ and $a_{22}^{(12)}$. However, while diagrams (b) and (c) of the lower row contribute to $a_{22}^{(12)}$, increasing its value, the corresponding diagrams of the upper row do not contribute to $a_{22}^{(11)}$. One can prove that, in the absence of other interactive couplings at the fixed point, it will be $a_{22}^{(11)} < a_{22}^{(12)} = a_{22}^{(22)}$, which leads to $\theta_{12} = 0$. Yukawa coupling y_{12} will in this case be marginal. However, by considering solutions with more interactive couplings at the fixed point it is possible to push θ_{12} to negative values, while θ_{11} remains positive. As a result, y_{12} will follow an irrelevant direction and will be forbidden to appear in the Lagrangian at any scale. Even if the two Yukawa couplings are identically allowed by the gauge symmetry, quantum scale symmetry is forbidding one of them.

One interesting consequence of the described mechanism is that sectors of the theory that are in principle allowed to mix by the gauge symmetry, remain in fact secluded. For example, suppose scalar fields S_1, S_2 acquire vacuum expectation values (vevs), v_1 and v_2 respectively, thus breaking the gauge symmetry. A schematic expression for the mass matrix of fermions $f_{L,1}, f_{L,2} \in G$, $f_{R,1} \in F_1$, $f_{R,2} \in F_2$ will read

$$M_f = \frac{1}{\sqrt{2}} \begin{pmatrix} y_{11} v_1 & y_{12} v_2 \\ y_{21} v_1 & y_{22} v_2 \end{pmatrix}. \quad (9)$$

If quantum scale invariance protects $y_{12} = y_{21} = 0$ from appearing at any scale, the fermion mass matrix becomes diagonal and mixing is forbidden. Incidentally, according to the discussion above, since y_{11} is relevant, it is a free parameter of the theory. $y_{22} \neq 0$, on the other hand, is a fundamental prediction of the UV completion.

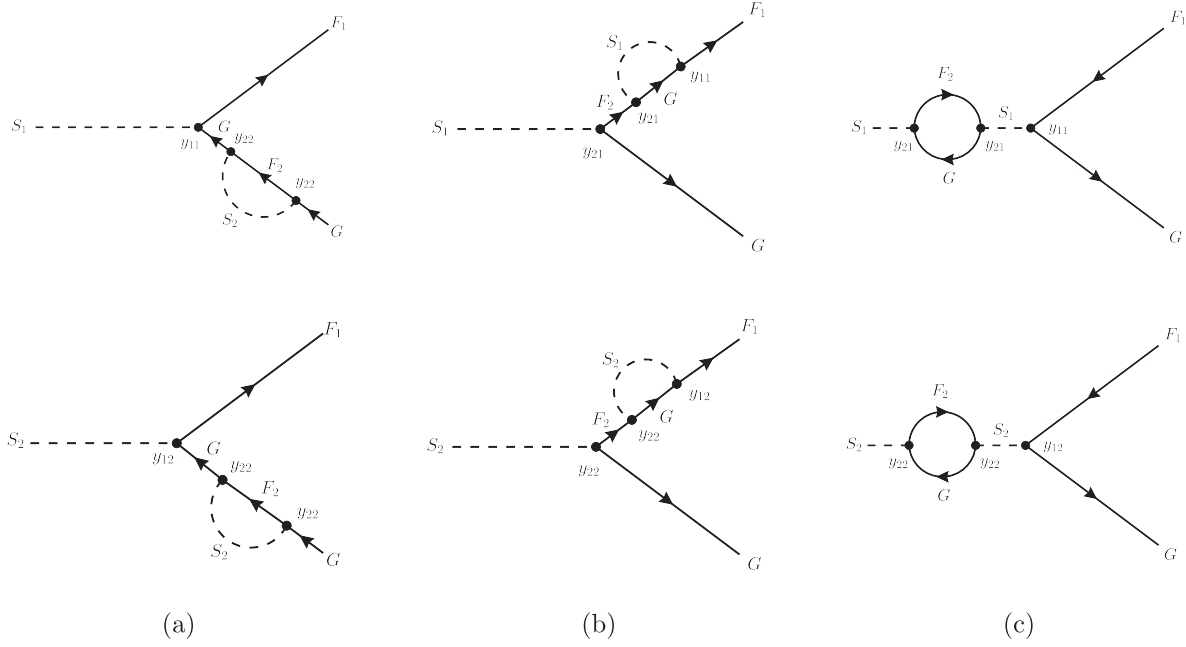


FIG. 1. Diagrams (a), (b), and (c) of the upper row contribute to the beta function of Yukawa coupling y_{11} . Diagrams (a), (b), and (c) of the lower row contribute to the beta function of Yukawa coupling y_{12} .

IV. SU(6) GUT

A. General introduction

The mechanism described in Sec. III applies to any UV model featuring more particles in copies of the same representation. This is typical in GUTs, where the requirement of anomaly cancellation often leads to the presence of a number of fermion multiplets indistinguishable from one another under the gauge symmetry. Moreover, as a means to seclude different sectors of the Yukawa theory, quantum scale symmetry finds a particularly useful realization in nonsupersymmetric SU(N) GUTs, as they typically are not endowed with a dark sector [65]. Once the requirement of anomaly cancellation is imposed, all SU(N) groups can be given an equivalent fermionic content, for example: $N - 4$ copies of the antifundamental and one antisymmetric representation per generation. This implies that the secluding strategy described in Sec. III can be applied in principle to the Yukawa sector of SU(N) theories of any rank. On the other hand, the low-scale phenomenology of the model will be strongly characterized by the mass hierarchies in the scalar sector and thus has to be analyzed on a case-by-case basis. For illustrative purposes, in this section we focus on one particular example, SU(6), which is the most minimal SU(N) group that can accommodate a DM candidate.

B. The model

Let us consider a simple modification of the SU(6) DM scenario introduced originally in Ref. [70]. The fermionic content of the model is determined by anomaly cancellation: for each fermion generation it includes three Weyl

multiplets, $\bar{\mathbf{6}}_1^{(F)}$, $\bar{\mathbf{6}}_2^{(F)}$, and $\mathbf{15}^{(F)}$. To make sure that all fermions acquire masses after the gauge symmetries are spontaneously broken, at least three scalar multiplets need to be introduced. In the following we are going to consider four scalar multiplets, $\mathbf{6}_1^{(S)}$, $\mathbf{6}_2^{(S)}$, $\mathbf{15}^{(S)}$, and $\mathbf{21}^{(S)}$, which is a simpler scalar content than in Ref. [70] where the two scalar 6's were replaced by two scalar 84's.¹ One also needs the adjoint $\mathbf{35}^{(S)}$, which breaks $\text{SU}(6) \rightarrow \text{SU}(5)_{\text{SM}} \times \text{U}(1)_X$.

The model in Ref. [70] admits the existence of a SM-singlet, Majorana fermion weakly interacting massive particle (WIMP) belonging to a combination of the multiplets $\bar{\mathbf{6}}^{(F)}$. The WIMP in Ref. [70] is not stable, but its lifetime is longer than the age of the Universe. Such a long lifetime is a consequence of decay channels mediated by loop-induced couplings with the SM that are naturally suppressed by powers of the GUT-scale mass. On the other hand, the long lifetime is also facilitated by, first, the author's choice of an extra $\mathbb{Z}_2$ symmetry that forbids a

¹It is certainly advantageous to be able to obtain the same DM as in Ref. [70] with a smaller number of fields, however, the most important reason to replace the multiplets has to do with the nature of our fixed points. Not all systems of gauge-Yukawa RGEs admit solutions that are real. In particular, the system obtained from Lagrangian terms involving the 84 scalars admits fixed points with complex Yukawa couplings. While the implications for DM of CP violation in the Yukawa sector would be an interesting subject of investigation *per se*, its thorough analysis exceeds the purpose of this paper. Real fixed-point solutions with the properties we are seeking emerge from the RGE system with the scalar 6 multiplets. As we shall see in Sec. V, we also differ from Ref. [70] in our choice of DM annihilation mechanism.

certain number of gauge-allowed Yukawa couplings; and second, by the author adjusting the size of some of the remaining $\mathbb{Z}_2$ -allowed Yukawa coupling to very small values. As we shall see below, in the AS-based framework that we described in Sec. III the same outcome may be obtained without the need of introducing a $\mathbb{Z}_2$ symmetry, or of adjusting any of the gauge-Yukawa parameters, which instead are either forbidden by quantum scale symmetry or

emerge as naturally suppressed from the UV completion. However, as in any GUT construction, loop-induced effective couplings are present, and for a large enough GUT mass they guarantee that the WIMP DM particle is metastable.

Let us now move to define the gauge-Yukawa sector of the SU(6) GUT. For each fermion generation, the Yukawa part of the Lagrangian reads,

$$\begin{aligned} \mathcal{L} \supset & y_{11} \mathbf{15}^{(F)} \bar{\mathbf{6}}_1^{(F)} \bar{\mathbf{6}}_1^{(S)} + y_{12} \mathbf{15}^{(F)} \bar{\mathbf{6}}_1^{(F)} \bar{\mathbf{6}}_2^{(S)} + y_{21} \mathbf{15}^{(F)} \bar{\mathbf{6}}_2^{(F)} \bar{\mathbf{6}}_1^{(S)} + y_{22} \mathbf{15}^{(F)} \bar{\mathbf{6}}_2^{(F)} \bar{\mathbf{6}}_2^{(S)} \\ & + \tilde{y}_{11} \bar{\mathbf{6}}_1^{(F)} \bar{\mathbf{6}}_1^{(F)} \mathbf{15}^{(S)} + \tilde{y}_{12} \bar{\mathbf{6}}_1^{(F)} \bar{\mathbf{6}}_2^{(F)} \mathbf{15}^{(S)} + \tilde{y}_{22} \bar{\mathbf{6}}_2^{(F)} \bar{\mathbf{6}}_2^{(F)} \mathbf{15}^{(S)} \\ & + \hat{y}_{11} \bar{\mathbf{6}}_1^{(F)} \bar{\mathbf{6}}_1^{(F)} \mathbf{21}^{(S)} + \hat{y}_{12} \bar{\mathbf{6}}_1^{(F)} \bar{\mathbf{6}}_2^{(F)} \mathbf{21}^{(S)} + \hat{y}_{22} \bar{\mathbf{6}}_2^{(F)} \bar{\mathbf{6}}_2^{(F)} \mathbf{21}^{(S)} \\ & + y_u \mathbf{15}^{(F)} \mathbf{15}^{(F)} \mathbf{15}^{(S)} + \text{H.c.} \end{aligned} \quad (10)$$

We assume for simplicity that the three SM fermion generations do not mix.

The scalar potential of the model is presented in Appendix A. In this work, we do not investigate the scalar sector of the theory from the point of view of AS. Suffices to say that masses and trilinear couplings are canonically relevant, therefore we will treat them as free parameters that can assume any needed value, positive or negative. In the context of fundamental scale invariance these couplings, rescaled by their mass dimension, have the role of effective parameters that break the quantum scale symmetry and dynamically generate a physical scale when they reach values greater than 1 in their flow to the IR.

We adopt the following SU(6) GUT breaking chain:

- (i) $v_6 = \langle \mathbf{35}^{(S)} \rangle \approx 10^{16} \text{ GeV}$ breaks $\text{SU}(6) \rightarrow \text{SU}(5)_{\text{SM}} \times \text{U}(1)_X$
- (ii) $v_5 = \langle \mathbf{24}_0^{(S)} \rangle \approx 10^{16} \text{ GeV}$ breaks $\text{SU}(5)_{\text{SM}} \rightarrow \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$
- (iii) $\bar{\mathbf{6}}_1^{(S)} \supset \bar{\mathbf{5}}_{-1}^{(S)} \supset (\mathbf{1}, \bar{\mathbf{2}}, -\frac{1}{2}; -1) + (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}; -1)$, where we have indicated in parentheses the quantum numbers in the extended SM group, $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_X$. Doublet-triplet splitting follows from the tuning of some scalar potential couplings (see Appendix A 1) and it guarantees that the SU(2) doublet field remains light
- (iv) $\bar{\mathbf{6}}_2^{(S)} \supset \mathbf{1}_5^{(S)} = (\mathbf{1}, \mathbf{1}, 0; 5)$ is a SM-singlet scalar field that remains light

- (v) $\mathbf{15}^{(S)} \supset \mathbf{5}_{-4}^{(S)} \supset (\mathbf{1}, \mathbf{2}, \frac{1}{2}; -4) + (\mathbf{3}, \mathbf{1}, -\frac{1}{3}; -4)$. Doublet-triplet splitting guarantees that the SU(2) doublet field remains light
- (vi) $\mathbf{21}^{(S)} \supset \mathbf{1}_{-10}^{(S)} = (\mathbf{1}, \mathbf{1}, 0; -10)$ is a SM-singlet scalar that remains light.

At low energy one is thus left with a two-Higgs doublet model (2HDM) extended by the addition of two complex scalar SM-singlets.

We rename the light scalar fields in the following way:

$$H_d = \left(\mathbf{1}, \bar{\mathbf{2}}, -\frac{1}{2}; -1 \right), \quad H_u = \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}; -4 \right),$$

$$s_6 = (\mathbf{1}, \mathbf{1}, 0; 5), \quad s_{21} = (\mathbf{1}, \mathbf{1}, 0; -10), \quad (11)$$

where we shall henceforth indicate weak isospin doublets with capital letters, and weak singlets in lowercase. Since no scalar field is neutral under the $\text{U}(1)_X$ group, the heaviest of the acquired vevs will approximately determine the mass of the Z' gauge boson, modulo the size of gauge coupling g_X and $\text{U}(1)_X$ charge of the scalars. The vevs of the light scalar fields, $v_d, v_u, v_{s_6}, v_{s_{21}}$, give mass to the light fermions in the model. As is customary in the 2HDM, we define $\tan \beta \equiv v_u/v_d$.

We can define the low-scale, left-chiral Weyl fermion multiplets of the extended SM group $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_X$. For each generation,

$$Q: \left(\mathbf{3}, \mathbf{2}, \frac{1}{6}; 2 \right), \quad u: \left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}; 2 \right), \quad d_1, d_2: \left(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}; -1 \right), \quad d': \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}; -4 \right), \quad (12)$$

$$L_1, L_2: \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}; -1 \right), \quad L': \left(\mathbf{1}, \bar{\mathbf{2}}, \frac{1}{2}; -4 \right), \quad e: (\mathbf{1}, \mathbf{1}, 1; 2), \quad \nu_1, \nu_2: (\mathbf{1}, \mathbf{1}, 0; 5), \quad (13)$$

where, again, we have indicated weak isospin doublets in capital letters, and isospin singlets in lowercase. For completeness, we indicate the original SU(6) representations from which the fermions of Eqs. (12) and (13) emerge upon the GUT-symmetry breaking,

$$\begin{aligned}
\bar{\mathbf{6}}_1^{(F)} \supset \bar{\mathbf{5}}_{-1}^{(\text{SM})} \supset d_1, L_1 & \quad \bar{\mathbf{6}}_1^{(F)} \supset \mathbf{1}_5^{(F)} \supset \nu_1 \\
\bar{\mathbf{6}}_2^{(F)} \supset \bar{\mathbf{5}}_{-1}^{(F)} \supset d_2, L_2 & \quad \bar{\mathbf{6}}_2^{(F)} \supset \mathbf{1}_5^{(F)} \supset \nu_2 \\
\mathbf{15}^{(F)} \supset \mathbf{10}_2^{(\text{SM})} \supset Q, u, e & \quad \mathbf{15}^{(F)} \supset \mathbf{5}_{-4}^{(F)} \supset d', L'. \quad (14)
\end{aligned}$$

We do not explore the issue of gauge-coupling unification in detail in this work. With the matter content given in Eqs. (11)–(13) the couplings unify at about $M_{\text{GUT}} \approx 10^{14}$ GeV, which is too low a scale to avoid fast proton decay [98,99]. Unification can be achieved at the scale $M_{\text{GUT}} \approx 4 \times 10^{16}$ GeV, if one adds to the low-energy model a color-octet and a weak isospin-triplet fermion multiplets. These could emerge if we extended the matter content by one or more adjoints 35 of SU(6). We neglect in Eq. (10) possible Yukawa couplings involving the adjoints needed exclusively to guarantee unification. In the context of AS, this is equivalent to assuming that those Yukawa couplings are zero in the trans-Planckian regime, and they are thus either protected from appearing at the low scale (irrelevant) or fine tuned to negligible values (relevant).

C. Trans-Planckian fixed points

The full set of trans-Planckian RGEs of the gauge and Yukawa couplings of the SU(6) model is presented in Appendix B. They are derived with the public tool `PyR@TE` 3 [100,101]. As was explained in Sec. II, the current status of quantum gravity calculations, using both the FRG and other schemes, strongly favors $f_g \geq 0$. As a consequence of this choice, $g_6^* = 0$ must be a relevant Gaussian fixed point. The RGE system admits several fixed points. Some of them are presented in Appendix C. Since we expect, by construction, the UV completion to be maximally predictive, we present here the fixed point with the maximal number of irrelevant couplings in the Yukawa sector. It features eight irrelevant eigendirections, two marginal, and one relevant. The fixed-point values are reported, together with their critical exponents, in Table I.

In Fig. 2(a) we show the flow of the Yukawa couplings from the deep trans-Planckian UV down to the EWSB scale for chosen values of the parameters: $\tan \beta = 1$, $f_y = 0.016$, and $f_g = 0.05$. Below the GUT scale, the RGEs of the low-energy model are computed with the public tool `RGBeta` [102].

In Fig. 2(b) we show the flow of the Yukawa couplings for $f_g = 0.01$. A comparison with Fig. 2(a) shows that the size of the Yukawa couplings at the fixed point is essentially independent of the value of the gravitational correction f_g , which can even be zero. This is because near the Planck scale the Yukawa-coupling flow is mostly driven by the (relevant) gauge coupling g_6 , which is pinned in the IR by the measured value of the SM gauge couplings. This RG “focusing” behavior of irrelevant couplings driven by a relevant one is well known in BSM theories with UV fixed points (for a discussion see, e.g., Sec. 3.1 of Ref. [79]). Note also that the sub-GUT flow of the BSM Abelian gauge coupling g_X (dark dotted gray) is very steep, due to the large $U(1)_X$ charges of the low-energy fields. It leads to a small value of the coupling at the low scale

$$g_X(\mu = 1 \text{ TeV}) = 0.07. \quad (15)$$

We do not show in Fig. 2 the running of the (Abelian) kinetic mixing parameter g_e . We have checked that the boundary condition $g_e(M_{\text{GUT}}) = 0$ leads to a negligible low-scale value, $g_e(\mu = 1 \text{ TeV}) \lesssim 10^{-3}$.

In Fig. 2(c) we show the flow of y_u for $f_g = 0$ and different choices of f_y . While changing the f_y value leads to different fixed points for y_u , the low-scale DM phenomenology does not, to a great extent, depend on the Yukawa size at the fixed point. This ceases to be true, however, if one takes a very small f_y , which leads to a very small y_u^* . In that case, the pull of the gauge coupling is not strong enough to drag the top quark Yukawa coupling to its measured low-scale value (inefficient focusing). All in all, the lesson we draw from Fig. 2 is that the low-scale Yukawa couplings are very mildly dependent on the outcome and details of a quantum-gravity calculation. This is true as long as the couplings’ scaling behavior at the fixed point is consistent with the critical exponents presented in Table I.

The low-scale value of irrelevant Yukawa couplings are predicted uniquely. The only remaining free coupling is y_{11} (relevant), which, as we shall see below, is adjusted to the mass of the bottom quark. Note also that there are two marginal couplings, thus needing higher-loop corrections to determine their fate.

One can see in Table I that six Yukawa couplings are prevented from appearing in the Lagrangian by trans-

TABLE I. Upper line: Trans-Planckian fixed points of the SU(6) Yukawa couplings for $f_y = 0.016$. Lower line: The corresponding critical exponents times $16\pi^2$. Critical exponents grouped together in the same box identify nondiagonal eigendirections in theory space. The diagonal directions are presented with the corresponding exponent in individual boxes.

y_u^*	y_{22}^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	y_{11}^*	$\tilde{y}_{11}^*$	$\tilde{y}_{12}^*$	$\tilde{y}_{22}^*$	$\hat{y}_{12}^*$	y_{12}^*	y_{21}^*
0.25	0.35	0.38	0.32	0.0	0.0	0.0	0.0	0.0	0.0	0.0
θ_u	θ_{22}	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	θ_{11}	$\tilde{\theta}_{11}$	$\tilde{\theta}_{12}$	$\tilde{\theta}_{22}$	$\hat{\theta}_{12}$	θ_{12}	θ_{21}
-5.1	-1.1	-4.7	-3.7	0.63	-0.27	-0.27	-0.27	-3.7	0	0

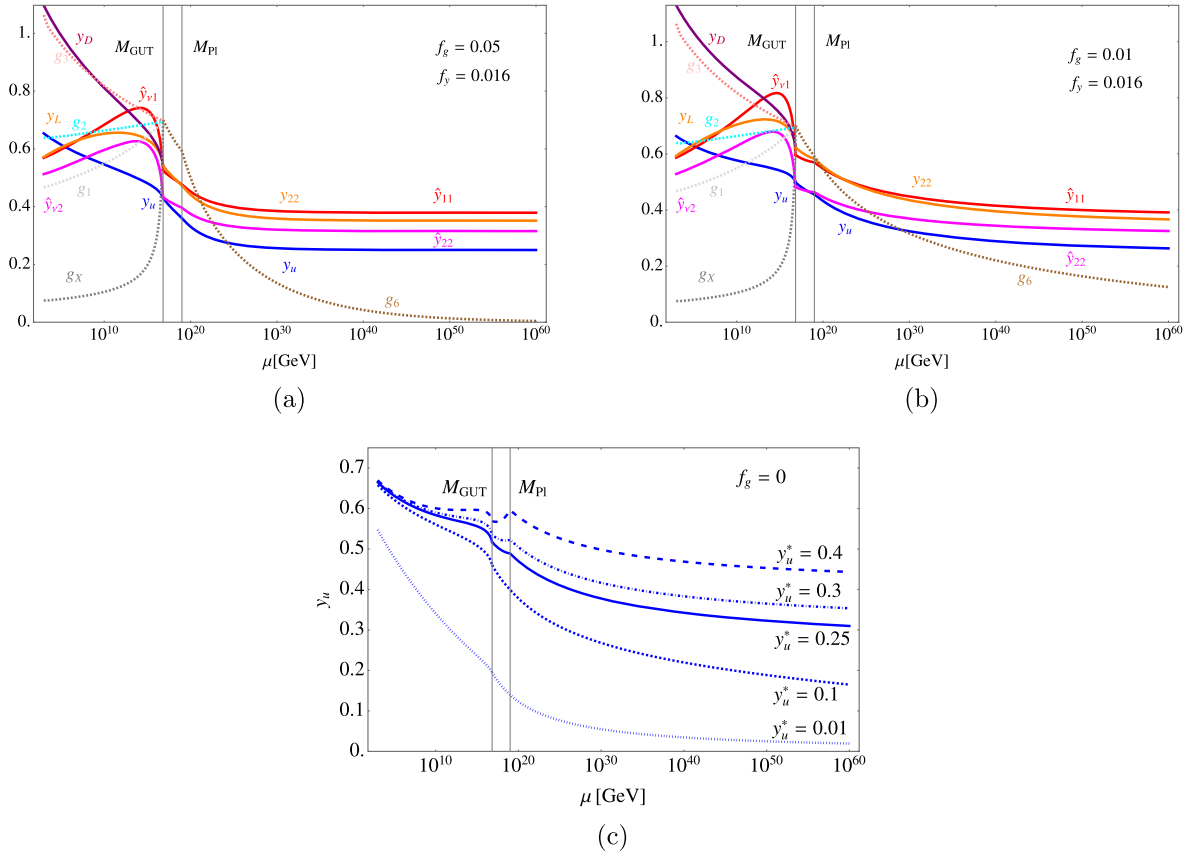


FIG. 2. (a) The RG flow of the SU(6) Yukawa (solid) and gauge (dotted) couplings from the deep trans-Planckian UV to the EWSB scale for $f_g = 0.05$ and $f_y = 0.016$. We have selected $\tan\beta = 1$ at the EWSB scale. (b) The flow of the SU(6) Yukawa (solid) and gauge (dotted) couplings from the deep trans-Planckian UV to the EWSB scale for $f_g = 0.01$ and $f_y = 0.016$. (c) The flow of the top-quark Yukawa coupling from the deep trans-Planckian UV to the EWSB scale for $f_g = 0$ and different choices of the fixed point, corresponding to different f_y .

Planckian scale symmetry, as their critical exponents are negative (we also include in this group the marginal couplings y_{12} and y_{21}). Due to the chiral symmetry of the sub-Planckian gauge theory, they remain zero even when gravity decouples at M_{Pl} , and through the process of GUT symmetry-breaking.² Thus, the Lagrangian (10) reduces at the DM mass scale to

$$\begin{aligned} \mathcal{L}_{\text{IR1}} \supset & 2y_u u H_u^{\dagger} Q + y_d d_1 H_d Q + y_e e H_d L_1 + y_\nu L' H_d^{\dagger} \nu_1 \\ & + y_D d_2 d' s_6 + y_L L' L_2 s_6 + y_{\nu_1} \nu_1 \nu_1 s_{21} + y_{\nu_2} \nu_2 \nu_2 s_{21} \\ & + \text{H.c.}, \end{aligned} \quad (16)$$

²For a comparison with the literature, we note that the couplings y_{12} , y_{21} , $\tilde{y}_{12}$, and $\hat{y}_{12}$ are forbidden in Ref. [70] by an imposed $\mathbb{Z}_2$ symmetry. However, the $\mathbb{Z}_2$ symmetry is not sufficient to forbid also the appearance of $\tilde{y}_{11}$ and $\tilde{y}_{22}$. It can be checked that, in order to prevent the DM particle from decaying too fast via the tree-level decays $\text{DM} \rightarrow h_1 \nu_{\text{SM}}$, $\text{DM} \rightarrow Z \nu_{\text{SM}}$, and $\text{DM} \rightarrow W^\pm J^\mp$, an upper bound must be imposed, $\tilde{y}_{11} \approx \tilde{y}_{22} < 10^{-9}$. In the AS-inspired framework considered here, $\tilde{y}_{11}$ and $\tilde{y}_{22}$ are set to zero automatically due to the presence of an irrelevant trans-Planckian fixed point.

where spinor and weak isospin indices are contracted trivially following matrix multiplication rules, we have defined $H_{u,d}^c \equiv i\sigma_2 H_{u,d}^*$, and we restrict the analysis to the third SM generation only. Couplings y_d , y_e , and y_ν in Eq. (16) originate from the relevant UV parameter y_{11} , after following the RG flow down to the EWSB scale. Couplings y_D and y_L originate from the irrelevant y_{22} and are thus uniquely predicted at every scale of phenomenological interest. We show in Table II, as an example, the predicted values of the irrelevant Yukawa couplings at the scale $\mu = 1$ TeV for $\tan\beta = 1$, $f_g = 0.05$, and $f_y = 0.016$ (we reiterate that the numerical values depend very mildly on the actual choice of the gravitational parameters). Similar considerations allow one to derive from the RG flow the values of the irrelevant couplings that are important for the DM phenomenology at an arbitrary renormalization scale.

Because some of the Lagrangian couplings allowed by gauge invariance are forbidden by the quantum scale symmetry, low-energy particles that belong to GUT multiplets with the same quantum numbers do not mix with one another. In fact, the mass matrices for the bottomlike quarks

TABLE II. Left-hand side: the predicted values of the irrelevant Yukawa couplings at the scale $\mu = 1$ TeV for $\tan\beta = 1$, $f_g = 0.05$, and $f_y = 0.016$. The upper line indicates the corresponding SU(6) Yukawa couplings. The numerical values depend very mildly on f_g and f_y . Right-hand side: relevant couplings y_d and y_ν are related to each other by the RG flow. Their value at the low scale is adjusted so that $y_d v_d / \sqrt{2}$ matches the bottom quark mass.

	y_u	y_{22}	$\hat{y}_{11}$	$\hat{y}_{22}$	y_{11}		
SU(6)	y_u	y_D	y_L	y_{ν_1}	y_{ν_2}	y_d	y_ν
$\mu = 1$ TeV	0.69	1.1	0.55	0.57	0.51	0.027	0.014

and tau-like leptons read, in the basis $\langle d_1, d_2 |, |d_Q, d' \rangle$, and $\langle e, e_{L'} |, |e_{L_1}, e_{L_2} \rangle$, respectively,

$$M_b = \frac{1}{\sqrt{2}} \begin{pmatrix} y_d v_d & 0 \\ 0 & y_D v_{s_6} \end{pmatrix}, \quad M_\tau = \frac{1}{\sqrt{2}} \begin{pmatrix} y_e v_d & 0 \\ 0 & y_L v_{s_6} \end{pmatrix}, \quad (17)$$

where $d_Q \in Q$, $e_{L'} \in L'$, $e_{L_1} \in L_1$, and $e_{L_2} \in L_2$.

The mass matrix for the neutral fermions, in the basis $\langle \nu_{L_1}, \nu_{L_2}, \nu_{L'}, \nu_1, \nu_2 |, | \nu_{L_1}, \nu_{L_2}, \nu_{L'}, \nu_1, \nu_2 \rangle$ reads

$$\frac{1}{2} M_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y_L v_{s_6} & 0 & 0 \\ 0 & y_L v_{s_6} & 0 & y_\nu v_d & 0 \\ 0 & 0 & y_\nu v_d & y_{\nu_1} v_{s_{21}} & 0 \\ 0 & 0 & 0 & 0 & y_{\nu_2} v_{s_{21}} \end{pmatrix}, \quad (18)$$

where $\nu_{L_1}, \nu_{L_2}, \nu_{L'} \in L_1, L_2, L'$, respectively. One can see that the lightest neutrino of one generation remains massless (this is still a phenomenologically allowed possibility [103]), while the heavy Majorana neutrinos of the same generation comprise a dark sector prevented by the scale symmetry from decaying into the lightest neutrino. The lightest dark particle is thus a good candidate for weakly interacting massive particles (WIMP) DM (albeit metastable once GUT interactions mediated by heavy states are factored in, see Ref. [70]).

D. Naturally small Yukawa couplings

As we have discussed above, quantum scale symmetry—codified by the negative critical exponents of the fixed points of the SU(6) Yukawa couplings—has the effect of both providing unique predictions for the interactions of the low-scale theory, and of preventing the couplings that correspond to a Gaussian fixed point from appearing in the Lagrangian. On the other hand, some of us have shown in a couple of recent papers [44,57] that the mere presence of an irrelevant Gaussian fixed point of the RG flow does not always lead to a zero coupling in the IR. In fact, if the RGE system admits also a relevant fixed point at much higher values of μ , some couplings may flow from the relevant UV fixed point down to the basin of attraction of the Gaussian fixed point lying in the trans-Planckian IR, without ever reaching zero. The IR attractor induces an exponential suppression of the Yukawa coupling y , whose run becomes approximately parametrized as [44]

$$y(t, \kappa) = \left[\frac{16\pi^2 c_X (f_{\text{crit}} - f_y)}{e^{2c_X(f_{\text{crit}} - f_y)(16\pi^2 \kappa - t)} + \alpha_Z} \right]^{1/2}, \quad (19)$$

where f_{crit} is a critical f_y value below which a Gaussian irrelevant fixed point for y appears, c_X is a linear combination of the coefficients of the one-loop beta functions of Yukawa couplings other than y , and α_Z is the y^3 coefficient of the one-loop beta function of y . Once the theory and the UV completion are set, the suppression depends on one single parameter κ , which measures, in e-folds, the distance between the UV and the IR fixed points.

The RGEs of the SU(6) theory do feature a relevant UV fixed point at high values of μ . Its critical exponents are presented in Table III. As a consequence, some of the irrelevant Gaussian Yukawa couplings of Table I are now allowed to take arbitrarily small values that are naturally suppressed according to Eq. (19).

In Fig. 3, we show this effect for coupling $\tilde{y}_{11}$ (in red, dot-dashed). The size of $\tilde{y}_{11}$ can be made arbitrarily small, depending on where in the flow couplings $y_u, y_{22}, \hat{y}_{11}$, and $\hat{y}_{22}$ reach their interactive fixed point. One of the interesting properties of such a solution for $\tilde{y}_{11}$ is that its low-scale value can induce a small left-handed neutrino mass via a light seesaw [104]. We show for completeness, in Eq. (E3) of Appendix E, the form of the full Majorana fermion mass matrix when all of the Gaussian irrelevant couplings of

TABLE III. SU(6) Yukawa couplings and their critical exponents times $16\pi^2$ at the extreme UV fixed point for $f_y = 0.016$.

y_u^*	y_{22}^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	y_{11}^*	y_{12}^*	y_{21}^*	$\tilde{y}_{11}^*$	$\tilde{y}_{12}^*$	$\tilde{y}_{22}^*$	$\hat{y}_{12}^*$
0.0	0.54	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
θ_i	θ_{22}	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	θ_{11}	θ_{12}	θ_{21}	$\tilde{\theta}_{11}$	$\tilde{\theta}_{12}$	$\tilde{\theta}_{22}$	$\hat{\theta}_{12}$
1.9	-5.0	2.5	1.0	2.2	0	0	2.5	1.8	1.0	1.8

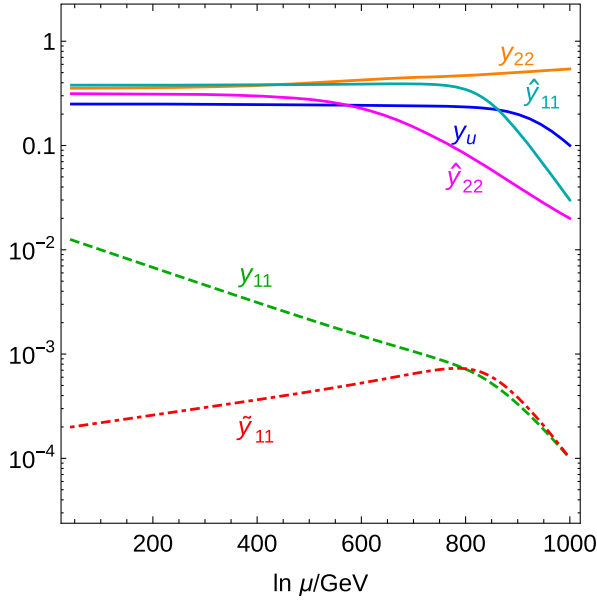


FIG. 3. Trans-Planckian RG flow of the Yukawa couplings from the deep UV fixed point of Table III. Solid lines show the couplings that are predictions of the theory, while dashed and dot-dashed are the couplings that can be freely adjusted.

Table I are allowed to appear. Assuming the lightest neutrino mass is $\mathcal{O}(10^{-10} \text{ GeV})$, for $y_{\nu_1} v_{s_{21}} \approx 1 \text{ TeV}$ one needs $\tilde{y}_{11}(\mu = 1 \text{ TeV}) \approx 10^{-6}$ at $\tan \beta = 1$.

E. Scalar potential

As was mentioned in Sec. IV A, we will not investigate the fixed points of the scalar sector of the SU(6) model. Given the complexity of the scalar potential in Eq. (A1) of Appendix A, doing so would amount to a formidable task even at one loop. In this subsection, we limit ourselves to addressing qualitatively the expected scaling behavior of the scalar-potential parameters in relation to the current state of calculations in asymptotically safe quantum gravity.

Let us define the dimensionless running parameters of the scalar potential: mass parameters $\tilde{M}_i^2 = M_i^2/\mu^2$, trilinear couplings $\tilde{\kappa}_j = \kappa_j/\mu$, and quartic couplings α_k , for i, j, k spanning all the couplings given in Eq. (A1) of Appendix A. The RGEs of the scalar potential are modified in the trans-Planckian regime with a “correction” due to the gravity fixed points, in analogy to Eqs. (1) and (2). Following several studies in the literature [105–107] one can approximate the RGEs schematically,

$$\frac{d\alpha_k}{dt} = \eta_{\alpha_k} \alpha_k + \beta_{\alpha_k, \text{add}} - f_\lambda \alpha_k, \quad (20)$$

$$\frac{d\tilde{\kappa}_j}{dt} = (-1 + \eta_{\tilde{\kappa}_j}) \tilde{\kappa}_j + \beta_{\tilde{\kappa}_j, \text{add}} - f_\lambda \tilde{\kappa}_j, \quad (21)$$

$$\frac{d\tilde{M}_i^2}{dt} = (-2 + \eta_{\tilde{M}_i^2}) \tilde{M}_i^2 + \beta_{\tilde{M}_i^2, \text{add}} - f_\lambda \tilde{M}_i^2, \quad (22)$$

where η_{α_k} , $\eta_{\tilde{\kappa}_j}$, and $\eta_{\tilde{M}_i^2}$ indicate the matter anomalous dimensions. The terms with an “add” subscript parametrize additive contributions to the matter beta functions not included in the anomalous dimensions; they potentially depend on any of the couplings of the theory with the exception of the very one indicated in the subscript. f_λ is the universal multiplicative correction analogous to f_g and f_y , which typically depends on the fixed points of the gravitational action. Note that for practicality we neglect in Eqs. (20)–(22) terms that parametrize additive contributions potentially arising from nonminimal direct couplings of the scalar potential to gravitational operators, see, e.g., the truncation introduced in Ref. [80].

One may envision the outcome of the eventual UV calculation of f_λ broadly in three ways:

Case A. $f_\lambda \ll -2$. Under this condition, the set of Eqs. (20)–(22) admit a fully irrelevant Gaussian fixed point. It was shown in Refs. [105–107] that $f_\lambda \ll -2$ may emerge in an FRG calculation of the Higgs potential and Einstein-Hilbert truncation of the gravitational action, for certain values of the running Planck mass. Under this assumption the scalar potential of the matter theory features complete quantum scale symmetry. Its dimensionful couplings may tend, when gravity decouples rapidly below the Planck scale, to specific predicted values that are naturally small [105–107]. For the same reason, this assumption is in strong tension with the hierarchical ladder of physical scales typical of GUT theories with a realistic phenomenology.

Case B. $-1 \lesssim f_\lambda < 0$. An eventual outcome of the UV calculation within this range would allow the dimensionful couplings of the scalar potential to remain relevant, while the fixed points of the quartic couplings may be irrelevant [86]. While in this framework one can easily accommodate the generation of physical scales below Planck, it is certainly hard to envision, without a detailed fixed-point analysis, whether the requirement of doublet-triplet splitting, Eq. (A9) in Appendix A 1, and other scale separations could be implemented at all.

Case C. $f_\lambda \gg 0$. In this case the fixed points of the scalar potential are all relevant. Dimensionful and dimensionless couplings cannot be predicted, as any chosen IR value is eventually consistent with AS. It was shown in Ref. [106] that $f_\lambda > 0$ cannot be an outcome of the FRG calculation with the action comprising the SM Higgs potential and gravity in the Einstein-Hilbert truncation, independently of the fixed-point value of the gravitational parameters and of the choice of regulator. If this negative outcome were to be confirmed by a detailed analysis of the fixed points of the gravitational and scalar potential of the SU(6) model presented here, it would place it squarely outside of

the “landscape” of realistic theories that can be derived from asymptotically safe quantum gravity (see [83,108–111] for a discussion).

On the other hand, it was recently shown in Ref. [55] for the SM Higgs, that the situation may be different if one keeps track in the FRG calculation of several higher-order operators arising from a Taylor expansion of the scalar potential. In the specific case of the SM coupled to Einstein-Hilbert gravity, one can observe the emergence of a second relevant direction at the Gaussian fixed point, in addition to the one typically associated with the Higgs mass squared. This additional relevant direction, which remains hidden in calculation performed in a $\mathcal{O}(H^4)$ truncation, is indeed welcome as it allows a phenomenologically viable connection between the UV fixed point and the physical SM at low energies.

A complete FRG analysis of the SU(6) scalar potential exceeds the purposes of this paper. Here we work under the assumption that $f_\lambda \gg 0$, so that all the quartic couplings of Eq. (A1) in Appendix A are relevant at the Gaussian fixed point. An interesting exercise for future research would be to determine the minimal truncation of the SU(6) + gravity effective action that is consistent with the requirement that $f_\lambda \gg 0$.

V. PREDICTIONS FOR DARK MATTER

A. Two-component dark matter

With the value of all BSM Yukawa couplings fixed by the UV completion at every scale of phenomenological interest (cf. Table II), BSM fermion masses become uniquely determined by two relevant parameters, the vevs v_{s_6} and $v_{s_{21}}$. In particular, the masses of the heavy Majorana fermions, derived by diagonalizing Eq. (18), read, in the limit $v_d \ll v_{s_6}, v_{s_{21}}$,

$$\begin{aligned} m_{N_1} &\simeq \sqrt{2} y_{\nu_2} v_{s_{21}}, \\ m_{N_2} &\simeq \sqrt{2} \left(y_L v_{s_6} - \frac{1}{2} \frac{y_L^2 v_d^2}{y_{\nu_1} v_{s_{21}}} \right), \\ m_{N_3} &\simeq \sqrt{2} \left(y_L v_{s_6} + \frac{1}{2} \frac{y_L^2 v_d^2}{y_{\nu_1} v_{s_{21}}} \right), \\ m_{N_4} &\simeq \sqrt{2} y_{\nu_1} v_{s_{21}}. \end{aligned} \quad (23)$$

Depending on the hierarchy between v_{s_6} and $v_{s_{21}}$, different DM scenarios are possible.

- (i) $v_{s_{21}} < v_{s_6}$. This case results in the mass ordering $m_{N_1} < m_{N_4} < m_{N_{2,3}}$. The DM candidate is in this case N_1 . On the other hand, since N_2, N_3 , and N_4 do not mix with N_1 at the tree level, the lightest of these three heavy neutrinos, N_4 , is stable as well. As a consequence, this scenario features a *two-component* DM sector, which is a SM-singlet.
- (ii) $v_{s_6} < v_{s_{21}}$. This case leads to the mass ordering $m_{N_{2,3}} < m_{N_1} < m_{N_4}$. The stable particles are in this case N_1, N_2 , and N_3 , resulting in two-component Majorana DM with an SU(2)_L doublet and a singlet as constituents.

The situation changes if one considers the scenario discussed in Sec. IV D. In that case very small, yet nonzero couplings $\tilde{y}_{11}, \tilde{y}_{22}$ can be generated dynamically at the low-energy scale. The presence of these extra interactions facilitates the decay of the heaviest of the two-component DM particles into either the SM neutrinos (if $\tilde{y}_{11}$ is turned on), or the lightest BSM Majorana fermion (if $\tilde{y}_{22}$ is turned on). This is illustrated in Fig. 4(a) for $v_{s_{21}} < v_{s_6}$, and in Fig. 4(b) for $v_{s_6} < v_{s_{21}}$. As a result, DM is in this case one-component: either a SM-singlet (provided $v_{s_{21}} < v_{s_6}$), or a weak isospin doublet (provided $v_{s_6} < v_{s_{21}}$), reminiscent of a supersymmetric Higgsino-like neutralino. Note that, in the latter case, $\tilde{y}_{11}$ has to either remain zero or be very small

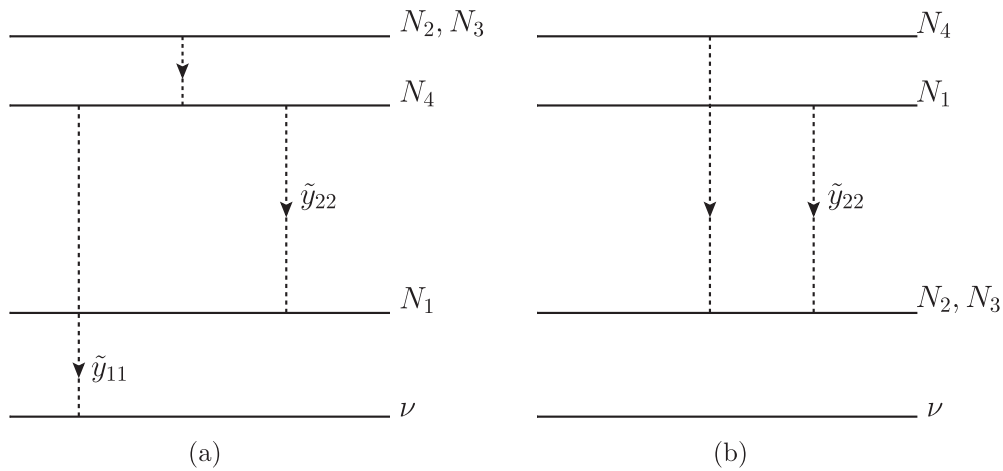


FIG. 4. Decay chains of the heavy Majorana fermions N_1, N_2, N_3 , and N_4 in the presence of non-zero (yet arbitrarily small) Yukawa couplings $\tilde{y}_{11}$ and $\tilde{y}_{22}$. (a) $v_{s_{21}} < v_{s_6}$ and (b) $v_{s_6} < v_{s_{21}}$.

in order to make the DM candidate stable over cosmological timescales.

Finally, one may consider the possibility that $SU(6)$ coupling y_{21} , which is marginal at the one loop order (see Table I) will eventually emerge as relevant once a higher-order analysis of the stability matrix is performed. The DM sector would be then equivalent to the case of a nonzero $\tilde{y}_{22}$ illustrated in Fig. 4, where the role of $\tilde{y}_{22}$ is replaced by y'_ν (see Eq. (E1) in Appendix E). Incidentally y_{21} may also generate a small y'_d , which would in turn induce mixing of the heavy down-type quark with its SM counterpart, thus opening a two-body decay channel of the heavy quark which is otherwise absent.³

B. Singlet fermion DM ($v_{s_{21}} < v_{s_6}$)

Let us briefly discuss the current bounds on the single-particle DM cases discussed in the previous two paragraphs. In this subsection we consider the case where the DM candidate is a SM-singlet Majorana fermion with mass

$$m_{\text{DM}} \simeq \sqrt{2} y_{\nu_2} v_{s_{21}}. \quad (24)$$

Its relic abundance at freeze-out is determined by pair annihilation into SM particles via an s -channel exchange of heavy BSM scalars, or an s -channel exchange of the Z' vector boson (note that t -channel “bulklike” annihilation would require mediators with masses of the order of 100 GeV [112–114]). Annihilation via the s -channel exchange of a scalar was discussed in Ref. [70]. Following our discussion in Sec. IV E, we assume in this work that the spectrum in the scalar sector is entirely controlled by relevant parameters in the Lagrangian (mass terms and trilinear couplings), which do not emerge as predictions of the fixed points. For this reason, we focus here on the Z' resonance, which is instead entirely determined by gauge and Yukawa couplings that can be predicted from UV considerations. (In other words, we work under the assumption that the physical masses of all CP -even and odd scalar fields of the model, bar the Higgs boson at 125 GeV, are larger than $m_{Z'}$, cf., Eqs. (A13) and (A16) in Appendix A.)

Upon $U(1)_X$ symmetry breaking, the Z' boson acquires a mass,

$$m_{Z'} \simeq 5g_X \sqrt{v_{s_6}^2 + 4v_{s_{21}}^2}. \quad (25)$$

The resonance condition states $m_{Z'} \approx 2m_{\text{DM}}$, where the approximate sign accounts for the effects of a finite decay width. We show in Fig. 5 the region corresponding to the

³Absent the two-body decay channel, one may have to adjust the mass of a color triplet component in $\mathbf{21}^{(S)}$ to allow a three-body decay of the heavy down-type quark into the bottom quark and two singlet BSM Majorana fermions [70].

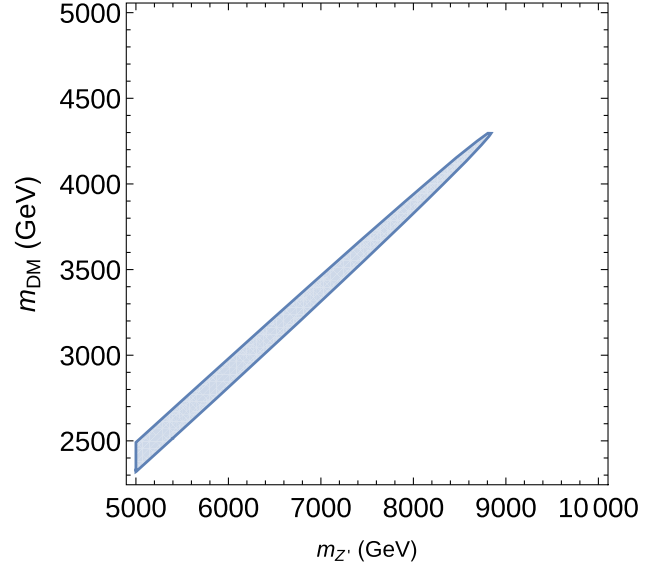


FIG. 5. The region of under-abundant relic density (in blue) in the SM-singlet DM case.

condition $\langle \sigma v \rangle \gtrsim 2.15 \times 10^{-26} \text{ cm}^3/\text{s}$ [115] in the $(m_{Z'}, m_{\text{DM}})$ plane, where the thermally averaged pair annihilation cross section is defined, following, e.g., Ref. [116,117], in Eq. (D1) of Appendix D. The lower bound on the Z' mass, $m_{Z'} \gtrsim 5 \text{ TeV}$, is set by the null results of searches for narrow resonances with sequential SM couplings at $\sqrt{s} = 13 \text{ TeV}$ and a total integrated luminosity of up to 140 fb^{-1} by CMS [118]. The upper bound on the Z' mass, $m_{Z'} \lesssim 9 \text{ TeV}$ emerges when the annihilation cross section starts to become kinematically suppressed by the mass of the mediator. Note that, since the Yukawa coupling y_{ν_2} and the gauge coupling $g_X(1 \text{ TeV}) \approx 0.07$ are entirely predicted by the UV completion, the relic-abundance condition indirectly constrains two of the relevant parameters of scalar potential (A1) (M_{62}^2 and M_{21}^2 , related to vevs v_{s_6} and $v_{s_{21}}$, respectively) to a very narrow slice of the parameter space.

Incidentally, the possibility of precisely constraining the physical mass scales of the theory via the measurement of an observable or an effective operator at low energy is one of the most attractive features of UV completions based on AS because it drastically increases the testability of the theory. For a discussion of the uncertainties involved in the process see, e.g., Sec. 3 of Ref. [79].

C. Doublet DM ($v_{s_6} \ll v_{s_{21}}$)

The DM is the $SU(2)_L$ doublet Majorana fermion, similar to a supersymmetric Higgsino-like neutralino (see, e.g., Refs. [115,119,120] for reviews). As in the previous case, we work under the assumption that the masses of all CP -even and odd scalar fields of the model, bar the Higgs boson at 125 GeV, are much heavier than the DM particle. It is well known that condition

$\langle\sigma v\rangle \approx 2.15 \times 10^{-26} \text{ cm}^3/\text{s}$ requires in this case $m_{\text{DM}} = m_{N_2} \approx 1.1 \text{ TeV}$, see also Eq. (D8) in Appendix D.

As in the singlet DM case, all the Yukawa couplings and the gauge coupling g_X are uniquely predicted. Indirect constraints can be derived on the heaviest vev of the DM sector, $v_{s_{21}}$, or, equivalently, on the mass $m_{Z'}$. The lower bound on the Z' mass is established again by the null results of searches for narrow resonances with sequential SM couplings at $\sqrt{s} = 13 \text{ TeV}$ and a total integrated luminosity of up to 140 fb^{-1} by CMS [118].

The upper bound on the Z' mass, $m_{Z'} \lesssim 50 \text{ TeV}$, is extracted from the inelastic scattering limit of the spin-independent DM-nucleon cross section. One can derive from Eq. (23) the mass splitting of the neutral components of the doublet in the limit $v_{s_{21}} \gg v_{s_6}$

$$\delta m_{\text{DM}} \equiv m_{N_3} - m_{N_2} = \sqrt{2} \frac{y_\nu^2 v_d^2}{y_{\nu_1} v_{s_{21}}}. \quad (26)$$

In Ref. [121] it was estimated that the lower bound on the mass splitting should be $\delta m_{\text{DM}} \geq 200 \text{ keV}$. For smaller values of δm_{DM} inelastic scattering $N_2 p \rightarrow N_3 p$, with exchange of a SM Z boson in the t -channel, becomes extremely constrained in DM direct-detection experiments.

In Fig. 6 we plot the mass splitting δm_{DM} in red solid as a function of $m_{Z'}$. The region excluded by the inelastic scattering limit is shown in blue. In similar fashion to the case of singlet DM, relevant parameters M_{62}^2 and M_{21}^2 of the scalar potential, related to the vevs v_{s_6} and $v_{s_{21}}$ are indirectly constrained to a narrow range of the parameter space.

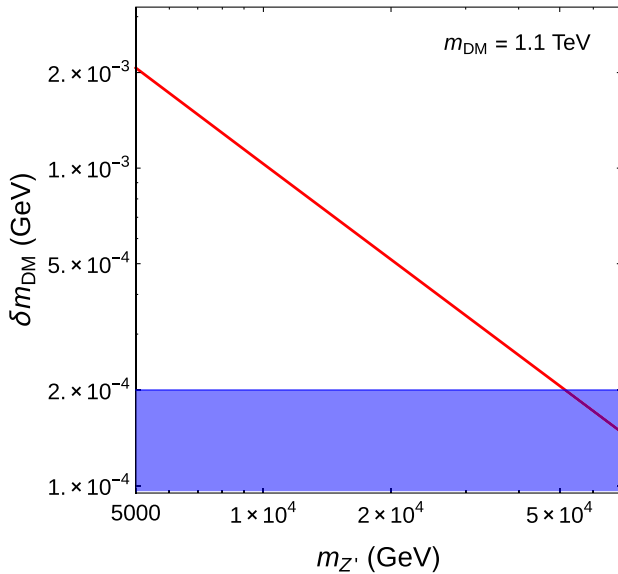


FIG. 6. The upper bound on $m_{Z'}$ from inelastic scattering limits on the spin-independent DM-nucleon cross section in the case of doublet DM. The exclusion bound on δm_{DM} (in blue) is taken from Ref. [121].

We conclude by emphasizing that our goal in this section is to highlight different ways in which UV boundary conditions based on quantum scale symmetry can constrain the DM sector of the SU(6) theory. It should thus be noted that we are not trying to constrain the free parameters of the low-energy theory by means of a very precise quantitative analysis of all relic-density channels and DM-detection bounds (direct and indirect). It goes without saying that a full numerical analysis of the DM sector (including two-component DM) would be required to improve on the approximate treatment used for Figs. 5 and 6, which is based on formulas summarized in Appendix D. Such full analysis should also involve the details of the scalar potential, which may in turn open up additional mediator and final-state channels for the DM particle(s) to decay to. We leave these interesting aspects of investigation for future work.

VI. SUMMARY AND CONCLUSIONS

In this study, we have investigated the possibility of using trans-Planckian quantum scale symmetry, defined as the existence of Gaussian IR-attractive fixed points of the RG flow, to prevent the appearance in a gauge-Yukawa theory of couplings that would otherwise be allowed by the gauge symmetry. In particular, we have shown that this feature can supply a BSM model with a dark sector hosting a DM candidate, and it thus provides a viable alternative to the introduction of global or discrete symmetries “by hand.”

As an example, we have analyzed the gauge-Yukawa sector of an SU(6) GUT theory spontaneously broken to $\text{SU}(5) \times \text{U}(1)_X$. SU(5) is subsequently broken spontaneously to the SM group. Incidentally, once the minimal anomaly-free fermionic content is determined the Yukawa sector shows remarkable similarities in all SU(N) GUTs. As a consequence, our discussion can be generalized straightforwardly to SU(N) groups of rank higher than 6. The low-energy phenomenology, on the other hand, strongly depends on the details of the scalar sector and has to be investigated on a case-by-case basis. In particular, in our example the SU(6) UV completion gives rise to an extended version of the 2HDM at scales about or slightly above EWSB. The gauge-Yukawa RGE system admits a trans-Planckian fixed point at which all but one Yukawa couplings are irrelevant (or marginal). Of those irrelevant couplings, the nonzero ones provide unique predictions for the strength of the interactions of the extended 2HDM, whereas the Gaussian ones prevent the appearance of interactions that can jeopardize the stability of DM. Predictivity is thus maximally enhanced and the only parameters of the theory that remain unconstrained in the UV are the masses of the BSM particles. Importantly, however, once the interaction strengths are known, physical mass scales can be constrained quite

precisely measuring a low-scale observable, which in this case is the relic density of DM.

An important feature of the asymptotically safe SU(6) GUT discussed in this paper is that the low-scale predictions for the gauge and Yukawa couplings depend only very mildly on the outcome of a quantum-gravity calculation of the effective parameters f_g and f_y . While $f_y \neq 0$ is required to obtain the negative critical exponents enforcing the quantum scale symmetry dynamically, close to the Planck scale the RG flow of the irrelevant Yukawa couplings deviates from the scaling solution, driven predominantly by the relevant gauge coupling g_6 , whose IR value is fixed by the measured values of the SM gauge couplings. As a result the DM phenomenology is much more sensitive to measured/measurable parameters like the strong coupling constant and $\tan\beta$, than it is to the actual value of UV parameters like f_y , whose computation is notoriously ridden with large uncertainties.

The analysis presented in this work can be extended in several directions. First of all, the low-scale constraints on the DM sector were derived here under the assumption that the BSM scalar sector of the 2HDM decouples from the lightest (meta-)stable fermions (by acquiring an appropriately large mass). This is not automatically warranted and, even more importantly, it is not known whether this assumption is in itself consistent with an FRG calculation in AS (in the form of asymptotically safe quantum gravity). We leave it for future work to perform a full fixed-point analysis of the quartic couplings of the SU(6) scalar potential (A1), either in itself or in relation to simple truncations of the gravitational action.

It would also be instructive to perform a detailed numerical analysis of the parameter space of the DM sector of our model (including the quartic and trilinear couplings and the masses of the scalar potential) over several mass scales, in order to identify nontrivial regions and annihilation mechanisms for the correct relic abundance that go beyond the simple cases considered in Sec. V.

In conclusion, in this work we demonstrated in yet another concrete example the promising role of trans-Planckian AS in shaping predictive UV-complete extensions of the SM, without relying on imposed symmetries to justify hard-to-achieve observational features like the extreme smallness of certain couplings or the stability of DM. This adds to the large body of work that has investigated potential measurable imprints of quantum gravity effects on particle phenomenology.

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DATA AVAILABILITY

The data that support the findings of this article are openly available.

APPENDIX A: SU(6) SCALAR POTENTIAL

The model introduced in Sec. IV contains four scalar multiplets which mediate the Yukawa interactions of Eq. (10): $\mathbf{6}_1^{(S)}$, $\mathbf{6}_2^{(S)}$, $\mathbf{15}^{(S)}$, $\mathbf{21}^{(S)}$, as well as an adjoint $\mathbf{35}^{(S)}$ whose vev breaks the SU(6) symmetry. The full SU(6)-symmetric scalar potential reads (the superscript (S) is omitted)

$$\begin{aligned}
 V_{\text{SU}(6)} = & -\frac{1}{2}M_{35}^2\text{Tr}(\mathbf{35}^2) + \frac{1}{4}A_{35}[\text{Tr}(\mathbf{35}^2)]^2 + \frac{1}{2}B_{35}\text{Tr}(\mathbf{35}^4) \\
 & - M_{21}^2\text{Tr}(\mathbf{21}^\dagger\mathbf{21}) + A_{21}[\text{Tr}(\mathbf{21}^\dagger\mathbf{21})]^2 + B_{21}\text{Tr}[(\mathbf{21}^\dagger\mathbf{21})^2] \\
 & - M_{15}^2\text{Tr}(\mathbf{15}^\dagger\mathbf{15}) + A_{15}[\text{Tr}(\mathbf{15}^\dagger\mathbf{15})]^2 + B_{15}\text{Tr}[(\mathbf{15}^\dagger\mathbf{15})^2] \\
 & - M_{61}^2(\mathbf{6}_1^\dagger\mathbf{6}_1) - M_{62}^2(\mathbf{6}_2^\dagger\mathbf{6}_2) - M_{12}^2(\mathbf{6}_1^\dagger\mathbf{6}_2 + \text{H.c.}) + B_{61}(\mathbf{6}_1^\dagger\mathbf{6}_1)^2 + B_{62}(\mathbf{6}_2^\dagger\mathbf{6}_2)^2 \\
 & + C_3(\mathbf{6}_1^\dagger\mathbf{6}_1)(\mathbf{6}_2^\dagger\mathbf{6}_2) + C_4(\mathbf{6}_1^\dagger\mathbf{6}_2)(\mathbf{6}_2^\dagger\mathbf{6}_1) \\
 & + (C_5(\mathbf{6}_1^\dagger\mathbf{6}_2)^2 + C_6(\mathbf{6}_1^\dagger\mathbf{6}_1)(\mathbf{6}_1^\dagger\mathbf{6}_2) + C_7(\mathbf{6}_2^\dagger\mathbf{6}_2)(\mathbf{6}_1^\dagger\mathbf{6}_2) + \text{H.c.}) \\
 & + \alpha_1(\mathbf{6}_1^\dagger\mathbf{6}_1)\text{Tr}(\mathbf{35}^2) + \beta_1\mathbf{6}_1^\dagger(\mathbf{35}^2)\mathbf{6}_1 + \alpha_2(\mathbf{6}_2^\dagger\mathbf{6}_2)\text{Tr}(\mathbf{35}^2) + \beta_2\mathbf{6}_2^\dagger(\mathbf{35}^2)\mathbf{6}_2 \\
 & + \alpha_{12}(\mathbf{6}_1^\dagger\mathbf{6}_2)\text{Tr}(\mathbf{35}^2) + \beta_{12}\mathbf{6}_1^\dagger(\mathbf{35}^2)\mathbf{6}_2 + \alpha_{21}(\mathbf{6}_2^\dagger\mathbf{6}_1)\text{Tr}(\mathbf{35}^2) + \beta_{21}\mathbf{6}_2^\dagger(\mathbf{35}^2)\mathbf{6}_1 \\
 & + \gamma_1(\mathbf{6}_1^\dagger\mathbf{6}_1)\text{Tr}(\mathbf{21}^\dagger\mathbf{21}) + \delta_1\mathbf{6}_1^\dagger(\mathbf{21}^\dagger\mathbf{21})\mathbf{6}_1 + \gamma_2(\mathbf{6}_2^\dagger\mathbf{6}_2)\text{Tr}(\mathbf{21}^\dagger\mathbf{21}) + \delta_2\mathbf{6}_2^\dagger(\mathbf{21}^\dagger\mathbf{21})\mathbf{6}_2
 \end{aligned}$$

$$\begin{aligned}
& + \gamma_{12}(\mathbf{6}_1^\dagger \mathbf{6}_2) \text{Tr}(\mathbf{21}^\dagger \mathbf{21}) + \delta_{12} \mathbf{6}_1^\dagger (\mathbf{21}^\dagger \mathbf{21}) \mathbf{6}_2 + \gamma_{21}(\mathbf{6}_2^\dagger \mathbf{6}_1) \text{Tr}(\mathbf{21}^\dagger \mathbf{21}) + \delta_{21} \mathbf{6}_2^\dagger (\mathbf{21}^\dagger \mathbf{21}) \mathbf{6}_1 \\
& + \rho_1(\mathbf{6}_1^\dagger \mathbf{6}_1) \text{Tr}(\mathbf{15}^\dagger \mathbf{15}) + \sigma_1 \mathbf{6}_1^\dagger (\mathbf{15}^\dagger \mathbf{15}) \mathbf{6}_1 + \rho_2(\mathbf{6}_2^\dagger \mathbf{6}_2) \text{Tr}(\mathbf{15}^\dagger \mathbf{15}) + \sigma_2 \mathbf{6}_2^\dagger (\mathbf{15}^\dagger \mathbf{15}) \mathbf{6}_2 \\
& + \rho_{12}(\mathbf{6}_1^\dagger \mathbf{6}_2) \text{Tr}(\mathbf{15}^\dagger \mathbf{15}) + \sigma_{12} \mathbf{6}_1^\dagger (\mathbf{15}^\dagger \mathbf{15}) \mathbf{6}_2 + \rho_{21}(\mathbf{6}_2^\dagger \mathbf{6}_1) \text{Tr}(\mathbf{15}^\dagger \mathbf{15}) + \sigma_{21} \mathbf{6}_2^\dagger (\mathbf{15}^\dagger \mathbf{15}) \mathbf{6}_1 \\
& + a_1 \text{Tr}(\mathbf{15}^\dagger \mathbf{15}) \text{Tr}(\mathbf{21}^\dagger \mathbf{21}) + a_2 \text{Tr}(\mathbf{15}^\dagger \mathbf{15}) \text{Tr}(\mathbf{35}^2) + a_3 \text{Tr}(\mathbf{35}^2) \text{Tr}(\mathbf{21}^\dagger \mathbf{21}) \\
& + b_1 \text{Tr}(\mathbf{15}^\dagger \mathbf{15} \mathbf{21}^\dagger \mathbf{21}) + b_2 \text{Tr}(\mathbf{15}^\dagger \mathbf{15} \mathbf{35}^2) + b_3 \text{Tr}(\mathbf{35}^2 \mathbf{21}^\dagger \mathbf{21}) \\
& + [\kappa_1(\mathbf{6}_1 \mathbf{15}^\dagger \mathbf{6}_1) + \kappa_2(\mathbf{6}_2 \mathbf{15}^\dagger \mathbf{6}_2) + \kappa_3(\mathbf{6}_1 \mathbf{15}^\dagger \mathbf{6}_2) \\
& + \kappa_4(\mathbf{6}_1 \mathbf{21}^\dagger \mathbf{6}_1) + \kappa_5(\mathbf{6}_2 \mathbf{21}^\dagger \mathbf{6}_2) + \kappa_6(\mathbf{6}_1 \mathbf{21}^\dagger \mathbf{6}_2) \\
& + \lambda_1(\mathbf{6}_1^\dagger \mathbf{35} \mathbf{6}_1) + \lambda_2(\mathbf{6}_2^\dagger \mathbf{35} \mathbf{6}_2) + \lambda_3(\mathbf{6}_1^\dagger \mathbf{35} \mathbf{6}_2) + \lambda_4(\mathbf{6}_2^\dagger \mathbf{35} \mathbf{6}_1) \\
& + \eta_1(\mathbf{15}^\dagger \mathbf{35} \mathbf{15}) + \eta_2(\mathbf{15} \mathbf{15} \mathbf{15}) + \eta_3(\mathbf{21}^\dagger \mathbf{35} \mathbf{21}) + \eta_4(\mathbf{21}^\dagger \mathbf{35} \mathbf{15}) + \text{H.c.}] \\
& + \xi_1 \text{Tr}(\mathbf{21} \mathbf{21}^\dagger \mathbf{21} \mathbf{15}^\dagger) + \xi_2 \text{Tr}(\mathbf{15} \mathbf{15}^\dagger \mathbf{15} \mathbf{21}^\dagger) + \xi_3 \text{Tr}(\mathbf{15} \mathbf{15} \mathbf{15} \mathbf{35}) \\
& + (\xi_4(\mathbf{6}_1^\dagger \mathbf{15} \mathbf{21}^\dagger \mathbf{6}_1) + \xi_5(\mathbf{6}_2^\dagger \mathbf{15} \mathbf{21}^\dagger \mathbf{6}_2) + \xi_6(\mathbf{6}_1^\dagger \mathbf{15} \mathbf{21}^\dagger \mathbf{6}_2) + \text{H.c.}) \\
& + \xi_7(\mathbf{6}_1 \mathbf{15}^\dagger \mathbf{35} \mathbf{6}_1) + \xi_8(\mathbf{6}_2 \mathbf{15}^\dagger \mathbf{35} \mathbf{6}_2) + \xi_9(\mathbf{6}_1 \mathbf{15}^\dagger \mathbf{35} \mathbf{6}_2) + \xi_{10}(\mathbf{6}_2 \mathbf{15}^\dagger \mathbf{35} \mathbf{6}_1) \\
& + \xi_{11}(\mathbf{6}_1 \mathbf{21}^\dagger \mathbf{35} \mathbf{6}_1) + \xi_{12}(\mathbf{6}_2 \mathbf{21}^\dagger \mathbf{35} \mathbf{6}_2) + \xi_{13}(\mathbf{6}_1 \mathbf{21}^\dagger \mathbf{35} \mathbf{6}_2) + \xi_{14}(\mathbf{6}_2 \mathbf{21}^\dagger \mathbf{35} \mathbf{6}_1). \tag{A1}
\end{aligned}$$

In Eq. (A1) the parameters M_i have dimension mass², the trilinear couplings κ_i , λ_i and η_i have dimension mass¹, and all the quartic couplings ($A_i, B_i, C_i, \alpha_i, \beta_i, \gamma_i, \delta_i, \rho_i, \sigma_i, a_i, b_i, \xi_i$) are dimensionless.

1. GUT symmetry breaking

The SU(6) symmetry is spontaneously broken to SU(5) $\times$ U(1)_X by $\langle \mathbf{35} \rangle = v_6 \text{diag}(-5, 1, 1, 1, 1, 1)$. 10 gauge bosons acquire masses of the order of v_6 . The vev is given by

$$v_6^2 = \frac{M_{35}^2}{30A_{35} + 42B_{35}}, \tag{A2}$$

where $M_{35}^2 > 0$, $A_{35} > 0$, and $B_{35} > 0$. We decompose the 35 in the following way:

$$\mathbf{35} - \langle \mathbf{35} \rangle = \begin{pmatrix} -\frac{5}{\sqrt{30}}A_0 & 0 \\ 0 & \mathbf{24}_0 + \frac{A_0}{\sqrt{30}}\delta_{ij} \end{pmatrix}, \tag{A3}$$

where A_0 is a singlet of SU(5), $i, j = 1, \dots, 5$, and we neglect to write the massless Goldstone bosons in the off-diagonal terms. Given Eqs. (A1) and (A3), the potential for the $\mathbf{24}_0$ becomes

$$\begin{aligned}
V_{\text{SU}(5) \times \text{U}(1)} = & -\frac{1}{2} M_{35}^2 \left(\frac{6B_{35}}{5A_{35} + 7B_{35}} \right) \text{Tr}(\mathbf{24}_0^2) \\
& + \frac{1}{4} A_{35} [\text{Tr}(\mathbf{24}_0^2)]^2 + \frac{1}{2} B_{35} \text{Tr}(\mathbf{24}_0^4). \tag{A4}
\end{aligned}$$

The SU(5) symmetry is spontaneously broken by $\langle \mathbf{24}_0 \rangle = v_5 \text{diag}(1, 1, 1, -3/2, -3/2)$, where

$$v_5^2 = \frac{12B_{35}M_{35}^2}{(5A_{35} + 7B_{35})(15A_{35} + 7B_{35})}. \tag{A5}$$

12 gauge bosons acquire masses of the order of v_5 . We decompose the $\mathbf{24}_0$ in the standard way

$$\mathbf{24}_0 - \langle \mathbf{24}_0 \rangle = \begin{pmatrix} \mathbf{8} + \frac{2}{\sqrt{30}}P_0\delta_{ij} & 0 \\ 0 & \mathbf{3} - \frac{3}{\sqrt{30}}P_0\delta_{lm} \end{pmatrix}, \tag{A6}$$

where we isolate an octet of color, $\mathbf{8}$, and a weak isospin triplet, $\mathbf{3}$; P_0 is a singlet of the SM group and $i, j = 1, \dots, 3$, $l, m = 1, 2$. As before, we neglect to write the massless Goldstone bosons in the off-diagonal terms. Note that $M_{35}^2 > 0$, $A_{35} > 0$, and $B_{35} > 0$ imply that the octet, triplet, and singlet fields have positive mass terms.

The model features two scalar multiplets in the fundamental representation, $\mathbf{6}_{1,2} \supset \mathbf{1}_{1,2}(-5) + \mathbf{5}_{1,2}(1)$, where we have indicated the U(1)_X charge in parentheses. The model also features scalars $\mathbf{15}$ and $\mathbf{21}$. After the breaking of SU(6), we can extract from the fourth and seventh lines in Eq. (A1) the mass terms for the scalar components,

$$\begin{aligned}
V_{\text{SU}(5) \times \text{U}(1)} \supset & [(30\alpha_2 + 25\beta_2)v_6^2 - M_{62}^2]|\mathbf{1}_2|^2 \\
& + [(30\alpha_2 + \beta_2)v_6^2 - M_{62}^2](\mathbf{5}_2)^\dagger \mathbf{5}_2. \tag{A7}
\end{aligned}$$

If the relation $30\alpha_2 + 25\beta_2 = 0$ holds, v_6 will not contribute to the mass of the SM singlet field $s_6 \equiv \mathbf{1}_2(-5)$, which, if $M_{62}^2 \ll v_6^2$, will remain light as required in Sec. III.

Equivalently, after the breaking of SU(5) we can extract from the fourth and seventh lines in Eq. (A1) the mass terms

$$\begin{aligned}
V_{\text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \times \text{U}(1)} \supset & [(30\alpha_1 + 25\beta_1)v_6^2 - M_{61}^2]|\mathbf{1}_1|^2 \\
& + \left[\left(30v_6^2 + \frac{15}{2}v_5^2 \right) \alpha_1 + (v_6^2 + v_5^2)\beta_1 - M_{61}^2 \right] \text{Tr}(T^\dagger T) \\
& + \left[\left(30v_6^2 + \frac{15}{2}v_5^2 \right) \alpha_1 + \left(v_6^2 + \frac{9}{4}v_5^2 \right) \beta_1 - M_{61}^2 \right] H_d H_d^\dagger,
\end{aligned} \tag{A8}$$

where doublet H_d was defined in Eq. (11) and we have introduced a color triplet $T \equiv (\bar{\mathbf{3}}, \mathbf{1}, 1/3; -1)$. If the relation

$$\left(30v_6^2 + \frac{15}{2}v_5^2 \right) \alpha_1 + \left(v_6^2 + \frac{9}{4}v_5^2 \right) \beta_1 = 0 \tag{A9}$$

holds, neither v_5 nor v_6 contribute to the mass term of the weak isospin doublet.

Similar relations can be derived in order to make sure that the other isospin doublet, H_u , is the only multiplet

remaining light inside the 15, and that the singlet s_{21} is the only field remaining light inside the 21.

2. Scalar mass matrices

After the SU(6) and SU(5) GUT symmetries are broken, we are left with the following potential for the SU(2)_L doublets H_u , H_d and the singlets s_6 , s_{21} :

$$\begin{aligned}
V_{\text{low}} = & \mu_d^2 H_d H_d^\dagger + \mu_u^2 H_u H_u^\dagger + \mu_6^2 |s_6|^2 + \mu_{21}^2 |s_{21}|^2 \\
& + z_d (H_d H_d^\dagger)^2 + z_u (H_u H_u^\dagger)^2 + z_6 |s_6|^4 + z_{21} |s_{21}|^4 \\
& + z_{d6} H_d H_d^\dagger |s_6|^2 + z_{d21} H_d H_d^\dagger |s_{21}|^2 + z_{u6} H_u H_u^\dagger |s_6|^2 + z_{u21} H_u H_u^\dagger |s_{21}|^2 \\
& + z_{du} (H_d H_d^\dagger) (H_u H_u^\dagger) + z_{duu} H_d H_u (H_d H_u)^\dagger + z_{621} |s_6|^2 |s_{21}|^2 \\
& + (z_{ud621} H_d H_u s_6^\dagger s_{21}^\dagger + w_{du6} H_d H_u s_6 + w_{2166} s_{21} s_6^2 + \text{H.c.}),
\end{aligned} \tag{A10}$$

where the couplings z_i , w_j should be expressed in terms of the parameters of the potential in Eq. (A1). The fields H_u , H_d , s_6 , and s_{21} are expanded around their vacuum states,

$$\begin{aligned}
H_u = & \begin{pmatrix} h_u^+ \\ \frac{1}{\sqrt{2}}(v_u + h_u^0 + i\sigma_u) \end{pmatrix}, & H_d = & \begin{pmatrix} h_d^- \\ \frac{1}{\sqrt{2}}(v_d + h_d^0 + i\sigma_d) \end{pmatrix}^T, \\
s_6 = & \frac{1}{\sqrt{2}}(v_{s_6} + s_6^0 + i\sigma_6), & s_{21} = & \frac{1}{\sqrt{2}}(v_{s_{21}} + s_{21}^0 + i\sigma_{21}).
\end{aligned} \tag{A11}$$

The CP -even scalar mass matrix in the basis $(h_u^0, h_d^0, s_6^0, s_{21}^0)$ — $\mathbf{M}_{\text{CP-even}}^2$ —can be diagonalized by an orthogonal matrix R_h ,

$$\text{diag}\{M_{h_1}^2, M_{h_2}^2, M_{h_3}^2, M_{h_4}^2\} = R_h (\mathbf{M}_{\text{CP-even}}^2) R_h^T. \tag{A12}$$

The lightest eigenvalue, h_1 , corresponds to the SM Higgs boson. The masses of the remaining neutral scalars, in the limit $v_{s_6}, v_{s_{21}} \gg v_u, v_d$, are approximately given by

$$\begin{aligned}
M_{h_2}^2 & \simeq -z_{ud621} v_{s_{21}} v_{s_6} - \sqrt{2} w_{du6} v_{s_6}, \\
M_{h_3}^2 & \simeq z_{21} v_{s_{21}}^2 + z_6 v_{s_6}^2 - w_{2166} \frac{v_{s_6}^2}{\sqrt{2} v_{s_{21}}} - \sqrt{A}, \\
M_{h_4}^2 & \simeq z_{21} v_{s_{21}}^2 + z_6 v_{s_6}^2 - w_{2166} \frac{v_{s_6}^2}{\sqrt{2} v_{s_{21}}} + \sqrt{A},
\end{aligned} \tag{A13}$$

where the factor under square root reads

$$\begin{aligned}
A = & (z_{21} v_{s_{21}}^2 - z_6 v_{s_6}^2)^2 + v_{s_6}^2 (\sqrt{2} w_{2166} + z_{621} v_{s_{21}})^2 \\
& - z_{21} w_{2166} \frac{v_{s_{21}} v_{s_6}^2}{\sqrt{2}} + w_{2166}^2 \frac{v_{s_6}^4}{8 v_{s_{21}}^2} + w_{2166} z_6 \frac{v_{s_6}^4}{\sqrt{2} v_{s_{21}}}.
\end{aligned} \tag{A14}$$

The CP -odd scalar mass matrix in the basis $(\sigma_u, \sigma_d, \sigma_6, \sigma_{21})$ — $\mathbf{M}_{\text{CP-odd}}^2$ —after diagonalization with an orthogonal matrix R_a , leads to the physical spectrum with two massless Goldstone bosons and two massive pseudo-scalars, a_1 and a_2 ,

$$\text{diag}\{0, 0, M_{a_1}^2, M_{a_2}^2\} = R_a (\mathbf{M}_{\text{CP-odd}}^2) R_a^T, \tag{A15}$$

where, in the limit $v_{s_6}, v_{s_{21}} \gg v_u, v_d$, one finds

$$\begin{aligned} M_{a_1}^2 &\simeq -z_{du621} v_{s_{21}} v_{s_6} - \sqrt{2} w_{du6} v_{s_6}, \\ M_{a_2}^2 &\simeq -\sqrt{2} w_{2166} \frac{4v_{s_{21}}^2 + v_{s_6}^2}{2v_{s_{21}}}. \end{aligned} \quad (\text{A16})$$

Finally, the charged scalar mass matrix— $\mathbf{M}_{\text{Charged}}^2$ —can be diagonalized with a mixing matrix R_β ,

$$\text{diag}\{0, M_{h^\pm}^2\} = R_\beta (\mathbf{M}_{\text{Charged}}^2) R_\beta^T. \quad (\text{A17})$$

In the physical basis, one is then left with a massless charged Goldstone boson and a charged Higgs boson, whose mass squared reads

$$M_{h^\pm}^2 = z_{dudu} \frac{v_u^2 + v_d^2}{2} - (v_{s_{21}} z_{du621} + \sqrt{2} w_{du6}) \frac{v_{s_6} (v_u^2 + v_d^2)}{2v_d v_u}. \quad (\text{A18})$$

One should notice that, had we not relied on AS to seclude different sectors of the theory, we would need to impose a $\mathbb{Z}_2$ or other symmetry to ensure the (meta-)stability of DM [70]. The quartic and trilinear terms in parentheses at the bottom of Eq. (A10), which are necessary to give the desired masses to the two pseudoscalars and the charged Higgs boson, would then break the $\mathbb{Z}_2$ symmetry. Thus,

with quantum scale invariance we also avoid soft-breaking terms.

APPENDIX B: RENORMALIZATION GROUP EQUATIONS

In this appendix, we present the leading-order RGEs of the SU(6) gauge-Yukawa sector described in the main text. For a generic coupling c_i they take the following form in the trans-Planckian regime:

$$\frac{dc_i}{dt} = \frac{1}{16\pi^2} \beta^{(1)}(c_i) - f_c c_i, \quad (\text{B1})$$

where $f_c = f_g$ (for the gauge coupling) and $f_c = f_y$ (for all Yukawa couplings) are gravitational corrections at the UV fixed point, which rapidly tend to zero below the Planck scale (see Sec. 3.2 of Ref. [79] for a discussion of the impact of the position of the Planck scale on the predictions of AS). The superscript (1) means that we work at the 1-loop order. All matter RGEs are derived with the public tool PyR@TE 3 [100,101].

$$\beta^{(1)}(g_6) = -\frac{38}{3} g_6^3$$

$$\begin{aligned} \beta^{(1)}(y_{11}) &= \left(\frac{17}{2} y_{11}^2 + \frac{17}{2} y_{12}^2 + \frac{17}{2} y_{21}^2 + y_{22}^2 - 10 \tilde{y}_{11}^2 + \frac{5}{2} \tilde{y}_{12}^2 + 7 \hat{y}_{11}^2 + \frac{7}{4} \hat{y}_{12}^2 + 12 y_u^2 - \frac{91}{4} g_6^2 \right) y_{11} \\ &\quad + \frac{15}{2} y_{12} y_{21} y_{22} - 5 \tilde{y}_{11} \tilde{y}_{12} y_{21} - 5 \tilde{y}_{12} \tilde{y}_{22} y_{21} + \frac{7}{2} \hat{y}_{11} \hat{y}_{12} y_{21} + \frac{7}{2} \hat{y}_{12} \hat{y}_{22} y_{21} \end{aligned}$$

$$\begin{aligned} \beta^{(1)}(y_{12}) &= \left(\frac{17}{2} y_{11}^2 + \frac{17}{2} y_{12}^2 + y_{21}^2 + \frac{17}{2} y_{22}^2 - 10 \tilde{y}_{11}^2 + \frac{5}{2} \tilde{y}_{12}^2 + 7 \hat{y}_{11}^2 + \frac{7}{4} \hat{y}_{12}^2 + 12 y_u^2 - \frac{91}{4} g_6^2 \right) y_{12} \\ &\quad + \frac{15}{2} y_{11} y_{21} y_{22} - 5 \tilde{y}_{11} \tilde{y}_{12} y_{22} - 5 \tilde{y}_{12} \tilde{y}_{22} y_{22} + \frac{7}{2} \hat{y}_{11} \hat{y}_{12} y_{22} + \frac{7}{2} \hat{y}_{12} \hat{y}_{22} y_{22} \end{aligned}$$

$$\begin{aligned} \beta^{(1)}(y_{21}) &= \left(\frac{17}{2} y_{11}^2 + y_{12}^2 + \frac{17}{2} y_{21}^2 + \frac{17}{2} y_{22}^2 + \frac{5}{2} \tilde{y}_{12}^2 - 10 \tilde{y}_{22}^2 + \frac{7}{4} \hat{y}_{12}^2 + 7 \hat{y}_{22}^2 + 12 y_u^2 - \frac{91}{4} g_6^2 \right) y_{21} \\ &\quad + \frac{15}{2} y_{11} y_{12} y_{22} + 5 \tilde{y}_{11} \tilde{y}_{12} y_{11} + 5 \tilde{y}_{12} \tilde{y}_{22} y_{11} + \frac{7}{2} \hat{y}_{11} \hat{y}_{12} y_{11} + \frac{7}{2} \hat{y}_{12} \hat{y}_{22} y_{11} \end{aligned}$$

$$\begin{aligned} \beta^{(1)}(y_{22}) &= \left(y_{11}^2 + \frac{17}{2} y_{12}^2 + \frac{17}{2} y_{21}^2 + \frac{17}{2} y_{22}^2 + \frac{5}{2} \tilde{y}_{12}^2 - 10 \tilde{y}_{22}^2 + \frac{7}{4} \hat{y}_{12}^2 + 7 \hat{y}_{22}^2 + 12 y_u^2 - \frac{91}{4} g_6^2 \right) y_{22} \\ &\quad + \frac{15}{2} y_{11} y_{12} y_{21} + 5 \tilde{y}_{11} \tilde{y}_{12} y_{12} + 5 \tilde{y}_{12} \tilde{y}_{22} y_{12} + \frac{7}{2} \hat{y}_{11} \hat{y}_{12} y_{12} + \frac{7}{2} \hat{y}_{12} \hat{y}_{22} y_{12} \end{aligned}$$

$$\beta^{(1)}(\tilde{y}_{11}) = \left(5 y_{11}^2 + 5 y_{12}^2 - 24 \tilde{y}_{11}^2 + 12 \tilde{y}_{12}^2 - 4 \tilde{y}_{22}^2 + 14 \hat{y}_{11}^2 + \frac{7}{2} \hat{y}_{12}^2 + 12 y_u^2 - \frac{35}{2} g_6^2 \right) \tilde{y}_{11} + 5 \tilde{y}_{12} \tilde{y}_{22}$$

$$\begin{aligned}
\beta^{(1)}(\tilde{y}_{12}) &= \left(\frac{5}{2}y_{11}^2 + \frac{5}{2}y_{12}^2 + \frac{5}{2}y_{21}^2 + \frac{5}{2}y_{22}^2 - 24\tilde{y}_{11}^2 + 7\tilde{y}_{12}^2 - 24\tilde{y}_{22}^2 - 20\tilde{y}_{11}\tilde{y}_{22} + 7\hat{y}_{11}^2 + \frac{7}{2}\hat{y}_{12}^2 + 7\hat{y}_{22}^2 \right. \\
&\quad \left. + 12y_u^2 - \frac{35}{2}g_6^2 \right) \tilde{y}_{12} + 5\tilde{y}_{11}y_{11}y_{21} + 5\tilde{y}_{11}y_{12}y_{22} + 7\tilde{y}_{11}\hat{y}_{11}\hat{y}_{12} + 7\tilde{y}_{11}\hat{y}_{12}\hat{y}_{22} \\
&\quad + 5\tilde{y}_{22}y_{11}y_{21} + 5\tilde{y}_{22}y_{12}y_{22} + 7\tilde{y}_{22}\hat{y}_{11}\hat{y}_{12} + 7\tilde{y}_{22}\hat{y}_{12}\hat{y}_{22} \\
\beta^{(1)}(\tilde{y}_{22}) &= \left(5y_{21}^2 + 5y_{22}^2 - 4\tilde{y}_{11}^2 + 12\tilde{y}_{12}^2 - 24\tilde{y}_{22}^2 + \frac{7}{2}\hat{y}_{12}^2 + 14\hat{y}_{22}^2 + 12y_u^2 - \frac{35}{2}g_6^2 \right) \tilde{y}_{22} + 5\tilde{y}_{11}\tilde{y}_{12}^2 \\
\beta^{(1)}(\hat{y}_{11}) &= \left(5y_{11}^2 + 5y_{12}^2 - 20\tilde{y}_{11}^2 + 5\tilde{y}_{12}^2 + 16\hat{y}_{11}^2 + 8\hat{y}_{12}^2 + 2\hat{y}_{22}^2 - \frac{35}{2}g_6^2 \right) \hat{y}_{11} \\
&\quad + \frac{5}{2}\hat{y}_{12}y_{11}y_{21} + \frac{5}{2}\hat{y}_{12}y_{12}y_{22} + \frac{7}{2}\hat{y}_{12}^2\hat{y}_{22} \\
\beta^{(1)}(\hat{y}_{12}) &= \left(14\hat{y}_{11}\hat{y}_{22} + \frac{5}{2}y_{11}^2 + \frac{5}{2}y_{12}^2 + \frac{5}{2}y_{21}^2 + \frac{5}{2}y_{22}^2 - 10\tilde{y}_{11}^2 + 5\tilde{y}_{12}^2 - 10\tilde{y}_{22}^2 + 16\hat{y}_{11}^2 + \frac{9}{2}\hat{y}_{12}^2 \right. \\
&\quad \left. + 16\hat{y}_{22}^2 - \frac{35}{2}g_6^2 \right) \hat{y}_{12} - 10\tilde{y}_{11}\tilde{y}_{12}\hat{y}_{11} - 10\tilde{y}_{11}\tilde{y}_{12}\hat{y}_{22} - 10\tilde{y}_{12}\tilde{y}_{22}\hat{y}_{11} - 10\tilde{y}_{12}\tilde{y}_{22}\hat{y}_{22} \\
&\quad + 5\hat{y}_{11}y_{11}y_{21} + 5\hat{y}_{11}y_{12}y_{22} + 5\hat{y}_{22}y_{11}y_{21} + 5\hat{y}_{22}y_{12}y_{22} \\
\beta^{(1)}(\hat{y}_{22}) &= \left(5y_{21}^2 + 5y_{22}^2 + 5\tilde{y}_{12}^2 - 20\tilde{y}_{22}^2 + 2\hat{y}_{11}^2 + 8\hat{y}_{12}^2 + 16\hat{y}_{22}^2 - \frac{35}{2}g_6^2 \right) \hat{y}_{22} \\
&\quad + \frac{7}{2}\hat{y}_{11}\hat{y}_{12}^2 + \frac{5}{2}\hat{y}_{12}y_{11}y_{21} + \frac{5}{2}\hat{y}_{12}y_{12}y_{22} \\
\beta^{(1)}(y_u) &= (2y_{11}^2 + 2y_{12}^2 + 2y_{21}^2 + 2y_{22}^2 - 4\tilde{y}_{11}^2 + 2\tilde{y}_{12}^2 - 4\tilde{y}_{22}^2 + 36y_u^2 - 28g_6^2)y_u
\end{aligned}$$

APPENDIX C: ALTERNATIVE FIXED-POINT SOLUTIONS

For the sake of completeness, we present in this appendix a few fixed-point solutions to the trans-Planckian RGE system, alternative to the one selected in Table I of Sec. IV C. They are reported, together with their critical exponents, in Table IV. Each of the featured fixed points corresponds to a different discrete symmetry of the Yukawa sector in the deep UV. Note that FP2-FP5 are all characterized by a larger number of relevant directions than the fixed point in Table I, and are thus less predictive. In contrast, FP1 features only one extra marginal direction with respect to the point selected in Sec. IV C. However, it is typically excluded on phenomenological grounds, since $y_{11}^* = y_{22}^* = 0.33$ predicts a low-scale value of the bottom quark Yukawa larger than the top Yukawa value.

APPENDIX D: RELIC ABUNDANCE

In this appendix, we collect useful formulas that pertain to Figs. 5 and 6.

Vector resonance. Following Refs. [116,117], we write down a solution for the Boltzmann equation in the case of SM-singlet Majorana fermion pairs annihilating to the SM

d.o.f.'s via the Z' resonance, see Sec. V. The thermally averaged annihilation cross section reads [122]

$$\langle \sigma v \rangle = \frac{1}{16\pi^4} \left(\frac{m_{\text{DM}}}{x_f} \right) \frac{1}{n_{\text{eq}}^2} \int_{4m_{\text{DM}}^2}^{\infty} ds \hat{\sigma}(s) \sqrt{s} K_1 \left(\frac{x_f \sqrt{s}}{m_{\text{DM}}} \right), \quad (\text{D1})$$

where $x_f = m_{\text{DM}}/T \approx 30$ and $K_1(z)$ is the modified Bessel function of the second kind. The number density of DM, n_{eq} , the entropy density of the thermal plasma, $S(m_{\text{DM}})$, and the thermal equilibrium DM yield, $Y(m_{\text{DM}})$, are given by

$$n_{\text{eq}} = S(m_{\text{DM}})Y(m_{\text{DM}})/x_f^3, \quad (\text{D2})$$

$$S(m_{\text{DM}}) = \frac{2\pi^2}{45} g_* m_{\text{DM}}^3, \quad (\text{D3})$$

$$Y(m_{\text{DM}}) = \frac{1}{\pi^2} \frac{x_f^2 m_{\text{DM}}^3}{S(m_{\text{DM}})} K_2(x_f), \quad (\text{D4})$$

where g_* is the effective total number of degrees of freedom for the particles in thermal equilibrium. $g_* = 106.75$ is the

TABLE IV. A sample of trans-Planckian fixed points alternative to the one given in Table I, together with their critical exponents.

FP1										
y_u^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	$\tilde{y}_{11}^*$	$\tilde{y}_{22}^*$	y_{11}^*	y_{22}^*	y_{12}^*	y_{21}^*	$\tilde{y}_{12}^*$	$\hat{y}_{12}^*$
0.25	0.37	0.38	0.15	0.17	0.33	0.33	0.0	0.0	0.0	0.0
θ_u	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	$\tilde{\theta}_{11}$	$\tilde{\theta}_{22}$	θ_{11}	θ_{22}	θ_{12}	θ_{21}	$\tilde{\theta}_{12}$	$\hat{\theta}_{12}$
-5.1	-4.7	-3.6	0	0.62	-3.6	-1.1	0	0	-0.87	-3.8
FP2										
y_u^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	$\tilde{y}_{11}^*$	$\tilde{y}_{22}^*$	y_{11}^*	y_{12}^*	y_{22}^*	y_{21}^*	$\tilde{y}_{12}^*$	$\hat{y}_{12}^*$
0.26	0.37	0.41	0.17	0.15	0.19	0.29	0.0	0.0	0.0	0.0
θ_u	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	$\tilde{\theta}_{11}$	$\tilde{\theta}_{22}$	θ_{11}	θ_{12}	θ_{22}	θ_{21}	$\tilde{\theta}_{12}$	$\hat{\theta}_{12}$
-5.1	-3.6	-5.2	0.64	0	0	-1.0	0	0.61	0.18	-3.7
FP3										
y_u^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	$\tilde{y}_{11}^*$	$\tilde{y}_{22}^*$	y_{11}^*	y_{12}^*	y_{22}^*	y_{21}^*	$\tilde{y}_{12}^*$	$\hat{y}_{12}^*$
0.28	0.41	0.41	0.17	0.17	0.0	0.0	0.0	0.0	0.0	0.0
θ_u	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	$\tilde{\theta}_{11}$	$\tilde{\theta}_{22}$	θ_{11}	θ_{12}	θ_{22}	θ_{21}	$\tilde{\theta}_{12}$	$\hat{\theta}_{12}$
-5.7	-3.7	-5.1	0	0.66	0.69	0.69	0.69	0.69	0	-3.7
FP4										
y_u^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	$\tilde{y}_{11}^*$	$\tilde{y}_{22}^*$	y_{21}^*	y_{22}^*	y_{11}^*	y_{12}^*	$\tilde{y}_{22}^*$	$\hat{y}_{12}^*$
0.26	0.41	0.37	0.15	0.17	0.29	0.19	0.0	0.0	0.0	0.0
θ_u	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	$\tilde{\theta}_{11}$	$\tilde{\theta}_{22}$	θ_{21}	θ_{22}	θ_{11}	θ_{12}	$\tilde{\theta}_{22}$	$\hat{\theta}_{12}$
-5.1	-5.2	-3.6	0	0.64	0	-1.0	0	0.61	0	-3.5
FP5										
y_u^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	$\tilde{y}_{11}^*$	$\tilde{y}_{22}^*$	$\tilde{y}_{12}^*$	$\hat{y}_{12}^*$	y_{11}^*	y_{12}^*	y_{21}^*	y_{22}^*
0.26	0.37	0.41	0.17	0.15	0.19	0.29	0.0	0.0	0.0	0.0
θ_u	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	$\tilde{\theta}_{11}$	$\tilde{\theta}_{22}$	$\tilde{\theta}_{12}$	$\hat{\theta}_{12}$	θ_{11}	θ_{12}	θ_{21}	θ_{22}
-5.1	0.86	0.86	0.35	0	-6.4	1.5	1.5	0.48	1.5	0.48

number of d.o.f.'s of the SM, and $K_2(z)$ is the modified Bessel function of the second kind. The reduced cross section $\hat{\sigma}(s)$ is defined as

$$\hat{\sigma}(s) = 2(s - 4m_{\text{DM}}^2)\sigma_{\text{SM}}(s), \quad (\text{D5})$$

which in our model leads to the following expression,

$$\sigma_{\text{SM}}(s) = \frac{25 \cdot 135\pi}{3} \frac{g_X^4}{16\pi^2} \frac{\sqrt{s(s - 4m_{\text{DM}}^2)}}{(s - m_{Z'}^2)^2 + m_{Z'}^2 \Gamma_{Z'}^2}, \quad (\text{D6})$$

with the Z' boson decay width given by

$$\Gamma_{Z'} = \frac{135}{6} \frac{g_X^2}{4\pi} m_{Z'}. \quad (\text{D7})$$

Weak doublet DM. In the limit of all four components (neutral and charged) of the two weak isospin doublets L_2 and L' being degenerate in mass, the effective thermally averaged cross section takes into account their coannihilation into final states involving the gauge bosons of the SM. As long as the scalars of the theory are much heavier than the DM, it is well approximated by (see, e.g., Ref. [123])

$$\langle \sigma v \rangle_{\tilde{H}}^{(\text{eff})} \approx \frac{21g_2^4 + 3g_2^2 g_Y^2 + 11g_Y^2}{512\pi m_{\text{DM}}^2}, \quad (\text{D8})$$

where $g_Y(\mu = 1 \text{ TeV}) \approx 0.33$ is the $U(1)_Y$ gauge coupling and $g_2(\mu = 1 \text{ TeV}) \approx 0.65$ is the $SU(2)_L$ gauge coupling. $\langle \sigma v \rangle_{\tilde{H}}^{(\text{eff})} = 2.15 \times 10^{-26} \text{ cm}^3/\text{s}$ implies $m_{\text{DM}} = 1.1 \text{ TeV}$.

APPENDIX E: FULL YUKAWA LAGRANGIAN AND FERMION MASSES

When the couplings that in Table I correspond to Gaussian irrelevant directions of the RG flow are allowed to appear via the mechanism discussed in Sec. IV D, more terms are featured in the low-scale Yukawa Lagrangian,

$$\begin{aligned} \mathcal{L}_{\text{IR}2} \supset \mathcal{L}_{\text{IR}1} &+ [y'_d d_2 H_d Q + y'_e e H_d L_2 + y'_\nu L' H_d^{c\dagger} \nu_2 \\ &+ y'_D d_1 d' s_6 + y'_L L' L_1 s_6 + 2\tilde{y}_{11} \nu_1 H_u^{c\dagger} L_1 + 2\tilde{y}_{22} \nu_2 H_u^{c\dagger} L_2 \\ &+ \tilde{y}_{12} (\nu_1 H_u^{c\dagger} L_2 + \nu_2 H_u^{c\dagger} L_1) + \hat{y}_{12} \nu_1 \nu_2 s_{21} + \text{H.c.}], \end{aligned} \quad (\text{E1})$$

where $\mathcal{L}_{\text{IR}1}$ was defined in Eq. (16). Couplings y'_d, y'_e, y'_ν depart from the common value, y_{21} , after the breaking of $SU(6)$. Couplings y'_D, y'_L stem from y_{12} .

The mass matrices of the bottom quark and tau lepton read,

$$M_b = \frac{1}{\sqrt{2}} \begin{pmatrix} y_d v_d & y'_D v_{s_6} \\ y'_d v_d & y_D v_{s_6} \end{pmatrix}, \quad M_\tau = \frac{1}{\sqrt{2}} \begin{pmatrix} y_e v_d & y'_e v_d \\ y'_L v_{s_6} & y_L v_{s_6} \end{pmatrix}. \quad (\text{E2})$$

The mass matrix for the neutral fermions, in the basis $\langle \nu_{L_1}, \nu_{L_2}, \nu_{L'}, \nu_1, \nu_2 |, | \nu_{L_1}, \nu_{L_2}, \nu_{L'}, \nu_1, \nu_2 \rangle$ reads

$$\frac{1}{2} M_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & y'_L v_{s_6} & 2\tilde{y}_{11} v_u & \tilde{y}_{12} v_u \\ 0 & 0 & y_L v_{s_6} & \tilde{y}_{12} v_u & 2\tilde{y}_{22} v_u \\ y'_L v_{s_6} & y_L v_{s_6} & 0 & y_\nu v_d & y'_\nu v_d \\ 2\tilde{y}_{11} v_u & \tilde{y}_{12} v_u & y_\nu v_d & y_{\nu_1} v_{s_{21}} & \hat{y}_{12} v_{s_{21}} \\ \tilde{y}_{12} v_u & 2\tilde{y}_{22} v_u & y'_\nu v_d & \hat{y}_{12} v_{s_{21}} & y_{\nu_2} v_{s_{21}} \end{pmatrix}. \quad (\text{E3})$$

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